Improving Airfoil Drag Prediction

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An improved formulation of drag estimation for thick airfoils is presented. Drag underprediction in XFOIL-like viscous–inviscid interaction methods can be quite significant for thick airfoils used in wind turbine applications (up to 30%, as seen in the present study). The improved drag formulation predicts the drag accurately for airfoils with reasonably small trailing-edge thicknesses. The derivation of drag correction is based on the difference between the actual momentum loss thickness based on freestream velocity and that based on the velocity at the edge of the boundary layer. The improved formulation is implemented in the most recent versions of XFOIL and RFOIL (an aerodynamic design and analysis method based on XFOIL, developed by a consortium of the Energy Research Centre of the Netherlands, the National Aerospace Laboratory/NLR, and the Delft University of Technology after the Energy Research Centre of the Netherlands maintained and improved the tool), and the results are compared with experimental data, results from commercial computational fluid dynamics methods like ANSYS CFX, as well as other methods like the Technical University of Denmark Aeroelastic Design Section (AED)'s EllipSys2D and the National Renewable Energy Centre (CENER)'s Wind Multiblock. The improved version of RFOIL shows good agreement with the experimental data.

Nomenclature

\( A, B \) = \( G \beta \) equilibrium locus coefficients

\( C_{\mu 0} \) = equilibrium maximum shear-stress coefficient

\( c \) = airfoil chord length

\( c_d \) = sectional drag coefficient

\( c_l \) = sectional lift coefficient

\( D \) = drag

\( g \) = correction factor for mass flow shape factor

\( H \) = shape factor

\( H_k \) = kinematic shape factor

\( H_l \) = mass flow shape factor

\( H^* \) = kinetic energy shape factor

\( h_{\text{TE}} \) = airfoil trailing-edge thickness

\( L \) = length scale reference parameter

\( M_{\infty} \) = freestream Mach number

\( N_{\text{crit}} \) = critical amplification factor

\( R_{\text{eq}} \) = chord-length based Reynolds number

\( R_{\theta} \) = momentum thickness Reynolds number

\( U_e \) = boundary-layer edge velocity

\( U_{\text{inc}} \) = incident freestream velocity

\( u \) = velocity

\( x, y \) = Cartesian space coordinates

\( x_{\mu 0} \) = equilibrium shear-stress coefficient multiplier in wake

\( \alpha \) = angle of attack

\( \Delta \theta \) = error in \( \theta \)

\( \delta \) = boundary-layer thickness

\( \delta^* \) = boundary-layer displacement thickness

\( \theta \) = boundary-layer momentum thickness

\( \xi, \eta \) = streamline space coordinates

\( \rho \) = density of fluid

Subscripts

\( \text{airfoil} \) = airfoil parameters

\( e \) = boundary-layer edge condition

\( \text{effective} \) = overall domain parameters

\( \text{fp} \) = flat plate

\( \text{RFOIL} \) = RFOIL parameters

\( \text{wake} \) = wake parameters

\( \text{XFOIL} \) = XFOIL parameters

\( \infty \) = incident freestream condition

I. Introduction

METHODS for accurate estimation of drag are important for the analysis and design of airfoils. Since, for an airfoil, the drag is usually two orders of magnitude smaller than lift, even small errors in drag values can cause a significant change in airfoil performance (lift–to-drag ratio). Thick airfoils are commonly used in wind turbine blades, with the thickness varying from 15% of the chord at the tip section to about 50% of the chord at the root section. It is thus important to have an accurate prediction for drag in order to determine the performance of the wind turbine. With the trend of the size of wind turbines increasing, thicker airfoils are becoming more and more important, as the structural requirements need to be satisfied while maintaining optimal aerodynamic performance. Present-day drag estimation models appear to underpredict the drag by a significant margin, ranging from 10% for thin airfoils to as high as 30% or more for thick airfoils with negligible trailing-edge thickness \( h_{\text{TE}} < 3\% \) of chord, as can be seen in the results of the present study. In this paper, the method currently used for the calculation of drag for airfoils is studied in detail and the cause of inaccuracy is analyzed. The limitations of the present method are also investigated in order to explain why it gives acceptable results for a certain range of airfoil types. Finally, a correction is proposed to improve the prediction of drag, and the results based on the correction are discussed.
II. Aerodynamic Methods

For the analysis presented in this paper, XFOIL [1] and RFOIL [2] are used. XFOIL, a viscous–inviscid interaction method for predicting flow about airfoils developed by Mark Drela at the Massachusetts Institute of Technology [1]. It uses a linear-vorticity panel method with a Karman–Tsien compressibility correction for analysis in direct and mixed-inverse modes. Source distributions superimposed on the airfoil and wake permit the modeling of viscous layer effects on potential flow results. A two-equation lagged dissipation integral method is used to represent the viscous layers. Both laminar and turbulent flows are treated with an \( \nu \)-type amplification formulation determining the transition point. The boundary-layer and transition equations are solved simultaneously with the inviscid flowfield by a global Newton method. The procedure is especially suitable for rapid analysis of low-Reynolds-number flows around airfoils with transitional separation bubbles. RFOIL is a modified version of XFOIL featuring an improved prediction for the maximum lift coefficient and includes a method for predicting the effect of rotation on airfoil characteristics. Regarding the maximum lift factor of the boundary layer for deviation from the equilibrium flow observed at high values of the shape factor. From the assumptions:

1) The incident flow direction (\( \theta_e \)) is in the freestream direction. Thus, in the far wake of an airfoil, the viscous effects in that region. This assumption is only valid when \( U_e \), which is the flat-plate case. For an airfoil, the assumption that velocity outside the boundary layer is constant in the \( \eta \) direction; hence, we can neglect the viscous effects in that region. This assumption is only valid when \( U_e \) is in the far wake of an airfoil, and the airfoil is considered to have zero thickness. There is also an important effect of the angle of attack that is zero for the flat-plate theory. As a consequence, the outer region (outside the boundary layer), which is governed by the potential flow equations, has a significant pressure gradient in the \( \eta \) direction (normal to streamline direction). This pressure gradient causes the velocity outside the boundary layer to vary in the \( \eta \) direction. For the analysis, we assume that the velocity outside the boundary layer is constant in the \( \eta \) direction; hence, we can neglect the viscous effects in that region. This assumption is only valid when \( U_e \) is in the far wake of an airfoil, which is the flat-plate case. For an airfoil, the assumption that velocity outside the boundary layer is \( U_e \) at all points in the normal direction results in an underprediction of the integral quantities displacement thickness \( \delta' \) and momentum thickness \( \theta \). This also leads to an overprediction of \( U_e \). In effect, the underprediction of \( \theta \) reduces the predicted drag and overprediction of \( U_e \) increases lift. This behavior is very commonly seen in viscous–inviscid interaction methods like XFOIL and RFOIL.

III. Momentum Thickness and Drag: Current Formulation

The currently used formulation for estimation of the drag on an airfoil is based on flat-plate boundary-layer theory. The momentum conservation in the \( x \) direction (direction of incident freestream) yields the drag on a body immersed in the flow. The expression for the drag of a flat plate [6,7] is given in terms of the momentum thickness as follows:

\[
D_{fp} = \rho U_e^2 \left[ \theta_e \right]_{\infty = \infty} \tag{1}
\]

where the subscript \( \infty = \infty \) indicates the value of the parameter at the far wake of the airfoil, and \( \theta_e \) is the momentum thickness (for incompressible flow) based on freestream velocity (indicated by subscript \( \infty \)) given by

\[
\theta_e = \int_0^\infty \frac{u}{U_\infty} \left( 1 - \frac{u}{U_\infty} \right) \, dy \tag{2}
\]

The preceding expression of drag involves the following assumptions:
1) The incident freestream velocity \( U_\infty \) is in the \( x \) direction.
2) \( U_\infty \) is a constant in space and time coordinates.
3) The velocity outside the boundary layer (the edge velocity) equals \( U_\infty \), i.e., zero pressure gradient along the flow.
4) The flow is in a steady state.
5) The flow is incompressible.

In dimensionless form, the drag coefficient is given by

\[
c_d = \frac{2[\theta_e]_{\infty = \infty}}{\Delta \theta} \tag{3}
\]

where \( \Delta \theta \) is a typical length scale parameter. For airfoils, the chosen length scale parameter is the chord length.

In RFOIL (as in XFOIL), the drag coefficient is calculated as given in Eq. (3) but the definition of momentum thickness is different. The momentum thickness is given in terms of streamwise coordinates instead of Cartesian coordinates, as defined in Eq. (2), and the reference velocity is the velocity at the edge of the boundary layer \( U_e \), instead of the freestream velocity \( U_\infty \). The edge velocity \( U_e \) on the airfoil surface is obtained by coupling the boundary-layer equations to the potential flow equations describing inviscid, irrotational flow in the outer region using a strong interaction scheme. The expression for momentum thickness in RFOIL is as follows:

\[
\theta_e = \int_0^\infty \frac{u}{U_e} \left( 1 - \frac{u}{U_e} \right) \, dy \tag{4}
\]

and the drag coefficient is given by

\[
c_{d,\text{RFOIL}} = \frac{2[\theta_e]_{\infty = \infty}}{c} \tag{5}
\]

where the subscript \( e \) denotes the momentum thickness calculated by using \( U_e \) as the reference velocity.

The two definitions of the drag coefficient are not the same. Equation (3) gives the force in the \( x \) direction, which is the definition of drag, whereas Eq. (5) gives the force along the \( \xi \) direction (streamwise direction), which may not necessarily be equal to the actual drag force on the airfoil. But, this assumption is reasonable at a location far downstream of the airfoil, where the flow is almost in the freestream direction. Thus, in the far wake of an airfoil, the streamwise and Cartesian coordinates are almost coincident as seen in Fig. 1:

\[
\xi \sim x, \quad \eta \sim y \tag{6}
\]

However, the use of \( U_e \) instead of \( U_\infty \) as the reference velocity in the momentum thickness calculation would cause a significant difference in the estimation of the drag. In the next section, this difference is analyzed in detail and a correction is proposed.

IV. Drag Correction

The relation for drag follows from momentum conservation along the incident flow direction (\( x \) direction). However, there is a significant difference between a flat plate and an airfoil in terms of the pressure distribution. There exists a pressure gradient along and normal to the surface of the airfoil that is a result of the airfoil being of finite thickness, as opposed to a flat plate, which is considered to have zero thickness. There is also an important effect of the angle of attack that is zero for the flat-plate theory. As a consequence, the outer region (outside the boundary layer), which is governed by the potential flow equations, has a significant pressure gradient in the \( \eta \) direction (normal to streamline direction). This pressure gradient causes the velocity outside the boundary layer to vary in the \( \eta \) direction. For the analysis, we assume that the velocity outside the boundary layer is constant in the \( \eta \) direction; hence, we can neglect the viscous effects in that region. This assumption is only valid when \( U_e \) is in the far wake of an airfoil, which is the flat-plate case. For an airfoil, the assumption that velocity outside the boundary layer is \( U_e \) at all points in the normal direction results in an underprediction of the integral quantities displacement thickness \( \delta' \) and momentum thickness \( \theta \). This also leads to an overprediction of \( U_e \). In effect, the underprediction of \( \theta \) reduces the predicted drag and overprediction of \( U_e \) increases lift. This behavior is very commonly seen in viscous–inviscid interaction methods like XFOIL and RFOIL.

To calculate the error in drag, we need to estimate the error in the momentum thickness \( \Delta \theta \) at the end of the wake (taken to be one chord length downstream of the airfoil in XFOIL and RFOIL). \( \Delta \theta \) is the difference between the actual momentum loss thickness \( \theta_e \), which is based on the freestream velocity \( U_\infty \), and the predicted momentum loss thickness \( \theta_e \) which is based on the calculated value of the velocity at the edge of the boundary layer \( U_e \). It is important to note that this error \( \Delta \theta \) is a deviation of the physical model used by viscous–inviscid interaction methods from the real physical process; it is not an error of a numerical origin. For the following part of this section, all boundary-layer variables are taken to be at the end of the wake:

\[
\Delta \theta = \theta_e - \theta_e \tag{7}
\]
As mentioned earlier in Eq. (6), \( \eta \sim \gamma \) at the end of wake as seen in Fig. 1. This allows us to write the expression of \( \theta_e \) as an integration in the \( \gamma \) direction. Also, the limits of integration can be taken as the boundary-layer thickness \( \delta \), from zero to \( \theta_e \) (the lower limit is taken as zero in XFOIL and RFOIL, since the wake is considered as symmetric and the initial values of the boundary-layer variables are taken as the sum of the upper and lower airfoil surface trailing-edge values). Based on this, we have

\[
\Delta \theta = \int_0^\delta \left( 1 - \frac{u}{U_e} \right) \, dy - \int_0^\delta \left( 1 - \frac{u}{U_e} \right) \, dy
\]

\[
= \int_0^\delta \left( \left( \frac{u}{U_e} - \frac{u}{U_e} \right) - \left( \frac{u^2}{U_e^2} - \frac{u^2}{U_e^2} \right) \right) \, dy
\]

\[
= \left( \frac{U_e}{U_e} - 1 \right) \int_0^\delta \frac{u}{U_e} \, dy + \int_0^\delta \frac{u^2}{U_e^2} \, dy
\]

\[
= \left( \frac{1}{U_e} - 1 \right) \left( \frac{1}{U_e} + 1 \right) \int_0^\delta \frac{u^2}{U_e^2} \, dy
\]

\[
= \left( \frac{1}{U_e} - 1 \right) \left( \theta_e + \frac{U_e}{U_e} \left( \delta_e + \theta_e - \delta \right) \right)
\]

where

\[
\delta_e \equiv \int_0^\delta \left( 1 - \frac{u}{U_e} \right) \, dy
\]

Since

\[
H_1 = \frac{\delta - \delta_e}{\theta_e}
\]

in XFOIL and RFOIL, we have

\[
\Delta \theta = \theta_e \left( 1 - \frac{1}{U_e} \right) \left( \frac{U_e}{U_e} \left( H_1 - 1 \right) - 1 \right)
\]

(8)

where the value of \( H_1 \) is obtained using the correlation from Green et al. \([5]\) and simplified by Drela and Giles \([8]\):

\[
H_1 = 3.15 + \frac{1.72}{\delta_e - 1}
\]

(9)

This expression is based on experimental results and correlates \( \delta \) to \( \delta_e \) and \( \theta_e \) as

\[
H_1 = \frac{\delta - \delta_e}{\theta_e}
\]

Since \( \delta_e \) and \( \theta_e \) are larger than \( \delta_e \) and \( \theta_e \), respectively (as evidenced from drag underprediction and lift overprediction in XFOIL and RFOIL), using the edge velocity-based values will result in an overprediction of \( H_1 \). As seen from Eq. (9), the value of \( H_1 \) depends only on the value of kinematic shape factor \( H_1 \), which for incompressible flow is the same as the shape factor \( H(\equiv \delta / \theta) \). The rate of change of \( H_1 \) with respect to \( H \) becomes extremely high for values of \( H \) tending to unity (\( H \rightarrow 1 \); see Fig. 2). Thus, in the far wake, where the value of \( H \) is close to unity, the value of \( H_1 \) is very sensitive to the prediction of \( H \); and even small changes in the boundary-layer model can cause a significant difference in the results. This difference is seen in the results of RFOIL compared to XFOIL. In the next section, this difference in prediction is explored in order to obtain a generalized expression for the \( \theta \) correction.

To correct the value of \( H_1 \), we introduce a constant \( g \) as a multiplier to \( H_1 \) with the condition that \( g < 1 \). The value of \( g \) depends on the turbulent boundary-layer method used in the solver and will be determined later by trial and error. Using this in Eq. (8) and rearranging the terms, we have

\[
\Delta \theta = \theta_e \left( 1 - \frac{1}{U_e} \right) \left( \frac{U_e}{U_e} \left( gH_1 - 1 \right) - 1 \right)
\]

(10)

From Eqs. (7) and (10), we have

\[
\theta_e = \theta_e + \Delta \theta = \theta_e \left( 1 + \left( 1 - \frac{1}{U_e} \right) \left( \frac{U_e}{U_e} \left( gH_1 - 1 \right) - 1 \right) \right)
\]

(11)

Once \( \Delta \theta \) is calculated, it can be added to \( \theta_e \) to obtain \( \theta_e \), as shown in Eq. (11). Applying the Squire and Young correction to the end wake \( \theta_e \) (one chord length downstream in XFOIL and RFOIL) will give us the value of momentum loss thickness at downstream infinity, which can be used to calculate drag coefficient. The corrected value of the drag coefficient is given by

\[
c_d = \frac{2[\theta_e]_{x=0}}{c} = \frac{2[\theta_e + \Delta \theta]_{x=\infty}}{c}
\]

(12)

V. Determination of Correction Factor \( g \) for Mass Flow Shape Factor

Equation (10) gives an expression for the correction of the momentum thickness at the end of the wake. This expression has the empirical parameter we need to determine, i.e., mass flow shape factor correction \( g \). To determine the value of \( g \), XFOIL results are used as the baseline. A set of airfoils is analyzed for a wide range of angles of attack, and the value of \( g \) is varied to determine the value that results in good agreement with the experimental data for the drag. As a first guess, it is assumed that \( g \) is a constant and not a function of some parameters. The correction factor \( g \) is a consequence of the assumptions made in the boundary-layer equations (described in Sec. III), and hence need not necessarily depend on the boundary-layer variables themselves, given that the boundary-layer model remains
the same when comparing two different solvers. At present, the
analysis is confined to XFOIL and RFOIL only, and the correction
factor $g$ is related to the results from these two solvers.

The analysis to determine the value of $g$ was carried out by
evaluating the drag coefficients of two airfoils: NACA 0012 and DU
97-W-300. The choice was made in order to have a value of $g$
that predicts drag well for thin airfoils, as well as for thick
airfoils. The analysis presented in this section is computed for incompressible
flow conditions ($M_{\infty} = 0$) for conditions of natural transition with a
freestream turbulence intensity of 0.07% ($N_{\text{crit}} = 9$). For thin
airfoils, there is not much of a problem in the predicted drag in the
present model of XFOIL and RFOIL; thus, the correction should not
affect the results greatly. For thick airfoils, the underprediction in
drag is noticeable and the correction is expected to improve the
prediction. Preliminary results from XFOIL indicate the value of $g$
to be 0.4. Figures 3 and 4 show the predicted lift and drag characteristics
for NACA 0012 and DU 97-W-300 airfoils for the case of the most
recent XFOIL version, experimental data, and XFOIL with the
proposed drag correction using $g = 0.4$.

The case of NACA 0012 is analyzed as a benchmark case to make
sure that the proposed correction does not affect drag for cases for
which the XFOIL prediction is accepted to be accurate. As can be
seen in Fig. 3, the drag correction improves the drag prediction and
the curve moves closer to the curve with experimental data. The
magnitude of the correction is comparable to the difference between
the XFOIL prediction and the experimental data. The experimental
data for the NACA 0012 airfoil used for comparison were obtained
from the airfoil data catalog of Miley [9].

For the case of a thick airfoil without a significant trailing-edge
thickness, the drag prediction appears to improve by using the correction. Figure 4 shows the results for the DU 97-W-300 airfoil,
which is a 30% thick airfoil with a small trailing-edge thickness
($\approx 1.8\%$ of chord). The corrected lift–drag polars are still not on top of
the experimental data, as there is an overprediction of lift as well. This
is due to the underprediction of $\delta^*$. The correction for drag deteri-
orates as the angle of attack becomes larger. In deep stall, the correc-
tion is no longer valid, as the considered wake length of one chord is
no longer sufficient to resolve a fully developed wake. In such cases,
the edge velocity $U_e$ becomes higher than the freestream velocity and the drag correction then yields a negative value. However, this is due to the failure of the solver to predict the boundary-layer parameters accurately in deep stall regions due to the massive separation present in such a case. The experimental data for the DU 97-W-300 airfoil used for comparison were obtained from the technical report by Timmer [10].

At high angles of attack, the drag prediction deteriorates, as the predicted drag with the correction is somewhat higher than experimental data, as seen in Fig. 4. For this airfoil, the calculation has been carried out at five different angles of attack (denoted by five points on the curves) and compared to experimental results. This behavior can also be attributed to the overall drop in the accuracy of prediction in XFOIL due to the strong adverse pressure gradients and trailing-edge flow separation that are commonly observed near stall, which cannot be coped with by the strong-interaction method. The same behavior is observed near stall for the FX 69-274 and AH 94-W-301 airfoils. The drag correction is sensitive to the change in the shape factor $H_e$ at the far-field wake location, and proper care must be taken to refine the grid (mesh density) in order to obtain a reliable result. The value of $g = 0.4$ is also verified in calculations for several other airfoils, and the results are presented in Sec. VI.

Once the value of the shape factor correction is established for XFOIL, the same procedure is repeated for RFOIL. For the case of RFOIL, $g = 0.4$ causes an overprediction in drag. Figure 5 shows this behavior for the case of the NACA 0012 airfoil. There is a noticeable overprediction in drag in RFOIL when the proposed drag correction formula is used. Since the only empirical parameter in the drag correction formula is $g$, changing its value may improve the value of the predicted drag. The most significant difference between RFOIL and XFOIL is the $G − \beta$ equilibrium locus coefficients. Another difference is the quadrupling of the equilibrium shear-stress level in the wake in RFOIL.

The aforementioned differences between the formulation of RFOIL and that of XFOIL, will certainly have an effect on the magnitude of the drag correction. This means the mass flow shape factor correction $g$ will no longer be a constant and will need to be related to the differences in the two models. To quantify the effect of these differences in the model and relate them to $g$, we need to look at how these differences affect the solution. The solution domain can be split into two parts, namely, the airfoil surface and the wake. We will analyze the difference in the equilibrium shear-stress levels in these two domains and then quantify the change as a weighted average of the equilibrium shear-stress levels for the airfoil surface and that for the wake. The rationale behind such a weighted average is that the equilibrium shear-stress levels are indicators of the level of dissipation, and the weighted average gives an equivalent value of the change dissipation level in the whole domain.

On the airfoil surface, the only significant difference between XFOIL and RFOIL is in the $G − \beta$ equilibrium locus, namely, the values of the coefficients $A$ and $B$. These coefficients affect the magnitude of the equilibrium shear stress. The equilibrium shear stress is described in the paper from Drela and Giles [8] as

$$C_{eq} = \frac{0.5H^*(H_1 - 1)^3}{A^2B(1 - U_s)HH_1}$$

(13)

where the definitions of $H^*$, $H_1$, and $U_s$ can be found in the reference from Drela and Giles [8].

In the preceding relation, the term $A^2B$ in the denominator is the most significant term that is different between RFOIL and XFOIL. Table 1 shows the values of the $G − \beta$ equilibrium locus coefficients $A$ and $B$ in XFOIL and RFOIL on the airfoil surface. Since the equilibrium locus is the most significant change between RFOIL and XFOIL, the difference in the equilibrium shear-stress levels will play a major role in determining the level of error in momentum thickness. From Eq. (12), we have

$$C_{eq} \propto \frac{1}{A^2B}$$

(14)

assuming that the rest of the terms do not have a significant dependence on $A$ and $B$. Comparing the equilibrium shear-stress levels in XFOIL and RFOIL on the airfoil surface results in the following:

$$\begin{bmatrix} C_{eq, RFOIL} \\ C_{eq, XFOIL} \end{bmatrix} \begin{bmatrix} C_{eq, \text{airfoil, RFOIL}} \\ C_{eq, \text{airfoil, XFOIL}} \end{bmatrix} = \begin{bmatrix} A^2B_{\text{XFOIL}} \\ A^2B_{\text{RFOIL}} \end{bmatrix} \begin{bmatrix} C_{eq, RFOIL} \\ C_{eq, XFOIL} \end{bmatrix} = 0.89$$

(15)

![Fig. 5 Predicted lift-drag polars for NACA 0012 in RFOIL. Experimental data from Ref. [9].](image-url)
In the wake, the equilibrium shear stress $C_{\tau EQ}$ is quadrupled in RFOIL, whereas it remains the same in XFOIL. Thus, for the wake region, we have

$$\frac{[C_{\tau EQ}]_{RFOIL}}{[C_{\tau EQ}]_{XFOIL}}_{\text{wake}} = 4\frac{[C_{\tau EQ}]_{RFOIL}}{[C_{\tau EQ}]_{XFOIL}} = 4\frac{[A^2B]_{XFOIL}}{[A^2B]_{RFOIL}} = 3.56$$  \hspace{1cm} (16)$$

Thus, the effective ratio of the equilibrium shear stress predicted by RFOIL and that predicted by XFOIL can be taken as the weighted average of the equilibrium shear-stress ratio on the airfoil surface and that on the wake. Thus, the ratio of the equilibrium shear stress in RFOIL and XFOIL is given by

$$\frac{[C_{\tau EQ}]_{RFOIL}}{[C_{\tau EQ}]_{XFOIL}}_{\text{effective}} = \frac{1}{2} \left( \frac{[C_{\tau EQ}]_{RFOIL}}{[C_{\tau EQ}]_{XFOIL}}_{\text{airfoil}} + \frac{[C_{\tau EQ}]_{RFOIL}}{[C_{\tau EQ}]_{XFOIL}}_{\text{wake}} \right) = 2.225$$  \hspace{1cm} (17)$$

From the preceding equation, we see that the equilibrium shear-stress levels are higher in RFOIL, which will result in a higher dissipation level. This means that the predicted value of $\theta_e$ will be higher in RFOIL; hence, the correction in momentum thickness should be smaller in RFOIL compared to the one in XFOIL:

$$[\Delta \theta]_{RFOIL} < [\Delta \theta]_{XFOIL}$$  \hspace{1cm} (18)$$

Since $\theta_\infty$ should be the same for an airfoil at a given angle of attack in both RFOIL and XFOIL, $\Delta \theta$ for XFOIL and RFOIL depends directly on the predicted value of $\theta_e$. RFOIL has a higher dissipation level (higher effective $C_{\tau EQ}$) compared to that in XFOIL, resulting in a higher $\theta_e$. Hence, the correction $\Delta \theta$ should be smaller in RFOIL compared to that in XFOIL. This implies that $\Delta \theta$ is inversely proportional to the effective $C_{\tau EQ}$:

$$\Delta \theta \propto \frac{1}{C_{\tau EQ}}$$  \hspace{1cm} (19)$$

This expression is only a qualitative relationship of the error $\Delta \theta$ with respect to the equilibrium shear stress $C_{\tau EQ}$ for both methods. A quantitative relationship between the two quantities is difficult to establish, as the actual expression for the error $\Delta \theta$ should be based on the differences in the mathematical models on which two methods are based and their deviation from the true solution. Nonetheless, the qualitative relationship used here yields a satisfactory result, indicating that the true relationship between the two quantities is not significantly different.

Looking at the magnitude of the terms in Eq. (10), $H_1 \equiv 50$ to 100 at the end of the wake, as observed in the results for several airfoils, where $H_1 \to 1$ and $U_e/U_\infty \to 1$. Thus, Eq. (10) can be approximated as

$$\Delta \theta \approx \theta_e \left( 1 - \frac{U_e}{U_\infty} \right) \left( \frac{U_e}{U_\infty} (gH_1) \right)$$  \hspace{1cm} (20)$$

From this equation, we can infer that

$$\Delta \theta \propto g$$  \hspace{1cm} (21)$$

Using Eqs. (17), (19), and (21), we have

$$\frac{[\Delta \theta]_{RFOIL}}{[\Delta \theta]_{XFOIL}}_{\text{effective}} = \frac{[g]_{RFOIL}}{[g]_{XFOIL}} = \frac{[C_{\tau EQ}]_{RFOIL}}{[C_{\tau EQ}]_{XFOIL}}_{\text{effective}} = \frac{1}{2.225}$$  \hspace{1cm} (22)$$

Rearranging the terms and using the value of $g$ obtained for XFOIL results ($g = 0.4$) in the following:

$$[g]_{RFOIL} = \frac{[g]_{XFOIL}}{2.225} = 0.18$$  \hspace{1cm} (23)$$

Figure 5 shows the lift–drag polars using RFOIL for two values of $g$ for the case of the NACA 0012 airfoil. The value of $g = 0.18$ yields a good agreement with the experimental data. An expression for the mass flow shape factor correction $g$ can be obtained by combining Eqs. (15–17), (22), and (23), resulting in the following:

$$[g]_{\text{any solver}} = \frac{[A^2B]_{\text{any solver}}}{[A^2B]_{XFOIL}} \left( \frac{2}{1 + x_{\tau EQ}} \right)$$  \hspace{1cm} (24)$$

where $x_{\tau EQ}$ is the equilibrium shear-stress coefficient multiplier for the wake. Substituting the known values into the equation gives

$$[g]_{\text{any solver}} = \frac{[A^2B]_{\text{any solver}}}{42.084(1 + x_{\tau EQ})}$$  \hspace{1cm} (25)$$

Equation (25) gives a generalized expression for the mass flow shape factor correction $g$ for any XFOIL-like method. This formula is based on the equilibrium shear stress. It can be used for any XFOIL-like method, given that the difference between that method and XFOIL is only in the equilibrium shear stress (as in the case of RFOIL). Any other differences will require further modification of the expression for $g$. The value of $x_{\tau EQ}$ in RFOIL is four, in accordance with the experimental results obtained by Narasimha and Prabhav[11], who suggested a quadrupling of the eddy viscosity in the wake. The expression for the drag coefficient including the proposed correction is as follows:

$$c_d = \frac{2}{c} \theta_e \left( 1 + \left( 1 - \frac{U_e}{U_\infty} \right) \left( \frac{U_e}{U_\infty} \left( \left( \frac{A^2B}{42.084(1 + x_{\tau EQ})} \right) (H_1 - 1) - 1 \right) \right) \right)$$  \hspace{1cm} (26)$$

VI. Results

The proposed drag correction method has been implemented in XFOIL and RFOIL, and it has been tested for a range of airfoils in order to verify the accuracy of drag prediction, as shown by Ramamujam[12]. A significant improvement was observed for all the test cases considered. The analysis presented in this section is computed for incompressible flow conditions ($M_\infty = 0$) for conditions of natural transition with a freestream turbulence intensity of 0.07% ($N_{crit} = 9$). Only, for the case of the Innovative Wind Conversion Systems (10-20MW) for Offshore Applications (INNWIND.EU) analysis (FFA-W3-301 airfoil), the freestream turbulence is set to 0.1% ($N_{crit} = 8.145$). The level of freestream turbulence used in this study is within the range found in wind tunnels. Since experimental data in the public domain are only available from certain wind tunnels, the corresponding levels of freestream turbulence are used. It is important to note that, in XFOIL-like methods, the freestream turbulence levels only affect the transition locations, whereas the increased mixing in the turbulent boundary layer that leads to a delay of flow separation is not accounted for[12]. Thus, at elevated freestream turbulence levels, the quality of prediction is expected to deteriorate anyway.

Even for thinner airfoils, there is a noticeable improvement in drag prediction, as seen from the case of NACA 0012. A similar trend is also observed for the NACA 63 - 418 airfoil, as seen in Figs. 6a and 6b. For this airfoil, the drag underprediction is approximately 12% at the minimum drag point when using the original XFOIL and RFOIL results. For thicker airfoils, the underprediction in drag becomes more severe. The levels of underprediction in drag can become as high as 30% for very thick airfoils, such as DU 99-W-405LM, which has a maximum thickness of 40.5%. Figure 6 gives an illustration of

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\(^{\text{1Data available online at http://www.innwind.eu/About-INNWIND.}}\)
the levels of drag underprediction for thick wind turbine airfoils of varying thicknesses.

The drag correction yields excellent results in the linear lift regime, although the drag in the poststall regime is not very well predicted. This is to be expected, since the viscous–inviscid interaction method of XFOIL and RFOIL is not designed to handle massively separated flows.

For the AH 94-W-301 airfoil, the experimental and the predicted data (with improved drag) do not match satisfactorily as seen in Fig. 6e. This is due to the overprediction of lift observed in this case, as shown in Fig. 7. For DU 99-W-405LM, the same issue of lift overprediction exists, although it is necessary to note that the experimental data used for this airfoil are synthesized from low-Reynolds-number airfoil data (using the Aërodynamische Tabel...
Generator (ATG) tool. The ATG tool computes the geometry of the selected airfoil in its database and interpolates the experimental data from the database to generate (synthesize) the aerodynamic coefficients of the input airfoil. Lack of open-access experimental data on airfoils at higher Reynolds numbers is the reason why the synthesized data have been used in this case. The experimental data for Wortmann (FX series) and Althaus (AH series) airfoils were obtained from the University of Stuttgart [14]. The experimental data for the NACA 633−418 airfoil were obtained from Abbott and von Doenhoff [15].

The significance of the improvement in drag prediction using the developed correction can be realized by a comparison of the results from various commercial and in-house aerodynamic software with the results from the improved version of RFOIL. For this purpose, the FFA-W3-301 airfoil is chosen, as this airfoil was analyzed using various solvers in the INNWIND EU [16]. The solvers used for computing the aerodynamic characteristics of the selected airfoil are: 1) EllipSys 2D, which is CFD software developed by the Technical University of Denmark; 2) Wind Multi-Block, which is the CFD software developed by CENER and the University of Liverpool; and 3) CFX, which is the commercial CFD software developed by ANSYS.

A detailed description of these solvers and their analysis results can be found in the INNWIND EU deliverable report [16].

The improved version of RFOIL performs better for both Reynolds numbers as seen in Fig. 8. For a Reynolds number of 10×10⁶, EllipSys3D overpredicts the drag coefficient. The drag coefficient usually decreases with increasing Reynolds numbers, whereas the values predicted by EllipSys3D at a 10×10⁶ Reynolds number are larger than the experimental values at 3×10⁶.

VII. Conclusions

To enable accurate prediction of drag for thick wind turbine airfoils, a modification to the existing drag calculation method in XFOIL-like methods has been developed. The general methodology of the correction methods was applicable to a wide range of viscous–inviscid interaction methods with a two-equation k-ε dissipation integral boundary-layer method. The proposed method was applicable in regions of angle of attack before stall, as the correction could become negative in the deep stall region where, at the end of the wake, the
predicted value of velocity at the edge of the boundary layer exceeded the magnitude of the freestream velocity. This happened in the case of massively separated flows over the airfoil, which was outside the range of applicability of these computational methods. The correction was applicable to airfoils with a relatively small trailing-edge thickness ($h_{TE} < 3\%$ of chord). The present analysis was performed for a wake length of one chord, which was adequate for most airfoils. Increasing the wake length would require a recalibration of the value of the mass flow shape factor correction $\gamma$, as the end wake values of $H_1$ would be quite high due to the rapid increase in $H_1$ at values of $H$ close to unity.

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