Model Checking Dependability Attributes of Wireless Group Communication

Mieke Massink
C.N.R.-ISTI, Via Moruzzi 1,
I-56124 Pisa, Italy,
M.Massink@isti.cnr.it

Joost-Pieter Katoen
Dept. of Comp. Science, Univ. of Twente,
P.O. Box 217, 7500 AE Enschede, The Netherlands
katoen@cs.utwente.nl

Diego Latella
C.N.R.-ISTI, Via Moruzzi 1,
I-56124 Pisa, Italy,
D.Latella@isti.cnr.it

Abstract

Models used for the analysis of dependability and performance attributes of communication protocols often abstract considerably from the details of the actual protocol. These models often consist of concurrent sub-models and this may make it hard to judge whether their behaviour is faithfully reflecting the protocol. In this paper, we show how model checking of continuous-time Markov chains, generated from high-level specifications, facilitates the analysis of both correctness and dependability attributes. We illustrate this by revisiting a dependability analysis [8] of a variant of the central access protocol of the IEEE 802.11 standard for wireless local area networks. This variant has been developed to support real-time group communication between autonomous mobile stations. Correctness and dependability properties are formally characterised using Continuous Stochastic Logic and are automatically verified by the ETMCC model checker. The models used are specified as Stochastic Activity Nets.

1. Introduction

Continuous-time Markov chains (CTMCs) are widely used to analyse important system dependability and performance issues. Usually such CTMCs are generated automatically from higher-level specifications such as e.g. Stochastic Activity Nets (SAN) [21] for which mature tool-support is available. Such tools support both the development of high-level specifications and the calculation of relevant measures, e.g. steady-state and transient state probabilities.

The high-level specifications used for the analysis of dependability and performance aspects of communication protocols often abstract considerably from the details of the actual protocol. In many cases, these models, in their turn, consist of concurrently composed sub-models. This may complicate the judgement whether their behaviour is faithfully reflecting the protocol. The extension of dependability and performance analysis tools with model-checking capabilities and a temporal logic allows for the verification of behavioural aspects as well as for the convenient, concise and unambiguous specification and automated verification of dependability and performance measures.

In this paper, we illustrate these advantages in practice by revisiting a dependability analysis of a variant of the centralised medium access protocol of the IEEE 802.11 standard for wireless local area networks [4, 8, 9]. This variant has been developed to provide reliable real-time group communication within teams of autonomous mobile robotic systems over wireless (radio) networks [22, 19]. In the IEEE 802.11 standard, the problem of message loss is addressed by defining two alternating periods of medium access control; a centralised one suitable for the exchange of time-critical messages, and a distributed one, suitable for less or non-time critical messages. The distributed medium access control mechanism for non time-critical communication over wireless networks has been studied in [16] applying probabilistic model checking techniques.

Group communication between autonomous mobile stations via wireless local area networks presents particular problems due to the locomotion of the mobile stations and the unshieldedness of wireless communication. It is therefore susceptible to a high degree of message losses in a bursty fashion.

The variant of the protocol that we consider in this paper proposes to reduce the number of retransmissions required to guarantee reliable communication in order to improve the real-time performance of the protocol. The reduction of reliability due to fewer retransmissions is compen-
icated for by a mechanism of active acknowledgments and the distribution of decision information that is included in the header of broadcasted messages. For many applications on real-time mobile stations the reduced reliability does not cause a serious problem as long as all mobile stations in the network agree in time not to deliver a message to their application when there is some station that did not receive the user message, viz. the property of agreement is satisfied.

In this paper, we analyse the models developed in [4, 8, 9] to determine the probability that a station misses a decision message and the probability that a user message is never delivered. First we check the correctness of the analytic model by generating the CTMC using the UltraSAN tool [21] and verify correctness properties of the concurrent model by the prototype stochastic model checker ETMCC (Erlangen Twente Markov Chain Checker) [13]. This model checker allows for the verification of both qualitative and quantitative (stochastic time) properties expressed as formulas of the (stochastic) branching time logic CSL (Continuous Stochastic Logic) [1, 2]. UltraSAN is a software package for model-based evaluation of systems represented as SAN’s. It provides analytic solvers as well as discrete-event simulators but has no model-checking facilities.

The contribution of this paper is threefold. We use a model checking approach on a case study on which numerical and experimental results are available in the literature. We show that the model checking capability to verify both qualitative and quantitative properties of concurrent models can greatly enhance the effectiveness of existing dependability and performance analysis tools to increase the confidence in the accuracy and faithfulness of the models on which the analyses are based. In fact, its automatic verification reveals serious problems of the existing model and gives rise to the development of a new model that more faithfully reflects the synchronous broadcast aspects of the protocol. We show this by comparing the verification results for qualitative and quantitative properties for both models. Finally, the direct link between the high-level specification in SAN and the derived CTMCs on which the analysis by both UltraSAN and ETMCC are based gives an opportunity to compare the results and to obtain feedback on the performance of the tools. An extended and more detailed version of the present paper can be found in [17].

2. Model Checking Dependability

In the model checking approach to dependability analysis a model of the system under consideration is required together with a desired property or dependability measure. Model checking provides a systematic check whether the given model satisfies the property. Effective, optimised model checking algorithms have been developed to dramatically reduce the state space that needs to be searched, and to keep its representation compact as well [7]. Typically, models are finite-state automata, where transitions model the evolution of the system while moving from one state to another. These automata are usually generated from a high-level description language. In the case of stochastic modelling, such models are typically CTMCs and languages such as stochastic Petri nets, stochastic process algebras or SANs are used to generate them. In the model checking approach, the properties are usually expressed in some form of temporal logic. In this paper the Continuous Stochastic Logic [1, 2] is used, which is a stochastic variant of the well-known Computational Tree Logic (CTL) (see e.g. [7]). CTL allows for stating properties over states, and over paths. CSL extends CTL by two probabilistic operators that refer to the steady-state and transient behaviour of the system being studied. Whereas the steady-state operator refers to the probability of residing in a particular set of states (specified by a state-formula) in the long run, the transient operator allows us to refer to the probability of the occurrence of particular paths in the CTMC. In order to express the time-span of a certain path, the path-operators until $\U$ and next $\X$ are extended with a parameter that specifies a time-interval. Let $I$ be an interval on the real line, $p$ a probability value and $\p$ a comparison operator, i.e., $\p \in \{<,\leq,\geq,>\}$. The syntax of CSL is:

<table>
<thead>
<tr>
<th>State-formulas</th>
</tr>
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<tbody>
<tr>
<td>$\Phi ::= a$</td>
</tr>
<tr>
<td>$\neg \Phi$</td>
</tr>
<tr>
<td>$\Phi \land \Phi$</td>
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<tr>
<td>$S_{\text{op}}(\Phi)$</td>
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<tr>
<td>$P_{\text{op}}(\varphi)$</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Path-formulas</th>
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</thead>
<tbody>
<tr>
<td>$\varphi ::= \X I \Phi$</td>
</tr>
<tr>
<td>$\Phi U I \Phi$</td>
</tr>
<tr>
<td>$\Phi U I \Psi$</td>
</tr>
</tbody>
</table>

The meaning of atomic propositions ($a$), negation ($\neg$) and disjunction ($\lor$) is standard; note that using these operators, other boolean operators such as conjunction ($\land$), implication ($\Rightarrow$), true (true) and false (false), and so forth, can be defined. The state-formula $S_{\text{op}}(\Phi)$ asserts that the steady-state probability for the set of $\Phi$-states meets the bound $p$. The operator $P_{\text{op}}(\varphi)$ replaces the usual CTL path quantifiers $\exists$ and $\forall$. In CTL, the state-formula $\exists \varphi$ is valid in state $s$ if there exists some path starting in $s$ and satisfying $\varphi$ and $\forall \varphi$ is valid if all paths satisfy $\varphi$. In CSL, $\exists \Phi$ can be written as $P_{\geq 0}(\Phi)$ and $\forall \Phi$ as $P_{\geq 1}(\Phi)$. This allows for the expression of qualitative as well as stochastic properties in CSL. We shall frequently use this aspect. $P_{\text{op}}(\varphi)$ asserts that the probability measure of the set of paths satisfying $\varphi$ meets the bound $p$. In CTL, a path satisfies an until-formula $\Phi U \Psi$ if there is a state on the path where $\Psi$ holds, and at every preceding state on the path, if any, $\Phi$ holds. In CSL,
temporal operators like $\Diamond$, $\Box$ and their real-time variants $\Diamond^t$ or $\Box^t$ can be derived, e.g., $P_{\Box^p}(\Diamond^t \Phi) = P_{\Box^p}(\text{true} \ U^t \Phi)$ and $P_{\Diamond^p}(\Box^t \Phi) = P_{<1-p}(\Diamond^t \Phi)$. The untimed next- and until-operators are obtained by $X \Phi = X \ U^t \Phi$ and $\Phi_1 \ U^t \Phi_2 = \Phi_1 \ U^t \Phi_2$ for $I = [0, \infty)$.

CSL allows for the expression of four different types of performance and dependability measures, viz. steady-state measures, transient-state measures, path-based measures, and nested measures. In this paper we shall use several transient-state measures and nested measures.

The ETMCC model checker [13] is a prototype tool that supports the verification of CSL-properties over CTMCs. The model checker takes as input a model file with a textual representation of a CTMC, a label file associating each state to the atomic propositions that hold in that state and a given accuracy. ETMCC is based on sparse matrix representations of CTMCs. Alternative model checkers for CSL include PRISM [15], Prover [23] and the APNN (Abstract Petri Net Notation) toolbox [5].

3. Group Communication Protocols for Wireless Local Area Networks

Real-time group communication protocols for wireless local area networks are very important for applications where autonomous mobile stations are expected to cooperate and synchronise their behaviour in order to accomplish a common goal.

A real-time group communication protocol needs to (a) guarantee real-time communication, i.e., it needs to guarantee an upper bound on the delay of message communication, (b) provide reliable communication, (c) be able to handle failure of mobile stations in a group and keep the stations informed about the status of each station, and finally, (d) guarantee that all stations receive the same messages in the same order.

The main problem that a real-time group communication protocol for wireless networks needs to overcome is the high degree of message losses. This high degree of losses is caused by the unshieldedness of the wireless medium, and partially also by the velocity of the mobile stations. A further characteristic of these losses is their occurrence in bursts, which means that often series of consecutive messages are lost.

In the IEEE 802.11 standard [14], the problem of message losses and the real-time communication requirement have been addressed by the introduction of two alternating periods of medium access control. In the Contention Period (CP), distributed medium arbitration takes place and collisions may occur. The arbitration scheme used during CP is standard carrier sense multiple access with collision avoidance (CSMA/CA). This period is useful for the exchange of non time-critical or less time-critical messages. The Contention Free Period (CFP) has a centralised medium arbitration and the group members get exclusive access right to communicate over the shared medium. The CFP is specifically designed for real-time communication. The two periods, CP and CFP, are activated in an alternating way under the control of a central Access Point (AP). This is a special fixed node in the network with a central position with respect to the mobile stations with which it communicates to obtain an optimal reachability. Both periods can have variable length.

During the CFP the AP coordinates the medium access for all stations that are reachable over the wireless network. At the beginning of the CFP all stations remain silent, except for the AP that transmits a polling message to some station in the group. When a station receives a polling message it may broadcast a message over the network. The polling strategy is decided by the AP which is also in charge of assigning a sequence number to the broadcasted message in order to make total ordering of messages possible.

The real-time group communication protocol that we analyse in this paper is a variant of the protocol used for the CFP in the IEEE 802.11 standard and has been developed by Mock et al. [19].

3.1. Basic operation of the real-time group communication protocol

The protocol is based on a dynamic redundancy scheme. In this scheme a message is only retransmitted upon the detection of its loss. Such a scheme needs the explicit recognition of communication failures and an acknowledgment strategy that reports the status of a transmission.

The protocol is based on the following fault assumptions about the wireless network [8]: (a) if a message is delivered (during the CFP), it is delivered correctly and within a fixed time bound $(t_m)$, (b) messages may be lost, possibly in an asymmetric way, i.e. some stations may receive a broadcast message while others do not. In any case, the number of consecutive losses of the same message is assumed to be bounded by the so-called omission degree $OD$, (c) the AP is reliable, i.e. it is not subject to any kind of error and finally (d) stations may suffer from crash failures or leave the reach of the AP.

The group communication protocol is structured into rounds and it is assumed that there is a maximum number $n_{\text{max}}$ of stations in a group and that $N \leq n_{\text{max}}$ is the current size of the group. A round is composed of a series of slots, one for each station in the group, where each slot consists of a triple of message transmissions; a polling message from the AP to the station, a broadcast request message from the station to the AP and a broadcast message by which the AP sends the user message of the sending station to all stations. Each round is identified by a unique round.
number, starting from 0 and incremented by 1 at the beginning of each new round. The AP polls each station of the group exactly once in each round, and polls the stations always in the same order, sending them the round number in the polling message. After being polled, station \( s \) (denoted as originator in the following) sends a broadcast request message to the AP. This message is composed of an acknowledgment field, a local sequence number and a user message \( m \). The acknowledgment scheme implies that exactly one round after broadcasting a user message of a certain station, the AP is able to decide whether each group member has received the user message or not. For details we refer to [22] and [17].

3.2. Further enhancement of the protocol

The improvement of the protocol proposed in [22] aims at a further decrease in the latency of real-time messages by reducing the maximum number of message retransmissions from \( OD \) to a user-specified number lower than \( OD \). This number is the so-called resilience degree, \( res \). With this reduction of the number of retries full reliability of the protocol can no longer be guaranteed under the assumptions about the network (as long as \( res < OD \)). This means that it may happen that a user message that is broadcasted is not received by all stations in the group within \( res \) retransmissions. This is not a serious problem for many applications as long as all stations agree in time not to deliver that message to their respective application processes.

Thus, the stations are allowed to deliver a user message to the application only if the message is received by all stations. This is decided by the AP, based on a positive acknowledgment for that user message from every station. The decision of the AP is to be communicated in a reliable and timely way. This is achieved by means of the transmission of the decision for each user message through a field in the header of each broadcast message composed of \( OD + 1 \) bit-pairs. Every decision is retransmitted \( OD + 1 \) times, so there is no need for an acknowledgment of the reception of the decision by the station (given the network assumptions).

The functional correctness of the group communication protocol has been treated extensively in [22] where also a specification of the protocol is given in SDL (Specification and Design Language [18]). An analysis of the real-time performance of the protocol is provided in [19]. None of these works have used automatic verification tools for the verification of the properties.

3.3. Dependability measures

A number of dependability and performance measures for the protocol are addressed in [8], where a numerical analysis has been carried out by means of the UltraSAN tool [21]. We revisit two of the dependability measures in this paper, but follow a model-checking approach for their analysis. We start from the high-level SAN specifications in [8] and formulate the measures as CSL formulas. The two dependability measures that we address are:

\[ P_{D > OD} \]: The probability that a decision message (i.e. a message issued by the AP to commit or abort the delivery of a broadcast message) is not received by at least one station in the group, within \( T_{CFP} \) (duration of the CFP phase). This measure represents an estimate of the probability for the protocol to fail in an undetected and undesirable way with possible catastrophic consequences on the system and its users. Therefore, this probability should be sufficiently low.

\[ P_{UM} \]: The probability that the AP does not receive acknowledgments for a user message by all the stations within \( res \) retransmissions within \( T_{CFP} \). In this case, the AP broadcasts to all stations in the group the decision not to deliver that message to their applications. In other words, \( P_{UM} \) is the probability that some station in the group has not acknowledged a user message sent by the AP after \( res \) retransmissions. This property gives an indication to which extent the validity property is violated.

4. A Dependability Model

In [8], a model is developed that covers relevant aspects of the protocol and its environment that are necessary to analyse the dependability measures of interest. A single model is used to analyse several dependability measures by varying the values of its parameters.

4.1. Fading model

In modelling the environment, the interference between different versions of the transmitted signal and the Doppler shift caused by the relative motion of receiving and sending stations, has been taken into account. Both effects cause the so-called signal fading phenomenon. The probability of message loss resulting from fading signals has been approximated by the first-order discrete time Markov chain (DTMC) [24] depicted in Fig. 1. The DTMC has two states, \( S \) and \( F \), standing for (previous) success and failure of a communication respectively. If the previous communication has been successful, with probability \( p \) the next communication will also be successful. With probability \( 1 - p \), the next communication will be a failure. If the previous communication has failed, then with probability \( q \) the next communication fails, and with probability \( 1 - q \) it is successful. This behaviour can be presented as a transition probability matrix in a standard way. The probability of success or failure of a number of consecutive message losses (success) can be obtained by matrix multiplication. The param-
eters \( p \) and \( q \) have been derived considering the communication between the AP and the stations as Rayleigh fading channels and using experimental data available to calculate the approximate values [8]. In particular, \( p \) and \( q \) are functions of the steady state probability that a communication fails (PE) and the normalized Doppler frequency. For details we refer to [8, 24].

4.2. Station model

The fading model has been integrated into the model of a station defined using the SAN formalism [21] which is shown in Fig. 2. SANs are a high-level modelling formalism for the specification of dependability and performance models. SAN models consist of four primitive objects: places, activities, input gates and output gates. Places represent the state of the system and are marked by tokens, like in Petri nets. Activities represent transitions or actions of the system. Input gates are used to control the enabling of activities, and output gates are used to change the state of the model on completion of an activity.

Let us briefly explain the SAN model. The place \( POLL \) models the station that is polled by the AP. Initially it has one token. The input gate \( chk \) enables the timed activity \( nprb \) only if there is a token in place \( POLL \) and no token at place \( FAIL \). If this condition is satisfied, \( chk \) removes the token from place \( POLL \). The exact interpretation that is given to the failing of a station depends on the dependability measure that is analysed. For the analysis of \( P_{D>OD} \) a station fails if it missed more than \( OD \) consecutive decision messages within \( T_{CFP} \). In the case of \( P_{UM} \) it fails if it missed more than \( res \) consecutive user messages within \( T_{CFP} \). The timed activity \( nprb \) (probabilistic broadcast) models the performance aspects of the wireless network and forms the central part of the model.

Model for the analysis of \( P_{D>OD} \). When the model is used to analyse property \( P_{D>OD} \), the time distribution function is chosen to be exponential with a rate being the reciprocal of the duration of one slot, i.e. the sum of the transition time of one polling message, a broadcast request message and a broadcast message. Let \( TP \) be the mean time required for the polling message to be transferred from the AP to a station, and \( TM \) the same for a broadcast message. Then the exponential distribution rate of a slot is \( 1/(2*TM+TP) \).

The timed activity \( nprb \) has two cases, represented as two small circles attached to the hollow oval in Fig. 2. The probability distribution of the two cases is defined by the case distribution and may also depend on the marking of the network at the moment of completion of the activity. In this model, the distribution depends on the marking of place \( SUCCESS \). A token in place \( SUCCESS \) means that the previous triple of polling, broadcast request and broadcast messages, has been a success. We obtain the fading characteristics as the outcome of the product of three matrices \( M'.M.M \). Here \( M' \) represents the matrix defining the fading characteristics of the short polling message, with its characteristic probabilities \( p' \) and \( q' \), and \( M \) defines the fading characteristics of a broadcast message with its respective probabilities. Let \( P \) and \( Q \) be the resulting probabilities of this matrix multiplication, i.e. \( P \) is the probability in that resulting matrix of the self-loop from state \( S \) to itself, \( 1-P \) is the probability in that matrix of the transition from \( S \) to \( F \), etc. The probabilities associated with the two cases in the timed activity \( nprb \) are then derived from the DTMC in Fig. 1 where \( p \) and \( q \) are now \( P \) and \( Q \). So case 1, denoting a broadcast failure, connected to output gate \( FAIL_{BC} \) becomes \( 1-P \) and case 2, denoting a successful broadcast, becomes \( P \). If there is no token on place \( SUCCESS \), the probabilities for the two cases are \( Q \) and \( 1-Q \), respectively.

The output gate \( FAIL_{BC} \) removes any token from place \( SUCCESS \), increments the number of tokens on place \( COUNTER \) by one, and if the number of tokens on \( COUNTER \) exceeds the omission degree \( OD \), it puts a token on place \( FAIL \). Otherwise, it puts a token on place \( POLL \), modelling that the station is ready for the next communication (triple). The \( COUNTER \) represents the number of consecutive failed communications for a given station. The output gate \( SUCCESS_{BC} \) changes the state of the model after a successful broadcast has taken place. It puts a token on place \( SUCCESS \), resets \( COUNTER \) to zero (i.e. removes all its tokens) and puts a token on place \( POLL \). Initially, there is one token on place \( SUCCESS \) and on place \( POLL \), and all other places are empty.

UltraSAN provides a mechanism for replicating a single station. This allows for the easy generation of a model with \( N \) stations that may share places. In this case the stations share place \( FAIL \).

Model for the analysis of \( P_{UM} \). The model used for analysing property \( P_{UM} \) is the same as that for \( P_{D>OD} \) except for the values of \( P \) and \( Q \) and the rate of the exponential distribution of the timed activity. In fact, for \( P_{UM} \) we
are interested in the probability that a user message is not received by a station within \( \text{res} \) retransmissions and within the duration of the CFP. This means that we need to set the rate of the timed activity to the reciprocal of the mean duration of one round, i.e. to \( \frac{1}{(N \cdot (2 \cdot TM + TP))} \) where \( N \) is the number of stations in the group, which is equal to the number of slots in a round. The probabilities \( P \) and \( Q \) have now to be based on a round as well. They can be obtained as the result of the matrix multiplication \( [M'.MA]^N \) in the same way as for the model for \( P_{D>OD} \).

5. Model Checking Properties

The format of the model file generated by UltraSAN is different from that required by the ETMCC model checker [13], but contains all the information needed to construct the proper model file for ETMCC.

The association between the markings of the SAN model and the CTMC facilitates setting up a proper labelling file that is also part of the input for the ETMCC model checker. The labelling file defines the atomic propositions that hold in the various states of the CTMC. The atomic propositions are used to state interesting properties of the model in a precise and formal way using CSL. We developed two programs to transform the model file and the marking file generated with UltraSAN into proper model and label files for ETMCC. UltraSAN provides two different ways for composing subnets using the repeat operator (REP) or the join operator (JOIN). The REP operator is used to replicate the same subnet a specified number of times. It also allows for the selection of shared places. The advantage of this operator is that it allows for the generation of a reduced CTMC that exploits the symmetry in the specification. The reduced CTMC is lumping-equivalent to the non-reduced model [11, 20]. In the reduced model the identity of the stations is, however, not maintained. The generated marking file only reports how many stations have which marking. The JOIN operator joins subnets while maintaining their identity. In order to join four stations we need to make four copies of the station subnet, each with its own name, and join them together. Also in this case shared places can be selected, but no reduction of the state space takes place. In both cases it is not difficult to automatically generate a label file for ETMCC by encoding markings into proper atomic propositions.

In the following we address a selection of qualitative and quantitative properties of the multi-station model. For further properties we refer to [17].

**Qualitative properties.** For the verification of the qualitative properties we used the CTMC derived from a four station SAN model composed using the JOIN operator. We call the stations \( a, b, c, \) and \( d \). In the following formulas the variables \( i \) and \( j \) range over the set of stations. We introduce the atomic propositions \( \neg @ \text{fail} \) or \( \neg @ \text{suc} \) to indicate that station \( i \) has a token on place \( \text{FAIL} \) or \( \text{SUCCESS} \) respectively. Furthermore, we use \( c_i = k \) (with \( k \) a natural number) to denote that the \( \text{COUNTER} \) of station \( i \) has value \( k \). The model of individual stations is, to a limited extent, keeping track of the history of the success or failure of receiving broadcast messages from the AP. If the previous broadcast by the AP to the station was successfully received, it uses one probability distribution for the next success or failure. If it was a failure it uses another distribution. This requires that in the global model every station needs to deal with every broadcast message sent by the AP. Therefore, the model should not allow traces (paths) in which e.g. only one station deals with the broadcasts, and another does not perform any transition. Properties like this cannot be formulated for the reduced model, because in that model the identities of the stations are no longer maintained, but they can be formulated for the full state (JOIN) model. For instance, we consider it undesirable if from any state in which the counter of station \( i \) is zero and that of station \( j \) \((i \neq j)\) is one there exists a path in which station \( j \) remains in the state with its counter on one (i.e. does not perform any transition) and station \( i \) proceeds to a state in which its counter has become 3 (i.e. proving the fact that it made at least 3 transitions). This can be formalised in CSL as:

\[
P_{>0}(\Diamond (c_i = 0 \land c_j = 1 \land P_{>0}(c_j = 1 \cup c_i = 3)))
\]

where \( i, j \in \{a, b, c, d\} \) and \( i \neq j \). Model checking for the case that \( i = a \) and \( j = b \) shows that the formula is satisfied by 81 out of 189 states, including the initial state.

This nested path-based property clearly shows that the current model is not properly capturing the broadcast-aspect of the protocol. In Section 6 we shall therefore propose an alternative model that does address this as-

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**Figure 2. UltraSAN model for one station**

\[
\begin{align*}
&\text{POL} \quad \text{chks} \quad \text{nptb} \\
&\text{FAIL} \quad \text{FAIL-BC} \\
&\text{SUCCESS} \quad \text{SUCCESS}
\end{align*}
\]
expect properly.

Quantitative properties. Although we have seen from the qualitative properties that this model has some problems, it is nevertheless worth to have a look at the results for quantitative analysis. In particular for the property $P_{D>OD}$, which gives us an occasion to compare the results in the literature obtained with UltraSAN with those obtained with ETMCC. For the verification of quantitative properties we used a CTMC derived from a SAN model with four stations composed by means of the REP operator. The atomic propositions can therefore only address the number of stations that are in a state in which variables and places have certain values. Consequently, in the following we slightly change the names of atomic propositions and write $\text{@fail} = k$ with $k$ a natural number, to indicate that $k$ stations have a token on place $FAIL$.

$P_{D>OD}$: The property that a station does not receive the decision message after $OD + 1$ retransmissions can be generalised to the multi-station case. It is easy to verify that always first one station reaches the $FAIL$ before other stations do [17]. Therefore $P_{D>OD}$ can be formalised as:

$P_{\text{fail}}(\text{fail} = 1)$

Fig. 3 shows the results for $P_{D>OD}$ under the assumptions and parameters given in Table 1, for omission degrees of 2, 4, 6 and 8 resp., for various packet loss probabilities (PE), with normalized Doppler frequency (FDT) equal to 3.0E-03 and a duration of $T_{CFP}$ equal to 2400 seconds.

The values obtained with UltraSAN correspond very well to those obtained by ETMCC if the on-the-fly steady state analysis of ETMCC is turned off. The latter is needed because the models are very stiff 1 (i.e. the ration between the largest and the smallest rate in the CTMC is very high.)

$P_{UM}$: With the modification of the values of the variables $P$ and $Q$ and the rate of the exponential distribution in the way described in Section 4, the same formula as for $P_{D>OD}$ in this case reflects the probability that a station does not receive the retransmitted user message for $res$ times in a row:

$P_{\text{fail}}(\text{fail} = 1)$

We postpone the analysis of $P_{UM}$ to the next section, where we develop a more faithful model of the wireless protocol behaviour.

6. Including Synchrony in the Model

In this section, we develop an alternative model. The key point in the new model is that we want to make sure that every broadcast by the AP to the stations is processed by every station in the model within the same slot. Moreover, we aim again at a model in which all stations are modelled in the same way and can be composed by REP for taking advantage of the reduction strategies of UltraSAN. Finally, it would be preferable not to introduce further subnets in order to avoid the generation of large state-spaces. Therefore, we develop a model that takes care of the synchronisation of stations in a fully distributed way.

6.1. New model

Typically, in the new model, each station processes one broadcast message and then waits until all stations in the group have done so. When the last station has processed the broadcast, it notifies the other stations about this by means of a shared variable. After this, the stations are ready for processing the next broadcast from the AP.

We point out that there is no prescribed order in which the stations deal with the broadcast in every slot. This al-

<table>
<thead>
<tr>
<th>FDT</th>
<th>$T_{CFP}$</th>
<th>$T_P$</th>
<th>$T_M$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0E-03</td>
<td>2400 sec.</td>
<td>7.646 ms</td>
<td>2.380 ms</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1. Parameter values for results of Fig. 3

1 In earlier analyses ETMCC gave incorrect results because of a premature detection of a steady state during transient analysis. See [17] for details.
allows for the abstraction of the identity of the stations, which is an advantage for state-space reduction techniques. Secondly, the rate assigned to the timed activity is the same as in the previous model for $P_{D>OD}$. This is allowed because all rates are exponential, so they enjoy the memoryless property. The stations perform their timed activity one after the other in the model. But, by following a similar reasoning as in ([12], p.63), the delay of the second station represents the distribution of the ‘remaining delay’ after the first station processed its broadcast, which is again exponentially distributed with the same rate as that of the first station. The same holds for any further stations in the group.

In order to model the synchronisation, we need to introduce two more shared places in the model (see Fig. 4). The place WAITING is a counter that records the number of stations in the slot that have processed the broadcast. The place TURN is a simple boolean that communicates the change of slot to all stations. Both places and place POLL are initialised to zero. Each station can perform the timed activity as long as $\text{MARK}(\text{TURN}) == \text{MARK}(\text{POLL})$ holds. At the start of the timed activity the station flips the value of place POLL. After the completion of the timed activity the station behaves as in the previous model for $P_{D>OD}$, but it also increases the counter WAITING and checks whether it is the last station that performed the timed activity. If so, it flips the value of TURN and resets the value of WAITING to zero. Since now both POLL and TURN have flipped their value, each station is again able to perform the next timed activity, i.e. deal with the next broadcast from the AP.

Henceforth we call the model of Section 5 model A and the new model described in the current section model B. As before, models for $P_{D>OD}$ and $P_{UM}$ can be obtained by selecting proper values for the model parameters.

6.2. Properties of the new model

In order to formulate properties for model B, which has additional variables, we introduce the following names for atomic properties much in the same way as we have done for model A. Let $i$ and $j$ range over the set of stations $\{a, b, c\}$ and $k$ a natural number, then we mean by $\text{wait}_i = k$ that the value of variable WAITING of station $i$ is equal to $k$. Further we use $\text{turn}_i = k$ ($\text{poll}_i = k$) to denote that there is ($k = 1$) or is not ($k = 0$) a token on place TURN (POLL).

Qualitative properties. For the verification of the qualitative properties we used a CTMC derived from a SAN model composed of three stations by means of the JOIN operator. For model B we can now verify the qualitative property of Sect. 5, namely

$$\mathcal{P}_{>0}(\diamond(c_i = 0 \land c_j = 1 \land \mathcal{P}_{>0}(c_j = 1 \land c_i = 3)))$$

which is, as expected, not satisfied by any of the 594 states.

We can also provide evidence that the synchronisation has been modelled correctly, i.e. whenever for all stations $\text{MARK}(\text{TURN}) == \text{MARK}(\text{POLL})$, all stations are ready to start a new slot, i.e. WAITING is equal to 0.

$$(\forall i.\text{turn}_i = 1 \land \text{poll}_i = 1) \lor (\forall i.\text{turn}_i = 0 \land \text{poll}_i = 0) \Rightarrow (\forall i.\text{wait}_i = 0)$$

The property is satisfied by all states (for details see [17]).

Quantitative properties. At this point we are of course interested in the difference between the results for model A and B. We address the results for $P_{D>OD}$ and $P_{UM}$. In the following we present only the results obtained with UltraSAN; those obtained with ETMCC coincide to a high degree.

$P_{D>OD}$: Fig. 5 shows the difference between the two models for the case that $PE = 5.0e^{-4}$ and for several values of $OD$ ranging from 0 to 8. It is clear that model B estimates the probability of an error lower than is the case with model A. Of course, in both models, the higher the value of $OD$ the lower the probability of errors.

$P_{UM}$: Fig. 6 shows several results for the property $P_{UM}$ using the values for the parameters $P$ and $Q$ and the rate of the exponential distribution of the timed activity as described in Sect. 4. The results have been obtained for $PE = 1.6E - 4$ and the resilience $res$ varying between $[0, ..., 3]$. The upper two curves are the results for $TCFP$ equal to 2400 seconds. The upper of the two giving the results for model A and the lower of the two those for model

![Figure 4. Station in model B](image_url)
B. The third curve from above shows the experimental data obtained for a similar setting, as has been reported in [9]. Actually, the value measured for res = 3 was equal to 0, so the value could not be established with sufficient precision in Fig. 6. Below the third curve, two curves show the results for model A and B resp., but for T_{CFP} equal to 2400 milliseconds, similar to those reported in [4]. Finally, the curve at the bottom shows the results of an earlier model developed by Coccoli et al. [9] that did not consider correlation between communication failures due to fading effects. The largest models used for the quantitative analysis were composed of 660 states and 3135 transitions for model A and 13260 states and 43320 transitions for model B, both for four stations, OD = 8 and in reduced base model format. For more details we refer to [17].

6.3. Discussion

It is clear that neither model A nor model B for P_{UM} are matching exactly the experimental data, although model B gives a better approximation than model A. For small values of res (i.e., res = 0 or res = 1) both models considerably over-estimate the probability of error, while the prediction becomes better for higher values of res (res = 2), even if there is not enough experimental data available to give a well-informed judgement. Maybe that the correlation between transmission errors during the experiments was lower than that assumed for the model, or, more likely, an explanation could be that user messages are retransmitted only once per round and are therefore much less susceptible to the bursty nature of a fading channel. In other words, the loss of user messages is much less correlated than for example, the loss of consecutive decision messages. Unfortunately, the number of losses of decision messages has not been established in an experimental way.

Nevertheless, model B is preferred over an earlier model developed in [9] that did not account for fading effects. The latter considerably under-estimates the error probabilities [9].

7. Conclusions

In this paper we have revisited an earlier dependability analysis of a variant of the centralised medium access protocol of the IEEE 802.11 standard for wireless local area networks [8]. We have analysed some of the models used in that work both from a behavioural (qualitative) and from a dependability (quantitative) point of view by means of the prototype stochastic model checker ETMCC. Both qualitative and quantitative properties have been formalised using CSL.

The qualitative analysis of the concurrent behaviour of the models showed a discrepancy between the expected behaviour of the model and its actual behaviour. The use of model checking allowed for the clear and unambiguous specification and verification of the desired behavioural properties. Some of these concerned properties over state sequences, that can in general only be analysed in an indirect way by means of path-automata by current state-of-the-art dependability analysis tools such as UltraSAN or Mönbsius [6]. Extending these tools with (stochastic) model checking capabilities would allow model developers to assess also the often intricate concurrent behaviour of dependability models.

Based on the results of the qualitative analysis we have developed a new model, that has been shown to reflect more faithfully the assumed synchronisation aspects of the protocol that is induced by the concept of broadcasts within single slots and rounds of the protocol.
Two of the main dependability measures, introduced informally in [8], have been formalised as formulas of the Continuous Stochastic Logic and assessed by means of the stochastic model checker ETMCC. The results corresponded very well when on-the-fly steady state analysis in ETMCC was turned off. This can be explained by the fact that the models under analysis were very stiff.

We believe that this paper provides further evidence of the potential advantages of the integration of (stochastic) model checking capabilities and advanced tools for model-based dependability and performance analysis and its application to realistic case-studies.

In this paper we have discussed only part of the interesting properties and models for the analysis of the real-time wireless protocol. Our future research aims at a more complete formal analysis of the protocol, using proper abstraction techniques and forms of compositionality in order to address further qualitative and quantitative properties in a coherent and systematic way.

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