Modeling of strain in multifilamentary wires deformed by thermal contraction and transverse forces

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A previously published analytical model that describes a simplified wire geometry with three stacked cylinders is compared with finite element model calculations. The thermal strain from the matrix on the superconducting filaments is considered first. It appears that the analytical model is able to describe the strain that occurs in the filaments relatively accurate. Especially the radial dependence of the strain if a central core of normal material is present, is described quit well by the analytical model. The strain inside a wire surrounded by epoxy and subjected to a transverse load is almost uniform and can be approximated with an analytical model too. When yielding is involved to simulate a more localised transverse load inside a multifilamentary wire it is necessary to consider a numerical model.

INTRODUCTION

The critical current density in brittle Nb₃Sn and BSCCO-2212/Ag superconductors highly depends on the strain state of the material. This strain sensitivity is a serious restriction for the operating current of superconductors in applications where large stresses are generated. In order to predict the critical current of a superconducting wire in its operating environment a combination of a mechanical and an electrical model is required to describe the maximum current of the wire. In a previous paper an analytical description for the strain state in combination with a description for the integral critical current is considered. In this model two types of transverse loads can act on a conductor that consists of three stacked cylinders [1]. The limitations of this analytical description are investigated by comparing it with a finite element analyses applied to a few specific load cases. In the first part a thermally induced prestrain in a round multifilamentary wire is considered. After that the influence of epoxy that surrounds the conductor is considered, in the case of a transverse load. Finally the strain state of a realistic multifilamentary wire model with plastic properties for the matrix and subjected to a transverse load is calculated.

THERMAL STRAIN IN A MULTIFILAMENTARY WIRE

The analytical model

The analytical wire model consists of three elements: A central normal conducting core has a radius r_1 , the superconductor is present in a cylinder from r_1 to r_2 and the outer normal conducting core is a cylinder from r_2 to radius 1. All the elements are perfectly connected and elastic with a constant Young's modulus (E) and Poisson's ratio (ν). If the difference in thermal contraction between the normal material and the superconducting cylinder introduces a net thermal contraction in the axial direction (δ), then the strain at a position r between r_1 and r_2 is described by [1]:

$$\begin{pmatrix} \varepsilon_r \\ \varepsilon_\phi \\ \varepsilon_z \end{pmatrix} = \begin{pmatrix} (1-3\nu+r^{-2}(1+\nu)f_a)(2-2\nu)^{-1} \\ (1-3\nu-r^{-2}(1+\nu)f_a)(2-2\nu)^{-1} \\ 1 \end{pmatrix} \cdot \delta \quad \text{with: } f_a = \frac{r_1^2}{1-r_2^2+r_1^2}.$$

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In order to model a multifilamentary wire the superconducting cylinder between r_1 and r_2 should be divided into separated filaments with matrix material in between. The mechanical behaviour of such a system is treated here as an extension of the three cylinder model with the filaments concentrated in the cylinder between r_1 and r_2 . This area is then called the *filamentary region*. This region is described as a collection of separate filaments surrounded by tubes of normal conducting material. The thermal strain in each single filament is then described by the three cylinder model in the limit $r_1 = 0 = f_a$. In a second step the three cylinder model is applied to a “new” set of cylinders. The central core and the outer ring contains again matrix material but the filamentary region has a mean thermal contraction based on the local normal to superconductor ratio. It is found that the thermal contraction inside the superconducting filaments is again described by equation 1 but with a modification for the factor f_a :

$$f_a = \frac{A_{sc}}{A_n} \frac{r_1^2}{r_2^2 - r_1^2}, \text{ where } A_{sc} \text{ is the superconducting and } A_n \text{ the normal conducting area.}$$

The numerical model

A realistic 192 filaments Nb_3Sn wire (ECN type powder in tube process) is modelled in a finite element structure. Each filament is represented by a single hexagon that contains six triangular elements. The matrix material surrounding the filaments is build up with two layers of rectangular elements. The interface between the elements is modelled by two layers of very thin rectangular elements that surround each filament. The filaments and the first interface layer are made of superconductor with: $E = 80 \text{ GPa}$ and $\nu = 0.3$, which describes the mixed properties of a $\text{Nb}+\text{Nb}_3\text{Sn}$ filament. The properties of the Cu matrix are $E = 124 \text{ GPa}$ and $\nu = 0.3$. The integral thermal contraction difference between the materials is taken as 0.5%, in combination with a Cu area fraction of 51%. This results in a thermal contraction in the filaments of 0.34% in the axial direction. The geometry of the model is drawn in figure 1a.

The thermally contracted wire is depicted in figure 1b. Especially the inner circle of filaments is deformed by the thermal contraction of the matrix. A comparison is made with the analytical model, for $\nu = 0.3$, $E = 100 \text{ GPa}$ and $\delta = -0.25\%$, inside the six adjacent filaments along the vertical symmetry line. The strain along this line is drawn in figure 2. The resemblance between the two models is good. The major difference is a small error of 0.1% in the mean value of the r - and ϕ -strain. In both models there occurs a large difference between the r - and ϕ -strain near the central core, which pulls the filaments towards the centre. This is equivalent to a large deviatoric strain component close to r_1 . This deviatoric strain can influence the critical current of the wire significantly [1], if the superconducting material is very sensitive to deviatoric strain, as in the A15 superconductors [2]. These deviatoric strains are minimised when all materials with a thermal contraction larger than that of the superconductor are moved to the outer core.

A TRANSVERSE LOAD

A cylinder surrounded by epoxy

Brittle wires used in practical applications are often surrounded by epoxy which is soft, compared to the strength of the wire. The influence of the epoxy on the strain state is investigated in a numerical model for a cylinder surrounded by epoxy and pressed between two stiff plates. The width of the press and the epoxy is 10% larger than the wire thickness. The epoxy is not constrained at both sides, otherwise a hydrostatic pressure cell would be the result. If the Young's modulus of the epoxy is zero then the strain state inside the cylinder coincides with the analytical model for a perfect line load [1]. If the elastic properties of the epoxy are equal to those of the cylinder, a uniform strain is induced in the entire system. These two extreme cases are compared with intermediate values for the Young's modulus in figure 3. The top of the cylinder touches the press. If a small distance of the order of 0.01 is present and filled with epoxy, then the strains are even further reduced towards the uniform limit that is represented by the dotted line. It is concluded that for realistic properties of the epoxy ($E \approx 10 \text{ GPa}$) the extreme influence of a point load is diminished and an almost uniform strain pattern is obtained.

A line load on a multifilamentary wire

A transverse line force acting on a bare wire without epoxy, cannot be approximated accurately by an elastic description. The effect of yielding of the matrix is considered in the numerical wire model. The

mechanical properties of the Cu in the finite element model of the wire with 192 filaments that is described before are modified. The yield point is defined at 124 MPa, and for higher pressures a constant slope of $d\sigma/d\varepsilon = 2.5$ GPa is applied. A press working in the transverse direction, as drawn in figure 3, is deforming the conductor. The strain pattern that is induced on the symmetry axis is depicted in figure 4. For a small pressure ($P < 25$ MPa) the influence of plastic deformation is small and the resulting strain is similar to the strain state that is calculated in an elastic cylinder with a line force working on the sides. If the pressure is increased the influence of the plastic deformation causes a large difference with the analytical description. The strains that occur in this model are far beyond the limit that is acceptable in brittle superconductors as Nb_3Sn or Bi based copper-oxide materials. The large strains of more than 0.5% that occur here will drastically reduce the current carrying capacity of the conductor. The obvious and essential solution is to embed the conductor in epoxy material by impregnation or wet winding.

CONCLUSIONS

1. The radial dependent thermal strain induced in a multifilamentary wire with the filaments located in a tube is described satisfactory with the analytical model of three stacked cylinders.
2. A significant deviatoric thermal strain appears inside filaments that are placed in matrix around a central core with a different thermal contraction. If the critical current of the filament is sensitive for deviatoric strains, as for example Nb_3Sn , then the critical current of the wire will increase if the size of the central core is reduced.
3. The strain state in a round wire that is pressed in the transverse direction reduces considerably and becomes more uniform when epoxy with a moderate Young's modulus ($10 > \text{GPa}$) encloses the wire.
4. In the case of a bare wire placed in a transverse press, plastic deformation of the different materials will seriously influence the strain state in the wire. A good knowledge of the material properties is then required to set-up a realistic model and to predict the critical current.

REFERENCES

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Figure 1: The finite element model for a multifilamentary wire. **A:** The unloaded wire with the separate elements indicated in one filament. **B:** The thermally contracted wire with the strain magnified 50 times. The arrow indicates the deformation of a single filament.

Figure 2: The strain induced inside the superconducting filaments of a multifilamentary wire. A comparison is made between the analytical (----) and the finite element model (—) of a Cu wire with 192 Nb/Nb₃Sn filaments.

Figure 3: An elastic cylinder surrounded by epoxy and pressed between two stiff plates with 100 MPa. The induced strain along the path is indicated for various values of the Young's modulus in the epoxy. The dotted line shows the strain induced by a perfectly distributed force.

Figure 4: The strain in a transversely pressed multifilamentary wire. The strain in the wire with a plastically deformed matrix is compared to what is found by the analytical model of an elastic cylinder. Only one strain component is indicated and traced along the path that is indicated in figure 3.