Port-Hamiltonian Passivity-Based Control on $SE(3)$ of a Fully-Actuated UAV for Aerial Physical Interaction near-Hovering

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Abstract—In this work, we approach the control problem of fully-actuated UAVs in a geometric port-Hamiltonian framework. The UAV is modeled as a floating rigid body on the special Euclidean group $SE(3)$. A unified near-hovering motion and impedance controller is derived by the energy-balancing passivity-based control technique. A detailed analysis of the closed-loop system's behavior is presented for both the free-flight stability and contact stability of the UAV. The robustness of the control system to uncertainties is validated by several experiments, in which the UAV is controlled near its actuator limits. The experiments show the ability of the UAV to hover at its maximum allowed roll angle and apply its maximum allowed normal force to a surface, without the input saturation destabilizing the system.

I. INTRODUCTION

There is an increasing interest in the aerial robotics community to transition the use of unmanned aerial vehicles (UAVs) from passive tasks, like visual inspection and remote sensing, to active interaction tasks, like contact-based inspection and manipulation. There are two common approaches in the literature for aerial manipulation. The first approach is to endow conventional UAVs with a robotic manipulator arm [1]. However, these systems suffer from several drawbacks like inertial coupling, variable center of mass, no exertion of lateral forces, in addition to the limited payload, force exertion, and operation time.

The second common approach for aerial manipulation is the use of fully-actuated UAVs, where the UAV itself is considered as a flying end-effector which can interact with its environment. Although there have been several novel fully-actuated UAV designs introduced in the literature, most of them were introduced as a solution to the underactuation problem of conventional UAVs, e.g. [2]. The research works that focused on aerial interaction using fully-actuated UAVs include [3], where an omnidirectional bar-shaped hexarotor with asymmetrically-aligned rotors was introduced. The same authors extended their work in [4] and introduced an omnidirectional bar-shaped octarotor. In [5], an omnidirectional hexarotor with tiltable rotors was introduced and demonstrated for wall interaction. In [6], [7], a planar hexarotor with fixed-tilt rotors was used for contact-based inspection scenarios.

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Fig. 1: Fully-actuated hexarotor hovering at a non-zero pitch and roll angle.

Several techniques have been applied in the literature for the interaction control problem. In the work of [3], [4], a hybrid pose/wrench technique was used, whereas pure motion-controllers were used for interaction in [5], [7]. The drawback of these control approaches is that they both require an accurate model of the aerial robot and the contact properties of the environment. In addition, the hybrid pose/wrench technique suffers from the problem of geometric inconsistency related to the reciprocity of wrenches and twists [8]. An admittance control approach was used for the interaction in [6], which relied on a geometric tracking controller with the rotational and translational dynamics separately considered. However, as shown in [9], a controller derived in this manner will consequently suffer from not being invariant to changes of the inertial coordinate frame.

In this work, we consider the problem of interaction control of fully-actuated UAVs in the port-Hamiltonian framework. The interaction behavior of an aerial robot can be modeled effectively and elegantly by power ports in port-Hamiltonian systems theory. In this paradigm the interaction is perceived as an exchange of energy between the aerial robot and the environment, instead of an independent control of pose or wrench. This is the underlying basic idea of the impedance control technique introduced in [10], and was implemented for underactuated UAVs in [11], [12].

The concepts considered in this work are presented in a geometrically consistent manner by modeling the UAV as a floating rigid body with the special Euclidean group $SE(3)$ as its configuration manifold, without separating the rotational dynamics on special orthogonal group $SO(3)$ and the translational dynamics on $\mathbb{R}^3$, as in [11], [12]. Our work is inspired by [13]–[15] in which the geometric impedance control problem of rigid manipulators is studied. In [13] a geometric impedance controller was designed in

Fig. 2: Commutative Diagram relating the Lie group $SE(3)$ to the Lie algebra $\mathfrak{se}(3)$ and its dual space $\mathfrak{se}^*(3)$. The maps π and π_f represent the canonical and fiber projections, respectively.

the Lagrangian framework for a rigid manipulator, which only allowed diagonal stiffness and damping parameters. The work of [13], [14] was generalized in [15] in the port-Hamiltonian framework as an interconnection of mechanical structures, for robot grasping. The impedance controller of [15] enabled achieving arbitrary stiffness control and was invariant to changes of the inertial coordinate frame.

The novelty of this paper is the reformulation of the geometric impedance controller of [15] as an energy-balancing passivity-based control (EB-PBC) problem for a fully-actuated UAV modeled geometrically on $SE(3)$. We extend the work of [15] by applying it for both the motion and interaction control of a UAV, in addition to a rigorous mathematical derivation and analysis of the closed loop system. Moreover, we validate the presented controller experimentally on a fully-actuated planar hexarotor to show that its passivity-based nature allows full exploitation of the UAV's capabilities. This is demonstrated by controlling the hexarotor near its control input limits.

In our recent work [16], the impedance controller, derived in this article using EB-PBC, was used in parallel with an observer-based wrench regulator to set the desired interaction wrench. The main focus of [16] was to restore the system's passivity using energy-tanks. The main contributions of this article with respect to [16] include the systematic design procedure to derive the motion and interaction controller, in addition to detailed analysis and experimental validation of the system's free-flight behavior.

II. MATHEMATICAL PRELIMINARIES

The motion of objects is described in this work by the Lie group approach. The reader is referred to [17] for an introduction of the topic in robotics. Let $\{\Psi_I : o_I, \hat{x}_I, \hat{y}_I, \hat{z}_I\}$ denote an orthonormal inertial frame and $\{\Psi_B : o_B, \hat{x}_B, \hat{y}_B, \hat{z}_B\}$ denote a body-fixed frame, as shown in Fig. 3. The pose of the UAV is represented by the homogeneous matrix H_B^I , which is an element of the matrix Lie group $SE(3)$ ¹.

The instantaneous relative motion of the UAV $\dot{H}_B^I$ is an element of the tangent space $T_{H_B^I} SE(3)$, which can be mapped to an element (called twist) of the Lie algebra $\mathfrak{se}(3)$ of the Lie group $SE(3)$. As shown in the commutative

diagram in Fig 2, the map from a twist $\tilde{T}_B^{B,I} \in \mathfrak{se}(3)$ to $\dot{H}_B^I \in T_{H_B^I} SE(3)$ is performed by using the pushforward of the left translation map, at the identity, $(L_{H_B^I})_*$ such that

$$(L_{H_B^I})_*(\tilde{T}_B^{B,I}) = H_B^I \tilde{T}_B^{B,I} = \dot{H}_B^I. \quad (1)$$

The twist $\tilde{T}_B^{B,I}$ describes the motion of Ψ_B with respect to Ψ_I expressed in Ψ_B , and is given by

$$\tilde{T}_B^{B,I} := \begin{pmatrix} \tilde{\omega}_B^{B,I} & \mathbf{v}_B^{B,I} \\ \mathbf{0} & 0 \end{pmatrix} \in \mathfrak{se}(3),$$

where $\mathbf{v}_B^{B,I}$ is the UAV's linear velocity, and $\tilde{\omega}_B^{B,I} := (\mathbf{R}_B^I)^\top \dot{\mathbf{R}}_B^I$ is an element of the Lie algebra $\mathfrak{so}(3)$ of the Lie group $SO(3)$, with $\mathbf{R}_B^I \in SO(3)$ denoting the orientation of Ψ_B with respect to Ψ_I .

The Lie algebra $\mathfrak{so}(3)$ can be identified with $\mathbb{R}^3$ through the Lie algebra isomorphism

$$\begin{aligned} \sim: \mathbb{R}^3 &\rightarrow \mathfrak{so}(3) \\ \boldsymbol{\omega} := \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} &\mapsto \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} := \tilde{\boldsymbol{\omega}}. \end{aligned}$$

This allows us to represent the vector product $\wedge$ in $\mathbb{R}^3$ as $\tilde{\boldsymbol{\omega}} \mathbf{x} = \boldsymbol{\omega} \wedge \mathbf{x}$, $\forall \boldsymbol{\omega}, \mathbf{x} \in \mathbb{R}^3$. Similarly, the Lie algebra $\mathfrak{se}(3)$ can be identified with $\mathbb{R}^6$ through the vector space isomorphism

$$\begin{aligned} \sim: \mathbb{R}^6 &\rightarrow \mathfrak{se}(3) \\ \mathbf{T} := \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix} &\mapsto \begin{pmatrix} \tilde{\boldsymbol{\omega}} & \mathbf{v} \\ \mathbf{0} & 0 \end{pmatrix} := \tilde{\mathbf{T}}, \end{aligned} \quad (2)$$

where, with an abuse of notation, we denote both isomorphisms by $\sim$, as it will be clear from the context. Moreover, we will call both $\mathbf{T}$ and $\tilde{\mathbf{T}}$ a twist for simplicity.

Since the Lie algebra $\mathfrak{se}(3)$ is a vector space, its dual vector space, denoted by $\mathfrak{se}^*(3)$, represents the space of wrenches (generalized forces). A wrench applied to the UAV's body expressed in Ψ_I is denoted by $\tilde{\mathbf{W}}_B^I$. Moreover, the dual² map of (2) can be used to identify each element $\tilde{\mathbf{W}} \in \mathfrak{se}^*(3)$ with an element $\mathbf{W} = (\boldsymbol{\tau}^\top, \mathbf{f}^\top)^\top \in \mathbb{R}^6$ where $\mathbf{f}, \boldsymbol{\tau} \in \mathbb{R}^3$ represent the force and torque components, respectively. Similar to the case of twists, we will call both $\mathbf{W}$ and $\tilde{\mathbf{W}}$ a wrench.

As shown in Fig. 2, using the pullback of the left translation map $(L_{H_B^I})^*$, we can map any covector $\tilde{\mathbf{W}} \in T_{H_B^I}^* SE(3)$ to a wrench $\tilde{\mathbf{W}} \in \mathfrak{se}^*(3)$. This map is defined such that

$$\langle (L_{H_B^I})^*(\tilde{\mathbf{W}}) | \tilde{\mathbf{T}} \rangle := \langle \tilde{\mathbf{W}} | \dot{H}_B^I \rangle, \quad (3)$$

where $\langle \cdot | \cdot \rangle$ denotes the vector-space dual product. The dual product on $\mathfrak{se}(3)$ can be related to the usual dual product on $\mathbb{R}^6$ through (2) such that

$$\langle \tilde{\mathbf{W}} | \tilde{\mathbf{T}} \rangle = \mathbf{W}^\top \mathbf{T} = \boldsymbol{\tau}^\top \boldsymbol{\omega} + \mathbf{f}^\top \mathbf{v}. \quad (4)$$

The change of coordinates of twists and wrenches between any two frames Ψ_j and Ψ_k is performed by

$$\mathbf{T}_{\bullet}^{k,*} = \text{Ad}_{H_j^k} \mathbf{T}_{\bullet}^{j,*}, \quad \mathbf{W}_{\bullet}^j = \text{Ad}_{H_j^k}^\top \mathbf{W}_{\bullet}^k, \quad (5)$$

¹ $SE(3)$ is, properly speaking, the group of motions [18].

Fig. 3: Schematic view of the reference frames used.

where the map $Ad_H : \mathbb{R}^6 \rightarrow \mathbb{R}^6$ is the adjoint map of $H \in SE(3)$, given by

$$Ad_H := \begin{pmatrix} R & 0 \\ \tilde{\xi}R & R \end{pmatrix}, \quad H := \begin{pmatrix} R & \xi \\ 0 & 1 \end{pmatrix},$$

where $\xi \in \mathbb{R}^3$ denotes the displacement vector.

Finally, the following identities will be used throughout the paper. Consider the twist $T_{j,k}^{j,k}$, matrices $A, B \in \mathbb{R}^{3 \times 3}$, $R \in SO(3)$, and vectors $v, w \in \mathbb{R}^3$. Then the following hold:

$$T_{j,k}^{j,k} = -T_{j,k}^{j,k}, \quad (6)$$

$$\text{tr}(\tilde{v}\tilde{w}) = -2v^\top w, \quad (7)$$

$$\tilde{v}w = -\tilde{w}v = \tilde{w}^\top v, \quad (8)$$

$$(Rv)^\sim = R\tilde{v}R^\top, \quad (9)$$

$$\text{tr}(AB) = \text{tr}(\text{sym}(A)\text{sym}(B)) + \text{tr}(\text{sk}(A)\text{sk}(B)), \quad (10)$$

$$v = (\text{tr}(A)I_3 - A^\top)w \Leftrightarrow \tilde{v} = 2\text{sk}(A\tilde{w}), \quad (11)$$

where $\text{tr}(\cdot)$ denotes the matrix trace, while $\text{sym}(\cdot)$ and $\text{sk}(\cdot)$ denote the symmetric and anti-symmetric parts of a matrix, respectively.

III. UAV DESCRIPTION

In this work, similar to [6], [7], we consider a hexarotor UAV with fixed-tilt rotor as a case study, chosen due to its mechanical simplicity compared to other designs. To modify a traditional hexarotor in order to be fully-actuated, the six rotors should point in different directions. Each rotor's orientation is fixed and parametrized by a cant angle. Next, we derive the aerodynamic wrench applied by the six rotors to the UAV's body, followed by the choice of cant angles.

A. Static Wrench Analysis

The collectively applied wrench, which represents the control input to move the body in space, is produced by six rotors attached to the UAV's body. Let $\{\Psi_{p_k} : o_{p_k}, \hat{x}_{p_k}, \hat{y}_{p_k}, \hat{z}_{p_k}\}$ denote the frame associated with the k -th propeller, where $\hat{z}_{p_k}$ is the direction of generated thrust, and the origin o_{p_k} coincides with the CoM of the k -th propeller. We denote the displacement vector of the k -th propeller in Ψ_B by

$$\xi_k := o_{p_k}^B = R_z(\psi_k)[L, 0, 0]^\top,$$

where $R_z(\cdot) \in SO(3)$ is a rotation matrix about $\hat{z}_B$, L is the distance from the hexarotor's central axis $\hat{z}_B$ to each rotor, and the angle $\psi_k := (k-1)\frac{\pi}{3}$. Moreover, we denote

the thrust generation direction of the k -th propeller by $u_k := \hat{z}_{p_k} = R_{p_k}^B \hat{e}_3$, where $\hat{e}_j$ is a vector of zeros with one at the j -th element, and $R_{p_k}^B$ denotes the orientation of Ψ_{p_k} with respect to Ψ_B given by

$$R_{p_k}^B = R_z(\psi_k)R_x(\alpha_k),$$

where α_k denotes the cant angles respectively.

The control wrench produced by the rotors can be derived based on a static wrench analysis. It is well known from literature that the aerodynamic thrust and drag torque of a single propeller are both proportional to the square of the propeller's spinning velocity in quasi-static flights. The thrust magnitude generated by the k -th propeller in Ψ_{p_k} will be denoted by λ_k , while the drag torque will be expressed as $\tau_{d,k} = \gamma\sigma_k\lambda_k$, where γ is the propeller's drag-to-thrust ratio, and $\sigma_k \in \{-1, 1\}$ specifies the direction of the propeller's rotation. The wrench generated from each propeller on the UAV's body is expressed in Ψ_B as

$$W_{p_k}^B = \lambda_k Ad_{H_{p_k}^B}^\top(0, 0, \gamma\sigma_k, 0, 0, 1)^\top.$$

By summing over k , the cumulative aerodynamic control wrench W_c^B expressed in Ψ_B can be written as

$$W_c^B = \begin{pmatrix} \tau_c^B \\ f_c^B \end{pmatrix} = \sum_k \lambda_k \begin{pmatrix} \tilde{\xi}_k u_k + \gamma\sigma_k u_k \\ u_k \end{pmatrix} =: M\lambda, \quad (12)$$

where $\lambda = [\lambda_1, \dots, \lambda_6]^\top$, and $M \in \mathbb{R}^{6 \times 6}$ is the control allocation matrix.

B. Choice of the cant angles

The fully-actuated hexarotor considered in this work is optimized following the work of [7]. The objective of the optimization process is to maximize the aerodynamic wrench that can be generated by the propellers on the body of the UAV. This in turn will maximize the agility of the UAV, the maximum angle the UAV can hover at, and the interaction wrench that the UAV can apply to the environment in physical contact.

The optimized² cant-angles, for the choice of objective, are $\alpha_k = (-1)^{k+1}\alpha^*$, where $\alpha^* = 47^\circ$, while the rotation directions of the rotors are given by $\sigma_k = (-1)^k$.

IV. UAV PORT-HAMILTONIAN MODELING

Now we present the port-Hamiltonian dynamic model of the UAV. The state of the system comprises of the UAV's pose H_B^I and its generalized momentum given by $P^B := \mathcal{I}T_B^{B,I}$, where $\mathcal{I} \in \mathbb{R}^{6 \times 6}$ denotes the generalized inertia tensor expressed in Ψ_B . Thus, the state is given by

$$\bar{x} = (H_B^I, P^B) \in SE(3) \times \mathbb{R}^6. \quad (13)$$

The Hamiltonian of the system is given by the sum of the UAV's kinetic and potential energies:

$$\begin{aligned} \mathcal{H}(\bar{x}) &= \mathcal{H}_{\text{kin}}(P^B) + \mathcal{H}_{\text{grv}}(H_B^I) \\ &= \frac{1}{2}(P^B)^\top \mathcal{I}^{-1} P^B + m(\xi_B^I)^\top g, \end{aligned} \quad (14)$$

²The details of the optimization process are omitted as the optimization-based design is out of the scope of this paper.

where m is the UAV's mass, $\mathbf{g} := g\hat{\mathbf{z}}_I$ is the gravity vector (inverse), and g is the gravitational acceleration constant.

The UAV's dynamic model in the port-Hamiltonian framework [19], is given by

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} \mathbf{H}_B^I \\ \mathbf{P}^B \end{pmatrix} &= \begin{pmatrix} \mathbf{0} & \chi_{\mathbf{H}_B^I} \\ -\chi_{\mathbf{H}_B^I}^* & (\mathbf{P}^B)^\times \end{pmatrix} \begin{pmatrix} \partial\mathcal{H}/\partial\mathbf{H}_B^I \\ \partial\mathcal{H}/\partial\mathbf{P}^B \end{pmatrix} + \begin{pmatrix} \mathbf{0} \\ \mathbf{I}_6 \end{pmatrix} \mathbf{W}^B, \\ \mathbf{T}_B^{B,I} &= (\mathbf{0} \quad \mathbf{I}_6) \begin{pmatrix} \partial\mathcal{H}/\partial\mathbf{H}_B^I \\ \partial\mathcal{H}/\partial\mathbf{P}^B \end{pmatrix}, \end{aligned} \quad (15)$$

where the map χ and its dual χ^* are the compositions defined in Fig 2. The skew-symmetric matrix $(\mathbf{P}^B)^\times$ is given by

$$(\mathbf{P})^\times := \begin{pmatrix} \tilde{\mathbf{P}}_\omega & \tilde{\mathbf{P}}_v \\ \tilde{\mathbf{P}}_v & \mathbf{0} \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} \mathbf{P}_\omega \\ \mathbf{P}_v \end{pmatrix} \in \mathbb{R}^6.$$

Finally, The total wrench applied to the UAV's rigid body is given by

$$\mathbf{W}^B = \mathbf{W}_c^B + \mathbf{W}_{\text{int}}^B, \quad (16)$$

with $\mathbf{W}_c^B$ denoting the aerodynamic control wrench (12), and $\mathbf{W}_{\text{int}}^B$ denoting the interaction wrench applied to the UAV by the environment.

V. ENERGY-BALANCING PASSIVITY-BASED CONTROL

Now we present the stabilization control system design of the port-Hamiltonian system (15) for both motion and interaction control. The control approach used is the Energy-Balancing Passivity-Based Control (EB-PBC) approach [20]. The formulation of the UAV geometric control design as an EB-PBC problem is one of the contributions of this work.

A. Change of Coordinates

With reference to Fig. 3, let Ψ_D denote the (virtual) desired frame of the UAV. For simplicity of presentation, it is useful to introduce another body-fixed frame, called the *center of stiffness* frame Ψ_{CS} , which has the same orientation as Ψ_B but a different location. The location of Ψ_{CS} will vary according to the control task³. For motion control Ψ_{CS} coincides Ψ_B (i.e. $\xi_{CS}^B = 0$), while for interaction control Ψ_{CS} coincides with the end-effector frame Ψ_E .

Now we introduce a change of coordinates of the state (13) such that the new system's state $\mathbf{x}$ is given by

$$\mathbf{x} = \Phi(\bar{\mathbf{x}}) := (\mathbf{H}_I^D \mathbf{H}_B^I \mathbf{H}_{CS}^B, \mathbf{P}^B) = (\mathbf{H}_{CS}^D, \mathbf{P}^B).$$

The time derivative of $\mathbf{H}_{CS}^D$ can be written in the new coordinates as

$$\begin{aligned} \dot{\mathbf{H}}_{CS}^D &= \chi_{\mathbf{H}_{CS}^D}(\mathbf{T}_{CS}^{CS,D}) = \chi_{\mathbf{H}_{CS}^D}(\mathbf{Ad}_{\mathbf{H}_B^{CS}} \mathbf{T}_{CS}^{B,D}) \\ &= \chi_{\mathbf{H}_{CS}^D}(\mathbf{Ad}_{\mathbf{H}_B^{CS}} \mathbf{T}_B^{B,I}), \end{aligned}$$

where (5) was used. The equality of $\mathbf{T}_{CS}^{B,D}$ and $\mathbf{T}_B^{B,I}$ is due to the fact that the twist of Ψ_B is the same as Ψ_{CS} , in addition to the assumption that $\mathbf{T}_D^{D,I} = 0$ for regulation control.

³In general, the Ψ_{CS} frame can be made variable [21].

By rewriting the time derivative of $\mathbf{P}^B$ in the new coordinates similar to the aforementioned argument, the open-loop port-Hamiltonian system (15) can be rewritten now as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{J}(\mathbf{x}) \frac{\partial\mathcal{H}(\mathbf{x})}{\partial\mathbf{x}} + \mathbf{G}\mathbf{W}^B, \\ \mathbf{y} &= \mathbf{G}^\top \frac{\partial\mathcal{H}(\mathbf{x})}{\partial\mathbf{x}}, \end{aligned} \quad (17)$$

where $\mathbf{W}^B$ is given by (16), $\mathbf{y} = \mathbf{T}_B^{B,I}$ is the output, $\mathbf{G}^\top = (\mathbf{0} \quad \mathbf{I}_6)$ is the input matrix, and the skew-symmetric interconnection operator $\mathbf{J}(\mathbf{x})$ is given by

$$\mathbf{J}(\mathbf{x}) = \begin{pmatrix} \mathbf{0} & \chi_{\mathbf{H}_{CS}^D} \circ \mathbf{Ad}_{\mathbf{H}_B^{CS}} \\ -\mathbf{Ad}_{\mathbf{H}_B^{CS}}^\top \circ \chi_{\mathbf{H}_{CS}^D}^* & (\mathbf{P}^B)^\times \end{pmatrix},$$

with $\circ$ denoting the composition operator.

B. Control Objective

Considering the port Hamiltonian system (17), the control objective (in the absence of $\mathbf{W}_{\text{int}}^B$) is to find a control law

$$\mathbf{W}_c^B = \mathbf{W}_{\text{es}}^B(\mathbf{x}) + \mathbf{W}_{\text{di}}^B, \quad (18)$$

such that the closed loop system will retain the port-Hamiltonian structure:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{J}(\mathbf{x}) \frac{\partial\mathcal{H}_{\text{cl}}(\mathbf{x})}{\partial\mathbf{x}} + \mathbf{G}\mathbf{W}_{\text{di}}^B, \\ \mathbf{y} &= \mathbf{G}^\top \frac{\partial\mathcal{H}_{\text{cl}}(\mathbf{x})}{\partial\mathbf{x}}, \end{aligned} \quad (19)$$

with the closed-loop Hamiltonian $\mathcal{H}_{\text{cl}}$ presenting a strict minimum at $\mathbf{x}^* = (\mathbf{I}_4, \mathbf{0})$. The first term of the control law $\mathbf{W}_{\text{es}}^B$ is the state feedback component responsible for the energy shaping, while $\mathbf{W}_{\text{di}}^B$ is the output feedback component responsible for injecting damping such that asymptotic stabilization is achieved.

C. Matching Equation

For the UAV's model (17), in which internal dissipation is not present, the closed loop energy $\mathcal{H}_{\text{cl}}$ is chosen as

$$\mathcal{H}_{\text{cl}}(\mathbf{x}) = \mathcal{H}(\mathbf{x}) + \mathcal{H}_a(\mathbf{x}), \quad (20)$$

where $\mathcal{H}_a$ is the energy added by the controller to the system. By substituting (18) in (17), and (20) in (19), then comparing both systems together, we get the matching equation

$$\mathbf{J}(\mathbf{x}) \frac{\partial\mathcal{H}_a(\mathbf{x})}{\partial\mathbf{x}} = \mathbf{G}\mathbf{W}_{\text{es}}^B(\mathbf{x}), \quad (21)$$

which should be solved to find the energy shaping control term $\mathbf{W}_{\text{es}}^B$. It can be shown that the matching equation reduces to

$$\partial\mathcal{H}_a/\partial\mathbf{P}^B = 0, \quad (22)$$

$$-\mathbf{Ad}_{\mathbf{H}_B^{CS}}^\top \circ \chi_{\mathbf{H}_{CS}^D}^* (\partial\mathcal{H}_a/\partial\mathbf{H}_{CS}^D) = \mathbf{W}_{\text{es}}^B. \quad (23)$$

The solution of (22) implies that $\mathcal{H}_a(\mathbf{x}) = \mathcal{H}_a(\mathbf{H}_{CS}^D)$. Now the control problem reduces to solving (23) for a choice of $\mathcal{H}_a$, which will be discussed next.

D. Energy Shaping Control Procedure

From the definition of $\chi_{H_{CS}^D}^*$, we can expand (23) as

$$\mathbf{W}_{\text{es}}^B = \mathbf{A} d_{H_B^{CS}}^\top \mathbf{W}_{\text{es}}^{CS}, \quad (24)$$

$$\tilde{\mathbf{W}}_{\text{es}}^{CS} = -(L_{H_{CS}^D})^*(d\mathcal{H}_a(\mathbf{H}_{CS}^D)), \quad (25)$$

where the covector $d\mathcal{H}_a(\mathbf{H}_{CS}^D) \in T_{H_{CS}^D}^* SE(3)$ denotes the differential of the smooth function $\mathcal{H}_a: SE(3) \rightarrow \mathbb{R}$ at $\mathbf{H}_{CS}^D$. Similar to (3), the action of the pullback in (25) is defined as

$$\langle \tilde{\mathbf{W}}_{\text{es}}^{CS} | \delta \tilde{\mathbf{T}}_D^{CS} \rangle = \langle d\mathcal{H}_a(\mathbf{H}_{CS}^D) | d\mathbf{H}_{CS}^D \rangle, \quad (26)$$

with $d\mathbf{H}_{CS}^D$ expressed, by using (6), as

$$d\mathbf{H}_{CS}^D = \mathbf{H}_{CS}^D \delta \tilde{\mathbf{T}}_{CS}^{CS,D} = -(L_{H_{CS}^D})_*(\delta \tilde{\mathbf{T}}_D^{CS}), \quad (27)$$

with $\delta \tilde{\mathbf{T}}_D^{CS}$ as a shorthand notation for $\delta \tilde{\mathbf{T}}_D^{CS,CS} \in \mathfrak{se}(3)$ which is an infinitesimal twist displacement.

Remark 1: Note that the pairing in (26) represents infinitesimal virtual work, in contrast to the pairing in (3) which represents mechanical power, which is the first-order time-derivative of the energy. Similar to [14], we choose to follow the variational approach for the controller derivation.

The right-hand side of (26) can be calculated by

$$\langle d\mathcal{H}_a(\mathbf{H}_{CS}^D) | d\mathbf{H}_{CS}^D \rangle = \mathcal{H}_a(\mathbf{H}_{CS}^D + d\mathbf{H}_{CS}^D) - \mathcal{H}_a(\mathbf{H}_{CS}^D). \quad (28)$$

From (4), we can express the left-hand side of (26) as

$$\langle \tilde{\mathbf{W}}_{\text{es}}^{CS} | \delta \tilde{\mathbf{T}}_D^{CS} \rangle = (\boldsymbol{\tau}_{\text{es}}^{CS})^\top \delta \boldsymbol{\theta}_D^{CS} + (\mathbf{f}_{\text{es}}^{CS})^\top \delta \boldsymbol{\xi}_D^{CS}, \quad (29)$$

where $\delta \boldsymbol{\theta}_D^{CS}, \delta \boldsymbol{\xi}_D^{CS} \in \mathbb{R}^3$ are components of $\delta \mathbf{T}_D^{CS} \in \mathbb{R}^6$, related to the components of $d\mathbf{H}_{CS}^D$ by

$$d\mathbf{R}_{CS}^D = -\mathbf{R}_{CS}^D \delta \tilde{\boldsymbol{\theta}}_D^{CS}, \quad d\boldsymbol{\xi}_{CS}^D = -\mathbf{R}_{CS}^D \delta \boldsymbol{\xi}_D^{CS}. \quad (30)$$

Using identity (7), it will be useful to express (29) as

$$\langle \tilde{\mathbf{W}}_{\text{es}}^{CS} | \delta \tilde{\mathbf{T}}_D^{CS} \rangle = -\frac{1}{2} \text{tr}(\tilde{\boldsymbol{\tau}}_{\text{es}}^{CS} \delta \tilde{\boldsymbol{\theta}}_D^{CS}) - \frac{1}{2} \text{tr}(\tilde{\mathbf{f}}_{\text{es}}^{CS} \delta \tilde{\boldsymbol{\xi}}_D^{CS}). \quad (31)$$

E. EB-PBC Control Law

In this work, the controller's added energy is chosen as

$$\mathcal{H}_a(\mathbf{H}_{CS}^D) = \mathcal{H}_p(\mathbf{H}_{CS}^D) - \mathcal{H}_{\text{grv}}(\mathbf{H}_{CS}^D), \quad (32)$$

where $\mathcal{H}_{\text{grv}}$ is given by

$$\mathcal{H}_{\text{grv}}(\mathbf{H}_{CS}^D) = m(\boldsymbol{\xi}_D^I + \mathbf{R}_D^I \boldsymbol{\xi}_{CS}^D + \mathbf{R}_D^I \mathbf{R}_{CS}^D \boldsymbol{\xi}_B^{CS})^\top \mathbf{g},$$

and $\mathcal{H}_p$ is the elastic potential energy [15] given by

$$\begin{aligned} \mathcal{H}_p(\mathbf{R}, \boldsymbol{\xi}) &= \mathcal{H}_{t,1}(\boldsymbol{\xi}) + \mathcal{H}_{t,2}(\mathbf{R}, \boldsymbol{\xi}) + \mathcal{H}_o(\mathbf{R}) \\ &:= \frac{1}{4} \boldsymbol{\xi}^\top \mathbf{K}_t \boldsymbol{\xi} + \frac{1}{4} \boldsymbol{\xi}^\top \mathbf{R} \mathbf{K}_t \mathbf{R}^\top \boldsymbol{\xi} - \text{tr}(\mathbf{G}_o(\mathbf{R} - \mathbf{I}_3)). \end{aligned} \quad (33)$$

The potential energy function (33) is parameterized by the translational stiffness matrix $\mathbf{K}_t \in \mathbb{R}^{3 \times 3}$ and the orientational co-stiffness matrix $\mathbf{G}_o \in \mathbb{R}^{3 \times 3}$. Stiffness and co-stiffness matrices are related to each other by

$$\mathbf{K}_x = \text{tr}(\mathbf{G}_x) \mathbf{I}_3 - \mathbf{G}_x, \quad \mathbf{G}_x = \frac{1}{2} \text{tr}(\mathbf{K}_x) \mathbf{I}_3 - \mathbf{K}_x, \quad (34)$$

for $x \in \{t, o\}$. Both $\mathbf{K}_x, \mathbf{G}_x$ are symmetric and their eigenvectors coincide with one another i.e.

$$\mathbf{K}_x = \mathbf{R}_x \boldsymbol{\Lambda}_x \mathbf{R}_x^\top, \quad \mathbf{G}_x = \mathbf{R}_x \boldsymbol{\Gamma}_x \mathbf{R}_x^\top, \quad (35)$$

where $\mathbf{R}_x = (e_{x_1}, e_{x_2}, e_{x_3}) \in SO(3)$ represents the principal axes of stiffness, while $\boldsymbol{\Gamma}_x = \text{diag}(\gamma_{x_1}, \gamma_{x_2}, \gamma_{x_3})$ and $\boldsymbol{\Lambda}_x = \text{diag}(\lambda_{x_1}, \lambda_{x_2}, \lambda_{x_3})$ represent the principal stiffness and co-stiffness values along the axes, respectively.

Assumption 1: For $i = \{1, 2, 3\}$, the principal stiffness gains γ_{x_i} are strictly positive and chosen such that the principal costiffness gains λ_{x_i} are distinct.

Remark 2: Note that in (33), the first term is the usual potential energy considered for a translational spring, while the third term represents a misalignment of $\mathbf{R}$ from the identity [9]. The second term in (33) is the term that allows for arbitrary non-diagonal stiffness $\mathbf{K}_t$ such that the controller maintains invariance to changes of the inertial coordinate frame Ψ_I [9], [15]. This term can be only included by considering the dynamics fully on $SE(3)$ without splitting the rotational and translational dynamics.

For the choice of the added energy (32), the EB-PBC control law $\mathbf{W}_c^B$ is given by

$$\mathbf{W}_c^B = \mathbf{A} d_{H_B^{CS}}^\top (\mathbf{W}_p^{CS} - \mathbf{W}_{\text{grv}}^{CS}) + \mathbf{W}_{\text{di}}^B, \quad (36)$$

$$\tilde{\boldsymbol{\tau}}_p^{CS} = -2\text{sk}(\mathbf{G}_o \mathbf{R}_{CS}^D) - \text{sk}(\mathbf{G}_t \mathbf{R}_{CS}^D \tilde{\boldsymbol{\xi}}_{CS}^D \tilde{\boldsymbol{\xi}}_{CS}^D \mathbf{R}_{CS}^D), \quad (37)$$

$$\tilde{\mathbf{f}}_p^{CS} = -\mathbf{R}_{CS}^D \text{sk}(\mathbf{G}_t \tilde{\boldsymbol{\xi}}_{CS}^D) \mathbf{R}_{CS}^D - \text{sk}(\mathbf{G}_t \mathbf{R}_{CS}^D \tilde{\boldsymbol{\xi}}_{CS}^D \mathbf{R}_{CS}^D), \quad (38)$$

$$\boldsymbol{\tau}_{\text{grv}}^{CS} = -m \tilde{\boldsymbol{\xi}}_B^{CS} \mathbf{R}_{CS}^D \mathbf{R}_I^D \mathbf{g}, \quad \mathbf{f}_{\text{grv}}^{CS} = -m \mathbf{R}_{CS}^D \mathbf{R}_I^D \mathbf{g}, \quad (39)$$

$$\mathbf{W}_{\text{di}}^B = -\mathbf{K}_d \mathbf{T}_B^{B,I}, \quad (40)$$

where $\mathbf{K}_d \in \mathbb{R}^{6 \times 6}$ is a symmetric positive definite matrix.

F. Control Law Derivation

Now, we present the derivation of the energy-shaping control law (37-39) for each separate energy term in (32).

1) *Orientalional energy:* For the orientational energy $\mathcal{H}_o(\mathbf{R}_{CS}^D)$, from the definition of $\mathcal{H}_o$ and (28), we get

$$\begin{aligned} \langle d\mathcal{H}_o(\mathbf{R}_{CS}^D) | d\mathbf{R}_{CS}^D \rangle &= -\text{tr}(\mathbf{G}_o(\mathbf{R}_{CS}^D + d\mathbf{R}_{CS}^D - \mathbf{I}_3)) \\ &\quad + \text{tr}(\mathbf{G}_o(\mathbf{R}_{CS}^D - \mathbf{I}_3)), \\ &= -\text{tr}(\mathbf{G}_o d\mathbf{R}_{CS}^D), \\ &= \text{tr}(\mathbf{G}_o \mathbf{R}_{CS}^D \delta \tilde{\boldsymbol{\theta}}_D^{CS}), \\ &= \text{tr}(\text{sk}(\mathbf{G}_o \mathbf{R}_{CS}^D) \delta \tilde{\boldsymbol{\theta}}_D^{CS}), \end{aligned} \quad (41)$$

where the second equality results from the linearity of the trace map, the third equality results from (30), and the last equality results from identity (10).

By comparing equations (41) and (31), we conclude that the corresponding control wrench for $\mathcal{H}_o(\mathbf{R}_{CS}^D)$ is given by

$$\tilde{\boldsymbol{\tau}}_o^{CS} = -2\text{sk}(\mathbf{G}_o \mathbf{R}_{CS}^D), \quad \tilde{\mathbf{f}}_o^{CS} = \mathbf{0}. \quad (42)$$

2) *Translational energy:* By following the same steps for the first term of the translational energy $\mathcal{H}_{t,1}(\boldsymbol{\xi})$, we get

$$\begin{aligned} \langle d\mathcal{H}_{t,1}(\boldsymbol{\xi}_{CS}^D) | d\boldsymbol{\xi}_{CS}^D \rangle &= \frac{1}{4} (\mathbf{R}_{CS}^D \delta \boldsymbol{\xi}_D^{CS})^\top \mathbf{K}_t (\mathbf{R}_{CS}^D \delta \boldsymbol{\xi}_D^{CS}) \\ &\quad - \frac{1}{4} (\mathbf{R}_{CS}^D \delta \boldsymbol{\xi}_D^{CS})^\top \mathbf{K}_t \boldsymbol{\xi}_{CS}^D - \frac{1}{4} (\boldsymbol{\xi}_{CS}^D)^\top \mathbf{K}_t \mathbf{R}_{CS}^D \delta \boldsymbol{\xi}_D^{CS}, \end{aligned}$$

where the first can be neglected since it is a second order term [14], thus we get

$$\langle d\mathcal{H}_{t,1}(\xi_{CS}^D) | d\xi_{CS}^D \rangle = -\frac{1}{2}(\delta\xi_{CS}^D)^\top (R_{CS}^D)^\top K_t \xi_{CS}^D.$$

By comparison to (29), we conclude that

$$f_{t,1}^{CS} = -\frac{1}{2}R_{CS}^{CS} K_t \xi_{CS}^D, \quad \tau_{t,1}^{CS} = 0. \quad (43)$$

By using (34) and the identity (11), we can show that $(K_t \xi_{CS}^D)^\sim = 2\text{sk}(G_t \tilde{\xi}_{CS}^D)$, which allows us to rewrite (43) as

$$\tilde{f}_{t,1}^{CS} = -R_{CS}^{CS} \text{sk}(G_t \tilde{\xi}_{CS}^D) R_{CS}^D, \quad \tilde{\tau}_{t,1}^{CS} = 0, \quad (44)$$

where the identity (9) was used. To simplify the control derivation for the second term of the translational energy $\mathcal{H}_{t,2}(\xi)$, it is useful [15] to rewrite it as

$$\mathcal{H}_{t,2}(H_{CS}^D) = \mathcal{H}_{t,2}(R_{CS}^D, \xi_{CS}^D) = -\frac{1}{4}\text{tr}(\tilde{\xi}_{CS}^D R_{CS}^D G_t R_{CS}^{CS} \tilde{\xi}_{CS}^D).$$

By following the same line of thought and some lengthy calculations [15], we can express the infinitesimal work as

$$\begin{aligned} \langle d\mathcal{H}_{t,2}(H_{CS}^D) | dH_{CS}^D \rangle &= \frac{1}{2}\text{tr}(\text{sk}(G_t R_{CS}^{CS} \tilde{\xi}_{CS}^D \tilde{\xi}_{CS}^D R_{CS}^D) \delta\tilde{\theta}_D^{CS}) \\ &\quad + \frac{1}{2}\text{tr}(\text{sk}(G_t R_{CS}^{CS} \tilde{\xi}_{CS}^D R_{CS}^D) \delta\tilde{\xi}_D^{CS}). \end{aligned}$$

By comparison to (31), we conclude that

$$\begin{aligned} \tilde{f}_{t,2}^{CS} &= -\text{sk}(G_t R_{CS}^{CS} \tilde{\xi}_{CS}^D R_{CS}^D), \\ \tilde{\tau}_{t,2}^{CS} &= -\text{sk}(G_t R_{CS}^{CS} \tilde{\xi}_{CS}^D \tilde{\xi}_{CS}^D R_{CS}^D). \end{aligned} \quad (45)$$

By using (8,11,34), we can rewrite the force component as

$$f_{t,2}^{CS} = -\frac{1}{2}K_t R_{CS}^{CS} \xi_{CS}^D. \quad (46)$$

3) *Gravity compensation*: Finally, by evaluating the infinitesimal work (28) for the gravitational energy $\mathcal{H}_{\text{grv}}(H_{CS}^D)$ we get

$$\begin{aligned} \langle d\mathcal{H}_{\text{grv}}(\xi_{CS}^D) | d\xi_{CS}^D \rangle &= -m[(R_D^I R_{CS}^D \delta\xi_D^{CS})^\top + (R_D^I R_{CS}^D \delta\tilde{\theta}_D^{CS} \xi_{CS}^B)^\top] g \\ &= -m(\delta\xi_D^{CS})^\top R_{CS}^D R_D^I g - m(\delta\theta_D^{CS})^\top \tilde{\xi}_{CS}^D R_{CS}^D R_D^I g, \end{aligned}$$

where identity (8) was used. By comparison to (29), we get

$$f_{\text{grv}}^{CS} = -m R_{CS}^{CS} R_D^I g, \quad \tau_{\text{grv}}^{CS} = \tilde{\xi}_B^{CS} f_{\text{grv}}^{CS}. \quad (47)$$

This concludes the control law derivation.

VI. CLOSED LOOP SYSTEM BEHAVIOR

In this section, we examine the closed loop system's behavior. Using the control law (36) we can write the closed loop system as

$$\dot{x} = (J(x) - R) \frac{\partial \mathcal{H}_{\text{cl}}(x)}{\partial x} + G W_{\text{int}}^B, \quad (48)$$

with the damping matrix R given by

$$R = \begin{pmatrix} 0 & 0 \\ 0 & K_d \end{pmatrix}, \quad (49)$$

and the closed loop Hamiltonian given by $\mathcal{H}_{\text{cl}} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{t,1} + \mathcal{H}_{t,2} + \mathcal{H}_o$. In what follows, we present the *almost*⁴ global asymptotic stability of the system in free flight ($W_{\text{int}}^B = 0, H_{CS}^B = I_4$), followed by the contact stability and impedance behavior of the system in contact ($W_{\text{int}}^B \neq 0, H_{CS}^B = H_E^B$)

A. Free-flight Stability

The boundedness of $\mathcal{H}_{\text{cl}}$ can be proven by the fact that its first three terms are quadratic in P^B, ξ_{CS}^D , and $(R_{CS}^{CS} \xi_{CS}^D)$ respectively, thus are bounded from below. As for $\mathcal{H}_o$, due to its smoothness and the compactness of $SO(3)$ as a topological space, $\mathcal{H}_o$ is also bounded from below [22].

As a consequence of assumption 1, the closed loop system has *four* isolated equilibrium points $x_i^* = (H_i^*, P_i^*)$, where for all four points $\xi_i^* = 0$ and $P_i^* = 0$. The equilibrium orientations are given by [22]

$$R_i^* = \exp(\pi \tilde{e}_{o_i}), \quad i = \{1, 2, 3\}, \quad R_4^* = I_3, \quad (50)$$

where e_{o_i} are the principal axes of K_o , as shown in (35).

The time derivative of $\mathcal{H}_{\text{cl}}$ is given by

$$\begin{aligned} \dot{\mathcal{H}}_{\text{cl}} &= \frac{\partial \mathcal{H}_{\text{cl}}}{\partial x}^\top (J - R) \frac{\partial \mathcal{H}_{\text{cl}}}{\partial x}, \\ &= -(P^B)^\top \mathcal{I}^{-\top} K_d \mathcal{I}^{-1} P^B \leq 0, \end{aligned} \quad (51)$$

which follows from the skew-symmetry of J , and the positive definiteness of K_d and $\mathcal{I}$. Thus, we conclude the system's passivity using $\mathcal{H}_{\text{cl}}$ as a storage function.

By applying LaSalle's invariance principle, it can be shown that the largest invariance set consists of the union of all equilibrium points x_i^* , which results from

$$\dot{\mathcal{H}}_{\text{cl}} = 0 \Rightarrow P^B = 0 \Rightarrow \dot{P}^B = 0 \Rightarrow W_p^{CS}(H_{CS}^D) = 0.$$

Thus, all solutions of the system (48), converge to one of the four points. However, it will turn out that only the equilibrium $x_4^* = (I_4, 0)$ is asymptotically stable, while the rest are unstable. This can be shown by examining the local linearized behavior of the control term $W_p^{CS}(H_{CS}^D)$ around the equilibrium points. By substituting

$$R_{CS}^D = R_i^* - R_i^* \delta\tilde{\theta}_D^{CS}, \quad \xi_{CS}^D = 0 - R_i^* \delta\xi_D^{CS},$$

in equations (42,43,45,46), and discarding high order terms, we get

$$\begin{aligned} \tilde{\tau}_o^{CS} &= 2\text{sk}(G_o R_i^* \delta\tilde{\theta}_D^{CS}), & \tilde{\tau}_{t,2}^{CS} &= 0, \\ f_{t,1}^{CS} &= \frac{1}{2}(R_i^*)^\top K_t R_i^* \delta\xi_D^{CS}, & f_{t,2}^{CS} &= \frac{1}{2}K_t \delta\xi_D^{CS}, \end{aligned}$$

which allows rewriting the equivalent wrench in (37,38) as

$$W_p^{CS} = W_p^B = -K_i^* \delta T_{B,I}^B = K_i^* \delta T_{D,I}^{CS}, \quad (52)$$

with $K_i^* \in \mathbb{R}^{6 \times 6}$ given by

$$K_i^* = \begin{pmatrix} \text{tr}(G_o R_i^*) I_3 - (G_o R_i^*)^\top & 0 \\ 0 & \frac{1}{2}[(R_i^*)^\top K_t R_i^* + K_t] \end{pmatrix}.$$

⁴As is it impossible topologically to achieve global asymptotic stability on $SO(3)$ and consequently $SE(3)$.

By evaluating the stiffness matrix K_i^* at $R_4^* = I_3$, we get the positive definite matrix

$$K_s := K_4^* = \begin{pmatrix} K_o & 0 \\ 0 & K_t \end{pmatrix}, \quad (53)$$

which shows that the potential control term (52) resembles a linear elastic wrench near the equilibrium R_4^* . Similar to the analysis in [13], it can be shown that the stiffness matrix evaluated at the remaining equilibrium points is not positive definite. Finally, this concludes the *almost global* asymptotic stability of the equilibrium point $x_4^* = (I_4, 0)$.

B. Interaction behavior

In the interaction control mode, the passivity of the system with respect to the interaction power port $(T_B^{B,I}, W_{\text{int}}^B)$ can be analyzed by rewriting (51) as

$$\begin{aligned} \dot{\mathcal{H}}_{\text{cl}} &= -(P^B)^\top \mathcal{I}^{-\top} K_d \mathcal{I}^{-1} P^B + (\mathcal{I}^{-1} P^B)^\top W_{\text{int}}^B \\ &\leq (W_{\text{int}}^B)^\top T_B^{B,I}. \end{aligned} \quad (54)$$

This guarantees the contact stability of the UAV with any passive arbitrary environment.

The impedance behavior of the UAV's end effector with respect to the environment can be calculated by representing all terms of the momentum dynamics in (48) in frame Ψ_E using (5). The linearized impedance behavior in terms of an infinitesimal twist displacement $\delta T_E^{E,D}$ can be shown to be

$$\mathcal{I}^E \delta \ddot{T}_E^{E,D} + K_d^E \delta \dot{T}_E^{E,D} + K_s \delta T_E^{E,D} = W_{\text{int}}^E, \quad (55)$$

where the apparent inertia $\mathcal{I}^E$ and damping K_d^E matrices are given, respectively, by

$$\mathcal{I}^E = A d_{H_E^B}^\top \mathcal{I} A d_{H_E^B}, \quad K_d^E = A d_{H_E^B}^\top K_d A d_{H_E^B},$$

while the stiffness matrix K_s is given by (53).

To conclude, during contact with the environment, the EB-PBC control law (36) acts as an impedance controller with the impedance behavior (55).

Remark 3: Note that the passivity result in (54) actually holds for any external power port to the system. Consider the *actual* control wrench in (16) which is equal to the *desired* control wrench in (36) only if $(\Delta W_c = 0)$ which represents the wrench due to modeling uncertainties like unmodeled dynamics or input constraints. In case $\Delta W_c \neq 0$, the closed loop system (48) would have an additional term $(G \Delta W_c)$, and the system's passivity can be easily shown as in (54).

VII. EXPERIMENTAL VALIDATION

The experimental setup used to validate the proposed control approach consists of a fully-actuated hexarotor UAV with unidirectional rotors. Details of the UAV and software architecture used to implement the control law (36-40) can be found in [16]. The experiments are conducted in an indoor lab with a motion-capture system. The experiments consist of two scenarios, one for interaction control and the other for motion control. The controller gains in both experiments are chosen as $K_t = \text{diag}(12, 12, 12)$, $K_o = \text{diag}(0.8, 0.8, 0.5)$, and $K_d = \text{diag}(3.6, 3.5, 2.1, 7, 7, 7)$. The results are presented next, while recordings of the experiments can be found in the supplementary video.

Fig. 4: Experiment 1: Snapshot ($t = 215$ s) showing the UAV hovering with one rotor off and applying an ≈ 10 N force to a vertical surface rigidly connected to a force/torque sensor.

Fig. 5: Experiment 1: Measured normal force applied by the UAV to the vertical surface (top). The commanded rotor PWM signals (bottom).

A. Experimental Results

The first experiment comprises of the UAV applying a normal force to a vertical surface rigidly connected to an ATI mini40 force/torque sensor (ATI Industrial Automation), as shown in Fig. 4. The measured normal force and commanded rotor PWM signals are displayed in Fig. 5. The applied force is increased by slowly increasing ξ_E^P , until its maximum allowed normal force is achieved, as shown by the force measurement in the top plot. In the bottom plot, it can be seen that this maximum force is achieved with λ_2 at its lower limit. Therefore, there are instants in which the hexarotor interacts while hovering with its second motor turned off, as shown in the snapshot in Fig. 4.

The second experiment consists of the UAV hovering at different roll angles. In Fig. 6, snapshots of the UAV hovering at an angle of 0, 15, and 25 deg are shown. The UAV is commanded to hover at the roll angle $\phi_B = 25$ deg for $t \geq 130$ s. As shown in Fig. 7, for the UAV to hover at this maximum angle, λ_4 is commanded to be zero for an interval about 40 seconds. Thus, the UAV hovers at this maximum angle with *five* rotors only. However, due to the inability of the UAV to maintain its desired position against wind disturbances, the UAV drifts which then requires a non-zero λ_4 to be able to reduce the position error.

Fig. 6: Experiment 2: Snapshots of the UAV hovering at different roll angles ϕ_B . The right snapshot shows the UAV hovering at its maximum allowed angle with one of its rotors off.

Fig. 7: Experiment 2: The commanded rotor PWM signals where the fourth rotor hits its lower limit.

The main feature of the controller that the two aforementioned experiments demonstrate is its robustness to the actuator limits and in general any uncertainties in the model as discussed in Remark 3. Therefore, near the UAV's maximum physical capabilities, the nonlinear saturation of the motor inputs did not destabilize the system.

VIII. CONCLUSION

In this work we presented a geometric port-Hamiltonian controller for a class of fully actuated UAVs. The controller was designed using the EB-PBC approach. Although the port-Hamiltonian control of fully-actuated mechanical systems is a well studied topic, the problem considered in this work was not trivial, due to the non-symplectic structure of the dynamic model. In addition, the stiffness parameters can be chosen arbitrary without losing geometric consistency as discussed in Remark 2. Moreover, the geometric notation allowed for a compact derivation and rigorous mathematical analysis of the control system. The experiments performed show the validity and robustness of the control approach especially to unmodeled uncertainties like input saturation.

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