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An analytical model of wave overtopping flow velocities on dike crests and landward slopes

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Abstract

An accurate description of the maximum flow velocity across flood-protective structures is necessary in order to determine the stability of the landward slope and the amount of cover erosion. In this paper, two new formulas are derived to describe the change in the maximum overtopping flow velocity along the dike crest and the adjacent landward slope. These formulas are coupled in an effort to accurately predict the velocities in wave overtopping events. This analytical model is validated using 244 data points from flume tests and 300 data points from field tests on river dikes in the Netherlands. The modelled flow velocity shows good agreement with the measured flow velocity, with a Nash-Sutcliffe efficiency factor varying from 0.49 to 0.87. Also, the derived formulas are compared with existing formulas for wave overtopping flow velocities and overall showed a better performance for a wide range of geometries and covers.

Keywords: Wave overtopping, Flow velocity, Analytical overtopping model, Dike crest, Landward slope

1. Introduction

Wave overtopping is one of the main failure mechanisms of flood-protective structures such as dikes, embankments, spillways, seawalls and dams. The average overtopping discharge is often used to determine the design height of

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5 flood-protective structures. However, the stability of the slope and the cover
 6 erosion are determined by the maximum overtopping flow velocity on the
 7 crest and landward slope, and not solely by average values [1]. For example,
 8 the slope stability of a dike depends on the instantaneous loads caused by
 9 wave overtopping, such as the maximum flow velocity and layer thickness
 10 [2, 3]. Also, in the case of dam spillways, the size of the rock revetment for
 11 slope stability is determined by the maximum flow velocity [4].

12 Erosion of earthen spillways occurs if the maximum overtopping flow
 13 velocity exceeds the critical flow velocity [5]. The critical velocity is often
 14 low which means that the overtopping flow velocity needs to be lowered, for
 15 instance by adapting the crest shape of the spillway [6]. The cover erosion
 16 of earthen dikes is determined by the exceedance of a critical velocity by
 17 the maximum overtopping flow velocity [7, 8, 9]. The critical flow velocity
 18 depends on the type and quality of the (dike) cover [5, 9, 10].

19 It is common to use one value for the critical flow velocity along the entire
 20 dike profile [8] or the spillway [5]. However, the maximum overtopping flow
 21 velocity varies across the structure. The flow velocity decreases on the crest
 22 due to bottom friction, and in some cases due to a small slope towards the
 23 waterside. The overtopping wave accelerates along the landward slope due
 24 to gravity. Therefore, the flow velocity is usually highest at the end of the
 25 landward slope, which is also the location at which most erosion by wave
 26 overtopping is observed [11, 12]. It is important to know how the maximum
 27 flow velocity varies along both the crest and the landward slope to be able
 28 to accurately predict erosion and slope stability.

29 Flood-protective structures are highly variable in geometry and in cover
 30 type. A model is needed that integrates cross-structure profiles consisting
 31 of a combination of horizontal parts, slopes and multiple cover types. For
 32 this reason, the model needs to be adjustable to a variety of configurations.
 33 Several formulas are available to calculate the overtopping flow velocity along
 34 the dike crest or landward slope for various dike geometries. However, the
 35 existing overtopping formulas have several shortcomings. They (a) include
 36 unclear empirically determined constants; (b) are limited to a part of the
 37 cross-dike profile; or (c) are not applicable to other cover types. Van Gent
 38 [13, 14] defined a relationship between the flow velocity at the start of the
 39 crest and the flow velocity at the end of the dike crest, based on the results of
 40 flume experiments. This relationship was validated for various wave condi-
 41 tions, dike geometries and two different dike covers. However, this relation-
 42 ship cannot be used to calculate the flow velocity at every point on the dike

crest. A combination of studies by Schüttrumpf, Van Gent and Oumeraci [15, 14, 2, 1] resulted in formulas for the flow velocity on the dike crest and the landward slope. The formula for the flow velocity on the landward slope [1] is time-dependent and can only be solved in an iterative manner. Furthermore, changes in cover or geometry along the slope cannot be included, because they can lead to instabilities. The formula for the flow velocity on the dike crest includes an empirically determined constant of either 0.4 or 0.75 depending on the dataset [13, 1]. No clear explanation has been found for the varying values of the constant, although Bosman et al. [16] concluded that the velocities obtained by Schüttrumpf [15] were too high. Bosman et al. [16] introduced a formula for the flow velocity on the crest that depends on the offshore wave length. That formula was adapted by Van der Meer et al. [8] by removing an unknown friction factor and including the geometry of the waterside slope. However, despite the fact that the formula of Van der Meer et al. [8] accounts for various geometries and wave conditions, it does not account for changes in cover type. From the above, it becomes clear that several formulas are available; however, these formulas are not widely applicable nor easy adjustable to different dike geometries. Therefore, the objective of this study was to derive two analytical formulas to calculate the maximum flow velocity on (1) the crest and (2) the landward slope. Both formulas must take geometry and cover types changes into account.

The analytical formulas are derived in Section 2. The first formula describes the change in the maximum flow velocity along a horizontal part of the dike profile, and the second formula calculates the variation along the landward slope. The formulas are coupled in an analytical model for the flow velocity variation along the dike profile for various dike geometries and dike covers (Section 3). The model is validated using 544 data points from several overtopping tests and compared with existing formulas in Section 4. The analytical model performed well for all tests, and was also able to accurately simulate the flow velocity for smooth and rough dike covers as well as slope transitions unlike the existing formulas. The results are discussed in Section 5 and the conclusions are drawn in Section 6.

2. Theory

2.1. Wave overtopping parameters

During storms, high waves can overtop dikes. These waves transform during the wave overtopping process. The wave breaks and runs up on the

Figure 1: Schematization of the overtopping flow with the maximum flow velocity U_0 at the start of the crest ($x = x_0$) and the maximum flow velocity $U_{s,0}$ at the start of the slope ($x = x_1$). The dike geometry is characterized by the crest width B_c , the angle of the landward slope ϕ , the horizontal length of the landward slope B_s , the cross-dike coordinate x and the along-slope coordinate $s = (x - B_c)/\cos(\phi)$ for $x \geq B_c$.

Figure 2: A typical shape of the layer thickness $h(x, t)$, flow velocity $u(x, t)$ and discharge $q(x, t)$ as a function of time based on measurements from Hughes [17] together with the overtopping duration T_0 . The maximum flow velocity $U(x)$ occurs at time $t = t_{max}(x)$ simultaneously with the momentary layer thickness $h_U(x)$ and the momentary discharge Q .

waterside slope. Part of the wave runs down along the waterside slope while the remaining part overtops, flows over the dike crest and runs down the landward slope (Figure 1).

The study domain covers the dike crest and the landward slope. The wave overtopping flow is characterized by the overtopping flow velocity $u(x, t)$ in m/s, the layer thickness $h(x, t)$ in m and the specific discharge $q(x, t)$ in $\text{m}^3/\text{s}/\text{m}$. These parameters vary in time and space (Figure 2). When the wave front arrives, the parameters rapidly increase to maximum followed by a slow decrease to zero once the wave has completely passed, resulting in a saw-tooth shape [8, 18]. The flow velocity $u(x, t)$ is at its maximum $U(x)$ at time $t = t_{max}(x)$. At $t = t_{max}(x)$, we define the momentary layer thickness $h_U(x)$ and the momentary discharge Q . The maximum flow velocity $U(x)$ and the momentary layer thickness $h_U(x)$ change along the cross-dike profile. The momentary discharge Q is assumed to be constant along the crest and the landward slope. Using the continuity of discharge, the momentary discharge Q is written as

$$Q = U(x) \cdot h_U(x) = U(0) \cdot h_U(0) = \text{constant} \quad (1)$$

Numerous waves overtop the crest during a storm. Instead of using the maximum of one wave, the extreme values of overtopping parameters during a storm are used for the design of flood-protective structures. These parameters have a low probability of exceedance, for which the 2% exceedance flow velocity $u_{2\%}(x)$ - that is the flow velocity $u(x, t)$ exceeded by 2% of the incident waves - is commonly used.

2.2. Derivation of the formulas

Two formulas for the overtopping flow velocity are derived from the 1D shallow water equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} + \frac{\tau}{h} = 0 \quad (2)$$

with the depth-averaged flow velocity $u(x, t)$, the cross-dike direction x , the time t , the gravitational acceleration g , the layer thickness $h(x, t)$ and the bottom shear stress τ (Figure 2). The bottom shear stress is written as

$$\tau(x, t) = \frac{1}{2} f u(x, t)^2 \quad (3)$$

107 with f the bottom friction coefficient. The bottom friction coefficient f is
108 related to the commonly used Manning's roughness coefficient n as

$$f = \frac{2 g n^2}{h(x, t)^{1/3}} \quad (4)$$

109 The maximum flow velocity $U(x)$ is defined as the maximum of the flow
110 velocity $u(x, t)$ with respect to time, so the maximum flow velocity $U(x)$ and
111 the momentary layer thickness $h_U(x)$ become time-independent. The change
112 in the maximum flow velocity $U(x)$ of the overtopping wave is described by

$$U \frac{\partial U}{\partial x} + g \frac{\partial h_U}{\partial x} + \frac{1}{2} \frac{f U^2}{h_U} = 0 \quad (5)$$

113 with the momentary layer thickness $h_U(x)$.

114 2.2.1. Flow velocity on the crest

115 The run-down on the waterside slope is neglected, so the variations in
116 the momentary layer thickness along the dike crest are assumed to be small.
117 Equation 5 results in a balance between advection and bottom friction.

$$U \frac{\partial U}{\partial x} + \frac{f U^2}{2 h_U} = 0 \quad (6)$$

118 The momentary layer thickness $h_U(x)$ is removed from this equation using
119 the momentary discharge Q (Equation 1). The balance becomes

$$U \frac{\partial U}{\partial x} + \frac{f U^3}{2 Q} = 0 \quad (7)$$

120 Since the momentary discharge Q and the friction coefficient f are indepen-
121 dent of the cross-dike coordinate x , partial integration results in

$$\frac{1}{U} = \frac{f x}{2 Q} + C \quad (8)$$

122 The integration constant C is determined from the boundary condition at
123 the start of the dike crest ($x = x_0$)

$$U(x = x_0) = U_0 \quad (9)$$

124 Leading to

$$\frac{1}{U} = \frac{f x}{2Q} + \frac{1}{U_0} \quad (10)$$

125 Resulting in the formula for the maximum flow velocity on the dike crest

$$U(x) = \frac{1}{\frac{f x}{2Q} + \frac{1}{U_0}} \quad (11)$$

126 This formula describes the decrease in the maximum flow velocity due to
127 friction and is also applicable to other horizontal parts of the dike profile, for
128 example the horizontal part of a berm on the landward slope.

129 2.2.2. Flow velocity on the landward slope

130 To derive a formula for the maximum flow velocity $U(s)$ along the land-
131 ward slope, the horizontal cross-dike coordinate x in Equation 5 is replaced
132 by the along-slope coordinate s , where $s = (x - B_c)/\cos(\varphi)$ (see Figure 1).

$$U \frac{dU}{ds} + g \left[\cos(\varphi) \frac{dh_U}{ds} - \sin(\varphi) \right] + \frac{f U^2}{2h_U} = 0 \quad (12)$$

133 with the maximum flow velocity along the slope $U(s)$ the momentary layer
134 thickness along the slope $h_U(s)$ and the angle of the slope φ . The additional
135 gravity term is a result of the coordinate transformation, since the gravity
136 force is not perpendicular to the along-slope coordinate.

137 Assuming that the along slope variation in the momentary layer thickness
138 is smaller than the steepness of the slope ($|dh_U/ds| \ll \tan(\varphi)$) and using the
139 continuity of discharge (Equation 1), Equation 12 reduces to

$$U \frac{dU}{ds} + g \sin(\varphi) + \frac{f U^3}{2Q} = 0 \quad (13)$$

140 The maximum flow velocity on the landward slope $U(s)$ can be written in
141 terms of a mean velocity $\bar{U}$ - that is constant along the slope - and a variation
142 in flow velocity along the slope $U^*(s)$.

$$U(s) = \bar{U} + U^*(s) \quad (14)$$

143 An expression for the mean velocity $\bar{U}$ is found by substituting $U(s) = \bar{U}$ in

Equation 13, resulting in

$$g \sin(\varphi) + \frac{f \bar{U}^3}{2Q} = 0 \quad (15)$$

From this equation, an expression for the mean velocity $\bar{U}$ is found

$$\bar{U} = \sqrt[3]{\frac{2 g Q \sin(\varphi)}{f}} \quad (16)$$

Experiments have shown that the maximum flow velocity increases exponentially on the landward slope. This exponential increase in maximum flow velocity along the slope is expressed by the varying flow velocity $U^*(s)$

$$U^*(s) = \mu \exp(-3 \alpha \beta^2 s) \quad (17)$$

where the parameters are given by

$$\begin{aligned} \mu &= U_{s,0} - \frac{\alpha}{\beta} \\ \alpha &= \sqrt[3]{g \sin \varphi} \\ \beta &= \sqrt[3]{f/2Q} \end{aligned} \quad (18)$$

with the maximum flow velocity at the start of the landward slope $U_{s,0}$. Combining the Equations 16 and 17, the maximum flow velocity $U(s)$ along the landward slope is given by

$$U(s) = \frac{\alpha}{\beta} + \mu \exp(-3 \alpha \beta^2 s) \quad (19)$$

This formula describes the increase in the maximum flow velocity along the slope until a balance is reached between the acceleration due to gravity and the deceleration due to friction (see also [13, 14]). The maximum flow velocity along the landward slope can be expressed in the horizontal cross-dike coordinate x using $s = (x - B_c)/\cos(\varphi)$ for $x \geq B_c$.

2.3. Momentary layer thickness

The momentary layer thickness $h_U(x)$ along the dike profile is determined from the continuity of discharge

$$h_U(x) = \frac{Q}{U(x)} = \frac{h_U(0) U(0)}{U(x)} \quad (20)$$

Note that this represents the layer thickness at the moment the flow velocity is at its maximum ($t = t_{max}(x)$), not the maximum layer thickness of the overtopping wave, since the maximum flow velocity and layer thickness do not necessarily occur simultaneously. A time lag between the maximum flow velocity and maximum layer thickness is observed on the crest [19], leading to a large difference between the maximum layer thickness and the momentary layer thickness $h_U(x)$. However, on the slope, the difference between the momentary layer thickness $h_U(x)$ and the maximum layer thickness is small since the layer thickness and the flow velocity are maximum approximately at the same time.

Equation 20 was not validated because the measured momentary layer thickness is not reported for the experiments, since this variable is not used in existing overtopping equations [20]. The (maximum) layer thickness is a necessary input variable for some overtopping equations, for example the equations of Schüttrumpf and Oumeraci [1] which also use Equation 20 for the layer thickness on the slope (Appendix A.1). The derived formulas in this study do not require a layer thickness as input, because it is assumed that the momentary discharge Q is constant.

2.4. Assumptions

In this study, it is assumed that all the overtopping water arriving at the crest flows towards the landward slope. Under this assumption, the time-averaged discharge is constant and the decrease of the layer thickness over the dike crest can be assumed to be small [1]. Further, the momentary discharge Q is assumed constant, so the saw-tooth shape of the discharge $q(x, t)$ does not change along the dike profile. This assumption implies that diffusion of the overtopping is small compared to its advection, and that the overtopping duration is constant along the dike profile (Figure 2), which is valid for high-velocity wave overtopping events. Hughes [17] performed a combined overflow and wave overtopping experiment and showed that the momentary discharge is indeed constant along the dike profile. Although the

191 tests were not performed solely for wave overtopping, the tests with low surge
192 levels resulted in individual overtopping waves [17]. These tests resemble the
193 case of wave-only overtopping, which is what this study is concerned with.

194 Additional assumptions made during derivation of the formula of the
195 maximum flow velocity $U(x)$ on top of the dike crest were that

$$g \frac{\partial h_U(x)}{\partial x} \ll U \frac{\partial U}{\partial x} \quad \text{and} \quad g \frac{\partial h_U}{\partial x} \ll \frac{f U^2}{2 h_U} \quad (21)$$

196 During the derivation for the formula of the maximum flow velocity $U(s)$ on
197 the landward slope, it is assumed that

$$\frac{\partial h_U}{\partial s} \ll \tan(\varphi) \quad (22)$$

198 These three additional assumptions are expressed by three ratios R_1 , R_2 and
199 R_3

$$R_1 = \frac{U \frac{\partial U}{\partial x}}{g \frac{\partial h_U}{\partial x}}, \quad R_2 = \frac{f U^2}{2 g h_U \frac{\partial h_U}{\partial x}} \quad \text{and} \quad R_3 = \frac{\tan(\varphi)}{\frac{\partial h_U}{\partial s}} \quad (23)$$

200 where ratios R_1 and R_2 hold for the dike crest and ratio R_3 only holds for
201 the landward slope. The formulas derived in this paper are only valid if

$$R_1 \gg 1, \quad R_2 \gg 1 \quad \text{and} \quad R_3 \gg 1 \quad (24)$$

202 These assumptions are validated in Section 5.1. Ratios R_1 and R_2 are calcu-
203 lated using the variation in the modelled maximum flow velocity $U(x)$ and
204 momentary layer thickness $h_U(x)$ over the crest using $\Delta x = x_1 - x_0 = B_c$
205 (Figure 1). The average of the modelled maximum flow velocity $U(x)$ on the
206 crest is used in ratio R_1

$$R_1 = \frac{\frac{1}{2} [U(x_0) + U(x_1)] \cdot [U(x_1) - U(x_0)]}{g [h_U(x_1) - h_U(x_0)]} \quad (25)$$

207 Furthermore, the average of the modelled momentary layer thickness $h_U(x)$
208 is used in ratio R_2

$$R_2 = \frac{f B_c \left[\frac{1}{2} [U(x_0) + U(x_1)] \right]^2}{g [h_U(x_0) + h_U(x_1)] \cdot [h_U(x_1) - h_U(x_0)]} \quad (26)$$

Ratio R_3 is calculated with the modelled momentary layer thickness $h(s, t_{max})$ variation over the landward slope using $\Delta x = x_2 - x_1 = B_s$

$$R_3 = \frac{\tan(\varphi)B_s}{[h_U((x_2 - B_c)/\cos\varphi) - h_U((x_1 - B_c)/\cos(\varphi))]} \quad (27)$$

3. Methodology

3.1. Model formulation

The analytical wave overtopping model couples the formula for the maximum flow velocity on the dike crest with the formula for maximum flow velocity on the landward slope to calculate the maximum flow velocity along the dike profile. The maximum flow velocity variation over the dike crest is calculated using the derived formula (Equation 11). The calculated flow velocity at the end of the dike crest is used as boundary condition for the maximum flow velocity at the start of the landward slope where the maximum flow velocity along the slope is calculated using the formula for the slope (Equation 19). The momentary discharge is assumed to be constant along the crest and the slope, and equal to the momentary discharge at the start of the dike crest.

The input of the model consists of the dike profile (geometry and surface roughness) and two boundary conditions. The dike profile is described by the crest width B_c , the length of the landward slope B_s , the slope angle φ and the friction coefficient of the dike cover f (Figure 1). The maximum flow velocity at the start of the dike crest and the momentary overtopping discharge are the two necessary boundary conditions. The model simulates the maximum flow velocity at any point along both the dike crest and the landward slope.

3.2. Data

For the validation of the analytical model, 544 data points are used from a set of five flume experiments at Delft Hydraulics [13] and two overtopping field experiments on grass-covered river dikes [21, 19]. The flume tests cover a wide range of wave conditions and five different dike configurations resulting in 244 data points for the model validation. The tests on river dikes were performed with the Wave Overtopping Simulator (WOS) [22]. The simulator was located on top of the dike crest and released overtopping volumes according to an overtopping spectrum. The tests were performed at two locations

in the Netherlands: at Tholen in 2010 [21] and at Millingen a/d Rijn in 2013 [19] resulting in 104 and 196 data points, respectively.

3.2.1. Flume tests at Delft Hydraulics

Van Gent [13] performed overtopping tests in the Scheldt flume ($55 \times 1.0 \times 1.2$ m) at Delft Hydraulics for various dike configurations and wave conditions. The 2% exceedance flow velocity and the 2% exceedance discharge were measured at 5 locations: the start of the dike crest ($x = x_0$), the end of the dike crest ($x = x_1$) and three locations on the landward slope. The 2% exceedance flow velocity and 2% exceedance discharge at $x = x_0$ are used as boundary conditions. The measured flow velocity at the other four points is used to validate the model. The crest width, the steepness of the slope and the roughness of the five configurations in Table 1 are used to build the cross-dike profile in the model. Configurations A, B, C and D are built of smooth wood with a friction coefficient $f = 0.005$ [13]. The cover roughness of configuration D2 was increased to $f = 0.025$ by placing a layer of stone material on top of the wooden base [13].

For each configuration, 18 tests were performed using multiple combinations of water depth, wave height and wave period. For the five tests, the layer thickness or flow velocities were so low that no data was reported at the waterside of the crest. Since this position provides the boundary condition for the model, those tests were not taken into account. Also, the data of four additional series were discarded for configuration D2 because of missing data at the first measurement position. In total, the selected flume tests result in 244 data points for model validation excluding the points used as boundary conditions.

Table 1: The steepness of the landward slope $\cot(\varphi)$, the crest width and the bottom friction coefficient f of the five dike configurations in the flume experiment of Van Gent [13]. Four validation points are available for each tests resulting in 244 data points for validation.

Configuration	$\cot(\varphi)$	Crest width (m)	Friction coefficient f	Number of selected tests
A	2.5	0.2	0.005	13
B	4	0.2	0.005	13
C	2.5	1.1	0.005	13
D	4	1.1	0.005	13
D2	4	1.1	0.025	9

Figure 3: The set-up of the tests at Tholen with the dike profile (solid line) and the five measurement locations P1, P2, P3, P4 and P5 as squares.

266 3.2.2. Field tests at Tholen

267 The dike at Tholen is characterized by a slope steepness $\cot(\varphi) = 2.4$ and
 268 was completely covered in grass with a roughness coefficient $f = 0.01$ [21].
 269 The maximum flow velocity and the maximum layer thickness were measured
 270 along the landward slope using paddle wheels and surfboards [21]. Figure 3
 271 shows the measurement locations at the end of the dike crest ($x = x_1$) and
 272 four locations along the slope at $x = 2.2$ m, 5.01 m, 7.81 m, 10.6 m and
 273 13.41 m from the outlet of the simulator. The measured maximum flow
 274 velocity at the start of the slope ($x = x_1$) is used as boundary condition $U_{s,0}$
 275 in the model. The momentary discharge Q is determined from the product
 276 of the measured maximum flow velocity and the measured layer thickness at
 277 the first measurement point ($x = x_1$). The maximum flow velocity at the
 278 other four points along the slope are used for validation. The maximum flow
 279 velocity and the maximum layer thickness were measured for 13 different
 280 overtopping volumes varying from 400 l/m up to 5500 l/m. Each volume
 281 was released twice resulting in 26 tests. Since the model is independent of
 282 the wave volume, the tests with a similar overtopping volume are handled as
 283 two separate tests as they give slightly different initial velocities. In total,
 284 104 data points are available for model validation excluding the points used
 285 as boundary conditions. Since the flow velocity and layer thickness are only
 286 measured on the slope, the data of the tests at Tholen can only be used for
 287 validation on the landward slope.

Figure 4: The set-up of the tests at Millingen a/d Rijn with the dike profile (solid line) and the measurement locations as squares.

3.2.3. Field tests at Millingen a/d Rijn

The tests at Millingen a/d Rijn were performed on a grass-covered dike with a slope steepness $\cot(\varphi) = 3$. The maximum flow velocity and layer thickness were measured halfway the crest, at the end of the crest and at 6 locations on the landward slope. Figure 4 shows the measurement locations at $x = 2$ m, 4 m, 5 m, 6 m, 7 m, 8 m, 9 m and 16 m from the outlet of the WOS. At each location, the maximum flow velocity and layer thickness were measured using paddle wheels and surfboards. The model domain starts halfway the dike crest at the location of the first measurement point ($x = x_0$). The flow velocity at this measurement point is used as boundary condition for the model. Again, the discharge is determined from the product of the flow velocity and layer thickness at the first measurement point ($x = x_0$). The measured flow velocity at the other seven locations are used for validation. In total, 28 tests were reported with varying volumes from 400 l/m up to 5500 l/m, equivalent to $Q \approx 325 - 2500$ l/s/m. This results in 196 data points for validation of the analytical model.

3.3. Comparison with existing formulas

The analytical model is compared with two existing formulas for the flow velocity on the crest and with one formula for the flow velocity on the slope as reported in Appendix A. These selected formulas are the only formulas that are able to calculate the maximum flow velocity at every point along

the dike profile for both the flume tests and the field tests. The formulas of Schüttrumpf and Oumeraci [1] for the crest and the slope were also coupled to study the performance for the combined profile of the crest and the landward slope.

In this study, the spatial step in the cross-dike direction was set to 0.01 m. Since the flow velocity on the slope from Schüttrumpf and Oumeraci [1] is iteratively calculated, the spatial step needs to be sufficiently small. The formulas derived in this paper for the flow velocity and the layer thickness are independent of the spatial step and likewise the analytical model.

The effectiveness of the new and the selected formulas is measured using the Nash-Sutcliffe model efficiency factor. The Nash-Sutcliffe model efficiency factor E is a measure for the predictive power of models [23]. The factor compares the model output with the observations and calculates the deviation from the one-to-one relationship. In the case the model output perfectly matches the data, the efficiency factor is 1. If the efficiency factor is 0, the model predictions are as accurate as the mean of the observed data. The model efficiency factor is calculated as

$$E = 1 - \frac{\sum_{i=1}^N (U_{model} - U_{data})^2}{\sum_{i=1}^N (U_{data} - \overline{U_{data}})^2} \quad (28)$$

with the modelled flow velocity U_{model} , the measured flow velocity U_{data} , the average of the measured flow velocity $\overline{U_{data}}$ and the number of data points N . The model efficiency factor is calculated on the slope separately ($N = 468$) using the new formula for the slope (Equation 19) and the formula for the slope of Schüttrumpf and Oumeraci [1]. The model efficiency is also calculated for the crest and the slope of the analytical model and the coupled formulas of Schüttrumpf and Oumeraci [1] using the 544 data points.

Furthermore, the model efficiency factor is calculated separately for the crest using the measurements at the end of the dike crest of the flume tests ($N = 48$) and the tests at Millingen a/d Rijn ($N = 28$). The offshore wave length L_0 in the formula of Van der Meer et al. [8] is determined from the peak period T_p as

$$L_0 = \frac{g T_p^2}{2\pi} \quad (29)$$

Figure 5: Comparison of the formulas: The normalized modelled flow velocity against the normalized measured flow velocity U_{data} together with the one-to-one relationship for the analytical model U_{model} , the formula of Schüttrumpf and Oumeraci [1] U_{SO} and the formula of Van der Meer et al. [8] U_{vdM} for (a) the crest, (b) the slope, (c) combined profile.

338 The peak periods of the flume tests varied between 1.6 and 2.5 s [13]. The
 339 overtopping spectrum used in the WOS at Millingen a/d Rijn was based on
 340 a peak period of 4 s [21].

For better comparison between the flume tests and the field tests, the flow velocity is normalized by the significant wave height H_s as

$$U_n(x) = \frac{U(x)}{\sqrt{gH_s}} \quad (30)$$

341 with $U_n(x)$ the normalized flow velocity, which is dimensionless. The signif-
 342 icant wave height of the field test was 1 m and 2 m for the experiments at
 343 Millingen a/d Rijn and Tholen, respectively [21, 19]. The significant wave
 344 height varied between 0.12 m and 0.15 m for the flume tests [13].

345 4. Results

346 4.1. Comparison of the formulas

347 The performance of the analytical model is compared with the perfor-
 348 mance of the existing formulas separately for the crest, the slope and the
 349 combined profile, in which case the formulas for the crest and slope are cou-
 350 pled (Figure 5). The difference between the combined profile and the results
 351 on the slopes are caused by the deviation from the modelled flow velocity and
 352 the measured flow velocity at the end of the crest. If the formulas are only

Figure 6: Model validation: The normalized modelled flow velocity against the normalized measured flow velocity U_{data} together with the one-to-one relationship of all data points of (a) the analytical model U_{model} , (b) the formula of Schüttrumpf and Oumeraci [1] U_{SO} , (c) the formula of Van der Meer et al. [8] U_{vdM} with only data on the crest.

353 applied to the slope, the measured flow velocity at the start of the slope is
 354 used as boundary condition instead of the model output from the crest in the
 355 case of the combined profile. Most of the validation points of the analytical
 356 model are close to the one-to-one relationship, while the formula of Van der
 357 Meer et al. [8] slightly overestimates the flow velocity on the crest and the
 358 formulas of Schüttrumpf and Oumeraci [1] underestimate the flow velocity
 359 on the crest, the slope and the combined profile (Figure 5). The formula of
 360 Van der Meer et al. [8] overestimates the flow velocity for the flume tests, but
 361 shows good agreement with the data for the tests at Millingen (Figure 6c).
 362 The underestimation of the flow velocity using the formula of Schüttrumpf
 363 and Oumeraci [1] holds for the flume as well as the field data (Figure 6b).

364 The Nash-Sutcliffe model efficiency factor was determined for the slope
 365 and the combined profile for the analytical model and the formula of Schüttrumpf
 366 and Oumeraci [1]. Also, the efficiency factor was determined separately for
 367 the crest for comparison with the formula of Van der Meer et al. [8]. The
 368 efficiency factors are shown in Table 2, in which the bold values correspond
 369 to the highest efficiency factor. On the dike crest, the new formula (Equa-
 370 tion 11) and the formula of Van der Meer et al. [8] perform the best in an
 371 equal amount of cases, but both show negative Nash-Sutcliffe values for con-
 372 figuration B. The formula of Van der Meer et al. [8] was calibrated for smooth
 373 slopes and contains no parameter for the dike cover roughness so the formula
 374 can not be applied to the rough stone cover of configuration D2. On the
 375 slope, the new formula (Equation 19) performs better than the formula of

Table 2: The Nash-Sutcliffe efficiency factor E for the dike crest, the slope and the combined dike profile using the analytical model, the formula of Schüttrumpf and Oumeraci [1] (*SO*), the formula of Van der Meer et al. [8] (*vdM*). The bold values indicate the highest efficiency factor.

Data	Crest			Slope		Combined	
	Model	SO	vdM	Model	SO	Model	SO
Tholen	-	-	-	0.80	0.61	-	-
Millingen a/d Rijn	0.91	0.82	0.95	0.89	0.45	0.81	0.37
Configuration A	0.84	0.79	0.84	0.74	0.05	0.87	0.08
Configuration B	-0.60	-0.67	-0.54	0.50	-0.02	0.62	-0.21
Configuration C	0.50	0.95	0.21	0.66	0.13	0.72	0.21
Configuration D	0.87	0.63	0.60	0.57	0.08	0.80	0.09
Configuration D2	0.93	0.57	-	0.69	0.06	0.49	0.58

Schüttrumpf and Oumeraci [1] for all tests. In case of the coupled formulas, the analytical model performs best, except for configuration D2 where the formulas of Schüttrumpf and Oumeraci [1] perform better. The deviation between the modelled flow velocity and the measured flow velocity at the end of the crest has a positive effect on the flow velocity on the slope in case of Schüttrumpf and Oumeraci [1], since the analytical formulas derived in this study perform much better than the formulas of Schüttrumpf and Oumeraci [1] in the separate cases.

The efficiency factor of configuration B is negative on the crest for all three formulas. The data of configuration B show an increase in flow velocity over the crest, while all three formulas only describe a flow velocity decrease over the dike crest. The increase in flow velocity might be caused by the small crest width of 0.2 m which can lead to violation of the continuity of discharge. Even though the analytical model is not able to predict a flow velocity increase on the dike crest, the efficiency factor for the combined profile is good. This means that the model is able to calculate the maximum flow velocity on slope for dikes with a small crest width despite the fact that the model is not able to accurately simulate the flow velocity on the crest itself.

4.2. Model validation

In this section, the deviations between the model results and the data are discussed in more detail. The analytical model overpredicts the flow velocity for some points of the field tests at Tholen and Millingen (Figure 6a). At first, it was hypothesized that the deviations from the one-to-one rela-

Figure 7: Model results of the experiment at Tholen: (a) The modelled flow velocity U_{model} against the measured flow velocity U_{data} for the grouped overtopping volumes V released from the WOS together with the one-to-one relationship. (b) The modelled flow velocity $U(x)$ (lines) as a function of the cross-dike distance x with the measured flow velocity as marker for four overtopping volumes. (c) The modelled flow velocity U_{model} against the measured flow velocity U_{data} for the four measurement points used for validation (see Figure 3).

400 tionship might be the result of measurements for small overtopping volumes.
 401 It is hard to measure the layer thickness and flow velocity for small over-
 402 topping volumes because the layer thickness is very small. As an example,
 403 Figure 7a shows the modelled flow velocity against the measured flow velocity
 404 for groups of overtopping volumes at Tholen. The deviation from the one-
 405 to-one relationship is slightly larger for smaller overtopping volumes, but the
 406 deviation is still large for a volumes of 2500 - 3500 l/m (yellow diamonds).

407 Figure 7b shows the modelled flow velocity along the cross-dike profile
 408 for four overtopping volumes at Tholen where the markers indicate the mea-
 409 sured flow velocities. The difference between the modelled and the measured
 410 flow velocity is largest for the measurement point at the end of the slope
 411 ($x = 13.4$ m). This is also observed in Figure 7c where the deviation from
 412 the one-to one relationship is largest for the measurement location at the
 413 end of the slope $P5$ (Figure 3). The difference is caused by a flow velocity
 414 decrease at the lower half of the slope, which is not expected from the ex-
 415 isting wave overtopping theory. The decrease in flow velocity on the lower
 416 half of the slope is probably caused by an increase in overtopping duration
 417 [19]. Increase in overtopping duration violates the continuity of momentary
 418 discharge resulting in a decrease in flow velocity while the layer thickness
 419 stays the same. The formula of Schüttrumpf and Oumeraci [1] is also not
 420 able to simulate the decrease in flow velocity at the lower part of the slope.

Figure 8: Model results for the slope transition at Millingen a/d Rijn: The modelled flow velocity $U(x)$ (lines) as a function of the cross-dike distance x with the measured flow velocity plotted as markers for four released volumes. The slope changes from a steepness $\cot(\varphi) = 3$ to a steepness $\cot(\varphi) = 6$ at $x = 13.4$ m.

4.3. Slope change along the landward slope

At the experiment at Millingen a/d Rijn, the slope steepness changes halfway the landward slope. The upper slope has a much steeper slope $\cot(\varphi) = 3$ compared to the lower slope of $\cot(\varphi) = 6$ (Figure 4). This slope change can easily be incorporated in the model by locally changing the slope angle φ . Figure 8 shows the modelled flow velocity along the dike profile for four released volumes at Millingen a/d Rijn. The flow velocity decreases at the slope transition ($x = 13.4$ m), because the acceleration due to gravity is smaller as a result of the gentle lower slope. At the slope transition, the flow velocity is high resulting in a larger friction term. The friction term is larger than the acceleration term due to gravity leading to a decrease in flow velocity until a balance is reached between the friction and the gravity term. The formula of Schüttrumpf and Oumeraci [1] was not able to simulate the slope change along the landward slope and showed unphysical behaviour at the slope transition.

Table 3: The ratios R_1 , R_2 and R_3 determined from the modelled flow velocity and layer thickness according to Equations 25-27, with $R_1 = R_2$.

Data	R_1, R_2	R_3
Millingen a/d Rijn	8.35	31.43
Tholen	-	20.19
Configuration A	8.10	98.15
Configuration B	10.38	106.24
Configuration C	8.51	88.70
Configuration D	3.40	89.13
Configuration D2	3.80	56.79

5. Discussion

5.1. Validation of assumptions

The formulas of the analytical model are derived under three assumptions which were written as three ratios in Section 2.4. The ratios are calculated for the different tests and the minimum value is reported in Table 3. The value of ratios R_1 and R_2 are equal because of the balance between advection and bottom friction on the dike crest (see Equation 6). The ratios are larger than 1 for all cases, although ratio R_1 and R_2 for configuration D and D2 are small. The formula for the flow velocity on the crest is derived for flow where the advection process is more important than the diffusive process. The importance of the advection process relative to the diffusive process is calculated using ratio R_1 and this ratio should be larger than 1. In case of configuration D and D2, the diffusive process is relatively more important compared to the other tests. Although the diffusive process is neglected, the model performs well with a Nash-Sutcliffe efficiency factor of 0.87 and 0.80 on the crest for configuration D and D2, respectively.

5.2. Model limitations

The analytical model has two limitations. Firstly, continuity of momentary discharge was assumed during derivation of the formulas. For this reason, the model is not able to accurately capture the observed decrease in flow velocity on the lower part of the landward slope.

Secondly, the model is generally not applicable for all values of the friction coefficient. From theory (Section 2), it is expected that the acceleration at the start of the slope is larger for smaller values of the friction coefficient because of a smaller friction term. For very low friction factors, the model does not show the expected physical behaviour (Figure 9a). The acceleration

at the start of the slope decreases significantly when the friction coefficient is lowered to $f = 0.001$. Thus, the model is not applicable for values smaller than the limiting friction coefficient f_{lim} . This lower limit of the friction coefficient is also seen in the gradient of the flow velocity at the start of the slope $\partial U(s)/\partial s(s = 0)$ (Figure 9b). The gradient increases with decreasing values of the friction coefficient f until the limiting friction coefficient (dashed line) is reached and the gradient rapidly decreases for smaller values of the friction coefficient. The model is not applicable for values smaller than the limiting friction coefficient f_{lim} .

The limitation of the model is calculated from the geometry and the boundary conditions of the model (see Appendix B for the derivation of the formula).

$$f_{lim} = \frac{gQ \sin(\varphi)}{4U_{s,0}^3} \quad (31)$$

The lower limit of the friction coefficient increases with decreasing flow velocity at the start of the slope $U_{s,0}$, increasing momentary discharge Q and increasing slope steepness $\cot(\varphi)$ (Figure 9 c,d). The boundary conditions of the tests at Millingen are also plotted in Figure 9c. The friction coefficient of grass ($f = 0.01$) is used for the tests at Millingen. The model is valid for the tests at Millingen since the limiting friction factor is smaller than 0.01 for the test. The model is also valid for the flume tests and the test at Tholen. The friction values used in this study were close to the limit. Thus, the limiting friction coefficient needs to be tested before applying the analytical model to a new dike configuration.

5.3. Application of the model

In this paper, an analytical overtopping model was built and tested for a range of conditions. The model was applied to scale models with multiple crest widths, slope angles and roughness. The model was also applied to two river dikes in the Netherlands. The variety in data shows that the analytical model can be applied to a wide range of geometries and covers. The advantage of an analytical model is that the role of the parameters is directly clear so that, in this model, the role of the friction coefficient is clear from the start.

The input parameters of the model are the maximum flow velocity at the start of the dike profile U_0 and the momentary discharge Q . In this

Figure 9: (a) The modelled flow velocity on the slope $U(x)$ for various values of the friction coefficient f using slope steepness $\cot(\varphi) = 3$, $U_{s,0} = 4.5$ m/s and discharge $q = 1.0$ $\text{m}^3/\text{s}/\text{m}$, corresponding to an overtopping volume of 1500 l/m at Millingen. (b) The velocity gradient at the start of the slope $\partial U(s)/\partial s(s=0)$ as a function of the friction coefficient f together with the limiting friction coefficient f_{lim} (dashed) for the same parameters as (a). (c) Contour lines of the limiting friction coefficient f_{lim} as function of the momentary discharge Q and the velocity at the start of the landward slope $U_{s,0}$ together with the data of the tests at Millingen as squares. (d) As (c), Contour lines of the limiting friction coefficient f_{lim} as function of the momentary discharge Q and the slope steepness $\cot(\varphi)$.

study, the input parameters were obtained from measurements. The model is designed for individual overtopping waves with maximum flow velocity $U(x)$. However, the model was also able to calculate the variation in the 2% exceedance flow velocity in the scale test. To apply the model to a case where no measurements are available, the overtopping discharge and overtopping flow velocity can be calculated using formulas in literature, for example the formula of Van der Meer et al. [8] (Equation A.7). Coupling of the model with a formula for the flow velocity on the waterside of the dike is possible through the boundary condition for the flow velocity U_0 .

The formulas derived in this paper can be applied to any dike profile without calibration of parameters and no iterative calculation is necessary as is the case for the formulas of Schüttrumpf and Oumeraci [1]. The formula for the maximum flow velocity on the dike crest showed similar performance as the formulas of Van der Meer et al. [8] and performed better than the formula of Schüttrumpf and Oumeraci [1]. The formula for the maximum flow velocity on the crest derived in this paper can be used to study the effect of dike cover roughness on the flow velocity, which is not possible using the existing formulas.

The model application is limited to non-diffusive overtopping (see Section 2.4). Also, the friction coefficient in the model shows unphysical behaviour for values smaller than the limiting friction coefficient. If the geometry and the boundary conditions are known, the lower limit f_{lim} can easily be calculated using Equation 31.

The main focus of this study is the change in maximum overtopping flow velocity on the dike crest and the landward slope, but the formulas can also be applied to other flood-protective structures such as embankments, spillways and dams. However, the analytical model is not validated for other flood protective structures, so further testing of these structures is recommended.

Several numerical models for wave overtopping are available [24]. Complex Navier-Stokes models are usually computationally expensive so that probabilistic calculations are infeasible. NLSW models have lower computational costs and can be used for probabilistic calculations, for example to determine the mean overtopping discharge during a storm for various flood-protective structures. Van Gent [25] showed that a NLSW model can provide estimates of the layer thickness and velocities at the landward slope with similar accuracy as the analytical expressions. However, the analytical model is computationally much faster so it can more easily be used for probabilistic calculations. An example of a probabilistic calculation in which a model for

the maximum flow velocity across the dike profile is needed, is the probability of dike failure due to wave overtopping during a storm [10].

6. Conclusions

In this study, two formulas are derived to describe the change in the overtopping flow velocity along the crest and the landward slope of flood-protective structures. Applying these formulas, it is possible to analytically calculate the maximum flow velocity at any point along the profile. The formulas for the flow velocity on the crest and the landward slope were coupled in an analytical model for wave overtopping on dikes. The validation using 544 data points from overtopping tests showed that the model performed well for a wide range of dike configurations both at flume and field scales, and is able to accurately simulate slope transitions along the landward slope. In addition, the two new formulas were validated separately for the crest and the slope, which means that the formulas can also be used independently. The presented formula for the velocities at the crest and the formula of Van der Meer et al. [8] showed similar performance for the data used in this study. Moreover, the presented analytical model performs better than the formulas of Schüttrumpf and Oumeraci [1]. However, the application of the model is limited to non-diffusive overtopping. Furthermore, the model is only valid for friction coefficients larger than the limiting friction coefficient.

The analytical model is validated for a wide range of dike geometries and dike covers. Recently, various cover layers have been developed to reduce overtopping erosion, for example geotextile, armour slope, grass blocks and many other rock revetments [24, 26, 27]. The proposed model can be used to obtain a first estimate of the reduction in maximum flow velocity of such innovative dike covers. Accurate predictions of the maximum flow velocity will reduce the uncertainty in the calculations of the failure mechanisms, in turn leading to more accurate calculations of the dike strength, which can be used to further improve the assessment and design of flood-protective structures.

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645 **Appendix A. The selected formulas for comparison**

646 *Appendix A.1. Formulas of Schüttrumpf and Oumeraci [1]*

647 The formula for the maximum layer thickness h_{max} on the dike crest was
648 empirically determined by Schüttrumpf and Oumeraci [1] resulting in

$$h_{max}(x) = h_0 \exp\left(-c_1 \frac{x}{B_c}\right) \quad (\text{A.1})$$

649 with h_0 the layer thickness at the start of the dike crest, B_c the crest width
650 and c_1 a calibration constant. Schüttrumpf [15] determined c_1 to be 0.75 for
651 wave spectra and regular waves. However, Van Gent [13] determined $c_1 = 0.4$
652 for irregular waves. Bosman et al. [16] remarked that the flow velocities

of Schüttrumpf [15] were too high, which might be an explanation for the difference in values for the calibration constant. The value of Schüttrumpf and Oumeraci [1] for c_1 is used in this analysis.

The maximum flow velocity $U(x)$ on the dike crest depends on the friction coefficient f and the maximum layer thickness $h_{max}(x)$

$$U(x) = U_0 \exp\left(-\frac{f x}{2 h_{max}}\right) \quad (\text{A.2})$$

with U_0 the maximum flow velocity at the start of the dike crest.

The maximum flow velocity on the dike slope $U(s)$ is given by

$$U(s) = \frac{U_{s,0} + \frac{k_1 h_U}{f} \tanh\left(\frac{k_1 t}{2}\right)}{1 + \frac{f U_{s,0}}{h_U k_1} \tanh\left(\frac{k_1 t}{2}\right)} \quad (\text{A.3})$$

with $U_{s,0}$ the maximum flow velocity at the start of the slope, $h_U(s)$ the momentary layer thickness along the slope, the along-slope coordinate $s = (x - B_c)/\cos(\varphi)$ and the time t is determined from the motion of a mass point on a slope without friction.

$$t(s) \approx -\frac{U_{s,0}}{g \sin(\varphi)} + \sqrt{\frac{U(s)^2}{g^2 \sin^2(\varphi)} + \frac{2s}{g \sin(\varphi)}} \quad (\text{A.4})$$

and the coefficient k_1

$$k_1 = \sqrt{\frac{2f g \sin(\varphi)}{h_U}} \quad (\text{A.5})$$

The momentary layer thickness $h_U(s)$ on the slope is determined from the continuity of discharge (Equation 1 and 20). Since the maximum flow velocity and the momentary layer thickness on the slope depend on each other, the maximum flow velocity on the slope can only be solved in an iterative manner.

Appendix A.2. Formulas of Van der Meer et al. [8]

The formula Van der Meer et al. [8] for the change in the maximum flow velocity on the dike crest is given by

$$U(x) = U_0 \exp\left(\frac{-1.4x}{L_0}\right) \quad (\text{A.6})$$

with the maximum flow velocity at the start of the crest U_0 and the offshore wave length L_0 . The formula of Van der Meer et al. [8] is adapted from Bosman et al. [16]. To overcome the discrepancy in the calibration constant of Equation A.1, Bosman et al. [16] introduced a formula for the flow velocity on the dike crest as a function of the offshore wave length and a friction factor. The formula of Bosman et al. [16] was only validated for smooth surfaces, thus it is unknown how the performance is for other friction factors. Van der Meer et al. [8] adapted the formula of Bosman et al. [16] by removing the friction factor using the geometry of the waterside slope. The formula was calibrated for smooth slopes, so it can not be applied to rough dike covers such as stone revetments. The parameters on the waterside slope are combined to the velocity at the start of the dike crest U_0

$$U_0 = 0.35 \cot(\theta) \sqrt{g(R_{u,2\%} - R_c)} \quad (\text{A.7})$$

with the angle of the waterside slope θ , the wave run-up height $R_{u,2\%}$ and the crest freeboard R_c .

Appendix B. Limiting friction coefficient

The gradient of the maximum flow velocity $U(s)$ at the start of the slope ($s = 0$) is given by

$$\frac{\partial U}{\partial s}(s = 0) = -3\alpha\beta^2\mu \quad (\text{B.1})$$

$$= \frac{3}{2^{-2/3}}\alpha \left(\frac{f}{Q}\right)^{1/3} \left(2^{1/3}\alpha - \left(\frac{f}{Q}\right)^{1/3} U_{s,0}\right) \quad (\text{B.2})$$

The gradient depends on the friction factor f . The limiting friction factor f_{lim} occurs when the gradient is maximum (Figure 9b). The limiting friction coefficient is calculated from

$$\frac{\partial}{\partial f} \frac{\partial U}{\partial s}(s = 0) = 0 \quad (\text{B.3})$$

Resulting in the formula for the limiting friction factor

$$f_{lim} = \frac{gQ \sin(\varphi)}{4U_{s,0}^3} \quad (\text{B.4})$$

Highlights

- Derivation of an analytical model for flow velocities on the dike crest and slope
- Insights on the velocity variation on the crest and the landward slope are presented
- The model is applicable to various dike geometries and multiple cover types
- The analytical model performs better compared with existing formulas