NOISE-BASED TRANSMIT REFERENCE MODULATION

A FEASIBILITY ANALYSIS

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Summary

Wireless sensor networks (WSNs) receive huge research interest for a multitude of applications, ranging from remote monitoring applications, such as monitoring of potential forest fires, floods and air pollution, to domestic and industrial monitoring of temperature, humidity, vibration, stress, etc. In the former set of applications, a large number of nodes can be involved which are usually deployed in remote or inaccessible environments. Due to logistic and cost reasons, battery replacement is undesired. Energy-efficient radios are needed, with a power-demand so little that batteries can last the lifetime of the node or that the energy can be harvested from the environment. Coherent direct-sequence spread spectrum (DSSS) based radios are widely employed in monitoring applications, due to their overall resilience to channel impairments and robustness against interference. However, a DSSS rake receiver has stringent requirements on precise synchronization and accurate channel knowledge. To obviate the complexity of a coherent DSSS receiver, particularly for low data rate sensor networks, a DSSS scheme that has fast synchronization and possibly low power consumption, is much desired.

In this regard, this thesis studies a noncoherent DSSS scheme called transmit reference (TR), which promises a simple receiver architecture and fast synchronization. In traditional TR, the modulated information signal is sent along an unmodulated reference signal, with a small time offset between them. In this thesis, we present and investigate a variant of TR, called noise-based frequency offset modulation (N-FOM), which uses pure noise as the spreading signal and a small frequency offset (instead of a time offset) to separate the information and reference signals. The detection is based on correlation of the received signal with a frequency-shifted version of itself, which collects the transmitted energy without compromising the receiver simplicity.

Analytical expressions on performance metrics, supplemented by simulation results, improve understanding of the underlying mechanisms and provide insights into utility of N-FOM in low-power WSNs. In point-to-point line-of-sight (LOS) communication, it was observed that the communication scheme has a minimal utility. The energy-detector type of receiver mixes all in-band signals, which magnifies the overall noise. Particularly, the self-mixing of the transmitted signal also elevates the noise level, which increases with a further increase in the received signal energy. Therefore, for a fixed set of system pa-

rameters, the performance attains an asymptote with increasing transmission power. The phenomenon also establishes a non-monotonic relation between performance and the spreading factor. It was observed that a higher spreading factor in N-FOM is beneficial only in a high-SNR regime.

After developing an understanding of the performance degrading mechanisms, few design considerations are listed. It is found that a suitable choice of the receiver front-end filter can maximize the SNR. However, the optimal filter depends on received signal and noise levels. A practically feasible – albeit sub-optimal – filter is presented which gives close to the optimal performance. Next, timing synchronization is considered. The implications of synchronization errors are analyzed, and a synchronization strategy is devised. The proposed synchronization strategy has little overhead and can be easily implemented for symbol-level synchronization.

The N-FOM LOS link model is extended to assess the performance degradation due to interference. Performance metrics are derived which quantify the effects of multiple-user interference, as well as that from external interferers, such as WiFi. Since the correlation operation mixes all in-band signals, the total interfering entities are quadratically increased. The research shows the vulnerability of N-FOM to interference, which makes it optimistic to operate in a crowded shared spectrum (such as the ISM 2.4 GHz band). We also observe an upper limit on the number of mutually interfering links in a multiple access (MA) network, that can be established with an acceptable quality.

The scheme is further investigated for its resilience against impairments introduced by a dense multipath environment. It is observed that despite the noise enhancement, the N-FOM system performs reasonably well in a non-line-of-sight (NLOS) communication. The detection mechanism exploits the multipath channel diversity and leads to an improved performance in a rich scattering environment. An analytical expression for outage probability is also derived. The results indicate that a healthy N-FOM link with very low outage probability can be established at a nominal value of the received bit SNR. It is also found that the choice of the frequency offset is central to the system design. Due to multiple practical implications associated with this parameter, the maximum data rate and the number of usable frequency offsets are limited, particularly in a MA NLOS communication scenario. The analysis evolves into a rule-of-thumb criterion for the data rate and the frequency offset.

It is deduced that, due to its limited capability to coexist in a shared spectrum, N-FOM is not a replacement for coherent DSSS systems. The scheme is mainly suited to a low data rate network with low overall traffic, operating in an interference-free rich scattering environment. Such a niche of sensor applications could benefit from N-FOM where the design goal requires a simple detection mechanism and immunity to multipath fading.

Samenvatting

Wireless sensor networks (WSNs) staan in een sterke wetenschappelijke belangstelling vanwege verscheidene toepassingsmogelijkheden, variërend van het op afstand detecteren van mogelijke bosbranden, overstromingen en luchtvervuiling, tot aan het huiselijk en industrieel monitoren van temperatuur, luchtvochtigheid, vibratie, spanning, etc. In de eerdergenoemde toepassingen kan een groot aantal modules worden gebruikt in afgelegen of lastig tot nauwelijks bereikbare omgevingen. Het vervangen van batterijen of accus is ongewenst vanuit het oogpunt van logistiek en kosten. Energie-efficiënte radios zijn nodig, die een dusdanig laag energieverbruik hebben dat de batterij tijdens de gehele levensduur van de module werkt of dat er voldoende energie uit de omgeving kan worden geëxtraheerd. *Direct sequence spread spectrum* (DSSS) technieken worden veelal ingezet voor monitorende toepassingen, voornamelijk vanwege hun robuustheid tegen kanaalgebreken en storingen. De DSSS *rake*-ontvangerarchitectuur legt desalniettemin strikte eisen op aan de synchronisatieprecisie en vereist nauwgezette kennis van het kanaal. Om de complexiteit van de coherente DSSS-ontvanger te omzeilen, specifiek voor sensornetwerken met een lage datasnelheid, is een DSSS-architectuur met een snelle synchronisatie en mogelijk een laag energieverbruik gewenst.

Dit proefschrift beschrijft een niet-coherent DSSS-schema genaamd *transmit reference* (TR). Het heeft een vereenvoudigde ontvangerarchitectuur en tevens een versnelde synchronisatie. In traditionele TR wordt het gemoduleerde informatiesignaal samen met een ongemoduleerd referentiesignaal verstuurd, waartussen een kleine tijdverschuiving bestaat. Dit proefschrift presenteert een variant op TR genaamd *noise-based frequency offset modulation* (N-FOM), welke gebruik maakt van pure ruis als een verspreidingssignaal en een kleine frequentieverschuiving (in plaats van een tijdverschuiving) om het informatiesignaal van het referentiesignaal te scheiden. De detectie is gebaseerd op het correleren van het ontvangen signaal met een frequentie-verschoven versie van zichzelf, wat zorgt voor het ontvangen van de verzonden energie zonder in te boeten op de eenvoud van de ontvanger.

Analytische uitdrukkingen voor prestatiestatistieken, ondersteund door simulatieresultaten, vergroten het begrip van het onderliggende mechanisme en verschaffen inzicht in de bruikbaarheid van N-FOM in WSNs met een laag energieverbruik. In punt- naar- puntcommunicatie met een *line of sight* (LOS) is

dit communicatieschema beperkt bruikbaar. Ontvangers van het type energie-detector mengen alle signaaltermen die zich in dezelfde band bevinden, waardoor de totale ruis toeneemt. Vooral het mengen van het verzonden signaal met zichzelf zorgt voor een verhoogd ruisniveau, dat toeneemt bij een verhoging van de ontvangen signaalenergie. Hierdoor is het voor een gegeven verzameling systeempparameters op een gegeven moment niet meer mogelijk de prestaties verder te verbeteren door het toevoegen van meer zendvermogen. Dit fenomeen zorgt ook voor een niet-monotone relatie tussen prestatie en spreidingsfactor. Daarnaast is vastgesteld dat een toenemende spreidingsfactor in N-FOM alleen nuttig is in een hoog SNR-regime.

Na het verkrijgen van inzicht in prestatieverminderende mechanismen worden een aantal ontwerpoverwegingen behandeld. Het blijkt dat de signaal-ruisverhouding kan worden gemaximaliseerd door een geschikte front-end filter in de ontvanger te kiezen. Het optimale filter is echter afhankelijk van de ontvangen signaal- en ruisniveaus. Een praktisch realiseerbaar filter, hoewel suboptimaal, komt dichtbij de optimale prestatie. Hierna wordt tijdssynchronisatie beschouwd. De implicaties van synchronisatiefouten worden onderzocht en een synchronisatiestrategie wordt geconstrueerd. De voorgestelde strategie heeft nauwelijks overhead en kan gemakkelijk worden geïmplementeerd voor synchronisatie op symboolniveau.

Het N-FOM LOS linkmodel wordt uitgebreid om de prestatievermindering ten gevolge van storing te onderzoeken. Prestatiestatistieken worden afgeleid om de storingseffecten van meerdere gebruikers en externe storingen zoals WiFi te kwantificeren. Aangezien de correlatieoperatie alle signalen in dezelfde band mengt, nemen de totale storingstermen kwadratisch toe. Het onderzoek legt de gevoeligheid van N-FOM voor storing bloot, waardoor men kan stellen dat het optimistisch is het in een druk bezet deel van het frequentiespectrum zoals de 2.4 GHz band (ISM) te gebruiken. Bovendien identificeren we een bovengrens aan het aantal onderling versturende verbindingen dat in een *multiple access* (MA) netwerk met een acceptabele kwaliteit tot stand kan worden gebracht.

Het schema is verder onderzocht op zijn weerbaarheid tegen verstoringen die worden geïntroduceerd door een dichte multipadomgeving. Ondanks de ruisversterking werkt het N-FOM systeem redelijk goed in *non-line-of-sight* (NLOS) communicatie. In sterk verstrooiende omgevingen maakt het detectiemechanisme gebruik van de diversiteit van het multipadkanaal, hetgeen resulteert in een verbeterde prestatie. Ook wordt een analytische uitdrukking afgeleid voor de *outage probability*, de kans dat het systeem uitvalt. Een gezonde N-FOM verbinding met lage uitvalkans kan worden gerealiseerd met een nominale waarde van de ontvangen signaalruisverhouding per bit. De keuze van de frequentieverschuiving blijkt een centrale rol te spelen in het systeemontwerp. Vanwege verschillende praktische implicaties die gepaard gaan met de keuze van deze parameter zijn de maximale datasnelheid en het aantal bruikbare frequentieverschuivingen beperkt, zeker in een MA NLOS communicatiescenario. Uit deze analyse kan een vuistregel worden afgeleid voor de datasnelheid en de frequentieverschuiving.

Uiteindelijk kan worden geconcludeerd dat door de beperkte mogelijkheden van een gedeeld spectrum N-FOM geen vervanger is voor coherente DSSS-systemen. Het schema is voornamelijk geschikt voor een netwerk met weinig dataverkeer en lage datasnelheid dat opereert in een storingvrije maar zeer sterk verstrooiende omgeving. Een dergelijke niche sensortoepassing zou baat kunnen hebben bij N-FOM, mits de ontwerpdoelen stellen dat er een simpel detectiemechanisme en immuniteit voor multipaden moet zijn.

خُلاصہ

آلات جس کے وائرلیس نیٹ ورکس (WSNs) پر دلچسپی سے سائنسی تحقیق انکی افادیت کے باعث ہوتی ہے، جیسا کہ جنگلات میں آگ کی پیشگی اطلاع، سیلاب کی اطلاع، ہوا میں آلودگی، درجہ حرارت، نمی، ارتعاش اور کشیدگی وغیرہ کی نگرانی۔ ایسے اعمال میں مواصلاتی آلات کی کثیر تعداد کو کام کرنا پڑتا ہے اور انہیں عام طور پر دور دراز یا ناقابل رسائی ماحول میں نصب کیا جاتا ہے۔ علاقائی دوری اور زیادہ خرچے کے سبب بیٹری کو تبدیل کرنا دشوار ہوتا ہے۔ لہذا بجلی کی کم مانگ والے ایسے مواصلاتی آلات کی اشد ضرورت ہے، جن کی بیٹریاں آلات کو انکی حیات تک توانائی میسر کر سکیں، یا یہ کہ ماحول سے حاصل شدہ توانائی سے ہی آلات چل سکیں۔ وائرلیس چینل کی خرابی کے خلاف مضبوطی، اور مدخلی شعاعوں کے خلاف لچک کے باعث ڈائریکٹ سیگنل سپریڈ شیڈیٹم (DSSS) کو نگرانی والے اعمال میں اکثر استعمال کیا جاتا ہے۔ تاہم DSSS کے ریک (rake) ریسپور کے ادراک کے لئے ہابنگ سازی اور چینل کا درست علم جیسے سخت ضروریات کا سامنا ہے۔ ایک مربوط DSSS ریسپور کی پیچیدگیوں کو زائل کرنے کے لئے، اور خاص طور پر کم ڈیٹا کی شرح والے WSNs کے لئے ضرورت اس امر کی ہے کہ ایک ایسی DSSS سکیم تیار کی جائے جو تیز رفتار ہابنگ سازی اور حتی الامکان کم بجلی استعمال کرنے کے قابل ہو۔

اس حوالے سے اس مضمون میں ایک غیر مربوط DSSS سکیم کو زیر بحث لایا گیا ہے جسے ٹرانسمٹ ریفرنس (TR) کہا جاتا ہے۔ یہ ایک غیر پیچیدہ ریسپور اور تیز رفتار ہابنگ سازی فراہم کرتا ہے۔ ایک روایتی TR میں ماڈیولیڈ معلوماتی سگنل کو ایک غیر ماڈیولیڈ ریفرنس سگنل سے ایک لمحہ فرق کے ساتھ ملا کر بھیجا جاتا ہے۔ اس مضمون میں ہم TR کی ایک مختلف قسم کو پیش کر کے اس کی تحقیق کریں گے، جسے شور پر مبنی فریکوئنسی موڈولیشن آفسیٹ (N-FOM) کہا جاتا ہے۔ اس میں خالص شور کو سپریڈنگ سگنل کے طور پر استعمال کیا جاتا ہے اور (لمحہ آفسیٹ کی بجائے) ایک چھوٹے سے فریکوئنسی آفسیٹ کے ذریعے معلوماتی اور ریفرنس سگنل کو الگ کیا جاتا ہے۔ ریسپور کا تشخیصی عمل موصولہ سگنل کو اپنی فریکوئنسی شفٹڈ ورژن سے کوریلیٹ (correlate) کرتا ہے۔ یہ عمل ریسپور کی سادگی پر کسی قسم کا سمجھوتہ کئے بغیر منتقل شدہ توانائی کو حاصل کرتا ہے۔

کارکردگی کو پرکھنے والے ریاضی مساوات اور بناوٹی نتائج N-FOM کے بنیادی عمل کو سمجھنے میں معاون ہوتے ہیں اور کم توانائی والے WSNs میں N-FOM کی افادیت کو اجاگر کرتے ہیں۔ پوائنٹ ٹو پوائنٹ حد نظر (LOS) مواصلات میں اس سسٹم کی محدود افادیت دیکھی گئی ہے۔ انرجی ڈیٹیکٹر (energy-detector) قسم کے ریسپور تمام وصول شدہ سگنل کو آپس میں ضرب کر دیتا ہے، جس سے مجموعی شور بہت بڑھ جاتا ہے۔ مخصوصاً، منتقل شدہ سگنل کا خود سے اختلاط شور کی سطح بلند کرتا ہے، جس میں بھیجے گئے سگنل کی توانائی بڑھانے سے مزید اضافہ ہوتا ہے۔ لہذا جب منتقلی توانائی ایک حد سے بڑھ جاتی ہے تو سسٹم کی

کارکردگی میں اضافہ بھی محدود ہو جاتا ہے۔ یہ عمل کارکردگی اور سپریڈنگ فیکٹر کے درمیان ایک بے ربط تعلق قائم کرتا ہے۔ دیکھا گیا ہے کہ سپریڈنگ فیکٹر میں اضافہ اسی صورت میں مفید ہے جب شور و سگنل کا تناسب (SNR) بھی بہت عمدہ ہو۔

کارکردگی میں خرابی کے طریقہ کار کو سمجھ لینے کے بعد کچھ آراء پیش کی گئی ہیں۔ یہ نتیجہ اخذ کیا گیا ہے کہ ایک موزوں فرنٹ اینڈ ریسیور فلٹر SNR کو بہترین بناتا ہے۔ تاہم، ایک بہترین فلٹر کا انحصار موصولہ سگنل اور شور کی سطح پر ہے۔ اسی سلسلے میں ایک عملی مگر غیر مستقیم فلٹر بھی پیش کیا گیا ہے، جو بہترین کارکردگی کے قریب قریب نتائج دیتا ہے۔ اس کے بعد ہابنگ سازی کے موضوع کو اٹھایا گیا ہے۔ ہابنگ سازی میں غلطیوں کے اثرات پہ تجزیہ کرنے کے بعد اس کیلئے حکمت عملی وضع کی گئی ہے۔ مجوزہ حکمت عملی کے نظام کا سسٹم پہ بوجھ بہت کم دیکھا گیا ہے، اور اس لئے بسولت لاگو کیا جاسکتا ہے۔

مداخلتی شعاعوں کی وجہ سے کارکردگی میں خرابی کا اندازہ لگانے کے لئے سسٹم ماڈل کی توسیع کی گئی ہے۔ ریاضیاتی مشق سے کارکردگی کے پیرامیٹرز حاصل کئے گئے ہیں، جو اندرون صارفین کی مداخلت (MUI) کے ساتھ ساتھ بیرونی مداخلت جیسا کہ WiFi کے اثرات کا علم دیتے ہیں۔ چونکہ تشخیصی عمل تمام موصولہ سگنلوں کو آپس میں ضرب دیتا ہے، اس لئے مداخلتی عناصر کی تعداد میں مریع اضافہ ہو جاتا ہے۔ یہ تحقیق N-FOM کی مداخلتی شعاعوں کے سامنے کمزوری کو ظاہر کرتی ہے، جس سے مشترکہ سپیکٹرم، جیسا کہ ISM بیڈز، میں کام کرنے کی امید کم نظر آتی ہے۔ اسی طرح ایک نیٹ ورک میں باہمی مداخلت کرنے والے ایسے صارفین کی تعداد محدود ہے، جو ایک قابل قبول معیار کے مطابق رابطہ قائم رکھ سکیں۔

مزید تحقیق ایک گنجان ملپاتھ (multipath) ماحول میں کی گئی ہے، تاکہ ملپاتھ ماحول کی خرابیوں کے خلاف اس سکیم کی چلک کو پرکھا جائے۔ دیکھا یہ گیا ہے کہ شور بڑھنے کے باوجود غیر مرئی خط (NLOS) مواصلات میں N-FOM نظام ایک مناسب اچھی کارکردگی کا مظاہرہ کرتا ہے۔ تشخیصی نظام ایک ملپاتھ ماحول کا فائدہ اٹھاتا ہے، جس کی وجہ سے شعاعوں کو منتشر کرنے والے ماحول میں بہتر کارکردگی دیکھی گئی ہے۔ اس مطالعہ میں غیر فعالیت کے امکان کے ریاضی مساوات بھی اخذ کئے گئے ہیں۔ نتائج سے پتہ چلتا ہے کہ دو N-FOM صارفین کا رابطہ کٹنے کا امکان ایک عام قدر SNR سے ہی بہت کم رکھا جاسکتا ہے۔ مزید یہ اکتشاف ہوا ہے کہ فریکوئنسی آفسیٹ کا انتخاب نظام کے ڈیزائن میں مرکزی اہمیت کا حامل ہے۔ اس پیرامیٹر کے کئی عملی اثرات ہیں۔ اس کا انتخاب زیادہ سے زیادہ ڈیٹا کی شرح اور قابل استعمال فریکوئنسی آفسیٹ کی تعداد کو محدود کرتا ہے، جو بطور خاص نیٹ ورک کے صارفین کے مابین NLOS مواصلات کے پس منظر میں کافی اہمیت رکھتا ہے۔ یہ تجزیہ ہمیں ڈیٹا کی شرح اور فریکوئنسی آفسیٹ کے بیچ ایک ناپختہ اصول فراہم کرتا ہے۔

انجام یہ نکلتا ہے کہ مشترکہ سپیکٹرم میں اس کی محدود صلاحیت کی وجہ سے N-FOM ایک مربوط DSSS نظام کا متبادل نہیں ہے۔ یہ اسکیم بنیادی طور پر کم سے کم ٹریفک والے اُن نیٹ ورک کے لئے موزوں ہے، جو مداخلت سے آزاد اور شعاعوں کو منتشر کرنے والے ماحول میں نصب ہوں۔ WSN کی وہ اپیلی کیشنز N-FOM سے فائدہ اٹھا سکتی ہے جہاں مانگ ایسی سکیم کی ہو جو ایک سادہ تشخیصی نظام پر مبنی ہو اور ملپاتھ دھندلاہٹ سے محفوظ ہو۔

Acronyms

ACF	autocorrelation function
AFH	adaptive frequency hopping
APC	adaptive power control
AWGN	additive white Gaussian noise
BER	bit-error-rate
BLE	Bluetooth Low Energy
BPSK	binary phase shift keying
CIR	carrier-to-interference ratio
CLT	Central Limit Theorem
CNR	carrier-to-noise ratio
CSS	chirp spread spectrum
CW	continuous wave
DFS	dynamic frequency selection
DS	direct-sequence
DSP	digital signal processing
DSSS	direct-sequence spread spectrum
EIRP	effective isotropic radiated power
ETSI	European Telecommunications Standards Institute
FCC	Federal Communication Commission
FDMA	frequency division multiple-access
FH	frequency-hopping

FHSS	frequency-hopping spread spectrum
GFSK	Gaussian frequency shift keying
GPD	Green Power Devices
h.f.c.	high frequency components
ICD	Integrated Circuit Design
IDF	integrate-and-dump filter
IF	intermediate frequency
IoT	Internet-of-Things
IPI	inter-pulse interference
IR	impulse radio
ISI	inter-symbol interference
ISM	industrial, scientific and medical
ISO	International Organization for Standardization
ITU	International Telecommunication Union
LNA	low-noise amplifier
LO	local oscillator
LOS	line-of-sight
LPF	low-pass filter
LTI	linear time-invariant
MA	multiple access
MAC	medium access control
MPC	multipath component
MUI	multi-user interference
NB	narrowband
NBI	narrowband interference
N-FOM	noise-based frequency offset modulation
NLOS	non-line-of-sight
NOMAC	noise modulation and correlation system

NRZ	non-return-to-zero
OFDM	orthogonal frequency division multiplexing
PAPR	peak-to-average power ratio
PDF	probability density function
PDP	power delay profile
PHY	physical layer
PN	pseudo-noise
PRBS	pseudo-random binary sequence
PSD	power spectral density
QPSK	quadrature phase shift keying
RDE	reference decoherence effect
RF	radio frequency
RFE	receiver front-end
RRC	root raised-cosine
SIR	signal-to-interference ratio
SL	shifted-limiter
SNR	signal-to-noise ratio
SS	spread spectrum
TDL	tapped-delay line
TE	Telecommunication Engineering
ToA	time-of-arrival
TR	transmit reference
UWB	ultra-wideband
WBI	wideband interference
WSN	wireless sensor network
WSS	wide sense stationary
WSSUS	wide sense stationary uncorrelated scatterers
WPAN	wireless personal area network

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Introduction

1.1 Background

Wireless communication has fingerprints on nearly every modern technology. Whether it is a space station communicating with Earth or a smartphone communicating with a home theater system, life in the current era relies on services enabled by wireless transfer of information. It has changed the way how people around the world work, learn and communicate. Each passing day, new gadgets such as action cameras, smart watches, drones and virtual reality headsets are introduced to the market, all of which rely on seamless communication of data. Undeniably, the number of wireless devices is rapidly increasing [1]. The Internet of things (IoT), envisaged back in 1990s, is now becoming a reality. The idea is to create an intersection of communication for a plethora of “things” or electronic devices which can communicate over a variety of platforms. Realization of the IoT can significantly enhance public utility. The smart services offered by the IoT enable efficient transportation, enhance medical services, reduce carbon emissions, optimize energy consumption, increase public safety, and even improve shopping experience.

With advances in digital electronics, the design and development of power-efficient, low-cost circuits have become feasible. This has paved the way for wireless sensor networks (WSNs), which have become integral to our way of life. WSNs have a wide range of applications [2]. They have enabled or revolutionized applications, such as

- space exploration, where a swarm of sensors are deployed as nano-satellites for space-based radio astronomy;
- weather forecast and disaster monitoring, where measurements from sensors are used to report and predict weather events;
- smart homes, which enables consumers to automatize and/or control climate and home appliances;

- security, where one can detect and localize smoke, fire and movement with the help of sensor networks;
- intelligent transportation, which helps a coordinated and smooth flow of traffic;
- and telemedicine, where sensors provide imaging and health informatics data for remote diagnostics.

The vision behind WSNs is that they provide a cost-effective way of gathering information through a collective effort of a large number of small inexpensive devices. These devices sense physical phenomena such as motion, radiation, temperature, humidity, vibration or vital signs of a human body. The raw data is then sent to a central management node using single-hop or multi-hop transmission. The central management node, usually connected to the end user, uses algorithms to translate the raw physical data into useful information, such as the medical condition of a person, the state of congestion in a crowd, or the risk of flooding in a region. In this sense, the wireless sensor nodes themselves have limited processing capability, which allows them to operate on low power.

Remote monitoring is a key application of WSNs. For example, as preventive measure for forest fires, a large network of wireless sensors is deployed in a forest [3], as shown in Figure 1.1. The nodes sense temperature, smoke, and humidity levels, and send it to a central management node via multi-hop transmission. The management node interprets the collected data and warns the end-user against potential outbreak of a fire. Similarly, in wine-making, WSNs are used to monitor the health and growth of vines [4]. Using a combination of image and ambient sensors, the vine-grower monitors the microclimate of the vineyard. A potential pest infestation or a deficit of soil nutrients is timely predicted, helping the user to maintain a good health of the grapes.

A few key categories of monitoring applications are depicted in Figure 1.2. In military, Smart Dust was one of the first WSNs designed to help combatants

Figure 1.1: A wireless sensor network (WSN) in fire monitoring.

in hostile environments. The nodes can be deployed in a battlefield to provide information about enemy movements, infrastructure and airborne biochemicals [5]. In environmental monitoring, WSNs have been tested for volcano monitoring in Ecuador [6]. Using a set of seismic and acoustic sensors, the researchers were able to gather information about earthquakes near an active volcano. In agriculture, the WSNs have been used to optimize productivity in greenhouses by saving heat energy; to monitor the predicted climate change; and also for health of livestock, e.g., by monitoring the temperature and pH levels of the insides of the cattle [7].

A large number of monitoring applications require the nodes to be deployed in a hostile or an inaccessible environment. Depending on the practical scenario, replacement of faulty nodes or batteries can be costly or even infeasible. A design goal in such applications is to have a reliable sensor network that requires minimal or no human intervention. To reduce the energy consumption, the nodes sleep most of the time and wake up regularly to perform the designated tasks. In order to operate for longer durations without battery replacement or recharge, it is imperative that energy efficiency is maintained at all segments of the design. Ideally, one would like the nodes to operate on harvested energy. In certain applications, such as in military, the network has to be robust to jamming, electromagnetic interference or other such potential disruptions. Realization of these goals not only requires an extremely efficient network protocol, but also a low-power reliable communication scheme which can be easily implemented on a power-inexpensive chip. The communication scheme is central to the design of any wireless radio. An electromagnetic (EM) wave transmitted over a wireless medium is potentially subject to corruption by interference, noise, and impairments due to reflection, refraction and absorption of the EM wave. The need for reliable communication can lead to an increase in the power consumption of both the receiver and the transmitter, thus compromising the low-power design of the radio.

1.2 Communication in sensor networks

Wireless communication in monitoring applications is characterized by low data rate and low duty cycles. These applications do not demand continuous monitoring. Therefore, the nodes are in sleep mode most of the time in order to conserve energy, and wake up after long intervals to sense and transmit data.

In general, the nature of the application requires the WSN to be deployed in remote, often inaccessible locations where there can be multiple obstacles between the communicating nodes. The transmitted signal experiences multipath propagation; the received signal consists of multiple copies of the transmitted signal, each arriving at a different time and with a different phase and magnitude. Destructive interference between different paths or *multipaths* can partially suppress communication in a particular frequency band.

Moreover, such applications mostly operate in unlicensed frequency bands, sharing the frequency spectrum with other systems such as WiFi and Blue-

Figure 1.2: Key categories of monitoring applications using WSNs.

tooth. Multipath propagation can fade and distort the transmitted signal whereas other systems in the frequency band can cause significant interference. As such, communication in monitoring applications needs to be robust against impairments introduced by the propagation environment and in-band interferers.

Finally, the large number of nodes in a WSN provides further challenges for the design of a suitable communication scheme. Multiple communication links can be present at any given time in the network, which can mutually interfere with each other. This has two implications. First, the communication scheme should have a mechanism such that a receiver node can differentiate between signals from the different nodes. Secondly, the detection scheme should exhibit resistance to the overall interference. For a highly scalable network, the communication scheme should be able to handle a large number of nodes.

1.2.1 Spread-spectrum communication

A widely used communication scheme, which offers resistance to impairments from both the multipath channel and interferers, is spread-spectrum (SS) communication. An SS system is characterized by a couple of key elements:

- first, the transmitted signal occupies a bandwidth much larger than that of the information signal.

- Second, the spread in frequency is carried out by means of a spreading code.
- Third, the receiver synchronizes to the code in order to recover the information signal.

A prominent variant¹ of the SS systems is direct-sequence SS (DSSS), depicted in Figure 1.3. At the transmitter, a wideband spreading signal is generated which is usually known to the receiver. The spreading signal is a train of very narrow pulses or *chips*, with polarities defined by a spreading code. Typically, the spreading code is a pseudo-random binary sequence (PRBS), which, although generated by a deterministic algorithm, has spectral properties similar to truly random noise. The durations of the narrow pulses determine the operational bandwidth, which is much larger than that of the data signal. The spreading signal modulates the data signal, resulting in spreading of the information over the large frequency band. At the receiver, the demodulator “locks” on to the chips of the received signal, and then correlates it with the known spreading signal to retrieve the narrowband data signal. The ratio of the bandwidth of the spreading signal to that of the data signal is sometimes known as *spreading gain*.

Figure 1.3: Communication model of direct-sequence spread spectrum (DSSS).

This spreading over a wide bandwidth presents many advantages, one of which is illustrated in Figure 1.4. If a narrowband interferer (or a jammer) is present in the frequency spectrum, the correlation operation at the receiver spreads the interference over the wide bandwidth while simultaneously despreading the data signal. As a result, the interference caused to the demodulated signal is reduced by a factor proportionate to the spreading gain. Therefore, the spreading signal is sometimes also referred to as *processing gain*. The wideband operation of DSSS also provides resistance to frequency-selectivity of the propagation environment. As the transmitted signal is wideband, it is

¹Other well-known variants are frequency-hopping SS (FHSS) and time-hopping SS (THSS).

Figure 1.4: DSSS detection mechanism in the presence of an interferer.

highly unlikely that all frequency components are suppressed by the channel. Thus, the wideband system benefits from frequency diversity.

A key benefit of an SS system is its security. The unpredictable nature of the spreading code makes the communication cryptic and difficult to intercept. Additionally, if the information signal is finely and uniformly spread over a wide bandwidth, then the transmitted signal is indistinguishable from noise, making it difficult to be detected by a surveillance receiver. Although security is not a key concern in remote sensing, it is important to mention that security was the prime motivation behind development of the DSSS scheme [8]. During World War II, both Allies and Axis were in a technological race, in a desperate attempt to develop a reliable communication system with excellent anti-jamming capability. Although the concept of transmitting over a wide frequency range was known in those times, there was a significant gap in research and development of an effective receiver.

The earliest patent regarding “modulating a certain range of frequencies” was filed by Alfred N. Goldsmith in 1924 [9], who argued that such a communication method would be resistant to fading effects. The suggested communication scheme was a type of frequency modulation with the frequency varied over a certain range, but it lacked the correlation-based detection, typical to a DSSS receiver. It missed on full potential of an SS scheme, and, therefore, it was not SS in its true sense. At the height of World War II, a patent on FHSS was filed by actress Hedy Lamarr and composer George Antheil [10], who presented this “secret communication system” as a jam-proof radio guidance system for torpedoes. The idea was inspired by the working of a piano roll, which has 88 rows of perforations, and as such, the communication was suggested to switch between 88 different carrier frequencies. Although a significant invention, it received little attention at that time. Today, a variation of FHSS is used in the well-known Bluetooth standard [11].

The development of a DSSS system not only faced a challenging implementation but also a lack of understanding. In 1949, Claude E. Shannon provided a framework for delivering information over channels disturbed by additive Gaussian noise [12]. He revealed that even for low signal-to-noise ratios

(SNRs), one can maintain or even increase the *rate of transfer of information*² by expanding the transmission bandwidth. This landmark work shifted the paradigm in wireless communication and galvanized the communication engineers into action. As Robert Scholtz points out [8], this triggered a number of classified studies in the 1950s. One such workable system using noise-like carriers was NOMAC – noise modulation and correlation system [13]. This system used a noise-like spreading signal and had a correlation-based receiver for demodulation. The correlation at the receiver offered anti-jamming capability, but in multipath channels, the system was not able to fully benefit from the spreading gain, as the receiver was only able to lock onto one multipath.

In a multipath environment, the transmitted energy is distributed over the multipaths. To maximize efficiency, it is vital for the receiver to collect energy from as many paths as possible. Shannon had shown that any function of time that is limited to a bandwidth of W Hz can be accurately represented by samples that are $1/(2W)$ seconds apart. This meant that the wideband operation of DSSS enables excellent time-of-arrival (ToA) resolution of the receiver, i.e., the receiver can distinguish between signals from multipaths that are $1/(2W)$ seconds apart. It was soon realized that to fully achieve the benefits of DSSS, multiple correlators have to be implemented to gather energy from all multipaths. Additionally, the output of each correlator has to be added in phase after applying *proper* weights. Bob Price and Paul Green provided the answer to the optimal weights and this led to what we now know as the *rake receiver* [14], as depicted in Figure 1.5. A rake receiver consists of multiple sub-receivers or “fingers”, designed to independently decode the signal from the multipaths. In a rake receiver, the locally generated spreading signal locks and despreads the signal from each path. Using the complex weights $(\psi_1, \dots, \psi_N)$, the output of each sub-receiver is combined intelligently and added in phase, such that contributions from the strongest paths are accented the most, resulting in an overall enhanced SNR. In other words, the multipath channel provides *delay diversity* which is effectively exploited at the receiver.

In 1955, the rake receiver architecture was designed and implemented in a system known as F9C-A at Lincoln Laboratory [15]. With the rake receiver, the designers were able to gather energy from all the multipaths and make full use of the spreading gain. The F9C-A system was rushed into production and from thereon, used extensively in military applications.

It must be said that during its conception, DSSS was shrouded in cloak of secrecy and most of the studies were classified for decades. Although there is no single inventor of the DSSS system, Scholtz suggests that Shannon did have the initial insight. On the other front, Green mentions [15] that during his early talks (in Russian) with the Soviets about the rake receiver, he did not get any good questions. He implies that perhaps the Soviets knew about the DSSS, or perhaps it was “his lousy Russian”.

²This term is known as channel capacity, a principle term in wireless communication.

Figure 1.5: Rake receiver architecture with N fingers., and the optimal complex weights $\{\psi_1, \dots, \psi_N\}$.

1.2.2 Ultra-wideband communication

In 1998, ultra-wideband (UWB) was introduced by Win and Scholtz with their study of impulse radio (IR) in [16]. UWB is defined as a system with bandwidth either greater than 500 MHz or at least 20% of the carrier frequency. At the transmitter, a sequence of pulses with sub-nanosecond duration is generated. The short duration of the pulses (impulses) translates to ultra-large bandwidths. For modulation, classical SS approaches of direct-sequence (DS), frequency-hopping (FH) or time-hopping (TH) can be used to create a UWB signal [17]. The idea is to spread the signal spectrum using a combination of the data signal and the user-defined spreading sequence. With transmission power spread over a very large bandwidth, the UWB signal has very low power spectral density (PSD). This ensures that it appears *silent* to other systems, allowing operation alongside a licensed system without causing noticeable interference. Shortly after the conception of UWB, the Federal Communication Commission (FCC) made the frequency band between 3.1 and 10.6 GHz available to all unlicensed systems [18], under the limitation that the power emission level is not greater than -41.3 dBm/MHz.

In an ideally power-controlled environment, Win considered TH-IR in an MA scenario in an additive white Gaussian noise (AWGN) channel [19]. He modeled the UWB signal as a train of ultra-narrow pulses with relatively large pulse repetition time. Pulse position modulation was considered, and the position of each pulse in the train was further shifted by a unique TH sequence. A fully coherent receiver was proposed which locally generated the pulse train using the known TH sequence and correlated it with the received waveform. The research work provided a theoretical framework and showed that ideally, a network of more than 20,000 nodes can be deployed using TH-UWB for a data rate of 19.2 kbps. Due to its low transmission power and multiplexing capability, UWB was suggested as a viable candidate for short-range communication,

particularly for high-data-rate sensor networks [17].

The use of large bandwidths implies that the receiver has extremely high ToA resolution. It indicates that, to capture maximum energy in a multipath environment, one has to employ a rake receiver with a large number of fingers. Win et al. carried out semi-analytical and experimental analysis to evaluate the complexity of a rake receiver in a UWB system [20]. Based on maximum-likelihood estimation, optimal weights and delays were derived for implementation of the rake. In the experimental setup, a UWB system with 1 GHz transmission bandwidth was deployed in a typical office building. The results indicated that in a high-SNR regime, a rake receiver with 50 fingers sufficiently captures (about 60% of) the total transmitted energy. Increasing the number of fingers beyond that slightly improves the energy capture. However, beyond 100 fingers, there is negligible improvement.

It is worth noting that UWB is not confined to SS. In 2007, UWB was added to the physical layer (PHY) of the IEEE 802.15.4a standard [21]. UWB has been used in conjunction with other modulation techniques such as orthogonal frequency-division multiplexing (OFDM). Moreover, the short duration of UWB pulses allows high-data-rate communication. It has been widely suggested as potential scheme for efficient media transfer between peripherals such as smart televisions, personal computers, camcorders and imaging devices. Another application of UWB is localization. UWB has excellent ToA resolution, which makes it ideal for tracking, particularly for indoor environments [22].

1.2.3 Constraints

DSSS provides many opportunities. With the ability to suppress in-band interferers and robustness against channel fading, it seems to be a promising scheme for communication in monitoring applications. In conjunction with UWB, a DS-UWB system can operate in the licensed frequency bands without causing interference to the license users. Combined with unique time-hopping sequences, a UWB system exhibits excellent multiplexing, making a very promising approach for large sensor networks.

However, the benefits come at the cost of complexity and increased power consumption. For large spreading factors, a large number of fingers have to be employed. Furthermore, the coherent nature of the DSSS receiver requires locking onto the narrow pulses of the received signal. This demands precise timing synchronization; a tiny mismatch in synchronization time leads to imperfect despreading, resulting in high error rates [23]. Also, the RF oscillator at the receiver needs to be matched to the carrier in frequency and in phase; and imperfect carrier synchronization can lead to a substantial effect on the performance. Therefore, an efficient synchronization and tracking scheme is required for effective communication. The channel estimation and accordingly, adaptation of the fingers leads to an increased overall receiver complexity.

In monitoring applications, where transmissions are infrequent, the rake receiver has to initialize for every burst of traffic. Channel estimation, finger

adaptation and synchronization for each burst directly translates to an increase in energy consumption. Therefore, using a typical DSSS receiver in monitoring applications has a high energy requirement.

The design objective in monitoring applications is to communicate at low data rates at ultra-low power. Ideally, in order to reduce maintenance costs, one desires a sensor node which can solely run on solar energy. Therefore, a typical DSSS receiver is needlessly complex and unsuitable for monitoring applications. There is a need for a communication scheme which retains some or all of the benefits the traditional SS system provides, and is also suitable for applications where the traffic arrives in bursts. This pushes for research in a new communication scheme which can deliver low data rates at ultra-low power, while still providing MA capability, robustness to multipath fading and interference.

1.3 Transmit reference

Transmit reference or transmitted-reference (TR) is an SS technique, designed to have a simple receiver for SS communication. A simplified architecture of a TR system is shown in Figure 1.6. As the name TR suggests, the transmitter in a TR system sends the modulated spreading signal along with the unmodulated spreading signal as a *reference*, over the same channel. To separate the two signals, a tiny delay in time ΔT is introduced which is already known to the receiver. The idea is to provide the receiver with a template of the spreading signal – albeit noisy – for correlation. The transmitted signal can be considered as a pair of chips of the spreading signal, separated by time offset ΔT , where the polarity of each delayed chip in a bit interval is determined by the information bit. Detection is based on a *self-correlation* operation. The receiver realigns the two signal in time, using the known time offset, and then correlates the two to recover the information signal. The correlation is carried out by means of a mixer and a low-pass filter (LPF). The LPF is usually an integrator; integration can be carried out over a bit interval or a chip interval, the latter of which requires implementation of a bank of parallel correlators.

The self-correlation operation captures significant energy from multipath components without the need for channel adaptations. Unlike the traditional SS (rake) receiver, the TR receiver has no knowledge of the spreading signal. Due to the noncoherent nature of the detection mechanism, the TR receiver only requires bit-level synchronization. To retrieve the information bit, the TR receiver mainly needs to know the symbol time and time offset used at the transmitter. This eases the stringent requirements on synchronization of a DSSS receiver. In low duty cycle communication, where the transmissions are infrequent, TR can have reduced power consumption due to fast synchronizations and the simple detection mechanism.

The idea of “transmitting a reference” dates back to the now-declassified NOMAC system in 1952 [13]. It was one of the earliest DSSS systems using a correlation-based receiver, designed for military applications. Two methods

Figure 1.6: Concept architecture of the time-offset TR communication system.

were tested for correlation; one involved the set of possible reference waveforms to be stored at the receiver, whereas the other involved sending the reference waveform through an auxiliary channel (different frequency band). The “store and correlate” method resulted in better performance but had stringent requirement for synchronization and memory, particularly for technology of that time. On the other hand, sending the waveforms through an auxiliary channel significantly reduced the synchronization problem. However, this was at the expense of channel capacity, as the system expended twice the bandwidth.

Half a century later, a search for a simple UWB receiver led to TR-UWB [24, 25]. Hoorcar and Tomlinson suggested a TR-UWB system where the transmitted signal consisted of a pair of modulated and unmodulated pulses separated by a small time offset, already known to the receiver. Multiple access was enabled by varying the time offset according to a unique code. Using off-the-shelf components, the communication scheme was tested in [26]. A signal processing model for the scheme was provided by Trindade et al. in [27]. At the receiver, a bank of correlators was implemented, each of which correlated the received pulse sequence with a delayed version of itself. The outputs of the correlators were then combined using the known code to retrieve the information bit. Although the proposed system used many correlators, the synchronization and detection phase was moved from the receiver front-end to a stage following the despreading operation. Thus, the suggested system has less stringent requirements on synchronization compared to a standard UWB scheme. The authors suggested TR-UWB to be a suitable candidate for burst-mode communication where synchronizing every burst of data elevates the overall power consumption.

As a simple receiver for UWB, TR gained significant interest in the first half of the last decade. The correlation receiver in TR mixes all in-band signals, leading to multiple undesired terms which enhance the overall noise. The authors of [28–30] proposed TR signaling schemes based on weighted correlation at the receiver. In [28] and [30], the authors proposed to split the received signal in many intervals. Multiple correlators were used to self-correlate the received signal over the different intervals and the outcomes were combined using linear weights, derived by means of digital signal processing techniques. This helped to average out the effect of enhanced noise and inter-symbol-interference. How-

ever, on the other hand, this increased the receiver complexity.

An apparent disadvantage of TR is that it incurs a 3 dB loss of transmission power due to two-fold transmission. However, the rake receiver also demands transmission of pilot symbols for channel estimation. A comparison between a variation of the TR receiver and a rake receiver in dense multipath channels was made in [31]. A theoretical framework for performance analysis was provided and using a set of indoor channel measurements, the trade-off between complexity and performance was studied. As in line with previous studies, TR brought simplicity and easy synchronization to UWB. The performance of the rake receiver depends on the number of tracked multipaths (fingers) and the manner they are combined (maximal ratio combining, selection combining, square-law combining). In general, a rake receiver with optimal combining schemes performs better than TR, but it was found that in some cases, TR can perform as good as the rake receiver. The authors found that TR performs slightly better than a rake receiver which tracks only the strongest path, or a rake receiver which tracks the first four multipaths including the strongest. The self-correlation operation of the TR receiver gathers significant energy from the multipath channel, but the performance is limited by the noise enhancement. The results indicate that although TR is not a replacement for the rake receiver in multipath channels, it is generally a good compromise if the design goal is to have a simple receiver, especially in bursty communication where adaptivity to the channel is not desirable.

At the Telecommunication Engineering group in the University of Twente, the idea of TR was picked up in the last decade, and a number of small studies were carried out to investigate the communication scheme [32–34]. Since the noncoherent TR receiver does not require knowledge of the spreading signal, any type of spreading signal can be used. Motivated by a similar idea in optical networks [35], Haartsen and Meijerink proposed to use pure noise [36]. It might be convenient to generate pure noise than generating a PRBS with ideal statistical properties. The idea of pure noise carriers has also been independently studied in [37, 38] for radar and imaging systems based on UWB.

In [39], Haartsen and Meijerink suggested to replace the time offset by a frequency offset. Since it is much easier to implement a low-frequency local oscillator (LO), the complexity of the TR system is further reduced. In fact, the integration of the tiny time offset on a small chip has been a known implementation concern [40]. Realizing one nanosecond of a time offset requires a 30 cm delay element which is practically unfeasible for small circuits. In the frequency offset TR scheme, different frequency offsets can be used for different communication links for multiple access. Independently, this idea of using frequency offset was also studied by Goeckel in [41, 42] for UWB-IR, who claimed that the frequency-offset TR system has a simple receiver and is suitable for low-to-moderate data rate applications.

Combined with the noise-based carriers, this led to the term noise-based frequency-offset modulation (N-FOM), shown in Figure 1.7. As depicted, a frequency offset Δf separates the modulated and unmodulated carrier. To

Figure 1.7: Concept architecture of the N-FOM communication system.

ensure that the two signals face similar distortion by the multipath channel, the frequency offset should be much smaller than the coherence bandwidth of the channel³. This choice also minimizes the occupied bandwidth, and hence the noise captured by the receiver front-end filter. After reception, the receiver realigns the two signals in frequency, and correlates the two using a mixer and a low-pass filter (LPF). The LPF, as seen in the right-most inset, extracts the despread information signal that appears at baseband.

Shang derived an analytical framework for the performance analysis of N-FOM [43], and presented feasibility of the system in a MA scenario. Subsequently, Balkema developed an RF test bed for operation in the 2.4 GHz unlicensed band [33]. Owing to losses in the circuitry, a disparity between measurements and theory was observed, but they were in agreement in terms of performance trend. Wang provided a short study of the frequency offset TR system in a two-path fading channel. In 2009, a semi-analytical framework for performance of the N-FOM in dense multipath channels was provided [44].

Schellekens did an analytical study on the synchronization time of the DSSS and N-FOM systems [34]. In a DSSS receiver, synchronization is carried out using correlation of the known spreading sequence with the received signal. The delay of the spreading sequence is adjusted to obtain the maximum auto-correlation. The process is carried out chip-by-chip; the synchronization time depends on the length of the sequence, and hence the spreading factor. The process is further complicated by tracking the channel magnitude and phase information, leading to long synchronization times. On the other hand, the N-FOM system requires only symbol-level synchronization. Therefore, for high spreading factors, N-FOM has much shorter synchronization time in comparison. It was shown that at a spreading factor of 20 dB, synchronization in DSSS can take more than 100 times to that in N-FOM.

³This will be discussed in detail in Chapter 6

1.4 Research context

N-FOM is very different from the traditional SS systems. Many studies have pointed out at its faster synchronization time and a much simpler receiver architecture. Nevertheless, N-FOM has some fundamental drawbacks. For a start, the SNR after despreading is much lower than the SNR before despreading. This is partly because half of the transmission power is spent on the unmodulated reference signal, but mainly because the self-correlation at the receiver enhances the overall noise (further explained in Chapter 2). Secondly, for low received signal levels, the mixing of the desired signal with itself further lowers the signal strength. Although, it can be solved by implementation of a high gain amplifier prior to the mixing stage, it compromises the energy-efficiency feature of the N-FOM system. Finally, since all users in the network share the same spectrum, the interference from other users can be further enhanced by the self-mixing, which can restrict the scalability of the system.

To assess the feasibility of N-FOM in low-power WSNs, it is important to develop fundamental understanding of the system and all its different aspects. At the PHY of N-FOM, the theoretical limitations of the system have to be investigated. There can be a number of trade-offs involved which affect the overall link performance, particularly in the face of impairments introduced by the channel, random interferers and other users in the network.

Moreover, the integration of N-FOM on a fully integrated chip is essential before commercial realization. The studies so far have been mainly focused on analysis and simulations. Few of the studies which consider implementation, are based on printed circuit boards or off-the-shelf equipment [26, 42]. It needs to be investigated whether one can realize the N-FOM architecture on a small chip, and whether the chip is inexpensive in terms of both power and circuitry. Optimizing the corresponding electronics with low system noise while ensuring N-FOM does not lose its principal feature, such as the low synchronization time, is critically important.

Finally, low-power operation is not only limited to the electronics, but also the system design at the data link and the layers above. For energy conservation, the wake-up and sleep time of the nodes have to be optimized. For the system to be scaled to a large network, an efficient channel access scheme is equally important. At the data link layer, a suitable protocol has to be chosen or one has to be designed from scratch, which considers restrictions imposed by the other layers.

At the University of Twente, these research questions culminated in the initiation of a cross-disciplinary project called the Wireless Ad-hoc Links using robust Noise-based Ultra-wideband Transmission (WALNUT) project. Researchers from disciplines of circuit design (ICD group), computer networks (DACS group), and wireless systems (TE group) collaborated to investigate three different angles of the frequency-offset TR system. The research project was funded by the the Dutch Organization for Scientific Research (NWO) under its domain Applied and Technical Sciences (TTW, formerly STW). The

industry partners which supported the project are NXP, DevLab, Plantronics, Thales and TNO. The aim of the project was to carry out a feasibility study of the noise-based frequency-offset TR system. This includes investigating the physical layer of the system, design of suitable network protocols and integration on a power-inexpensive chip.

1.5 Competing technologies

As discussed earlier, the WALNUT project is aimed at investigating the N-FOM system for its feasibility in WSNs for low-duty-cycled monitoring applications, where fast synchronization and low-power operation are essential for the design. The outcome of the project can potentially lead to commercialization of the system. Since commercial products are widely available for sensor applications, many of which promise offer cost-effectiveness in terms of power and design, it is interesting to assess the current market landscape. Before we formally define the research goals, we wish to identify where N-FOM can benefit the consumer.

Figure 1.8 illustrates some popular standards used for wireless communication. Two powerful standards used for ad-hoc networking are ZigBee and Bluetooth [45]. At its PHY, classical Bluetooth employs FHSS over a range of 80 MHz, and can achieve data rates of up to 50 Mbps at a nominal transmission range of 10 m. However, with a transmission power of up to 100 mW, a Bluetooth node demands the battery to be regularly recharged. Furthermore, the scalability of Bluetooth is limited, as it supports only eight devices in a star topology network. Bluetooth was designed and intended for short-range data transfers with focus on cable replacement for device-to-device connections, and hence, is an uninteresting choice for low-data-rate sensor networks. Recently, a low-power variant of Bluetooth was launched under the name of Bluetooth Low Energy (BLE), with Bluetooth Core Specification 4.0 [46]. It boasts of a transmission power of as low as 1 mW with a nominal transmission range of 10 m. With relatively cheaper implementation costs, possibly for many-to-many communication, and a battery life spanning a couple of years [47], the technology has great potential for low data rate WSNs.

ZigBee is a low-energy standard [48] which adopts DSSS, based on standards defined in IEEE 802.15.4 for low-rate wireless personal area networks (WPANs) [49]. It makes use of both the sub-1 GHz and 2.4 GHz unlicensed bands, with bandwidths of 0.6 MHz and 2 MHz respectively. Nominally, the transmission range is from 10 to 100 m with data rates up to 250 kbps [50]. With the ability to support 65 thousand nodes in a mesh network topology, ZigBee is highly scalable, and is thus appealing to a wide variety of large scale data networks that do not demand high data rates. It is widely used for industrial and residential applications, such as industrial control, building automation, environmental monitoring and energy management [51, 52].

A comparison of the technologies for low data rate WSNs is made in Table 1.1. In [53], a BLE device was tested for energy efficiency. It was found that a BLE device not only has a power requirement proportionate to that

Figure 1.8: Prominent current technologies with nominal values of data rates and transmission range.

of a ZigBee device, it also has 2.5 times higher energy utility (kB/J). In mid 2017, mesh topology and multi-hop transmission was introduced to BLE, amid an increasing demand for sensor network technologies. The scalability of BLE depends on various factors such as the nature of traffic, density of the network, and, among others, the data rate.

With a strong drive to minimize power consumption, energy harvesting systems have gained attention in recent times. This has enormous potential in WSNs. For example, in a large building complex with a network of thousands of nodes, replacing batteries would be a costly endeavor. A WSN with battery-less operation would be ideal in such a scenario, and in all others where the nodes are placed in inaccessible environments such as volcanoes, forests, battlefields etc. In the first decade of the century, a number of self-powered sensors were introduced, most notably by EnOcean, a spin-off company of Siemens AG [54]. The energy harvesting techniques are mainly based on photoelectric, piezoelectric and thermoelectric phenomena. However, for implementation in a network, the energy consumed by the communication protocols is rather high. In 2012, the ZigBee Alliance saw the potential in self-powered sensors, and introduced a battery-less variant of its protocol, called ZigBee Green Power [55]. Green Power Devices (GPDs) are self-powered nodes with very limited processing capability, and rely on a standard ZigBee network to process and transport data to another GPD. For example, flipping a self-powered switch (a GPD) can issue a command, which will be tunneled through a ZigBee network, to flip another switch in a remote location.

At the same time, the EnOcean Alliance was formed, with the aim to standardize the technology. A new wireless standard [56] was developed and ratified by ISO. At its PHY, the standard uses GFSK modulation and narrowband transmission in sub-gigahertz unlicensed bands (868 MHz in Europe, 902 MHz

Table 1.1: Comparison* of low-power technologies for sensor networks [47, 50].

Standard	ZigBee	BLE
IEEE specification	802.15.4	802.15.1
frequency band	868/915 MHz, 2.4 GHz	2.4 GHz
data rate	20/40 kbps, 250 kbps	1 Mbps
bandwidth	0.3/0.6 MHz, 2 MHz	2 MHz
modulation	BPSK, OQPSK	GFSK
spreading technique	DSSS	FHSS
interference avoidance	dynamic freq. selection	adaptive FH
nominal Tx power	0 dBm	0 – 10 dBm
nominal range	10 – 100 m	10 – 150 m
topology	mesh	mesh

*Acronyms: BPSK (binary phase-shift keying), O-QPSK (offset quadrature phase-shift keying), GFSK (Gaussian frequency-shift keying).

in USA). It specifies a transmission range of up to 30 m indoors with $50 \mu\text{W}$ of harvested power [57]. A couple of years later, the 2.4 GHz version of the devices was introduced, based on BLE and ZigBee Green Power technologies. A wide range of battery-less actuators and sensors are available for home and building automation [58]. However, since they rely on battery-powered networks (such as ZigBee and BLE) for transportation and processing of data, a completely self-powered network is still a distant goal.

Although the N-FOM technique does not specifically aim at the self-powered network, nonetheless, the goal is to achieve a level of power efficiency such that the batteries can last the lifetime of the node. At the same time, the key benefits of using DSSS have to be secured, without compromising the energy-efficiency.

1.6 Research goals

This thesis deals with the research in the PHY, carried out in the Telecommunication Engineering (TE) group. By definition, the PHY deals with establishing a robust communication link between communicating nodes. This layer encompasses a plethora of research areas such as modulation format, channel coding, equalization, radio resource management, interference, synchronization, multiplexing, channel fading, etc. Not all of these fields are pertinent to this research. *The main aim of this Ph.D. thesis is to assess the potential of the N-FOM system by investigating its fundamental limitations and underlying mechanisms.* This goal can be broken down to the following research questions:

1. What are the underlying mechanisms that dictate the quality of a point-to-point line-of-sight link? What are the basic PHY considerations, that have to be made for practical realization of the system?
2. How scalable is N-FOM? Does the communication scheme exhibit high level of tolerance to a multi-user-interference (MUI)?

3. Can the N-FOM system coexist with other systems in unlicensed bands? How does the system behave when subjected to external interference in the same frequency spectrum?
4. Finally, how is the performance affected when there is no direct line-of-sight between the communicating nodes? What are the implications of multipath propagation on the design parameters?

The above-mentioned research questions, among others, must be addressed for the feasibility study. A couple of these questions have been partially dealt with in few of the earlier works in the TE group (mentioned in Section 1.3) [33, 34, 43]. Nonetheless, those works consider limited operational scenarios, and fall short of addressing underlying mechanisms and associated trade-offs that influence the utility of N-FOM. This thesis builds on those works, and extends the analysis in a comprehensive fashion. Different operation scenarios with practical considerations are considered, in order to build an overall picture of the system.

1.7 Research approach

In the design of any communication system, there is a limited power budget available at the transmitter and at the receiver. To calculate how far the nodes can *effectively* communicate with the budget they have, one has to account for all the gains and losses from the transmitter, the wireless medium, and the receiver. This practice, ubiquitous in link design, is called the *link budget calculation*, as shown in Figure 1.9. It specifies how much SNR at the receiver input is achieved, for certain values of the link parameters. Tuning the parameters values, the designers can maximize the SNR at the receiver input.

The definition of effective communication translates to the degree of reliability in the communication. This is represented by bit-error-rate (BER), and depending on the application, the desired BER (or desired link quality) can be different. For example, in banking applications, higher reliability is required, and therefore, the desired BER is 10^{-6} . On the other hand, in monitoring applications, a desired BER of 10^{-3} is widely accepted. In general, there is a one-to-one relation between the required SNR and BER, but the relation is different for different systems. Channel fading, interference, synchronization accuracy, spreading gain, etc., all affect the SNR and consequently the BER. An analytical relation for the SNR incorporates all these factors, and one can see how different channel and system parameters interplay with the BER and how one can tune these to improve the link quality. Therefore, the recurring approach in each chapter of this thesis is to find the BER-vs-SNR relation for different communication scenarios.

Figure 1.9: A pictorial representation of link budget analysis.

1.8 Outline of the thesis

In line with the research questions mentioned in Section 1.6, this thesis is organized as follows. Chapter 2 begins with a detailed overview of the N-FOM communication principle. A point-to-point N-FOM communication link in an additive white Gaussian noise channel is modeled and analyzed, and the underlying mechanisms and trade-offs are explained. The choice of noise carrier, receiver front-end filter, and frequency offset can have a pronounced effect on the performance. The practical considerations associated with these parameters are discussed in Chapter 3. Chapter 4 deals with the scalability of the system. The performance of the system is analyzed in an MA scenario, where multiple communication links are operating in the sensor network. The limitations on the inherent multiplexing capability of the system are outlined. The ability of the system to co-exist with other systems is considered in Chapter 5. First, the link performance is studied when the communication is subjected to a single-tone interferer. The model is then extended to incorporate a more general type of interferer. The performance parameters, that can be tuned to achieve a better resistance to interference are identified and analyzed. Since the nodes in a sensor network usually communicate in a non-line-of-sight (NLOS) propagation environment, the effect of the propagation impairments on link quality is detrimental to the link design. This is the subject of Chapter 6 where the performance of the N-FOM NLOS link is investigated. A number of underlying mechanisms are explored which influence the link quality, often antagonistically. Providing a framework for the probability of having a poor link quality, the chapter gives insights into the utility of N-FOM in practical scenar-

ios. Finally, Chapter 7 gives conclusions to the research and recommendations for further studies.

Basic principle and performance in AWGN

This chapter provides a comprehensive analysis of the N-FOM communication system. To begin with, a point-to-point N-FOM communication link is modeled in a frequency-flat AWGN channel. The N-FOM operation is explained and an analytical framework of the performance is derived, in order to get insight into the behavior of the system. Next, the system parameters that influence the performance are outlined and explained.

To derive a generic framework, all parameters and signals are represented in a general form. A specific form is chosen only to obtain a certain closed-form result, which is applicable to a certain scenario. It must be brought into knowledge that part of the analytical work in this chapter had been previously studied in [39, 43]. However, the studies were made with limited applicability. The added value of this chapter is providing the reader with a generalized analysis, taking the choice of system parameters into considerations.

We begin with a description of the system and its operation mechanism in Section 2.1. The analytical model and analysis is provided in Section 2.2, where we derive a close-form expression of the performance metrics in a point-to-point N-FOM link. Section 2.3 discusses results and performance trends, followed by conclusions in Section 2.4.

2.1 System architecture

The basic architecture of N-FOM is illustrated in Figure 2.1a. At the transmitter (Tx), a wideband spreading signal $x(t)$, modeled as a wide-sense-stationary (WSS) random bandpass process, is locally generated. The signal is sent over two branches of the transmitter; at one branch, it is sent as a *reference* signal, whereas on the other branch, it spreads a narrowband message signal $m(t)$. A small frequency offset f_{Tx} is introduced between the spread *information* signal

Figure 2.1: Architecture of an N-FOM system.

and the reference signal. The two signals are linearly combined and the TR signal $y(t)$ is sent over the wireless channel.

The received signal, disturbed by the wireless channel, is first filtered by a bandpass filter with impulse response $g(t)$, to suppress out-of-band signals. Analogous to the Tx, the receiver (Rx) sends the received signal $r(t)$ over two branches. In one branch, the signal is sent as it is, whereas in the other branch, the receiver shifts the frequency spectrum of the received signal by f_{Rx} . In order to realign the reference and information signals in frequency and phase, f_{Rx} must be equal to f_{Tx} , and ϕ_{Rx} must compensate for the phase offsets at the Tx (ϕ_{Tx}) as well as that introduced by the channel. However, for the sake of generality, we initially assume different values at the receiver. The signals from the two branches are mixed with each other, resulting in the signal $z(t)$. This mixing operation despreads the information signal to a baseband frequency $f_{Rx} - f_{Tx}$. The transmitted information signal can be recovered by applying an appropriate low-pass filter.

An alternate interpretation of the receiver architecture is depicted in Figure 2.1b. The self-correlation operation is represented by an energy detector, followed by a frequency offset local oscillator (LO) and a matched filter. The motivation for using a matched filter will be discussed in the next section. The output of the matched filter is sampled, giving the decision variable d_k , where k is the data index. Next, d_k is compared to a threshold; the outcome of the threshold detector gives an estimate $\hat{b}_k$ of the transmitted bit b_k .

2.2 System model and analysis

For mathematical convenience, we represent all bandpass signals and filters in their baseband equivalents, also known as complex envelope. Any bandpass signal $s(t)$, centered at any RF carrier frequency f_c , with baseband equivalent $\tilde{s}(t)$, can be written as

$$s(t) = \text{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}, \quad (2.1)$$

where $\text{Re}\{\cdot\}$ denotes the real part of the argument.

We take this moment to state a few key properties of WSS processes which will be used throughout this thesis. Any WSS process $s(t)$ with PSD $S_s(f)$ has an autocorrelation function (ACF)

$$R_s(\tau) = \mathbb{E}[s(t)s^*(t - \tau)] = \mathcal{F}^{-1}\{S_s(f)\},$$

where $\mathbb{E}[\cdot]$ denotes statistical averaging and $\mathcal{F}\{\cdot\}$ denotes Fourier transform. The ACF and PSD of a WSS process form a Fourier pair. A WSS process $s(t)$, filtered by a linear time-invariant (LTI) filter with impulse response $h(t)$, and a transfer function $H(f)$, results in another WSS process $s_{\text{out}}(t)$. The output signal satisfies

$$s_{\text{out}}(t) = s(t) \otimes h(t) \quad \Rightarrow \quad S_{s_{\text{out}}}(f) = S_s(f)|H(f)|^2 \quad (2.2a)$$

$$\tilde{s}_{\text{out}}(t) = \frac{1}{2}\tilde{s}(t) \otimes \tilde{h}(t) \quad \Rightarrow \quad S_{\tilde{s}_{\text{out}}}(f) = \frac{1}{4}S_{\tilde{s}}(f)|\tilde{H}(f)|^2, \quad (2.2b)$$

where the fraction $(1/2)$ follows from the properties of a filter operation in complex baseband [59]; and the symbol $\otimes$ represents convolution, i.e., $q(t) \otimes w(t) = \int q(\tau)w(t - \tau) d\tau$. Finally, we also state the ACF of a real-valued deterministic signal, which is given as

$$R_s(\tau) = \int_{-\infty}^{\infty} s(t)s(t - \tau) dt.$$

Spreading signal

We model the spreading signal $x(t)$ as a zero-mean WSS bandpass process, which has a PSD centered around a carrier frequency f_c . We define the complex envelope $\tilde{x}(t)$ of the spreading signal to have a two-sided PSD,

$$S_{\tilde{x}}(f) = \frac{2P_x}{B_x} \mathcal{S}\left(\frac{f}{B_x}\right), \quad (2.3)$$

where P_x is mean power in the bandpass signal $x(t)$, B_x is the bandwidth, and $\mathcal{S}(\nu)$ is a unit-width and a unit-area spectral shape, with ν being the normalized frequency. In practice, the spreading signal is generated by band-limiting WSS white noise. The transfer function of the band-limiting filter

specifies the spectral shape $\mathcal{S}(\nu)$. Note that the mean power in the bandpass signal is

$$P_x = \mathbb{E}[|x(t)|^2] = \frac{1}{2} \mathbb{E}[|\tilde{x}(t)|^2] = \frac{1}{2} \int_{-\infty}^{\infty} S_{\tilde{x}}(f) df, \quad (2.4)$$

which is aligned with the properties of WSS bandpass processes [60]. Although we have considered the spreading signal to be band-limited white noise, one can also consider different types of spreading signal. This detailed discussion is postponed till the next chapter.

Message signal

We model $m(t)$ as an antipodal message signal, i.e.,

$$m(t) = \sum_k b_k p(t - kT_b), \quad (2.5)$$

where T_b is the bit duration; $p(t)$ is a pulse shape, real-valued with energy T_b ; k is the data index; and $\{b_k\}$ are binary digits taking values of $+1$ and -1 with equal probabilities. The ACF of the pulse is $R_{pp}(\tau)$, with $\int_{-\infty}^{\infty} p^2(t) dt = R_{pp}(0) = T_b$. The data rate $R_b = 1/T_b$ is considered to be much smaller than the signal bandwidth, i.e., $1/T_b \ll B_x$. Higher-order modulation formats by modulating the phase of the offset carrier $\cos(2\pi f_{Tx}t + \phi_{Tx})$ could be used as well, but they are not considered in this thesis.

Transmitted and received signal

The transmitter linearly combines the reference and information signals, which are offset in frequency by the offset f_{Tx} . The total transmission power available to the transmitter can be distributed equally or unequally between the two signals. To find the optimal combination, we chose to scale the reference signal with a constant c . This will be optimized later in the analysis. The signal transmitted over the wireless channel is

$$y(t) = x(t)(c + m(t) \cos(2\pi f_{Tx}t + \phi_{Tx})), \quad (2.6)$$

We assume that the frequency offset f_{Tx} is much smaller than the bandwidth B_x of the spreading signal. It implies that the bandwidth of the transmitted signal $y(t)$ is almost equal to B_x .

The transmitted signal is perturbed by a frequency-flat AWGN channel¹. The receiver obtains

$$r(t) = g(t) \otimes y(t)/\sqrt{L} + g(t) \otimes n(t), \quad (2.7)$$

where L is the channel loss; and $n(t)$ is the AWGN with spectral density $S_n(f) = N_0/2$. The linear time-invariant (LTI) front-end bandpass filter with impulse

¹A frequency-selective fading channel is subject of Chapter 6.

response $g(t)$ filters the signal at the output of the receiver antenna. The bandwidth of the filter is considered equal to the bandwidth of the transmitted signal $y(t)$. The output of the filter then undergoes a self-correlation operation. The signal is shifted in frequency by an offset f_{Rx} , and then multiplied with itself. At the input of the detection filter, we have

$$\begin{aligned} z(t) &= r^2(t) \cos(2\pi f_{\text{Rx}}t + \phi_{\text{Rx}}) \\ &= \frac{1}{2} |\tilde{r}(t)|^2 \cos(2\pi f_{\text{Rx}}t + \phi_{\text{Rx}}) + \text{h.f.c.}, \end{aligned} \quad (2.8)$$

where we have used (2.1) and denoted the high-frequency RF components, around the frequency $2f_c$, as h.f.c. The h.f.c. will be suppressed by the low-pass detection filter later on. Therefore, we do not further consider this in our analysis. For brevity of the mathematical expressions, we define

$$\tilde{n}_g(t) = \frac{1}{2} \tilde{n}(t) \otimes \tilde{g}(t), \quad \tilde{x}_g(t) = \frac{1}{2} \tilde{x}(t) \otimes \tilde{g}(t). \quad (2.9)$$

$$a(t) = c + m(t) \cos(2\pi f_{\text{Tx}}t + \phi_{\text{Tx}}) \Rightarrow y(t) = a(t)x(t), \quad (2.10)$$

where $a(t)$ represents the narrowband (NB) function, modulating the wideband signal $x(t)$, in $y(t)$. From the properties of WSS processes (2.2), the PSDs of $\{\tilde{x}_g(t), \tilde{n}_g(t)\}$ are

$$S_{\tilde{x}_g} = \frac{1}{4} S_{\tilde{x}}(f) |\tilde{G}(f)|^2, \quad S_{\tilde{n}_g} = \frac{1}{4} S_{\tilde{n}}(f) |\tilde{G}(f)|^2 = \frac{N_0}{2} |\tilde{G}(f)|^2. \quad (2.11)$$

Since the bandpass filter is wideband, one can consider the NB function $a(t)$ to be almost constant over the filter impulse response. In other words, one can move the NB function out of the convolution $y(t) \otimes g(t)$. Substituting (2.7) in (2.8), and using the definitions in (2.9) and (2.10), $z(t)$ can be written as

$$\begin{aligned} z(t) &= \frac{1}{2} \left| \frac{1}{\sqrt{L}} a(t) \tilde{x}_g(t) + \tilde{n}_g(t) \right|^2 \cos(2\pi f_{\text{Rx}}t + \phi_{\text{Rx}}) \\ &=: z_y(t) + z_{yn}(t) + z_n(t), \end{aligned} \quad (2.12)$$

where we have respectively defined signal-signal, signal-noise and noise-noise cross-products, produced by the squaring operation, as

$$z_y(t) = \frac{1}{2L} |\tilde{x}_g(t)|^2 a^2(t) \cos(2\pi f_{\text{Rx}}t + \phi_{\text{Rx}}) \quad (2.13a)$$

$$z_{yn}(t) = \frac{1}{2\sqrt{L}} \left(\tilde{x}_g(t) \tilde{n}_g^*(t) + \tilde{x}_g^*(t) \tilde{n}_g(t) \right) a(t) \cos(2\pi f_{\text{Rx}}t + \phi_{\text{Rx}}) \quad (2.13b)$$

$$z_n(t) = \frac{1}{2} |\tilde{n}_g(t)|^2 \cos(2\pi f_{\text{Rx}}t + \phi_{\text{Rx}}). \quad (2.13c)$$

Note that the despread information signal is buried in $z_y(t)$, whereas $z_{yn}(t)$ and $z_n(t)$ are the undesired terms. Furthermore, since the bandwidth of $y(t)$ is approximately B_x , the (single-sided) bandwidth of $z(t)$ is B_x .

Detection filter

The self-mixing of the filtered version of $\tilde{y}(t)$ not only despreads the information signal but also creates undesired terms. The undesired terms in $z_{\tilde{y}\tilde{y}}(t)$ consist of a wideband noise term and a few bias terms. The desired term is narrowband, and an appropriate narrowband detection filter with bandwidth comparative to that of the data signal ($\sim 1/T_b$) has to be chosen. The problem of detection is akin to detection of a narrowband data signal in additive white² noise. It is well-known that the optimal linear filter which maximizes the SNR of a signal perturbed by additive white noise, is the one which is matched to the signal template [61]. For any real signal shape $s(t)$ of duration T_b , the impulse response of the matched filter is $s(T_s - t)$, where T_s is the sampling time.

To detect the information bit, a matched filter is used at the receiver. In Figure 2.1b, the detection filter is matched to the pulse $p(t)$ of the message signal. This is followed by a sample-and-hold node which samples at multiples of the bit time T_b to give the decision variables $\{d_k\}$. Since the pulse $p(t)$ is narrowband, the detection filter is equivalent to an LPF which will suppress the h.f.c. in (2.13). Therefore, we can ignore h.f.c. in further analysis. Using (2.7) and (2.13), the decision variable for a transmitted bit b_k can be written as

$$d_k = [z(t) \otimes p(T_s - t)]_{t=T_s+kT_b} = \int_{-\infty}^{\infty} z(u)p(u - kT_b) du. \quad (2.14)$$

Without losing generality, we will focus on detection of the information bit with index $k=0$.

2.2.1 Statistics of the decision variable

The input $z(t)$ to the detection filter is a combination of random processes. Therefore, the decision variable d_0 is a random variable with a certain probability density function (PDF). Since $B_x \gg 1/T_b$, it can be seen from (2.13) that the additive noise terms are signals with bandwidths much larger than the detection filter band. In other words, the detection filter sums up a large number of samples in each symbol duration. Each sample d_k is a sum of a large number of independent random variables. Therefore the Central Limit Theorem (CLT) applies and we can assume that the decision variable follows a Gaussian distribution. To derive the PDF of the decision variable, it is hence sufficient to find its conditional mean and variance. The statistics of the decision variable, for all subsequent analysis, will be conditioned on values of the transmitted bit b_k . For notational brevity, we drop the conditional variables, i.e., $\mathbb{E}[d_0] = \mathbb{E}[d_0|b_k]$.

²The whiteness of the additive noise will be discussed in the next subsection.

Mean

Taking the expectation of (2.14), the mean of the decision variable is

$$\mathbb{E}[d_0] = \int_{-\infty}^{\infty} \mathbb{E}[z(u)] p(u) du, \quad (2.15)$$

where $z(u)$ is the sum of the mixing products given in (2.13). Taking the expectation of the signal self-mixing product, we obtain

$$\begin{aligned} \mathbb{E}[z_y(t)] &= \frac{1}{2L} \mathbb{E}[|\tilde{x}_g(t)|^2] a^2(t) \cos(2\pi f_{\text{Rx}} t + \phi_{\text{Rx}}) \\ &= \frac{P_{x_g}}{L} a^2(t) \cos(2\pi f_{\text{r}} t + \phi_{\text{Rx}}) \end{aligned} \quad (2.16)$$

where we have denoted P_{x_g} as the mean power in the filtered spreading signal, i.e., $P_{x_g} = \mathbb{E}[x_g(t)] = \frac{1}{2} \mathbb{E}[|\tilde{x}_g(t)|^2]$. The mean value of the remaining cross-products are given as

$$\mathbb{E}[z_{yn}(t)] = \frac{1}{\sqrt{L}} \mathbb{E}[\text{Re}\{\tilde{x}_g(t) \tilde{n}_g^*(t)\}] a(t) \cos(2\pi f_{\text{r}} t + \phi_{\text{Rx}}) = 0, \quad (2.17a)$$

$$\mathbb{E}[z_n(t)] = \frac{1}{2} \mathbb{E}[\tilde{n}_g^2(t)] \cos(2\pi f_{\text{Rx}} t + \phi_{\text{Rx}}) = P_{n_g} \cos(2\pi f_{\text{Rx}} t + \phi_{\text{Rx}}), \quad (2.17b)$$

where (2.17a) follows from mutual independence of $x(t)$ and $n(t)$, and that noise is zero mean; and (2.17b) follows by defining P_{n_g} as the mean power in the filtered noise $n_g(t)$. Substituting (2.16) and (2.17) in (2.15), we get

$$\begin{aligned} \mathbb{E}[d_0] &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} a^2(u) \cos(2\pi f_{\text{Rx}} u + \phi_{\text{Rx}}) p(u) du \\ &\quad + P_{n_g} \int_{-\infty}^{\infty} \cos(2\pi f_{\text{Rx}} u + \phi_{\text{Rx}}) p(u) du \end{aligned} \quad (2.18)$$

The second term in the above expression is a bias term scaled by the mean power of the filtered noise. Suppression of this bias term is essential for efficient detection. It can be shown that under some conditions on the pulse shape $p(t)$ and the frequency offset f_{r} , one can suppress this bias term. For a better organization of the chapter, this problem is addressed later in Section 2.2.3. For now, we consider that this term is suppressed.

Substituting the value of $a(t)$ from (2.10) in (2.18), the mean of the decision variable can be rewritten as

$$\begin{aligned} \mathbb{E}[d_0] &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} (c + m(u) \cos(2\pi f_{\text{Tx}} u + \phi_{\text{Tx}}))^2 \cos(2\pi f_{\text{Rx}} u + \phi_{\text{Rx}}) p(u) du, \\ &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} \left[(c^2 + m^2(u) \cos^2(2\pi f_{\text{Tx}} u + \phi_{\text{Tx}})) \cos(2\pi f_{\text{Rx}} u + \phi_{\text{Rx}}) \right. \\ &\quad \left. + 2c m(u) \cos(2\pi f_{\text{Tx}} u + \phi_{\text{Tx}}) \cos(2\pi f_{\text{Rx}} u + \phi_{\text{Rx}}) \right] p(u) du. \end{aligned} \quad (2.19)$$

The first integrand consists of multiple bias terms appearing at DC and at $f = f_{\text{Rx}} \pm 2f_{\text{Tx}}$. The second integrand contains the desired information at frequencies $f_{\text{Rx}} \pm f_{\text{Tx}}$. Clearly, the message will be demodulated with maximum magnitude if $f_{\text{Tx}} = f_{\text{Rx}}$ and $\phi_{\text{Tx}} = \phi_{\text{Rx}}$, i.e., the LO at Tx and Rx are synchronized. We consider that the synchronization is ensured, and fix $f_{\text{Tx}} = f_{\text{Rx}} =: f_r$ and $\phi_{\text{Tx}} = \phi_{\text{Rx}} =: \phi_r$. Substituting $m(t)$ from (2.5), we solve

$$\begin{aligned} \mathbb{E}[d_0] &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} \left[c^2 + \frac{1}{2} m^2(u) \left(1 + \cos(4\pi f_r u + 2\phi_r) \right) \right] \cos(2\pi f_r u + \phi_r) p(u) du \\ &\quad + \frac{P_{x_g}}{L} c \sum_k b_k \int_{-\infty}^{\infty} p(u - kT_b) p(u) \left(1 + \cos(4\pi f_r u + 2\phi_r) \right) du \\ &=: \frac{P_{x_g}}{L} c \sum_k b_k R_p(kT_b) + \Xi(f_r, \phi_r, m(t)), \end{aligned} \quad (2.20)$$

where we have denoted $\Xi(f_r, \phi_r, m(t))$ as the bias terms. The variable $R_p(kT_b)$ represents ACF of $p(t)$ evaluated at a delay kT_b . This brings us again to the conditions on the pulse $p(t)$ and the frequency offset. First, the pulse should be chosen such that $R_p(kT_b)$ is zero at integer multiples of the bit interval T_b . In other words, there is no inter-symbol-interference (ISI) at the sampling instants³. In that case, the ACF in (2.20) will be nonzero only for $k = 0$. It has to be stressed that in order to effectively capture the desired bit, this condition should be satisfied regardless of the choice of the frequency offset. Once again, we postpone the analysis of the bias term $\Xi(f_r, \phi_r, m(t))$ to the next subsection, in which we show that under certain conditions on $p(t)$ and f_r , one can suppress these terms. With zero ISI, and suppression of the bias terms, (2.20) can be written as

$$\begin{aligned} \mathbb{E}[d_0] &= \frac{c b_0 T_b P_{x_g}}{L} = \frac{1}{2L} c b_0 T_b \mathbb{E}[|\tilde{x}_g(t)|^2] \\ &= \frac{1}{8L} c b_0 T_b \int_{-\infty}^{\infty} S_{\tilde{x}}(f) |\tilde{G}(f)|^2 df = c b_0 \frac{P_x T_b}{4B_x L} \int_{-\infty}^{\infty} \mathcal{S}\left(\frac{f}{B_x}\right) |\tilde{G}(f)|^2 df \\ &= \frac{c b_0 E_b}{(1/2 + c^2) B_x} \int_{-\infty}^{\infty} \mathcal{S}\left(\frac{f}{B_x}\right) |\tilde{G}(f)|^2 df = \frac{c b_0 E_b}{1/2 + c^2} \Lambda_1, \end{aligned} \quad (2.21)$$

where we have substituted the definition of $\tilde{x}_g(t)$ and $S_{\tilde{x}}(f)$ from (2.9) and (2.3) respectively; defined $\Lambda_1 = \frac{1}{4} \int \mathcal{S}(\nu) |\tilde{G}(\nu B_x)|^2 d\nu$; and denoted E_b as the average received energy per bit. The value of E_b is derived by finding expected

³This condition is one of the Nyquist criteria on a pulse shape with minimum bandwidth requirements for zero ISI.

energy in a bit interval of the received signal, that is

$$\begin{aligned}
 E_b &= \frac{1}{L} \int_{-T_b/2}^{T_b/2} \mathbb{E}[y^2(t)] dt = \frac{1}{L} \int_{-T_b/2}^{T_b/2} \mathbb{E}[x^2(t)] \mathbb{E}[(c + m(t) \cos(2\pi f_r t + \phi_r))^2] dt \\
 &= \frac{P_x}{L} \int_{-T_b/2}^{T_b/2} \left[c^2 + 2c \sum_k \mathbb{E}[b_k] p(t - kT_b) \cos(2\pi f_r t + \phi_r) \right. \\
 &\quad \left. + \sum_k \sum_l \mathbb{E}[b_k b_l] p(t - kT_b) p(t - lT_b) \cos^2(2\pi f_r t + \phi_r) \right] dt,
 \end{aligned}$$

where we have substituted $m(t)$ from (2.5) in the last equality.

The bits b_k follow discrete uniform distribution, taking values of $+1$ and -1 with equal probability. Therefore, its mean $\mathbb{E}[b_k] = 0$, whereas its ACF $\mathbb{E}[b_k b_l] = 0 \forall k \neq l$ and is unity when $k = l$. Using this information, the above expression is solved

$$\begin{aligned}
 E_b &= \frac{P_x}{L} \left[\int_{-T_b/2}^{T_b/2} c^2 dt + \frac{1}{2} \sum_k \int_{-T_b/2}^{T_b/2} p^2(t - kT_b) (1 + \cos(4\pi f_r t + 2\phi_r)) dt \right] \\
 &= \frac{P_x}{L} \left[c^2 T_b + \frac{1}{2} \int_{-\infty}^{\infty} p^2(t) dt + \frac{1}{2} \sum_k \int_{-T_b/2}^{T_b/2} p^2(t - kT_b) \cos(4\pi f_r t + 2\phi_r) dt \right] \\
 &= \frac{P_x T_b}{L} \left(c^2 + \frac{1}{2} \right) + \frac{1}{2} \sum_k \int_{-T_b/2}^{T_b/2} p^2(t - kT_b) \cos(4\pi f_r t + 2\phi_r) dt \\
 &\approx \frac{P_x T_b}{L} \left(c^2 + \frac{1}{2} \right), \tag{2.22}
 \end{aligned}$$

where the second equality follows because the integration of a function over an interval and then summing over a large number of contiguous intervals is equivalent to integration of the function over all intervals; the second last equality follows from $R_p(0) = T_b$; and the last equality follows from conditions that are addressed later in Section 2.2.3.

The expression in (2.21) shows that the mean of the decision variable contains the desired information bit b_0 scaled with the mean power in the filtered spreading signal. With a proper choice of the frequency offset and the pulse shape, all the bias terms are suppressed. This also ensures that the remaining mixing terms captured by the detection filter have zero mean.

Variance

The variance $\sigma_{d_0}^2$ of the decision variable in (2.14) is

$$\begin{aligned}
 \sigma_{d_0}^2 &= \mathbb{E}[(d_0 - \mathbb{E}[d_0])^2] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\mathbb{E}[z(u)z(v)] - \mathbb{E}[z(u)]\mathbb{E}[z(v)]) p(u)p(v) du dv \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C_{zz}(u, v) p(u)p(v) du dv \tag{2.23}
 \end{aligned}$$

where $C_{zz}(u, v)$ is the autocovariance function of $z(t)$. Since the processes $x(t)$ and $n(t)$ are mutually independent and have zero mean, it can be shown that all mixing products in (2.13) are mutually uncorrelated. The function $C_{zz}(u, v)$ can then be written as the sum of the autocovariance functions of the individual mixing products. Defining the contribution from each mixing product to the variance as

$$\Psi_{yy} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\mathbb{E}[z_y(u)z_y(v)] - \mathbb{E}[z_y(u)]\mathbb{E}[z_y(v)])p(u)p(v) du dv, \quad (2.24a)$$

$$\Psi_{yn} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\mathbb{E}[z_{yn}(u)z_{yn}(v)] - \mathbb{E}[z_{yn}(u)]\mathbb{E}[z_{yn}(v)])p(u)p(v) du dv, \quad (2.24b)$$

$$\Psi_{nn} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\mathbb{E}[z_n(u)z_n(v)] - \mathbb{E}[z_n(u)]\mathbb{E}[z_n(v)])p(u)p(v) du dv, \quad (2.24c)$$

we can rewrite $\sigma_{d_0}^2$ as

$$\sigma_{d_0}^2 = \Psi_{yy} + \Psi_{yn} + \Psi_{nn}. \quad (2.25)$$

For ease of notations, we define

$$\Lambda_0 = \frac{1}{16} \int_{-\infty}^{\infty} |\tilde{G}(\nu B_x)|^4 d\nu, \quad \Lambda_1 = \frac{1}{4} \int_{-\infty}^{\infty} \mathcal{S}(\nu) |\tilde{G}(\nu B_x)|^2 d\nu \quad (2.26a)$$

$$\Lambda_2 = \frac{1}{16} \int_{-\infty}^{\infty} \mathcal{S}^2(\nu) |\tilde{G}(\nu B_x)|^4 d\nu, \quad \Lambda_3 = \frac{1}{16} \int_{-\infty}^{\infty} \mathcal{S}(\nu) |\tilde{G}(\nu B_x)|^4 d\nu, \quad (2.26b)$$

as constants which depend on the choice of the front-end filter and the PSD of the noise carrier. These constants have values $\Lambda_i \geq 1$ for all i , and specify the extent to which the signal and noise power are captured by the front-end filter. Moreover, we define

$$\eta_1 := \frac{1}{T_b(1/2 + c^2)^2} \int_{-\infty}^{\infty} a^4(u) \cos^2(2\pi f_r u + \phi_r) p^2(u) du \quad (2.27a)$$

$$\eta_2 := \frac{1}{T_b(1/2 + c^2)} \int_{-\infty}^{\infty} a^2(u) \cos^2(2\pi f_r u + \phi_r) p^2(u) du. \quad (2.27b)$$

These constants specify the effect on total noise, in regards to the choice of $m(t)$, f_r and the scaling factor c . With these definitions, the terms in (2.24), are derived in Appendix A.1.2 till A.1.4. They are given as

$$\Psi_{yy} = \eta_1 \Lambda_2 E_b^2 / S, \quad \Psi_{yn} = 2\eta_2 \Lambda_3 E_b N_0, \quad \Psi_n = \frac{1}{2} \Lambda_0 N_0^2 S, \quad (2.28)$$

where $S := B_x T_b$ is the *spreading factor* and E_b follows from (2.22).

2.2.2 Signal-to-noise ratio

We have derived the mean and variance of the decision variable. Combining (2.21) with (2.25) and (2.28), the SNR of the decision variable is given by

$$\begin{aligned} \text{SNR} &= \frac{(\mathbb{E}[d_0])^2}{\sigma_{d_0}^2} = \frac{c^2 E_b^2 \Lambda_1^2 (1/2 + c^2)^{-2}}{\eta_1 E_b^2 \Lambda_2 / S + 2\eta_2 E_b N_0 \Lambda_3 + N_0^2 S \Lambda_0 / 2} \\ &= \frac{c^2 \Lambda_1^2 \gamma^2 (1/2 + c^2)^{-2}}{\eta_1 \Lambda_2 \gamma^2 / S + 2\eta_2 \gamma \Lambda_3 + S \Lambda_0 / 2}, \end{aligned} \quad (2.29)$$

where $\gamma := E_b/N_0$ is the received SNR per bit. For a large spreading factor, the PSD of the collective noise from the wideband terms is flat in the region of interest. By this nature, the collective noise exhibits whiteness thus justifying the use of a matched filter for white noise. Moreover, since the noise in the signal band can be considered Gaussian, the matched filter not only maximizes the SNR but also minimizes the BER. For a Gaussian distributed decision variable and an antipodal message signal, the BER can be shown to be

$$\text{BER} = Q(\sqrt{\text{SNR}}) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\text{SNR}}}^{\infty} \exp\left(-\frac{u^2}{2}\right) du, \quad (2.30)$$

where $Q(\cdot)$ is defined as the Gaussian tail probability. The equation in (2.29) represents a general expression for the SNR of the decision variable, in an AWGN channel.

2.2.3 Conditions and assumptions

We list and discuss assumptions made in the derivation of the SNR expression. First of all, the general requirement for arriving at (2.21) is that the frequency realigning followed by matched filter operation at the receiver, should suppress the bias terms in (2.20). This is guaranteed if the bias terms are moved to the nulls of the transfer function of the detection filter, or if they are moved out of the detection filter band. This is explored in Appendix B.1 for two examples of signaling schemes: i) a polar NRZ signaling scheme where $p(t) = \text{rect}(t/T_b)$ is a rectangular pulse, and ii) any polar signaling scheme with a real-valued $p(t)$ having an absolute bandwidth B_p . Sufficient conditions to suppress the bias terms are derived for these two signaling schemes, which are enumerated as

1. **Polar NRZ signaling scheme:** For the binary polar NRZ message signal $m(t)$, one should choose $f_r = n/T_b$ for any positive integer n . Such a choice of f_r ensures that the rectangular pulse $p(t)$ is orthogonal to $\cos(2\pi n f_r t)$ for any positive integer n . Analytically, this implies that the transfer function of the detection filter (IDF) has nulls at $f = n f_r$. The frequency-realigning at the receiver moves the biased terms to the nulls of the transfer function.

2. **Generic signaling scheme:** In this case, one should choose $f_r \gg B_p$. In other words, the frequency offset is much larger than the bandwidth of the matched filter. It is shown that $f_r > 3B_p$ is sufficient to move the bias terms out of the detection filter band.

These design criteria serve as sufficient conditions, and should be considered as two practical examples. More general conditions may exist. However, to keep the research direction on track, those are not thoroughly explored.

The first example provides little freedom in the choice of $p(t)$ and is sensitive to synchronization errors⁴. However, it allows the use of small frequency offsets. It will be shown later in Chapter 4 and 6 that small frequency offsets are beneficial for the scalability of the system and for operation in a fading environment. The second example is more flexible on the choice of $p(t)$. However, it should be considered with caution. For such a system design, the reference and information signals can occupy a rather large frequency band. This means that in order to collect both the reference and information signals, the front-end filter band has to be considerably wider than the bandwidth B_x of the spreading signal. It implies that the filter also collects additional noise; the strength of this noise increases proportionally with the filter bandwidth. Secondly, in a multipath fading environment, the channel frequency response can be different for the bands occupied by the two signals; thus they lose correlation between them, resulting in a weak despread signal. This will be discussed later in Chapter 6.

Despite the limitations with the condition $f_r \gg B_p = 1/T_b$ for the second example, we will show in Chapter 3 and in Chapter 4, that this indeed is the practical choice. Moreover, it is possible to choose f_r such that $1/T_b \ll f_r \ll B_x$ is satisfied. Since both choices have their advantages, we shall consider both of them in parallel in this thesis, wherever their significance arises. In summary, the fundamental assumptions made in deriving the SNR expression are:

1. The receiver is assumed to be perfectly synchronized with the received bit stream. The sampler at the output of the matched filter samples exactly at the peak of the output. Moreover, the phase and frequency of the receiver LO are locked to that of the transmitter LO.
2. An antipodal message signal is assumed as proof of principle. Although this can be extended to higher order modulation schemes, they are not considered in this thesis.
3. The pulse $p(t)$ is chosen such that it leads to zero ISI at the sampling instants, for a synchronized system.
4. The frequency offset is much smaller than the operational bandwidth i.e., $f_r \ll B_x$. Secondly, f_r is considered to be much larger than the absolute bandwidth of $p(t)$.

⁴This is discussed in Section 3.3.

5. The expression holds for a WSS spreading signal with any spectral shape, and any receiver front-end filter, given that the filter transfer function has a fairly flat magnitude over the signal bandwidth.

2.2.4 Simplified example

The relation in (2.29) is a general form of the SNR expression. This can be further simplified if specific choices are made for the system parameters. We simplify by considering an ideal and the most simplified setting. We consider $\tilde{x}(t)$ to have a perfect rectangular spectral shape, i.e.,

$$S_{\tilde{x}}(f) = \frac{2P_x}{B_x} \text{rect}\left(\frac{f}{B_x}\right) = \begin{cases} \frac{2P_x}{B_x}, & |f| \leq B_x/2, \\ 0, & \text{otherwise.} \end{cases} \quad (2.31)$$

Accordingly, we assume that $G(f)$ is a brick-wall filter with unity frequency response over the signal bandwidth and zero outside it. For $f_r \ll B_x$, such a filter collects the transmitted signal, more or less perfectly, and removes all out-of-band noise. With such a filter and spectral shape, one can show that $\Lambda_i = 1$ for all i in (2.26). This results in

$$\text{SNR} = \frac{c^2 \gamma^2 / (1/2 + c^2)^2}{\eta_1 \gamma^2 / S + 2\eta_2 \gamma + S/2}. \quad (2.32)$$

For further simplification, we consider $p(t)$ to have a rectangular shape, i.e.,

$$p(t) = \text{rect}\left(\frac{t}{T_b}\right) = \begin{cases} 1, & |t| \leq T_b/2, \\ 0, & \text{otherwise.} \end{cases} \quad (2.33)$$

It implies that $m(t)$ is a binary polar NRZ message signal. Moreover, the detection filter is a simple integrate-and-dump filter (IDF), which is known to be optimal for binary NRZ signals in AWGN. The constants η_1 and η_2 in (2.27) are computed in Appendix A.1.1 to be

$$\eta_1 = \frac{8c^4 + 36c^2 + 5}{16(1/2 + c^2)^2}, \quad \eta_2 = \frac{4c^2 + 3}{8(1/2 + c^2)}. \quad (2.34)$$

With this set of system parameters, we can rewrite (2.32) as

$$\text{SNR} = \frac{16c^2 \gamma^2}{(8c^4 + 36c^2 + 5)\gamma^2 / S + 4(c^2 + 1/2)(4c^2 + 3)\gamma + 2(2c^2 + 1)^2 S}. \quad (2.35)$$

We now find the optimum scaling factor c which results in the highest SNR. Finding the maxima of the above expression with respect to c , we get the optimal value of the scaling factor to be

$$c_{\text{opt}} = \left(\frac{5(\gamma/S)^2 + 6\gamma/S + 2}{8(\gamma/S)^2 + 16\gamma/S + 8} \right)^{1/4}. \quad (2.36)$$

One sees that c_{opt} varies with γ/S . However, it can be shown that the optimum value does not vary much. It remains between $\sqrt[4]{5/8} \approx 0.89$ and $1/\sqrt{2} \approx 0.71$. The latter is an intuitively appealing result; with $c = 1/\sqrt{2}$, one ensures equal power distribution between the reference and information signals in (2.6). This results in

$$\text{SNR} = \frac{8\gamma^2}{25\gamma^2/S + 20\gamma + 8S}, \quad (2.37)$$

which is the closed-form-expression for the SNR of the decision variable of a point-to-point N-FOM communication link in an AWGN channel, using a binary polar NRZ message signal, an ideal receiver front-end filter and a noise carrier with a perfectly rectangular PSD.

2.3 Results and discussions

We presented the communication model of an N-FOM system with an AWGN channel. The general expression for the BER is given in (2.30) which has one-to-one relation with the SNR of the decision variable. For a point-to-point LOS communication link, we derived the SNR in (2.29), with certain design restrictions on the message signal and the frequency offset. For a specific choice of the parameters involved, we derived the SNR in (2.37). In this section, we will begin with presenting and discussing numerical examples. This will be followed by simulation results. Throughout this section, we will use the specific case of the N-FOM system parameters presented in Section 2.2.4, i.e., the PSD of the spreading signal, the transfer function of the front-end filter and the pulse $p(t)$ have perfectly rectangular shapes.

We first plot the BER of the N-FOM system in Figure 2.2 as a function of the received SNR per bit $\gamma = E_b/N_0$, for different values of the spreading factor S . The BER was obtained by substituting (2.37) in (2.30). As a reference, we also plot the performance curve for a standard point-to-point DSSS link in AWGN, with coherent detection. The BER for such a link is given by $\text{BER} = Q(\sqrt{2\gamma})$, where 2γ is the SNR of the decision variable for DSSS with coherent reception. For the moment, we study the solid lines (the dashed line will be discussed later). A couple of observations are made.

- First, it is observed that the BER attains a floor or an asymptote as γ increases. In other words, the SNR does not improve by increasing the transmitted power beyond a certain point.
- Second, a small spreading factor gives a better performance for smaller values of γ , whereas for larger values of γ , it is preferable to choose a large spreading factor.
- Finally, the overall performance is poor for low-to-moderate values of γ (≤ 15 dB). Large values of γ are required for a reliable link ($\text{BER} \leq 10^{-3}$) in comparison with a DSSS link.

Figure 2.2: Bit-error-rate (BER) as a function of received SNR per bit ($\gamma = E_b/N_0$) of a point-to-point N-FOM link in an AWGN channel.

In order to understand these phenomena, we manipulate (2.37) to reflect the effects of each mixing product. We can write

$$\text{SNR} = (\text{SNR}_{yy}^{-1} + \text{SNR}_{yn}^{-1} + \text{SNR}_{nn}^{-1})^{-1}, \quad (2.38)$$

where $\text{SNR}_{yy} = 8S/25$, $\text{SNR}_{yn} = 2\gamma/5$ and $\text{SNR}_{nn} = \gamma^2/S$ represent the ratio of signal power to the noise power from signal self-mixing, signal-noise mixing, and noise self-mixing products respectively. It can be observed from the above expression that the overall SNR will be dominated by the lowest (worst) of $\{\text{SNR}_{yy}, \text{SNR}_{yn}, \text{SNR}_{nn}\}$. These SNRs are plotted in Figure 2.3 as functions of γ . We first observe that SNR_{yy} does not increase with γ and only increases with the spreading factor. This is due to the self-mixing at the detection stage. The transmitted signal also causes self-interference and the noise power of this self-interference increases with the increasing values of γ . The terms $\{\text{SNR}_{yn}, \text{SNR}_{nn}\}$ both logarithmically increase with γ , but the rate of increase of SNR_{nn} is twice as much as that of SNR_{yn} . This again is due to the fact that the noise in $z_{yn}(t)$ increases with increasing signal power.

The overall SNR, as mentioned earlier, follows the lowest of the three curves. One can increase S to enhance SNR_{yy} , nonetheless, the highest overall SNR will be $S_{\text{dB}} - 5 \text{ dB}$. In other words, for a very high γ , the total SNR is dominated by SNR_{yy} and hence it achieves an asymptote,

$$\lim_{\gamma \rightarrow \infty} \text{SNR} = \text{SNR}_{yy} = 8S/25 \quad [S_{\text{dB}} - 5 \text{ dB}]. \quad (2.39)$$

Figure 2.3: SNR components against increasing values of γ .

This is the maximum SNR of the N-FOM system for a fixed spreading factor S . In logarithmic scale, this is approximately equal to $S_{\text{dB}} - 5$ dB, where S_{dB} denote the spreading factor in dBs. Choosing a large or a small spreading factor will proportionally affect the SNR asymptote and hence the BER floor. Further observation of the figure reveals that enhancing S will also shift the point of intersection ($S_{\text{dB}} - 2.5$ dB) between SNR_{nn} and SNR_{yy} . This has two implications. First, for $\gamma < S_{\text{dB}} - 2.5$ dB, the total SNR is dictated primarily by SNR_{nn} . In other words, an increase in S magnifies the noise contribution from AWGN. Since a large S translates to a wide passband of the front-end filter and/or a large integration interval of the IDF, the noise power from the noise self-mixing product increases with increasing S . For low values of γ , one can see that the SNR closely follows $\text{SNR}_{nn} = \gamma^2/S$, which is small for large S .

The second implication of a high S is that one has to increase γ to benefit from a high spreading factor. This indicates that there is always a trade-off in performance between choosing high and low values of S . The SNR does not vary monotonically with varying spreading factor, but it has a contrasting behavior depending on the values of γ . For a fixed value of γ , there is an optimum value of S which results in the highest SNR for the N-FOM system. This optimal spreading factor S_{opt} is related to the point of intersection ($\gamma_{\text{dB}} = S_{\text{dB}} - 2.5$ dB), and can be found by minimizing the denominator of (2.37). The optimal spreading factor, and consequently, the maximum SNR using S_{opt} are

$$S_{\text{opt}} = 5\gamma\sqrt{2}/4 \quad \left[\gamma_{\text{dB}} + 2.5 \text{ dB} \right] \quad (2.40)$$

$$\text{SNR}_{\text{max}} = \frac{2}{5}(\sqrt{2} - 1)\gamma \quad \left[\gamma_{\text{dB}} - 7.8 \text{ dB} \right]. \quad (2.41)$$

The BER using the optimal spreading factor⁵ for each value of γ is depicted by the dotted curve in Figure 2.2. Using S_{opt} for each value of γ always results in the best achievable performance. This curve defines an upper-bound on the performance w.r.t. the spreading factor. It does not have a BER floor because it adapts S for each value of γ . Although adapting S to γ is not practical, the curve for optimal performance helps us choose the spreading factor, given a certain desired BER. For example, it can be seen that for achieving $\text{BER}=10^{-3}$ at the lowest possible γ , one should choose $S=100$ [20 dB].

The dependence of performance on γ/S is not unusual; in fact, the ratio (γ/S) is the carrier-to-noise ratio (CNR). The CNR is a ratio of power of the received transmitted signal to that of the noise at output of the receiver front-end filter. Since the in-band signals and noise determine which noise cross product is dominant, the CNR defines the performance regimes. From (2.40), we deduce that for $\gamma/S = 2\sqrt{2}/5 \approx 0.56$, there is no dominant noise term, and we refer to it as the *moderate-CNR regime*. The optimal S corresponds to this regime, i.e., $S_{\text{opt}} = \gamma/0.56$. For $\gamma/S \ll 0.56$, the SNR is limited by SNR_{nn} , and we identify it as the *low-CNR regime*. On the other hand, for $\gamma/S \gg 0.56$, the SNR is dictated by SNR_{yy} , and we call it the *high-CNR regime*. It must be stressed, that the CNR regimes are different if there are more noise products. For example, in the presence of an interferer, the self-mixing at the receiver will create interference cross-products. As a result, the optimal spreading factor and hence the moderate-CNR regime will also depend on the interference level.

The collective effect of the mixing products leads to a poor performance of the N-FOM system for low-to-moderate CNR regimes. The maximum achievable SNR, given by (2.41), is $\gamma_{\text{dB}} - 7.8$ dB. On the other hand, a coherent DSSS system in ideal setup has SNR of 2γ [$\gamma_{\text{dB}} + 3$ dB] for a point-to-point link in an AWGN channel [62]. This number marks the performance gap between the two systems, as seen from Figure 2.2. N-FOM in an ideal situation (optimal spreading factor) will require about 10.8 dB more power than coherent DSSS to achieve a certain BER. Another way to formulate it is that the N-FOM detection mechanism degrades the received SNR by at least 10.8 dB. N-FOM has a significantly poorer performance than a coherent SS system. The inferior performance is mainly due to the cross-terms produced by the mixer at the receiver. However, the benefit of using N-FOM over DSSS is that the simple receiver architecture significantly reduces cost on circuitry, synchronization and signal acquisition times — a desired feature in the design of any communication technique for monitoring applications.

Simulation results

So far, we have discussed numerical examples which were obtained directly from the analytical expressions in (2.30) and (2.37). As discussed at the end of Section 2.2, many assumptions were made while deriving this expression. To test

⁵Recall that $c = 1/\sqrt{2}$ is used, which is not optimal. However, it can be shown that choosing c_{opt} (2.36) hardly changes (2.41).

the validity of the approximations and hence the accuracy of the expression, a simulation of the N-FOM system is essential. We simulated a point-to-point N-FOM communication link in an AWGN channel, in MATLAB. For the message signal, a binary bipolar NRZ signal was used, and accordingly, an IDF was employed at the detection stage. Note that the Fourier transform of the pulse $p(t)$ has nulls at $f = n/T_b$ for any nonzero integer n . Therefore, the frequency offset was chosen equal to the symbol rate, i.e., $f_r = 1/T_b$. This is in line with the first condition on the relation between $p(t)$ and f_r , outlined in Section 2.2.3.

Since simulation in passband demands high sampling rates f_s and consequently high run times, all operations were carried out in complex baseband. For the spreading signal, a sequence of Gaussian distributed samples, with sampling rate $f_s = 1.25B_x$, was generated. The frequency spectrum of the signal was then limited to the operational bandwidth B_x by a filter. In order to have a more or less rectangular PSD, a type II Chebychev filter with a 40 dB stop-band ripple was used. Since $f_r \ll B_x$, the PSDs of the reference and information signals almost completely overlap. It implies that the transmitted as well as the spreading signal has the bandwidth B_x . Therefore, we used the same filter at the receiver front-end to collect the transmitted signal and suppress out-of-band noise. Finally, to operate at a certain spreading factor, both the symbol rate and bandwidth were manipulated.

The simulation results are plotted in Figure 2.4 for different spreading factors S . For comparison, BER values obtained from theoretical results are plotted in solid lines. There are two main observations. First, for low values

Figure 2.4: Comparison of BER between theoretical (in solid lines) and simulation (in dotted lines), for various spreading factors.

of the spreading factor, the theoretical curve is an inaccurate representation of the simulated BER. Second, the difference between theory and simulation further enlarges as $\gamma = E_b/N_0$ increases. While the differences could be due to a number of reasons, such as imperfect filtering, the fact that the simulated results give better results implies inaccuracies in key assumptions. We conjecture that these discrepancies arise primarily from the Gaussian assumption of the decision variable. The input to the detection filter (IDF), given in (2.13) is an unknown random process. However, each discrete sample of the input is a linear combination of generalized χ^2 distributed variables. The number of samples added up by the IDF is comparative to the spreading factor $S = B_x T_b$. We had assumed that $1/T_b \ll B_x$; the detection filter sums up a large number of independent random samples. The CLT was invoked and the decision variable was considered a Gaussian distributed random variable.

There are a couple of elements to the CLT assumption. A small S implies that the decision variable is sum of a fewer samples, and the CLT cannot be used. Therefore the analysis in this chapter and in this thesis is invalid for low values of S . However, in practice we will always use low data rates and high bandwidths, in order to benefit from the features of an SS system. For large spreading factors, the theoretical curves diverge slightly from the simulation in low-to-moderate CNR regimes $\gamma_{\text{dB}} \leq S_{\text{dB}} - 2.5$ dB. Nevertheless, the difference increases as γ increases beyond that value. For $S = 100$, the difference in γ (between theory and simulation) to achieve a BER of 10^{-3} and 10^{-6} is 1 and 3 dB respectively, whereas for $S = 200$ it is 0.2 and 1 dB respectively.

To get further understanding of the phenomena, we simulate the N-FOM system for $S = 100$ and a message signal consisting of 5000 symbols. The number of samples per symbol is 125, and hence each decision variable is a sum of 125 samples. We plot the histogram of the 5000 decision variables in Figure 2.5, for different values of γ . Moreover, in Figure 2.6, we plot the quantiles, which depict the decision variables (sketched in circles) stretched over a linear Gaussian line (sketched in solid red). In this graph, the readings on the x-axis are insignificant, as they depict values of a normal distributed decision variable. Therefore, we focus on the values on the y-axis. Note that a detection error is made whenever the decision variable crosses the decision threshold zero. The two figures show that the decision variable exhibits strong *Gaussianity* for the most part, except at the tails. However, for low-to-moderate CNR regimes ($\gamma_{\text{dB}} \leq S_{\text{dB}} - 2.5$), the tails have little significance, as seen from Figure 2.6. The difference between theory and simulation arises mainly where the tails of the PDF are meaningful, i.e., for $\gamma_{\text{dB}} \geq S_{\text{dB}} - 2.5$. In general, the true PDF of d_k is skewed, and the Gaussian PDF over-estimates the tails near the threshold.

It is well-known that the CLT approximation is reliable for modeling the center point (mean) of the PDF. For accuracy in the tails, the CLT requires a very large number of independent samples. The looseness of the Gaussian assumption in UWB systems was considered in [63, 64]. Chao and Scholtz verified the Gaussian assumption for a MA TR-UWB system [64], where there is a significant number of independent MA interferers. The convergence of a

Figure 2.5: Histogram of the decision variable d_k with Gaussian fitting, for different values of γ .

sum of χ^2 distributed random variables to a Gaussian random variable has been subject of a number of studies, which aim to find the error incurred due to looseness of the assumption. However, the studies find asymptotes of this error, which are uninteresting for the feasibility analysis of N-FOM.

Finding the exact distribution of the decision variable, although much desirable, is mathematically tedious, and is not considered in this thesis. Nevertheless, for moderate-to-high spreading factors ($S \geq 23 \text{ dB}$), the required γ values closely approximate the simulated values in the region of interest ($10^{-6} \leq \text{BER} \leq 10^{-3}$). This is exactly what we want to achieve from the theoretical models; to have a simplified expression which fairly approximates the true results for practical sets of system parameters. Therefore, for the rest of the thesis, the Gaussian assumption will be used, although with due diligence. Theoretical results will be supplemented with simulation results for rigor and accuracy.

2.4 Conclusions

In this chapter, we explained the basic principle of the N-FOM system and provided a comprehensive analysis for a point-to-point LOS link in an AWGN channel. A general analytical framework for the SNR of the decision variable was developed under certain fundamental assumptions. The mixing operation at the receiver creates multiple bias terms, and suppression of these terms imposes a few conditions on the choice of the frequency offset and the message

Figure 2.6: The decision variable plotted against a Gaussian fit (in red), for different values of γ .

signal. One can either exercise freedom in the choice of the message waveform, but will have to choose a frequency offset much larger than the symbol rate; or one can choose a rectangular pulse shape with a frequency offset that is an

integer multiple of the symbol rate. Both choices have practical implications in different operation scenarios.

As an example, the SNR expression was simplified by choosing a binary polar NRZ message signal, and a box shape for the spectrum of the spreading signal and transfer function of the front-end filter. The numerical examples were discussed and it was found that the cross terms created by the mixing operation elevate the overall noise floor. The detection mechanism is noisy, and as a result, the received SNR is decayed by at least 10.8 dB. A high spreading factor improves the error rate in the high-CNR regime $\gamma/S \gg 2\sqrt{2}/5$, but at the same time, degrades it in the low-CNR regime $\gamma/S \ll 2\sqrt{2}/5$. Therefore, an optimum spreading factor ($S_{\text{opt}} = 5\gamma\sqrt{2}/4$) exists, which gives the best possible BER for the lowest possible value of γ . For a BER of 10^{-3} , the optimum spreading factor is 20 dB. Furthermore, the simulation results were found to be deviating from the derived analytical results for a certain set of system parameters. However, in the region of interest ($10^{-6} \leq \text{BER} \leq 10^{-3}$) the analytical expression gives a close approximation of the performance, for large spreading factors.

In summary, in a point-to-point LOS scenario, using an N-FOM scheme has no advantage over a coherent DSSS scheme in terms of performance. A number of trade-offs are involved and the performance can be sensitive to the choice of key system parameters. However, this is expected from an energy-based detection scheme. In the next chapter, we explore the parameters and fundamental assumptions used in this chapter.

Implementation considerations¹

In the last chapter, we derived performance metrics of the N-FOM system, studied its behavior, and provided design criteria for a number of system parameters. To keep things as general as possible, a generalized expression for the SNR was derived in (2.29). However, some key assumptions were made. For a start, perfect symbol synchronization was considered at the receiver. Similarly, the LO at the receiver was assumed to be perfectly locked in phase and frequency to that at the transmitter. Finally, the numerical examples and simulations involved an N-FOM system with a rectangular shape of the PSD of the spreading signal and the transfer function of the receiver front-end (RFE) filter.

These assumptions, particularly regarding the spectral shape of the spreading signal and the RFE filter, are not always practical. As discussed in the last chapter, the noise from the signal-signal cross-product is performance-limiting in the moderate-to-high CNR regime. Therefore, the shape of the spreading signal (and hence the bandwidth) can have a pronounced effect on the performance. Similarly, the bandwidth and spectral shape of the RFE filter are equally important. An RFE filter with a large bandwidth will capture more noise, leading to a poor performance in the low-CNR regime. On the other hand, an RFE filter with a small bandwidth (compared to the spreading signal) risks rejection of the desired signal's information and energy. It is reasonable to conclude that, for any choice of the spreading signal, the RFE filter will also have to be appropriately adapted. In other words, there is a relation between the spectral shape of the spreading signal and that of the RFE filter. Secondly, the symbol and LO synchronization are crucial in the N-FOM system. Perfect synchronization is essential not only to get most of the desired signal energy, but also to ensure that the bias terms originating from the cross-mixing are moved out of the detection band. For any error in synchronization, the bias

¹Part of the contents of this chapter were published in [65]

terms can cause a significant nuisance and reduce the overall SNR.

In this chapter, we look into these considerations which are fundamental to the realization of the system. We begin with a brief discussion on the choice of the spreading signal in Section 3.1. Although this thesis considers pure noise as the spreading signal, other practical choices are also outlined. Next, we look into the relation between the spectral shapes of the spreading signal and the RFE. An optimal filter is derived and numerical examples are discussed in Section 3.2.2. Finally, the symbol and LO synchronization are investigated in Section 3.3, where the effects of synchronization errors are explored. Finally, the main findings of the chapter are summarized in Section 3.4.

3.1 Choice of the noise carrier

In the modeling and analysis of N-FOM, we have assumed WSS bandpass Gaussian noise as the noise carrier. It is arguable whether it is a suitable choice. In this section, we briefly discuss the possible choices and their implications.

3.1.1 Pure noise

Chaos-based communication has gained a lot of interest in recent times [66]. Chaos can be generated with a theoretically infinite number of independent signals, with excellent auto-correlation and cross-correlation properties, which has a huge potential in multi-user SS schemes. Furthermore, the unpredictability of chaotic signals has benefits in terms of security. A chaotic signal can be generated from a deterministic generator, such as a Chebychev polynomial function map or chaotic Markov map [67]. These are nonlinear dynamical systems whose state is highly sensitive to the initial conditions.

Another example of a chaotic signal is pure noise. Pure noise completely exhibits statistical randomness and can be generated as a current through a noisy resistor². This is in fact the electronic noise resulting from the thermal agitation of the charged particles in the resistor. The measured noise process can then be filtered to limit the noise to the operational bandwidth. Since the transmission is in RF bands, generating and processing signals in high frequency has a huge power penalty. However, the noise carrier can be completely generated at baseband. This can be accomplished by taking two independent noise processes and identically filtering them at baseband. Treating them as the in-phase and quadrature components ($x_I(t)$, $x_Q(t)$) of the noise carrier, they can be combined together using an I-Q modulator, to form the bandpass noise carrier, which is given as

$$x(t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t).$$

The basic architecture of an I-Q modulator is shown in Figure 3.1. One can completely eliminate the lower branch, and send only the in-phase component.

²Nonetheless, this turned out to be difficult to realize in practice. This is discussed in conclusions of the thesis.

Figure 3.1: Generation of a bandpass noise carrier using a quadrature modulator.

However, in this case, the bandpass noise would no longer be WSS, and WSS assumption is important for analytic convenience. The WSS assumption was outlined in Section 2.2, and was used in the subsequent analysis. Finally, it is important to mention here that both $x_I(t)$ and $x_Q(t)$ should have a mean power of P_x , not only to ensure that the total power in the bandpass signal $x(t)$ is P_x , but also for the validity of the WSS assumption.

Although we have used the transmitter architecture given in Figure 2.1, one can also have a different transmitter architecture. By combining the RF LO with the frequency offset LO, one can implement the transmitter as shown in Figure 3.2. Such an architecture was considered in an earlier work [43]. It was found that the performance for such a design suffers at least 1.8dB power penalty compared to that considered in Figure 2.1 in Chapter 2. Due to a potentially simpler transmitter architecture, it is an interesting option. However, it is not further considered in this thesis.

3.1.2 Pseudo-noise (PN) carrier

Another interesting choice for the noise carrier is a PN sequence which has spectral properties similar to that of filtered noise. It is commonly used in SS communication systems. A PN carrier is of the form

$$x(t) = \sqrt{P_x} \sum_{i=0}^{N_s-1} c_i w(t - iT_c), \quad (3.1)$$

where $\{c_i\}$ is a pseudo-random binary sequence (PRBS) taking values $\{+1, -1\}$; $T_c = 1/B_x = T_b/N_s$ is the chip duration; N_s is the number of chips in the PN sequence³; and $w(t)$ is pulse shape of each chip. The sequence $\{c_i\}$ is generated

³Ideally, the PN carrier should have an infinite length, but in practice, it is repeated periodically after a certain interval, although it is generally not desired.

Figure 3.2: Alternative transmitter architecture.

using a deterministic algorithm, but is termed “pseudo-random” due to the orthogonality of the sequence with time-shifted versions of itself. The PRBS is generated using linear feedback shift registers and the length of the sequence N_s depends on the number of shift registers in the circuitry. The longer the sequence, the higher the number of orthogonal sequences one can generate. For a large period N_s , the PN sequence exhibits good auto-correlation and cross-correlation properties. Such properties are important not only for maximizing the SNR of the despread signal but also in ensuring that an unintended user in a MA scenario is seen as noise, i.e., there is zero correlation between signals from different users.

If $w(t)$ is chosen to be a rectangular pulse, then we have constant instantaneous power $x^2(t) = P_x$. This means that the self-mixing of the desired signal $y(t)$ only produces the despread information signal and h.f.c., and does not add to the noise. The detection does not suffer from self-interference, so $\Psi_{yy} = 0$ in (2.29). The SNR of the point-to-point N-FOM link will be reduced to

$$\text{SNR}_{\text{PN}} = \frac{2c^2\Lambda_1^2\gamma^2(1/2 + c^2)^{-2}}{4\eta_2\gamma\Lambda_3 + S\Lambda_0}.$$

In ideal case, we have $\Lambda_i = 1$, for all i . Substituting the value of η_2 from (2.34) for the NRZ signaling scheme, with $c = 1/\sqrt{2}$, the above expression is solved to

$$\text{SNR}_{\text{PN}} = \frac{\gamma^2}{5\gamma/2 + S}, \quad (3.2)$$

which asymptotically converges to $5\gamma/2$ for large values of γ . This is a significant result; in contrast to N-FOM SNR asymptote in (2.39), the performance of the link using PN carrier improves with increasing γ . Since the self-mixing of the desired signal does not add to the total noise, there will be no BER floor.

Comparing with the SNR of BPSK (2γ), the above SNR expression is at least 7 dB worse. On the other hand, the maximum SNR of N-FOM, with optimal value of S for each value of γ , was shown to be 10.8 dB worse than that of BPSK. This implies that in ideal case, using a PN carrier in a LOS link has a 3.8-dB advantage over using a noise carrier, in terms of the E_b/N_0 required for a given BER.

However, with rectangular $w(t)$, the PSD of the spreading signal will leak some power beyond $B_x/2$. It is impossible to design a filter which perfectly collects the transmitted signal without distorting the shape of $w(t)$, while suppressing all out-of-band noise. For any other pulse shape, such as Gaussian or raised-cosine, the system will suffer from self-mixing to some extent. Furthermore, in practical scenarios, where the system will experience NLOS fading channels, the choice between a PN carrier and a pure noise carrier has little added value. We will also show this in Section 6.3.4 in Chapter 6.

3.1.3 Impulse radio and chirp

For minimal to no self-interference in a LOS N-FOM communication link, one requires a spreading signal with a constant envelope, and preferably a flat PSD. The peak-to-average power ratio (PAPR) of the carrier dictates the strength of the noise from the signal self-mixing. Therefore, a Gaussian noise carrier leads to a high self-interference, due to its Rayleigh distributed envelope and theoretically infinite PAPR.

A plausible option for the choice of the carrier is impulse radio (IR), which is typical to UWB schemes. IR consists of a train of sparsely-arranged narrow pulses. The sparseness of the pulses allows one to code between them. For instance, the pulses can be dithered to get desired spectral features. Pulse-position modulation can be used, which is beneficial in flat-fading scenarios. Moreover, to enable MA feature, the pulses can be made to hop between difference positions according to a TH sequence. The IR, although an interesting concept, is more suitable to UWB techniques, and its applicability to wideband schemes, such as N-FOM, is limited.

A family of random signals with constant envelopes and flat PSDs are chirp signals. A chirp signal is generated by sweeping the frequency of a harmonic carrier over a range of frequencies determined by the bandwidth. Chirp SS (CSS) technique are widely studied, with particular focus on their robustness to interference [68], and can be an interesting candidate for N-FOM. Whether the noncoherent detection mechanism of CSS conforms to the design goals of TR should be investigated. However, for the rest of the thesis, we will use pure noise, and outline plausible alternatives in the final conclusions.

3.2 Receiver front-end filter

The spectral shape in TR has mainly been studied in the context of UWB. Traditionally, the UWB technique employs sparse spreading sequences. The

UWB spectrum has undesired spectral lines, and a combination of techniques such as pulse-dithering, time-hopping and polarity randomization is used to shape the spectrum of the IR-UWB signal [69]. Such studies aim to make the UWB spectrum compliant to FCC rules, and do not cover the relation between the transfer function of the filter and the spectral shape of the spreading signal. This is partly because, unlike traditional UWB TR systems, noise from the signal-signal cross-product is specific to the N-FOM system. Therefore, a study of the filter in relation with the spreading signal is required, and can lead to significant insights.

3.2.1 Filter optimization

In Chapter 2, we considered a perfectly rectangular PSD of the spreading signal; accordingly, the RFE filter was considered to be a brick-wall filter. The SNR of the decision variable was derived in (2.37). In practice, such a rectangular spectrum and filter can only be approximated by other spectral shapes. A degradation in performance can be expected by making such approximations. To study this degradation alone, the rest of the system model is kept simple. Specifically, the message signal is assumed to be polar NRZ, the frequency offset is chosen as $f_r = 1T_b$, and the scaling parameter is considered to be $c = 1/\sqrt{2}$. Rewriting the general SNR expression in (2.29) by substituting Λ_i from (2.26), η_i from (2.34) and $c = 1/\sqrt{2}$, we get

$$\text{SNR} = \frac{8T_b \left[\int_{-\infty}^{\infty} S_{\tilde{x}}(f) |\tilde{G}(f)|^2 df \right]^2}{\int_{-\infty}^{\infty} |\tilde{G}(f)|^4 (25S_{\tilde{x}}^2(f) + 40N_0S_{\tilde{x}}(f)L + 32N_0^2L^2) df}. \quad (3.3)$$

Clearly, the choice for the RFE filter, with transfer function $G(f)$, depends on the spectrum $S_{\tilde{x}}(f)$ of the spreading signal. In (3.3), the numerator represents the power in the despread signal part whereas the denominator represents the collective noise contribution from signal-signal, signal-noise and noise-noise cross-products, respectively. One can observe that choosing a filter which minimizes the denominator might also suppress the numerator. Therefore, there is a trade-off in choosing $\tilde{G}(f)$. There exists an optimum RFE filter which, passes the signal part while suppressing the noise from the cross-terms, such that the SNR in (3.3) is maximized. Defining

$$W(f) = 25S_{\tilde{x}}^2(f) + 40N_0S_{\tilde{x}}(f)L + 32N_0^2L^2, \quad (3.4)$$

we can rewrite (3.3) as

$$\text{SNR} = 8T_b \frac{\left[\int_{-\infty}^{\infty} |\tilde{G}(f)|^2 S_{\tilde{x}}(f) df \right]^2}{\int_{-\infty}^{\infty} |\tilde{G}(f)|^4 W(f) df}. \quad (3.5)$$

To find the optimal filter transfer function $\tilde{G}_{\text{opt}}(f)$, we follow the same approach as that of the classical matched filter problem [60]. The difference is that, instead of detecting a known pulse in additive noise, we attempt to filter the received signal $r(t)$ in such a way that it leads to the maximum SNR at the output of the IDF. We invoke the Cauchy-Schwarz inequality, stating

$$\left| \int A(f)B(f)df \right|^2 \leq \int |A(f)|^2 df \int |B(f)|^2 df,$$

with equality if $A(f) = C B^*(f)$ for any arbitrary nonzero constant C . Supposing $A(f) = |\tilde{G}(f)|^2 \sqrt{W(f)}$ and $B(f) = S_{\tilde{x}}(f)/\sqrt{W(f)}$, the maximum SNR is found as

$$\text{SNR}_{\text{max}} = 8T_b \int_{-\infty}^{\infty} \frac{S_{\tilde{x}}^2(f)}{W(f)} df, \quad (3.6)$$

for an optimum filter with equivalent baseband transfer function

$$|\tilde{G}_{\text{opt}}(f)|^2 = C \frac{S_{\tilde{x}}(f)}{W(f)} = C \frac{S_{\tilde{x}}(f)}{25S_{\tilde{x}}^2(f) + 40N_0S_{\tilde{x}}(f)L + 32N_0^2L^2}. \quad (3.7)$$

The expression in (3.7) represents the squared-magnitude response of the filter which achieves the maximum SNR (3.6) in an AWGN channel for any spectral shape of the spreading signal. We observe from (3.7) that $\tilde{G}_{\text{opt}}(f)$ depends on the channel loss L as well as on the noise level N_0 . Note that this implies that the optimal filter will have to adapt dynamically to the received SNR.

3.2.2 Analysis of the optimal filter

In this section, we study the maximum SNR and the optimal RFE filter, derived in the last section. Different filter shapes and CNR regimes will be considered in order to get insight into the optimality of the filter. The constant C in (3.7) is irrelevant for the overall SNR expression; therefore, it will be omitted in all subsequent analysis.

Ideal case

To study $\tilde{G}_{\text{opt}}(f)$, we first consider a simple already known case. We assume the spreading signal $\tilde{x}(t)$ to have an ideal rectangular spectral shape as defined in (2.31). In this case, it follows from (3.7) that $\tilde{G}(f)$ should be a brick-wall filter, with unity frequency response between $-B_x/2$ and $B_x/2$. This also follows from intuition, as such a filter does not need to prefer or suppress one spectral component of the spreading signal over any other spectral component. The resulting SNR is the same as that in (2.37). The resulting performance and trade-offs are already discussed in Section 2.3 of Chapter 2.

One can prove that choosing a rectangular PSD for the spreading signal gives the best achievable SNR in an N-FOM system. Consider a non-rectangular spectrum $S_{\tilde{x}}(f)$ as shown in Figure 3.3a. One can approximate

Figure 3.3: A non-rectangular PSD (a), divided into rectangular blocks (b).

the spectrum using a staircase shape i.e., by dividing the spectrum into M non-overlapping rectangular spectral blocks $\{S_1(f), \dots, S_M(f)\}$ as depicted in Figure 3.3b. Each i th spectral block has a height $2P_i/B_i$ where B_i is its bandwidth and $2P_i$ is total power in the block. The factor 2 stems from the fact that we are considering the PSD of the complex baseband signal. The power in each block satisfies

$$P_x = \sum_{i=1}^M P_i.$$

From (3.6), we see that the $\text{SNR}_{\max}$ is the sum of SNR in each block, i.e.,

$$\text{SNR}_{\max} = \sum_{i=1}^M \text{SNR}_i = 8T_b \sum_{i=1}^M \int_{B_i} \frac{(2P_i/B_i)^2}{W_i(f)} df,$$

where $W_i(f)$ is the PSD of noise in each spectral block. Similar to the optimization w.r.t. S in Chapter 2, we can optimize each SNR_i with respect to B_i . The optimal bandwidths would scale proportionally to the received SNR per bit γ_i , and hence P_i in each block. Therefore, for optimal performance, the spectral blocks with small P_i should have smaller bandwidths, whereas those with large P_i should have larger bandwidths. As a result of this optimization, spectral blocks with large P_i will have their PSDs lowered, whereas those with small P_i will have raised PSDs. In the end, all blocks will have the same PSD but different (optimal) bandwidths. Using the optimal bandwidths, and redrawing the spectrum such that the blocks do not overlap, would reveal that the optimal spectral shape is rectangular. It means that for a given γ , the spectrum in (2.31) with optimized value for B_x given by (2.40) is *ideal*, resulting in the best achievable SNR (2.41). This allows us to define an upper bound on the performance with respect to the spectral shapes. Using any other spectral shape would result in an SNR at most as good as that in (2.41).

General case

In practice, we cannot have a perfectly rectangular $S_{\tilde{x}}(f)$. For any other spectral shape, choosing a brick-wall filter will not result in the maximum SNR. In this subsection, we consider a non-rectangular $S_{\tilde{x}}(f)$; analyze $\tilde{G}_{\text{opt}}(f)$ and SNR_{max} ; and compare the results with those of the ideal case (rectangular PSD). We wish to analyze how much performance penalty is suffered by using a non-ideal spectral shape and correspondingly the optimal filter. As stated in (2.3) we can write any given spectrum as

$$S_{\tilde{x}}(f) = \frac{2P_x}{B_x} \mathcal{S}\left(\frac{f}{B_x}\right), \quad (3.8)$$

where $\mathcal{S}(\nu)$ denotes a basic spectral shape with unit width and unit area, and ν is normalized frequency. The exact definition of the width⁴ will be addressed shortly. For further illustration, we introduce a running example of a spectrum $S_{\tilde{x}}(f)$ which is shaped by a Butterworth filter, i.e.,

$$S_{\tilde{x}}(f) = \frac{2P_x}{B} \mathcal{S}_B\left(\frac{f}{B_x}\right), \text{ with } \mathcal{S}_B(\nu) = \frac{\mu}{1 + (\nu/\bar{\nu})^{2n}}, \quad (3.9)$$

where $\mathcal{S}_B(\nu)$ is a Butterworth shape; n is the filter order; $\bar{\nu}$ is the normalized 3 dB bandwidth; and μ is a normalization variable. The variables $\bar{\nu}$ and μ ensure that $\mathcal{S}_B(\nu)$ has unit width and unit area; the exact values of the variables will be derived once B_x is defined. The Butterworth spectral shape is an interesting example since it has maximally flat magnitude and is physically realizable. Increasing the filter order n makes it more uniform over the passband with a sharp roll-off. For very large n , $\mathcal{S}_B(\nu)$ closely resembles a rectangular PSD.

To analyze $\tilde{G}_{\text{opt}}(f)$, we study (3.6) and (3.7) for the asymptotic cases. At first, we look at the low-CNR regime ($\gamma \ll S$). From (2.22) and (3.9), one can derive that $\gamma/S \ll 0.56$ implies $S_{\tilde{x}}(f)/L \ll N_0$ for all frequencies. Evaluating (3.4) for $S_{\tilde{x}}(f)/L \ll N_0$, we get $W(f) \approx 32L^2N_0^2$ for all frequencies, resulting in $|\tilde{G}_{\text{opt}}(f)|^2 \approx S_{\tilde{x}}(f)$. This is an important observation; $\tilde{G}_{\text{opt}}(f)$ in this case only depends on the PSD of the spreading signal and does not depend on the noise levels. This indicates that, at low values of the CNR, we can conveniently choose the RFE filter $|\tilde{G}(f)|^2 = S_{\tilde{x}}(f)$, i.e., the RFE filter is the same as the one used to shape the noise carrier at the transmitter. In Figure 3.4a, where we have plotted $\{S_{\tilde{x}}(f)/L, N_0, |\tilde{G}_{\text{opt}}|^2\}$ for a Butterworth spectrum in the low-CNR regime. The filter optimization in this scenario is similar to a matched filter problem; with the difference that the optimal filter does not require phase conjugation. Therefore, we see that the squared-magnitude response of the optimal filter is equal to the PSD $S_{\tilde{x}}(f)$. From (3.6), the SNR_{max} in the low-

⁴In Chapter 2, we had considered the spectral shape to have a width B_x , but had not defined the exact definition of the width for a non-rectangular PSD.

(a) Low-CNR regime, $\gamma/S = 0.1$ (b) High-CNR regime, $\gamma/S \approx 316$

Figure 3.4: $\tilde{G}_{\text{opt}}(f)$ in low and high CNR regimes for a Butterworth PSD $S_{\tilde{x}}(f)$, with filter order $n = 4$ and a 3 dB bandwidth B_c

CNR regime, with our notation in (3.8), is derived as

$$\text{SNR}_{\max} \approx \frac{\gamma^2}{S} \int_{-\infty}^{\infty} \mathcal{S}^2(\nu) d\nu. \quad (3.10)$$

The above result is exactly equal to that of the rectangular PSD in the low-CNR regime in (2.38), if we define the spectral shape $\mathcal{S}(\nu)$ such that

$$\int_{-\infty}^{\infty} \mathcal{S}^2(\nu) d\nu = \int_{-\infty}^{\infty} \mathcal{S}(\nu) d\nu = 1. \quad (3.11)$$

This property of the spectral shape will be used throughout this thesis. Using (3.8), this implies that the definition of bandwidth is

$$B_x = \frac{(2P_x)^2}{\int_{-\infty}^{\infty} S_{\tilde{x}}^2(f) df} = \frac{\left(\int_{-\infty}^{\infty} S_{\tilde{x}}(f) df \right)^2}{\int_{-\infty}^{\infty} S_{\tilde{x}}^2(f) df}. \quad (3.12)$$

Note that this is in accordance with (2.31); and this definition of bandwidth does not change the analysis carried out in Chapter 2.

With the notations in (3.8) and (2.22), $\text{SNR}_{\max}$ in (3.6) can be written as

$$\text{SNR}_{\max} = \int_{-\infty}^{\infty} \frac{8\gamma^2 \mathcal{S}^2(\nu) d\nu}{25(\gamma^2/S) \mathcal{S}^2(\nu) + 20\gamma \mathcal{S}(\nu) + 8S}, \quad (3.13)$$

The above expression represents the maximum achievable SNR for an N-FOM system in AWGN, using an optimal filter for any spectral shape $\mathcal{S}(\nu)$ and a fixed value for spreading factor S .

We now turn to our running example. Using the relations $\int \mathcal{S}(\nu) d\nu = 1$ and $\int \mathcal{S}^2(\nu) d\nu = 1$, we derive in Appendix B.3,

$$\mu = \frac{2n}{2n-1}, \quad \text{and} \quad \bar{\nu} = \frac{1}{2\mu} \text{sinc}(1/2n), \quad (3.14)$$

where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. To find the maximum achievable SNR for Butterworth PSD, we have to evaluate the integral in (3.13) for $\mathcal{S}(\nu) = \mathcal{S}_B(\nu)$. This can be evaluated with the help of Residue Theorem, resulting in

$$\text{SNR}_{\max} = \frac{4}{5} \gamma (2\kappa^2 - 2\kappa + 1)^{-\frac{1}{2\mu}} \sin \left(\frac{1}{\mu} \tan^{-1} \left(\frac{\kappa - 1}{\kappa} \right) \right), \quad (3.15)$$

where $\kappa := 1 + 5\gamma\mu/(4S)$. The performance results for an N-FOM LOS link with $S = 100$ is illustrated in Figure 3.5. The solid lines depict the performance using the optimal filter for our running example; the dashed lines will be discussed later. We first observe that, as expected, the results for the low-CNR regime ($\gamma_{\text{dB}} \ll S_{\text{dB}} - 2.5 \text{ dB}$) are similar for the system using a rectangular PSD and that using an optimal filter for a Butterworth spectrum. In the high-CNR regime, the system using a Butterworth spectrum with a small filter order

Figure 3.5: BER as a function of E_b/N_0 , for an N-FOM system using a Butterworth PSD of different orders n , with optimal and suboptimal filtering, and spreading factor $S=100$.

n outperforms the system using a large n . Interestingly, it also outperforms the system with a rectangular PSD in the regime. This may seem counter-intuitive, as one may expect the performance to improve as the PSD approaches a rectangular shape (by increasing n).

To investigate this phenomenon, we study the optimal filter for the other asymptotic case. Evaluating (3.4) and (3.7) for the high-CNR regime ($\gamma \gg S$), we get $W(f) \approx 25S_{\tilde{x}}^2(f)$ which leads to $|\tilde{G}_{\text{opt}}(f)|^2 \approx 1/S_{\tilde{x}}(f)$ over the frequencies where $S_{\tilde{x}}/L \ll N_0$. The optimum filter in the high-CNR regime has dips at frequencies where the PSD of the spreading signal has peaks and vice versa. This phenomenon is illustrated Figure 3.4b. For high received signal power, the noise from the signal-signal cross-product (corresponding to the first term in (3.4)) dominates the total noise. The PSD $W(f)$ will have more power concentrated at the center. The optimal filter attenuates the received signal where $W(f)$ is strong, and amplifies where the signal spectrum $S_{\tilde{x}}(f)$ is low, as long as it is above N_0 . We see that the optimal filter inverts $S_{\tilde{x}}(f)$ at frequencies, where $S_{\tilde{x}}(f)/L$ is larger than N_0 . As a result, the bandwidth of the filtered PSD ($|\tilde{G}_{\text{opt}}(f)|^2 S_{\tilde{x}}(f)$) increases. In other words, the optimal filter attempts to increase the bandwidth (and hence spreading factor) to improve the SNR. This explains the counter-intuitive result, we observed earlier. At large values of γ , optimal filtering increases the bandwidth of a spectrum with

a slow roll-off (small n) at a higher proportion than that of a spectrum with a sharp roll-off (large n). Therefore, the spectrum $|\tilde{G}_{\text{opt}}(f)|^2 S_{\tilde{x}}(f)$ of the filtered signal has a larger bandwidth than that for the rectangular case, thus resulting in an improved performance. Note that due to this bandwidth enhancement, the curve for the optimal filtering has no BER floor.

To see how close we can get to the upper bound (2.41) by using optimal filtering, we derive the achievable performance for an optimized spreading factor S_{opt} . For our running example, S_{opt} is found by numerically maximizing (3.15) with respect to κ . The derived S_{opt} , and SNR_{max} using S_{opt} are tabulated in Table 3.1, for different filter orders. The performance results for an N-FOM LOS link with optimized S are illustrated in Figure 3.6. We first direct attention to the solid lines, which depict the maximum achievable performance using the optimal filter for our running example (the dashed lines will be discussed later). As in the previous figure, we observe that the performance for the Butterworth spectrum with optimal filtering is similar to that for ideal rectangular PSD in the low-CNR regime. This is expected and is in line with our observation regarding (3.10). Furthermore, we see in the figure that the ideal case presents an upper bound on the performance. Finally, the degradation in performance for using a non-ideal (Butterworth) spectral shape is marginal. To achieve any desired BER using a Butterworth spectral shape of the spreading signal, optimal filtering has a power penalty of 1.22 dB for $n = 1$ and 0.29 dB for $n = 3$, given that an optimal spreading factor is employed.

3.2.3 Suboptimal filtering

We have seen that, in order to achieve the optimal performance, the receiver has to adapt the RFE filter to the received signal levels. For most practical scenarios, this is unfeasible. Moreover, the average performance in multipath channels is dominated by low-SNR conditions. Therefore, we may conveniently

Table 3.1: Parameter values for optimal and suboptimal filtering using a Butterworth PSD.

n	μ	optimal filtering		suboptimal filtering	
		S_{opt}	$\text{SNR}_{\text{max}}(S_{\text{opt}})$	(λ_1, λ_2)	$\text{SNR}_{\text{sub}}(S_{\text{opt}})$
1	2	1.83γ	0.125γ	(1.5, 2.5)	0.107γ
3	6/5	1.785γ	0.155γ	(1.1, 1.25)	0.149γ
6	12/11	1.776γ	0.161γ	(1.04, 1.11)	0.158γ
10	20/19	1.772γ	0.163γ	(1.02, 1.06)	0.161γ
∞	1	1.768γ	0.165γ	(1, 1)	0.165γ

choose $|\tilde{G}(f)|^2 = S_{\tilde{x}}(f)$ in all cases, i.e.,

$$|G_{\text{sub}}(f)| = \sqrt{\mathcal{S}\left(\frac{f-f_c}{B_x}\right)} + \sqrt{\mathcal{S}\left(\frac{f+f_c}{B_x}\right)} \Rightarrow |\tilde{G}_{\text{sub}}(f)|^2 = 4\mathcal{S}\left(\frac{f}{B_x}\right). \quad (3.16)$$

Although such a filter is optimal only for low received signal strengths, it will be interesting to see if this convenient choice of the filter is an overall good compromise. We therefore wish to find how close we can get to the optimal performance (SNR_{max}) by using this *suboptimal* filter. At first, we make a performance comparison of suboptimal filtering with that of the ideal case. Choosing $|\tilde{G}_{\text{sub}}(f)|^2$ for any spectral shape, it follows from (3.4), (3.5), (3.8) and with the definition of E_b from (2.22) that the resulting SNR of the decision variable is given by

$$\begin{aligned} \text{SNR}_{\text{sub}} &= \frac{8\gamma^2}{\int_{-\infty}^{\infty} [25(\gamma^2/S) \mathcal{S}^4(\nu) + 20\gamma \mathcal{S}^3(\nu) + 8S] d\nu}, \\ &= \frac{8\gamma^2}{25\lambda_2\gamma^2/S + 20\lambda_1\gamma + 8S}, \end{aligned} \quad (3.17)$$

where the last equality follows by defining

$$\lambda_1 = \int_{-\infty}^{\infty} \mathcal{S}^3(\nu) d\nu, \quad \lambda_2 = \int_{-\infty}^{\infty} \mathcal{S}^4(\nu) d\nu. \quad (3.18)$$

For our running example, these values are calculated in Appendix B.3.3 as

$$\lambda_1 = \frac{4n-1}{2(2n-1)}, \quad \lambda_2 = \lambda_1 \cdot \frac{6n-1}{3(2n-1)}. \quad (3.19)$$

The values are tabulated in Table 3.1 for a few filter orders of the Butterworth filter. We refer again to Figure 3.5, and observe the dashed lines this time. The curves depict the BER of N-FOM systems using the Butterworth spectral shape for different filter orders n and a fixed $S=100$. As a reference, we also plot the achieved error rates using a rectangular PSD. We see that in the low-CNR regime ($\gamma/S \ll 0.56$), suboptimal filtering leads to the same performance as that of an ideal case (2.37), as expected. In the high-CNR regime, the difference between the BER floor for the two scenarios is determined by λ_2 . Moreover for $n=6$, the performance of a suboptimal filter is comparable to that of an ideal rectangular spectrum. The trend shows that the difference further reduces as we increase the filter order; and for very large filter order, the performances will be identical, as expected.

For further comparison between the optimal and suboptimal filtering, we study the achievable performance with an optimized spreading factor. For suboptimal filtering, this optimization is carried out by finding minima of the denominator in (3.17) with respect to S . This results in $S_{\text{opt}} = 5\gamma\sqrt{2\lambda_2}/4$, and

$$\text{SNR}_{\text{sub}}(S_{\text{opt}}) = \frac{2\gamma}{5(\lambda_1 + \sqrt{2\lambda_2})}, \quad (3.20)$$

Figure 3.6: BER as a function of E_b/N_0 , for an N-FOM system using a Butterworth PSD, with optimal and suboptimal filtering, and an optimized spreading factor S_{opt} .

which is the maximum achievable performance using suboptimal filtering. In Figure 3.6, we compare an N-FOM system using an optimal filter to that using a suboptimal filter, with an optimized spreading factor S_{opt} for each system. As a reference, we also plot our upper bound (2.41). Compared to the optimal filter, we observe a slight performance degradation by using the suboptimal filter (penalty of 0.67 dB for $n = 1$ and 0.16 dB for $n = 3$). This degradation is further reduced if $S_{\tilde{x}}(f)$ is shaped by a filter with high filter order. In other words, for a spectral shape which closely resembles a rectangular shape, the suboptimal filter leads to optimal performance for all values of γ . The results reveal that in optimal sense, we do not lose much by choosing the suboptimal filter.

3.3 Synchronization

In the performance calculations so far, we have assumed that the receiver is perfectly synchronized to the incoming signal. This means that not only the phase and frequency of the receiver LO are perfectly matched to those in the transmitter, but the integration time intervals of the detection filter are also synchronized to the bit transitions. In practice, the radio transceivers have an internal clock which provides a timing reference to all components. The aim

is to have all operations, such as bit transitions, sampling, LOs, listening and transmission intervals, synchronized with each other. It is fair to consider that in an N-FOM receiver, the integration time of the detection filter is coupled with the phase of the LO. So far, we had assumed that the clocks at the transmitter and the receiver nodes are perfectly matched. In practical scenarios, this is not guaranteed and the communication risks performance degradation. In this section, we explore the effect of synchronization errors and present a potential synchronization strategy.

3.3.1 Timing synchronization

To quantify the effect of synchronization errors, we need to examine the demodulated bit. We consider that the receiver is not synchronized to the incoming bit stream. The IDF and the receiver LO are off in time by a random offset τ , as shown in Figure 3.7. Note that the offset oscillators are assumed to be locked in frequency, i.e., $f_{Tx} = f_{Rx} = f_r$. Furthermore, we also assume the initial phase of the LOs be matched, i.e., $\phi_{Rx} = \phi_{Tx} = \phi_r$. The offset τ is considered to be a fixed value in the interval $[0, T_b]$. Reproducing (2.18), the mean of the decision variable with a timing mismatch, is given by

$$\begin{aligned} \mathbb{E}[d_0] &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} a^2(u) \cos(2\pi f_r(u - \tau) + \phi_r) p(u - \tau) du \\ &\quad + P_{n_g} \int_{-\infty}^{\infty} \cos(2\pi f_r(u - \tau) + \phi_r) p(u - \tau) du \\ &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} a^2(t + \tau) \cos(2\pi f_r t + \phi_r) p(t) dt \\ &\quad + P_{n_g} \int_{-\infty}^{\infty} \cos(2\pi f_r t + \phi_r) p(t) dt, \end{aligned} \quad (3.21)$$

where the last equality follows from substitution of variable $t = u - \tau$. The second integral is analogous to that in (2.18). This bias will be suppressed if the conditions on the pulse shape and the frequency offset are satisfied, i.e., the Frequency transform of the pulse is zero at $f = f_r$. Substituting $a(t)$ from

Figure 3.7: N-FOM receiver with a synchronization mismatch τ .

(2.10) and $m(t)$ from (2.5), (3.21) is rewritten as

$$\begin{aligned}\mathbb{E}[d_0] &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} \left(\frac{1}{2} + m^2(t+\tau) \cos^2(2\pi f_r(t+\tau) + \phi_r) \right) \cos(2\pi f_r t + \phi_r) p(t) dt \\ &\quad + \frac{P_{x_g}}{L} \sum_k b_k \int_{-\infty}^{\infty} p(t - kT_b + \tau) p(t) \left(\cos(2\pi f_r \tau) + \cos(2\pi f_r(2t + \tau) + 2\phi_r) \right) dt \\ &= \frac{P_{x_g}}{L} \sum_k b_k \cos(2\pi f_r \tau) \int_{-\infty}^{\infty} p(t - kT_b + \tau) p(t) dt + \Xi(f_r, \tau, m(t), \phi_r), \quad (3.22)\end{aligned}$$

where we have denoted $\Xi(f_r, \tau, m(t))$ as the combination of bias terms. This term is similar to the bias term in (2.20). It is shown in Appendix B.2 that for any generic signaling scheme, it is suppressed if $f_r \geq 3B_p$, where B_p is the absolute bandwidth of the pulse. This condition is the same as that mentioned in Section 2.2.2. However, for a polar NRZ signaling scheme, the conditions are slightly different. In this case, the bias term $\Xi(\cdot)$ is suppressed if either $f_r \gg 1/T_b$, or if $f_r = n/T_b$ with $\phi_r = (2n+1)\pi/4$ for a positive integer n . We assume that these conditions are met, and proceed with the assumption $\Xi(f_r, \tau, m(t), \phi_r) \approx 0$.

The integral in (3.22) represents correlation of $p(t)$ with a delayed version of itself. We can rewrite

$$\begin{aligned}\mathbb{E}[d_0] &= \frac{P_{x_g}}{L} \cos(2\pi f_r \tau) \sum_k b_k R_{pp}(\tau - kT_b) \\ &= \frac{1}{\sqrt{2}T_b} E_b \Lambda_1 \cos(2\pi f_r \tau) \left(b_0 R_{pp}(\tau) + \sum_{k \neq 0} b_k R_{pp}(\tau - kT_b) \right), \quad (3.23)\end{aligned}$$

where we have substituted $P_{x_g} = \int S_{\tilde{x}}(f) |\tilde{G}(f)|^2 df$ as mean power of the filtered signal; E_b from (2.22); and Λ_1 from (2.26). The above result is in line with (2.21), for perfect synchronization (i.e., $\tau=0$). For a pulse shape that satisfies the zero ISI condition (i.e., $R_{pp}(kT_b) = 0$ for $k \neq 0$, and T_b for $k=0$), (3.23) solves to (2.21), for $\tau=0$ and $c=1/\sqrt{2}$.

The above expression shows that a synchronization mismatch τ has the following effects on the demodulated information bit:

1. First, there is an effect of the phase mismatch, characterized by the factor $\cos(2\pi f_r \tau)$. This factor is independent of the pulse shape.
2. Second, as a result of the error in sampling instants, the detection scheme fails to sample the matched-filter's output at its peak. This is manifested by $b_0 R_{pp}(\tau)$ in (3.23).
3. Finally, due to mismatch in the integration interval, the demodulated bit suffers interference from neighboring bits. The energy reduction strongly depends on the polarity of neighboring bits, as well as on how swiftly the ACF decays.

Although the summation in (3.23) implies an infinite sum, in practice, the ACF $R_{pp}(\tau)$ decays rapidly outside the interval $[-T_b, T_b]$. Therefore, the interference from neighboring bits is significantly lower for each consecutive neighboring bit. The normalized ACFs of a rectangular pulse, and a root raised-cosine (RRC) pulse with different roll-off factors α , are depicted in Figure 3.8. Compared to the ACF of the rectangular pulse, the ACF of the RRC pulse has a flatter magnitude for small values of τ . This implies that the RRC pulse is less sensitive to a timing mismatch and therefore, the first term in (3.23) is larger for the RRC pulse in comparison with the rectangular pulse. This comes at the price of interference from the neighboring bits, manifested by the second term in (3.23). Since the ACF of the rectangular pulse is zero outside the interval $[-T_b, T_b]$, it suffers interference from only one neighboring bit. On the other hand, depending on the roll-off factor α , using an RRC pulse can lead to interference from more than one neighboring bit.

In Figure 3.9, we plot the normalized $\mathbb{E}[d_0]$ from (3.23) as a function of τ for the different pulse shapes. The frequency offset is chosen $3R_b$; the desired bit is assumed to be a binary 1; and the surrounding two neighboring bits are both considered to be -1 . Note that the figure considers the worst case scenario, where each neighboring bit has an opposite polarity. For $\tau = 0$, the receiver captures the desired bit $b_0 = 1$, whereas for $\tau = T_b$, it captures the neighboring bit $b_1 = -1$. We further observe that the curves oscillate, due to the cosine term in (3.23). The oscillations increase for a large f_r , leading to an increased

Figure 3.8: Normalized ACF against normalized delays for a rectangular pulse and a root raised-cosine pulse.

sensitivity of N-FOM to the timing mismatch τ . For a large frequency offset, a small τ can potentially suppress the demodulated bit, as indicated by the zero crossings in the figure. Finally, it seems that the RRC pulse apparently has no added benefit over the rectangular pulse as far as sensitivity to synchronization errors is considered. However, it must be kept in mind that the suppression of bias terms for a rectangular pulse also entails conditions on ϕ_r .

3.3.2 Solution

Clearly, additional steps have to be taken to rectify the synchronization problem. Timing synchronization in IR-UWB has been widely studied, most notably by Yang [70], introducing the idea of using “dirty templates” for synchronization. The idea is similar to TR; the received UWB signal for one symbol is correlated with the neighboring symbol. The result gives a unique maximum for the correct synchronization time, $\tau = \tau_0$. The algorithm involves scanning the delay τ over one symbol period with an appropriate resolution. After finding the unique maximum, the correct symbol timing is fed back to the demodulation stage.

The mechanism behind the dirty template method is similar to TR. Therefore, one can implement the algorithm in N-FOM without compromising the receiver complexity. Goeckel studied a similar method for TR-UWB in [42],

Figure 3.9: Normalized magnitude of the demodulated bit ($b_0 = 1$, with neighboring bits of opposite polarity) as a function of the normalized synchronization offset τ/T_b , for a rectangular pulse and a root raised-cosine pulse, and $f_r = 3R_b$.

where he suggested to find the unique maximum from the variance of the decision variable. We propose to extract the delay τ from the mean of the decision variable (3.23), using a training sequence. If all ones are sent, i.e., $b_k = 1$ for all k , one can see from (3.23) that the maximum value will be at $\tau = 0$ and $\tau = T_b$ for $f_r = 1/T_b$. However, if f_r is a higher multiple of the symbol rate, then there will be no unique maximum in the interval $[0, T_b]$. This is depicted in Figure 3.10, which illustrates the normalized $\mathbb{E}[d_0]$ for $f_r = R_b$ (in dashed lines) and $f_r = 5R_b$ (in solid lines), with $b_k = 1$ for all k . On the other hand, if one sends a sequence with alternating polarity, i.e., $b_k = -b_{k-1}$, there is a unique maximum for all values of $f_r = n/T_b$, as seen from Figure 3.9. Scanning the parameter τ over one symbol period, one can find the unique maximum.

Note that, since the correlation is carried out over one symbol period, the synchronization algorithm is not affected by a fading channel, as long as the channel does not vary over a symbol period. Secondly, the training sequence does not need to be long. For one trial value of τ , a training sequence of two symbols is sufficient, as indicated by Figure 3.9. Although we have provided a concept of the timing synchronization in N-FOM, it is not investigated in detail. The goal was to show that one can correct for timing errors with simple methods, without increasing the receiver complexity. For the rest of the thesis, perfect synchronization will be assumed.

Figure 3.10: Normalized magnitude of the demodulated bit, with $b_k = 1 \forall k$, as a function of the normalized synchronization offset τ/T_b , for $f_r = R_b$ (in dashed lines) and $f_r = 5R_b$ (in solid lines).

3.4 Conclusions

For an N-FOM point-to-point link in an AWGN channel, we studied fundamental implementation considerations of the system. The use of pure noise and PN carriers were briefly discussed. The instantaneous power of the pure noise is random and fluctuates highly with respect to its mean. As a result, the self-mixing of the signal adds to the noise. Due to their fixed instantaneous power, PN carriers with rectangular pulses do not cause self-interference after the mixing operation. Thus it has an advantage over pure noise carriers, although a desired spectral shape can be difficult to achieve.

Further, the relation between the PSD of the pure noise carrier and the required RFE filter was explored. It was found that for a fixed set of system parameters, the brick-wall shape for both the signal's PSD and the transfer function of the RFE filter is ideal. However, for a non-rectangular spectral shape, the choice of the RFE filter is not straightforward in an N-FOM communication system. The expression for an optimal filter was derived and it was observed that the optimal filter depends on the received signal strength and noise levels. In the low-CNR regime, the total noise is dominated by the AWGN. Therefore the transfer function of the optimal filter is matched to the spectral shape of the spreading signal. In the high-CNR regime, the total noise is dominated by the signal self-mixing, resulting in a BER floor; hence, better performance can be achieved by increasing the spreading factor. Therefore, the optimal filter in the high-CNR regime is similar to an inverse filter, which increases the bandwidth (and hence the spreading factor) of the filtered signal.

A suboptimal solution for the filter was also suggested, which is practically more feasible than the optimal filter. The suggested filter is close to optimal for low-to-moderate CNRs. In the high-CNR regime, it is close to optimal if the spectral shape closely resembles a rectangular shape. For a carrier signal shaped by a Butterworth filter of order 1, the maximum achievable performance using a suboptimal filter has a penalty of 1.89 dB, compared to the maximum achievable performance using a rectangular PSD.

Finally, we studied the timing synchronization in N-FOM. The effect of the synchronization mismatch is manifested in the ACF of the used pulse shape. It was found that the N-FOM link performance is sensitive to timing errors, particularly for large values of the frequency offset. With rectangular pulses, and the frequency offset chosen three times the symbol rate, a timing error one tenth of the symbol time can suppress the magnitude of the demodulated bit by more than 50%. It was observed that for a pair of neighboring bits with opposite polarity, the decision variable has a unique maximum which corresponds to the correct timing. A simple timing synchronization algorithm was suggested based on this observation.

The study about the N-FOM mechanism, which began with fundamental analysis in Chapter 2, culminates in the implementation considerations with this chapter. In the next three chapters, the N-FOM system will be considered in different scenarios, namely, the LOS multiple-access environment, the LOS

interference-rich environment, and the NLOS dense multipath fading environment.

Multiple access and scalability¹

In monitoring applications, a large or small number of energy-efficient sensor nodes are scattered in a geographical area. The nodes work in tandem to transport measurements data from one point to another, and finally to a final management node, as depicted in Figure 4.1. With effective routing protocols, the information is reliably carried around the region via multihop transmissions. If a node is faulty or if a particular communication link has a poor quality, alternative routes can be taken to traverse the region. The reliability of a WSN is enhanced by having redundant nodes. Redundancy improves consistency of the measured data and boosts tolerance of the WSNs to node failures.

Figure 4.1: Fire monitoring WSN with a faulty node.

One aspect of a WSN is that at any given time, multiple communication links are established in the network. In other words, multiple nodes are simultaneously accessing the wireless channel. To enable MA in PHY, different

¹The main results in this chapter were published in [71].

strategies are adopted; for example in frequency division MA (FDMA), different frequency bands are allotted to the simultaneous links. A receiver node distinguishes between signals from different links by knowledge of the allotted frequency band. In DSSS systems, all links share the same frequency band; MA is enabled by using a unique spreading sequence for each link. A receiver node distinguishes between the different signals by knowledge of the unique spreading sequence associated with each link. N-FOM, on the other hand, is a noncoherent SS system, where the receiver node does not know the spreading sequence. In N-FOM, MA is enabled by using a unique frequency offset for each concurrent link [39].

Unlike in FDMA systems where users do not interfere with each other by virtue of using separate frequency bands, MUI is unavoidable in SS systems. Therefore, in WSNs using SS communication schemes, tolerance to MUI is essential. It determines whether the WSN is suitable for large scale implementation, where multiple nodes are communicating in an ad-hoc fashion. The higher the resistance to MUI, the larger the number of communication links that can be established simultaneously. Generally, coherent SS systems are tolerant to MUI. However, the same cannot be supposed about noncoherent SS techniques such as N-FOM. The scalability of N-FOM was partly addressed by Haartsen et al. in [39], where they considered a MA N-FOM system in an ideally power-controlled setup. In this chapter, we revise that model and consider a general setup. Particularly, we investigate the maximum MUI an N-FOM communication link can withstand, while still operating at an acceptable error rate.

The chapter is structured as follows. Section 4.1 begins with an overview of MA in N-FOM, where we study how multiple links can be established in an N-FOM network and what that means for the choice of the frequency offsets. Section 4.2 presents the MUI problem and provides the system model and performance analysis. The resulting performance trends are studied and discussed in Section 4.3. In the end, Section 4.4 briefly discusses MAC layer considerations, followed by conclusions in Section 4.5.

4.1 Multiple access mechanism

As discussed in Chapter 1, MA in N-FOM can be realized by using unique values of the frequency offset for different communication links. Without losing generality, let us consider two simultaneous communication links in the network as shown in Figure 4.2. The Tx1-Rx1 link is using a frequency offset f_1 , whereas the Tx2-Rx2 link is using $f_2 < f_1$. Perfect synchronization is assumed between each pair of communicating nodes, i.e., the Tx and Rx in a particular communication link are locked in frequency, phase and timing.

Let us focus on the Tx1-Rx1 link; the signal obtained by the receiver Rx1 is a linear combination of the desired signal from Tx1 and the undesired signal from Tx2. The detection mechanism is illustrated in Figure 4.3. The squaring operation at the receiver despreads both the desired and undesired signals. The

desired message signal, depicted in green, appears at f_1 , whereas the undesired message signal, depicted in red, appears at f_2 . Additionally, the self-mixing also gives rise to multiple bias terms, which are depicted in blue in the figure. Analytically, these terms can be written similarly to $\Xi(\cdot)$ in (2.20), which was discussed in Chapter 2. To detect the desired demodulated signal, the receiver shifts the outcome of the squarer by f_1 . This moves the desired signal to baseband, which can be recovered using an LPF. In this way, the receiver can extract the desired information. This analysis can be extended to any arbitrary number of concurrent links. Furthermore, a multipoint-to-point communication link is also possible. One can see from Figure 4.3 that if the receiver Rx1 wishes to decode the signal from Tx2, it can do so by either tuning the frequency of the LO to f_2 or by filtering the signal at f_2 . Given that the values of the corresponding frequency offsets are known, a single node can communicate with multiple users, or a transmitter can address a particular receiver.

Figure 4.2: Two simultaneous links in the network interfere with each other.

4.1.1 Conditions on frequency offset

For MA in N-FOM, the values of the frequency offsets must be carefully chosen. As discussed in Chapter 2, the suppression of the bias terms in the mean of the decision variable imposes a few conditions on the pulse shape and the frequency offset. If a polar NRZ message signal is chosen, then the frequency offsets (f_i) used in the network must be integer multiples of the symbol rate, i.e., $f_i = n/T_b$ for any positive integer n . For such a signaling scheme, the bias terms (depicted in blue) in Figure 4.3 are frequency-shifted impulse functions. One can notice from the figure, that for recovery of the desired signal, the frequency offset f_1 cannot be twice f_2 , i.e., $f_1 \neq 2f_2$. Violation of this condition will result in appearance of an undesired bias term along with the desired signal at baseband. As such, the received symbols will be displaced from its original constellation point by this bias term, which can significantly deter the link performance. This condition on frequency offset applies to all simultaneous (mutually interfering) links in the network. For N concurrent links in the network where each link is interfering with all other links, the condition $f_i = n/T_b \neq 2f_j$ must be satisfied²

²This rule has only one exception. The link with the lowest of the frequency-offsets can operate without experiencing crosstalk. It can be seen from Figure 4.3 that one can extract the signal at f_2 (by employing f_2 at the LO) even if $f_1 = 2f_2$.

Figure 4.3: Receiver operation in MA scenario. The insets depict narrowband signals (not to scale) before and after frequency-realignment at the receiver.

for all i and j .

On the other hand, if the message signal employs any arbitrary pulse shape with an absolute bandwidth B_p , then the suppression of the bias terms impose a different condition on f_1 and f_2 . In this case, ensuring $|f_1 - 2f_2| \geq 3B_p$ is *sufficient* to suppress the bias terms. This will be shown later in Section 4.2.2.

4.1.2 Usable frequency offsets

Theoretically, N-FOM has a large number of frequency offsets at its disposal. For a large geographic area, where two simultaneous links are operating without causing noticeable interference to each other, one can choose any value for f_i as long as the conditions mentioned in Section 2.2.2 are met, i.e., $f_i = n/T_b$ for a rectangular pulse, and $f_i \geq 3B_p$ for any general pulse shape with absolute bandwidth B_p .

For a rectangular pulse shape, $f_i = 1/T_b$ is the lower limit on the usable frequency offset, whereas $f_i \ll B_x$ imposes an upper limit. The upper limit is determined by the multipath environment; the frequency offset must be very small compared to the channel coherence bandwidth. Table 6.1 in Chapter 6 discusses this condition in detail. For now, let us assume a fixed upper limit on usable frequency offset and denote it by f_U . The maximum number of *available* frequency offsets is then $N_a = f_U T_b$. If we consider interference between the links, then we also have to impose the condition $f_i \neq 2f_j$. In that case, the maximum number of *usable* frequency offsets, N_u is

$$N_u = \sum_{n=1}^{n_{\max}} \left\lfloor \frac{N_a + 4^n}{2^{2n+1}} \right\rfloor + \left\lfloor \frac{N_a + 1}{2} \right\rfloor \approx 2N_a/3 \text{ (for a large } N_a). \quad (4.1)$$

where $\lfloor \frac{b}{a} \rfloor$ denotes floor operation, and $n_{\max} = \lfloor \log_4(N_a) \rfloor$. The expression (4.1) and the subsequent approximation is proved in Appendix C.1 by mathematical induction.

On the other hand, if any arbitrary pulse shape with an absolute bandwidth B_p is chosen, then computing the number of the usable frequency offsets is not straightforward. The upper limit on the usable frequency offset f_U remains the same. The lower limit is $f_i = 3B_p$. Two mutually interfering links with frequency offsets f_1 and f_2 should satisfy $|f_1 - 2f_2| \geq 3B_p$, such that the receiver can recover the information from a desired user, without crosstalk.

4.2 System model and analysis

As mentioned earlier, MUI is unavoidable in the SS system. On the other hand, coherent SS systems exhibit high resistance to MUI. In coherent systems, the receiver has knowledge of the spreading signal employed by a desired user. The spreading sequences used by different links in the MA are mutually orthogonal. To obtain the desired information, the receiver correlates the received waveform with the spreading signal associated with the particular link. The desired signal is despread, whereas all the undesired signals are mitigated which add to the noise level. Certainly, a large number of links in the MA will increase the overall noise level. This imposes a limit on the maximum number of links.

MUI hugely depends on the received signal strength from the interfering node. As illustrated in Figure 4.4, the nodes in an established link might be far away from each other. The signal transmitted by Tx1 will suffer a high propagation loss. In such a setup, an interferer Tx2 in the vicinity of the receiver can potentially inhibit the desired signal. The transmitter Tx1 has to significantly increase its transmission power to maintain a reliable link with Rx1. This is commonly referred to as the *near-far problem*, and is a frequently encountered phenomenon in SS networks.

In this section, we present a model of an MA N-FOM network. We first analyze and derive the performance metrics for two concurrent links in the network. This is later on generalized for any arbitrary number of mutually interfering links.

Figure 4.4: Near-far problem; a strong interferer clouds the desired signal.

4.2.1 System model

Consider a MA N-FOM communication system shown in Figure 4.5, where a desired signal from Transmitter 1 is interfered by an undesired signal from Transmitter 2. Each transmitting user locally generates a bandpass spreading signal with a complex envelope $\tilde{x}_i(t)$ where $i \in \{1, 2\}$ in this example. The spreading signals are modeled as independent zero-mean WSS Gaussian processes. The complex envelope of each spreading signal has a PSD

$$S_{\tilde{x}_i}(f) = \frac{2P_{x_i}}{B_x} \mathcal{S}\left(\frac{f}{B_x}\right), \quad (4.2)$$

where P_{x_i} is the mean power in the bandpass signal of User i ; B_x is the bandwidth; $\mathcal{S}(\nu)$ is the spectral shape which satisfies the criteria $\int \mathcal{S}(\nu) d\nu = \int \mathcal{S}^2(\nu) d\nu = 1$. The message signal $m_i(t)$ of each user is modeled as

$$m_i(t) = \sum_k b_{k,i} p(t - kT_b), \quad (4.3)$$

where $p(t)$ is any arbitrary pulse with energy T_b ; $\{b_{k,i}\}$ is a binary bit sequence of user i , taking values $\{+1, -1\}$ with equal probability. The pulse shape is chosen such that it satisfies the zero-ISI criterion, as discussed in Section 2.2.3. The frequency offsets used by transmitter 1 and 2 are f_1 and f_2 respectively; and both are assumed to satisfy the conditions mentioned in Section 4.1.1. The signal transmitted by each node i has a complex envelope

$$\tilde{y}_i(t) = \tilde{x}_i(t) \left(m_i(t) \cos(2\pi f_i t + \phi_i) + \frac{1}{\sqrt{2}} \right) =: \tilde{x}_i(t) a_i(t), \text{ for } i = 1, 2. \quad (4.4)$$

where $a_i(t)$ denotes the narrowband part of the signal, and is given by

$$a_i(t) = m_i(t) \cos(2\pi f_i t + \phi_i) + \frac{1}{\sqrt{2}}. \quad (4.5)$$

The wireless links between the transmitting nodes and the receiving node is modeled as flat-fading additive white Gaussian noise (AWGN) channels. The receiver obtains a linear combination of the transmitted signals, i.e.,

$$\tilde{r}(t) = \frac{1}{2} \tilde{g}(t) \otimes \left(\tilde{y}_1(t - \tau_1) / \sqrt{L_1} + \tilde{y}_2(t - \tau_2) / \sqrt{L_2} \right) + \frac{1}{2} \tilde{g}(t) \otimes \tilde{n}(t),$$

where $\tilde{g}(t)$ is the baseband equivalent impulse response of the front-end LTI filter; τ_i is transmission delay associated with each link; $\tilde{n}(t)$ is the baseband equivalent of the noise, independent from $\tilde{x}_i(t)$; and $\{L_1, L_2\}$ represent propagation losses experienced by the signals from Node 1 and Node 2 respectively. Under the assumption $f_i \ll B_x$, the transmitted signal $\tilde{y}_i(t)$ has approximately the same bandwidth as that of the spreading signal. For such a case, we use the key results regarding suboptimal filtering in Section 3.2.3. We use an RFE

Figure 4.5: A MA N-FOM system with two concurrent links in the network.

filter which ensures $|\tilde{G}(f)|^2 = 4\mathcal{S}(f/B_x)$, i.e., the transfer function of the filter is matched to the spectral shape of the spreading signal. Furthermore, since the filter is wideband, one can consider the NB function $a_i(t)$ to be almost constant over the filter impulse response. In other words, one can move the NB function out of the convolution $y(t) \otimes g(t)$. The above equation can be rewritten as

$$\tilde{r}(t) = \sum_{i=1}^2 \tilde{x}_{g,i}(t - \tau_i) a_i(t - \tau_i) / \sqrt{L_i} + \tilde{n}_g(t), \quad (4.6)$$

where we have defined $\{\tilde{x}_{g,i}(t), \tilde{n}_g(t)\}$ as the baseband equivalents of the filtered spreading signal and the filtered noise, analogous to the definitions in (2.9). The input of the detection filter, we have

$$z(t) = r^2(t) \cos(2\pi f_1 t + \phi_r) = \frac{1}{2} |\tilde{r}(t)|^2 \cos(2\pi f_1 t + \phi_r) + \text{h.f.c.}, \quad (4.7)$$

where the second term in the last equation represents high frequency components. The frequency re-aligning at the receiver moves the despread information signal to baseband, where it is recovered by a detection filter. The impulse response of the detection filter is matched to the pulse shape $p(t)$. The output of the matched filter is sampled, which gives us the decision sample

$$d_k = [z(t) \otimes p(T_s - t)]_{t=T_s+kT_b} = \int_{-\infty}^{\infty} z(u) p(u - kT_b) du. \quad (4.8)$$

The decision sample is passed through a comparator, resulting in a bit estimate $\hat{b}_{k,1}$. In traditional MA SS communication, signals from the communicating nodes raise the noise level. This is inherently different than N-FOM. The

squaring operation at the receiver produces multiple cross-terms; the noise level is raised by the self- and cross-mixing products of the signals from the communicating nodes. The strength of these mixing products varies differently with system parameters. In this sense, the near-far problem has a different effect on the N-FOM system compared to the standard SS system.

4.2.2 Performance analysis

We now derive a closed-form expression for the SNR of user 1 in the MA network shown in Figure 4.5. Without loss of generality, the derivation is carried out for the data index zero, i.e., the desired information bit is $b_{0,1}$. Furthermore, since the bandwidth of the pulse is much smaller than RF, the h.f.c. in (4.7) will be suppressed by the filter. Therefore, we ignore this term for further analysis. Moreover, using the properties of WSS processes, the signals $\{\tilde{x}_{g,i}(t), \tilde{n}_g(t)\}$ have PSDs and mean powers

$$S_{\tilde{x}_{g,i}}(f) = \left(\frac{2P_{x_i}}{B_x}\right) \mathcal{S}^2\left(\frac{f}{B_x}\right), \quad S_{\tilde{n}_g}(f) = 2N_0 \mathcal{S}\left(\frac{f}{B_x}\right) \quad (4.9a)$$

$$P_{x_{g,i}} := \frac{1}{2} \mathbb{E} [|\tilde{x}_{g,i}(t)|^2] = P_{x_i}, \quad P_{n_g} := \frac{1}{2} \mathbb{E} [|\tilde{n}_g(t)|^2] = 2N_0 B_x, \quad (4.9b)$$

where we have used the properties of $\mathcal{S}(\nu)$ from (3.11).

The input to the detection filter can be written as a linear combination of signal-cross-signal, MUI-cross-MUI, signal-cross-MUI, signal-cross-noise, MUI-cross-noise, and noise-cross-noise mixing products. Substituting (4.6) in (4.7), we define the mixing products

$$z_{y_i}(t) := \frac{1}{2L_i} |\tilde{x}_{g,i}(t - \tau_i)|^2 a_i^2(t - \tau_i) \cos(2\pi f_1 t + \phi_r), \quad (4.10a)$$

$$\begin{aligned} z_{y_1 y_2}(t) &:= \frac{1}{2\sqrt{L_1 L_2}} \left[\tilde{x}_{g,1}(t - \tau_1) \tilde{x}_{g,2}^*(t - \tau_2) + \tilde{x}_{g,2}^*(t - \tau_2) \tilde{x}_{g,1}(t - \tau_1) \right] \\ &\quad \cdot a_1(t - \tau_1) a_2(t - \tau_1) \cos(2\pi f_1 t + \phi_r), \end{aligned} \quad (4.10b)$$

$$\begin{aligned} z_{y_i n}(t) &:= \frac{1}{2\sqrt{L_i}} \left[\tilde{x}_{g,i}^*(t - \tau_i) \tilde{n}_g(t) + \tilde{x}_{g,i}(t - \tau_i) \tilde{n}_g^*(t) \right] \\ &\quad \cdot a_i(t - \tau_i) \cos(2\pi f_1 t + \phi_r), \end{aligned} \quad (4.10c)$$

$$z_n(t) := \frac{1}{2} |\tilde{n}_g(t)|^2 \cos(2\pi f_1 t + \phi_r), \quad (4.10d)$$

for $i = 1, 2$. Using these definitions, (4.7) is written as

$$z(t) = z_{y_1}(t) + z_{y_2}(t) + z_{y_1 y_2}(t) + z_{y_1 n}(t) + z_{y_2 n}(t) + z_n(t). \quad (4.11)$$

Before proceeding further, we assume that the receiver is synchronized³ to the desired user. This means that the receiver has acquired and compensated for the delay τ_1 ; furthermore, the phase of the receiver oscillator is assumed to be in phase with that of the LO at Node 1. Therefore, for further analysis, we set $\tau_1 = 0$, and $\phi_r = \phi_1$.

³A synchronization solution in N-FOM was discussed in Section 3.3.

Statistics of the decision variable

In line with the discussion in Section 2.2.1, we invoke the CLT and consider the decision variable to be a Gaussian distributed random variable. Therefore, it is sufficient to analyze the mean and variance of the decision variable. The mean of the decision variable is

$$\mathbb{E}[d_{0,1}] = \int_{-\infty}^{\infty} \mathbb{E}[z(u)] p(u) du. \quad (4.12)$$

We can solve the integration by separately calculating the contribution of each term in (4.10). Using (4.10)a and (4.12), the desired information in $z_{y_1}(t)$ is extracted as

$$\begin{aligned} \int_{-\infty}^{\infty} \mathbb{E}[z_{y_1}(u)] p(u) du &= \frac{1}{2L_1} \int_{-\infty}^{\infty} \mathbb{E}[|\tilde{x}_{g,1}(u)|^2] a_1^2(u) \cos(2\pi f_1 u + \phi_1) p(u) du \\ &= \frac{1}{L_1} P_{x_1} \int_{-\infty}^{\infty} a_1^2(u) \cos(2\pi f_1 u + \phi_1) du = \frac{1}{\sqrt{2}L_1} b_{0,1} P_{x_1} T_b, \end{aligned} \quad (4.13)$$

where the second last equality follows from (4.9b), and the last equality follow similar analysis as that in Section 2.2.1, given that the conditions on f_1 and $p(t)$ stated in Section 2.2.3 are satisfied.

The undesired MUI self-mixing product $z_{y_2}(t)$ contains narrowband terms at different frequencies, as illustrated earlier in Figure 4.3. Ideally, we would like these terms to be suppressed by the detection filter. However, the contribution of these narrowband terms to the mean of the decision variable is not necessarily zero, and thus results in crosstalk. This crosstalk is defined as

$$\chi := \int_{-\infty}^{\infty} \mathbb{E}[z_{y_2}(t)] dt. \quad (4.14)$$

The mechanism behind the crosstalk is visualized in Figure 4.6a. The detection filter captures part of the despread signal from the undesired user. Due to the delay τ_2 , the desired and undesired signals are not orthogonal in the integration interval. There is no crosstalk, if the neighboring bits of User 2 in the integration interval have the same polarity, or if the signals from User 1 and 2 are aligned in time, e.g., $\tau_2 = \tau_1 = 0$. However, since these two conditions cannot be guaranteed, there will be crosstalk, and we have to find suitable design choices that is sufficient to suppress or minimize χ .

It is shown in Appendix C.2 that χ can be modeled as a bias term. The crosstalk depends on the frequency offsets used by the different links, as well as the signaling scheme of the communication system. In the appendix, the crosstalk is analyzed in the context of two signaling schemes. For a generic signaling scheme that employs a pulse $p(t)$ with an absolute bandwidth B_p , it is shown that for the crosstalk-free reception, it is sufficient to choose the frequency offsets such that $|f_1 - 2f_2| \geq 3B_p$ and $f_L := f_1 - f_2 \geq 2B_p$. For a polar NRZ signaling scheme, note that the transfer function of the pulse

(a) Bit transition of an undesired user over the filter integration period.

(b) Normalized magnitude of crosstalk for different values of f_L .

Figure 4.6: Depiction of crosstalk in a MA scenario a) mechanism and b) effect.

decays slowly with frequency. In this case, the crosstalk is approximately zero if the frequency offsets are chosen such that $f_L = f_1 - f_2 \gg 1/T_b$.

For a polar NRZ signaling scheme, the expression for the crosstalk is given in (C.2). In Figure 4.6b, we plot the magnitude of the crosstalk (C.2) against transmission delay of User 2 (τ_2), for different values of f_L . Here, R_b is the data rate; $\tau_1 \neq \tau_2$; $b_{2,k-1} \neq b_{2,k}$; and $f_2 = 3R_b$. The magnitude of the crosstalk is normalized to the received bit energy ($E_{b,2}$) of User 2. The larger the $E_{b,1}$ compared to $E_{b,2}$, the smaller the impact of the crosstalk. Nonetheless, it causes nuisance. However, as seen from the graph, the crosstalk is negligible if one chooses the frequency offsets such that $|f_i - f_j| \gg 1/T_b$ for a polar NRZ signaling scheme. The difference does not necessarily has to be very large. However, larger frequency differences are preferred for a crosstalk-free reception. Introducing this new criterion reduces the number of usable frequency offsets given in (4.1), and hence limits the scalability of the system. If we set the criterion⁴ $f_i - f_j \geq 4/T_b$, then it can be shown that the number of usable frequency offsets is $N_{\bar{u}} = \lfloor (N_a + 3)/4 \rfloor$. For further analysis, we assume that the frequency offsets are chosen such that the bias term χ is suppressed.

Since $\{x_1(t), x_2(t), n(t)\}$ are mutually independent and zero mean processes, the wideband terms $\{z_{y_1 y_2}(t), z_{y_1 n}(t), z_{y_2 n}(t)\}$ have no contribution to the mean of the decision variable. Moreover, following similar analysis as in Section 2.2.1, one can show that the noise self-mixing product $z_{nn}(t)$ also has no mean contribution, as long as the conditions on the pulse shape and the frequency offset f_1 are satisfied. Therefore, the mean of the decision variable, conditioned on data bit $b_{0,1}$, follows from (4.13), and is given by

$$\mathbb{E}[d_{0,1}] = \int_{T_b} \mathbb{E}[z_{y_1}(t)|b_{0,1}] dt + \chi \approx \frac{1}{\sqrt{2}L_1} b_{0,1} P_{x_1} T_b. \quad (4.15)$$

⁴With this criterion, the crosstalk from a secondary user has a magnitude of less than or equal to $\gamma_2 - 10$ dB.

The variance of the overall noise in the MA N-FOM communication is

$$\sigma_{d_{0,1}}^2 = \mathbb{E} \left[\left(d_{0,1} - \mathbb{E}[d_{0,1}|b_{0,1}] \right)^2 \right] = \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} C_{zz}(t_1, t_2) dt_1 dt_2,$$

where $C_{zz}(t_1, t_2)$ is the covariance function of $z(t)$. The covariance function contains multiple cross-covariance and auto-covariance terms. However, due to the mutual independence of $\{\tilde{x}_1(t), \tilde{x}_2(t), \tilde{n}(t)\}$, one can prove that $C_{zz}(t_1, t_2)$ is the sum of the auto-covariance functions of each term in (4.10). Defining the noise contribution from any mixing product $z_{qs}(t)$ as Ψ_{qs} , one can write

$$\sigma_{d_{0,1}}^2 = \Psi_{y_1 y_1} + \Psi_{y_2 y_2} + \Psi_{y_1 y_2} + \Psi_{y_1 n} + \Psi_{y_2 n} + \Psi_{nn}.$$

The terms are analogous to those in Section 2.2.1. The terms $\{\Psi_{y_1 y_1}, \Psi_{y_2 y_2}\}$ are analogous to the noise contribution from the signal self-mixing product derived in Appendix A.1.2, whereas $\{\Psi_{y_1 n}, \Psi_{y_2 n}\}$ are analogous to noise contribution from the signal-noise mixing product, derived in A.1.3. Moreover, since $\{x_1(t), x_2(t)\}$ are bandlimited white noise processes, the term $\Psi_{y_1 y_2}$ follow similar analysis as that of a signal-noise mixing product. Furthermore, analogous to $\{\eta_1, \eta_2\}$ in (2.27), we define

$$\eta_{y_i} = \frac{1}{T_b} \int_{-\infty}^{\infty} a_i^4(t - \tau_i) \cos^2(2\pi f_1 t + \phi_1) dt \quad (4.16a)$$

$$\eta_{y_1 y_2} = \frac{1}{T_b} \int_{-\infty}^{\infty} a_1^2(t - \tau_1) a_2^2(t - \tau_2) \cos^2(2\pi f_1 t + \phi_1) dt \quad (4.16b)$$

$$\eta_{y_i n} = \frac{1}{T_b} \int_{-\infty}^{\infty} a_i^2(t - \tau_i) \cos^2(2\pi f_1 t + \phi_1) dt \quad (4.16c)$$

For NRZ signaling scheme, the terms $\{\eta_{y_1}, \eta_{y_1 n}\}$ are same as $\{\eta_1, \eta_2\}$ in (2.34) for $c=1/\sqrt{2}$. The terms $\{\eta_{y_2}, \eta_{y_1 y_2}, \eta_{y_2 n}\} = \{17/16, 5/4, 1\}$ can be calculated in a similar fashion. Using the definitions, the variance can be shown to be:

$$\begin{aligned} \sigma_{d_{0,1}}^2 &\approx (\eta_{y_1} E_{b,1}^2 + \eta_{y_2} E_{b,2}^2 + \eta_{y_1 y_2} E_{b,1} E_{b,2}) \lambda_2 / S \\ &\quad + 2(\eta_{y_1 n} E_{b,1} + \eta_{y_2 n} E_{b,2}) N_0 \lambda_1 + N_0^2 S / 2 \\ &= \left(25 E_{b,1}^2 + 17 E_{b,2}^2 + 20 E_{b,1} E_{b,2} \right) \frac{\lambda_2}{16S} + (5 E_{b,1} + 4 E_{b,2}) \frac{N_0 \lambda_1}{4} + \frac{N_0^2 S}{2}, \end{aligned} \quad (4.17)$$

where $\lambda_1 = \int \mathcal{S}^3(\nu) d\nu$ and $\lambda_2 = \int \mathcal{S}^4(\nu) d\nu$, and $E_{b,i}$ is the received bit energy of User i , given by

$$E_{b,i} = \frac{1}{L_i} \int_{-\infty}^{\infty} \mathbb{E}[y_i^2(t)] dt = \frac{P_{x_i} T_b}{L_i}, \quad (4.18)$$

which is derived similarly as that in (2.22). The expression in (4.17) represents the power of the total noise, experienced by an N-FOM receiver while detecting a desired user in the presence of one interfering user.

SNR of the decision variable

Using (4.15) and (4.17), the SNR of the decision variable of User 1 is

$$\text{SNR}_1 = \frac{\mathbb{E}^2[d_{0,1}]}{\sigma_{d_{0,1}|b_{0,1}}^2} = \frac{8\gamma_1^2}{(25\gamma_1^2 + 17\gamma_2^2 + 20\gamma_1\gamma_2)\lambda_2/S + (20\gamma_1 + 16\gamma_2)\lambda_1 + 8S}, \quad (4.19)$$

where $\gamma_i = E_{b,i}/N_0$ is the received bit SNR of User i , and $S = B_x T_b$ is the spreading factor. Compared to the total SNR in a point-to-point link (2.29), we see additional terms in the denominator of the above expression. The number additional terms quadratically increase with each additional interfering user; and the magnitude of these terms depends on the received bit SNR (γ_i) of the interferer. Therefore, the larger the number of interferers with a significant signal strength, the lower the total SNR. The parameters $\{\lambda_1, \lambda_2\}$ depend on the spectral shape of the spreading signal (see Table 3.1), and are both equal to one for a brick-wall PSD. If the information of User 2 is to be decoded, one can do so by choosing f_2 as frequency of the LO at the receiver. The resulting SNR of user 2 can be found from (4.19) by interchanging γ_1 and γ_2 , given that the receiver is synchronized to the the incoming bit stream of User 2.

For more than two transmitting nodes, the number of cross-terms increases quadratically. For a desired User 1 in N simultaneous links, the variance of the decision variable of the desired user can be written as

$$\sigma_{d_{0,1}|b_{0,1}}^2 = \sum_{i=1}^N \sum_{j=1}^N \Psi_{y_i y_j} + \sum_{i=1}^N \Psi_{y_i n} + \Psi_{nn}, \quad (4.20)$$

where the first and the second term, respectively represent summation of noise from all cross terms of the form $z_{y_i y_j}(t)$ and $z_{y_i n}(t)$ for all i and j . Compared with the variance (4.17) for an MA system with one interfering link, the above expression has a higher number of noise terms. A main difference between the two expressions is that (4.20) also includes noise from cross-products of signals of the undesired users, i.e., noise from the terms $z_{y_i y_j}(t)$ for $i, j \neq 1$ and $i \neq j$. Combining (4.15) with (4.20), the SNR of User 1 in N simultaneous links is

$$\text{SNR}_1 = \frac{\mathbb{E}^2[d_{0,1}|b_{0,1}]}{\sigma_{d_{0,1}|b_{0,1}}^2} := \left(\text{SNR}_{yy}^{-1} + \text{SNR}_{yn}^{-1} + \text{SNR}_{nn}^{-1} \right)^{-1}, \quad (4.21)$$

where we have defined

$$\text{SNR}_{yy} = \frac{8S}{25\lambda_2} + \sum_{i=2}^N \frac{2S\gamma_1}{5\lambda_2\gamma_i} + \sum_{i=2}^N \frac{8S\gamma_1^2}{17\lambda_1\gamma_i^2} + \sum_{i=2}^N \sum_{j \neq i, j \neq 1}^N \frac{\gamma_1^2 S}{\lambda_2 \gamma_i \gamma_j}, \quad (4.22a)$$

$$\text{SNR}_{yn} = \frac{2S\gamma_1}{5\lambda_2\gamma_i} + \sum_{i=2}^N \frac{\gamma_1^2}{2\gamma_i\lambda_1}. \quad (4.22b)$$

We have written SNR_1 in terms of multiple SNR expressions in order to gain insight. The expressions show how the SNR of User 1 is affected by MUI.

Analogous to the discussion related to Figure 2.3 in Chapter 2, we can say that SNR_1 will always follow the lowest of the SNR functions in (4.22). Having multiple interfering links can have a compounded effect. The SNR depends on the magnitude of not only the products of the desired and undesired signals, but also on that of all the undesired signals with each other. In order to achieve a certain BER for User 1, one has to increase γ_1 , such that $\gamma_1^2/(\gamma_i\gamma_j)$ is large for any i and j .

Furthermore, one can see that choosing $\gamma_i = 0$ in (4.22) for all $i \geq 2$, and substituting the result in (4.21), we get the result for a point-to-point communication link (2.37). On the other hand, the performance of a coherent DSSS MA system [62] is

$$\text{SNR}_1^{\text{ss}} = 2\gamma_1 / \left(1 + 2 \sum_{i=2}^N \gamma_i / S \right). \quad (4.23)$$

Evidently, the tolerance of coherent DSSS to MUI is much higher than that of the N-FOM system. In N-FOM, the receiver mixes the undesired signals with themselves as well as with each other, which increases the overall noise. It is expected that an N-FOM link will not be able to tolerate many concurrent links. Furthermore, similar to the SNR expression of a point-to-point N-FOM link, the SNR in an MA scenario is not a monotonic function of S . As observed from (4.21), an optimum S exists which maximizes the SNR. By finding the minima of (4.21) with respect to S , we derive

$$S_{\text{opt}} = \sqrt{\left(\frac{25}{8}\gamma_1^2 + \frac{17}{8} \sum_{i=2}^N \gamma_i^2 + 2 \sum_{i=2}^{N-1} \sum_{j=i+1}^N \gamma_i \gamma_j + \frac{5}{2}\gamma_1 \sum_{i=2}^N \gamma_i \right) \lambda_2}, \quad (4.24)$$

as the optimal spreading factor in an MA scenario. By substituting $S = S_{\text{opt}}$ for each value of γ_i in (4.21), we get

$$\text{SNR}_{\text{max}} = 2\gamma^2 / \Phi, \quad \text{where} \quad (4.25)$$

$$\Phi = \sqrt{\left(50\gamma_1^2 + 34 \sum_{i=2}^N \gamma_i^2 + 40\gamma_1 \sum_{i=2}^N \gamma_i + 32 \sum_{i=2}^{N-1} \sum_{j=i+1}^N \gamma_i \gamma_j \right) \lambda_2 + \left(5\gamma_1 + 4 \sum_{i=2}^N \gamma_i \right) \lambda_1},$$

which is the maximum SNR for a given link in a MA N-FOM network, with $N - 1$ multi-user interferers. It can be verified that SNR_{max} in the above expression solves to (3.20), if one substitutes $\gamma_i = 0$ for all $i \geq 2$.

Since the decision variable is Gaussian distributed, and the message signal is antipodal, the BER can be shown to be

$$\text{BER}_1 = Q\left(\sqrt{\text{SNR}_1}\right) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\text{SNR}_1}}^{\infty} \exp\left(-\frac{u^2}{2}\right) du, \quad (4.26)$$

where $Q(x)$ is the Gaussian tail probability.

4.3 Results and discussions

In the last section, we carried out the performance analysis of an MA N-FOM communication system. We derived a close-form expression for the SNR of a particular user, subject to MUI posed by other users. In this section, we study numerical examples and supplement them with simulation results, to understand the effect of the MUI in an N-FOM communication network. For all the subsequent analysis, the message signal is modeled as polar NRZ, and the frequency offsets are chosen such that they satisfy the conditions mentioned in Section 4.2.2. Finally, the simulation model is unchanged as compared to that discussed in Section 2.3.

We consider the scenario in which the receiver is simultaneously receiving signals from two transmitting users, as earlier depicted in Figure 4.4. Transmitter 1 is the intended user, whereas Transmitter 2 is the interferer. In Figure 4.7, we fix $S=200$ and plot the BER of User 1 as a function of γ_1 for different values of the received bit SNR of User 2. As a reference we also plot the BER of an N-FOM system in a single-user point-to-point link, which can be obtained by substituting $\gamma_2=0$ in (4.19).

The graph clearly shows the effect of MUI. The error rate of User 1 deteriorates as the strength of the signal received from Transmitter 2 increases. A strong signal from User 2 masks the weak signal from User 1, making it

Figure 4.7: BER of a desired user 1 as a function of the received SNR per bit of User 1 (γ_1), for different values of γ_2 and a fixed $S = 200$, in an N-FOM MA link.

difficult for the receiver to recover the desired information. The steep performance degradation is attributed to the elevated noise levels (resulting from the mixing operation) endured by the N-FOM detection mechanism. Ultimately, one needs to increase the transmit power of User 1 (and hence γ_1) to achieve an improved error rate. We also see a slight difference between simulated and theoretical values of the performance metric. This phenomenon, as discussed in Chapter 2, is due to the loose CLT approximation. Nonetheless, in the region of interest [$10^{-3} \leq \text{BER}_1 \leq 10^{-6}$], the difference between the theoretical and simulation results for the required γ_1 is less than 0.5 dB.

Alternatively, if it is desired to decode the signal from Transmitter 2, then the signal from Transmitter 1 will cause interference. If one wishes to establish a multipoint-to-point link, i.e., the receiver is communicating with both User 1 and User 2, then the nearby node (with a strong signal strength) will lead to the near-far problem, and as such, additional measures have to be taken to reduce or prevent the problem. One method is to reduce the transmit power of the nearby node (strong user) such that the receiver can listen to the signals from both users. This is called adaptive power control (APC) and is frequently implemented in coded division MA systems. The objective is to achieve an equilibrium state where the received energies of the signals from each user are almost the same.

In the following discussion, we consider an MA system where a receiver is simultaneously communicating with multiple nodes at the same time. We further assume that APC is implemented, such that the received SNR-per-bit of each user is the same, i.e., $\gamma_i = \gamma$ for all i . Accordingly, the error rate of each user is also the same, i.e., $\text{BER}_i = \text{BER}$. In Figure 4.8, we fix $S = 200$ and plot the BER as a function of γ . In this multipoint-to-point link, the receiver aims to extract information from each transmitting node, and, hence, we have implicitly assumed that the receiver is synchronized to each user.

The figure shows that an adequate performance can be achieved if there is a small number of concurrent links in the network. However, for more than three users, the cross-mixing at the receiver results in an increased overall noise at the input of the detection filter. Consequently, achieving an acceptable link quality (10^{-3}) demands an excessively high γ . This result was previously analyzed and discussed in [39]. Another observation from the figure is that the difference between the simulation and the theoretical results is low for a high number of interfering users. This can be explained with the help of two important points from our discussion in Section 2.3. Recall that the theoretical results are less consistent with the simulation results in the high-CNR regime (i.e., $\gamma/S \gg 0.56$). Secondly, the moderate-CNR regime is determined by S_{opt} , which corresponds to the value of S for which there is no dominant noise component. Since the optimal spreading factor S_{opt} in (4.24) is different for each additional interferer, the CNR regimes are also different. The simulation curves for $N = 3$ and $N = 4$ in the figure only span low-to-moderate CNR regimes, therefore we do not see substantial difference between simulation and theoretical results.

We now compare the performance of a MA N-FOM link with that of a MA

Figure 4.8: BER of N concurrent links in a network, where transmit power of each transmitter is adaptively controlled, for a fixed $S = 200$.

DSSS link that employs a coherent receiver (4.23). As in previous graph, we assume multipoint-to-point links in a scenario where APC is implemented. For $S=200$, we plot the BER for N simultaneous links in Figure 4.9. As expected, the figure shows a large difference between the performance of the two systems which increases with the number of users in the MA. This is because unlike the N-FOM receiver, the traditional DSSS receiver does not produce multiple cross-terms and hence has a much lower noise level. It is clearly evident from (4.23), which has fewer terms in the denominator compared to that of (4.21). In the coherent DSSS receiver, the number of noise terms scale with N , whereas in N-FOM, it scales with N^2 , thus resulting in a huge performance difference as N increases.

4.4 MA and channel access protocols

From the results so far, we have established that N-FOM is not highly scalable. Due to the high sensitivity of the receiver to MUI, only a modest number (~ 3) of simultaneous links can be established, even with perfect power control. Since N-FOM is aimed at ad-hoc networks, APC is extremely difficult to implement. Therefore, it is pertinent to avoid many simultaneous links in a given time slot. In this context, valuable contributions were made in the medium access control (MAC) track of the WALNUT project [72], which we summarize in this section.

Figure 4.9: BER comparison of N-FOM and a coherent DSSS system for N concurrent links in a MA scenario, assuming perfect power control and a fixed $S = 200$.

Medium access control (MAC) is at the heart of every communication network. It specifies the model of networking; it defines how and when should the nodes access the channel, when to sleep or wake up, and how to establish a wireless link. The N-FOM system has unique challenges and opportunities. It offers fast link establishment owing to the low synchronization times, but also risks performance degradation due to a noisy detection mechanism. Parallel to the PHY track of the project, Morshed designed a new MAC protocol, called the Transmit-Reference MAC (TR-MAC), for an N-FOM network [73]. The idea was to exploit the low synchronization times of N-FOM to reduce the energy consumption at the link establishment stage. With the help of a detailed mathematical model, it was shown that the suggested TR-MAC consumes little energy, and it was further optimized to ensure that energy-efficiency is maintained in varying data rate.

From the perspective of MA in N-FOM, efficient channel access schemes are essential for a decent performance of a link, particularly when considering MUI from simultaneous links. Considering the number of tolerable interfering links and the maximum number of usable frequency offsets, Morshed et al. proposed a channel access model for N-FOM. The idea was to ensure minimum simultaneous links in a given time slot. It was found that the expected throughput increases with the pool size of the usable frequency offsets. With a sizable

the pool (~ 25) of usable frequency offsets, more than 100 links can be established in a certain time-frame [72, Sec. 5.4]. It was concluded that TR-MAC is suitable for networks with a few devices with moderate traffic load, or many devices with very low traffic load, as long as the overall offered traffic is low.

4.5 Conclusions

We studied the N-FOM communication system in an MA scenario, where multiple users are communicating in the network. To enable MA in an N-FOM network, each link employs a different frequency offset. In order to ensure a crosstalk-free reception, the values of the frequency offsets have to satisfy certain conditions, discussed in Section 4.2.2. In summary, the crosstalk is negligible or suppressed altogether, if the difference between the offsets is much larger than the bandwidth of the pulse. These conditions impose a limit on the number of usable frequency offsets. For an NRZ signaling scheme, this was investigated and an expression for the maximum number of usable offsets was derived.

We further investigated the link performance in face of interference posed by other links in the MA. An expression of the BER was derived and numerical examples, supplemented by simulations, were discussed. The results reveal that the MUI critically limits the performance of a link. Each additional interfering link in the network enhances the total noise substantially, especially in comparison with a traditional DSSS system. This acute degradation is mainly attributed to the operation of the self-correlation receiver which produces multiple noise terms. If the nodes in an established link are far away from each other, then an interfering node in close proximity of the receiver can potentially prevent the communication. An adaptive power control can help counter this near-far problem in a multipoint-to-point link. However, even with perfect APC implementation, the number of simultaneous links in a MA is limited to less than four. One concludes from the analysis that N-FOM is ill-suited to networks with many concurrent transmissions, since the N-FOM link suffers greatly from MUI. The user scenario where N-FOM could be used, is a network where the transmissions are sporadic and the nodes are sparsely located.

This chapter considered interference from users in the N-FOM network. In the next chapter, we will consider how external interference affects the overall link performance, and how can the system be made more robust to its effects.

Interference and coexistence¹

Typically, SS communication schemes employ large spreading bandwidths. As discussed in Chapter 1, larger bandwidths in SS systems not only improve immunity to frequency-selective fading but also to interference. A large operational bandwidth demands a significant chunk of the scarce radio frequency spectrum. One may use the unlicensed bands which comprise of chunks of frequency spectrum, reserved by the International Telecommunication Union (ITU), to be used unlicensed for industrial, scientific and medical (ISM) purposes. The most notable ISM bands are 915 MHz, 2.45 GHz and 5.8 GHz bands.

Operation in an ISM band implies that signals from other devices in the band will pose interference. Coherent SS systems are generally robust to in-band interference. By using a high spreading factors, the effect of any interferer is diminished by a factor comparative to the spreading gain. This was illustrated in Figure 1.4 in Chapter 1. On the other hand, the effect of interference on TR systems is different. The self-correlation operation at the N-FOM receiver will create multiple cross-terms of interference; which can be detrimental to communication. In this chapter, we aim to gain insight into the effect of in-band interferers on the performance of an N-FOM system. We begin with an overview of potential interferers and theoretical background in Section 5.1. Section 5.2 presents the system model for an in-band interferer in an N-FOM communication link. Next, the analysis is provided in Section 5.3, followed by key results and performance trends in Section 5.4. A brief discussion on interference mitigation is provided in Section 5.5. In the end, Section 5.6 summarizes the main findings and results.

5.1 Background

A large number of industrial and home appliances operate in the ISM bands. Applications such as weather stations, RFID tags, navy surveillance radar and home automation systems use the 915 MHz band [76]. In some regions, the

¹The contents of this chapter were published in [74] and [75].

900 MHz band is licensed and is used for 2nd generation cellular services. In 1985, the 2.45 GHz band was opened up for the public and has been one of the most popular ISM bands. Most notable devices such as baby monitors, toys, microwave ovens, peripheral devices, cordless phones, Bluetooth and WiFi devices, all operate in the 2.4 GHz ISM band. Later in the 1990s, the ISM 5.8 GHz band was also made available for public communication. It allowed for transmission of higher data rates, albeit at the cost of reduced transmission range. Since all devices in the unlicensed bands share the frequency spectrum, any device operating in the ISM band has to tolerate potentially harmful interference from all neighboring devices. In order to reduce this interference, a restriction on maximum allowed transmission power and the type of modulation scheme was placed by the ITU. Particularly, spread spectrum must be used for operation at 2.45 GHz and 5.8 GHz, whereas in the 900 MHz band, there is a limitation on maximum transmit power of 1 mW for non-SS and 1 W for SS modulation [77]. Table 5.1 lists notable applications in different ISM bands, along with the restriction on their effective isotropic radiated power (EIRP).

In-band interferers can be classified in a number of ways. With respect to bandwidth of the interferer and the desired signal, it is either narrowband interference (NBI) or wideband interference (WBI). As the name suggests, NBI refers to an interferer whose bandwidth is much smaller than that of the desired signal. Similarly, WBI refers to interferer with a larger bandwidth than that of the desired signal. In this chapter, we mainly focus on NBI, and comment on the effect of WBI on the N-FOM link.

In the context of UWB, the robustness of TR system against NBI has been widely studied [78–81]. Pausini et al. investigated a time-offset TR system in the presence of a continuous wave (CW) or single-tone interferer [78, 79], with a bank of correlators for demodulation. They showed that the interference cross-terms, produced as a result of self-correlation, have a huge impact on the performance. Particularly, the self-mixing of interference can completely disrupt the communication. The nuisance from the interference self-mixing was modeled as a random bias term, whose strength was shown to be dependent

Table 5.1: Regulations on EIRP of appliances using unlicensed ISM band.

ISM Band	Applications	Sub-band(s)	Tx Power
900 MHz	Home automation Military radar	902–928 MHz	1 mW 300 kW
2.45 GHz	Bluetooth	2.4–2.484 GHz	100 mW
	WiFi 802.11b/g	2.412–2.462 GHz	100 mW
	Microwave oven Car alarms	2.45 GHz 2.447–2.452 GHz	5 mW* 500 mW
5 GHz	WiFi 802.11a/n	5.15–5.35 GHz 5.47–5.725 GHz	200 mW 1 W

*This number is the FCC limit on power radiated during the lifetime of the appliance.

on the center frequency of the NBI. A similar conclusion was reached by Quek and Win in [80], where they provided an analytical method for quantifying the effect of NBI on the performance of a time-hopping TR-UWB system. The effect of NBI on the frequency offset UWB-TR system was studied by Lai et al. in [81]. It was found that, while single-tone NBI introduces multiple terms of nuisance, the effect of the interference self-mixing term is negligible in a frequency offset TR system.

The extremely large bandwidth of UWB systems allows most interfering systems (e.g., WiFi, Bluetooth, etc.), to be modeled as single-tone interferers. For non-UWB systems such as N-FOM, the bandwidth of the interfering systems must be considered, since it can be large enough to squelch a significant chunk of the desired signal energy. This is explained with the help of Figure 5.1,

Figure 5.1: Spectrum of wideband signal ($\diamond$) interfered by single-tone NBI ($\dagger$) and modulated NBI ($\dagger$).

where we have shown spectrum of a desired wideband signal subject to the two types of NBI, single-tone and a rather wideband interferer. Due to lack of an appropriate terminology, the latter of the two types of NBI will be referred to as *modulated* NBI. This is one example of how a modulated interfering signal, such as an OFDM-based WiFi signal, is modeled. In the next few sections, we model an N-FOM link in presence of NBI; both the interference level and the bandwidth will be considered to study and quantify the effect of NBI.

5.2 System model

In this section, we present the communication model and outline the key assumptions. We consider a binary N-FOM communication system as shown in Figure 5.2, where a LOS link between two nodes is subject to interference. Perfect synchronization is assumed between the communicating nodes. The transmitter sends the TR signal

$$y(t) = x(t)(1/\sqrt{2} + m(t) \cos(2\pi f_r t)) =: x(t) a(t), \quad (5.1)$$

where $x(t)$ is the spreading signal, $m(t)$ is the message signal, and $a(t)$ is the NB function defined analogous to (4.5). To study the effects of the interference, we keep the system model simple as outlined in Section 2.2.4. This implies that the spreading signal is considered to have a perfectly rectangular spectrum, a mean power P_x and a bandwidth B_x , as given in (2.31); and $m(t)$ is modeled as an antipodal signal with rectangular pulse shape, as given in (2.33). Furthermore, the frequency offset is considered equivalent to the symbol rate, i.e., $f_r = 1/T_b$. Perfect synchronization is assumed between the communicating nodes.

The wireless link between the N-FOM transmitter and receiver node is modeled as frequency-flat AWGN channel. The received signal is passed through a

Figure 5.2: N-FOM communication system in the presence of an interferer.

front-end filter, that is assumed to have an ideal response, such that it perfectly collects the TR signal and suppresses all out-of-band signals. At the output of the receiver front-end filter, we obtain

$$r(t) = y(t)/\sqrt{L} + i(t) + n(t), \quad (5.2)$$

where L is propagation loss; $i(t)$ is the NBI; and $n(t)$ is filtered noise, which is modeled as a WSS Gaussian bandpass process with PSD,

$$S_n(f) = \begin{cases} N_0/2, & |f - f_c| \leq B_x/2, \\ 0, & \text{otherwise.} \end{cases} \quad (5.3)$$

where f_c is the carrier frequency.

5.2.1 NBI model

As discussed earlier, NBI can be modeled as a single-tone or a modulated signal. We will cover both models in our analysis. For the single-tone model, $i(t)$ is considered to be a sinusoid with a random phase, that is

$$i(t) = \sqrt{2P_i} \cos(2\pi f_i t + \theta_i), \quad (5.4)$$

is a WSS process with mean power P_i ; θ_i is the random phase uniformly distributed between 0 and 2π ; and f_i is the spectral position of the tone within the desired signal band, i.e., $|f_i - f_c| < B_x/2$. Note that the interference has a constant envelope; the complex baseband equivalent of the interference is $\tilde{i}(t) = \sqrt{2P_i} e^{j2\pi(f_i - f_c)t + j\theta_i}$.

For modulated interference, a common interference model [82], that we also adopt here, is to consider $i(t)$ as a WSS Gaussian random bandpass process with a PSD

$$S_i(f) = \begin{cases} \frac{P_i}{2B_i}, & f_i - B_i/2 \leq |f| \leq f_i + B_i/2, \\ 0, & \text{otherwise,} \end{cases} \quad (5.5)$$

where B_i is the bandwidth of the interferer. The bandwidth B_i and center frequency f_i are modeled such that $B_i < B_x$, and $|f_i| - B_i/2 \geq |f_c| - B_i/2$, i.e., the interference is relatively narrowband ($B_i < B_x$) and fully in-band to the desired received signal $y(t)$. Finally, we assume that, compared to the N-FOM system, the interferer is a high-data-rate system, that is $1/T_b \ll B_i$. This assumption follows from considering that the transmit-reference based scheme targets low-data-rate applications, whereas the interferer (e.g. WLAN) might be a high-data-rate system.

5.2.2 Receiver model

The receiver obtains $r(t)$ as given in (5.2), and mixes it with a frequency-shifted version of itself. The detection filter receives

$$\begin{aligned} z(t) &= r^2(t) \cos(2\pi f_r t) = \frac{1}{2} |\tilde{r}(t)|^2 \cos(2\pi f_r t) + \text{h.f.c.} \\ &= z_y(t) + z_i(t) + z_{yi}(t) + z_{ni}(t) + z_{yn}(t) + z_n(t) + \text{h.f.c.}, \end{aligned} \quad (5.6)$$

where h.f.c. denotes high frequency components, and the mixing-products are defined as

$$z_y(t) := \frac{1}{2L} |\tilde{x}(t)|^2 a^2(t) \cos(2\pi f_r t), \quad (5.7a)$$

$$z_i(t) := \frac{1}{2} |\tilde{i}(t)|^2 \cos(2\pi f_r t), \quad (5.7b)$$

$$z_{yi}(t) := \frac{1}{2\sqrt{L}} \left[\tilde{x}(t) \tilde{i}^*(t) + \tilde{x}^*(t) \tilde{i}(t) \right] a(t) \cos(2\pi f_r t), \quad (5.7c)$$

$$z_{yn}(t) := \frac{1}{2\sqrt{L}} \left[\tilde{x}^*(t) \tilde{n}(t) + \tilde{x}(t) \tilde{n}^*(t) \right] a(t) \cos(2\pi f_r t), \quad (5.7d)$$

$$z_{ni}(t) := \frac{1}{2} \left[\tilde{n}(t) \tilde{i}^*(t) + \tilde{n}^*(t) \tilde{i}(t) \right] \cos(2\pi f_r t), \quad (5.7e)$$

$$z_n(t) := \frac{1}{2} |\tilde{n}(t)|^2 \cos(2\pi f_r t), \quad (5.7f)$$

Compared with the mixing products in an interference-free scenario (2.13), the presence of an interferer introduces three additional terms to $z(t)$: the interference self-mixing product (5.7b), the interference-cross-signal product (5.7c), and the interference-cross-noise product (5.7e).

As in previous chapters, the detection filter is an integrate-and-dump filter. The decision variable for the bit with a data index $i = 0$ is given by

$$d_0 = \int_{-T_b/2}^{T_b/2} z(t) dt. \quad (5.8)$$

Since all mixing products are random processes, d_0 is a random variable, with certain probability distribution. To find the statistics of the decision variable, we need to study the additive noise terms introduced by the interference in (5.6). We conduct this analysis for each model of the NBI.

5.3 Performance analysis

In this section we derive the BER of the N-FOM system in the presence of in-band interference, for both single-tone and modulated NBI models. The statistics of the decision variable d_0 are conditioned on b_0 as well as the interference. However, for brevity of notation, we drop the conditional variables.

5.3.1 Single-tone interference

This type of interference has a constant envelope, even if there is some sort of phase of frequency modulation. The contribution of the interference self-mixing term $z_i(t)$ to the decision variable, is simplified as

$$\begin{aligned} \int_{-T_b/2}^{T_b/2} z_i(t) dt &= \frac{1}{2} \int_{-T_b/2}^{T_b/2} \left| \sqrt{2P_i} e^{j2\pi(f_i - f_c)t + j\theta_i} \right|^2 \cos(2\pi f_r t) dt \\ &= P_i \int_{-T_b/2}^{T_b/2} \cos(2\pi f_r t) dt = 0. \end{aligned}$$

This is an important observation: for a constant envelope interference, the disturbance from the NBI self-mixing product is suppressed by the N-FOM receiver. For such an interference model, $|\tilde{i}(t)|^2$ is a constant DC which is then moved to a frequency $f = f_r$. As long as the conditions (discussed in Section 2.2.3) on the pulse shape and the frequency offset are met, the bias term arising from squaring the NBI is moved out of the detection filter band. This is an advantage of using a frequency offset instead of a time offset in TR communication. While many energy-detector-based receivers use additional filtering blocks (e.g., Teager-Kaiser based filtering) to remove the NBI self-mixing component, the suppression of this term is an inherent trait of a frequency-offset TR receiver.

The rest of the terms in (5.7) are wideband terms. The interference-cross-signal product $z_{yi}(t)$ is in fact a collection of scaled versions of $y(t)$, centered at frequencies that are closer to $f = 0$. Same thing goes for $z_{ni}(t)$ which is a collection of scaled versions of $n(t)$. This is exemplified in Figure 5.3, which shows spectral constituents of the interference cross-term $z_{ni}(t)$. Note that since f_i is within the signal band, these mixing terms are effectively at baseband. The detection filter with bandwidth $1/T_b$ will capture a part of these cross-terms. Since the bandwidths

Figure 5.3: Spectral constituents of the interference cross-term $z_{ni}(t)$.

($\sim B_x$) of these terms is much larger than the detection filter band, therefore, as motivated in Chapter 2, the Central Limit Theorem still holds. The decision variable can be considered a Gaussian distributed variable with a certain mean and variance.

It is important to make some additional remarks. In case the interference is not in-band to the desired signal (i.e., if $|f_i - f_c| > B_x/2$), but still captured by (a non-ideal) front-end filter, then subject to the choice of the frequency offset, the interference cross-terms $\{z_{yi}(t), z_{ni}(t)\}$ may not affect the decision variable. As observed from the figure, if $|f_i - f_c| - f_r$ is larger than $B_x/2$, then the cross-terms will lie outside the detection filter band, and hence, they will be suppressed. In this chapter, we are looking at the worst-case scenario. We are considering that all cross-terms have a significant portion within the detection filter band.

Mean of the decision variable

The mean of the decision variable is given by

$$\mathbb{E}[d_0] = \int_{-T_b/2}^{T_b/2} \mathbb{E}[z(t)|b_0] dt = \int_{-T_b/2}^{T_b/2} \mathbb{E}[z_y(t) + z_{yn}(t) + z_n(t) + z_{yi}(t) + z_{ni}(t)] dt,$$

where the second equality follows from (5.6) and by substituting $z_i(t) = 0$, which follows from the discussion earlier in this section.

The first three terms in the last expression have been previously treated in Section 2.2.1. Therefore, we focus on the additional terms $\{z_{yi}(t), z_{ni}(t)\}$. Since $\{x(t), n(t), i(t)\}$ are mutually independent zero mean WSS processes, one can show that these interference cross-term $\{z_{ni}(t), z_{yi}(t)\}$ are also zero mean processes, and hence their contribution to $\mathbb{E}[d_0]$ is zero. Therefore, the mean of d_0 is the same as in (2.21), with $c = 1/\sqrt{2}$ and $\Lambda_1 = 1$, i.e.,

$$\mathbb{E}[d_0] = \int_{-T_b/2}^{T_b/2} \mathbb{E}[z(t)|b_0] dt = \int_{-T_b/2}^{T_b/2} \mathbb{E}[z_y(t)|b_0] dt = \frac{1}{\sqrt{2}} b_0 E_b, \quad (5.9)$$

where $E_b = P_x T_b / L$ is the received bit energy, and is derived similarly to (2.22).

Variance of the decision variable

The variance of d_0 is calculated as

$$\begin{aligned} \sigma_{d_0}^2 &= \mathbb{E}[(d_0 - \mathbb{E}[d_0])^2] = \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} C_{zz}(t_1, t_2) dt_1 dt_2, \\ &:= \Psi_{yy} + \Psi_{yn} + \Psi_{nn} + \Psi_{yi} + \Psi_{ni}, \end{aligned} \quad (5.10)$$

where $C_{zz}(t_1, t_2)$ is the covariance function of $z(t)$, and Ψ_{ab} represents power of the noise contribution from a mixing product $z_{ab}(t)$ for any signals $a(t)$ and $b(t)$. Due to the mutual independence of $y(t)$, $i(t)$ and $n(t)$, it can be shown

that the covariance function $C_{zz}(t_1, t_2)$ is the sum of the covariance functions of the individual mixing products. The variables $\{\Psi_{yy}, \Psi_{yn}, \Psi_{nn}\}$ were calculated earlier in Section 2.2.1 and follow from Appendix A.1.5. The noise contributions from the interference cross-terms $\{z_{yi}(t), z_{ni}(t)\}$ are calculated in Appendix A.2. This results in

$$\sigma_{d_0}^2 = \frac{25E_b^2}{16S} + \frac{5}{4}E_b N_0 + \frac{1}{2}N_0^2 S + P_i T_b \left(\frac{5E_b}{4S} + N_0 \right), \quad (5.11)$$

where $S = B_x T_b$ is the spreading factor. Using (5.9) and (5.11), the SNR of an N-FOM link in the presence of a single-tone interferer is given by

$$\begin{aligned} \text{SNR} &= \frac{\mathbb{E}^2[d_0]}{\sigma_{d_0}^2} = \frac{8\gamma^2}{25\gamma^2/S + 20\gamma + 8S + 4(5\gamma^2/S + 4\gamma)/\Phi}, \quad (5.12) \\ &:= \left[\text{SNR}_{yy}^{-1} + \text{SNR}_{yn}^{-1} + \text{SNR}_{nn}^{-1} + \text{SNR}_{yi}^{-1} + \text{SNR}_{ni}^{-1} \right]^{-1} \end{aligned}$$

where $\gamma = E_b/N_0$ is the received SNR per bit; $\Phi := P_x/(P_i L)$ is the ratio of the received power of the TR signal to that of the interference, also known as the carrier-to-interference ratio (CIR); and

$$\text{SNR}_{yy} = 8S/25, \quad \text{SNR}_{yn} = 2\gamma/5, \quad \text{SNR}_{nn} = \gamma^2/S, \quad (5.13a)$$

$$\text{SNR}_{yi} := 2\Phi S/5, \quad \text{SNR}_{ni} := \frac{1}{2}\gamma\Phi. \quad (5.13b)$$

The multiple terms in the denominator of (5.12) correspond to the noise power of the cross-terms that arise after the squaring block. The number of these terms is one fewer than that in (5.6). This is because the NBI self-mixing product $z_i(t)$ for a single-tone interferer is suppressed by the N-FOM receiver. Furthermore, we have written the overall SNR as a function of multiple SNR functions. The functions SNR_{yi} and SNR_{ni} scale with the CIR. As discussed in Section 2.3, the total SNR will follow the lowest of the the SNR_{ab} functions. This will be discussed later in Section 5.4.

5.3.2 Modulated interference

We now consider the modulated NBI model, which has a PSD specified by (5.5). Note that with reference to the desired signal, the complex envelope of the interference has a PSD

$$S_i(f) = \frac{2P_i}{B_i} \text{rect} \left(\frac{f - f_i + f_c}{B_i} \right). \quad (5.14)$$

Here we have implicitly assumed that $f_i > f_c$. The subsequent analysis applies to this assumption, nonetheless, the results do not change if $f_i < f_c$, as long as the interference is in-band to the desired signal.

For this NBI model, the spectra of interference self-mixing product $z_i(t)$ and the interference cross-mixing product $z_{ni}(t)$ are exemplified in Figure 5.4. The term $z_i(t)$ comprises of scaled versions of $S_{\tilde{i}}(f) \otimes S_{\tilde{i}}(f)$, centered around $f = \pm f_r$. One can see that if $f_r > B_i$, then an insignificant portion of $S_{z_i}(f)$ will be captured by the detection filter. Hence, its contribution to the total noise will be negligible. In this case, the effect of interference is analogous to that of the single-tone interferer, discussed in Section 5.3.1. On the other hand, if f_r is in order of B_i , then there will be some noise contribution from $z_i(t)$. In this chapter, we consider the worst-case scenario: we assume $f_r \ll B_i$ and $1/T_b \ll B_i$, such that the spectral constituents of $S_{z_i}(f)$ are approximately at baseband, and have a maximum contribution to the total noise. This is also analytically convenient. We can motivate the CLT approximation, and consider the decision variable to be a Gaussian distributed variable.

Mean of the decision variable

Following similar reasoning as that in Section 5.3.1, the cross-terms $z_{yi}(t)$ and $z_{ni}(t)$ do not contribute to the mean of the decision variable. The contribution from the interference self-mixing term to the mean of d_0 is given as

$$\begin{aligned} \int_{-T_b/2}^{T_b/2} \mathbb{E}[z_i(t)] dt &= \int_{-T_b/2}^{T_b/2} \mathbb{E}[|\tilde{i}(t)|^2] \cos(2\pi f_r t) dt \\ &= 2P_i \int_{-T_b/2}^{T_b/2} \cos(2\pi f_r t) dt = 0, \end{aligned}$$

which shows that the interference does not contribute to $\mathbb{E}[d_0]$, as long as the conditions on the frequency offset and the pulse shape (discussed in Section 2.2.3) are satisfied. As a result, the mean of the decision variable d_0 remains the same as that in (5.9).

Figure 5.4: Spectral constituents of the terms $z_i(t)$ and $z_{ni}(t)$, for an arbitrary value of f_r (figure not to scale).

Variance of the decision variable

The variance of the decision variable for the N-FOM link, in case of a modulated interferer, will be

$$\begin{aligned}\sigma_{d_0}^2 &= \mathbb{E}[(d_0 - \mathbb{E}[d_0])^2] = \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} C_{zz}(t_1, t_2) dt_1 dt_2, \\ &:= \Psi_{yy} + \Psi_{yn} + \Psi_{nn} + \Psi_{ii} + \Psi_{yi} + \Psi_{ni}.\end{aligned}\quad (5.15)$$

In presence of a modulated NBI, the interference self-mixing term contributes to the total noise. As per discussion above, this is one major difference between the effects of modulated NBI and single-tone NBI (5.10). Moreover, the total noise will also depend on the bandwidth of the interferer.

The variables $\{\Psi_{yy}, \Psi_{yn}, \Psi_{nn}\}$ remain unchanged, compared to that in (5.15). We derive the noise contribution from the interference mixing-terms $\{z_i(t), z_{yi}(t), z_{in}(t)\}$ in Appendix A.2. The variance is given by

$$\begin{aligned}\sigma_{d_0}^2 &= \Psi_{yy} + \Psi_{yn} + \Psi_{nn} + \Psi_{ii} + \Psi_{yi} + \Psi_{ni} \\ &= \frac{25E_b^2}{16S} + \frac{5}{4}E_b N_0 + \frac{1}{2}N_0^2 S + P_i T_b \left(\frac{P_i}{2B_i} + \frac{5E_b}{4S} + N_0 \right).\end{aligned}\quad (5.16)$$

Using (5.9) and (5.16), the SNR of an N-FOM link in the presence of a random interferer, modeled as Gaussian bandpass process with a certain bandwidth $B_i \gg f_r$, is computed as

$$\begin{aligned}\text{SNR} &= \frac{\mathbb{E}^2[d_0]}{\sigma_{d_0}^2} = \frac{8\gamma^2}{25\gamma^2/S + 20\gamma + 8S + 4(\frac{2\gamma}{B_i\Phi T_b} + 5\gamma/S + 4)\gamma/\Phi} \\ &:= \left[\text{SNR}_{yy}^{-1} + \text{SNR}_{yn}^{-1} + \text{SNR}_{nn}^{-1} + \text{SNR}_{ii}^{-1} + \text{SNR}_{yi}^{-1} + \text{SNR}_{ni}^{-1} \right]^{-1}\end{aligned}\quad (5.17)$$

where $\Phi = P_x/(P_i L)$ was earlier defined as the CIR; the individual SNR functions, barring SNR_{ii} , are given in (5.13); and the additional SNR function $\text{SNR}_{ii} := B_i \Phi^2 T_b$. Compared to the SNR in (5.12) for a single-tone interferer, the total SNR in (5.17) is a quadratic function of the CIR. Furthermore, it also depends on the interference bandwidth B_i . This will be explained later in Section 5.4 with the help of Figure 5.8.

For each NBI model, we reasoned that the decision variable follows Gaussian distribution. Therefore, the BER of the N-FOM link, for both of the NBI models, is found by substituting the corresponding SNR expression (5.12) or (5.17) in

$$\text{BER} = Q(\sqrt{\text{SNR}}).\quad (5.18)$$

5.4 Results and discussions

In this section, we investigate the effects of the interference on an N-FOM communication link. At first, we look at the single-tone NBI, with the SNR

expression given in (5.12). In Figure 5.5, we fix $S=200$ and plot the BER of the N-FOM system against received SNR per bit, in the presence of the single-tone NBI for different values of the CIR, $\Phi = P_x/(P_i L)$. As a reference, we also plot the interference-free case; the SNR expression for this case directly follows from (5.12) by substituting $1/\Phi=0$. A general observation is that an N-FOM link perturbed by a single tone interference can achieve an acceptable performance (i.e., $\text{BER} \leq 10^{-3}$) for medium-to-low values of the CIR. One further discovers that the BER curves for the NBI case follow a similar trend as that of the interference-free case. Note that, in the figure, we have fixed the CIR by adjusting the interference power for each value of γ . In practice, however, CIR increases as γ increases. Therefore, the BER curve of the interference case will converge to the interference-free case for increasing values of γ . Finally, as discussed in Chapter 2, slight differences exist between theoretical and simulation results, which stem from the loose CLT approximation. Nonetheless, the differences in γ are not significant for a desired range of the BER $[10^{-6}, 10^{-3}]$.

Clearly, the N-FOM link needs to operate at a higher E_b/N_0 to achieve a certain BER in the presence of NBI. To investigate this further, we wish to know the value of E_b/N_0 , the N-FOM receiver requires, to maintain an error rate of 10^{-3} . In Figure 5.6, we plot theoretical values of the required E_b/N_0 as a function of the CIR, for different values of the spreading factor S . We make two

Figure 5.5: BER against $\gamma = E_b/N_0$ of the N-FOM system in the presence of a single-tone interferer, with different values of CIR and a fixed $S = 200$. Theoretical curves are depicted in solids, whereas simulation results are depicted in circles.

observations. First, the required E_b/N_0 to achieve the given BER converges to an asymptote as the CIR increases above 0 dB. This asymptote corresponds to the interference-free case. As the SIR drops below 0 dB, the N-FOM system suffers a significant penalty in E_b/N_0 , and a vertical asymptote is observed. This indicates that the N-FOM link cannot tolerate very low values of CIR.

Secondly, we observe from the figure that for the depicted range of γ , it is better to choose a large S for operation in the low-CIR regime, and a small S for operation in the high-CIR regime. This phenomenon results from two mechanisms. First, a large spreading factor implies that the despreading at the receiver results in a low PSD of the interference-cross-signal term. Therefore, a large spreading factor is always beneficial against interference. This can also be seen from (5.12); a large S reduces the noise contribution from the interference-cross-signal term. Second, as discussed in Section 2.3, a large S is beneficial for high values of γ , and a small S for low values of γ . This will be further explained in Figure 5.8.

For interference analysis of non-UWB systems, the single-tone NBI model in (5.4) is inapplicable to most interferers, such as WiFi. Commonly, such an OFDM-like interferer is modeled [83] as a WSS Gaussian bandpass process, presented in (5.5). For the following discussions, we consider this type of interferer; and we choose the bandwidth B_i such that $f_r \ll B_i < B_x$.

Figure 5.6: Required $\gamma = E_b/N_0$ to achieve a $\text{BER} = 10^{-3}$, as a function of the CIR, $\Phi = P_x/(P_i L)$.

Particularly, we study how the interference parameters influence the performance metrics of the N-FOM link. For a fixed set of system parameters $\{B_i = B_x/10, S = 200, f_r = 100 \text{ kHz}\}$, Figure 5.7 illustrates the BER of the N-FOM link against the received SNR per bit, for different values of the CIR, Φ . As a reference, we also plot the BER of the interference-free scenario ($1/\Phi = 0$). It is observed that the BER degrades significantly as the mean power of the interference increases. Despite the reasonably high value of the spreading factor S , an interferer with modestly higher transmission power than that of the TR system, can potentially inhibit the information signal at the detector.

To investigate this further, we fix $B_i = B_x/10$ and $S = 200$, and plot the SNR components in (5.17), as well as the total SNR, as functions of $\gamma = E_b/N_0$ in Figure 5.8. The CIR is not fixed, and increases with an increase in γ . One can see that the total SNR follows the lowest of the individual SNR components in (5.17). For $\gamma \leq \gamma_b$, the component SNR_{ii} is the lowest and dictates the total SNR. The value of γ_b is the value of E_b/N_0 where

$$\Phi = \frac{2\sqrt{2}}{5} \sqrt{\frac{B_x}{B_i}}.$$

It is interesting to observe that, a change in the spreading bandwidth with respect to that of the interferer, will slide the curve SNR_{ii} along the x-axis. The smaller the ratio B_x/B_i (for a fixed Φ), the lower the value of γ_b , and

Figure 5.7: BER against γ of an N-FOM system with a fixed $S = 200$ in the presence of a modulated interference.

hence, the interference self-mixing product $z_i(t)$ will have smaller contribution to the total noise. This phenomenon will be explained in the next figure. Notice that reducing B_x can also lower the spreading factor S , which, in turn, lowers SNR_{yy} . Therefore, an optimal spreading factor, if exists, will have a number of trade-offs.

Finally, for a single-tone interferer, the product $z_i(t)$ does not contribute to the noise. Therefore, for low-to-moderate CNR regimes, the total SNR will be dominated by either $\{\text{SNR}_{yn}, \text{SNR}_{nn}\}$ or $\{\text{SNR}_{yi}, \text{SNR}_{ni}\}$, depending on which one is the lowest. One can see from (5.13) that

$$\begin{aligned}\text{SNR}_{yi} &= \frac{\Phi S}{\gamma} \text{SNR}_{yn} = \frac{\text{CIR}}{\text{CNR}} \text{SNR}_{yn} \\ \text{SNR}_{ni} &= \frac{\Phi S}{2\gamma} \text{SNR}_{nn} = \frac{1}{2} \cdot \frac{\text{CIR}}{\text{CNR}} \text{SNR}_{nn},\end{aligned}$$

where $\text{CNR} = \gamma/S$. The above expression indicates that the total SNR will be dictated by the interference cross-terms $\{\text{SNR}_{yi}, \text{SNR}_{ni}\}$ if $\text{CIR} < \text{CNR}$, and by $\{\text{SNR}_{yn}, \text{SNR}_{nn}\}$ if CIR is roughly above CNR . In other words, the SNR trend will depend on the ratio of power of the in-band noise to that of the interferer. This is an expected result; if interference has a higher power than the in-band noise, the SNR will be dictated by the interference. In this figure, we have considered $\text{CIR} < \text{CNR}$. For such a case, the total SNR will follow

Figure 5.8: SNR components against increasing value of $\gamma = E_b/N_0$, and hence the CIR, $\Phi = P_x/(P_i L)$, for a fixed $S = 200$

SNR_{ni} for $\text{CNR} \leq -1$ dB, and SNR_{yi} for $4S/5 \leq \gamma \leq \gamma_a$, where γ_a defines the value of E_b/N_0 , where $\Phi = 4/5$ [-1 dB].

One concludes from the above figure that the effect of the interfering cross-products $\{z_{yi}(t), z_{ni}(t)\}$ depend on whether the interference power is greater than the power for the noise from the AWGN channel. For a modulated interferer, the interference self-mixing term $z_i(t)$ can a huge impact, which depends on CIR as well as the ratio of the bandwidth of the N-FOM system to that of the interferer. Finally, as γ increases, the total SNR asymptotically converges to that of the interference-free case ($8S/25$), however, the convergence is comparatively slower.

We have seen that, for a fixed CIR, the effect of the modulated NBI is reduced if the interferer has a rather large bandwidth. This implies that the PSD (P_i/B_i) of a modulated interference plays a role in determining its effect on the total SNR. In Figure 5.9 we fix $\text{CIR} = 1/2$, $S = 200$, $B_x = 20$ MHz, and plot the BER of the N-FOM system for different bandwidths of the interferer. We observe that for a fixed CIR, the N-FOM link suffers less degradation from an interferer with a larger bandwidth than that with a smaller bandwidth. This is because increasing the bandwidth B_i flattens the spectrum of $i(t)$ in (5.5), and hence the spectrum of the interference self-mixing product. A low PSD of NBI implies that less interference power is concentrated in the detection filter band. Analytically, this is also observed from (5.17); a very large B_i will reduce the noise from the interference self-mixing product.

This shows that can N-FOM link exhibits resistance to NBI, for an interferer which is either ultra narrow band (single-tone) or fairly wideband. As per our discussion pertaining to Figure 5.4, one can also help reduce the performance deterioration caused by the self-mixing product by choosing a frequency offset that is much larger than the bandwidth of the interference, i.e., $f_r \gg B_i$. With this choice, the frequency shift applied by the self-correlation receiver will move the interference self-mixing term out of the detection filter band. However, choosing a large frequency offset leads to substantial performance limitation for operation in multipath channels; this will be discussed in the next chapter.

Finally, one can also infer that, compared with NBI, an interferer with a bandwidth larger than B_x will degrade the N-FOM link to a lesser extent. In fact, such wideband interference (WBI) can be modeled similar to AWGN, and will have a similar effect on the link. By choosing $B_i = B_x$ in Figure 5.9, we have exemplified the effect of a wideband interferer.

5.5 Interference mitigation

It is established in the last few sections that NBI, both modulated and single-tone, significantly limits the performance of an N-FOM link. It warrants design of suitable techniques to better cope with interference. These techniques were not formally investigated in relation with N-FOM, mainly due to the associated complexities. Nonetheless, in this section we aim to review relevant techniques that have been used to enhance interference robustness of TR-UWB systems.

Furthermore, we also provide an overview of the research work in the circuit design track of the WALNUT project, where they have designed a receiver which tolerates CIRs as low as -50 dB while ensuring BER of 10^{-3} [84, 85].

5.5.1 Interference avoidance

As the name suggests, the idea of this method is to avoid interference channels, either by withholding communication, or by finding alternative channel resource for communication. The latter involves both the transmitter and the receiver in a link, to adapt their modulation and demodulation techniques in the presence of an interferer.

Two of the most popular interference avoidance techniques are dynamic frequency selection (DFS) and adaptive frequency hopping (AFH). The latter is implemented in FHSS modulation, such as in Bluetooth; whereas the former is used in both WiFi and ZigBee technologies. The idea behind both techniques is to switch to a different frequency than the one used by the interferer. For example, in Europe, both 802.11a/c WiFi and weather radar operate in the unlicensed 5 GHz band. To protect the radar, the European Telecommunications Standards Institute (ETSI) requires all radios in the 5 GHz band to employ DFS. Upon detecting radar signals in a particular frequency channel, the DFS-enabled WiFi radio ceases transmissions on the channel and randomly selects

Figure 5.9: BER against γ of an N-FOM system in the presence of a modulated NBI, for different interferer bandwidths and a fixed CIR, $\Phi = 1/2$.

a different channel for further communication.

Due to rather large bandwidth of the N-FOM system compared with the BLE and ZigBee systems (see Table 1.1), the number of frequency channels are limited for feasibility of DFS. Therefore, the N-FOM has to coexist with other systems and tolerate the potentially harmful interference.

5.5.2 Interference suppression

This approach involves reducing the effect of the interference or suppressing interference altogether at the receiver. In the context of TR-UWB, many NBI suppression techniques have been suggested. Comprehensive research on NBI in TR-UWB has been carried out by Alemseged and Wirtisal [83, 86]. They considered delay-hopped TR-UWB, with a bank of correlators at the receiver, in the presence of modulated NBI. A training sequence was used to collect statistical information about the interference; and optimal weights were derived for a linear interference-mitigating filter. For a TR link affected by NBI, the outcome of the correlators were linearly combined using the optimal weights. Exploiting the statistical properties of the NBI, the suggested combining techniques were proven (with the help simulations) to nearly completely reject the noise from NBI self- and cross-mixing products.

The strength of such NBI *post-cancellation* techniques not only lies in the number of parallel correlators, but also in a digital signal processing (DSP) unit which can accurately obtain the statistical information about the NBI. Considering bursty transmissions, implementing post-cancellation techniques entails hefty overheads in term of complexity, synchronization and energy consumption; and as such, it is not suitable for the N-FOM system. *Blind* interference mitigation schemes, which do not require prior knowledge of the interferer or a DSP unit have also attracted research interest [87, 88]. These techniques are based on idea of moving the NBI self-mixing product to a specific intermediate frequency (IF), followed by rejection filters. The N-FOM receiver achieves this by principle. We have shown that, although, the NBI self-mixing product is the dominant nuisance, the performance degradation due to NBI cross-products is still unacceptable.

Suppressing the NBI before the despreading stage, without digital implementation, can be ideal for the N-FOM receiver. Typically, an NBI *pre-cancellation* scheme require a notch filter whose center frequency and bandwidth are matched to those of the NBI. Alemseged had found that, as long as the NBI is fairly narrowband, the performance of a notch-based NBI mitigation scheme comes as close as 2 dB to that of the linear interference mitigation techniques [83]. Notching implies that the desired signal energy in the notch band is also rejected. Therefore, there is a trade-off between improvements gained by suppressing an NBI and loss of the desired signal energy. In [79], the authors derived an upper bound on the bandwidth of the notch filter, which benefits the overall performance.

There are two main difficulties in implementing a notch filter. First, an RF

notch filter is power-expensive; one solution could be that the received signal be first moved to an IF band before applying an appropriate notch filter. Second and the perhaps the most difficult one, is obtaining the exact knowledge of the NBI center frequency and bandwidth. Evidently, the notch filtering also involves computational complexities, which do not resonate with the N-FOM design goals. To avoid such complexities, one may identify prominent interferers in the operating band (such as cordless phones in ISM 900 MHz) before implementation, and integrate appropriate non-adaptable notch filters. Such a model was discussed by Lai et al. for a frequency offset TR-UWB system [81]. The approach in such an implementation is rather pessimistic, since it assumes that the interferers are always present. However, considering the impact of NBI on the N-FOM system, it is more beneficial to sacrifice a bit of signal energy rather than to face a probable interferer [81].

5.5.3 Shifted limiters for NBI rejection

We mentioned in Chapter 1 that this thesis is part of the inter-disciplinary WALNUT project. In the ICD track of the project, significant research has been carried out, especially in the design of an interference-robust receiver circuit. In this subsection, we provide an overview of the research; interested readers are referred to their work [84, 85].

An obvious disadvantage of the N-FOM detection mechanism is that the mixer (squaring block) in Figure 5.2 will further weaken a weak input signal. A high-gain low-noise-amplifier (LNA) is required before the mixing stage for an adequate amplification of the desired signal. Figure 5.10(a) depicts voltages at the input and output of a regular LNA. In presence of strong interference, the desired signal is shown as the envelope of the received signal. At very low CIRs, the LNA will be saturated, and it will clip the received signal. To counter this, Ye et al. suggested to replace the LNA by a shifted-limiter (SL) in the presence of NBI. An SL can be thought of as a regular LNA which amplifies voltages above a certain threshold. This is shown in Figure 5.10(b). An SL will amplify the peaks for the most part, while effectively suppressing the NBI. The designed receiver was integrated in CMOS [84]. It was shown that for single-tone NBI, a BER of 10^{-3} or lower can be ensured for SIRs as low as -50 dB, in the absence of AWGN.

Most interferers can not be modeled as a single-tone. Depending on the bandwidth of the NBI, the peak of the interference also fluctuates. Ye further extended the above receiver design to consider certain band-limited NBI [85]. A tracking loop was added that adjusted the threshold of the SL to the peaks of the NBI. Since the speed of this adaptation is limited, this could only be tested for interferers with a bandwidth less than the frequency offset. For an amplitude shift keying (ASK) interferer, the SL-equipped receiver was shown to tolerate SIRs of -30 dB (for BER of 10^{-3}).

Figure 5.10: Regular LNA in (a) versus LNA with shifted operating point in (b).

5.6 Conclusions

The performance of an N-FOM communication link in the presence of NBI was investigated, where the NBI was modeled as either a single-tone random process or a band-limited (OFDM-like) random process with a bandwidth much larger than the frequency offset. A closed-form expression of the BER was derived and performance trends were discussed.

The mixer operation at the N-FOM receiver gives rise to NBI mixing products. The NBI self-mixing term is the dominant interfering term, with power that scales quadratically with that of the NBI. For single-tone NBI as well as for interferers with bandwidths much smaller than the frequency offset, this self-mixing term is moved out of the detection filter band, and hence the performance of the N-FOM link is completely unaffected by the NBI self-mixing. Nonetheless, the cross products of the interference with the desired signal and the noise, also have a significant impact. For CIRs of less than -5 dB, excessively high values of E_b/N_0 is required to achieve a BER of 10^{-3} .

For modulated interferers with bandwidths much larger than the frequency offset, the NBI self-mixing term is not suppressed by the receiver. The noise contribution from this term dictates the overall SNR. The power of the noise from this cross term depends on the PSD of the interferer. Hence, for a fixed value of CIR, an N-FOM link suffers smaller degradation by an interferer with a large bandwidth than that with a small bandwidth.

One concludes that in general, the N-FOM system is not very resistant to NBI due to the noise from the mixing products, produced by the detection mechanism. However, it can still exhibit a degree of immunity, especially in comparison to energy-detector-type receivers.

Propagation in dense multipath channels¹

In previous chapters, we considered a direct LOS between the nodes in an N-FOM communication link. Typically in WSNs, the nodes are distributed over a geographical area and LOS is usually not guaranteed. There can be multiple obstacles between the communicating nodes, as depicted in Figure 6.1. In such NLOS propagation environments, the transmitted signal arrives at the receiver after reflecting off multiple obstacles. There can also be local scatterers around each obstacle, such as windows, walls, and edges of buildings. The received signal consists of a linear combination of multiple copies of the transmitted signal, each with a different delay and magnitude.

In order to recover the transmitted signal energy, it is essential to decode the signal from each reflection, realign them in time, and combine them *intelligently*, such that the SNR is maximized. In DSSS systems, this is achieved by employing a rake receiver. However, as discussed in Chapter 1, the rake receiver has stringent requirements on synchronization and accurate channel estimation. One of the key motivations behind TR modulation is that the TR receiver can recover energy from each multipath component (MPC) without compromising the receiver complexity. Nonetheless, as concluded in the last few chapters, TR also suffers a performance penalty due to the elevated noise levels. Performance analysis of the N-FOM system in a dense multipath propagation environment is imperative. So far, we have seen that N-FOM is not a good choice in a LOS link, and whether it performs well in an NLOS environment is crucial to assess the overall utility of the communication scheme. This chapter addresses this question.

We begin with a description of the multipath propagation environment in Section 6.1. The system model in an NLOS link is presented in Section 6.2, followed by the derivation of a performance metric in Section 6.3. In some applications, the quantity of interest is the probability that a certain link quality

¹Parts of this chapter are published in [89].

Figure 6.1: A non-line-of-sight propagation environment.

will not be guaranteed in a certain channel environment. This is addressed in Section 6.4 where we derive an approximated expression for the *outage probability*. Combining the theoretical models with a simulation framework, the derived results and performance trends are discussed in Section 6.5. Soft limits on the system parameters are observed and discussed, and the overall findings are summarized in Section 6.6.

6.1 Multipath propagation environment

In a NLOS link, the energy radiated from a transmitter experiences scattering, diffraction, reflection and absorption. We will refer to it collectively as a multipath propagation environment. The MPCs in a multipath propagation environment arrive at the receiver with different phases and magnitudes, and give rise to multipath fading. Small changes (such as displacements of all interacting objects) in the propagation environment results in rapid changes in the received signal strength over a small area (e.g., ten by ten wavelengths [90]). This is called small-scale fading. A large scale fading, also termed as *shadowing*, arises from slow variations in the signal strength. This typically happens due to changes in diffraction conditions; the variations in signal strength occur over large distances (100 m to few km). For example, large buildings, hills and other structures between the communicating nodes can cause the received signal strength to vary slowly over changes in position.

The small-scale fading is important for the design of any modulation scheme. Since it is strongly frequency-dependent, it can influence the demodulated signal differently in different radio systems. On the other hand, the shadowing effect is typically frequency-flat, and its effect on each modulation scheme is calculated similarly. It is important mainly for predicting coverage over a geographical area. Typically, on the basis of the terrain involved, a fade margin is

calculated for shadowing effects and added to the link budget. Since the effect of shadowing is more or less similar to all modulation schemes, this chapter does not look into shadowing effects. We focus on the effects of the small-scale fading.

There are two main effects of small-scale fading in wideband systems; delay dispersion and fading. In wideband systems, the impulse response of the channel is not a delta function, but a combination of delayed delta functions, i.e., the channel is delay-dispersive. As a result, the received data symbol has a longer duration than the transmitted symbol. Consequently, neighboring symbols overlap each other, giving rise to inter-symbol-interference (ISI). If viewed in the frequency domain, certain frequency components of the transmitted signal experience larger attenuations than others. In other words, the transfer function of the channel is not flat over the signal bandwidth. Consequently, the channel is said to be frequency-selective; in fact, delay dispersion is equivalent to frequency selectivity. Finally, since each delta function has its own phase and magnitude, each reflection is attenuated. The received signal experiences fluctuations in strength, and the signal is said to be *faded*.

6.1.1 Channel description

The interaction of a transmitted signal with a multipath channel is similar to linear filtering. If the received signal field strength varies over time, for example, due to a moving radio, the channel is said to be time-variant. We consider the channel to be the response at the output of the receiver antenna in Figure 6.1, to an impulse sent from the transmitter antenna at time t . The baseband equivalent impulse response of a wideband channel is written as

$$\tilde{h}(t, \tau) = \sum_i c_i(t) \delta(\tau - \tau_i). \quad (6.1)$$

where $\delta(\tau)$ is the Dirac delta function, τ_i and $c_i(t)$ are respectively, the delay and complex coefficient associated with each MPC.

A time-variant channel can be characterized as fast fading or slow fading. A fast moving mobile node, reflector or obstacle, leads to channel variations faster than the baseband signal variations (fast fading channel). On the other hand, a more or less stationary node and stationary interacting objects, lead to a slow-fading channel. In some cases, a slow-varying channel can be considered *quasi-static* or time-invariant over one frame/burst of communication. The baseband equivalent impulse response of a linear time-invariant (LTI) channel can be written as

$$\tilde{h}(\tau) = \sum_i c_i \delta(\tau - \tau_i). \quad (6.2)$$

In monitoring applications, the sensor nodes as well as the interacting objects in the environment are usually static. There can be slow variations in the environment, but for bursty communication, the channel can be considered static over duration of one data block or a burst of transmission. Therefore,

for the performance analysis of N-FOM in multipath propagation environments, the quasi-static channel model is sufficient, and we ignore the time dependence in this thesis.

6.1.2 Statistical characterization

The variations in the received signal field strength over the absolute time t and the delay τ are modeled as random. To simplify the channel characterization, the most commonly used assumptions are WSS and uncorrelated scatterers (US), together abbreviated as WSSUS. The WSS assumption means that the statistical properties of the channel do not change² over time t . The US assumption means that the different MPCs do not carry information about each other and are thus uncorrelated. With the US assumption, the ACF of the LTI channel simplifies to

$$R_{\tilde{h}}(\tau, \tau') = \mathbb{E}_h[\tilde{h}(\tau)\tilde{h}^*(\tau')] = P_h(\tau)\delta(\tau - \tau'), \quad (6.3)$$

where $\mathbb{E}_h[\cdot]$ denotes averaging over the channel ensemble, and $P_h(\tau)$ denotes the power delay profile (PDP) of the channel. The PDP specifies the channel energy received in a certain delay bin $[\tau, \tau + d\tau]$. The total channel energy or *channel gain* P_h can be calculated by integrating the PDP, i.e.,

$$P_h = \int_0^\infty P_h(\tau) d\tau = \int_{-\infty}^\infty \mathbb{E}_h[|\tilde{H}(f)|^2] df = 4 \int_{-\infty}^\infty \mathbb{E}_h[|H(f)|^2] df = \frac{4}{L}, \quad (6.4)$$

where L is the average channel loss, $H(f)$ is the transfer function of the LTI channel, and the factor 4 follows from the properties of complex baseband equivalents. The average channel loss, also called the mean propagation loss, is range-dependent; it follows from the inverse-square law, and also depends on the absorption by water vapors, and ground reflection [91].

The PDP gives information about energy distribution and delay dispersion of the channel. However, it is preferable to have a single parameter instead of a function that can describe the channel behavior. Among all possible parameters that can be derived from the PDP, the normalized second-order central moment of the PDP, termed as the root-mean-square (rms) delay spread, is most commonly used in analysis [90]. It is defined as

$$\tau_{\text{rms}} = \sqrt{\frac{1}{P_h} \int_0^\infty (\tau - \bar{\tau})^2 P_h(\tau) d\tau}, \quad (6.5)$$

where $\bar{\tau} = \frac{1}{P_h} \int \tau P_h(\tau) d\tau$ is the mean delay of the channel. The rms delay spread is a measure of how much the delays τ_i vary around their mean. It has been shown that the effect of delay dispersion directly relates with τ_{rms} in most cases [90, Chapter 6]. Nonetheless, it must be kept in mind that τ_{rms} does not

²Although we have considered a quasi-static channel, the WSS assumption implies that the statistical properties of the channel remain the same over different transmissions.

completely specify the channel behavior. In practice, it is convenient to define *maximum excess delay* τ_m , which defines the delay after which the energy in the MPCs are insignificant. An example of PDP, with τ_{rms} and τ_m , is depicted in Figure 6.2a.

To better understand the frequency selectivity of a channel, it is convenient to study the transfer function $H(f) = \mathcal{F}\{h(\tau)\}$ of the channel. For an LTI channel, an example of $|H(f)|$ is depicted in Figure 6.2b. The nulls or dips in the transfer function will suppress the corresponding frequency components of a signal transmitted over the multipath channel. For a signal with a bandwidth narrower than the that of the dips, the transmitted signal risks being significantly suppressed by the channel. Therefore, a wideband system shows better resistance to frequency-selective fading, as not all frequency components of the signals are suppressed.

Figure 6.2: Delay and frequency domain channel characterization.

The frequency correlation function $R_H(\Delta f)$ of a channel is given as

$$\begin{aligned}
 R_{\tilde{H}}(\Delta f) &= \mathbb{E}_h \left[\tilde{H}(f) \tilde{H}^*(f - \Delta f) \right] \\
 &= \mathbb{E}_h \left[\int_0^\infty \tilde{h}(\tau) e^{-j2\pi f \tau} d\tau \int_0^\infty \tilde{h}^*(\tau') e^{j2\pi(f - \Delta f)\tau'} d\tau' \right] \\
 &= \int_0^\infty \mathbb{E}_h \left[|\tilde{h}(\tau)|^2 \right] e^{-j2\pi \Delta f \tau} d\tau = \int_0^\infty P_h(\tau) e^{-j2\pi \Delta f \tau} d\tau,
 \end{aligned}$$

where $P_h(\tau)$ was earlier defined as the PDP. The width of the frequency correlation function is termed as the coherence bandwidth B_c of the channel. For an exponential PDP, i.e., $P_h(\tau) = \frac{1}{P_h} \exp(-\tau/\tau_{\text{rms}})$, one can show that

$$R_{\tilde{H}}(\Delta f) = \frac{1/P_h}{1 + j2\pi \Delta f \tau_{\text{rms}}}.$$

If B_c is defined as the 3 dB bandwidth of this function, then the coherence bandwidth can be shown to be precisely equal to $\sqrt{3}/(2\pi\tau_{\text{rms}})$. The frequency correlation function and its 3 dB coherence bandwidth are depicted in Figure 6.3.

The coherence bandwidth is significant in the context of N-FOM. For a signal transmitted over the channel, its spectral components with frequency spacing of more than B_c , experience uncorrelated fading. Spectral components with frequency spacing much lower than B_c experience more or less similar fading. Therefore, in an N-FOM system, the frequency offset should be much smaller than B_c to ensure that the reference and the information signals do not lose coherence between them.

Figure 6.3: Normalized frequency correlation function, as a function of frequency difference, of an LTI channel that follows an exponential PDP.

The MPCs arrive at the receiver more or less continuously. In practice, however, a receiver cannot resolve all MPCs. For a system with a bandwidth B_x , the receiver can resolve signals that are $1/B_x$ apart. Therefore, in modeling, the MPCs arriving in the delay bin $[i, i-1]/B_x$ are represented by a single *tap*. This leads to the well-known tapped-delay line (TDL) representation [90], i.e.,

$$\tilde{h}(\tau) = \sum_{i=0}^{\infty} c_i \delta(\tau - iT_s), \quad (6.6)$$

where $T_s = 1/B_x$ follows from Nyquist's criterion. Since each channel tap c_i is summation of a number of MPCs, the CLT approximation holds, and the coefficients c_i are modeled as zero mean circularly-symmetric complex Gaussian variables. Since all channel taps are uncorrelated, they are also mutually-independent. The TDL representation above, although simple, is accurate only if B_x is the absolute bandwidth of the transmitted signal.

6.1.3 N-FOM in a multipath channel

In a coherent DSSS system, the receiver decodes each reflection of the transmitted signal and combine them coherently. Thus, if properly implemented, a coherent DSSS receiver corrects for channel effects. On the other hand, the N-FOM receiver does not correct for delay dispersion, and thus, a highly dispersive channel will introduce ISI. Secondly, the detection is based on correlation of the reference and information signals; it is imperative that the two signals arrive at the receiver with their coherence maintained. Employing a large frequency offset f_r may result in different impact of the frequency-selective channel on the two signals. The coherence can be lost, which can inhibit the SNR of the demodulated signal. In this sense, a dispersive channel not only introduces ISI but also decoherence between the information and the reference signals. We will refer to this phenomenon as the *reference decoherence* effect (RDE).

Since the TR receiver does not correct for the delay dispersion, it is not a replacement for a rake receiver in multipath channels. However, Choi et al. found that in a multipath propagation environment, a receiver which tracks only the strongest path, is outperformed by a time-offset TR-UWB receiver [31]. The authors suggest that TR can be a good compromise if the design goal is to have a simple receiver. A frequency-offset TR-UWB system in fading environment was studied in [42, 44, 81]. In [42], the BER expression conditioned on a channel realization, was derived. Ignoring both IPI and ISI, the performance was shown to be in par with that of a time offset TR-UWB system. While ISI-free communication can be easily ensured by low-data-rate operation, the IPI-free assumption is an oversimplification. In [81], IPI was incorporated in the model; the results indicated a significant limitation on performance. Since the collective noise from IPI is enhanced by the increase in transmission power, the SNR attains an asymptote for an increasing value of received signal power.

N-FOM is not a UWB technique, and its interaction with a propagation environment is different than that of a TR-UWB system. In the next few sections, we will model an N-FOM system in a dense wideband multipath channel and carry out performance analysis. The effects of multipath fading will be isolated from that of the delay dispersion, i.e., the model will not consider effects of ISI and reference-decoherence. Those will be considered later in Section 6.3.3 in which we explore the interplays between the channel effects and design parameters, with the aim to identify suitable system parameters for practical considerations.

6.2 System model

We consider a binary N-FOM communication system shown in Figure 6.4. For mathematical convenience, the bandpass signals are described in terms of their baseband equivalents. The transmitter generates a spreading signal which is modeled as a wide-sense-stationary Gaussian bandpass process centered around an RF frequency f_c . The complex envelope $\tilde{x}(t)$ of the spreading signal has a two-sided PSD

$$S_{\tilde{x}}(f) = \frac{2P_x}{B_x} \mathcal{S}\left(\frac{f}{B_x}\right), \quad (6.7)$$

where P_x is mean power in the bandpass signal $x(t)$, B_x is the bandwidth, and $\mathcal{S}(\nu)$ is the shape of PSD. The spreading signal spreads a message signal $m(t)$ at one branch of the transmitter. On the other branch, the spreading signal is sent as a *reference signal*. The message signal is modeled as

$$m(t) = \sum_k b_k p(t - kT_b), \quad (6.8)$$

where $b_k \in \{+1, -1\}$ is a binary bit sequence; $1/T_b$ is the data rate; and $p(t)$ is the pulse shape with energy T_b . A frequency shift $f_r \ll B_x$, already known

Figure 6.4: A non-line-of-sight N-FOM communication link in a dense multipath fading channel $h(t)$.

to the receiver, is applied to the information signal. The frequency offset f_r and $p(t)$ are chosen such that they satisfy the criteria discussed earlier in Section 2.2.3. This implies that, for rectangular $p(t)$, f_r is an integer multiple of the symbol rate. On the other hand, for an arbitrary pulse with absolute bandwidth B_p , we have $f_r \geq 3B_p$.

The transmitter linearly combines the information signal and the spreading signal to form the TR signal, which has a complex envelope

$$\tilde{y}(t) = \tilde{x}(t) \left[1/\sqrt{2} + m(t) \cos(2\pi f_r t) \right] = \tilde{x}(t) a(t), \quad (6.9)$$

where we have defined the narrowband term

$$a(t) = 1/\sqrt{2} + m(t) \cos(2\pi f_r t + \phi_r). \quad (6.10)$$

We assume that the LO at the transmitter and at the receiver are locked in phase and in frequency, i.e., $f_{Tx} = f_{Rx} = f_r$, and $\phi_{Tx} = \phi_{Rx} = \phi_r$. The TR signal is sent over the wireless channel. Since $f_r \ll B_x$, the bandwidth of the TR signal $y(t)$ is approximately equal to that of the spreading signal (B_x).

6.2.1 Channel model

We consider a dense multipath fading environment. The channel is assumed to be linear, and time-invariant over one burst of transmission. The channel impulse response is $h(\tau)$ with a transfer function $H(f)$, and a complex baseband impulse response $\tilde{h}(\tau)$ given in (6.2). The channel has a certain PDP, with a certain delay spread τ_{rms} and mean excess delay τ_m . The total gain in baseband equivalent of the channel is $P_h = 4/L$, as given in (6.4). Furthermore, from the properties of complex Gaussian variables [92], it follows

$$\mathbb{E}_h \left[|H(f)|^4 \right] = 2\mathbb{E}_h^2 \left[|H(f)|^2 \right] = 2P_h^2 = 32/L^2. \quad (6.11)$$

The delay spread τ_{rms} can vary from tens of nanoseconds to a few microseconds, depending on the operational environment. Since we operate at very low data rates, we assume that the symbol time is much larger than the rms delay spread of the channel, i.e., $\tau_{\text{rms}} \ll T_b$. We further assume that $f_r \ll 1/\tau_{\text{rms}}$. These assumption guarantees a scenario where the effects of ISI and RDE

are negligible. These assumptions are made in order to isolate the effect of fading from that of delay dispersiveness of the channel, and are valid only for a particular range of the system and channel variables. For a much larger delay spread or frequency offset, we can no longer consider an ISI- and RDE-free scenario. This will be discussed later in Section 6.3.3.

6.2.2 Received signal model

At the receiver, a front-end filter with an impulse response $g(t)$ processes the received signal. At the output of the RFE filter, we obtain

$$r(t) = h(\tau) \otimes g(t) \otimes y(t) + g(t) \otimes n(t),$$

where $n(t)$ is AWGN with a PSD $N_0/2$.

Since the effects of delay dispersion are negligible in the current model, one can consider the variations in the narrowband term $a(t)$ in (6.9) to be negligible over the impulse response of the channel $h(t)$. Therefore, using the definitions in (2.9), the above equation can be written as

$$r(t) = (h(\tau) \otimes x_g(t))a(t) + n_g(t). \quad (6.12)$$

As discussed in Chapter 3, the optimal choice of the front-end receiver filter $g(t)$ depends on the PSD of the spectral shape and the total noise levels. We also presented a practical but suboptimal filter, that is matched to the spectral shape of the spreading signal. It was shown that such a filter is optimal in the low-CNR regimes, and is close to optimal in other CNR regimes if the spectral shape $S(\nu)$ is more or less uniform. Although the suboptimal filter is not optimal in fading channels, it is a good compromise for the noncoherent system. In this analysis, we use the suboptimal filter given in (3.16) and assume that the spectral shape $S(\nu)$ has the properties as given in (3.11). The filtered signal and noise have the PSDs

$$S_{\tilde{x}_g}(f) = \frac{1}{4} S_{\tilde{x}_g}(f) |\tilde{G}(f)|^2 df = \frac{2P_x}{B_x} S^2\left(\frac{f}{B_x}\right) \quad (6.13a)$$

$$S_{\tilde{n}_g}(f) = \frac{1}{4} S_{\tilde{n}_g}(f) |\tilde{G}(f)|^2 df = 2N_0 S\left(\frac{f}{B_x}\right). \quad (6.13b)$$

Note that the power in $S_{\tilde{x}_g}(f)$ is $2P_x$; and that in $S_{\tilde{n}_g}(f)$ is $2N_0 B_x$. The receiver recovers the information bit by a matched-filter operation followed by sampling and threshold detection. The input to the detection filter is given by

$$\begin{aligned} z(t) &= r^2(t) \cos(2\pi f_r t + \phi_r) = \frac{1}{2} |\tilde{r}(t)|^2 \cos(2\pi f_r t + \phi_r) + \text{h.f.c.} \\ &= z_y(t) + z_{yn}(t) + z_n(t) + \text{h.f.c.}, \end{aligned} \quad (6.14)$$

where we have defined the mixing products as

$$z_y(t) := \frac{1}{8} \left| \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{x}_g(t-u) du \right|^2 a^2(t) \cos(2\pi f_r t + \phi_r) \quad (6.15a)$$

$$\begin{aligned} z_{yn}(t) := & \frac{1}{4} a(t) \cos(2\pi f_r t + \phi_r) \left[\tilde{n}_g^*(t) \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{x}_g(t-u) du \right. \\ & \left. + \tilde{n}_g(t) \int_{-\infty}^{\infty} \tilde{h}^*(u) \tilde{x}_g^*(t-u) du \right] \end{aligned} \quad (6.15b)$$

$$z_n(t) := \frac{1}{2} |n_g(t)|^2 \cos(2\pi f_r t + \phi_r), \quad (6.15c)$$

The fading channel affects the desired signal in $z_y(t)$, but also affects the undesired terms. In the next section, we derive the SNR of an N-FOM NLOS link, conditioned on a particular channel realization.

6.3 Performance analysis

As in earlier chapters, the decision variable d_k can be considered a Gaussian distributed variable using CLT approximation. Therefore, we need to find the mean and variance of the decision variable. Without loss of generality, we consider the data index 0. We derive statistics of d_0 , conditioned on the channel realization h and data bit b_0 . The decision variable can be written as

$$d_0 = \int_{-\infty}^{\infty} z(t) p(t) dt = \int_{-\infty}^{\infty} (z_y(t) + z_{yn}(t) + z_n(t)) p(t) dt. \quad (6.16)$$

In the above equation, we have used the fact that the matched filter will suppress the high frequency components (h.f.c.) in (6.14).

6.3.1 Statistics of the decision variable

We first find the mean of the decision variable. One can see from (6.16) that $\mathbb{E}[d_0]$ is a linear combination of mean contributions of the mixing products in (6.16). Using similar analysis as that in Chapter 2, one can show that the contribution from signal-noise ($z_{yn}(t)$) and noise-noise ($z_n(t)$) products to the mean of the d_0 is zero. The former stems from mutual independence and zero mean nature of noise and the spreading signal, whereas the later follows from relation of $p(t)$ and f_r , specified in Section 2.2.3.

The mean of the d_0 , given a particular channel realization, solves to

$$\begin{aligned} \mathbb{E}[d_0|h] &= \int_{-\infty}^{\infty} \mathbb{E}[z_y(t)] p(t) dt \\ &= \frac{1}{8} \int_{-\infty}^{\infty} a^2(t) \cos(2\pi f_r t + \phi_r) p(t) dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{\tilde{x}_g}(v-u) \tilde{h}(u) \tilde{h}^*(v) du dv. \end{aligned}$$

Given that the conditions on $\{f_r, p(t)\}$, mentioned in Section 2.2.3 are met, the first integral in the above expression can be solved following the steps from (2.19) till (2.21). This results in

$$\mathbb{E}[d_0|h] = \frac{b_0 T_b}{8\sqrt{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{\tilde{x}_g}(v-u) \tilde{h}(u) \tilde{h}^*(v) du dv, \quad (6.17)$$

where the last equality follows from similar analysis as in (2.21). One can see that, the mean of the decision variable is subject to variations in the channel, and in general, will have its own mean and variance. However, we study the mean of the decision variable for a given channel realization. Note that the double integral in the above expression gives the total power in the spreading signal, after filter and channel operations. Since the channel is an LTI system, and $x_g(t)$ is a WSS process, using the properties in 2.2, we write

$$\mathbb{E}[d_0|h] = \frac{b_0 T_b}{8\sqrt{2}} \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) |\tilde{H}(f)|^2 df = \frac{b_0 T_b}{8\sqrt{2}} \alpha_h P_h \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) df, \quad (6.18)$$

where we have defined

$$\begin{aligned} \alpha_h &= \frac{\int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) |\tilde{H}(f)|^2 df}{\mathbb{E}_h \left[\int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) |\tilde{H}(f)|^2 df \right]} = \frac{\int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) |\tilde{H}(f)|^2 df}{P_h \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) df} \\ &= \frac{1}{P_h} \int_{-\infty}^{\infty} \mathcal{S}^2(\nu) |\tilde{H}(\nu B_x)|^2 d\nu, \end{aligned} \quad (6.19)$$

as a channel variable with mean $\mathbb{E}_h[\alpha_h] = 1$. The last equality in the above expression follows from substituting $S_{\tilde{x}_g}(f)$ from (6.13a), and then using (3.11). The outcome of the integration operation in (6.18) is the total power in $S_{\tilde{x}_g}(f)$, which was earlier shown to be $2P_x$; this results in

$$\mathbb{E}[d_0|h] = \frac{1}{4\sqrt{2}} b_0 P_x T_b \alpha_h P_h = \frac{1}{\sqrt{2}L} b_0 P_x T_b \alpha_h, \quad (6.20)$$

where the last equality follows from substituting $P_h = 4/L$ from (6.4).

We now compute the variance of the decision variable. As in previous chapters, the variance of the decision variable can be written as

$$\sigma_{d_0|h}^2 = \Psi_{yy} + \Psi_{yn} + \Psi_{nn}. \quad (6.21)$$

Analogous to the definition of α_h in (6.19), we define

$$\beta_h = \frac{2 \int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) |\tilde{H}(f)|^4 df}{\mathbb{E}_h \left[\int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) |\tilde{H}(f)|^4 df \right]} = \frac{1}{\lambda_2 P_h^2} \int_{-\infty}^{\infty} \mathcal{S}^4(\nu) |\tilde{H}(\nu B_x)|^4 d\nu \quad (6.22a)$$

$$\begin{aligned} \vartheta_h &= \frac{\int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) |\tilde{H}(f)|^2 df}{\mathbb{E}_h \left[\int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) |\tilde{H}(f)|^2 |\tilde{G}(f)|^2 df \right]} \\ &= \frac{1}{\lambda_1 P_h} \int_{-\infty}^{\infty} \mathcal{S}^3(\nu) |\tilde{H}(\nu B_x)|^2 d\nu, \end{aligned} \quad (6.22b)$$

where $\{\lambda_1, \lambda_2\}$ are defined in (3.18), and depend on the spectral shape $\mathcal{S}(\nu)$. Note that $\mathbb{E}_h[\beta_h] = 2$ and $\mathbb{E}_h[\vartheta_h] = 1$. Using the definitions, we derive the noise contributions (6.21) in Appendix A.3, and are given as

$$\Psi_{yy} = \frac{25\beta_h}{16L^2B_x} P_x T_b \lambda_2, \quad \Psi_{yn} = \frac{5\vartheta_h}{4L} P_x T_b N_0 \lambda_1, \quad \Psi_{nn} = \frac{1}{2} N_0^2 B_x T_b, \quad (6.23)$$

where we have used $P_h = 4/L$. Comparing the above terms for the NLOS link with that for a LOS link (2.24), one infers that a multipath channel influences the noise experienced by the decision variable. This is indicated by dependency of Ψ_{yy} and Ψ_{yn} on channel parameters.

6.3.2 Signal-to-noise ratio

Substituting the above expressions in (6.21) and using (6.20), the SNR conditioned on a channel realization is

$$\text{SNR}(h) = \frac{\mathbb{E}^2[d_0|h]}{\sigma_{d_0|h}^2} = \frac{8\alpha_h^2\gamma^2}{25\lambda_2\beta_h\gamma^2/S + 20\lambda_1\vartheta_h\gamma + 8S}, \quad (6.24)$$

where S is the spreading factor; $\gamma = E_b/N_0$ is the received SNR per bit; and E_b is the average received bit energy given by

$$\begin{aligned} E_b &= \frac{1}{L} \int_{-\infty}^{\infty} \mathbb{E}[y^2(t)] dt = \int_{-\infty}^{\infty} \mathbb{E}[a^2(t)] \mathbb{E}[(x_g(t) \otimes h(t))^2] dt \\ &= T_b \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) \mathbb{E}_h[|H(f)|^2] df = \frac{1}{L} P_x T_b, \end{aligned} \quad (6.25)$$

where the second last equality follows from similar analysis as in (2.22) with $c = 1/\sqrt{2}$, and the last equality follows from (6.4). The equation in (6.24) represents the SNR of an N-FOM link in dense frequency-selective fading channels. Since we have chosen the parameters such that the ISI and RDE are negligible, the derived SNR expression mainly models the effect of fading. The other effects will be discussed in the next section.

In case, $\mathcal{S}(\nu)$ in (6.7) is perfectly rectangular, then one can see from (6.19) and (6.22b), that $\alpha_h = \vartheta_h$ with $\lambda_1 = \lambda_2 = 1$. Furthermore, as described in Section 6.1.2, one can accurately apply the TDL description as given in (6.6). The resulting SNR expression is

$$\text{SNR}(h) = \frac{8\alpha_h^2\gamma^2}{25\beta_h\gamma^2/S + 20\alpha_h\gamma + 8S}, \quad (6.26)$$

with the channel parameters given by

$$\alpha_h := \frac{1}{P_h} \sum_i |c_i|^2, \quad \beta_h := \frac{1}{P_h^2} \sum_i \left| \sum_j c_j c_{j-i}^* \right|^2. \quad (6.27)$$

Analogous to that in Chapter 2, the BER of the N-FOM communication system, for any given channel realization, is given by

$$\text{BER}(h) = Q\left(\sqrt{\text{SNR}(h)}\right), \quad (6.28)$$

where $Q(\cdot)$ is the Gaussian tail probability. Comparing the above relation with the SNR expression for a N-FOM LOS link in (2.37), we observe the influence of the channel on the demodulated signal energy (in the numerator) and on the total noise power (in the denominator). Furthermore, since $\mathbb{E}_h[\alpha_h] = 1$ and $\mathbb{E}_h[\beta_h] = 2$, one concludes that on the average, the link's performance in a multipath fading channel will always be lower than that in a LOS AWGN channel.

The SNR expressions above represent the error rate for a particular channel realization. To find the average BER, we have to compute the BER for all possible channel realizations, i.e.,

$$\overline{\text{BER}} = \mathbb{E}_h[\text{BER}(h)]. \quad (6.29)$$

6.3.3 RDE and ISI

As discussed earlier, the ability of the N-FOM receiver to effectively capture the received energy relies on coherence between the received reference and the information signal. We had assumed $f_r \ll 1/\tau_{\text{rms}}$ in conjunction with $T_b \gg \tau_{\text{rms}}$, which not only ensures ISI-free communication, but also guarantees that the channel transfer function has the same response for the two signals. The assumptions were incorporated in the system model (6.12) by considering that the channel does not affect the narrowband part ($a(t)$) of the transmitted signal.

For a large $f_r \tau_{\text{rms}}$ or a large τ_{rms}/T_b or both, the system model has to be extended. We do that by incorporating effect of the channel on $a(t)$ in (6.12). With this modification, the signal self-mixing product from (6.15a) can be rewritten as

$$z_{yy}(t) = \frac{1}{8} \left| \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{x}_g(t-u) a(t-u) du \right|^2 \cos(2\pi f_r t).$$

Here, we have implicitly assumed that the RFE filter has no effect on $a(t)$. The motivation for this assumption follows from the consideration $f_r \ll B_x$.

To analyze ISI and RDE, we need to look at the reduction in the energy of the recovered information, which lies in the mean of the decision variable d_0 . As explained in Section 6.3, the mixing products $z_{yn}(t)$ and $z_{nn}(t)$ do not contribute to the mean of d_0 . Therefore, using the above expression, the mean

of the decision variable is written as

$$\begin{aligned}\mathbb{E}[d_0|h] &= \frac{1}{8} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{h}^*(v) R_{\tilde{x}_g}(v-u) \\ &\quad \cdot a(t-u) a(t-v) \cos(2\pi f_r t) p(t) du dv dt \\ &\approx \frac{1}{8} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{h}^*(v) R_{\tilde{x}_g}(u-v) dv \int_{-\infty}^{\infty} a^2(t-u) \cos(2\pi f_r t) p(t) dt du,\end{aligned}$$

where the approximation follows from observing that the support of $R_{\tilde{x}_g}(u-v)$ is very narrow ($\sim 1/B_x$), and the function $a(t)$ hardly varies over the support of the ACF. Substituting $a(t)$ from (6.10), the above expression is solved to

$$\begin{aligned}\mathbb{E}[d_0|h] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{h}^*(v) R_{\tilde{x}_g}(v-u) dv \\ &\quad \cdot \frac{1}{8\sqrt{2}} \cos(2\pi f_r u) \int_{-\infty}^{\infty} m(t-u) dt du + \Xi,\end{aligned}$$

where Ξ denotes a bias term, similar to that in (2.20). It is suppressed if the conditions on $p(t)$ and f_r , mentioned in Section 2.2.3, are met. Therefore, we assume that this term is suppressed. Substituting $m(t)$ from (6.8) in the above expression, we get

$$\begin{aligned}\mathbb{E}[d_0|h, b_k] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{h}^*(v) R_{\tilde{x}_g}(v-u) dv \\ &\quad \cdot \frac{\cos(2\pi f_r u)}{8\sqrt{2}} \left(b_0 R_p(u) + \sum_{k \neq 0} b_k R_p(kT_b + u) \right) du, \quad (6.30)\end{aligned}$$

where $R_p(\tau)$ represents ACF of the pulse $p(t)$, with $R_p(0) = T_b$. Comparing the above expression with the mean of d_0 in (6.17) for the ISI- and RDE-free case, one observes a couple of differences.

First, due to delay dispersion, the received symbol is distorted. The summation in (6.30) represents interference from neighboring symbols. The longer the duration of the impulse response $\tilde{h}(u)$, the higher the number of neighboring symbols which interfere with the desired information bit b_0 . The magnitude in the recovered sample is reduced, if the neighboring symbols have a different polarity. A low symbol rate, i.e., a small τ_{rms}/T_b , reduces ISI.

Second, even for negligible ISI, i.e., when $\tau_{\text{rms}}/T_b \ll 1$, the magnitude of the recovered sample is reduced, as signified by the term $\cos(2\pi f_r u)$ in (6.30). The frequency offset oscillator at the receiver is not locked in phase to the arriving MPCs. Therefore, the demodulated signal from each MPC suffers from a small phase offset. The effect of the phase offset is magnified for a large value of $f_r \tau_{\text{rms}}$. In other words, the reference and information signals tend to be incoherent if one chooses a large f_r .

To explain this further, we consider negligible ISI. This can be ensured by duty-cycling³ or by choosing the data rate such that $\tau_{\text{rms}}/T_b \ll 1$. In order to say something about the maximum performance deterioration due to RDE, we consider that the channel has attained its average state⁴. Rewriting (6.30), and averaging over the channel ensemble, we get

$$\begin{aligned} \mathbb{E}_h [\mathbb{E}[d_0]] &= \frac{b_0 T_b}{8\sqrt{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbb{E}_h [\tilde{h}(u) \tilde{h}^*(v)] R_{\tilde{x}_g}(v-u) \cos(2\pi f_r u) du dv \\ &= \frac{b_0 T_b}{8\sqrt{2}} R_{\tilde{x}_g}(0) \int_{-\infty}^{\infty} P_h(u) \cos(2\pi f_r u) du \\ &= \frac{b_0 P_x T_b}{4\sqrt{2}} \text{Re} \left\{ R_{\tilde{H}}(f_r) \right\}, \end{aligned} \quad (6.31)$$

where the second equality follows from (6.3); and $R_{\tilde{H}}(f)$ is the frequency correlation function of the channel. One can directly compare the above expression to (6.20) for the RDE-free case. The above expression indicates that the higher the value of f_r/B_c , the poorer the performance of the link. In other words, a large value of $f_r \tau_{\text{rms}}$ gives rise to the reference-decoherence effect. We refer to Figure 6.3, which depicts $R_{\tilde{H}}(f)$ for a channel with an exponential $P_h(\tau)$. For an exponential PDP $P_h(\tau)$, one can solve that for a value $f_r \tau_{\text{rms}} = \sqrt{3}/2\pi \approx 0.27$, one risks losing half of the received energy. For $f_r \tau_{\text{rms}} \leq 0.077$, only 10% of the signal energy is lost. Nevertheless, RDE will always affect the signal.

Recall that the smallest recommended value of f_r is $1/T_b$ for NRZ signaling scheme, and $3B_p$ for a generic signaling scheme. Therefore, a large τ_{rms}/T_b also leads to large $f_r \tau_{\text{rms}}$; in other words, ISI is coupled with RDE. If ISI is experienced in a link, RDE will also be observed. However, one can isolate RDE (without experiencing ISI) by choosing a large frequency offset (large $f_r \tau_{\text{rms}}$), and a small data rate (small τ_{rms}/T_b). The implications of the two phenomena will be explained with the help of simulations in Section 6.5.

6.3.4 Pseudo-random noise (PN) carrier

As discussed in Chapter 3, a PN carrier in a LOS scenario gives a much better performance than using a pure noise carrier. The SNR of an N-FOM LOS link achieves an asymptote as E_b/N_0 increases above $0.56S$. On the other hand, the performance of an N-FOM LOS link, using a PN carrier, linearly increases with E_b/N_0 . However, we argued that in an NLOS link, the PN carrier has little added value compared to the noise carrier. This argument was also supported by Goeckel et al. in [94] in the context of a frequency offset TR-UWB system using impulse radio (IR).

As presented in (3.1), a PN carrier consists of train of chips (or pulses) $w(t)$, weighted by a PRBS. For an N-FOM LOS link using a PN carrier, the trans-

³Duty-cycling can have two interpretations. Here, duty-cycling refers to using guard intervals between symbols in a packet. A study conducted in this context showed that duty-cycling can reduce the effect of ISI [93].

⁴This is called channel hardening, and will be discussed in Section 6.5.2

mitted signal does not suffer delay dispersion. A PN carrier with polar NRZ pulses has a constant instantaneous power, i.e., $x^2(t) = P_x$ and therefore, the self-mixing of the TR signal at the receiver does not add noise. In a multipath environment, the receiver multiple delayed versions of the transmitted signal. For example, the received signal in a multipath environment may look like

$$x_h(t) = \sum_i h_i x(t - i\Delta\tau).$$

The self-mixing of the signal at the receiver results in

$$|x_h(t)|^2 = P_x \sum_i |h_i|^2 + \sum_i \sum_{j \neq i} h_i h_j^* x(t - i\Delta\tau) x^*(t - j\Delta\tau).$$

The first term on the right hand side corresponds⁵ to the power of the recovered information sample, whereas the second term represents cross-mixing of the different reflections with each other. Clearly, depending on properties of the channel and the PN carrier, the self-mixing of the PN carrier will add to noise. We will further discuss this with simulation results in Section 6.5.

Typically, in impulse radio TR-UWB systems, sparse coding is considered. This implies that, the spacing between consecutive chips is larger than the delay spread (or even maximum excess delay) of the channel [42]. This assumption gives rise to high PAPR and undesired spectral properties. Usually, further spectral shaping techniques are used to solve this problem [69]. Secondly, sparse coding is more suitable to UWB than wideband techniques.

6.4 Outage probability

The state of the multipath channel can vary with time. At a particular time t , the channel can be in a deep fade, thus carrying little signal energy from the transmitter to the receiver. At other times, the transmission link will experience a healthy channel. Due to variations in the channel state, the SNR is random which leads to a different BER at different instances. So far, we have studied the average BER, by taking the average of (6.28) over the channel ensemble.

The average BER is a meaningful metric when there are fast changes in the radio environment. For example, due to mobile interacting objects or transceiver pairs, the channel can vary within one burst (packet) of transmission, thus the received signal energy is different for different symbols in the packet. Here, finding the average link quality can be interesting for the link design. On the other hand, if the channel is static over one burst of transmission, then the received signal energy is the same for all symbols in a packet. However, packets sent at other time instances may experience different channels. Thus, the received signal energy may vary from one burst of transmission to other.

⁵This is akin to the signal power recovered by a maximum ratio combining technique.

In this case, one might be interested in answering the following question. “What is the percentage of time, a desired link quality (BER) will not be supported by the channel?” This is called outage probability $\mathbb{P}_{\text{out}}$, which specifies the probability that the BER drops below a certain *error threshold* $\mathcal{E}_T$, i.e.,

$$\mathbb{P}_{\text{out}} = \Pr [\text{BER}(h) \leq \mathcal{E}_T] = \Pr [\text{SNR}(h) \leq \Gamma_T], \quad (6.32)$$

where we have defined $\Gamma_T := [\mathbf{Q}^{-1}(\mathcal{E}_T)]^2$ as the SNR threshold, and denoted a probability operation w.r.t. the channel as $\Pr[\cdot]$.

The outage probability is an important metric, in the context of link budget calculations. For design of any transmission link, a key goal is to maintain the received power above a certain threshold. It is necessary to know the probability that the minimum receiver power will be ensured. A completely outage-free operation is theoretically impossible. Allowing some flexibility can reduce the strain on the link budget, therefore a certain level of outage probability ρ is associated with a transmission link. This means that a certain fraction of time, the link will be in outage. In that case, the metric of great interest is the required $\gamma = E_b/N_0$ to remain below a certain level of a tolerable outage probability ρ . We will denote this metric by γ_ρ . In this section, we find an expression for the outage probability $\mathbb{P}_{\text{out}}$, and the metric γ_ρ .

6.4.1 Outage probability analysis

One can see from (6.32) that in order to derive an exact expression of $\mathbb{P}_{\text{out}}$, one needs knowledge of the PDF of $\text{SNR}(h)$, which requires the joint PDF of the variables $\{\alpha_h, \beta_h, \vartheta_h\}$ in (6.24) to be known. However, since β_h is a complicated function of dependent and independent random variables, the derivation of a joint PDF is not straightforward, and may not necessarily lead to a closed-form expression. Allowing some approximations, we can derive a closed-form expression for the outage probability.

Approximations

First, we consider that the spectral shape $\mathcal{S}(\nu)$ is more or less uniform, such that $S_{\hat{x}}(f)$ can be approximated as a brick-wall spectrum. This implies that $\lambda_1 \approx \lambda_2 \approx 1$, and that $\vartheta_h \approx \alpha_h$. As a result of this approximation, the channel can be accurately modeled using the TDL description. The SNR expression can be written as that in (6.26), with the parameters $\{\alpha_h, \beta_h\}$ defined in (6.27).

Next, we remark on the relation between α_h and β_h . One can see from (6.27) that

$$\begin{aligned} \beta_h &= \alpha_h^2 + \frac{1}{P_h^2} \sum_{i \neq 0} \left| \sum_j c_j c_{j-i}^* \right|^2 = \alpha_h^2 + \frac{1}{P_h^2} \sum_{i \neq 0} \sum_j c_j c_{j-i}^* \sum_m c_m^* c_{m-i} \\ &= 2\alpha_h^2 + \frac{1}{P_h^2} \sum_{i \neq 0} \sum_{j \neq m} \sum_m c_j c_m^* c_{m-i} c_{j-i}^* \end{aligned} \quad (6.33)$$

The parameter α_h is proportional to the energy in a channel realization. The second term in (6.33) represents cross-energy of different channel taps, and it depends on the overall channel energy. It can be seen from (6.33), that when α_h is low, β_h also tends to be low, and vice versa. The channel is considered to be in *deep fade* (i.e., $\text{SNR}(h)$ in (6.26) is low) when α_h is low. Since an outage occurs when a deep fade occurs, we are only interested in low values of α_h . For such values, we approximate $\beta_h \approx 2\alpha_h^2$, i.e., we have approximated the second term in (6.33) to be negligible. We briefly postpone the discussion on the validity of the approximation, and apply it to see its utility. Using the approximations on the spectral shape $\mathcal{S}(\nu)$ and β_h , we can write (6.26) as

$$\text{SNR}(h) \approx \widehat{\text{SNR}}(h) = \frac{4\alpha_h^2\gamma^2}{25\alpha_h^2\gamma^2/S + 10\alpha_h\gamma + 4S}, \quad (6.34)$$

where $\widehat{\text{SNR}}(h)$ denotes approximated $\text{SNR}(h)$. The SNR is now a function of a single channel parameter (α_h), thereby considerably simplifying the subsequent analysis. Using the above expression in (6.32), we get

$$\mathbb{P}_{\text{out}} \approx \Pr \left[\alpha_h^2\gamma^2(4 - 25\Gamma_T/S) - 10\alpha_h\gamma\Gamma_T - 4\Gamma_TS \leq 0 \right]. \quad (6.35)$$

One can show from the above expression that as long as $4S/25 > \Gamma_T$, $\mathbb{P}_{\text{out}}$ is less than 100%, regardless of the value of γ . This follows from the fact that $\text{SNR}(h)$ achieves an asymptote as γ increases. In order to ensure $\mathbb{P}_{\text{out}} < 100\%$, one should choose the system parameters such that the asymptote is greater than Γ_T . This defines a lower limit on S for a particular error threshold. However, we are interested in low values of the outage probability; outage probabilities above 10% are less meaningful.

With respect to the variable α_h , the argument of $\Pr[\cdot]$ in (6.35) is a quadratic inequality. Applying algebra, the positive solution of the inequality, denoted by α_T , is found to be

$$\alpha_T = \frac{S}{\gamma} \cdot \frac{5 + 2\sqrt{(4S/\Gamma_T - 25) + 25/4}}{4S/\Gamma_T - 25}. \quad (6.36)$$

We refer to α_T as the threshold on the channel gain. The outage probability (6.35) can now be written as

$$\mathbb{P}_{\text{out}} = \Pr [\text{SNR}(h) \leq \Gamma_T] \approx \Pr [\alpha_h \leq \alpha_T]. \quad (6.37)$$

The approximation on β_h has enabled us to find $\mathbb{P}_{\text{out}}$ from the probability density function (pdf) of α_h , whereas previously, we had to derive it from joint statistics of α_h and β_h . We now scrutinize the approximation $\beta_h \approx \alpha_h^2$. We generate a channel ensemble of 20,000 realizations, and compare β_h with $2\alpha_h^2$ for different values of $C_D := B_x\tau_{\text{rms}}$. The metric C_D is defined as the *channel diversity factor*, and roughly indicates the number of independent frequency bins in the channel transfer function. (More discussion on C_D at the end of the

Figure 6.5: Comparison of $2\alpha_h^2$ with β_h , for different values of $C_D = B_x \tau_{rms}$.

section.) A dashed line represents the approximation. The difference from the approximation is due to the variations in the second term in (6.33). From the figure, we observe a high absolute error ($|\beta_h - 2\alpha_h^2|$) in the approximation for large values of β_h , but a small absolute error for small values of β_h . However, the relative error is somewhat similar for all values of β_h .

Since the approximations were used to simplify the expression of $\text{SNR}(h)$, it is more meaningful to compare $\widehat{\text{SNR}}(h)$ in (6.34) with $\text{SNR}(h)$ in (6.26). We define the relative approximation error in $\widehat{\text{SNR}}(h)$ as

$$\epsilon_h := \left(\text{SNR}(h) - \widehat{\text{SNR}}(h) \right) / \text{SNR}(h),$$

and plot it as a function of the channel variable α_h for different set of system parameters in Figure 6.6. It is observed that the relative approximation error tends to be small, if α_h is small. Secondly, since the error ϵ_h leans towards a positive value, the approximated $\mathbb{P}_{\text{out}}$ in (6.37) will tend to overestimate the outage probability for large values of α_h . Finally, the approximation is more accurate for low values of the CNR (γ/S). For a fixed transmission power, a small CNR implies a large bandwidth B_x , which, as earlier discussed, tends to *harden* the channel. The channel attains its average state; consequently, the SNR approaches a deterministic value and, hence, the approximation error tends to be negligible. To summarize, the approximations on $\mathcal{S}(\nu)$ and β_h are restrictive, and do not hold for all values of the system parameters. A large error can be expected for large values of γ/S . Nonetheless, $\beta_h = 2\alpha_h^2$ should hold fairly for small α_h ; it will be later shown with simulation results that one can find the outage probability with a reasonable accuracy in cases of interest.

Probability distribution of α_h

In order to solve (6.37), one requires knowledge of the PDF of α_h . The parameter α_h in (6.27) is a sum of independent exponential distributed random

Figure 6.6: Relative approximation error ϵ_h , plotted against α_h , for system parameters $\{B_x \tau_{rms}, \gamma/S\}$ clockwise from top-left as $\{2, 0.5\}$, $\{2, 1\}$, $\{4, 0.5\}$, and $\{4, 0.25\}$.

variables, with each Gaussian element c_k having a zero mean but a distinct variance σ_i^2 . The statistics of such a variable (α_h) have been previously treated in renewal theory [95, p. 17] and more recently in the studies of MIMO systems [96]. It follows a generalized χ^2 distribution with a PDF,

$$f_{\alpha_h}(u; N, \sigma_i^2) = \sum_{i=1}^N \frac{\exp(-u/\sigma_i^2)}{\sigma_i^2 \prod_{j=1, j \neq i}^N (1 - \sigma_j^2/\sigma_i^2)}, \forall u \geq 0 \quad (6.38)$$

where N is the maximum number of the exponential distributed variables. In channel modeling, the impulse response (6.6) is usually truncated to a duration, such that most of the energy is received in that duration. In such case, N represents index of the last tap of the truncated channel, and is directly related with the maximum excess delay τ_m .

For two different values of $C_D = B_x \tau_{rms}$, we generate channel ensembles and plot the histograms of α_h in Figure 6.7. Using (6.38), we estimate and plot the PDF of α_h . For reference, a PDF estimate of α_h using Gaussian distribution⁶ is also plotted. The Gaussian PDF estimate is found from the empirical mean and variance of α_h . Owing to the CLT, the PDF of α_h approaches a Gaussian

⁶In modeling α_h , we also used a Gaussian approximation for the PDF of α_h . However, the theoretical results, using the Gaussian approximation, deviated a lot from the simulation results. This can also be seen from Figure 6.7, that a Gaussian PDF is a poor approximation. Therefore, the idea was not further explored.

Figure 6.7: Generalized χ^2 and Gaussian PDF fits onto the histograms of α_h .

PDF for a very large C_D . Nonetheless, the figure shows that the generalized χ^2 distribution is a valid estimate for different values of C_D . Secondly, the Gaussian approximation poorly models the lower tail of the histogram, which is the most relevant part in estimating the outage probability.

Using the PDF in (6.38), one can write (6.37) as

$$\begin{aligned} \mathbb{P}_{\text{out}} &= \Pr[\alpha_h < \alpha_T] = \int_0^{\alpha_T} f_{\alpha_h}(u, N, \sigma_i^2) du \\ &= \sum_{i=1}^N \frac{1 - \exp(-\alpha_T/\sigma_i^2)}{\prod_{j=1, j \neq i}^N (1 - \sigma_j^2/\sigma_i^2)}, \end{aligned} \quad (6.39)$$

where α_T is given in (6.36) and σ_i^2 follows from the PDP of the channel. The equation in (6.39) represents the approximated outage probability of an N-FOM communication link in dense multipath fading channels.

6.4.2 Minimum required energy and outage

Theoretically, an outage-free communication link is impossible. In practice, a certain level of downtime (outage probability) is tolerated, and it is interesting for the link design to know how much should the transmitter increase its power to keep the outage probability below this threshold. The transmission power is directly related to the received signal level, as discussed in Figure 1.9. Therefore, we wish to find a relation between $\gamma = E_b/N_0$ and $\mathbb{P}_{\text{out}}$.

Let us consider that we wish to keep $\mathbb{P}_{\text{out}}$ less than a certain *tolerable outage* ρ . We wish to find the required $\gamma = E_b/N_0$ at the receiver to remain below a tolerable level of outage, ρ . We denoted this parameter by γ_ρ . Using (6.38), γ_ρ is the positive and real root of the function

$$f(\gamma) = \sum_{i=1}^N \frac{1 - \exp(-\alpha_T/\sigma_i^2)}{\prod_{j=1, j \neq i}^N (1 - \sigma_j^2/\sigma_i^2)} - \rho, \quad (6.40)$$

where γ is manifested in α_T . The equation does not seem to be analytically solvable. Therefore we will numerically evaluate the above function to find γ_ρ .

Channel diversity factor

In multiple antenna wireless systems, the channel diversity is defined as the number of distinct independent paths from a transmitter to a receiver. In single antenna systems, where multipath propagation is involved, this is roughly equal to number of significant channel taps which are usually considered independent. In many theoretical models, the number of significant channel taps is given by $N = B_x \tau_{\text{rms}}$. However, with such a definition of N , most but not all of the channel energy is captured in the delay window $[\Delta\tau, N\Delta\tau]$. For an exponential PDP, around 64% of the channel energy lies in this interval. In our analysis, we consider an exponential PDP, and the number of channel taps will be determined by the 20 dB τ_m . For a 20 dB τ_m in an exponential PDP, the maximum number of taps will be $N = B_x \tau_m = 5B_x \tau_{\text{rms}}$, which captures 99% of the total energy. Finally, in order to conveniently describe the channel behavior with a single parameter, we define $C_D = B_x \tau_{\text{rms}}$ in our analysis and simulations. Therefore, $C_D = 1$ in the next section should not be confused with a single tap channel.

6.5 Results and discussions

In this section, we discuss the performance of the N-FOM system in dense frequency selective fading channels. The goal of this section is to see how the channel impairments affect the overall link performance. Secondly, based on the interplays between propagation effects and design parameters, we aim to identify suitable values of the parameters for practical implementation.

In Section 6.5.1, we describe the simulation framework that we have adopted for the performance analysis. Since the propagation environment can have multiple effects on the link's performance, we isolate and discuss these effects separately. The effect of fading is studied in Section 6.5.2, and that of delay dispersion in Section 6.5.2. Next, we study performance of the system in terms of the outage probability in Section 6.5.4. Finally, based on the observed performance trends, we identify suitable values of system parameters in Section 6.5.5.

6.5.1 Simulation framework

The steps followed for achieving the results are depicted in Figure 6.8. The channel model used in the simulation framework is based on the TDL channel description. For a particular multipath environment (fixed τ_{rms}), we generate a set of NLOS channel realizations, with and exponential PDP, i.e.,

$$P_h(\tau) = C \exp(-i\Delta\tau/\tau_{\text{rms}}), \quad i \geq 1.$$

where C is a constant that ensures $\sum_i |c_i|^2 = P_h$. The taps c_i are modeled as circularly symmetric complex Gaussian variables, each having a zero mean, but a distinct variance σ_i^2 . We truncate the PDP to a length N . The total number of taps is based on 20 dB τ_m , i.e., $N = 5B_x\tau_{\text{rms}}$.

The PSD of the noise-based spreading signal is shaped by a type II Chebyshev filter with a 40 dB stopband ripple. A polar NRZ message signal is chosen, and unless specified otherwise, the frequency offset is chosen as a unit multiple of the symbol rate i.e., $f_r = 1/T_b$. The channel diversity factor is defined as $C_D = B_x\tau_{\text{rms}}$. Furthermore, we define $\varphi := \tau_{\text{rms}}/T_b = C_D/S$ and $\varpi := f_r\tau_{\text{rms}}$ as indicators of ISI and RDE respectively. For negligible ISI, φ should be much smaller than unity. Similarly, for negligible RDE, ϖ should be much smaller than unity. One can observe that for $f_r = 1/T_b$, we have $\varphi = \varpi$.

For a particular multipath environment (fixed τ_{rms} and PDP), we generate a set of NLOS channel realizations. We then simulate the N-FOM NLOS link, obtaining a BER value for each realization. The $\overline{\text{BER}}$ simulation results are found by averaging the BER over all channel realizations. As far as outage probability is concerned, the simulation results are obtained by finding percentage of the total channel realizations over which the simulated link results in $\text{BER}(h) > \mathcal{E}_T$. Next, we find the semi-analytical or theoretical results. After computing the parameters $\{\alpha_h, \beta_h\}$ for a given channel description, we evaluate (6.26) and (6.28) to derive *semi-analytical* results for $\text{BER}(h)$. By averaging $\text{BER}(h)$ over the channel ensemble, we obtain the semi-analytical $\overline{\text{BER}}$ results. The theoretical results for $\mathbb{P}_{\text{out}}$ are found by evaluating (6.39), for the given channel environment.

Finally, we add that the dimensionless parameters $\{S, C_D, \varphi\}$ are all coupled with each other. Varying one of the basic parameters $\{\tau_{\text{rms}}, B_x, T_b\}$ will alter two of the dimensionless parameters. Therefore, in the subsequent figures, some curves will be plotted as functions of two of the dimensionless parameters, although only one of the basic parameters is varied.

6.5.2 Effects of fading

Before we present the results, we comment on the accuracy of the $\text{SNR}(h)$ expression in (6.24). We simulate the N-FOM system in a channel ensemble of 1000 realizations, for two different value of S , and a particular value of γ . The semi-analytical $\text{BER}(h)$ and simulated BER per realization are plotted in Figure 6.9. One can see that $\text{BER}(h)$ in (6.28) is not precise for all channel realizations. This is explained in the following paragraph.

In Chapter 2, we discussed that the disparity between the simulation and theoretical results follow from the weak CLT approximation. The difference is especially magnified in the high-CNR regime ($\gamma/S \gg 0.56$). In a multipath channel, the problem is two-fold. The high-CNR regime is not determined by γ/S but by $\alpha_h\gamma/S$. For moderate values of γ (say 20 dB), a strong channel state ($\alpha_h \gg 1$) will result in a high $\gamma\alpha_h$. In other words, the NLOS link can be in a different CNR-regime for each channel realization. We observe from the figure

Figure 6.8: Work flow for generating semi-analytical and theoretical results, with additional steps for computing outage probability (depicted in blue).

Figure 6.9: Comparison of BER per channel realization between simulation and semi-analytical evaluation of the system.

that the semi-analytical $\text{BER}(h)$ is higher than the simulated BER for channel realizations with a high gain (large α_h). However, for small values of γ/S , the deviations are less prominent. Therefore, there will be disparity between the simulation and theoretical results in the following discussions. Nonetheless, we expect small differences between the results for the average BER. Since the average BER is dominated by realizations in deep fade (low $\text{BER}(h)$), the effect on the $\overline{\text{BER}}$ will be minimal.

We now study the effect of fading on the average BER of the N-FOM NLOS link. We simulate the system in a channel ensemble of 1000 realizations and find the BER by averaging the BER over all realizations. By semi-analytical evaluation of (6.29) over the channel ensemble, we get the *semi-analytical* BER. Figure 6.10 illustrates the BER for the N-FOM system in the frequency-selective fading channel for different values of C_D . To isolate the fading effects, we have chosen the system parameters such that $\varphi = \varpi \ll 1$. For reference, we also plot the performance curve for BPSK system in both Rayleigh fading (NLOS) and AWGN (LOS) channels. The SNR of a BPSK link in Rayleigh fading channels is $2|h|^2\gamma$, where $|h|^2$ is an exponential distributed variable [97] with a PDF $f_{|h|^2}(u) = \exp(-u)$ for $u \geq 0$. The BER is derived as

$$\text{BER} = \int_0^\infty Q(\sqrt{2|h|^2\gamma}) f_{|h|^2}(u) du = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma}{1-\gamma}} \right). \quad (6.41)$$

The figure shows a degradation in performance of the N-FOM system due to multipath fading. However, a small fading margin is required in comparison with a standard BPSK system, particularly at high values of E_b/N_0 . At $\overline{\text{BER}} = 10^{-3}$ in fading channels, the BPSK system can have up to a 5-dB performance penalty compared to the N-FOM system with $S=200$. Moreover, we observe that for the chosen sets of system parameters, the theoretical $\overline{\text{BER}}$ closely follows the the average performance of the simulated system. It indicates that one can fairly approximate $\overline{\text{BER}}$ from the semi-analytical results, as earlier discussed.

Finally, a higher channel diversity factor C_D further improves the perfor-

(a) For spreading factor $S = 100$. (b) For spreading factor $S = 200$.

Figure 6.10: Average BER against average $\gamma = E_b/N_0$ for different values of C_D , in an ISI- and RDE-free scenario.

mance. For $C_D \rightarrow \infty$, the channel is *hardened*, i.e., it behaves deterministically. The parameters α_h and β_h assume values of 1 and 2 respectively, i.e., (6.26) reduces to

$$\text{SNR}(h) = \frac{8\gamma^2}{50\gamma^2/S + 20\gamma/S + 8S} \quad (6.42)$$

The hardened channel case is a hypothetical scenario. As the channel tends to be hardened, the variations in $\text{SNR}(h)$ tend to be negligible. Therefore, the hardened channel case provides a lower bound on the average BER. For this hypothetical scenario, a performance curve is depicted in the figures. One concludes that with increasing C_D , the performance achieves an asymptote. To shed further light on the matter, we study the above expression for different values of the spreading factor S , in Figure 6.11. Analogous to (2.40), the curve for $S_{\text{opt}} = 5\gamma_{\text{dB}}/2$ is found from optimizing the above expression w.r.t. S , and substituting the result in (6.28). This curve represents the best possible error rate performance in dense multipath fading channels, for any set of values of the system parameters. The figure shows that, in favorable channel conditions, a spreading factor of approximately 200 is optimal for an average BER of 10^{-3} , which is achieved for $\gamma = 18.5$ dB. Although these values apply to a hypothetical scenario, this information serves as a guideline for further analysis.

We now compare the performance achieved using a pure noise carrier and that using a PN carrier. Without using any sort of spectrum shaping techniques, we simulate the two systems. The RFE filter was kept the same (Chebyshev type II) as in earlier simulations. For a spreading factor of 100, and $C_D = 2$,

Figure 6.11: Comparison between BER of N-FOM systems with different spreading factors S , in a hardened channel environment.

the average BER is depicted in Figure 6.12. The plot shows insignificant difference between the two systems, in terms of γ . As earlier discussed in Section 6.3.4, the TR system using a PN carrier also suffers from self-interference. Therefore, in dense multipath fading environments, using a PN carrier has little added value. Nonetheless, the PN carrier has a better PAPR than a pure noise carrier. Therefore it might be a preferred choice from an implementation perspective. Although this thesis does not look into implications of using PN carriers, it could be a potential research direction. We will comment on this in Chapter 7, but for the rest of the analysis, we will use the pure noise carrier.

Figure 6.12: Comparison of average BER between two N-FOM systems, one using a pure noise and the other using a PN carrier, with a fixed $S = 100$.

6.5.3 Effects of delay dispersion

As earlier discussed, the N-FOM system exhibits improved performance in a channel environments with a large $C_D = B_x \tau_{\text{rms}}$. However, for environments with large delay spreads (and hence large C_D), the delay dispersiveness of the channel can introduce ISI and RDE. In previous subsection, we had assumed the values of f_r and T_b such that $\varphi = \tau_{\text{rms}}/T_b \ll 1$, and $\varpi = f_r \tau_{\text{rms}} \ll 1$. To investigate the effect of delay dispersion, we fix the spreading factor and plot the average BER against different values of C_D and $\varphi = \varpi$ in Figure 6.13. Although, the curves are presented as functions of two dimensionless parameters $\{C_D, \varphi\}$, the only parameter varied on the x-axis is the delay spread τ_{rms} .

We observe that the average BER as a function of C_D is concave up, and has a minimal value for a certain range of the channel diversity factor. This can be

Figure 6.13: Average BER against a combination of C_D and φ .

explained as follows. Low channel diversity factor implies fewer channel taps. Therefore, a tap in a deep fade has a strong influence on the average BER. For large values of the channel diversity factor, the number of taps is large. A tap in a deep fade has a smaller influence on the total signal level since the other taps provide diversity. This results in an improved performance for a large C_D . However, for a much larger C_D , the ISI- and RDE-free assumptions ($\varphi \ll 1$, $\varpi \ll 1$) do not hold, and hence performance degradation is observed. This is clearly evident from the differences between the simulation and the semi-analytical results. In contrast with the simulated $\overline{\text{BER}}$ curve, the semi-analytical curve shows improved error rates as C_D increases. This is because

the semi-analytical results ignore ISI and RDE. Therefore, low error rates are observed due to improvement in C_D . Eventually, the curve approaches the hardened channel case.

Moreover, even for low values of φ , we observe differences between the simulation and theoretical results in Figure 6.13a. As discussed in Section 6.5.2, this is due the loose CLT approximation, particularly for large values of γ/S . One further observes in the figure, that the performance of the simulated system degrades roughly at $\varphi \geq 0.025$. The figure does not help in identifying whether the degradation is due to ISI or RDE, or both. To isolate the impairments due to RDE, one can choose a large frequency offset (large ϖ) and a small data rate $1/T_b$ (small φ). We fix $\varphi \ll 1$ and $C_D = 2$, and simulate an N-FOM NLOS link for different values of f_r . Since a large f_r implies a large operational bandwidth, the stopband of the receiver front-end filter is also adjusted accordingly.

Figure 6.14 illustrates the $\overline{\text{BER}}$ -vs- ϖ metric for a fixed $C_D = 2$ and $\gamma = 22$ dB, and different values of S in the ISI-free scenario. The ISI-free scenario is ensured by sending bits of same polarity, i.e., $b_i = b_{i-1}$, $\forall i$. We see that the performance degrades if one increases f_r . This was earlier explained in the context of (6.31); RDE impairs the link performance for any given value of f_r . The performance degradation for high values of ϖ is of serious concern. The MA capability of N-FOM relies on the use of different frequency offsets for different links. Moreover, as discussed in Chapter 4, larger frequency offsets are desired to reduce the effects of MUI. However, the performance curves indicate that large

Figure 6.14: Average BER against increasing values of ϖ , and a fixed $C_D = 2$ and $\gamma = 22$ dB.

values of ϖ (and hence f_r) lead to decoherence between the reference and the information signal, and the NLOS link performance appears to be sensitive to the impairments due to RDE.

In order to assess the penalty of using a high frequency offset on the received SNR per bit $\gamma = E_b/N_0$, we fix S and C_D , and simulate the N-FOM link for different values of f_r . Figure 6.15 illustrates the average BER as a function of γ for different values of $\varpi = f_r \tau_{\text{rms}}$. It is seen that for the given channel model, the penalty for a system using $\varpi = 0.1$ compared to a system with $\varpi = 0.01$ is around 2 dB, at an average BER of 10^{-3} . Secondly, the power penalty grows larger for each step increase in f_r (or ϖ); for instance, the penalty is a fraction of a dB between choosing $\varpi = 0.01$ and $\varpi = 0.03$, but it is more than a dB between choosing $\varpi = 0.07$ and $\varpi = 0.09$. Clearly, large values of f_r are not feasible. Therefore, one should choose f_r such that the power penalty can be supported by the link budget. For example, simulation results for different system parameters indicate that the penalty in required E_b/N_0 to achieve an average BER of 10^{-3} is less than a dB, as long as $\varpi \leq 0.05$.

One may employ DSP units to reduce the effects of RDE and ISI. However, the resulting complexity of the receiver does not resonate with the design goals of N-FOM. Therefore, one may carefully choose the system parameters such that ISI and RDE is negligible. In this regard, we conclude from the observations from Figure 6.13 till 6.15 that, as a rule-of-thumb, one should choose the data rate such that $\varphi \leq 0.025$, and the frequency offset such that $\varpi \leq 0.05$, in order to have negligible ISI and RDE. If this criterion is chosen, then the highest value of the frequency offset is $f_U = 0.05/\tau_{\text{rms}}$, and the number of available frequency offsets is $N_a = f_U T_b = .05/\varphi$ in (4.1). Table 6.1 presents the available and usable frequency offsets $\{N_a, N_u, N_{\bar{u}}\}$ for different data rates and channel delay spreads.

It is important to remind that N_u incorporates the criterion $f_i - f_j \neq 2f_i$ for the set of all possible users $\{i, j\}$. However, as concluded in Chapter 4, that in order to minimize cross-talk in an MA environment, one should choose the frequency offsets such that there is a large differences between the offsets used by two concurrent links. Therefore, if one chooses the offsets such that $f_i - f_j \geq 4/T_b$, then the number of usable offsets is reduced. This is denoted by $N_{\bar{u}}$ in the table. The values suggest that the MA capability of the N-FOM system comes with restriction on the highest data rate, for a given multipath environment. Moreover, the number of concurrent transmissions can be considerably limited. Finally, it must be stressed that the criteria on the data rate and the frequency offset are a rule-of-thumb, based on simulation results for a dense multipath channel that follows an exponential PDP. Therefore, the criteria can be different for different channel models and signaling schemes.

6.5.4 Outage probability performance

We now investigate the outage probability of N-FOM, which, as earlier discussed, can be a more meaningful metric in monitoring applications. We first

(a) For spreading factor $S = 200$. (b) For spreading factor $S = 400$.

Figure 6.15: Average BER versus $\gamma = E_b/N_0$ for a fixed $C_D = 4$, $\varphi \ll 1$, and different values of RDE parameter $\varpi = f_r \tau_{\text{rms}}$.

Table 6.1: Available and usable frequency offsets for different data rates and multipath environments, with criterion $f_r \tau_{\text{rms}} \leq 0.05$ and $\varphi \leq 0.025$.

data rate	τ_{rms}	φ	$f_U = 0.05/\tau_{\text{rms}}$	N_a	N_u	$N_{\bar{u}}$
10 kbps	100 ns	0.001	500 kHz	40	26	10
	200 ns	0.002	250 kHz	20	14	5
	500 ns	0.005	100 kHz	8	5	2
25 kbps	100 ns	0.0025	500 kHz	16	11	4
	200 ns	0.005	250 kHz	8	5	2
	500 ns	0.012	100 kHz	3	2	1
50 kbps	100 ns	0.005	500 kHz	8	5	2
	200 ns	0.01	250 kHz	4	3	1
	500 ns	0.025	100 kHz	1	1	1

study the outage probability $\mathbb{P}_{\text{out}}$ of the system, and then study the minimum energy required to remain below a certain value of $\mathbb{P}_{\text{out}}$.

As illustrated in Figure 6.8, the analysis of outage probability comprises of three parallel tracks. We begin with defining the basic system and channel parameters. For the theoretical values of $\mathbb{P}_{\text{out}}$, presented in Section 6.4; we find α_T in (6.36); generate the PDF of α_h using the generalized χ^2 model; and find the area of the PDF where $\alpha_h < \alpha_T$. For the semi-analytical results; we generate a channel ensemble; compute (6.28) for each channel realization; and calculate the occurrences of $\text{BER}(h) \geq \mathcal{E}_T$. In order to assess the validity of the theoretical and semi-analytical results, we find $\mathbb{P}_{\text{out}}$ from simulation. The N-FOM NLOS communication link is simulated for the given channel ensemble, and the outage probability is computed with 99% confidence interval. It was earlier established that our semi-analytical model is imprecise for large values of $\alpha_h \gamma / S$. However, in the context of $\mathbb{P}_{\text{out}}$, the differences in the results will be significant mainly if the semi-analytical $\text{BER}(h)$ poorly approximates the simulated $\text{BER}(h)$ values, close to the threshold $\mathcal{E}_T$ in Figure 6.9.

In all subsequent analysis, we consider $\mathcal{E}_T = 10^{-3}$. In line with the semi-analytical and theoretical $\mathbb{P}_{\text{out}}$ models, the effects of ISI and RDE are ignored by choosing small $\varphi = \varpi \ll 1$. Due to higher run times and memory issues associated with Monte Carlo simulations for accurate estimates of $\mathbb{P}_{\text{out}} < 1\%$, the system is not simulated for such low values of the outage probability. Furthermore, we also study the outage probability of a BPSK system for comparison. From (6.41), the outage probability of a BPSK system in Rayleigh fading (NLOS) channel can be solved to $\mathbb{P}_{\text{out}} = 1 - \exp(-\Gamma_T / (2\gamma))$.

In Figure 6.16, we plot $\mathbb{P}_{\text{out}}$ against γ for different values of the system parameters. A couple of observations are made. First, the theoretical and simulation results have some differences, which are more pronounced at high values of the outage probability ($\mathbb{P}_{\text{out}} \geq 20\%$). This is likely due to the approximation $\beta_h = 2\alpha_h^2$ made in Section 6.4. For high values of α_h (and hence high α_T and $\mathbb{P}_{\text{out}}$), the approximation is inaccurate. Moreover, better agreements are observed for a large spreading factor in Figure 6.16b. As earlier discussed, these

(a) For spreading factor $S = 200$. (b) For spreading factor $S = 400$.**Figure 6.16:** $\mathbb{P}_{\text{out}}$ against γ , for different system and channel parameters.

deviations are due to the inaccuracies in CLT approximations, particularly at high values of $\alpha_h \gamma / S$.

Nonetheless, the differences exist prominently at high values of $\mathbb{P}_{\text{out}}$, which are not interesting for link design. In the significant region ($1\% \leq \mathbb{P}_{\text{out}} \leq 10\%$), the difference in required γ for a certain $\mathbb{P}_{\text{out}}$ is a fraction of 1 dB. This indicates that for practical design, one can fairly approximate the outage probability of the N-FOM system from the provided theoretical model. Moreover, an enhanced channel diversity (large C_D) reduces the received energy required to attain a certain level of outage probability. This is an expected result, which follows from the fact that the N-FOM system exploits channel diversity.

To investigate this further, we analyze γ_ρ , which as defined earlier, is the $\gamma = E_b/N_0$ required to remain below a tolerable value of outage probability ρ . For $\mathcal{E}_T = 10^{-3}$, γ_ρ is plotted against C_D in Figure 6.17 for different values of ρ . As a reference, we also plot γ_ρ for BPSK, which can be shown to be $\gamma_\rho = -\Gamma_T / (2 \ln(1 - \rho))$. We see that a high channel diversity factor leads to reduction in γ_ρ but little gain is achieved beyond $C_D \geq 4$. In fact, for a very large C_D , it seems that γ_ρ attains an asymptote. This is a plausible result; for a fixed value of $\mathbb{P}_{\text{out}}$, γ cannot be further reduced by improving the channel diversity (for example, by increasing the bandwidth). The hard limit on γ_ρ is imposed by the performance of the N-FOM system in the hardened channel case, as per our discussion related to Figure 6.11. For outage-free communication, this asymptote is 18.5 dB for $S=200$, and is around 19 dB for $S=400$.

Finally, we comment on comparison of the N-FOM system with a BPSK system, in the context of outage probability. We observed from Figure 6.16 that for low-to-moderate values of γ (i.e., $\gamma \leq 21$ dB), a BPSK system can achieve a better outage probability in multipath propagation environments. However, the values of outage probability achieved for such values of γ might be undesired. This is further revealed in Figure 6.17. In multipath propagation environments, N-FOM has an advantage over BPSK only if $\mathbb{P}_{\text{out}} \leq 5\%$ is desired.

6.5.5 Suitable system parameters

We considered an N-FOM system – which employs a polar NRZ signaling scheme – in a dense multipath channel that follows an exponential PDP. The results reveal a number of propagation effects, that are linked with the system parameters. First, the channel fading varies the energy of the demodulated signal. The N-FOM system exhibits considerable robustness against frequency-selective fading, particularly at large values of $C_D = B_x \tau_{\text{rms}}$. The delay dispersiveness of the channel leads to ISI as well as RDE. The implication of these effects can be significant, and it is better to choose the system parameters such that the effects of these impairments are minimized. For the studied model, it was found that for negligible ISI, the symbol rate ($1/T_b$) should be roughly chosen such that $T_b \geq 40 \tau_{\text{rms}}$. Moreover, for a tolerable RDE, a frequency offset should be roughly chosen, such that $f_r \tau_{\text{rms}} \leq 0.05$.

This paints a complicated picture. A number of trade-offs are involved,

(a) For spreading factor $S = 200$. (b) For spreading factor $S = 400$.**Figure 6.17:** Required γ to operate at $\mathbb{P}_{\text{out}} \leq \rho$ against C_D , with $\varpi = \varphi \ll 1$.

and choosing optimal values of the system parameters depend on the design objective and the operational environment. If the multipath environment is considerably dispersive then the highest data rate and the number of concurrent MA links are limited. For a channel with an exponential PDP, the tentative values for data rate and the number of concurrent MA links were listed in Table 6.1. Moreover, a large spreading factor $S = B_x T_b$ enhances the channel diversity, and leads to an improved performance. In a hypothetical case where the channel is hardened, the optimal spreading factor is 200 at an error threshold of 10^{-3} . However, in practice, a slightly larger spreading factor might be preferred, for better resistance to frequency-selective fading or ISI. A very large spreading factor is also not preferred. This is because the noise contribution from AWGN is enhanced with an increase in the spreading factor. For a considerable improvement in the average BER and the outage probability, one should operate in the high-CNR regime ($\gamma/S \gg 0.56$).

6.6 Summary

In this chapter, the performance of the N-FOM communication system in a dense frequency-selective fading channel was investigated. A quasi-analytical expression for BER was derived and the channel impairments were studied in relation with the design parameters. In addition to fading and ISI, the N-FOM link also suffers from RDE, which arises due to dissimilar channel response over the reference and the information signals. On the one hand, the system is robust to frequency-selective fading; the self-correlation operation effectively collects the channel energy. At $S = 200$, less than 3 dB of fade margin is required to operate at an average error rate of 10^{-3} . On the other hand, ISI and RDE resulting respectively from using a large frequency offset f_r or a large data rate can significantly limit the performance, particularly in the high-CNR regime. For a given channel model, a rule-of-thumb criterion for the values of data rate and frequency offset was suggested, which ensures negligible RDE and ISI. However, these restrictions weaken the MA capability of N-FOM. For a data rate of 10 kbps in an office environment ($\tau_{\text{rms}} \approx 100$ ns), one can choose from a total of 40 available offsets, with a maximum of 10 concurrent links.

An approximated analytical expression for the outage probability $\mathbb{P}_{\text{out}}$ was also derived, which was shown to be fairly estimate γ in the region of interest $\mathbb{P}_{\text{out}} \leq 10\%$. For an error threshold of 10^{-3} , it was found that the N-FOM system can operate at $\mathbb{P}_{\text{out}} < 1\%$ for a nominal value of $\gamma = 23$ dB. A slight increase in γ or channel diversity factor can considerably improve the outage probability. Comparisons with a BPSK system suggest that although N-FOM suffers greatly from noise-enhancement, it can be beneficial in multipath environments where outage probabilities of less than 5% are desired.

Conclusions and research directions

In this thesis, we presented and studied a noise-based frequency offset TR system (N-FOM) in the context of its feasibility for low data rate WSNs. The motivation for N-FOM stems from its fast synchronization and simple receiver architecture which enables an inexpensive design, in terms of power and circuitry. Since these features are highly desired in monitoring applications where WSNs operate in remote or accessible terrains, the N-FOM communication scheme makes an interesting research subject. In this thesis, the N-FOM communication system was investigated from the physical layer perspective, and insights about the strengths and constraints of the scheme were presented.

In line with the research questions stated in Section 1.6, we summarize the conclusions that can be drawn from the research. Moreover, some directions for further research will be suggested.

7.1 Conclusions

The N-FOM receiver has a simple detection mechanism. Unlike the traditional DSSS systems, the receiver recovers the desired information without the need of complex rake receiver implementation and associated channel adaptation. Nevertheless, the simplified detection mechanism comes at the cost of noise enhancement. The mixing of the received signal with itself creates multiple wideband noise terms. Particularly, the self-mixing of the transmitted signal also elevates the noise level. Hence, with increasing transmission power, the link quality achieves an asymptote. In the low-CNR regime, the total SNR is dictated by the self-mixing of AWGN, whereas in the high-CNR regime, it is dominated by the self-mixing of the transmitted signal. Thus, the performance of an N-FOM link is limited in both low- and high-CNR regimes. In a LOS link, there is at least 10.8 dB of performance penalty compared to a coherent BPSK system. Nonetheless, in the high-CNR regime, the link performance

is substantially improved by using a high spreading factor, albeit, at the cost of a slight performance degradation in the low-CNR regime. Therefore, the link performance has a non-monotonic relation with the spreading factor. An optimal spreading factor exists which scales linearly with the received SNR per bit $\gamma = E_b/N_0$.

In a sensor network where concurrent transmissions are expected, MA is enabled by use of different frequency offsets. With the knowledge of a frequency offset used by each communicating node, one can establish point-to-multipoint links. However, the analytical and simulation results reveal interference-limited nature of N-FOM. It was found that, due to magnification of MUI by the self-mixing operation, a strong signal from a nearby interfering node can disrupt communication with a faraway node. In the best case scenario, where the transmission power of each node is adaptively controlled and the frequency offsets are suitably chosen, three to four simultaneous LOS links can be established with an acceptable quality ($\text{BER} \leq 10^{-3}$). Therefore, N-FOM has a limited potential in a dense network with a large amount of traffic.

Similarly, the N-FOM has limited tolerance to NBI, which is in contrast to a coherent DSSS receiver. The NBI self-mixing product, at the output of the mixing operation, is the dominant noise term. For an interferer with a bandwidth smaller than the data rate of the N-FOM system, the system exhibits better resilience to NBI, particularly in comparison with an energy detector. On the other hand, if modulated NBI with a bandwidth much larger than the data rate of the N-FOM system is considered, then the dominant noise term is not suppressed by the detection mechanism. In this case, the total SNR of the N-FOM link reduces quadratically with the CIR, and also depends on the bandwidth of the interferer. Nonetheless, for both type of interferers, maintaining a LOS link with a $\text{BER} \leq 10^{-3}$ becomes optimistic for CIRs below -5 dB.

In dense multipath fading environments, the self-correlation operation collects energy from all multipath components. Employing large operational bandwidths in a rich scattering environments enhances the channel diversity, and one can operate at $\text{BER} = 10^{-3}$ with less than 3 dB of fade margin. A semi-analytical expression for the average BER was derived. The results indicate that the system is robust to frequency-selective fading. Furthermore, the immunity of N-FOM to small-scale fading relies on coherence between the reference and information signal. A large frequency offset implies that the two signals will be affected differently by a frequency-selective fading channel. It was found that this leads to the reference-decoherence effect (RDE). Similarly, the delay dispersion can introduce ISI, which is further enhanced if high data rates are employed. A rule-of-thumb criterion on the frequency offset and the data rate was presented, in order to ensure negligible ISI and RDE.

In addition to the average BER, an expression for outage probability $\mathbb{P}_{\text{out}}$ was also derived, provided some approximations on the channel parameters are allowed. It was found that one can achieve an outage probability of less than 5% for a nominal value of the average received SNR per bit $E_b/N_0 = 22$ dB. In

a high-CNR regime, a small increase in the received signal energy significantly reduces the outage probability. Finally, although N-FOM is robust to fading, the added value of the system manifests mainly in those scenarios where $\mathbb{P}_{\text{out}} \leq 5\%$ is required.

7.1.1 Design parameters

1. *The frequency offset f_r* is a key design parameter, and its value has numerous practical implications. First, for coherence between the reference and the information signals, the frequency offset must be much smaller than the channel coherence bandwidth. Second, the design choice should enforce orthogonality between the two signals over one symbol time. Orthogonality is ensured by choosing the frequency offset from a specific set of values, and depends on the symbol rate and the signaling scheme. Finally, in order to minimize crosstalk in a MA link, it is preferable to have a large difference between the frequency offsets used by the different links. The conditions on the frequency offset limit not only the maximum data rate, but also the highest usable frequency offset, thus limiting the MA capability. As an example, an N-FOM network can support up to 10 mutually concurrent links, at a data rate of 10 kbps using BPSK modulation.
2. *Choice of the spreading factor $S = B_x T_b$* depends on the design goal. It was found that in a point-to-point LOS link, the optimal spreading factor for operating at BER of 10^{-3} and 10^{-6} , is approximately 20 dB and 23 dB respectively. In NLOS links, large bandwidths are preferred for resistance against frequency-selective fading. Therefore, the bandwidth should be chosen such that the channel diversity $C_D = B_x \tau_{\text{rms}}$ is large. A channel diversity of 3 or 4 is sufficient, if a BER of 10^{-3} is the desired link quality.

7.1.2 Overall conclusion

To summarize, N-FOM as a communication principle – although simplifies the receiver – has limited utility for sensor applications where scalability, immunity to interference, and ultra-low power design are indispensable. The noisy detection mechanism is central to most shortcomings. It is the prime performance limiting factor which makes it both a noise- and an interference-limited system. Although N-FOM retains some positive features of a coherent DSSS system, such as robustness to fading, it loses on interference-robustness. The risk of MUI can be reduced by employing efficient channel access schemes and MAC protocols. However, external interferers, such as WiFi or Bluetooth, can potentially prevent the communication. Despite its performance penalty compared to the coherent DSSS system, its faster synchronization and simpler receiver design might make it valuable for a niche of sensor applications. Sparse WSNs in remote, interference-free, rich scattering environments, for ex-

ample for monitoring of areal evaporation, plant-animal interactions, or other eco-physiological factors, could benefit from N-FOM.

For possible success of the communication scheme, it seems imperative that one treats the noise-enhancement problem of the detection mechanism. Additional post and pre-processing schemes can be applied, in order to suppress the noise from the mixing operation. Such schemes bring complexity; in order to explore the trade-offs between complexity and performance, a system-level design of N-FOM should prove beneficial for its utility.

7.2 Recommendations

In this thesis, the N-FOM principle was investigated for its potential in a low-power and low data rate sensor network. Without compromising the receiver complexity, for example by adding DSP modules, adaptive filtering or coding techniques, the N-FOM system was found to have limited applicability. With added receiver complexity, the underlying TR scheme has been subject of a number of studies, where suppression of interference is achieved by either coding, adaptive filtering or a combination of both [98, 99]. Nevertheless, in line with the vision of this project, this section provides recommendations and research directions which may improve the overall utility of N-FOM without considerably increasing the implementation and power costs.

First, we point at research directions, that builds on the work carried out in this thesis. Next, we discuss improvements in the system model and provide an outlook.

7.2.1 Extensions

Throughout this thesis, we have assumed perfect frequency lock between the frequency offset LO at the receiver and that at the transmitter. Although the low frequency offset is easy to track and lock-on to, it has to be further investigated. Since the frequency offset and symbol timing could be coupled, a mismatch in frequency offset could also lead to errors in synchronization and detection. This is particularly crucial for an MA scenario, where multiple users are communicating with different frequency offsets. A mismatch in frequency offset and correspondingly in sampling time, might lead to severe crosstalk. For phase synchronization in N-FOM, a first order phase-locked loop (PLL) was investigated in [100]. The research could be extended for frequency offset synchronization.

In this thesis, BPSK modulation was adopted for a proof of concept. Limitation on maximum data rate (or symbol rate in the case of BPSK) is imposed by the delay dispersive nature of the multipath environment. In order to increase the throughput, higher order modulation schemes can be investigated. One can also implement differential PSK (DBPSK), which encodes information onto the phase difference between two consecutive symbols. This is valuable to counter the effect of phase offset at the detection stage.

Moreover, the analysis in this thesis is based on the Gaussian assumption of the decision variable. This simplified the analysis and helped derive analytical expressions for performance metrics. However, the approximation leads to a poor estimation of the required received energy to achieve a certain BER, if either the spreading factor is too low, or if the CNR is too high. This is particularly problematic in fading channels, where each received packet can have a different CNR. Similar problems exist in chaos-based communication, and a study on coherent chaos-based CDMA has proposed a solution [101]. To find the exact PDF of the decision variable, the authors applied numerical methods as well as approximations to known distributions. Although the derived BER expression is not closed-form, it is shown to be accurate for low spreading factors. This work cannot be directly applied to N-FOM, since the statistics of the decision variable in N-FOM vary not only with S , but also with the received bit energy. Nevertheless, the study is valuable, as it points out a potential research direction to help derive mathematical expressions for an accurate BER.

Finally, in this thesis, we have derived closed-form-expressions for the BER in different operational environments. However, a closed-form-expression for operation in an NLOS environment was not derived. Since the multipath environment is the most relevant to WSNs, a closed-form-expression in this scenario is very helpful for design of the system. This requires derivation of a joint distribution of the channel variables in (6.24). This is hard to solve. However, as shown in the analysis of the outage probability, one might approximate a variable as a function of another variable in different CNR regimes. This analytical manipulation can simplify the analysis, and lead to a closed-form expression for the BER for particular CNR regimes.

7.2.2 Improvements

First and foremost, the choice of using pure noise has to be reconsidered. Compared to a PN carrier, using the noise-based carrier risks challenging implementation due to a high PAPR associated with it. Secondly, the PN carrier opens up possibilities for applying different coding techniques, such as a time-hopping scheme studied in [102], which can be helpful in reducing MUI. However, such techniques are more applicable to UWB systems, where the ultra-narrow and sparse pulses allow manipulation of their timings.

For wideband systems, one can also investigate the traditionally used chaotic signals, such as those based on the Chebychev polynomial map. This field has gained significant research interest over the last decade [66]. A recent study on a chaos-based differential TR system has shown that, in a power-line multipath channel, the scheme can outperform a rake receiver with erroneous channel information [103]. In order to reduce MUI, coding schemes can also be applied to chaotic signals [104]. Research in this direction can potentially lead to significant improvements in the N-FOM link performance.

A short study on N-FOM in a delay dispersive channel found that ISI can be reduced by introducing guard intervals (duty cycling) [93]. Further study

in this area can lead to promising improvements, and also help in easing the restrictions on the system parameters. Moreover, one can also investigate alternatives to using the frequency offset. The main purpose of the frequency offset is to enable a receiver to distinguish between different links in a MA environment. This *separation* of the signals can also be carried out using different separating waveforms. The optimal separating waveform should have a broad ACF [94]. As a special case of these waveforms, spreading waveforms based on Walsh codes can be used [105]. This implies that the reference and information signals are separated by codes instead of time or frequency. Code-shifted TR although requires digital implementation, the gains achieved in terms of MA capability and performance are valuable.

The noise-enhancement due to operation of the energy-detector type receiver was found to be the major performance-limiting factor. In addition to reconsidering the noise carrier, additional steps can be taken for possible success of the N-FOM system. One possible method is to apply pre- and post-processing schemes to suppress NBI as well as noise from the mixing operation. Since such techniques will have to adapt to MUI and NBI, the overall energy requirement of such a receiver may eclipse the benefits of using N-FOM. One can also find alternatives to the energy detector. Chen studied an energy-detector type receiver [106], where the squaring operation is replaced by an arbitrary positive power operation. For a UWB system, the author showed that the squaring operation is not optimum, and one can achieve better BERs with the improved energy detector, particularly at high values of E_b/N_0 . Adapting this method to the frequency offset TR model can lead to valuable improvements.

Finally, the design of N-FOM should be taken to the system-level. In this context, MAC and PHY models were partially integrated [107], and the system was simulated for WSNs with different node densities. The results showed that the PHY has significant impact on the logical level, particularly in the context of the overall throughput in multi-user environments. In order to assess the overall utility of the N-FOM system, the integration of PHY design with architectural and logical design is required. Enabling the designer to operate at a higher level can prove hugely significant for overall system optimization, requirement analysis, benchmarking, and help find opportunities for further improvements.

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Appendix A

Derivation of SNR parameters

This appendix provides derivation of the variance of the decision variable. Each of the following sections aims to derive this SNR parameter for a different operational scenario. First, in Section A.1, we derive the SNR parameters for a point-to-point LOS AWGN link, that was subject of Chapter 2. In Section A.2, an external interferer is considered, and finally in Section A.3, the parameter is derived for an NLOS link.

The analyses in each of the following sections share similar mathematical steps, particularly in the approximations that are made. The derivations in Section A.1 serve as a template for the rest of the appendix. For brevity, repetition of mathematical steps will be avoided in Section A.2 and Section A.3. Instead, a reference will be made to the relevant part in Section A.1.

A.1 LOS AWGN link

The parameters derived in this section leads to the SNR expression (2.29). We first derive the narrowband parameters in (2.27) and then the wideband parameters in (2.28).

A.1.1 Narrowband parameters

The derivation of $\{\eta_1, \eta_2\}$ is slightly different for a binary NRZ message signal, and for a message signal with a general pulse shape.

NRZ signaling scheme

For a rectangular pulse shape $p(t)$, η_1 from (2.27a) can be written as

$$\eta_1 = \frac{1}{T_b(1/2 + c^2)^2} \int_{-T_b/2}^{T_b/2} a^4(u) \cos^2(2\pi f_r u + \phi_r) du$$

$$\begin{aligned}
&\stackrel{(a)}{=} \frac{1}{T_b(1/2 + c^2)^2} \int_{-T_b/2}^{T_b/2} (c + b_0 \cos(2\pi f_r t + \phi_r))^4 \cos^2(2\pi f_r t + \phi_r) \\
&= \frac{1}{T_b(1/2 + c^2)^2} \int_{-T_b/2}^{T_b/2} \left(c^4 \cos^2(2\pi f_r t + \phi_r) + 4c^3 b_0 \cos^3(2\pi f_r t + \phi_r) \right. \\
&\quad \left. + 6c^2 \cos^4(2\pi f_r t + \phi_r) + 4c b_0 \cos^5(2\pi f_r t + \phi_r) + \cos^6(2\pi f_r t + \phi_r) \right) dt, \quad (A.1)
\end{aligned}$$

where (a) follows from substituting $a(t)$ from (2.10). From the power reduction identity of the cosine, we come to know that

$$\cos^n(\theta) = \begin{cases} \frac{2}{2^n} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \cos((n-2k)\theta), & n \text{ is odd,} \\ \frac{1}{2^n} \binom{n}{n/2} + \frac{2}{2^n} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \cos((n-2k)\theta), & n \text{ is even.} \end{cases} \quad (A.2)$$

The reduction of cosines in (A.1) will lead to a summation of constants and cosines. The arguments of the cosines will be non-zero multiples of $2\pi f_r t$ with a constant phase offset ϕ_r . Since f_r is a multiple of $1/T_b$, any cosine in the integrand of (A.1) will be suppressed by the IDF. One can hence infer from (A.2) that η_1 will be integration of the constant terms. Solving for the constant terms only, we can rewrite η_1 as

$$\eta_1 = \frac{8c^4 + 36c^2 + 5}{16(1/2 + c^2)^2},$$

which is equal to (2.34a). Similarly, we can write η_2 as

$$\begin{aligned}
\eta_2 &= \frac{1}{T_b(1/2 + c^2)} \int_{-T_b/2}^{T_b/2} a^2(t) \cos^2(2\pi f_r t + \phi_r) dt \\
&= \frac{1}{T_b(1/2 + c^2)} \int_{-T_b/2}^{T_b/2} \left(c^2 \cos^2(2\pi f_r t + \phi_r) + \cos^4(2\pi f_r t + \phi_r) \right. \\
&\quad \left. + 2c b_0 \cos^3(2\pi f_r t + \phi_r) \right) dt = \frac{4c^2 + 3}{8(1/2 + c^2)},
\end{aligned}$$

which is equal to (2.34b). This completes the derivation of $\{\eta_1, \eta_2\}$. For $c = 1/\sqrt{2}$, we have $\{\eta_1, \eta_2\} = \{25/16, 5/8\}$.

Generic signaling scheme

For a message signal with a general pulse shape, the derivation of $\{\eta_1, \eta_2\}$ is mathematically tedious. Given that f_r is much larger than the bandwidth of

the pulse shape $p(t)$, η_1 and η_2 can be simplified to

$$\eta_1 = \frac{1}{16T_b(1/2 + c^2)^2} \int_{-\infty}^{\infty} (8c^4 + 36c^2m^2(u) + 5m^4(u)) p^2(u) du$$

$$\eta_2 = \frac{1}{8T_b(1/2 + c^2)} \int_{-\infty}^{\infty} (4c^2 + 3m^2(u)) p^2(u) du.$$

Since the pulses in $m(t)$ could be partially overlapping, the above integrals are not straightforward. However, using numerical methods, we compute the above integral for a root-raised cosine (RRC) pulse with a roll-off factor of zero. Note that such an RRC pulse is in fact a sinc pulse. We first solve for the parameter η_2 , which can be written as

$$\begin{aligned} \eta_2 &= \frac{1}{8T_b(1/2 + c^2)^2} \int_{-\infty}^{\infty} (4c^2 + 3m^2(u)) p^2(u) du \\ &= \frac{1}{8T_b(1/2 + c^2)^2} \left(4c^2 \int_{-\infty}^{\infty} p^2(u) du + 3 \sum_k \int_{-\infty}^{\infty} p^2(u - kT_b) p^2(u) du \right. \\ &\quad \left. + 6 \sum_{\substack{k,l \\ k>l}} b_k b_l \int_{-\infty}^{\infty} p(u - kT_b) p(u - lT_b) p^2(u) du \right) \end{aligned}$$

The first integral gives the energy (T_b) of the pulse, whereas the second integral can be seen as the ACF of $p^2(t)$, evaluated at kT_b , for all integer values of k . Finally, the last integral can be seen as cross-correlation of $p^2(t)$ with a product of two differently shifted versions of $p(t)$. The outcome of this integral is small ($\approx 0.05T_b$), and is further weighted by combination of bits $\{b_k, b_l\}$ for $k \neq l$. Since, the bits are polar and equi-probable, we approximate that the last term in the above expression amounts to zero. Using numerical methods to find the ACF, the above expression is evaluated as

$$\eta_2 \approx \frac{4c^2 + 2.82}{8(1/2 + c^2)}.$$

Similarly, η_1 can be written as

$$\begin{aligned} \eta_1 &= \frac{1}{16T_b(1/2 + c^2)^2} \int_{-\infty}^{\infty} (8c^4 + 36c^2m^2(u) + 5m^4(u)) p^2(u) du \\ &\approx \frac{1}{16T_b(1/2 + c^2)^2} \left(8c^4T_b + 33.84c^2T_b + 5 \int_{-\infty}^{\infty} p^2(u) m^4(u) du \right) \\ &\approx \frac{8c^4 + 33.84c^2 + 4.67}{16(1/2 + c^2)^2}. \end{aligned}$$

For $c = 1/\sqrt{2}$, the values of $\{\eta_1, \eta_2\}$ for the sinc pulse are $\{23.6/16, 4.82/8\}$, which are slightly less than that for the rectangular pulse $\{25/16, 5/8\}$ computed earlier. However, it has to be stressed that these results are obtained

from crude approximations, and can be slightly different for RRC pulses with different roll-off factors. For consistency in this thesis, we will use those values of $\{\eta_1, \eta_2\}$ which correspond to the rectangular pulses.

A.1.2 Power of noise from signal self-mixing

The noise contribution from the signal self-mixing product, is defined in (2.24a). For better readability, we reproduce the expression below

$$\Psi_{yy} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mathbb{E}[z_y(u)z_y(v)] - \mathbb{E}[z_y(u)]\mathbb{E}[z_y(v)] \right) p(u)p(v) du dv, \quad (\text{A.3})$$

The term $\mathbb{E}[z_y(t)]$ was calculated in (2.16). Using the definition of $z_y(t)$ from (2.13a), the first term in the integrand is calculated as

$$\begin{aligned} \mathbb{E}[z_y(u)z_y(v)] &= \frac{1}{4L^2} \mathbb{E}[\tilde{x}_g(u)\tilde{x}_g^*(u)\tilde{x}_g(v)\tilde{x}_g^*(v)] \\ &\quad \cdot a^2(u)a^2(v) \cos^2(2\pi f_r u + \phi_r) \cos^2(2\pi f_r v + \phi_r) \\ &= \frac{1}{4L^2} \left(\mathbb{E}[\tilde{x}_g(u)\tilde{x}_g^*(v)] \mathbb{E}[\tilde{x}_g(v)\tilde{x}_g^*(u)] + \mathbb{E}[|\tilde{x}_g(u)|^2] \mathbb{E}[|\tilde{x}_g(v)|^2] \right) \\ &\quad \cdot a^2(u)a^2(v) \cos(2\pi f_r u + \phi_r) \cos(2\pi f_r v + \phi_r), \end{aligned}$$

where the last equality follows from the moment theorem of complex Gaussian processes [92], which states that four jointly complex Gaussian random variables $\{s_1, s_2, s_3, s_4\}$ satisfy

$$\mathbb{E}[s_1 s_2^* s_3 s_4^*] = \mathbb{E}[s_1 s_2^*] \mathbb{E}[s_3 s_4^*] + \mathbb{E}[s_2^* s_3] \mathbb{E}[s_1 s_4^*]. \quad (\text{A.4})$$

Substituting the expression for $\mathbb{E}[z_y(u)z_y(v)]$ in (A.3) in combination with (2.16), we get

$$\begin{aligned} \Psi_{yy} &= \frac{1}{4L^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbb{E}[\tilde{x}_g(u)\tilde{x}_g^*(v)] \mathbb{E}[\tilde{x}_g(v)\tilde{x}_g^*(u)] \\ &\quad \cdot a_1^2(u)a_1^2(v) \cos(2\pi f_r u + \phi_r) \cos(2\pi f_r v + \phi_r) p(u)p(v) du dv \\ &= \frac{1}{4L^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |R_{\tilde{x}_g}(u-v)|^2 a^2(u)a^2(v) \cos(2\pi f_r u + \phi_r) \\ &\quad \cdot \cos(2\pi f_r v + \phi_r) p(u)p(v) du dv \\ &= \frac{1}{4L^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |R_{\tilde{x}_g}(\tau)|^2 a^2(u-\tau)a^2(u) \cos(2\pi f_r(u-\tau) + \phi_r) \\ &\quad \cdot \cos(2\pi f_r u + \phi_r) p(u-\tau)p(u) d\tau du, \end{aligned}$$

where the last equality follows from substituting $\tau = u - v$. Since $\tilde{x}_g(t)$ is a wideband random process, the support of its ACF $R_{\tilde{x}_g}(\tau)$ is very narrow ($\sim 1/B_x$). The narrowband functions in the above expression hardly vary over

τ . Therefore they can be taken outside the integral over τ with $\tau \approx 0$. This results in the approximation

$$\begin{aligned}\Psi_{yy} &\approx \frac{1}{4L^2} \int_{-\infty}^{\infty} |R_{\tilde{x}_g}(\tau)|^2 d\tau \int_{-\infty}^{\infty} a^4(u) \cos^2(2\pi f_r u + \phi_r) p^2(u) du \\ &= \frac{\eta_1 T_b (1/2 + c^2)^2}{4L^2} \int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) df,\end{aligned}$$

where the last equality follows from Parseval's theorem and the Wiener-Khinchin relations; and the definition of η_1 from (2.27a). Substituting $S_{\tilde{x}_g}(f)$ using (2.11) and (2.3), the above expression is written as

$$\begin{aligned}\Psi_{yy} &= \frac{\eta_1 T_b (1/2 + c^2)^2}{64L^2} \left(\frac{2P_x}{B_x} \right)^2 \int_{-\infty}^{\infty} \mathcal{S}^2 \left(\frac{f}{B_x} \right) |\tilde{G}(f)|^4 df \\ &= \eta_1 \frac{E_b^2}{16SB_x} \int_{-\infty}^{\infty} \mathcal{S}^2 \left(\frac{f}{B_x} \right) |\tilde{G}(f)|^4 df\end{aligned}$$

where we have used definition of E_b from (2.22) and $S = B_x T_b$ is the spreading factor. Substituting $\nu = f/B_x$ in the above expression, and then using the definition of Λ_2 in (2.26), we get

$$\Psi_{yy} = \eta_1 E_b^2 \Lambda_2 / S.$$

This completes the derivation of Ψ_{yy} .

A.1.3 Power of noise from signal-AWGN mixing

The noise contribution from the signal-noise mixing product, in (2.24b), is

$$\begin{aligned}\Psi_{yn} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mathbb{E}[z_{yn}(u)z_{yn}(v)] - \mathbb{E}[z_{yn}(u)]\mathbb{E}[z_{yn}(v)] \right) p(u)p(v) du dv, \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbb{E}[z_{yn}(u)z_{yn}(v)] p(u)p(v) du dv,\end{aligned}$$

where the last equality holds due to zero contribution from $\mathbb{E}[z_{yn}(t)]$ to the decision variable. Substituting $z_{yn}(t)$ from (2.13b), Ψ_{yn} solves to

$$\begin{aligned}\Psi_{yn} &= \frac{1}{4L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbb{E} \left[(\tilde{x}_g(u)\tilde{n}_g^*(u) + \tilde{x}_g^*(u)\tilde{n}_g(u)) (\tilde{x}_g(v)\tilde{n}_g^*(v) + \tilde{x}_g^*(v)\tilde{n}_g(v)) \right] \\ &\quad \cdot a(u)a(v) \cos(2\pi f_r u + \phi_r) \cos(2\pi f_r v + \phi_r) p(u)p(v) du dv \\ &= \frac{1}{4L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mathbb{E}[\tilde{x}_g(u)\tilde{n}_g^*(u)\tilde{x}_g^*(v)\tilde{n}_g(v)] + \mathbb{E}[\tilde{x}_g^*(u)\tilde{n}_g(u)\tilde{x}_g(v)\tilde{n}_g^*(v)] \right) \\ &\quad \cdot a(u)a(v) \cos(2\pi f_r u + \phi_r) \cos(2\pi f_r v + \phi_r) p(u)p(v) du dv \\ &= \frac{1}{4L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mathbb{E}[\tilde{x}_g(u)\tilde{x}_g^*(v)] \mathbb{E}[\tilde{n}_g^*(u)\tilde{n}_g(v)] + \mathbb{E}[\tilde{x}_g^*(u)\tilde{x}_g(v)] \right. \\ &\quad \cdot \mathbb{E}[\tilde{n}_g(u)\tilde{n}_g^*(v)] \left. \right) a(u)a(v) \cos(2\pi f_r u + \phi_r) \cos(2\pi f_r v + \phi_r) p(u)p(v) du dv,\end{aligned}$$

where in the last two equalities, we have used the moment theorem of complex Gaussian processes (A.4), and the relation $\mathbb{E}[\tilde{x}_g(u)\tilde{n}_g^*(u)] = 0$ due to their mutual independence and zero mean. Denoting $R_{\tilde{n}_g}(\tau)$ as the ACF of $\tilde{n}_g(t)$, and using similar approximations as in the last subsection, we can write

$$\begin{aligned}
\Psi_{yn} &= \frac{1}{4L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(R_{\tilde{x}_g}(u-v)R_{\tilde{n}_g}(v-u) + R_{\tilde{x}_g}(v-u)R_{\tilde{n}_g}(u-v) \right) \\
&\quad \cdot a(u)a(v) \cos(2\pi f_r u + \phi_r) \cos(2\pi f_r v + \phi_r) p(u)p(v) du dv, \\
&\approx \frac{1}{4L} \int_{-\infty}^{\infty} \left(R_{\tilde{x}_g}(\tau)R_{\tilde{n}_g}^*(\tau) + R_{\tilde{x}_g}^*(\tau)R_{\tilde{n}_g}(\tau) \right) d\tau \\
&\quad \cdot \int_{-\infty}^{\infty} a^2(u) \cos^2(2\pi f_r u + \phi_r) p^2(u) du \\
&\stackrel{(a)}{=} \frac{1}{2L} \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f)S_{\tilde{n}_g}(f) df \int_{-\infty}^{\infty} a_1^2(u) \cos^2(2\pi f_r u + \phi_r) p^2(u) du \\
&\stackrel{(b)}{=} \frac{\eta_2 T_b(1/2 + c^2)}{2L} \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f)S_{\tilde{n}_g}(f) df,
\end{aligned}$$

where (a) follows from Plancherel's theorem and the Wiener-Khinchin relations; and (b) follows from the definition of η_2 from (2.27b). Using (2.11) and (2.3), we substitute the PSDs $S_{\tilde{x}_g}(f)$ and $S_{\tilde{n}_g}(f)$. This results in

$$\begin{aligned}
\Psi_{yn} &= \frac{\eta_2 T_b(1/2 + c^2)N_0}{8L} \frac{P_x}{B_x} \int_{-\infty}^{\infty} \mathcal{S}\left(\frac{f}{B_x}\right) |\tilde{G}(f)|^4 df \\
&= \eta_2 \frac{E_b N_0}{8B_x} \int_{-\infty}^{\infty} \mathcal{S}\left(\frac{f}{B_x}\right) |\tilde{G}(f)|^4 df = 2\eta_2 E_b N_0 \Lambda_3,
\end{aligned}$$

where we have used the definition of E_b from (2.22); and the last equality follows by substituting $\nu = f/B_x$ and then using the definition of Λ_3 in (2.26). This completes the derivation of Ψ_{yn} .

A.1.4 Power of noise from AWGN self-mixing

The noise contribution from the noise self-mixing product is given by

$$\Psi_{nn} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mathbb{E}[z_n(u)z_n(v)] - \mathbb{E}[z_n(u)]\mathbb{E}[z_n(v)] \right) p(u)p(v) du dv.$$

To solve the above expression, we substitute $z_n(t)$ from (2.13c), and then follow similar steps as in Section A.1.2. Recall from (2.18) that $\mathbb{E}[z_n(t)]$ has zero contribution to the decision variable. One can show that

$$\begin{aligned}
\Psi_{nn} &= \frac{1}{4} \int_{-\infty}^{\infty} |R_{\tilde{n}_g}(\tau)|^2 d\tau \int_{-\infty}^{\infty} p^2(u) \cos^2(2\pi f_r u + \phi_r) du \\
&\stackrel{(a)}{=} \frac{T_b}{8} \int_{-\infty}^{\infty} S_{\tilde{n}_g}^2(f) df \stackrel{(b)}{=} \frac{1}{32} N_0^2 T_b \int_{-\infty}^{\infty} |\tilde{G}(f)|^4 df, \\
&\stackrel{(c)}{=} \frac{1}{2} N_0^2 S \Lambda_0,
\end{aligned}$$

where (a) follows from $\int p^2(t) dt = T_b$ and if either i) the bandwidth of $p(t)$ is much smaller than f_r , or ii) for a rectangular pulse shape with duration T_b , we choose $f_r = n/T_b$ or $\phi_r = (2n+1)\pi/4$ for any nonzero integer n . Finally, (b) in above relations follows from (2.11), and (c) follows from definition of Λ_0 in (2.26). This completes the derivation of Ψ_{nn} .

A.1.5 Simplified case

For a simplified case, described in Section 2.2.4, the parameters $\{\Psi_{yy}, \Psi_{yn}, \Psi_{nn}\}$ can be derived from the results in the above subsections. For a rectangular PSD of the spreading signal, we have a rectangular front-end filter $G(f)$, which has a unit magnitude in the interval $\{f_c - \frac{B_x}{2}, f_c + \frac{B_x}{2}\}$. Accordingly $\tilde{G}(f) = 2$; this results in $\Lambda_i = 1, \forall i$ in (2.26). Substituting this outcome in (2.28), we get

$$\Psi_{yy} = \eta_1 E_b^2 / S, \quad \Psi_{yn} = 2\eta_2 E_b N_0, \quad \Psi_{nn} = N_0^2 S / 2.$$

Furthermore, if one considers a polar NRZ message signal and $c = 1/\sqrt{2}$, then using values of $\{\eta_1, \eta_2\}$ from Section A.1.1, the above parameters solve to

$$\Psi_{yy} = \frac{25E_b^2}{16S}, \quad \Psi_{yn} = \frac{5E_b N_0}{4}, \quad \Psi_{nn} = \frac{1}{2} N_0^2 S.$$

A.2 LOS link with external interference

In this section, we derive the SNR parameters for a point-to-point LOS link subject to an external NB interferer, for both modulated and single-tone NBI models. The parameters derived in this section lead to the SNR expressions in (5.12) and (5.17). In the analysis below, we will use a few key properties of WSS Gaussian bandpass processes. Any zero-mean bandpass process, with a PSD that is symmetric around a carrier frequency, has uncorrelated but identical quadrature components. Furthermore, the PSDs of the components are also symmetric. If the process is Gaussian, then the quadrature components are jointly-Gaussian and mutually-independent. In conclusion, a zero-mean WSS Gaussian process $s(t)$, centered around f_c satisfies

$$\begin{aligned} R_s(\tau) &= R_{s_I}(\tau) \cos(2\pi f_c \tau) - R_{s_I s_Q}(\tau) \sin(2\pi f_c \tau) = R_{s_I}(\tau) \cos(2\pi f_c \tau) \\ \Rightarrow \quad S_{s_I}(f) &= S_{s_Q}(f) = \begin{cases} 2S(f + f_c), & f + f_c > 0 \\ 0, & f + f_c \leq 0 \end{cases} \end{aligned}$$

A.2.1 Power of noise from NBI self-mixing

We denote the noise contribution from the NBI self-mixing product as Ψ_{ii} . For single-tone NBI, this term is zero since the receiver operation moves it out of the detection band, as discussed in Section 5.3.1. For a modulated (OFDM-like)

interferer, this can be written as

$$\begin{aligned}\Psi_{ii} &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} C_{z_i z_i}(t_1, t_2) dt_1 dt_2 \\ &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \left(\mathbb{E}[z_i(t_1)z_i(t_2)] - \mathbb{E}[z_i(t_1)]\mathbb{E}[z_i(t_2)] \right) dt_1 dt_2.\end{aligned}$$

Recall from Section 5.3.2 that $\mathbb{E}[z_i(t)]$ has zero contribution the decision variable. Substituting $z_i(t)$ from (5.7b), we get

$$\begin{aligned}\Psi_{ii} &= \frac{1}{4} \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \mathbb{E}[|\tilde{i}(t_1)|^2 |\tilde{i}(t_2)|^2] \cos(2\pi f_r t_1) \cos(2\pi f_r t_2) dt_1 dt_2 \\ &= \frac{1}{4} \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} |R_{ii}^-(t_2 - t_1)|^2 \cos(2\pi f_r t_1) \cos(2\pi f_r t_2) dt_1 dt_2,\end{aligned}$$

where the last equality follows from (A.4), and from the definition of ACF. Since the interference bandwidth was assumed to be much larger than the frequency offset, the sinusoids in the above integral are almost constant over the narrow supports of the ACF of the NBI. Using similar steps of derivation as in Section A.1.2, the above expression solves to

$$\begin{aligned}\Psi_{ii} &= \frac{1}{4} \int_{-\infty}^{\infty} |R_{ii}^-(\tau)|^2 d\tau \int_{-T_b/2}^{T_b/2} \cos^2(2\pi f_r t) dt = \frac{T_b}{8} \int_{-\infty}^{\infty} S_i^2(f) df \\ &= \frac{T_b}{8} \left(\frac{2P_i}{B_i} \right)^2 = \frac{P_i^2 T_b}{2B_i},\end{aligned}$$

where the second line of expressions follow from substituting complex baseband equivalent of $S_i(f)$ from (5.5). This completes the proof of Ψ_{ii} . Although, we had assumed an interferer with a rectangular spectral shape, we would have obtained a similar result for any spectral shape as long as the spectrum is more or less uniform.

A.2.2 Power of noise from NBI-signal mixing

The NBI-cross-signal product in both (5.11) and (5.16) is defined as

$$\begin{aligned}\Psi_{yi} &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \left(\mathbb{E}[z_{yi}(t_1)z_{yi}(t_2)] - \mathbb{E}[z_{yi}(t_1)]\mathbb{E}[z_{yi}(t_2)] \right) dt_1 dt_2 \\ &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \mathbb{E}[z_{yi}(t_1)z_{yi}(t_2)] dt_1 dt_2,\end{aligned}\tag{A.5}$$

where the last equality follows because we had shown that $\mathbb{E}[z_{yi}(t)]$ does not contribute to the decision variable. For any NBI model, we can write the

NBI-cross-signal product as

$$\begin{aligned}
 z_{yi}(t) &= \frac{1}{2\sqrt{L}} \left(\tilde{y}(t)\tilde{i}^*(t) + \tilde{i}(t)\tilde{y}^*(t) \right) \cos(2\pi f_r t) \\
 &= \frac{1}{\sqrt{L}} \operatorname{Re} \left[\tilde{x}(t)\tilde{i}^*(t) \right] a(t) \cos(2\pi f_r t) \\
 &= \frac{1}{\sqrt{L}} \left(x_I(t)i_I(t) + x_Q(t)i_Q(t) \right) a(t) \cos(2\pi f_r t). \tag{A.6}
 \end{aligned}$$

Using (A.6) and (A.5), we find the noise power from $z_{yi}(t)$ for the two models of interferers.

Unmodulated NBI:

The complex baseband equivalent of single-tone NBI is $\sqrt{2P_i} e^{j(2\pi(f_i - f_c) + \theta_i)}$. We define $f_{\text{IF}} := f_i - f_c \leq B_x/2$ as the intermediate frequency. Using (A.6), the NBI-cross-signal mixing product is

$$z_{yi}(t) = \sqrt{\frac{2P_i}{L}} \left[x_I(t) \cos(2\pi f_{\text{IF}}t + \theta_i) + x_Q(t) \sin(2\pi f_{\text{IF}}t + \theta_i) \right] a(t) \cos(2\pi f_r t),$$

where $\{x_I(t), x_Q(t)\}$ are the quadrature components of the bandpass signal $x(t)$. Since $x(t)$ is a zero mean WSS Gaussian process with rectangular PSD, both $x_I(t)$ and $x_Q(t)$ have a power of P_x which follows from the properties of bandpass processes. Solving (A.5), the noise contribution from this term is solved as

$$\begin{aligned}
 \Psi_{yi} &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \mathbb{E}[z_{yi}(t_1)z_{yi}(t_2)] dt_1 dt_2 \\
 &= \frac{P_i}{L} \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \left(R_{x_I}(t_1 - t_2) + R_{x_Q}(t_1 - t_2) \right) \\
 &\quad \cdot \cos(2\pi f_{\text{IF}}(t_1 - t_2)) a(t_1)a(t_2) \cos(2\pi f_r t_1) \cos(2\pi f_r t_2) dt_1 dt_2,
 \end{aligned}$$

where we have used the fact that θ_i is uniformly distributed between 0 and 2π , and is independent from $x(t)$. Using similar approximations as that in Section A.1.3, we simplify

$$\begin{aligned}
 \Psi_{yi} &\approx \frac{P_i}{L} \int_{-\infty}^{\infty} \left(R_{x_I}(\tau) + R_{x_Q}(\tau) \right) \cos(2\pi f_{\text{IF}}\tau) d\tau \int_{-T_b/2}^{T_b/2} a^2(t_1) \cos^2(2\pi f_r t_1) dt_1 \\
 &\stackrel{(a)}{=} \frac{\eta_2 P_i T_b}{2L} \left(S_{x_I}(f_{\text{IF}}) + S_{x_I}(-f_{\text{IF}}) + S_{x_Q}(f_{\text{IF}}) + S_{x_Q}(-f_{\text{IF}}) \right) \\
 &\stackrel{(b)}{=} \frac{2\eta_2 P_i T_b}{L} S_{x_I}(f_{\text{IF}}),
 \end{aligned}$$

where (a) follows from Plancherel's theorem, and from the definition of η_2 from (2.27b) for $c = 1/\sqrt{2}$, $\phi_r = 0$, and a rectangular $p(t)$; and (b) follows from

the symmetry of the PSDs of the quadrature components. The above result indicates that the noise from the interference-cross-signal term depends on the carrier frequency of the interferer, as well as the PSD of the spreading signal. If $x(t)$ has a rapidly decaying PSD, then noise from $z_{yi}(t)$ can be small, if $f_i - f_c$ is large. However, in practice, the signal is shaped such that its PSD is more or less uniform. In our model, $x(t)$ has a brick-wall spectrum; solving the above expression, we get

$$\Psi_{yi} = \frac{2\eta_2 P_i T_b}{L} \cdot P_x = \frac{5P_i T_b E_b}{4S},$$

where the last equality follows from value of η_2 in Section A.1.1 for $c = 1/\sqrt{2}$; and by substituting $S = B_x T_b$ and $E_b = P_x T_b / L$. This completes the derivation of the noise contribution from $z_{yi}(t)$ product for a single-tone interferer.

Modulated NBI:

The noise power from NBI-signal mixing product, for modulated NBI, is derived in a similar fashion. In this case $i_I(t)$ and $i_Q(t)$ in (A.6) are wideband signals, with bandwidths much larger than both $1/T_b$ and f_r . These quadrature components of $i(t)$ are mutually independent, and their first and second order statistics are the same. Using similar steps of derivation as the last subsection, one can show that

$$\begin{aligned} \Psi_{yi} &= \frac{\eta_2 T_b}{L} \int_{-\infty}^{\infty} \left(S_{x_I}(f) S_{i_I}(f) + S_{x_Q}(f) S_{i_Q}(f) \right) df \\ &\stackrel{(a)}{=} \frac{2\eta_2 T_b}{L} \int_{-\infty}^{\infty} S_{x_I}(f) S_{i_I}(f) df \stackrel{(b)}{=} \frac{5}{4L} T_b \int_{-\infty}^{\infty} S_{x_I}(f) S_{i_I}(f) df, \end{aligned}$$

where (a) follows from the properties of bandpass Gaussian processes; and (b) follows from the value of η_2 in Section A.1.1 for $c = 1/\sqrt{2}$. The PSD of $\tilde{i}(t)$ was given in (5.14). From the properties mentioned in the beginning of Section A.2, it follows that $i_I(t)$ has a PSD $S_{i_I}(f) = \frac{P_i}{B_i} \text{rect}((f - f_{IF})/B_i)$. Using this in the above expression, we get

$$\begin{aligned} \Psi_{yi} &= \frac{5}{4L} T_b \int_{f_{IF}-B_i/2}^{f_{IF}+B_i/2} \frac{P_i}{B_i} S_{x_I}(f) df = \frac{5}{4L} T_b \int_{f_{IF}-B_i/2}^{f_{IF}+B_i/2} \frac{P_i P_x}{B_i B_x} df \\ &= \frac{5P_i P_x T_b}{4B_x L} = \frac{5P_i T_b E_b}{4S}. \end{aligned}$$

This completes the derivation of the noise contribution from the NBI-cross-signal product for modulated NBI. It is not surprising that the result is the same as that of a single-tone interferer.

A.2.3 Power of noise from NBI-AWGN mixing

The derivation of the noise contribution from NBI-cross-noise product is similar to that of the NBI-cross-signal product in Section A.2.2. For single-tone NBI,

we can solve

$$\begin{aligned}\Psi_{ni} &= 2P_i \int_{-\infty}^{\infty} R_{n_I}(\tau) \cos(2\pi f_{IF} \tau) d\tau \int_{-T_b/2}^{T_b/2} \cos^2(2\pi f_r t) dt \\ &= P_i T_b S_{n_I}(f_{IF}) = P_i N_0 T_b,\end{aligned}$$

For the OFDM-like NBI, it is solved as

$$\begin{aligned}\Psi_{ni} &= \int_{-\infty}^{\infty} \left(S_{n_I}(f) S_{i_I}(f) + S_{n_Q}(f) S_{i_Q}(f) \right) df \int_{-T_b/2}^{T_b/2} \cos^2(2\pi f_r t) dt \\ &= T_b \int_{-\infty}^{\infty} S_{n_I}(f) S_{i_I}(f) df = T_b \int_{f_{IF}-B_i/2}^{f_{IF}+B_i/2} \frac{P_i}{B_i} N_0 df = P_i N_0 T_b,\end{aligned}$$

which completes the derivation of the noise contribution from NBI-cross-noise mixing product.

A.3 NLOS link

In this section, we derive the variance of the decision variable for an N-FOM point-to-point link in a dense frequency-selective channel. The expression for the variance is given in (6.21), which is a sum of noise contribution from three mixing products $\{z_y(t), z_{yn}(t), z_n(t)\}$ defined in (6.15). The power of noise from these terms were given in (6.23); in this section, we derive those values. Note that the term Ψ_{nn} is the same as that in Section A.1.4, with $\Lambda_0 = 1$. Below, we find the power of noise from the remaining two terms.

A.3.1 Power of noise from signal self-mixing

The noise contribution Ψ_{yy} is defined as

$$\begin{aligned}\Psi_{yy} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C_{z_y z_y}(t_1, t_2) p(t_1) p(t_2) dt_1 dt_2 \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mathbb{E}[z_y(t_1) z_y(t_2)] - \mathbb{E}[z_y(t_1)] \mathbb{E}[z_y(t_2)] \right) p(t_1) p(t_2) dt_1 dt_2.\end{aligned}\tag{A.7}$$

The second term in the integrand can be solved similarly as that in (6.17). Using the definition of $z_y(t)$ from (6.15a), the first term in the above integrand can be written as

$$\begin{aligned}& \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbb{E}[z_y(t_1) z_y(t_2)] p(t_1) p(t_2) dt_1 dt_2 = \\ &= \frac{1}{64} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \iiint \int_{\mathcal{U}} \tilde{h}(u) \tilde{h}^*(v) \tilde{h}(w) \tilde{h}^*(s) a^2(t_1) a^2(t_2) \cos^2(2\pi f_r t_1 + \phi_r) p(t_1) \\ &\cdot \mathbb{E}[\tilde{x}_g(t_1 - u) \tilde{x}_g^*(t_1 - v) \tilde{x}_g(t_2 - w) \tilde{x}_g^*(t_2 - s)] \cos^2(2\pi f_r t_2 + \phi_r) p(t_2) d\mathcal{U} dt_1 dt_2,\end{aligned}$$

where, for notational brevity, we have represented $\iiint_{\mathcal{U}} d\mathcal{U}$ as the integration over the set of variables $\{u, v, w, s\}$ with limits $[-\infty, \infty]$. Using the theorem (A.4) in the last expression, and substituting the result in (A.7), we get

$$\Psi_{yy} = \frac{1}{64} \iiint_{\mathcal{U}} \tilde{h}(u) \tilde{h}^*(v) \tilde{h}(w) \tilde{h}^*(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{\tilde{x}_g}(t_1 - t_2 + s - u) a^2(t_1) a^2(t_2) \\ \cdot R_{\tilde{x}_g}^*(t_1 - t_2 + w - v) \cos^2(2\pi f_r t_1 + \phi_r) \cos^2(2\pi f_r t_2 + \phi_r) p(t_1) p(t_2) d\mathcal{U} dt_1 dt_2.$$

In the integrands, the NB terms $\{a(t), p(t), \cos(2\pi f_r t)\}$ hardly vary over the supports of the ACF $R_{\tilde{x}_g}(\tau)$, as well as the channel impulse response. Therefore, one can use similar approximations as that in Section A.1.2, which leads to

$$\begin{aligned} \Psi_{yy} &\approx \frac{1}{64L^2} \iiint_{\mathcal{U}} \tilde{h}(u) \tilde{h}^*(v) \tilde{h}(w) \tilde{h}^*(s) \int_{-\infty}^{\infty} R_{\tilde{x}_g}(\tau + s - u) \\ &\quad \cdot R_{\tilde{x}_g}^*(\tau + w - v) d\mathcal{U} d\tau \int_{-\infty}^{\infty} a^4(t_1) \cos^4(2\pi f_r t_1 + \phi_r) p^2(t_1) dt, \\ &\stackrel{(a)}{=} \int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) \iiint_{\mathcal{U}} \tilde{h}(u) \tilde{h}^*(v) \tilde{h}(w) \tilde{h}^*(s) e^{j2\pi f(s-u+w-v)} d\mathcal{U} df \\ &\quad \cdot \frac{1}{64} \int_{-\infty}^{\infty} a^4(t_1) \cos^4(2\pi f_r t_1 + \phi_r) p^2(t_1) dt_1 \\ &\stackrel{(b)}{=} \frac{1}{64} \int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) |\tilde{H}(f)|^4 df \int_{-\infty}^{\infty} a^4(t_1) \cos^4(2\pi f_r t_1 + \phi_r) p^2(t_1) dt_1 \\ &\stackrel{(c)}{=} \frac{T_b}{64} \eta_1 \int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) |\tilde{H}(f)|^4 df \end{aligned}$$

where (a) follows from the Plancherel's theorem and the Wiener-Kinchin relations; (b) follows from the definition of Fourier transform ($H(f) = \mathcal{F}[h(\tau)]$); and (c) follows from the definition of η_1 with $c = 1/\sqrt{2}$ in (2.27a). For a polar NRZ signaling scheme, it was shown in Section A.1.1 that $\eta_1 = 25/16$. The above expression reduces to

$$\begin{aligned} \Psi_{yy} &= \frac{25T_b}{1024} \int_{-\infty}^{\infty} S_{\tilde{x}_g}^2(f) |\tilde{H}(f)|^4 df \stackrel{(a)}{=} \frac{25T_b}{256} \cdot \left(\frac{P_x}{B_x}\right)^2 \int_{-\infty}^{\infty} \mathcal{S}^4\left(\frac{f}{B_x}\right) |\tilde{H}(f)|^4 df \\ &\stackrel{(b)}{=} \frac{25P_h^2}{256B_x} \beta_h P_x^2 T_b \lambda_2, \end{aligned}$$

where (a) follows from (6.13a); and (b) follows from variable substitution $\nu = f/B_x$, and the definition of β_h from (6.22a). This completes the proof of the noise from the signal self-mixing product $z_y(t)$, in a dense multipath operational environment.

In case, one can model the spectral shape $\mathcal{S}(\nu)$ as perfectly rectangular, then one can accurately model the channel using TDL description. In that case, $\int \mathcal{S}^n(\nu) d\nu = 1$ for any positive integer n . One can show that $\lambda_2 = 1$ and

that β_h from (6.22a) simplifies to

$$\begin{aligned}\beta_h &= \frac{1}{P_h^2} \sum_k \sum_l \sum_m \sum_n c_k c_l^* c_m c_n^* \delta[n-k-m+l] \\ &= \frac{1}{P_h^2} \sum_k \sum_l c_l c_{l-k}^* \sum_m c_{m-k} c_m^* = \frac{1}{P_h^2} \sum_{k=1} \left| \sum_{l=1} c_l c_{l-k}^* \right|^2,\end{aligned}$$

where c_i is the weight of the i^{th} tap of the baseband equivalent of $h(\tau)$.

A.3.2 Power of noise from signal-AWGN mixing

Similarly, the noise contribution from the signal-noise mixing product is

$$\begin{aligned}\Psi_{yn} &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} C_{z_{yn} z_{yn}}(t_1, t_2) p(t_1) p(t_2) dt_1 dt_2 \\ &= \int_{-T_b/2}^{T_b/2} \int_{-T_b/2}^{T_b/2} \mathbb{E}[z_{yn}(t_1) z_{yn}^*(t_2)] dt_1 dt_2,\end{aligned}$$

where the last equality follows because $z_{yn}(t)$ has a zero mean. Using the definition of $z_{yn}(t)$ from (6.15b), the above expression is solved as

$$\begin{aligned}\Psi_{yn} &= \frac{1}{16} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a(t_1) a(t_2) \cos(2\pi f_r t_1 + \phi_r) \cos(2\pi f_r t_2 + \phi_r) p(t_1) p(t_2) \\ &\quad \cdot \iint_{\mathcal{V}} \left(\tilde{h}(u) \tilde{h}^*(v) \mathbb{E}[\tilde{x}_g(t_1 - u) n_g^*(t_1) \tilde{x}_g^*(t_2 - v) n_g(t_2)] \right. \\ &\quad \left. + \tilde{h}^*(u) \tilde{h}(v) \mathbb{E}[\tilde{x}_g^*(t_1 - u) n_g(t_1) \tilde{x}_g(t_2 - v) n_g^*(t_2)] \right) d\mathcal{V} dt_1 dt_2,\end{aligned}$$

where we have represented $\iint_{\mathcal{V}} d\mathcal{V}$ as the integration over the set of variables $\{u, v\}$ with limits $[-\infty, \infty]$. Since $x_g(t)$ and $n_g(t)$ are independent processes, the expectations in the above expression can be written as products of ACFs of $\tilde{x}_g(t)$ and $\tilde{n}_g(t)$. Using similar steps of derivation as in the previous subsection, we get

$$\begin{aligned}\Psi_{yn} &= \frac{1}{16} \int_{-\infty}^{\infty} a^2(t) \cos(2\pi f_r t + \phi_r) p^2(t) dt \\ &\quad \cdot \left(\iint_{\mathcal{V}} \int_{-\infty}^{\infty} \tilde{h}(u) \tilde{h}^*(v) R_{\tilde{x}_g}(\tau + v - u) R_{\tilde{n}_g}^*(\tau) d\mathcal{V} d\tau \right. \\ &\quad \left. + \iint_{\mathcal{V}} \int_{-\infty}^{\infty} \tilde{h}^*(u) \tilde{h}(v) R_{\tilde{x}_g}^*(\tau + v - u) R_{\tilde{n}_g}(\tau) d\mathcal{V} d\tau \right) \\ &= \frac{1}{8} \int_{-\infty}^{\infty} a^2(t) \cos^2(2\pi f_r t) dt \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) S_{\tilde{n}_g}(f) |\tilde{H}(f)|^2 df,\end{aligned}$$

where the last equality follows from the Plancherel's theorem, the Wiener Kinchin relations, and the definition of Fourier transform. From (2.27b) we can see that the first integral in the above expression can be written as $\eta_2 T_b$ for $c = 1/\sqrt{2}$, and is solved to $5T_b/8$ for a polar NRZ signaling scheme, as shown earlier in Section A.1.1. This results in

$$\Psi_{yn} = \frac{5T_b}{64} \int_{-\infty}^{\infty} S_{\tilde{x}_g}(f) S_{\tilde{n}_g}(f) |\tilde{H}(f)|^2 df.$$

The integral in the above expression is solved using the definitions of the integrands in (6.13). We get

$$\Psi_{yn} = \frac{5N_0 P_x T_b}{16B_x} \int_{-\infty}^{\infty} \mathcal{S}^3\left(\frac{f}{B_x}\right) |\tilde{H}(f)|^2 df = \frac{5P_h}{16} \vartheta_h P_x T_b N_0 \lambda_1,$$

where the last equality follows from the variable substitution $\nu = f/B_x$, and the definition of ϑ_h in (6.22b). This completes the proof of the power of noise from the signal-noise mixing product $z_{yn}(t)$, in a dense multipath operational environment.

For a brick-wall spectrum $\mathcal{S}(\nu)$, we get $\lambda_1 = 1$ and $\vartheta_h = \alpha_h$. This results in $\Psi_{yn} = 5\alpha_h P_h P_x T_b N_0 / 16$.

Appendix B

Derivation of design parameters

In this appendix, key parameters for realization of the N-FOM system are derived. Recall that the derivation of the SNR expressions throughout this thesis assume that some conditions on the pulse shape and the frequency offset are met. Link performance can suffer severe deterioration if the conditions are not satisfied. It was shown that the effect of this performance degradation can be modeled as a bias term in the mean of the decision variable. Section B.1 and Section B.2 analyze these bias terms for a synchronized and unsynchronized point-to-point link respectively. In these sections, sufficient conditions are derived under which the bias terms are suppressed for a given signaling scheme. Finally, in Section B.3, we derive the parameters for a Butterworth filter, that were used throughout Chapter 3.

B.1 Bias in a synchronized link

In this section, we consider an N-FOM point-to-point link, in which the receiver is synchronized to the transmitter, in terms of symbol timing and phase of the frequency offset LO. We prove that under the conditions enumerated in Section 2.2.3, the bias term $\Xi(\cdot)$ in (2.20) is suppressed. The bias term is

$$\begin{aligned} \Xi = & \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} [c^2 \cos(2\pi f_r u + \phi_r) + m^2(u) \cos^3(2\pi f_r u + \phi_r)] p(u) du \\ & + \sum_k b_k \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} p(u - kT_b) \cos(4\pi f_r u + 2\phi_r) p(u) du. \end{aligned} \quad (\text{B.1})$$

We will derive sufficient conditions in the context of the two design choices, mentioned in Section 2.2.3. In the end, we will give a summary of the findings.

B.1.1 Case 1: NRZ signaling scheme

We will first solve the bias Ξ for a rectangular pulse, i.e., $m(t)$ is a NRZ pulse train. In that case, $m^2(t)$ is a constant DC, and $p(t)$ has a unit amplitude in the interval $[-T_b/2, T_b/2]$ and zero outside it. We can write (B.1) as

$$\begin{aligned}\Xi &= \frac{P_{x_g}}{L} \int_{-T_b/2}^{T_b/2} (c^2 \cos(2\pi f_r u + \phi_r) + \cos^3(2\pi f_r u + \phi_r)) du \\ &\quad + \frac{P_{x_g}}{L} \int_{-T_b/2}^{T_b/2} \cos(4\pi f_r u + 2\phi_r) du.\end{aligned}$$

It can be noticed that the first term (involving c^2) and the last term (i.e., the second integral) is suppressed if $f_r = n/T_b$ for a positive integer n . For any arbitrary choice of f_r , one can suppress the first term if $\phi_r = (2n+1)\pi/2$. However, this choice does not suppress the last term, which is suppressed if $\phi_r = (2n+1)\pi/4$. Since there is no single value of ϕ_r that ensures that both bias terms are suppressed, we will not further investigate the choice of ϕ_r . The bias term can be written as

$$\begin{aligned}\Xi &= \frac{P_{x_g}}{L} \int_{-T_b/2}^{T_b/2} \cos^3(2\pi f_r u + \phi_r) du \\ &= \frac{P_{x_g}}{L} \int_{-T_b/2}^{T_b/2} (3 \cos(2\pi f_r u + \phi_r) + \cos(6\pi f_r u + 3\phi_r)) du,\end{aligned}$$

where we have used the power reduction identity of cosine, presented in (A.2). It can be seen that as long as the frequency offset is chosen as $f_r = n/T_b$ for a positive integer n , this bias term will be suppressed.

B.1.2 Case 2: Generic signaling scheme

In this case, we consider $m(t)$ to be a train of some generic real-valued pulses $p(t)$, where the pulse has an absolute bandwidth B_p . In the subsequent analyses, we will frequently use a couple of identities derived from Fourier properties and Plancherel's theorem. For any real signals $g(t)$ and $p(t)$, one can derive

$$\begin{aligned}\int_{-\infty}^{\infty} \cos(2n\pi f_r t + \phi) p(t) dt &= \frac{1}{2} (e^{-j\phi} P(nf_r) + e^{j\phi} P(-nf_r)) \\ &= \text{Re}\{P(nf_r)e^{-j\phi}\},\end{aligned}\tag{B.2a}$$

$$\begin{aligned}\int_{-\infty}^{\infty} g(t+T)p(t) \cos(2n\pi f_r t + \phi) dt &= \frac{1}{2} \int_{-\infty}^{\infty} G(f)P^*(f+nf_r)e^{j(2\pi fT+\phi)} df \\ &\quad + \frac{1}{2} \int_{-\infty}^{\infty} G(f)P^*(f-nf_r)e^{j(2\pi fT-\phi)} df,\end{aligned}\tag{B.2b}$$

where (B.2a) follows from the definition and symmetry property of Fourier transform for real functions; and (B.2b) follows from applying Plancherel's theorem.

For the generic signaling scheme, we write the bias in (B.1) as sum of three bias terms, i.e.,

$$\Xi = \Xi_1 + \Xi_2 + \Xi_3,$$

and treat each of them separately. The bias term Ξ_1 is

$$\Xi_1 = \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} c^2 \cos(2\pi f_r u + \phi_r) p(u) du = \frac{c^2 P_{x_g}}{L} \text{Re}\{P(f_r) e^{-j\phi_r}\},$$

where the last equality follows from (B.2a). It can be seen that if the pulse has an absolute bandwidth $B_p \ll f_r$, then there is no spectral component of $p(t)$ at $f = f_r$. Hence, the bias term Ξ_1 will be suppressed. The bias term Ξ_2 in (B.1) is written as

$$\begin{aligned} \Xi_2 &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} m^2(u) \cos^3(2\pi f_r u + \phi_r) p(u) du \\ &= \frac{P_{x_g}}{4L} \int_{-\infty}^{\infty} m^2(u) \left(3 \cos(2\pi f_r u + \phi_r) + \cos(6\pi f_r u + 3\phi_r) \right) p(u) du. \end{aligned}$$

We denote $g(t) := m^2(t)$; using (B.2b), Ξ_2 is solved to

$$\Xi_2 = \frac{P_{x_g}}{2L} \sum_{l=1,3} c_l \int_{-\infty}^{\infty} G(f) \left(P^*(f + l f_r) e^{jl\phi_r} + P^*(f - l f_r) e^{-jl\phi_r} \right) df$$

where c_l is a constant taking value of 1 when $l = 3$, and 3 when $l = 1$. It can be noticed that the bias term Ξ_2 will be suppressed if there is no spectral overlap between $G(f)$ and $P(f \pm f_r)$. In other words, if $p(t)$ has a bandwidth B_p , then $G(f)$ should have no or insignificant spectral components for $|f| \geq f_r - B_p$. Since $G(f)$ represents Fourier transform of $m^2(t)$, the bandwidth of $g(t)$ is subject to the pulse $p(t)$. Depending on the used signaling scheme, it can be shown that the bandwidth of $g(t)$ is in the order of $2B_p$. Therefore, in order to suppress the bias term Ξ_2 , it is sufficient to choose $f_r \geq 3B_p$.

Finally, the bias term Ξ_3 from (B.1) is written as

$$\begin{aligned} \Xi_3 &= \sum_k b_k \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} p(u - kT_b) \cos(4\pi f_r u + 2\phi_r) p(u) du \\ &= \frac{P_{x_g}}{L} \sum_k b_k \int_{-\infty}^{\infty} P(f) \left(P^*(f + 2f_r) e^{j2\phi_r} + P^*(f - 2f_r) e^{-j2\phi_r} \right) e^{-j2\pi f k T_b} df. \end{aligned}$$

Analogous to the reasoning for Ξ_2 , the bias term Ξ_3 is suppressed if $f_r \geq B_p$. With such a choice, there is no spectral overlap between $P(f)$ and $P(f \pm 2f_r)$.

B.1.3 Conclusion

For perfectly synchronized point-to-point communication link, we looked into two possible signaling schemes. We derived sufficient conditions which ensure

that the bias term Ξ in (2.20) is suppressed. We found that, in case one chooses an NRZ signaling scheme, it is sufficient to choose the frequency offset as an integer multiple of the data rate, i.e., $f_r = n/T_b$ for a positive integer n . If one chooses a generic signaling scheme, where the pulse has an absolute bandwidth B_p , then one should choose the frequency offset as $f_r \geq 3B_p$. In both cases, the choice $f_r \gg B_p$ ensures that the decision variable is bias-free.

It must be stressed that we only looked at two examples, and other kinds of signaling schemes may lead to different conditions. Finally, the list of sufficient conditions is not exhaustive, and more conditions may exist.

B.2 Bias in an unsynchronized link

In this section, we consider an N-FOM point-to-point link, where there is a synchronization mismatch between the receiver and the transmitter. The mismatch gives rise to a bias term at the detection stage, denoted as $\Xi(\cdot)$ in (3.22). We derive conditions under which this bias term is suppressed. The term is given as

$$\begin{aligned} \Xi = & \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} \left(\frac{1}{2} + m^2(t+\tau) \cos^2(2\pi f_r(t+\tau) + \phi_r) \right) \cos(2\pi f_r t + \phi_r) p(t) dt \\ & + \frac{P_{x_g}}{L} \sum_k b_k \int_{-\infty}^{\infty} p(t - kT_b + \tau) p(t) \cos(2\pi f_r(2t + \tau) + 2\phi_r) dt. \end{aligned} \quad (\text{B.3})$$

We will first analyze the above expression for an NRZ signaling scheme, and then for a generic signaling scheme. The identities in (B.2) will be frequently applied, and some derivation steps from Section B.1 will be used without a detailed explanation.

B.2.1 Case 1: NRZ signaling scheme

Similar to the analysis in Section B.1.1, we can write (B.3) as

$$\begin{aligned} \Xi = & \frac{P_{x_g}}{L} \int_{-T_b/2}^{T_b/2} \left(\frac{1}{2} + \cos^2(2\pi f_r(t + \tau) + \phi_r) \right) \cos(2\pi f_r t + \phi_r) dt \\ & + \frac{P_{x_g}}{L} \sum_k b_k \int_{-T_b/2}^{T_b/2} p(t - kT_b + \tau) \cos(2\pi f_r(2t + \tau) + 2\phi_r) dt \\ = & \frac{P_{x_g}}{L} \sum_k b_k \int_{-T_b/2}^{T_b/2} p(t - kT_b + \tau) \cos(2\pi f_r(2t + \tau) + 2\phi_r) dt, \end{aligned}$$

where the last equality follows if $f_r = n/T_b$: as long as f_r is an integer multiple of the data rate ($1/T_b$), the first integral in the first equality will result in zero. Due to the synchronization mismatch, the integration interval in the last

expression will partly overlap one neighboring bit. Using basic calculus, and the property in (B.2a), one can solve

$$\begin{aligned}\Xi &= \frac{P_{x_g}}{L} \sum_k b_k \int_{-T_b/2}^{T_b/2} p(t - kT_b + \tau) \cos(2\pi f_r(2t + \tau) + 2\phi_r) dt \\ &= b_0 \int_{-T_b/2}^{T_b/2 - \tau} \cos(2\pi f_r(2t + \tau) + 2\phi_r) dt + b_1 \int_{T_b/2 - \tau}^{T_b/2} \cos(2\pi f_r(2t + \tau) + 2\phi_r) dt \\ &= b_0(T_b - \tau) \text{sinc}(2f_r(T_b - \tau)) \cos(2\phi_r) + b_1 \tau \text{sinc}(2f_r \tau) \cos(4\pi f_r T_b + 2\phi_r).\end{aligned}$$

The above expression is summation of two sinc functions at particular values. Given that $f_r = n/T_b$, one can show that the bias term is suppressed if $\tau = 0$ or if $b_1 = b_0$. Furthermore, using basic trigonometry, one can solve that under such a design choice ($f_r = n/T_b$), the bias term reduces to

$$\Xi = \left(b_0(T_b - \tau) \text{sinc}(2f_r(T_b - \tau)) + b_1 \tau \text{sinc}(2f_r \tau) \right) \cos(2\phi_r). \quad (\text{B.4})$$

If $b_0 \neq b_1$, this bias term is not suppressed for small values of f_r . However, the term is approximately zero if $f_r \gg 1/T_b$, and precisely zero if $\phi_r = \pm\pi/4$.

B.2.2 Case 2: Generic signaling scheme

We now look at the bias term $\Xi(\cdot)$ in (B.3) for a generic signaling scheme. We write $\Xi(\cdot)$ as summation of three bias terms,

$$\Xi = \Xi_1 + \Xi_2 + \Xi_3,$$

and treat each of them separately. The derivation steps are analogous to that in Section B.1.2. The bias term Ξ_1 is solved as

$$\Xi_1 = \frac{P_{x_g}}{2L} \int_{-\infty}^{\infty} \cos(2\pi f_r t + \phi_r) p(t) dt = \frac{P_{x_g}}{2L} \text{Re}\{P(f_r) e^{-j\phi_r}\},$$

which is suppressed if $B_p < f_r$. The bias term Ξ_2 is written as

$$\begin{aligned}\Xi_2 &= \frac{P_{x_g}}{L} \int_{-\infty}^{\infty} m^2(t + \tau) \cos^2(2\pi f_r(t + \tau) + \phi_r) \cos(2\pi f_r t + \phi_r) p(t) dt \\ &= \frac{P_{x_g}}{2L} \int_{-\infty}^{\infty} m^2(t + \tau) \left(\cos(2\pi f_r t + \phi_r) + \frac{1}{2} \cos(2\pi f_r(t + 2\tau) + \phi_r) \right) p(t) dt \\ &\quad + \frac{P_{x_g}}{4L} \int_{-\infty}^{\infty} m^2(t + \tau) \cos(6\pi f_r(t + 2\tau/3) + 3\phi_r) p(t) dt\end{aligned}$$

Denoting $g(t) := m^2(t)$, and using identity in (B.2b), Ξ_2 is solved as

$$\begin{aligned}\Xi_2 &= \frac{P_{x_g}}{4L} \int_{-\infty}^{\infty} G(f) \left(P^*(f+f_r) e^{j\phi_r} + P^*(f-f_r) e^{-j\phi_r} \right) e^{j2\pi f\tau} df \\ &\quad + \frac{P_{x_g}}{8L} \sum_{l=1,3} \int_{-\infty}^{\infty} G(f) \left(P^*(f+lf_r) e^{j(4\pi f_r\tau+l\phi_r)} \right. \\ &\quad \left. + P^*(f-lf_r) e^{-j(4\pi f_r\tau+l\phi_r)} \right) e^{j2\pi f\tau} df.\end{aligned}$$

It can be seen that in order to suppress the bias term Ξ_2 , it is sufficient to choose $P(f)$ such that there is not spectral overlap between $P(f \pm f_r)$ and $G(f)$. The bandwidth of $g(t)$ is in the order of $2B_p$ where B_p is the bandwidth of $p(t)$. Therefore, the design choice $f_r > 3B_p$ ensures that the bias term is suppressed. Finally, the bias term Ξ_3 in (B.3) is written as

$$\begin{aligned}\Xi_3 &= \frac{P_{x_g}}{L} \sum_k b_k \int_{-\infty}^{\infty} p(t-kT_b+\tau) p(t) \cos(2\pi f_r(2t+\tau)+2\phi_r) dt \\ &= \frac{P_{x_g}}{L} \sum_k b_k \int_{-\infty}^{\infty} P(f) \left(\sum_{l=1,-1} P^*(f+2lf_r) e^{j2l(\pi f_r\tau+\phi_r)} \right) e^{-j2\pi f(kT_b-\tau)} df.\end{aligned}$$

Analogous to the derivation in Section B.1.2, the bias Ξ_3 is suppressed if we choose a frequency offset such that $f_r > B_p$.

B.2.3 Conclusion

In this section, we analyzed the bias (Ξ) in the mean of the decision variable for a point-to-point N-FOM link, in the context of a timing mismatch τ between the receiver and the transmitter. The mismatch gives rise to a bias term at the detection stage. For two possible signaling schemes, we derived sufficient conditions which ensure that the bias term Ξ in (3.22) is suppressed. In case one chooses an NRZ signaling scheme, it was found sufficient that one chooses either the frequency offset $f_r \gg 1/T_b$, or $f_r = n/T_b$ and $\phi_r = (2n+1)\pi/4$ for any positive integer n . For a generic signaling scheme, we conclude that $\Xi(\cdot)$ is suppressed if the bandwidth of the pulse satisfies $B_p \ll f_r$. More specifically, if the pulse has an absolute bandwidth B_p , then $f_r \geq 3B_p$ is sufficient.

Once again, we looked only at two examples of signaling schemes and derived sufficient conditions for those schemes. The reader must be informed that the derived list of conditions may not be exhaustive.

B.3 Parameters of a Butterworth function

In this section, we derive the parameters used in Section 3.2 for a Butterworth filter. In the first two subsections, we derive the integrals of a Butterworth

function and the associated parameters $\bar{\nu}$ and μ . Finally, in Section B.3.3, we derive the parameter λ for suboptimal filtering, discussed in Section 3.2.3.

We first present the squared magnitude of a Butterworth function, which was defined in (3.9) as

$$\mathcal{S}_B(\nu) = \frac{\mu}{1 + (\nu/\bar{\nu})^{2n}}, \quad (\text{B.5})$$

where n is the filter order, $\bar{\nu}$ is the 3 dB cutoff frequency, and μ is a normalization variable.

B.3.1 Integral of the function with unit power

Using integration identities, the integral of a function (B.5) is solved to a form

$$\int_{-\infty}^{\infty} \frac{1}{1 + x^{2n}} dx = \frac{\pi/n}{\sin(\frac{\pi}{2n})} = 2/\text{sinc}\left(\frac{1}{2n}\right), \quad (\text{B.6})$$

where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. For the function to have a unit area and unit width, we had introduced variables $\bar{\nu}$ and μ in (3.9). Equating the integral over $\mathcal{S}_B(\nu)$ to 1, we find

$$\begin{aligned} \int_{-\infty}^{\infty} \mathcal{S}_B(\nu) d\nu &= 1 = \int_{-\infty}^{\infty} \frac{\mu}{1 + (\nu/\bar{\nu})^{2n}} d\nu = 2\mu\bar{\nu} \left[\text{sinc}\left(\frac{1}{2n}\right) \right]^{-1} \\ \Rightarrow \bar{\nu} &= \frac{1}{2\mu} \text{sinc}\left(\frac{1}{2n}\right), \end{aligned} \quad (\text{B.7})$$

which gives a relation between the normalization factor μ and the 3 dB cut-off frequency $\bar{\nu}$, for any filter order n .

B.3.2 Integral of the function with arbitrary power

To find the area under $\mathcal{S}_B^m(\nu)$ for any arbitrary positive integer m , note that

$$\frac{d}{dx} \frac{x}{(1 + x^{2n})^m} = \frac{1 + x^{2n}(1 - 2nm)}{(1 + x^{2n})^{m+1}} = \frac{1 - 2mn}{(1 + x^{2n})^m} + \frac{2mn}{(1 + x^{2n})^{m+1}},$$

where the last equality follows from partial fractions. Taking the definite integral of both sides with limits $[-\infty, \infty]$, we get

$$\begin{aligned} \left. \frac{x}{(1 + x^{2n})^m} \right|_{-\infty}^{\infty} &= \int_{-\infty}^{\infty} \frac{1 - 2mn}{(1 + x^{2n})^m} dx + \int_{-\infty}^{\infty} \frac{2mn}{(1 + x^{2n})^{m+1}} dx \\ \Rightarrow 0 &= \int_{-\infty}^{\infty} \frac{1 - 2mn}{(1 + x^{2n})^m} dx + \int_{-\infty}^{\infty} \frac{2mn}{(1 + x^{2n})^{m+1}} dx \\ \Rightarrow \int_{-\infty}^{\infty} \frac{dx}{(1 + x^{2n})^{m+1}} &= \frac{2mn - 1}{2mn} \int_{-\infty}^{\infty} \frac{dx}{(1 + x^{2n})^m}. \end{aligned}$$

This shows that the area under $\mathcal{S}_B^m(\nu)$ can be found using *integration by reduction*. Substituting $x = \nu/\bar{\nu}$, we can write the integral of $\mathcal{S}_B^m(\nu)$ as

$$\begin{aligned}
 \int_{-\infty}^{\infty} \mathcal{S}_B^m(\nu) d\nu &= \bar{\nu} \mu^m \int_{-\infty}^{\infty} \frac{1}{(1+x^{2n})^m} dx \\
 &= \bar{\nu} \mu^m \cdot \frac{2(m-1)n-1}{2(m-1)n} \int_{-\infty}^{\infty} \frac{dx}{(1+x^{2n})^{m-1}}, \quad \forall m \geq 2 \\
 &= \bar{\nu} \mu^m \int_{-\infty}^{\infty} \frac{dx}{1+x^{2n}} \prod_{k=1}^{m-1} \left(1 - \frac{1}{2kn}\right) \\
 &\stackrel{(a)}{=} 2\bar{\nu} \mu^m \left[\text{sinc}\left(\frac{1}{2n}\right) \right]^{-1} \prod_{k=1}^{m-1} \left(1 - \frac{1}{2kn}\right) \\
 &\stackrel{(b)}{=} \mu^{m-1} \prod_{k=1}^{m-1} \left(1 - \frac{1}{2kn}\right), \quad \forall m \geq 2
 \end{aligned} \tag{B.8}$$

where (a) follows from (B.6), and (b) follows from (B.7). This completes the derivation of area under a Butterworth PSD with arbitrary power m . With the criterion $\int \mathcal{S}_B^2(\nu) d\nu = 1$, one can find the parameter μ . It follows from (B.8) as

$$\int_{-\infty}^{\infty} \mathcal{S}_B^2(\nu) d\nu = 1 = \mu \left(1 - \frac{1}{2n}\right) \Rightarrow \mu = \frac{2n}{2n-1}. \tag{B.9}$$

B.3.3 Values for λ_1 and λ_2

Using (B.8), we can derive the variable λ_1 as

$$\begin{aligned}
 \lambda_1 &= \int_{-\infty}^{\infty} \mathcal{S}_B^3(\nu) d\nu = \mu^2 \left(1 - \frac{1}{2n}\right) \left(1 - \frac{1}{4n}\right) \\
 &= \left(1 - \frac{1}{2n}\right)^{-1} \left(1 - \frac{1}{4n}\right) = \frac{4n-1}{2(2n-1)},
 \end{aligned}$$

where the second equality follows from (B.9). Similarly, we can find

$$\lambda_2 = \int_{-\infty}^{\infty} \mathcal{S}_B^4(\nu) d\nu = \lambda_1 \mu \left(1 - \frac{1}{6n}\right) = \lambda_1 \cdot \frac{6n-1}{3(2n-1)}.$$

Derivation of MA parameters

In Chapter 4, we studied the N-FOM system in a MA environment. This appendix contains key derivations and proofs relevant to the analyses in that chapter. First, in Section C.1, we derive the maximum number of usable frequency offsets in an MA network. The final outcome serves as proof for (4.1). Next, in Section C.2, we derive the crosstalk, which was modeled as a bias term in (4.14). We derive expression for the crosstalk, and comment on its suppression in the context of design parameters.

C.1 Maximum number of usable frequency offsets

In this section, we prove (4.1) by mathematical induction. The equation represents the maximum number of usable frequency offsets N_u , which entails a few assumptions. First, this is only applicable to a polar NRZ message signal. Second, the receiver is considered to be perfectly synchronized with the transmitter. Finally, the transmission delays associated with each interfering link are ignored. While the first two assumptions are plausible, the last one is impractical. Therefore, N_u derived below should be seen as a theoretical upper bound on the number of usable frequency offsets.

For a polar NRZ signaling scheme, we have the criterion $f_r = n/T_b$ for positive integer n . If f_U is the maximum usable frequency offset, then we have $N_a = f_U T_b$ as the total number of available frequency offsets. Each number in the set $\{1, 2, 3, \dots, N_a\}$ represents the multiple of the symbol rate $1/T_b$. We can divide N_a in a number of subsets $\mathbb{R}_1, \mathbb{R}_2, \mathbb{R}_3, \dots$, such that

$$\begin{array}{rclclcl}
 \mathbb{R}_1 & = & 1 & 3 & 5 & 7 & \dots & 2m+1 \\
 \mathbb{R}_2 & = & 2 & 6 & 10 & 14 & \dots & 2(2m+1) \\
 \mathbb{R}_3 & = & 4 & 12 & 20 & 28 & \dots & 4(2m+1) \\
 \mathbb{R}_4 & = & 8 & 24 & 40 & 56 & \dots & 8(2m+1) \\
 \mathbb{R}_5 & = & 16 & 48 & 80 & 112 & \dots & 16(2m+1) \\
 \vdots & & \vdots & \vdots & \vdots & \vdots & \dots & \vdots
 \end{array}$$

for any non-negative integer m . Note that the elements in each subset $\mathbb{R}_{i+1}$ are twice those in the previous subset, i.e., $\mathbb{R}_{i+1} = 2\mathbb{R}_i$. For a large N_a , there will be a large number of subsets. For $N_a = 40$, all numbers from 1 to N_a will be encompassed by $\mathbb{R}_1$ to $\mathbb{R}_6$, with each subset having a different number of the elements.

Let us focus on the first two subsets. The subset $\mathbb{R}_1$ contains all odd numbers. If we use the elements of $\mathbb{R}_1$, then we cannot use the elements of $\mathbb{R}_2$, since the frequency offset of one user cannot be chosen as twice of that used by another user, i.e., $f_i \neq 2f_j$. This can be extended to all subsequent subsets; starting with $\mathbb{R}_1$, we can only use elements of consecutive odd subsets, i.e., $\{\mathbb{R}_3, \mathbb{R}_5, \mathbb{R}_7, \dots\}$.

For a fixed N_a , the total count of elements in $\mathbb{R}_1$ is $N_a/2$ if N_a is even, and $(N_a+1)/2$ if N_a is odd. One can see that the count of elements in $\mathbb{R}_3$ is $\lfloor \frac{N_a+4}{8} \rfloor$, where $\lfloor \frac{a}{b} \rfloor$ denotes floor operation, and the number 8 represents the difference between the consecutive elements in the subset. This can be extended to the rest of the usable subsets as

$$\left\lfloor \frac{N_a+4}{2 \times 4} \right\rfloor + \left\lfloor \frac{N_a+16}{2 \times 16} \right\rfloor + \left\lfloor \frac{N_a+64}{2 \times 64} \right\rfloor + \dots = \sum_{n=1}^{n_{\max}} \left\lfloor \frac{N_a+4^n}{2 \times 4^n} \right\rfloor,$$

where $n_{\max} = \lfloor \log_4(N_a) \rfloor$ is a limit that ensures that we stay within the count of available frequency offsets N_a . Combining the above outcome with the count of elements in $\mathbb{R}_1$, we get

$$N_u = \sum_{n=1}^{n_{\max}} \left\lfloor \frac{N_a+4^n}{2^{2n+1}} \right\rfloor + \left\lfloor \frac{N_a+1}{2} \right\rfloor.$$

To analyze the above expression, we plot N_u as a function of N_a in Figure C.1. One can see that for any value of N_a , the usable number of frequency offsets N_u can be roughly approximated as $2N_a/3$. This completes the proof of (4.1).

C.2 Crosstalk in MA

In this section, we derive the crosstalk caused by the User 2, in a MA communication link depicted in Figure 4.5. The crosstalk was analytically defined in (4.14). We consider two examples of signaling schemes and derive conditions which ensure minimization or suppression of the crosstalk.

C.2.1 NRZ signaling scheme

We first consider a system design where the message signal is a train of polar rectangular pulses. Furthermore, the frequency offset is chosen as a multiple of the symbol rate. In such a design, the detection filter is an integrate-and-dump

Figure C.1: Usable number of frequency offsets N_u plotted against number of available frequency offsets N_a .

filter. The contribution of the crosstalk in (4.14) is given by

$$\begin{aligned} \chi &= \int_{T_b} \mathbb{E}[z_{y_2}(t)] dt = \frac{1}{2L_2} \int_{T_b} \mathbb{E}[|\tilde{x}_{g,2}(t - \tau_2)|^2] a_2(t - \tau_2) \cos(2\pi f_1 t + \phi_1) dt \\ &= \frac{P_{x_2}}{L_2} \int_{T_b} \left(m_2(t - \tau_2) \cos(2\pi f_2(t - \tau_2) + \phi_2) + \frac{1}{\sqrt{2}} \right)^2 \cos(2\pi f_1 t + \phi_1) dt, \quad (\text{C.1}) \end{aligned}$$

where the last equality follows from using (4.5) and (4.9b). Since, we have assumed a polar NRZ message signal, with f_i as a non-zero multiple of the integration period, the above equation reduces to

$$\begin{aligned} \int_{T_b} \mathbb{E}[z_{y_2}(t)] dt &= \frac{P_{x_2}}{L_2} \int_{-T_b/2}^{T_b/2} \left(\frac{1}{2} \cos(4\pi f_2(t - \tau_2) + 2\phi_2) \cos(2\pi f_1 t + \phi_1) \right. \\ &\quad \left. + \sqrt{2} m_2(t - \tau_2) \cos(2\pi f_2(t - \tau_2) + \phi_2) \cos(2\pi f_1 t + \phi_1) \right) dt \\ &= \frac{P_{x_2}}{4L_2} \sum_{c=-1,1} \int_{-T_b/2}^{T_b/2} \cos(2\pi(f_1 - 2cf_2)t + cf_2\tau_2 + \phi_1 - 2c\phi_2) dt \\ &\quad + \frac{P_{x_2}}{\sqrt{2}L_2} \int_{-T_b/2}^{T_b/2} m_2(t - \tau_2) \cos(2\pi f_2(t - \tau_2) + \phi_2) \cos(2\pi f_1 t + \phi_1) dt. \end{aligned}$$

Adhering to the design criteria $f_i \neq 2f_j$ and $f_i = m/T_b$, one ensures that the first integral results in zero. In other words, the integration interval is an integer multiple of the period of $\cos(2\pi(f_1 \pm f_2)t + \phi)$, for any constant phase ϕ . In the third term, a possible bit transition in the message signal of User 2 will

result in a non-zero outcome. Defining $f_H := f_1 + f_2$ and $f_L := f_1 - f_2$, we solve

$$\begin{aligned} \chi = \frac{P_{x_2}}{\sqrt{2}L_2} & \left(\int_{-T_b/2}^{\tau_2 - T_b/2} b_{k-1,2} \cos(2\pi f_2(t - \tau_2) + \phi_2) \cos(2\pi f_1 t + \phi_1) dt \right. \\ & \left. + \int_{\tau_2 - T_b/2}^{T_b/2} b_{k,2} \cos(2\pi f_2(t - \tau_2) + \phi_2) \cos(2\pi f_1 t + \phi_1) dt \right) \end{aligned}$$

Note that we can substitute $E_{b,2} = P_{x_2} T_b / L_2$, in accordance with (4.18). After some tedious math, the integrals above are solved to

$$\begin{aligned} \chi = \frac{\tau E_{b,2}}{2T_b} b_{k-1,2} & \left[\text{sinc}(f_L \tau) \cos(\phi_1 - \phi_2 + \pi f_H \tau - \pi f_L T_b) \right. \\ & \left. + \text{sinc}(f_H \tau) \cos(\phi_1 + \phi_2 + \pi f_L \tau - \pi f_H T_b) \right] \\ & + \frac{E_{b,2}(T_b - \tau)}{2T_b} b_{k,2} \left[\text{sinc}(f_L(T_b - \tau)) \cos(\phi_1 - \phi_2 + \pi f_H \tau) \right. \\ & \left. + \text{sinc}(f_H(T_b - \tau)) \cos(\phi_1 + \phi_2 + \pi f_L \tau) \right]. \quad (\text{C.2}) \end{aligned}$$

This is a slightly complicated expression; nonetheless, it gives some insights. For any arbitrary values of $\{\tau, \phi_1, \phi_2\}$, one can minimize the crosstalk, if one chooses the offsets such that $f_L \gg 1/T_b$. This is further explained with the help of a figure in Section 4.2.2.

C.2.2 Generic signaling scheme

If the message signal employs an arbitrary pulse shape $p(t)$ with an absolute bandwidth B_p , then the derivation of the bias term in (C.1) follows a similar analysis as that of the bias term $\Xi(\cdot)$ in Appendix B.1. As in that appendix, we define $g(t) := m^2(t)$, which has a bandwidth in the order of $2B_p$. We write the crosstalk (4.14) as sum of three individual terms, i.e.,

$$\begin{aligned} \chi &= \frac{P_{x,2}}{L_2} \int_{-\infty}^{\infty} \left(\frac{1}{2} + g(t - \tau_2) \cos^2(2\pi f_2(t - \tau_2) + \phi_2) \right. \\ & \quad \left. + \sqrt{2} \sum_k b_{k,2} p(t - kT_b - \tau_2) \cos(2\pi f_2(t - \tau_2) + \phi_2) \right) p(t) \cos(2\pi f_1 t + \phi_1) dt \\ &= \chi_1 + \chi_2 + \chi_3, \end{aligned} \quad (\text{C.3})$$

and we will analyze the three terms separately. We will frequently use the relations in (B.2), that are derived from Fourier properties and Plancherel's

theorem. The term χ_1 can be written as

$$\chi_1 = \frac{P_{x,2}}{2L_2} \int_{-\infty}^{\infty} p(t) \cos(2\pi f_1 t + \phi_1) dt = \frac{P_{x,2}}{2L_2} \text{Re}\{P(f_1)e^{-j2\phi_1}\}.$$

It can be seen that a condition $B_p < f_1$ is sufficient to suppress this term. The second crosstalk term χ_2 is written as

$$\begin{aligned} \chi_2 &= \frac{P_{x,2}}{L_2} \int_{-\infty}^{\infty} g(t-\tau_2) \cos^2(2\pi f_2(t-\tau_2+\phi_2)) \cos(2\pi f_1 t + \phi_1) p(t) dt \\ &= \frac{P_{x,2}}{2L_2} \int_{-\infty}^{\infty} g(t-\tau_2) \cos(2\pi f_1 t + \phi_1) p(t) dt \\ &\quad + \frac{P_{x,2}}{2L_2} \int_{-\infty}^{\infty} g(t-\tau_2) \cos(4\pi f_2(t-\tau_2)+2\phi_2) \cos(2\pi f_1 t + \phi_1) p(t) dt. \end{aligned}$$

With analysis similar to that in Section B.2.2, one can show that the condition $f_1 \geq 3B_p$ is sufficient to suppress the first term in the last expression. Denoting $\theta_2 := \phi_2 - 2\pi f_2 \tau_2$ for brevity, the second integral is solved as

$$\begin{aligned} \chi_2 &= \frac{P_{x,2}}{2L_2} \int_{-\infty}^{\infty} g(t-\tau_2) \left[\cos(2\pi(f_1 - 2f_2) + \phi_1 - 2\theta_2) \right. \\ &\quad \left. + \cos(2\pi(f_1 + 2f_2) + \phi_1 + 2\theta_2) \right] p(t) dt \\ &= \frac{P_{x,2}}{4L} \sum_{c=-1,1} \int_{-\infty}^{\infty} G(f) \left[P^*(f + f_1 - 2cf_2) e^{j(\phi_1 + 2c\theta_2)} \right. \\ &\quad \left. + P^*(f - (f_1 - 2cf_2)) e^{-j(\phi_1 - 2c\theta_2)} \right] e^{-j2\pi f \tau_2} df. \end{aligned}$$

In order to suppress this term, it is sufficient to ensure that $G(f)$ is orthogonal to both $P(f \pm (f_1 - 2f_2))$ and $P(f \pm (f_1 + 2f_2))$. One can see that this results in the sufficient condition $|f_1 - 2f_2| \geq 3B_p$, which ensures zero spectral overlap between $G(f)$ and $P(f \pm (f_1 - 2f_2))$. Since $f_1 + 2f_2$ is always greater than $|f_1 - 2f_2|$, the condition also ensures zero overlap between $G(f)$ and $P(f \pm (f_1 + 2f_2))$.

Finally, the bias term χ_3 in (C.3) is written as

$$\begin{aligned} \chi_3 &= \frac{P_{x,2}}{\sqrt{2}L_2} \sum_k b_{k,2} \int_{-\infty}^{\infty} p(t - kT_b - \tau_2) \left(\cos(2\pi f_H t + \phi_1 + \theta_2) \right. \\ &\quad \left. + \cos(2\pi f_L t + \phi_1 - \theta_2) \right) p(t) dt \\ &= \frac{P_{x,2}}{2\sqrt{2}L_2} \sum_k b_{k,2} \int_{-\infty}^{\infty} P(f) \left[P^*(f + f_H) e^{j(\phi_1 + \theta_2)} + P^*(f - f_H) e^{-j(\phi_1 + \theta_2)} \right. \\ &\quad \left. + P^*(f + f_L) e^{j(\phi_1 - \theta_2)} + P^*(f - f_L) e^{-j(\phi_1 - \theta_2)} \right] e^{-j2\pi f(kT_b + \tau_2)} df, \end{aligned}$$

where we had earlier defined $f_H = f_1 + f_2$ and $f_L = f_1 - f_2$. Analogous to analysis in Section B.2.2, one can see that a zero spectral overlap between $P(f)$ and $P(f \pm f_L)$ is sufficient to ensure the χ_3 is suppressed. In other words, $\chi_3 = 0$ for $|f_1 - f_2| \geq 2B_p$.

C.2.3 Conclusion

We looked at a MA scenario where a received node is simultaneously receiving signals from two users. The signal from the undesired user causes crosstalk, which was modeled as a bias term. In this section, we analyzed the crosstalk in the context of two signaling schemes and deduced sufficient conditions which ensure that there is no crosstalk.

For any generic signaling scheme that employs a pulse with an absolute bandwidth B_p , the condition $|f_1 - 2f_2| \geq 3B_p$ along with $|f_1 - f_2| \geq 2B_p$ is sufficient to ensure crosstalk-free reception. For a polar NRZ signaling scheme, the crosstalk has a slightly complicated expression. In that case, one requires $|f_1 - f_2| \gg 1/T_b$ to ensure no or negligible crosstalk.

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چہ آخر کار کے جی تہ شوم -

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List of publications

Publications related to this thesis

1. I. Bilal, A. Meijerink and M. J. Bantum, "A frequency offset transmit reference system in dense multipath environments: propagation effects and design considerations," *IEEE Transactions on Wireless Communications*, [in submission]
2. I. Bilal, S. Morshed, A. Meijerink, R. A. R. van der Zee, G. Heijenk, M. J. Bantum, "A frequency offset based transmit reference system for wireless sensor networks," *IEEE Communications Magazine*, [in review]
3. I. Bilal, A. Meijerink and M. J. Bantum, "Optimum receiver filter for a noise-based frequency-offset modulation system," *IEEE 27th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Valencia, Spain, 2016, pp. 1-7
4. I. Bilal, A. Meijerink and M. J. Bantum, "Performance analysis of noise-based frequency offset modulation in dense frequency-selective fading channels," *9th International Conference on Signal Processing and Communication Systems (ICSPCS)*, Cairns, Australia, 2015, pp. 1-7.
5. B. I. Bitachon, I. Bilal, A. Meijerink and M. J. Bantum, "Near-far problem in noise-based frequency-offset modulation," *IEEE Symposium on Communications and Vehicular Technology in the Benelux (SCVT)*, Luxembourg City, 2015, pp. 1-6.
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1. I. Bilal, A. A. Zaidi, T. J. Oechtering and M. Skoglund, "Feedback stabilization over a Gaussian interference relay channel," *14th IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Darmstadt, Germany, 2013, pp. 560-564.
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