The background of the entire image is an abstract visualization of Taylor-Couette turbulence. It features a vertical grey cylinder on the left side, representing the inner rotating cylinder. The surrounding fluid is depicted with various swirling patterns and structures. There are large, complex vortices with multiple concentric and intersecting loops in shades of blue, purple, red, and white. Numerous smaller, light blue, oval-shaped structures, resembling droplets or bubbles, are scattered throughout the fluid region. The overall color palette is dominated by blues and purples, with accents of red and white in the swirling patterns.

Towards boiling Taylor-Couette turbulence

Rodrigo Ezeta Aparicio

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TOWARDS BOILING TAYLOR-COUETTE TURBULENCE

DISSERTATION

to obtain
the degree of doctor at the University of Twente,
on the authority of the rector magnificus,
Prof. Dr. T. T. M. Palstra,
on account of the decision of the graduation committee,
to be publicly defended
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Introduction

A short Preface

Dear reader, welcome to the introduction of my research as a PhD student in the field of turbulent fluid dynamics. As you have probably already noticed, the title of my thesis is *Towards boiling Taylor-Couette turbulence*, which is, in all fairness, a rather suitable title considering the content of what is about to be presented. As a very first introductory part, I would like to use the following sections to present to you, the reader, the basic fundamental scientific concepts which will be developed in the following chapters. Thus, without anything further due, let us start from the very beginning: What is turbulence?

Turbulence is omnipresent

The study of turbulence—referred to by the great Richard Phillips Feynman (1918-1988) as one the most important unsolved problem in classical physics—deals with the study of flows (liquid, gases) whose patterns are characterized by being far from deterministic. Chaos is often the main ingredient which makes physical predictions hard to come by. The mere act of defining it is a highly non-trivial challenge all by itself. Nonetheless, it is known by everyone because it is embedded in the everyday life. A very common example (experiment), that should be familiar to everyone can be found in the household. A water jet that comes out from a faucet, leaves the nozzle in a rather cylindrical shape provided of course, one opens the valve just the right amount. However, as experience tells us, the more we open the valve, the more distorted and

messy the flow becomes. This example allows us to rationalize the following observations. (i) When the flow moves slowly, its structure is well-organized and follows the same pattern over time.

In the context of fluid science, this flow is said to be *laminar*. (ii) However, past a certain position of the valve, which effectively increases the water flow rate, the flow pattern becomes *non-regular, chaotic, complicated*, and thus its trajectory cannot be described easily; put in another words, it is in a *turbulent* state. A plethora of other different examples can be found throughout, where turbulent flows are responsible of the governing physics: geophysical flows, which are responsible for the climate in the planet (see Fig. 1a); the air in the atmosphere, above our heads (see Fig. 1b); the flow of stardust within accretions disks (see Fig. 1c) which is responsible for the formation of galaxies in the universe; the flow inside Earth's mantle; plasma formation in our Sun; the airflow around an airplane; the airflow inside a brass instrument; the smoke rising from a lit cigarette (see Fig. 1d) and so on. Notice in all these examples, the difference of the length-scales involved.

In a more quantitative way, how can we readily identify whether a flow is turbulent or not. The answer comes in the form of a dimensional number, the Reynolds number $Re = UD/\nu$, where U is a typical velocity length scale of the flow, D defines a length scale intrinsic to the problem and ν is the kinematic viscosity. The Reynolds number—which takes its name from the famous Osborne Reynolds (1842-1912)—measures the magnitude of the inertial forces with respect to viscous forces. In this way, if Re is small, the flow is laminar. Conversely, if $Re \gg 1$ the flow can become turbulent. Thus, there exists a certain critical Re beyond which the flow undergoes a transition from laminar to turbulent state. In the very simple example of the water faucet presented above, should U increase by means of an increment in the flowrate (D and ν remain reasonably unchanged), it follows that Re increases as well, and as a consequence the flow will eventually transition into turbulence.

The phenomenology of turbulence is best described when looking at the seminal work of Lewis Fry Richardson (1881-1953). Here, a turbulent flow is presented as a set of vortices (just as the ones shown in Fig. 1) which transport energy to smaller eddies in a rather systematic way [1]. The process stops whenever the length scales of the eddies is so small that molecular viscosity becomes dominant. This concept of *cascading energy* only depends on the rate at which energy is dissipated into the smaller scales, i.e. the dissipation rate ϵ and ν in the inertial range and it is supposed to be universal. Many different models attempt to describe this phenomenology and perhaps

(a)

(b)

(c)

(d)

Figure 1: (a) Satellite image of a von Kármán vortex street near Alexander Selkirk Island in the southern Pacific Ocean. Credits: USGS EROS Data Center. (b) Turbulent atmospheric flow as imagined by Vincent W. van Gogh in his famous “De sterrennacht” ca. 1889 (Public domain). (c) Conceptual illustration of a supermassive black hole, which is surrounded by an accretion disk. Credits: NASA/JPL-Caltech, (d) Smoke rising from a cigarette. Credits: Christine Daniloff (MIT News). Notice in all images, the presence of dominant vortical structures (“eddies”), characteristic of turbulent flows.

the most important one is that of Andrey Kolmogorov (1903-1987), which was introduced in 1941. Often referred to as the K41 theory [2], it takes into consideration the energy cascade scenario and predicts how the energy will be distributed throughout all length scales within the inertial regime, i.e. $E(K) \sim k^{-5/3}$, with wavenumbers k within the inertial range of turbulence $2\pi/(15\eta_k) > k > 2\pi/\mathcal{L}$. Here, η_k is the Kolmogorov length scale and $\mathcal{L}$ is the so-called integral length scale. Kolmogorov's model, as it turns out, works remarkably well in describing homogenous isotropic turbulence (HIT), where the statistics of the flow are said to be invariant to both translation and rotation. Unfortunately HIT is an idealized model and in most practical cases (in both nature and industry), flows are found to be far from being in a HIT state. It is, nonetheless, a powerful tool because it serves as a benchmark for comparing different types of canonical flows.

In a more realistic scenario however, a flow is most commonly found to be confined by solid walls, i.e. transport of oil/gas through a pipeline, water running in our houses, nuclear reactors, chemical reactors, mixing milk in a cup of coffee, etc. The presence of a wall makes the prediction of the turbulence even more challenging. In his seminal work of 1933, Ludwig Prandtl (1875-1953) introduces the concept of a boundary layer (BL) [3]. Here, he presents the phenomenology of these regions, which are close to a wall, where the effect of the viscosity is dominant due to large shear that takes place there. As a result, one can work out from the Navier-Stokes equations (NS)—which are the governing equations for a flow (assuming a *self-similar* solution and for sufficient large Re)—that the velocity profile in the vicinity of a wall is a linear function. Further away from the wall however, the velocity profile develops into a logarithmic relation. The challenge is then to take into account the presence of the BL and make predictions of a particular flow configuration, which would certainly not be possible to do with a simple HIT model. The study of BLs is responsible for various outstanding technological developments over the past century, i.e. the aerodynamics behind the production of automobiles and commercial airliners. And up to this date, the study of wall-bounded flows is still a topic of active research in the community due to its vast application in industry.

In order to study wall-bounded turbulence for high-Reynolds number flows, i.e. $Re \gg 1$, one could in principle design experimental facilities where the physical variables can be precisely controlled and monitored. Selected examples of these facilities are the Boundary Layer Wind Tunnel [4] to study turbulent boundary layer flow, the Princeton Superpipe [5] to study pipe flow, the

Figure 2: (a) Taylor-Couette (TC) geometry, where the flow is driven by the rotation of the cylinders (Sketch taken from Ref. [8]). (b) Rayleigh-Bénard (RB) geometry, where the flow is driven by the temperature difference between the bottom and top plates.

Barrel of Ilmenau [6] and the High Pressure Convective Facility [7] to study Rayleigh-Bénard (RB) convection. However, one could also take inspiration from the original work of Sir Geoffrey Ingram Taylor (1886-1975) and Maurice Marie Alfred Couette (1858-1943) and study wall-bounded turbulence in the so-called Taylor-Couette (TC) geometry. From an engineering point of view, studying turbulence in TC flow is appealing since the (mechanical) driving that is required to rotate the cylinders, is far more efficient than, for instance, the (thermal) driving in RB flow.

Taylor-Couette flow turbulence

Taylor-Couette flow is the flow in-between two concentric cylinders which can rotate independently from each other. The TC geometry is completely characterized by two important parameters. The first one is the radius ratio $\eta = r_i/r_o$, where r_i and r_o are the radii of the inner and outer cylinders respectively; and the second one is the aspect ratio $\Gamma = L/d$, where L is the

height of the cylinders and $d = r_o - r_i$ is the gap (see Fig. 2a). The inner cylinder (IC) rotates with an angular frequency ω_i , while the outer cylinder (OC) rotates with an angular frequency ω_o , and thus, the driving of the flow is provided by $\Delta\omega = (\omega_o - \omega_i)$. The driving can be expressed in dimensionless form with the Taylor number, which is effectively $\propto \text{Re}^2$, i.e.

$$\text{Ta} = \frac{(1 + \eta)^4}{64\eta^2} \left(\frac{d(r_i + r_o)\Delta\omega}{\nu} \right)^2. \quad (1)$$

TC flow gives rise to a velocity field, which is better described in cylindrical coordinates (r, θ, z) as $\vec{u}(\vec{r}, t) = u_r(\vec{r}, t)\hat{e}_r + u_\theta(\vec{r}, t)\hat{e}_\theta + u_z(\vec{r}, t)\hat{e}_z$. Here, u_r , u_θ and u_z are the radial, azimuthal, and axial velocity components respectively with their corresponding unit vectors $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_z$. TC flow is characterized by a rich parameter space $(\text{Re}, \omega_i, \omega_o)$ in which different flow structures can be observed; whose creation and stability are mainly a function of the driving [9–11]. G. I. Taylor showed that these secondary flow structures (“Taylor rolls”) arise because the flow becomes predominantly unstable beyond a certain threshold of Re [12]. In addition to this, he used the TC geometry to show that the NS equations coupled with the no-slip boundary condition (i.e. zero relative velocity between the fluid and the walls) are the right equations that describe a fluid. With increasing driving, the morphology of the rolls is observed to change. As the driving increases, the rolls are observed to undergo *wavy* motion, which can be modulated if the driving is large enough. Past a certain Re however, the rolls transition naturally into turbulence [9].

TC flow has been paradigmatically used over the lapse of a century to study flow instabilities, pattern formation and the link between boundary layers and bulk. Together with RB flow, i.e. the convective flow in between two plates, one heated from below and cooled from above (see Fig. 2b), both flows, have been cited as ideal twin-models to study turbulence [13]. It is rather remarkable that despite they may appear as very different flows, in fact, they are governed by the same physics. In this way, one can probe the turbulence in either system and compare it to what would one get from the other. This entanglement is not a result of empirical findings. Since both systems are closed, one can actually work out from the NS equations, analogous relations (and scalings) between their driving, dissipation and response parameters [14]. One of the key ingredients is that there is a conserved quantity in both systems which is transported throughout the flow. In TC turbulence, this quantity is the angular momentum flux J^ω ; while in RB flow it is the thermal flux J^θ . The

response of TC flow is then the angular momentum flux, which is expressed in dimensionless form with the Nusselt number

$$\text{Nu}_\omega = \frac{J^\omega}{J_{\text{lam}}^\omega} = \frac{r^3(\langle u_r \omega \rangle_{A,t} - \nu \partial_r \langle \omega \rangle_{A,t})}{J_{\text{lam}}^\omega}, \quad (2)$$

with the angular velocity $\omega = u_\theta/r$, and the angular momentum flux for the laminar non-vortical case $J_{\text{lam}}^\omega = 2\nu r_i^2 r_o^2 \Delta\omega / (d(r_i + r_o))$. The symbol $\langle \cdot \rangle_{A,t}$ denotes an average over concentric cylindrical surfaces and time. In addition, the global normalized dissipation rate $\tilde{\epsilon}_{\text{global}}$ can be written as

$$\tilde{\epsilon}_{\text{global}} - \tilde{\epsilon}_{\text{lam}} = \sigma_{\text{TC}}^{-2} (\text{Nu}_\omega - 1) \text{Ta}, \quad (3)$$

where $\tilde{\epsilon}_{\text{lam}} = (2r_i r_o (\omega_i - \omega_o) / (\nu(r_i + r_o)))^2 = \sigma_{\text{TC}}^{-2} \text{Ta}$ is the normalized global dissipation rate in the laminar case and $\sigma_{\text{TC}} = (1 + \eta)^4 / (16\eta^2)$ is a Pseudo Prandtl number in close analogy with RB flow. Similar relations as the ones just described above can be found for RB flow [14]. In this case the driving becomes the Rayleigh number Ra and it quantifies the temperature difference between the plates. One of the key questions is, how is the transport of angular momentum related to the driving. If one knows this relation $\text{Nu}_\omega = \text{Nu}_\omega(\text{Ta})$, then the dissipation rate in Eq. (3) can be readily calculated. This relation can be effectively expressed as a power-law $\text{Nu}_\omega \propto \text{Ta}^\alpha$, although care must be taken since α depends on Ta and thus, no pure power-law exists that can describe the response throughout all decades of Ta . In 1962, Robert Kraichnan (1928-2008) predicted that in RB flow, the flow would enter an *ultimate* state provided the driving was large enough [15]. In this ultimate regime, the heat transport becomes effectively independent of the viscosity [15]. Using the analogy with TC flow, there exists then a Ta beyond which, the angular momentum flux becomes independent of the viscosity. The predicted scaling is $\text{Nu}_\omega \propto \text{Ta}^{1/2}$. As it turns out, this transition is close to $\text{Ta} \approx \mathcal{O}(10^8)$, where the presence of the BLs induce corrections to the scaling, i.e. $\text{Nu}_\omega \propto \text{Ta}^{0.4}$ [16]. Recently, this has been confirmed both by numerical simulations [17] and experiments [18–20]. Another interesting property of the angular momentum flux is, that for a given configuration of the rotating ratio of the cylinders ($a = -\omega_o/\omega_i$), a maximum of angular momentum transport is observed. This optimal momentum transport is found to be mainly a function of both η and Ta . In the case of $\eta \approx 0.7$ and $\text{Ta} \approx \mathcal{O}(10^{12})$, the location of the maximum is

observed at $a \approx 0.4$ [8, 19, 21]. For more detailed reviews of TC-flow—which span different decades of Ta —we refer the reader to Ref. [9, 11, 22].

All the aforementioned studies in TC flow consist of a single phase flow, i.e. only the carrier flow is considered. In more realistic situations however, this is not the case. What is the effect then of having a second phase in high-Reynolds TC flow?

Multiphase TC flow and boiling

In TC flow, the driving of the cylinders produces sheared layers throughout the flow. The amount of shear stress in the system is locally proportional to the velocity gradients. At the walls, however, the presence of the BL increases the slope of the velocity and the shear stress is maximized. At the inner cylinder for example, the wall shear stress there (force per unit area) has to be balanced with the torque (force times distance) $\times$ the radius r_i . This torque $\mathcal{T}$ is the one required to drive the IC at constant speed. From this force (torque) balance, we find that the averaged wall shear stress evaluated at the inner wall is simply $\tau_{w,i} = \mathcal{T}/(2\pi r_i^2 L)$. This relation is particularly useful since it states that given the measurement of the torque $\mathcal{T}$, the magnitude of the friction at the wall can be calculated. Note that an analogous relation can also be made with the OC. In multiphase TC flow, the presence of a dispersed phase can dramatically affect the wall shear stress and thus, the friction in the flow. Examples of dispersed phases are fibers, finite-sized particles, polymers, and bubbles [23–28]. The study of the interactions between the dispersed and carrier phase is of particular interest in the industry due to its potential applications. It has been shown for example, that a small amount of air bubbles injected at the hull of a ship can dramatically decrease the drag that is created between the ship and the ocean [29]. Naturally, this translates into less fuel consumption which brings economical and environmental benefits. On the other hand, one could think of cavitation in propellers, where it is known that these high-pressure events damage mechanical parts which is of course undesirable. Therefore, a deep understanding of the mechanisms that govern the physics of multiphase flows is very much desired. In particular, the experiments of Ref. [30] in high- Ta TC flow revealed that in the presence of air bubbles, a considerable amount of drag can be reduced. The drag reduction (DR) obtained here is observed to be a function of Ta , the amount of air i.e. the volume fraction in the system, and the deformability of the bubbles

characterized by the Weber number $We = \rho u_\theta'^2 d_b / \sigma$. Here, ρ is the liquid density, $u_\theta'^2$ is the variance of the azimuthal velocity, d_b is the bubble diameter and σ is the surface tension of the air/liquid interface. The Weber number, named after Moritz Gustav Weber (1871-1951), measures the inertial force of the bubbles with respect to surface tension, and its crucial, as it turns out for achieving a large amount of DR. It is useful since it quantifies just how much deformable a bubble is. If $We > 1$, the bubbles are expected to be deformable. Numerical simulations have also addressed the interaction between bubbles and the carrier phase [31–34]. Although at a lower Re than the experiments, they can provide additional information due to the accessibility of the full velocity field.

Air injection is however not the only way to introduce bubbles into TC flow. An alternative way—which is familiar to virtually everyone—is through boiling. The creation of vapor bubbles occurs when a liquid is heated above its saturation temperature. Explicitly, as the liquid temperature increases, the vapor pressure of the liquid increases as well. Boiling occurs when the vapor pressure of the liquid equals the surrounding pressure, and only then, vapor bubbles are created in nucleation sites. Vapor bubbles and air bubbles are different. Their creation, growth and stability rely on mass diffusion for the former and heat diffusion for the latter [35]. The question is then, how efficient are vapor bubbles in achieving DR as compared to air bubbles in TC flow? In order to address this question, a state-of-the-art TC experiment has been designed and constructed [36]. The Boiling Twente-Taylor Couette facility (see Fig. 3a) is a fully temperature controlled TC apparatus in which boiling can be precisely controlled and monitored. Just as its big brother, the Turbulent Twente-Taylor Couette facility (T³C) (Fig. 3b), it can operate in the ultimate regime of turbulence where both boundary layers (IC and OC) are turbulent. Both air and vapor bubbles interacting with a flow can be observed in a vast range of different industrial and natural processes, and the fundamental physical mechanisms underlying them, are up to this date, a topic of active research in the community [37–47].

A guide through this thesis

The goal of this thesis—as its title suggests—is to investigate boiling TC flow. However, prior to address this problem it is useful also to look at some other properties of TC flow for single phase. A very fundamental understanding of

Figure 3: (a) Rendering of the Boiling Twente Taylor-Couette (BTTC) facility. (b) Rendering of the Twente Turbulent Taylor-Couette (T³C) facility. Both experiments can achieve $\text{Re} \approx \mathcal{O}(10^6)$, with the appropriate working fluid; water in the case of T³C and a fluorinated liquid (3 times less viscous than water) for the BTTC.

the turbulence in the absence of the bubbles is mandatory in order to fully understand the effect of the dispersed phase. This thesis however, also deals with an additional ingredient that has not been mentioned thus far: roughness. The study of roughness in turbulent flows is also of great importance in industry since practically all wall-bounded flows possess a certain degree of roughness. In general, the presence of the roughness increases the friction in the flow, which is the opposing effect of having bubbles in the flow. However, as we will show later it can also be tailored in order to manipulate the flow structures, and thus control the drag increase. In addition to the presence of the roughness, we have also studied the effect of the curvature of the cylinders by studying TC flow for a very narrow gap $d = 20$ mm, for $\eta \approx 0.9$. Here, we will show that the transport of angular momentum is far more complex than at the commonly studied $\eta \approx 0.7$ case in experiments. The structure of the thesis is then as follows:

In chapter 1, we characterize the strength of the turbulence with the so-called Taylor-Reynolds number Re_λ as a function of the driving. By describing the strength of the driving with Re_λ , we use local quantities of the flow which allow us to compare the local turbulence generated in the bulk flow with other canonical flows where Re_λ is more commonly used, i.e. pipe flow, channel flow, von Kármán flow, etc.

In chapter 2, we focus on the statistics of TC flow and reveal yet again the powerful analogies between RB and TC flow. Here, we look at fine scale statistics and identify how the small structures (plumes) embedded in the flow are connected to the large scale structures (Taylor rolls) when the driving is close to the ultimate regime.

In chapter 3, we study the effect of spanwise roughness in high-Reynolds number TC flow. Here, we show that by a systematic positioning of the roughness throughout the inner cylinder, the Taylor rolls and thus, the angular momentum transport can both be tailored and manipulated.

In chapter 4, we look at what effect does the reduction of curvature have on the transport of angular momentum. In this scenario, we will show that within a particular range of Ta , two optimal angular momentum transports can be attained for $\eta \approx 0.9$. While one maximum is found in the regime of corotating cylinders, the second one appears only with sufficient driving, and in the counter-rotating case. These optimal peaks are originated from two very different physical mechanisms.

Finally, in chapter 5 we study boiling TC flow, where we quantify just how much vapor bubbles can reduce the drag in a well-controlled temperature

experiment. We study the effect of deformability of the vapor bubbles and compare it to the case of air bubbles.

Turbulence strength in ultimate Taylor–Couette turbulence[°]

We provide experimental measurements for the effective scaling of the Taylor-Reynolds number within the bulk $\text{Re}_{\lambda,\text{bulk}}$, based on local flow quantities as a function of the driving strength (expressed as the Taylor number Ta), in the ultimate regime of Taylor-Couette flow. The data are obtained through flow velocity field measurements using Particle Image Velocimetry (PIV). We estimate the value of the local dissipation rate $\epsilon(r)$ using the scaling of the second order velocity structure functions in the longitudinal and transverse direction within the inertial range—without invoking Taylor’s hypothesis. We find an effective scaling of $\epsilon_{\text{bulk}}/(\nu^3 d^{-4}) \sim \text{Ta}^{1.40}$, (corresponding to $\text{Nu}_{\omega,\text{bulk}} \sim \text{Ta}^{0.40}$ for the dimensionless local angular velocity transfer), which is nearly the same as for the global energy dissipation rate obtained from both torque measurements ($\text{Nu}_{\omega} \sim \text{Ta}^{0.40}$) and Direct Numerical Simulations ($\text{Nu}_{\omega} \sim \text{Ta}^{0.38}$). The resulting Kolmogorov length scale is then found to scale as $\eta_{\text{bulk}}/d \sim \text{Ta}^{-0.35}$ and the turbulence intensity as $I_{\theta,\text{bulk}} \sim \text{Ta}^{-0.061}$. With both the local dissipation rate and the local fluctuations available we finally find that the Taylor-Reynolds number effectively scales as $\text{Re}_{\lambda,\text{bulk}} \sim \text{Ta}^{0.18}$ in the present parameter regime of $4.0 \times 10^8 < \text{Ta} < 9.0 \times 10^{10}$.

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1.1 Introduction

Taylor-Couette (TC) flow, the flow between two coaxial co- or counter-rotating cylinders, is one of the idealized systems in which turbulent flows can be paradigmatically studied due to its simple geometry and its resulting accessibility through experiments, numerics, and theory. In its rich and vast parameter space, various different flow structures can be observed [9, 10, 12, 18, 21, 48, 49]. For recent reviews, we refer the reader to Ref. [22] for the low Ta range and Ref. [11] for large Ta.

The driving strength of the system is expressed through the Taylor number defined as

$$\text{Ta} = \frac{1}{4} \sigma_{\text{TC}} d^2 (r_i + r_o)^2 (\omega_i - \omega_o)^2 / \nu^2,$$

where $r_{i,o}$ are the inner and outer radii, $d = r_o - r_i$ the gap width, $\omega_{i,o}$ the angular velocities of the inner and outer cylinders, ν the kinematic viscosity of the fluid, $\sigma_{\text{TC}} = (1+\rho)^4/(4\rho)^2 \approx 1.06$ a pseudo-Prandtl number employing the analogy with Rayleigh-Bénard (RB) flow [14], and $\rho = r_i/r_o$ the radius ratio. The response of the system is generally described by the two response parameters Nu_ω and Re_w . The first is the Nusselt number $\text{Nu}_\omega = J_\omega/J_{\omega,\text{lam}}$, with the angular velocity transfer $J_\omega = r^3 \langle (u_r \omega - \nu \partial_r \omega) \rangle_{A,t}$, where $\langle \rangle_{A,t}$ denotes averaging over a cylindrical surfaces of constant radius and over time. $\omega = u_\theta/r$ is the angular velocity and $J_{\omega,\text{lam}} = 2\nu(r_i r_o)^2 (\omega_i - \omega_o)/(r_o^2 - r_i^2)$ is the angular velocity transfer from the inner to the outer cylinder for laminar flow. Nu_ω describes the flux of angular velocity in the system, and is directly linked to the torque through the Navier-Stokes equations. The second response parameter of the flow is the so-called wind Reynolds number $\text{Re}_w = \sigma_{\text{bulk}}(u_r) d / \nu$, where $\sigma_{\text{bulk}}(u_r)$ is the standard deviation of the radial component of the velocity inside the bulk. Re_w quantifies the strength of the secondary flows. In the ultimate regime of turbulence, where both the boundary layers (BL) and the bulk are turbulent ($\text{Ta} \geq 3 \times 10^8$), it was experimentally found that $\text{Nu}_\omega \sim \text{Ta}^{0.40}$, in the Taylor number regime of 10^9 to 10^{13} , independent of the rotation ratio $a = -\omega_o/\omega_i$ and radius ratio ρ [10, 18, 19, 21]. This scaling has been identified, using the analogy with RB flow, with the ultimate scaling regime $\text{Nu}_\omega \sim \text{Ta}^{1/2} \mathcal{L}(\text{Ta})$, where the log-corrections $\mathcal{L}(\text{Ta})$ are due to the presence of the BLs. [16]. The wind Reynolds number Re_w was found experimentally to scale as $\text{Re}_w \sim \text{Ta}^{0.495}$ within the bulk flow [20]; very close to the

1/2 exponent that was theoretically predicted by Ref. [16]. Here, remarkably, the log-corrections cancel out.

In this study we characterize the local response of the flow with an alternate response parameter based on the standard deviation of the azimuthal velocity $\sigma(u_\theta)$ and the microscales of the turbulence, i.e. the Taylor-Reynolds number which is defined as $\text{Re}_\lambda = u' \lambda / \nu$, where u' is the rms of the velocity fluctuations and λ is the Taylor micro-scale.

Re_λ is often used in the literature to quantify the level of turbulence in a given flow, ideally for homogeneous and isotropic turbulence (HIT), where it should be calculated from the full 3D velocity field. In experiments however, the entire flow field is generally not accessible. Assuming isotropy (which is most of the time not strictly fulfilled), the dissipation rate ϵ (in Cartesian coordinates) can be reduced to $\epsilon = 15\nu \langle (\partial u / \partial x)^2 \rangle_t$, where u is the component of the velocity in the streamline direction x . In this way, the Taylor micro-scale is then redefined as $\lambda^2 = \langle u^2 \rangle / \langle (\partial u / \partial x)^2 \rangle$. Examples where this procedure has been followed in spite of the lack for perfect isotropy include turbulent RB flow [50], the flow between counter-rotating disks [51], von Kármán flow [52], or channel flow [53]. In all cases the isotropic form of Re_λ is still chosen as a robust way to quantify the strength of the turbulence. It is in this spirit that we aim to calculate Re_λ in turbulent Taylor-Couette flow, albeit in a region sufficiently far away from the BLs (bulk). Such a calculation allows for a quantitative comparison between the turbulence generated in TC flow and the one produced by other canonical flows, i.e. pipe, channel, RB, von Kármán flow, etc. Following this route, we define the bulk Taylor-Reynolds number for TC flow as

$$\text{Re}_{\lambda, \text{bulk}} \equiv (\sigma_{\text{bulk}}(u_\theta))^2 \left(\frac{15}{\nu \epsilon_{\text{bulk}}} \right)^{1/2}, \quad (1.1)$$

$$\sigma_{\text{bulk}}(u_\theta) \equiv \langle \sigma_{\theta, t}(u_\theta(r, \theta, t)) \rangle_{r_{\text{bulk}}}, \quad (1.2)$$

$$\epsilon_{\text{bulk}} \equiv \langle \epsilon(r, \theta, t) \rangle_{\theta, t, r_{\text{bulk}}}, \quad (1.3)$$

where $\sigma_{\theta, t}(u_\theta(r, \theta, t))$ is the standard deviation of the azimuthal velocity in the azimuthal direction and over time. $\sigma_{\text{bulk}}(u_\theta)$ is then the average of the azimuthal velocity fluctuations profile over the bulk and ϵ_{bulk} the bulk-averaged dissipation rate. Note that the subscript r_{bulk} means that we average in the radial direction but only for $0.35 < (r - r_i)/d < 0.65$, i.e. the middle 30% of the gap (see also §1.3.1).

Multiple prior estimates of Re_λ in TC flow can be found in the literature: Ref. [54] calculated it using a combination of the local velocity fluctuations

and the global energy dissipation rate ϵ_{global} , where the latter is obtained from torque measurements denoted by τ through $\epsilon = \tau\omega_i/m$, where m is the total mass. Ref. [55], however, estimated Re_λ at midgap ($\tilde{r} = (r - r_i)/d = 0.5$) with the local velocity fluctuations and a local dissipation rate estimated indirectly through the velocity spectrum $E(k)$ in wave number space k , i.e. $\epsilon = 15\nu \int k^2 E(k) dk$. In this calculation, Taylor's frozen flow hypothesis was used to get the θ -dependence for the azimuthal velocity u_θ , i.e. $u(\theta + d\theta, t) = u(\theta, t - rd\theta/U)$, where U is the mean azimuthal velocity. To the best of our knowledge, however, a truly bulk-averaged calculation of $\text{Re}_{\lambda, \text{bulk}}$ (based on local quantities) has hitherto never been reported in the literature. Of particular interest is how this quantity scales with Ta in the ultimate regime, and how this scaling is connected to that of Nu_ω and Re_w .

As TC flow is a closed flow system, the global energy dissipation rate ϵ_{global} is connected to both the driving strength Ta and Nu_ω by [14]

$$\tilde{\epsilon}_{\text{global}} = \frac{d^4}{\nu^3} \epsilon_{\text{global}} = \sigma_{\text{TC}}^{-2} \text{Nu}_\omega \text{Ta}. \quad (1.4)$$

In the ultimate regime this implies an effective scaling of the global energy dissipation rate $\tilde{\epsilon}_{\text{global}} \sim \text{Ta}^{1.40}$. A calculation of Re_λ in the bulk does not require the global energy dissipation rate $\tilde{\epsilon}_{\text{global}}$, but the bulk-averaged energy dissipation rate, ϵ_{bulk} in combination with the bulk averaged velocity fluctuations $\sigma_{\text{bulk}}(u_\theta)$, see Eq. (1.2). In general, velocimetry techniques like Particle Image Velocimetry (PIV) can provide $\sigma_{\text{bulk}}(u_\theta)$ directly, thus the challenge of the calculation is to correctly estimate ϵ_{bulk} . While the global energy dissipation rate ϵ_{global} (Eq. (1.4)) can be obtained from torque measurements, an estimate of ϵ_{bulk} requires the knowledge of the local dissipation rate $\epsilon(r, \theta, t)$ as it is shown in Eq. (1.3). For fixed height along the cylinders, the dissipation rate profile $\epsilon(r) = \langle \epsilon(r, \theta, t) \rangle_{\theta, t}$ is connected to the global energy dissipation rate through $\epsilon_{\text{global}} = (\pi(r_o^2 - r_i^2))^{-1} \int_{r_i}^{r_o} \epsilon(r) 2\pi r dr$. We note that due to the non-trivial interplay between bulk and turbulent BLs in the ultimate regime, it is not known a priori that ϵ_{bulk} and ϵ_{global} will scale in the same way: local measurements are needed to confirm this assumption.

The energy dissipation rate ϵ is key for Kolmogorov's scaling prediction of the velocity structure functions (SFs) in HIT, namely $D_{LL}(s) = C_2(\epsilon s)^{2/3}$ for the second order longitudinal structure function and $D_{NN}(s) = C_2(4/3)(\epsilon s)^{2/3}$ for the second order transverse structure function within the inertial range, neglecting intermittency corrections [56, 57]. The Kolmogorov constant was measured to be $C_2 \approx 2.0$ and is believed to be universal [58]. The exponents for the scaling of the p -th order SFs (ζ_p^*) have been measured and found to

differ from Kolmogorov's original prediction $p/3$: the difference between them are attributed to the intermittency of the flow [54, 55, 59, 60]. However, second order SFs along with the classical Kolmogorov scaling $\zeta_2 = 2/3$ have been successfully used to estimate ϵ in fully developed turbulence [51, 52, 61, 62]. One can then expect only a moderate underestimation of ϵ since the intermittency correction to the exponent of the second order SFs is small $\zeta_2^* - 2/3 \approx 0.03$, where ζ_2^* is the measured exponent of the second order SFs in TC flow using extended self-similarity (ESS) [54, 55].

In this paper we make use of local flow measurements using planar Particle Image Velocimetry (PIV) to find $\sigma_{\text{bulk}}(u_\theta)$ and using the scaling of the second order ($p = 2$) SFs we estimate ϵ_{bulk} . The advantage of PIV over other flow measuring technique such as Laser- Doppler or Hot-wire anemometry is the possibility to access the whole velocity field at the same time in the $r - \theta$ plane, i.e. $\vec{u} = u_r(r, \theta, t)\hat{e}_r + u_\theta(r, \theta, t)\hat{e}_\theta$, from which we can obtain directly the θ -dependence of the velocities. Unlike in the calculation of Ref. [55] and [54], in this work, we do not need to invoke Taylor's hypothesis in the calculation of $\text{Re}_{\lambda, \text{bulk}}$. We only explore the case of inner cylinder rotation ($a = 0$), where there is virtually no stable structures (Taylor rolls) left when the driving strength is sufficiently large ($\text{Ta} \geq 10^8$) [21]. In this way, the calculation is independent of the axial height z and thus there is no need for an axial average [8].

1.2 Experimental apparatus

The PIV experiments were performed in the Taylor-Couette apparatus as described in Ref. [36]. This facility provides an optimal environment for PIV experiments in TC flow, due to its transparent outer cylinder and top plate. The radii of the setup are $r_i = 75$ mm and $r_o = 105$ mm, and thus $\rho = r_i/r_o = 0.714$, which is very close to $\rho = 0.724$ and $\rho = 0.716$ from Ref. [55] and Ref. [54], respectively. The height ℓ equals 549 mm, resulting in an aspect ratio $\Gamma = \ell/d = 18.3$. The excellent temperature control of the setup allows us to perform all the experiments at a constant temperature of 26.0 °C with a standard deviation of 15 mK. The measurements are done at midheight $z = \ell/2$ in the $r - \theta$ plane. The flow is seeded with fluorescent polyamide particles with diameters up to 20 μm and with an average particle density of ≈ 0.01 particles/pixels. The laser sheet we use for illumination is provided by a pulsed laser (Quantel Evergreen 145 laser, 532 nm)

Figure 1.1: (color online). (a) Vertical cross section of the experimental setup. (b) A sketch of the binning process on the $r - \theta$ plane for the calculation of the SFs. Here we show an exaggeration of how the velocity fields are binned in both the radial and azimuthal directions. $\hat{e}_r$ and $\hat{e}_\theta$ are the unit vectors in polar coordinates. The orange dashed line represents the streamline direction s for a fixed radius.

and has a thickness of ≈ 2.0 mm. The measurements are recorded using a high-resolution camera at a framerate of $f = 1$ Hz. The camera we use is an Imager sCMOS ($2560 \text{ pixels} \times 2160 \text{ pixels}$) 16 bit with a Carl Zeiss Milvus 2.0/100. The camera is operated in double frame mode which leads to an inter-frame-time $\Delta t \ll 1/f$. In Fig. 5.1a a schematic of the experimental setup is shown. In order to obtain a large amount of statistics, we capture 1500 fields for each of the 12 different Taylor numbers explored. The velocity fields are calculated using a “multi-pass” method with a starting window size of $64 \text{ pixels} \times 64 \text{ pixels}$ to a final size of $24 \text{ pixels} \times 24 \text{ pixels}$ with 50% overlap. This allows us to obtain a resolution of $dx = 0.01d$. When using the local Kolmogorov length scale in the flow (see §1.3.3), we find that dx/η_{bulk} ranges from ≈ 1.6 ($\text{Ta} = 4.0 \times 10^8$) to ≈ 10 ($\text{Ta} = 9.0 \times 10^{10}$).

1.3 Results

1.3.1 Identifying the bulk region

The profiles of the velocity fluctuations for both components of the velocity as a function of Ta are shown in Fig. 5.3a. The distance from the inner cylinder is represented by the normalized radius $\tilde{r} = (r - r_i)/d$. When normalized with the velocity of the inner cylinder $r_i\omega_i$, both profiles collapse for all Ta numbers in most of the gap width around the value of 0.03. Only very close the inner and outer cylinder, the fluctuations increase (decrease) for the azimuthal (radial) component. In our calculation of $Re_{\lambda, \text{bulk}}$ (Eq. (1.1)), we use $\sigma_{\text{bulk}}(u_\theta)$ as our velocity scale as u_θ is the primary flow direction. Here, we are essentially assuming that the radial and axial velocity fluctuations, on average, have the same order of magnitude, i.e. $\sigma_{\text{bulk}}(u_\theta) \approx \sigma_{\text{bulk}}(u_r)$ (the result is z -independent). In order to give an impression of how valid this assumption is, in Fig. 5.3b we show the ratio of the velocity fluctuations throughout the gap. We notice that within the bulk region, the ratio is between 1.0 and 1.6 for all analyzed Ta numbers; consistent with what one would expect for reasonably isotropic flows. Surprisingly, the ratio within the bulk increasingly deviates from unity as the driving is increased. The same observation is also observed in turbulent TC-flow ($Ta \in [5.8 \times 10^7, 6.2 \times 10^9]$) for a wider gap $\eta = 0.5$, where also the ratio within the bulk increasingly deviates from unity with increasing Ta . In that case however, it seems to reach a value of ≈ 1.8 for the largest Ta [63]. Since the same observation is found in two different studies (with two different experimental setups), we believe this is a feature of TC-flow; however, a more rigorous theoretical explanation has yet to be provided. Another interesting feature of the profiles in Fig. 5.3b is that they become flatter as the turbulence level is increased, reflecting an increase in spatial homogeneity. Note that these results do not suggest readily that the flow is in a HIT state. What this merely shows is that there is a special region (bulk) where the flow becomes more homogeneous as compared to regions close to the solid boundaries and it is reasonably isotropic. This justifies that our calculation is based on an isotropic form of Re_λ as was also used in other studies [50–53, 55].

Next, we define the bulk region as $r_{\text{bulk}} \equiv r - r_i \in [0.35d, 0.65d]$, wherein the magnitude of the velocity fluctuations for both u_r and u_θ are roughly

constant. This definition of the bulk was previously used by Ref. [20] who measured the scaling of Re_w in the ultimate regime. The same definition is also consistent with other studies [55, 64], where the bulk region is identified as the r domain wherein the normalized specific angular momentum remains constant ($\tilde{L}_\theta = r\langle u_\theta \rangle_{\theta,t}/(r_i^2 \omega_i) \approx 0.5$) for all Ta. In Fig. 5.3c we show $\tilde{L}_\theta(r)$ and we find a good collapse of the profiles within our definition of the bulk. Here, it is seen that the value of $\tilde{L}_\theta$ is indeed around 0.5 within the bulk.

1.3.2 Structure functions and energy dissipation rate profiles

Having defined the bulk region, we bin the velocity data in the azimuthal (streamwise) direction with a bin width $d\theta = 0.2^\circ$ for every r and Ta. Now we calculate the second order structure functions in both longitudinal (LL) and transverse (NN) directions for every radial bin,

$$\delta_{LL}(r, s) = \langle (u_\theta(r, \theta + s/r, t) - u_\theta(r, \theta, t))^2 \rangle_{\theta,t} \quad (1.5)$$

$$\delta_{NN}(r, s) = \langle (u_r(r, \theta + s/r, t) - u_r(r, \theta, t))^2 \rangle_{\theta,t}, \quad (1.6)$$

where s is the distance along the streamwise direction. Since $s = r\theta$, the azimuthal binning guarantees a constant spatial resolution $ds = rd\theta$ along the direction of s , when the radial variable r is fixed (see the sketch in Fig. 5.1b). The choice of ds is limited by the resolution of the PIV experiments dx and it is chosen such as to not filter out any intermittent fluctuations in the flow. The energy dissipation rate profiles for both directions are calculated as follows. For fixed r and Ta, ϵ_{LL} is chosen as the maximum of $s^{-1}(\delta_{LL}(r, s)/C_2)^{-2/3}$ such that s lies inside the inertial range. In the same manner, ϵ_{NN} is taken as the maximum of $s^{-1}(\delta_{NN}(r, s)/(4C_2/3))^{-2/3}$ with the same restriction for s . This operation is repeated for every r and Ta, leading to the dissipation rate profiles shown in Fig. 5.4. In this figure, the ϵ -profiles are made dimensionless as $\tilde{\epsilon}(r) = \epsilon(r)/(d^{-4}\nu^3)$. Near the solid boundaries, this figure shows that the dissipation rates (LL and NN) differ from each other: ϵ_{LL} increases while ϵ_{NN} decreases, which is consistent with the measurement of the velocity fluctuations (figures 5.3ab). However, as one moves into the bulk region, the discrepancy between them decreases until eventually both dissipation rates intersect. The crossing remains within the bulk region, independent of Ta, and does not seem to occur at any particular radial position. Only in the case of

Figure 1.2: (color online). (a) Normalized velocity fluctuations profiles for various Ta: (dashed lines) azimuthal, (solid lines) radial. (b) The profiles of the velocity fluctuations ratio (radial/azimuthal) for various Ta. (c) Normalized specific angular momentum profile for various Ta. In all figures, the bulk region $\tilde{r} \in [0.35, 0.65]$ is highlighted as the blue region. The different colors represent different Ta as described in Fig. 5.4.

Figure 1.3: (color online). Dimensionless energy dissipation rate profile $\tilde{\epsilon}(r) = \epsilon(r)/(d^{-4}\nu^3)$ for various Ta : (dashed lines) longitudinal direction $\tilde{\epsilon}_{LL}(\tilde{r})$, (solid lines) transversal direction $\tilde{\epsilon}_{NN}(\tilde{r})$. Ta is increasing from bottom to top, the lines correspond to the following Ta numbers: $Ta = 4.0 \times 10^8, 1.6 \times 10^9, 3.6 \times 10^9, 6.4 \times 10^9, 1.0 \times 10^{10}, 1.4 \times 10^{10}, 2.0 \times 10^{10}, 2.6 \times 10^{10}, 3.2 \times 10^{10}, 4.0 \times 10^{10}, 5.7 \times 10^{10}, 9.0 \times 10^{10}$. For every Ta , both ϵ -profiles cross within the bulk region ($\tilde{r} \in [0.35, 0.65]$) which is highlighted in blue. The black solid line is the total energy dissipation rate obtained from DNS for $Ta = 2.15 \times 10^9$ [65].

HIT, the dissipation rates obtained from both SFs are exactly the same. However, as indicated in figures 5.3ab, the flow tends to be more homogeneous within the bulk. We expect then that, regardless of the structure function (longitudinal or transverse) used, the energy dissipation rate obtained from either direction should, on average, be nearly the same within the bulk. In this study we will show that this is indeed the case, which means that ϵ_{bulk} can be obtained either from the dissipation rate in the LL direction ϵ_{LL} or from that in the NN direction ϵ_{NN} . A similar approach is followed in Ref. [66], where both SFs are calculated in RB flow within the sub-Kolmogorov regime where the flow is found to be nearly homogeneous and isotropic at the center of the cell.

In Fig. 5.4, we have included the dimensionless dissipation rate

$$\tilde{\epsilon}_u = (d^4/\nu^3) \langle (\nu/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)^2 \rangle_{V,t},$$

obtained from Direct Numerical Simulations (DNS) for $\rho = 0.714$, $\Gamma = 2$ and $Ta = 2.15 \times 10^9$ from Ref. [65]. Here, the $\langle \rangle_{V,t}$ denotes the average over the entire volume and time respectively. This includes the boundary layers, that we explicitly avoid in our r_{bulk} definition. When comparing the profile obtained from numerics and from our data for $Ta = 3.6 \times 10^{10}$ we notice that both agree rather well, thus mutually validating each other.

By averaging the ϵ -profiles in the bulk (Fig. 5.4), we finally find the bulk-averaged dissipation rates $\tilde{\epsilon}_{LL,\text{bulk}} = \langle \tilde{\epsilon}_{LL}(\tilde{r}) \rangle_{r_{\text{bulk}}}$ and $\tilde{\epsilon}_{NN,\text{bulk}} = \langle \tilde{\epsilon}_{NN}(\tilde{r}) \rangle_{r_{\text{bulk}}}$. In order to validate the calculation, in Fig. 5.5 we show the bulk-averaged longitudinal D_{LL} and transverse D_{NN} SFs for every Ta . Here, we compensate the SFs as $s^{-1}(D_{LL}(s)/C_2)^{2/3}$ and $s^{-1}(D_{NN}(s)/(4/3)C_2)^{2/3}$ such that their units match that of the dissipation rate. The horizontal axis is normalized with the corresponding bulk-averaged Kolmogorov length scale (see §1.3.3). According to Kolmogorov's scaling, within the inertial regime ($s \in [15\eta, L_{11}]$), where L_{11} is the integral length scale obtained from the azimuthal velocity, each compensated curve (fixed r and Ta) should be proportional to the dissipation rate in the bulk. Here we see that our estimates for the bulk-averaged dissipation rates are located within the plateau regions, demonstrating the self-consistency of the calculation. In the same figure, the separation of length scales in the flow can also be seen. Note in particular how such separation between η and L_{11} increases with Ta . The integral length scale $L_{11}(Ta)$ in Fig. 5.5 is calculated using the integral of the autocorrelation of the azimuthal velocity in the azimuthal direction and averaged over the bulk region.

Figure 1.4: (color online). Compensated time-bulk averaged structure functions for various Ta : (a) longitudinal, (b) transverse. The colors represent the variation in Ta as described in Fig. 5.4. In both figures, the black dashed line is 15η while the colored short vertical lines are located at L_{11}/η for each Ta : The inertial range is approximately bounded by these two lines. The colored stars show the maxima of each curve which correspond to $\langle \epsilon(r) \rangle_{r_{\text{bulk}}}$.

1.3.3 The dissipation rate in the bulk

In Fig. 5.2a we show the scaling of both $\tilde{\epsilon}_{LL,bulk}$ and $\tilde{\epsilon}_{NN,bulk}$. We find that the dissipation rate extracted from both directions scale effectively as $\tilde{\epsilon}_{bulk} \sim Ta^{1.40}$, with a nearly identical prefactor. This shows that the local energy dissipation rate scales in the same way as the global energy dissipation rate $\tilde{\epsilon} \sim Ta^{1.40}$. Correspondingly, this implies that the local Nusselt number scales as $Nu_{\omega,bulk} \sim Ta^{0.40}$. In the same figure (Fig. 5.2a), we include $\tilde{\epsilon}$ of [17], obtained from both DNS, and [21] torque measurements from the Twente Turbulent Taylor-Couette (T³C) experiment. The compensated plot (Fig. 5.2b) reveals that both the local and global energy dissipation rate scale indeed as $Ta^{1.40}$ with the ratio $\epsilon_{bulk}/\epsilon_{global} \approx 0.1$. In the regime of ultimate TC turbulence, it was suggested that both turbulent BLs extend throughout the gap until they meet around $d/2$ [16]. The turbulent BLs give rise to the logarithmic correction $\mathcal{L}(Ta)$ in the scaling of the Nusselt number, which changes the scaling from $Nu_{\omega} \sim Ta^{1/2}$ to effectively $Nu_{\omega} \sim Ta^{1/2}\mathcal{L}(Ta) \sim Ta^{0.40}$ [18,20]. With Eq. (1.4) one obtains the effective scaling of the global energy dissipation rate $\tilde{\epsilon}_{global} \sim Ta^{3/2}\mathcal{L}(Ta) \sim Ta^{1.40}$. It is remarkable how our local measurements of the local energy dissipation rate reveal the very same scaling due to $\mathcal{L}(Ta)$ as the global energy dissipation rate. In contrast, in RB flow it is shown that when the driving is in the order of $10^8 < Ra < 10^{11}$, i.e. far below the transition into the ultimate regime (BLs are still laminar), $\tilde{\epsilon}_{bulk} \sim Ra^{1.5}$ [66,67]. Note, however, that in that regime the global energy dissipation rate $\tilde{\epsilon}_{global}$ is still determined by the BL contributions, $\tilde{\epsilon}_{BL} \gg \tilde{\epsilon}_{bulk}$ and $\tilde{\epsilon}_{BL} \approx \tilde{\epsilon}_{global}$. Our measurements are thus consistent with the prediction of Ref. [16], where even at such large Ta numbers, a rather intricate interaction between turbulent BLs and bulk flow prevails through the entire gap.

In order to further show the quality of the scaling, we show in Fig. 1.6 the same ϵ -profiles shown in Fig. 5.4 but now compensated with $Ta^{-1.40}$. For both the LL and NN direction, the dissipation rates for different Ta collapse throughout most of the gap, far away from the inner and outer cylinder. Within the bulk however, they are nearly constant and very close to the prefactors ($\approx 5 \times 10^{-4}$) found from the scaling in Fig. 5.2a. When looking at the compensated data from DNS, we notice that the prefactor is in that case twice as large as ours ($\approx 10^{-3}$). The reason is that the nature of both calculations is different: While the data from DNS is obtained from averaging the 3D velocity gradients over the entire volume, we rely on the scaling of the second order SFs (without

Figure 1.5: (color online) (a) Dimensionless bulk-averaged energy dissipation rate: longitudinal $\tilde{\epsilon}_{LL,bulk}$ (blue open triangles), transverse $\tilde{\epsilon}_{NN,bulk}$ (red open circles). Dimensionless global energy dissipation rate ($\tilde{\epsilon}_{global}$): DNS [17] (solid black circles), torque measurements [21] (black line). (b) Compensated plot of the bulk-averaged dissipation rate, where an effective scaling of $\tilde{\epsilon}_{bulk} \sim \tilde{\epsilon} \sim Ta^{1.40}$ is revealed for both the global and the dissipation rate in the bulk. In both figures, the green star corresponds to the bulk-average dissipation rate data of Ref. [65] for $Ta = 2.15 \times 10^9$.

intermittency corrections) to approximate the local energy dissipation rate in the bulk at the maximum peak in the compensated curves (see §1.3.2).

In order to further characterize the turbulent scales in the flow, we calculate the Kolmogorov length scale in the bulk. Since there are two dissipation rates available, we define their corresponding Kolmogorov length scales as $\eta_{LL,\text{bulk}} = (\nu^3/\epsilon_{LL,\text{bulk}})^{1/4}$ and $\eta_{NN,\text{bulk}} = (\nu^3/\epsilon_{NN,\text{bulk}})^{1/4}$. Because $\tilde{\epsilon}_{\text{bulk}} \sim \text{Ta}^{1.40}$, the scaling of $\tilde{\eta}_{\text{bulk}} = \eta_{\text{bulk}}/d \sim \text{Ta}^{-0.35}$, which can be seen in Fig. 1.7a. Obviously, here we find a similar prefactor in both directions LL and NN too. The inset of the figure shows the corresponding compensated plot. For comparison, we include in the same figure the scaling from Ref. [55]. When comparing it with our data we notice some differences in magnitude. While we average in the bulk and make use of PIV to obtain the spatial dependence of the velocities directly, the data from Ref. [55] was measured at a single point ($\tilde{r} = 0.5$) using Hot-wire anemometry and Taylor's frozen flow hypothesis.

When fitting data to a power law, confidence bounds for every coefficient in the regression can be obtained, given a certain confidence level. In this paper, we use the standard 95% confidence for every fit, from which the uncertainty in the power law exponents (figures 5.2,1.6) were chosen as the middle point between the lower and upper bound of its corresponding confidence bound. This procedure is done for all the exponents reported throughout this paper.

1.3.4 The turbulent intensity in the bulk

The final step in the calculation of $\text{Re}_{\lambda,\text{bulk}}$ is to look at the azimuthal velocity fluctuations. Thus we average $\sigma_{\theta,t}(u_{\theta}(r, \theta, t))$ (see Eq. (1.2)) from Fig. 5.3a in the bulk and find a good description by the effective scaling law $(d/\nu)\sigma_{\text{bulk}}(u_{\theta}) \approx 11.3 \times 10^{-2} \text{Ta}^{0.44 \pm 0.01}$. In Fig. 1.7b, we show the turbulence intensity $I_{\theta,\text{bulk}} = \langle \sigma_{\theta,t}(u_{\theta}) / \langle u_{\theta} \rangle_{\theta,t} \rangle_{r_{\text{bulk}}}$ as a function of Ta. In this way, we are able to compare our data to the turbulence intensity scaling from Ref. [55]. We find that the effective scaling $I_{\theta,\text{bulk}} \sim \text{Ta}^{-0.061 \pm 0.003}$ reproduces our data well. In the inset of the same figure we show the compensated plot throughout the Ta range. Similarly as with the Kolmogorov length scale described in section §1.3.3, we include in the same figure the scaling of Ref. [55]. In this case, the exponent in our scaling is nearly identical to the one found by [55] with a slightly larger prefactor. We remind the reader once again that our average is done over the bulk region while the data of Ref. [55] is obtained at a single point at midgap.

Figure 1.6: (color online). Compensated dimensionless dissipation rate profiles calculated with both structure functions for different Ta : (dashed lines) longitudinal, (solid lines) transverse. The colors represent the variation in Ta as shown in Fig. 5.4. In both figures, the bulk region is highlighted in blue. The black solid line corresponds to the DNS data from Ref. [65] for $Ta = 2.15 \times 10^9$.

Figure 1.7: (color online) (a) Dimensionless bulk-averaged Kolmogorov length scale: longitudinal (blue open triangles), transverse (red open circles). Local scaling at $\tilde{r} = 0.5$ from Ref. [55] (black dashed line). The inset shows the compensated plots for the local quantities where the effective scaling of $\tilde{\eta}_{bulk} \sim Ta^{-0.35}$ is found to reproduce both directions. (b) Bulk-averaged azimuthal turbulent intensity. The data reveals an effective scaling of $I_{\theta,bulk} \sim Ta^{-0.061}$. The (dashed black line) represents the local scaling $I_{\theta} = 0.1Ta^{-0.062}$ at $\tilde{r} = 0.5$ as it was obtained from Ref. [55]. The inset in (b) shows the corresponding compensated plot.

1.3.5 The scaling of the Taylor-Reynolds number $\text{Re}_{\lambda,\text{bulk}}$

Finally, with both the local dissipation rate and the local velocity fluctuations in the bulk, we calculate the corresponding Taylor-Reynolds number as a function of Ta , using both $\epsilon_{LL,\text{bulk}}$, $\epsilon_{NN,\text{bulk}}$ and $\sigma_{\text{bulk}}(u_\theta)$. The results can be seen in Fig. 1.8a where an effective scaling of $\text{Re}_{\lambda,\text{bulk}} \sim \text{Ta}^{0.18 \pm 0.01}$ is found for both directions. The compensated plot in Fig. 1.8b reveals the good quality of the scaling throughout the range of Ta . In order to highlight the difference between the different calculations, we also include the estimate of Ref. [54] for $\text{Ta} = 1.49 \times 10^{12}$ ($\text{Re}_\lambda = 106$). We emphasize that our calculation is based entirely on local quantities (fluctuations and dissipation rate) whilst the estimate of Ref. [54] is done using a single point in space, $\tilde{r} = 0.5$, in combination with the global energy dissipation rate (Eq. (1.4)). Our scaling predicts that the local Taylor-Reynolds number at that Ta is approximately $\text{Re}_{\lambda,\text{bulk}} \approx 217$, roughly twice the value estimated by Ref. [54] for the same Ta .

1.4 Summary and conclusions

To summarize, we have measured local velocity fields using PIV in the ultimate regime of turbulence. We showed that both structure functions (longitudinal and transverse) yield similar energy dissipation rate profiles that intersect within the bulk, similarly as what is observed in Rayleigh-Bénard convection. When averaging these profiles within the bulk, this leads to an effective scaling of $\tilde{\epsilon}_{\text{bulk}} \sim \text{Ta}^{1.40 \pm 0.04}$, which is the same scaling as obtained for the global quantity $\tilde{\epsilon}$ measured from the torque scaling [17, 21]. This result reveals the dominant influence of the turbulent BLs over the entire gap. Future work will show whether this also holds for higher-order velocity structure functions, as it does hold in other turbulent wall-bounded flows [68].

Next, we showed that the Kolmogorov length scale scales as $\tilde{\eta}_{\text{bulk}} \sim \text{Ta}^{0.35 \pm 0.01}$ and the azimuthal turbulent intensity scales as $I_{\theta,\text{bulk}} \sim \text{Ta}^{-0.061 \pm 0.003}$. In order to evaluate the turbulence level in the flow, we showed that with both local quantities at hand (dissipation rate and turbulent fluctuations), the bulk Taylor-Reynolds number scales as $\text{Re}_{\lambda,\text{bulk}} \sim \text{Ta}^{0.18 \pm 0.01}$. Our calculation can be generalized by inserting our result for the ratio between the local and global energy dissipation rate $\tilde{\epsilon}_{\text{bulk}}/\epsilon_{\text{global}} = \alpha \approx 0.1$ back into Eq. (1.1) and using Eq. (1.4) to relate ϵ_{global} and Nu_ω . The latter yields

Figure 1.8: (color online) (a) $Re_{\lambda,bulk}$ as a function of Ta . The blue open triangles (red open circles) show the calculation using $\epsilon_{LL,bulk}$ ($\epsilon_{NN,bulk}$). The black solid point is the calculation using the global energy dissipation rate from Ref. [54]. (b) Compensated plot of $Re_{\lambda,bulk}$ where an effective scaling of $Re_{\lambda,bulk} \sim Ta^{-0.18}$ is found to be in good agreement with both LL and NN directions.

$$\text{Re}_{\lambda,\text{bulk}}(\text{Ta}) = \sqrt{1/\alpha} \left(\frac{\sqrt{15}\sigma_{\text{TC}}d^2}{\nu^2} \right) \frac{(\sigma_{\text{bulk}}(u_\theta))^2}{\sqrt{\text{Ta}\text{Nu}_\omega}}. \quad (1.7)$$

Thus, given the local variance of the velocity fluctuations and the global Nusselt number, the response parameter $\text{Re}_{\lambda,\text{bulk}}(\text{Ta})$ can be calculated in the bulk flow ($\tilde{r} \in [0.35, 0.65]$) for the case of pure inner cylinder rotation ($a = 0$). In order to extend the calculation to the case $a \approx a_{\text{opt}} \approx 0.36$, i.e. close to the rotation ratio for optimal Nu_ω , where pronounced Taylor rolls exist [10, 21], an extra averaging process in axial direction for both the velocity fluctuations and the dissipation rates would be needed.

2

Small-scale statistics of momentum transport and underlying flow structures in turbulent Taylor-Couette flow[°]

We experimentally study the influence of large scale Taylor rolls on the small-scale statistics and the flow organization in fully turbulent Taylor-Couette flow. We show that the local angular momentum transport (expressed in terms of a Nusselt number) mainly takes place in the regions of the vortex in- and outflow, where the radial and azimuthal velocity components are highly correlated. The efficient momentum transfer is reflected in intermittent bursts, which becomes visible in the exponential tails of the PDFs of the local Nusselt number. In addition, by calculating azimuthal energy co-spectra, small scale plumes are revealed to be the underlying structure of these bursts. These flow features are very similar to the ones observed in Rayleigh-Bénard convection, which emphasizes the analogies of these both systems. By performing a Proper Orthogonal Decomposition (POD), we remarkably detect azimuthally traveling waves superimposed on the turbulent Taylor vortices, not only in the classical but also in the ultimate regime. This very large-scale flow pattern—which is most pronounced at the axial location of the vortex center—is similar to the well-known wavy Taylor vortex flow, which has comparable wave speeds, but much larger azimuthal wave numbers.

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2.1 Introduction

The flow in between two independently rotating cylinders, known as Taylor-Couette (TC) flow, is a commonly used model for general rotating shear flows. It features rich and diverse flow states, which have been explored for nearly a century [9, 12, 69–71]. More recent reviews on the hydrodynamic instabilities can be found in Ref. [22], and on fully turbulent Taylor-Couette flows in Ref. [11]. The geometry of a Taylor-Couette system is defined by the gap width $d = r_2 - r_1$, where r_1 and r_2 are the inner and outer radii respectively; the radius ratio $\eta = r_1/r_2$, and the aspect ratio $\Gamma = \ell/d$, with ℓ the height of the cylinders. The external driving of the flow can be quantified by the shear Reynolds number according to Ref. [72]

$$\text{Re}_S = \frac{2r_1r_2d}{(r_1 + r_2)\nu}|\omega_2 - \omega_1| = \frac{u_S d}{\nu}, \quad (2.1)$$

with the cylinder angular velocities $\omega_{1,2}$, ν is the kinematic viscosity of the fluid, and u_S is the shear velocity. A further dimensionless control parameter is the ratio of the angular velocities

$$\mu = \frac{\omega_2}{\omega_1}, \quad (2.2)$$

implying $\mu > 0$ for co-rotation of the cylinders, $\mu = 0$ for pure inner cylinder rotation and $\mu < 0$ for counter-rotation.

The most important response parameter of the TC system to the cylinder driving is the angular velocity transport [14]

$$J^\omega = r^3 (\langle u_r \omega \rangle_{\varphi, z, t} - \nu \partial_r \langle \omega \rangle_{\varphi, z, t}), \quad (2.3)$$

where r denotes the radial coordinate, φ the azimuthal coordinate, t the time coordinate and $\langle \cdot \rangle_{\varphi, z, t}$ the azimuthal-axial-time average. This quantity is conserved along r and can be directly measured by the torque $\mathcal{T}$ acting on either the inner (IC) or the outer cylinder wall (OC). Normalizing J^ω with its corresponding laminar non-vortical value $J_{\text{lam}}^\omega = 2\nu r_1^2 r_2^2 (\omega_1 - \omega_2) / (r_2^2 - r_1^2)$ yields a quasi Nusselt number $\text{Nu}_\omega = J^\omega / J_{\text{lam}}^\omega$ [14], which is analogous to the Nusselt number Nu in Rayleigh Bénard flow (RB) flow, i.e. the buoyancy driven flow which is heated from below and cooled from above. There, Nu is a measure for the amount of transported heat flux normalized by the purely conductive heat transfer. Ref. [73] worked out the fundamental similarities between TC

Figure 2.1: Sketch of the ejecting regions bounded by the inner (IC) and outer (OC) cylinders within a Taylor roll: (a) vortex inflow, (b) vortex center, (c) vortex outflow.

and RB flow in terms of the Nusselt number and the energy dissipation rate, which we will use in this paper.

The dependence of the Nusselt number on the shear Reynolds number, commonly expressed in terms of an effective power law $Nu_\omega \sim Re_S^\alpha$, and on the rotation ratio μ has been widely investigated [8, 10, 11, 18, 19, 55, 74–76]. For pure inner cylinder rotation ($\mu = 0$), a change in the local scaling exponent α with increasing Re_S has been found which is caused by a transition from laminar (classical regime) to turbulent boundary layers (ultimate regime) [10]. The transition point depends on the radius ratio η and is located around $Re_{S,crit} \approx 1.6 \times 10^4$ for $\eta = 0.714$. In the ultimate regime, the scaling exponent becomes $\alpha \approx 0.76$ [77] independent on η .

Another feature of TC flow is a maximum in the angular momentum transport (Nu_ω) when the driving (Re_S) is kept constant and only μ is changed. For $\eta = 0.714$, the maximum is located in the counter-rotating regime at $\mu_{max} \approx -0.36$ and originates from a strengthening of the turbulent Taylor vortices [17, 78]. This finding reflects that turbulent Taylor vortices play a prominent role in the fully turbulent regime. Hence, the morphology and physical mechanisms behind these roll structures have been the focus of different studies throughout the literature. Ref. [77] showed that the large-scale rolls consist and are driven by small-scale unmixed plumes, in analogy to RB flow. They calculated the radial profiles of the angular velocity ω at specific axial positions of the large scale Taylor vortices; namely at the vortex inflow, vortex center and vortex outflow.

The vortex inflow is characterized by the ejection of plumes from the OC in conjunction with a mean radial velocity that points away from the OC (see the sketch in Fig. 2.1a). In contrast, the outflow features plume ejections from the IC with a mean radial velocity component directed from the IC to the OC (see the sketch in Fig. 2.1c). In between the in- and outflow, the radial

velocity component becomes zero in the middle of the gap, which denotes the location of the center of the vortex, as shown in Fig. 2.1b. In regions where plumes are ejected from the cylinder walls, i.e. in the vortex inflow at the outer wall and in the vortex outflow at the inner wall, in the ultimate regime the radial profiles of ω have a logarithmic shape in the corresponding boundary layer [79, 80]. For $\eta = 0.5$, Ref. [63] calculated the velocity of such plumes based on planar Particle Image Velocimetry (PIV) measurements performed at different heights, which further confirms the connection between small scale plumes and large scale vortices. The contribution of the large scale Taylor rolls to the angular momentum transport has been evaluated numerically by Ref. [78] and experimentally by Ref. [81] by means of the decomposition of Nu_ω into its turbulent fluctuation and large scale circulation contributions. They find that the large scale contribution dominates the transport in the region of the torque maximum.

Within the context of both TC and RB flow, many studies in the literature focus on establishing a distinct connection between the transport of angular momentum (or heat in RB flow) and the structures inherent to these flows, both for large scale rolls and small scale plumes. A comparative study of the probability density functions (PDFs) of the Nusselt number in TC and RB calculated over cylindrical surfaces and horizontal planes respectively has been performed by Ref. [76]. As a reference point for the comparison, they choose $Re_S = 2 \times 10^4$ for TC and a Rayleigh number of $Ra = \alpha_p g H^3 \Delta T / (\kappa \nu) = 10^7$ for RB, where the Nusselt number is identical for both flows. Here, Ra is the dimensionless driving parameter in RB flow with α_p the thermal expansion coefficient, g the gravitational acceleration, H the height of the RB cell, ΔT the temperature difference, and κ the thermal diffusivity. They find that the PDFs of the net transport have the same asymmetric shape with differences in the width of the tails in the boundary layer regions. These differences can be attributed to different shapes and detachment frequencies of the plumes. Moreover, the PDFs of the angular momentum and temperature fluctuations depict a cusp-like shape with pronounced exponential tails as a consequence of the effect of intermittent bursting plumes. Similar analyses of heat flux PDFs in RB flow have been performed by Ref. [82]. They find that the instantaneous heat flux fluctuates around zero and not around the volume-averaged Nusselt number, along with a broadening of the tails with increasing Ra . Ref. [83] revealed that the asymmetry of the PDFs of Nu , which mainly occurs in the tails, arises from correlated temperature and velocity signals produced by thermal plumes. These plumes lead to large but rare positive events of heat flux. How-

ever, the results of Ref. [83] are only based on point-wise measurements. For TC flow, the statistics of Nu_ω was analyzed by Ref. [36]. This overview shows, that more studies are required to fully understand the interaction between the large scale Taylor rolls and the small scale plumes.

Another approach to investigate structures in TC flow is to analyze the kinetic energy spectra. Ref. [84] performed direct numerical simulations (DNS) for $\eta = 0.5$, $Re = 8000$ and the OC at rest. Strikingly, he finds a small scale peak in the axial spectra of the radial velocity component. According to Ref. [84] the underlying structures can be specified as herringbone streaks. The DNSs of Ref. [80] in the boundary layer regions reveal a peak in the azimuthal and axial spectra of the radial velocity component at large wavenumbers, which indicates the existence of small scale plumes. Their simulations were done for $\eta = 0.909$ and $\mu = 0$ at $Re_S \geq 10^5$. We stress that the energy spectra in TC flow do not follow the classical Kolmogorov scaling for homogeneous and isotropic turbulence (HIT) of $-5/3$ [54, 55, 80, 84, 85].

Another interesting phenomenon regarding Taylor vortices, which has been reported in the literature, is the reappearance of azimuthal waves in the turbulent Taylor vortex regime. Ref. [86] measured the point-wise radial velocity component close to the OC boundary layer for $\mu = 0$ at $\eta = 0.875$ and for different aspect ratios. They find a regime of reappearance for $28 \leq Re/Re_C \leq 36$ and for $\Gamma \geq 25$, based on sharp peaks in the power spectrum. Here, Re_C is the critical Reynolds number for the onset of Taylor vortex flow. Later, Ref. [87] acquired time-resolved axial profiles of the axial velocity component using an ultrasonic measurement technique for $\eta = 0.904$ and $\Gamma = 20$. The azimuthal waves are identified based on Fourier analysis and Proper Orthogonal Decomposition (POD) in the range of $23 \leq Re/Re_C \leq 36$. Their results show good agreement with those of Ref. [86]. In another study, Ref. [88] performed planar PIV measurements in a meridional plane for $\eta = 0.733$ and $\Gamma = 34$. They capture the reappearance of azimuthal waves for $20 \leq Re/Re_C \leq 38$ based on spatial correlations. Note that the three aforementioned studies are all based on pure inner cylinder rotation ($\mu = 0$). More recently, Ref. [89] performed flow visualizations in TC flow with $\eta = 0.5$ at $Re_S = 5000$. They find an axial oscillation of the turbulent Taylor vortices in the range of $\mu \in [-0.15, -0.3]$, which includes the rotation ratio for optimum transport $\mu_{max} = -0.2$. In summery, the large scale turbulent Taylor rolls seem to feature an instability mechanism similar to the one in the laminar regime.

Within our study, we aim to experimentally analyze the Taylor-Couette flow in the presence of large scale Taylor rolls. More specifically, we look at the

statistics of the flow via the PDFs of the local angular momentum flux over cylindrical surfaces for various radial locations. Within this framework, we analyze the contributions of the vortex inflow, the vortex center and the vortex outflow. Thereby, we can reveal the contributions of rare and strong events of momentum flux to the overall transport due to the ejecting plumes. Furthermore, a detailed investigation of the kinetic velocity co-spectra will shed light on the energy distribution over different scales. In addition, we will demonstrate that azimuthal waves in TC flow will also reappear at high Re_S . To answer these questions, we make use of PIV measurements in horizontal planes at different cylinder heights for $\eta = 0.714$, in the range of $Re_S \in [9.3 \times 10^3, 3.5 \times 10^5]$ and $\mu \in [0, -0.36]$.

2.2 Experimental setup

The PIV experiments were performed in the Boiling Twente Taylor-Couette facility (BTTC) at the University of Twente. The BTTC is an ideal facility to perform PIV experiments due to its transparent outer cylinder and top plate. The inner and outer radius of the setup are $r_1 = 75$ mm and $r_2 = 105$ mm, respectively and thus the radial gap is $d = r_2 - r_1 = 30$ mm. The height of the cylinders is $\ell = 549$ mm, which gives an aspect ratio of $\Gamma = \ell/d = 18.3$. The radius ratio is then $\eta = r_1/r_2 = 0.714$. A more detailed overview of the setup can be found in Ref. [36]. The flow consists of water whose viscosity ν and density ρ can be controlled throughout the experiments due to the temperature control of the BTTC. We fix the temperature of the experiments to 20°C , leading to $\nu = 1.002 \text{ mm}^2/\text{s}$ and $\rho = 0.998 \text{ g/cm}^3$. The standard deviation of the temperature is 15 mK . The flow is seeded with fluorescent polyamide particles with diameters up to $20 \mu\text{m}$ with an average particle density of approximately 0.01 particles/pixel. These particles are coated with Rhodamine B which has a maximum emission centered at around 565 nm . The illumination is provided by a laser sheet from a double pulsed cavity laser (Quantel Evergreen 145 laser, 532 nm). The thickness of the laser sheet is $\approx 1 \text{ mm}$. The laser is mounted to a traverse system (Dantec lightweight traverse) which allows us to precisely change the location of the laser sheet along the vertical direction. The camera we used for the recordings is an Imager sCMOS ($2560 \times 2160 \text{ px}$) 16 bit with a Carl Zeiss Milvus 2.0/100 lens. We capture the velocity fields with a framerate of $f = 15 \text{ Hz}$. Since the camera is operated in double frame mode, we can have very small interframe times, i.e. $\Delta t \ll 1/f$.

In order to maximize the contrast of the images, we use a long-pass filter in front of the lens (Edmund High Performance Longpass Filter, 550 nm), which collects only the emitted light from the fluorescent particles. In Fig. 2.2a, we show a sketch of the experimental setup.

The velocity fields are calculated with a commercial software (Davis 8.0) using a multi-pass method. The algorithm uses windows of size 64×64 px for the first pass and windows of 24×24 px for the last iteration with a 50% overlap of the windows. This process yields the velocity fields in Cartesian coordinates. In order to have access to the velocity fields in polar coordinates, we map the Cartesian velocity fields onto a polar grid using bilinear interpolation. The mapping is done such that the radial Δr and azimuthal $\Delta \varphi$ resolution is the same as the spatial resolution in Cartesian coordinates Δx , i.e. $\Delta r = \Delta x$ and $r\Delta\varphi = \Delta x$. In this way, the resultant velocity fields are of the form $\mathbf{u} = u_r(r, \varphi, t)\mathbf{e}_r + u_\varphi(r, \varphi, t)\mathbf{e}_\varphi$, where u_r and u_φ are the radial and azimuthal velocity components which depend on the radial coordinate r , the azimuthal coordinate φ and the time coordinate t . $\mathbf{e}_r$ and $\mathbf{e}_\varphi$ are the unit vectors in the radial and azimuthal direction respectively.

In table 2.1, we present a summary of the measurements we performed. In total, 8 cases were investigated which will be addressed in the following sections. Each case contains measurements done at 23 different heights, and each height contains 1500 different velocity fields which are separated by $\Delta z = 4$ mm each. Thus, the height of the experiments spans a length of $\Delta \ell = 22\Delta z = 88$ mm. The resolution of the velocity fields Δx depends on the height but lies within $\Delta x \in [0.607, 0.752]$ mm, where the smallest value corresponds to the height closest to the camera at $(z - \ell/2)/d = 1.5$ and the smallest to $(z - \ell/2)/d = -1.5$.

2.3 Flow states and velocity profiles

We investigate flow states for pure inner cylinder rotation as a reference and for the rotation ratio that corresponds to the torque maximum (see table 2.1). Case 1 and 2 are in the so-called classical regime ($\text{Re}_S < 1.6 \times 10^4$), where the boundary layers are still of laminar type and only the bulk flow is turbulent. There, μ_{max} changes with the shear Reynolds number, which is why $\mu_{max} = -0.15$ is different to the cases in the ultimate regime ($\text{Re}_S > 1.6 \times 10^4$) [77]. For the flow states in the ultimate regime, the torque maximum is located around $\mu_{max} = -0.36$.

Figure 2.2: (a) Sketch of the experimental apparatus (BTTC). Shown is a vertical section. A mirror set at 45 degrees is used so the camera captures the velocity field in the $r - \varphi$ plane. The picture shows an exaggeration of the 23 positions of the laser sheet along $\Delta\ell$. (b) Temporally averaged flow field in a horizontal plane at the axial height of the vortex center for $\text{Re}_S = 3.51 \times 10^5$ and $\mu = 0$. Colors represent the azimuthal velocity component and arrows the velocity field. Only each tenth vector is shown. Black solid lines indicate the position of the inner and outer cylinder.

Case	Regime	Re_S	μ	Abbreviation	Linestyle
1	classical	9.32×10^3	0	C1 _{CR,0}	---
2	classical	9.30×10^3	-0.15	C2 _{CR,M}	—
3	ultimate	2.98×10^4	-0.36	C3 _{UR,M}	—
4	ultimate	6.68×10^4	-0.36	C4 _{UR,M}	—
5	ultimate	9.46×10^4	-0.36	C5 _{UR,M}	—
6	ultimate	2.15×10^5	0	C6 _{UR,0}	---
7	ultimate	2.14×10^5	-0.36	C7 _{UR,M}	—
8	ultimate	3.51×10^5	0	C8 _{UR,0}	---

Table 2.1: Overview of investigated flow states, defined by the regime, the shear Reynolds number Re_S , and the rotation ratio μ . The penultimate column depicts the abbreviations for the different flow states and the last column depicts the linestyles used in the study. The transition point from the classical to the ultimate regime is located at $Re_S \approx 1.6 \times 10^4$.

The flow at μ_{max} is dominated by turbulent Taylor vortices [18, 21, 77, 78]. This is also reflected in our experiments, as seen from the azimuthally and time averaged azimuthal velocity component for the 23 investigated heights (see Fig. 2.3).

For both flows in the classical regime (see Figs. 2.3a,b), large scale rolls are visible in the flow filling the whole gap. Apparently, the turbulent fluctuations are not strong enough at low shear Reynolds numbers to suppress these large scale rolls. Further, the rolls are more pronounced at $\mu_{max} = -0.15$, indicating an increase in strength. In the ultimate regime at $Re_S = 2.15 \times 10^5$ and $\mu = 0$, the turbulent Taylor rolls disappear and nearly no axial dependence of the azimuthal velocity is visible in the mean field, just as also found numerically (see the phase diagram, Fig. 6 in Ref. [10]). When the rotation rate is changed to μ_{max} , pronounced Taylor vortices are formed again, which are much more pronounced than in the classical regime. The contour plot reveals a mushroom-like structure with distinct in- and outflow regions. The axial profile of the radial velocity component corresponding to Fig. 2.3d is shown in Fig. 2.3e. This representation exemplifies the further analysis. The recorded 23 heights capture more than one vortex pair, which is why we exclude the grayly marked data points for flow quantities calculated over the axial coordinate. The axial length of evaluation therefore starts and ends at a vortex center. In between, we use the minimum of the azimuthally-temporally averaged axial profile of

Figure 2.3: Representation of the flow states in terms of temporal and azimuthally averaged velocities. The contour plots depict the azimuthal velocity component for (a) C1_{CR,0}, (b) C2_{CR,M}, (c) C6_{UR,0} and (d) C7_{UR,M}. (e) Axial profile of the radial velocity component for C7_{UR,M} with marked location of vortex inflow, vortex center and vortex outflow. Gray dots represent data points, which are excluded for flow quantities calculated over the axial coordinate.

Figure 2.4: Radial profiles of the (a) azimuthal and (b) radial velocity component, averaged over space (cylindrical surfaces) and time. Tilde symbols denote normalized quantities. The radial coordinate is normalized as $\tilde{r} = (r - r_1)/(r_2 - r_1)$, where 0 means the location of the inner cylinder (r_1) and 1 the location of the outer cylinder (r_2). The azimuthal velocity is normalized using the cylinder speeds as $\tilde{u}_\varphi = (u_\varphi - u_{\varphi,2})/(u_{\varphi,1} - u_{\varphi,2})$ with $u_\varphi = r\omega$. To normalize the radial velocity component, the shear velocity $u_S = \text{Re}_S \nu/d$ is used: $\tilde{u}_r = u_r/u_S$.

the radial velocity as the location of the vortex inflow and correspondingly, the location of the vortex outflow is obtained with its maximum. The data point which is closest to a value of zero is defined as the axial location of the vortex center. In Fig. 2.4, we show the normalized radial profiles of the azimuthal and radial velocity components averaged over space (cylindrical surfaces) and time ($\langle \cdot \rangle_{\varphi,z,t}$). The tilde symbols denote normalized quantities. The radial coordinate is normalized as $\tilde{r} = (r - r_1)/(r_2 - r_1)$, where 0 means the location of the inner cylinder (r_1) and 1 the location of the outer cylinder (r_2). Further, the azimuthal velocity is normalized using the cylinder speeds as $\tilde{u}_\varphi = (u_\varphi - u_{\varphi,2})/(u_{\varphi,1} - u_{\varphi,2})$, with $u_\varphi = r\omega$. In order to normalize the radial velocity component, the shear velocity is used $\tilde{u}_r = u_r/u_S$. The slopes of the profiles in the bulk nearly vanish at $\mu = \mu_{max}$, while for pure inner cylinder rotation they show a small negative slope in good agreement with other studies [76, 81, 90]. With increasing shear Reynolds number, the difference of the data points close to the wall from the wall velocity becomes larger, which is due to the steepness of the velocity gradients. The averaged radial profiles of the radial velocity component are nearly zero all over the gap, as the radial velocity has to vanish when averaged over one vortex pair. The

Figure 2.5: Radial profiles of the standard deviation of the (a) azimuthal and (b) radial velocity component, calculated over space (cylindrical surfaces) and time. The profiles are normalized by the shear velocity u_S .

deviation results from the restricted axial resolution of the individual heights. Next, the radial profiles of the standard deviation of the azimuthal and radial velocity component calculated over space (cylindrical surfaces) and time are plotted in Fig. 2.5. In terms of the azimuthal velocity component, the radial profiles of the standard deviation show a nearly constant low value in the center of the gap and increasing values close to the wall. This increase is more pronounced at μ_{max} ; while for the lowest shear Reynolds number, a peak close to the outer cylinder wall is visible. In case of the radial velocity component, the standard deviation depicts a maximum in the center of the gap and decreases towards the cylinder walls. The maximum of $\sigma_{\varphi,z,t}(\tilde{u}_r)$ for both flow cases in the classical regime is noticeable higher than the one in the ultimate regime. The overall shape of the profiles agrees well with the ones for pure inner cylinder rotation of Ref. [91], measured with PIV in the same facility at midheight.

2.4 Probability Density Function (PDF) of velocity components

To provide a basis for the investigation of the local PDFs of the angular momentum transport (section 2.6), we first analyse the PDFs of the azimuthal and radial velocity components. In Fig. 2.6, we show the PDFs of the azimuthal and radial velocity components at $\tilde{r} = 0.5$ and $\text{Re}_S = 2.1 \times 10^5$ for

Figure 2.6: Probability density functions of the (a,b) azimuthal and (c,d) radial velocity component calculated over space (cylindrical surfaces) and time at $\tilde{r} = 0.5$ for $\text{Re}_S = 2.1 \times 10^5$ ($\mu = 0, \mu_{max}$). Dark gray, gray and bright gray lines indicate the local PDFs for the vortex inflow, vortex center and vortex outflow, respectively. The black dashed line represents a Gaussian distribution with zero mean and unit variance.

$\mu = 0$ and $\mu = -0.36$ as representatives. In addition, the underlying PDFs at the locations of the vortex inflow, vortex center and vortex outflow are included. The local PDFs at the specific vortex locations are close to Gaussian distributions, as already shown by Ref. [54] for the azimuthal velocity component measured via LDV at midheight and $\tilde{r} = 0.5$. When all data at different heights are included into the PDFs, they still follow a Gaussian shape at $\mu = 0$ for both velocity components. At μ_{max} however, the PDF of the azimuthal velocity depicts a cusp-like form centered at the origin and approximately exponential tails in accordance with the numerical simulations at a lower Reynolds number of $\text{Re}_S = 2 \times 10^4$ by Ref. [76]. There and in the study of Ref. [92], a nearly identical shape was reported for the temperature PDF in RB flow, which reveals yet another clear evidence of the analogies between both systems, even on small scale statistics. In RB flow, the specific PDF shape of the temperature is induced by a combination of bursting plumes and large scale rolls [93–95]. By using the TC-RB flow analogy, we can identify here a clear fingerprint of plumes transporting angular momentum in TC flow. In addition, the vortex dominated TC flow in the region of the torque maximum in the fully turbulent regime shows a similar behavior to the flow

Figure 2.7: Probability density functions of the azimuthal velocity component calculated over space (cylindrical surfaces) and time for the radial locations (a) $\tilde{r} = 0.1$, (b) $\tilde{r} = 0.3$ and (c) $\tilde{r} = 0.5$

organization in RB flow. In the case of the radial velocity at μ_{max} , the PDF also deviates from the ideal Gaussian shape, which can be attributed again to the intermittent bursting plumes.

As the PDFs of u_r for flow cases at pure inner cylinder rotation depict a nearly Gaussian shape, which is also valid for different radial locations in the bulk, we omit the radial velocity component for the following analysis within this section. In Fig. 2.7, we calculate the PDF of the azimuthal velocity component for different radial locations. When the point of evaluation approaches from the gap center into the direction of the inner cylinder wall for μ_{max} , the right hand tail of the initial cusp-like PDF becomes more pronounced and its width increases with the shear Reynolds number. This change is caused by the dominance of the vortex outflow in the inner gap region, where the mean azimuthal velocity exhibits a large positive value. In addition, very close to the inner wall at $\tilde{r} = 0.1$, both tails become increasingly exponential. This behavior is even more pronounced at higher Re_S , which is another sign of the ejection of coherent plumes from the cylinder wall. Next to these local changes, with decreasing distance to the wall, the global asymmetry of the PDFs seem to increase.

To account for such global properties of the PDFs, we show in Fig. 2.8 the

Figure 2.8: Radial profiles of the (a) skewness and (b) kurtosis of the azimuthal velocity component, calculated over space (cylindrical surfaces) and time.

radial profiles of skewness and kurtosis of the azimuthal velocity component for different Reynolds numbers. When the outer cylinder is at rest, the skewness is close to zero and the kurtosis close to three, confirming the nearly Gaussian shape. The small radius dependent deviations in skewness for large Re_S may result from remnants of turbulent Taylor vortices [21, 49, 74]. At μ_{max} the skewness increases from the inner cylinder wall to a maximum around $\tilde{r} = 0.16$, then decreases to negative values with a minimum around $\tilde{r} = 0.86$ for $\text{Re}_S = 2.14 \times 10^5$ and then increases again in the direction of the outer cylinder. Furthermore, the absolute skewness increases with the shear Reynolds number. This behavior is similar to the one of the temperature field in RB flows reported by Ref. [92]. The kurtosis profiles at μ_{max} depict two maxima, one in the inner gap region at $\tilde{r} = 0.22$ and one in the outer gap region at $\tilde{r} = 0.68$ for the highest Re_S . This reflects that the non-Gaussianity of the PDFs is most pronounced in the regions dominated by the vortex in- and outflows due to coherent plumes.

2.5 Angular momentum transport

In this section we analyze the angular momentum transport. The Nusselt number is composed of a convective and a viscous part [14]:

$$\text{Nu}_\omega = \text{Nu}_\omega^c(r) + \text{Nu}_\omega^\nu(r) = \frac{r^2}{J_{lam}} \langle u_\varphi u_r \rangle_{\varphi,z,t} - \frac{\nu r^3}{J_{lam}} \partial_r \left\langle \frac{u_\varphi}{r} \right\rangle_{\varphi,z,t}. \quad (2.4)$$

While the viscous term Nu_ω^ν dominates in the boundary layers, the convective term Nu_ω^c dominates in the bulk. Since the focus of our investigation is mainly in the bulk region, we neglect thus the viscous part. Furthermore, as it shown in Fig. 2.4b, the radial velocity component nearly vanishes when averaged over one vortex pair. Therefore only the velocity fluctuations of the azimuthal velocity component contribute to the net momentum flux [76]. Accordingly, the net convective flux in the bulk flow can be calculated as

$$\begin{aligned} \text{Nu}_\omega^{c,net} &= \frac{r^2}{J_{lam}} \langle u'_\varphi u'_r \rangle_{\varphi,z,t}, \quad \text{with} \\ u'_r(r, \varphi, z, t) &= u_r(r, \varphi, z, t) - \langle u_r(r, \varphi, z, t) \rangle_{\varphi,z,t}, \\ u'_\varphi(r, \varphi, z, t) &= u_\varphi(r, \varphi, z, t) - \langle u_\varphi(r, \varphi, z, t) \rangle_{\varphi,z,t}. \end{aligned} \quad (2.5)$$

To avoid confusion, in the following, we will call the Nusselt number $\text{Nu}_\omega = \text{Nu}_\omega^c + \text{Nu}_\omega^\nu$ shown in Eq. (2.4), the *total* Nusselt number. In Fig. 2.9a, we show the radial profiles of Nu_ω indicated by lines, and the net convective momentum flux $\text{Nu}_\omega^{c,net}$, indicated by diamonds (μ_{max}) and circles ($\mu = 0$). The analysis is restricted to $\tilde{r} \in [0.1, 0.9]$, where the profiles are nearly flat in good agreement with the physical property of the momentum transport to be conserved along $\tilde{r}$ according to Ref. [14]. The net convective momentum flux dominates the total Nusselt number and is valid to evaluate the momentum transport in the bulk region [20]. Furthermore, in Fig. 2.9b, we show the radially averaged total Nusselt number as a function of Re_S , which is also compared to both torque measurements of Ref. [55] ($\mu = 0, \eta = 0.724$) and DNS of Ref. [10] ($\mu = -0.4, \eta = 0.714$). We find a very good agreement with these data, which enables a more detailed analysis of the momentum transport.

In order to get deeper insight into the relation between the convective and net convective Nusselt number, we plot in Figs. 2.10a,b both local quantities in a meridional plane ($r - z$ plane) for $\text{Re}_S = 2.14 \times 10^4$ and $\mu = -0.36$. The local Nusselt number can be much larger than its average value as already noticed by Ref. [20] and Ref. [17]. In the case of the convective Nusselt number Nu_ω^c (Fig. 2.10a), the momentum transport in the area of the vortex outflow is positive and strongly concentrated to a small axial region, while in the area of the vortex inflow, the transport is negative and less focused. The magnitude

Figure 2.9: (a) Azimuthal and time averaged profiles of the total Nusselt number indicated by lines and of the net convective Nusselt number indicated by diamonds (μ_{max}) and circles ($\mu = 0$). The evaluation is restricted to the interval $0.1 \leq \tilde{r} \leq 0.9$, where the profiles show no pronounced dependence on $\tilde{r}$. Colors are chosen according to table 1. (b) Radially averaged total Nusselt number as function of Re_S . Errorbars represent the standard deviation of the total Nusselt number along the radial coordinate. Data are compared to torque measurements of Ref. [55] ($\mu = 0$, $\eta = 0.724$) and DNS of Ref. [10] ($\mu = -0.4$, $\eta = 0.714$)

Figure 2.10: Contour plot of the dimensionless (a) convective and (b) net convective angular momentum transport in a meridional plane for $Re_S = 2.14 \times 10^5$ and $\mu = -0.36$. Black lines indicate the zero line of the depicted quantities. The color codes are given in the legends. (c) Joint PDF of the radial and azimuthal velocity fluctuations for the same flow state at $\tilde{r} = 0.5$. The colors give the probability (in log-scale), see legend.

Figure 2.11: Radial profiles of the contribution of the net convective momentum transport of the (a) inflow and (b) outflow to the total transport. n_z represents the number of heights included into the average over cylindrical surfaces.

of positive momentum flux is twice as large as the negative one, which yields in average a positive net transport. Alternatively, when the net transport is plotted (Fig. 2.10b), a much clearer picture of the transport process is revealed. While in the in- and outflow region the net transport is positive, only in the sheared regions in between negative net transport can be detected. In order to explain this difference, in Fig. 2.10c we show the joint PDF of the radial and azimuthal velocity fluctuations at $\tilde{r} = 0.5$. In the inflow region, where fluid is transported strongly in the negative radial direction, the azimuthal velocity depicts a high probability for negative fluctuation values. As a consequence, the net transport has to be positive. In the outflow region, both velocities are mainly positive, resulting also in a positive correlation. However, when the mean azimuthal velocity component is not subtracted, the joint PDF is shifted to the right, leading to negative correlations in the inflow region (not shown). In summary, the difference between Nu_{ω}^c and $Nu_{\omega}^{c,net}$ is the transport of the mean azimuthal velocity by the turbulent Taylor vortices, which vanishes when averaged over cylindrical surfaces. By neglecting this fraction, a much clearer picture of the transport process is revealed. In addition, the representation of the Nusselt number in the meridional plane demonstrates the importance of the in- and outflow regions for the net convective transport.

The contribution of the vortex in- and outflow as function of the radial location and Re_S is depicted in Fig. 2.11. As a global feature, the contribution of the vortex inflow to the total net convective momentum transport is especially

pronounced in the outer gap region ($\tilde{r} > 0.5$) while the opposite is true for the outflow. For the two flow states in the classical regime, where the values of the total Nusselt numbers are comparable (see Fig. 2.9), the contributions of the in- and outflow are much more dominant at μ_{max} . This reflects the strong correlation of the enhanced momentum flux at μ_{max} and the turbulent Taylor vortices. Further, in the ultimate regime at $\mu = 0$, again the effect of remnants of these large vortices becomes visible in the slight dependence of the Nusselt number on $\tilde{r}$. When the ultimate regime is reached at $Re_S = 2.98 \times 10^4$ at μ_{max} , the contributions of the vortex in- and outflow are small and become significant again at $Re_S = 6.68 \times 10^4$. For even higher shear Reynolds numbers, the contribution of the inflow stays nearly constant with a fraction slightly above 30% in the outer gap region; while the outflow contribution continuously increases up to approximately 60% in the inner gap region. This value is strikingly high and demonstrates that at very high Reynolds numbers, the net convective transport shrinks to very small axial regions, where most of the momentum transport takes place.

2.6 Probability density function of the net convective angular momentum flux

With the knowledge of the total net convective transport and the relative contributions of the in- and outflow, we analyze now the PDFs of $Nu_\omega^{c,net}$. According to Fig. 2.6, the PDFs of the radial and azimuthal velocity components are nearly Gaussian for pure inner cylinder rotation. In that case, the PDF of their product can be described according to Ref. [96] and Ref. [97] based on a Gaussian distribution. The PDF of two jointly Gaussian random variables x and y is given by

$$P(x, y) = \frac{1}{\pi \sigma_x \sigma_y \sqrt{1 - \rho_P^2}} \exp \left[\frac{1}{2(1 - \rho_P^2)} \left(\frac{x^2}{\sigma_x^2} - \frac{2xy\rho_P}{\sigma_x \sigma_y} + \frac{y^2}{\sigma_y^2} \right) \right], \quad (2.6)$$

with the individual standard deviations σ_x and σ_y and the correlation coefficient $\rho_P = \langle xy \rangle / (\sigma_x \sigma_y)$. Eq. (2.6) represents an inclined elliptic shape for the joint PDF in contrast to the one shown in Fig. 2.10c. Furthermore, the PDF of the product $z = xy$ is [96, 97]

Figure 2.12: Correlation coefficient ρ_P of the radial and azimuthal velocity components calculated over space (cylindrical surfaces) and time as function of the radial coordinate. Note that the correlation coefficient related to Fig. 2.10c for $\text{Re}_S = 2.14 \times 10^5$ and $\mu = -0.36$ becomes $\rho_P = 0.502$ at $\tilde{r} = 0.5$.

$$P(z) = \frac{1}{\pi \sigma_x \sigma_y \sqrt{1 - \rho_P^2}} \exp \left(\frac{\rho_P z}{(1 - \rho_P^2) \sigma_x \sigma_y} \right) K_0 \left(\frac{|z|}{(1 - \rho_P^2) \sigma_x \sigma_y} \right), \quad (2.7)$$

with K_0 the modified Bessel function of the second kind. The prefactor of the exponential function in Eq. (2.7) are used to normalize the different PDFs of the angular momentum flux, i.e to standardize the width of their tails. It is worth mentioning that the correlation coefficient ρ_P cannot be normalized in a proper way, which leads to different slopes of the exponential tails of the prediction depending on ρ_P . Therefore, in our calculation, we use the prediction according to Eq. (2.7) for the case $\text{C8}_{\text{UR},0}$. In Fig. 2.12, ρ_P is depicted for all flow cases as a reference. The correlation between the azimuthal and radial velocity component in the ultimate regime decreases with the radial coordinate for μ_{max} , while a local minimum at $\tilde{r} \approx 0.6$ can be seen in case of $\mu = 0$. We observe that ρ_P in a flow with strong Taylor rolls, is much larger than in a flow without them. Note that ρ_P is used to normalize the width of the PDFs of the local Nusselt numbers.

In Fig. 2.13, we show the PDFs of $\text{Nu}_\omega^{c, \text{net}}$, which are normalized with the factor $\sigma_{u_r} \sigma_{u_\theta} \sqrt{1 - \rho_P^2}$ in order to compare their shapes, at $\tilde{r} = 0.5$. For pure inner cylinder rotation, the PDFs of the net convective momentum transport agree very well with the proposed jointly Gaussian prediction. However, at μ_{max}

Figure 2.13: Probability density functions of the local net convective angular momentum transport calculated over space (cylindrical surfaces) and time for $\tilde{r} = 0.5$. (a) PDFs of all investigated flow states are depicted. (b) PDF for $C6_{UR,0}$ with the corresponding PDF at the axial height of vortex inflow, vortex center and vortex outflow. (c) Same plot as in (b) for $C7_{UR,M}$. The dashed black line indicates the prediction according to Eq. (2.7).

the PDFs become highly skewed due to a change of shape in the positive tails. This deviation results in an increasing but still rare number of positive extreme events of momentum flux, which becomes more pronounced with increasing Re_S . According to Fig. 2.13c, these rare and extreme events can be almost completely attributed to the vortex in- and especially the vortex outflow. Here, a strong correlation between the radial and azimuthal velocity component exists due to the coherence of the plumes (see also Fig. 2.10c). In addition, all PDFs have in common that the highest probable value is zero instead of the average value of the Nusselt number. The overall shape of the PDFs at μ_{max} is in good agreement with the findings of Ref. [76] and comparable to the corresponding PDFs of the heat flux in RB flow. Ref. [83] reported similar PDF shapes in the case of RB convection. They argue that such large rare events are footprints of heat flux fluctuations induced by thermal plumes. This feature is obviously shared with TC flows and supports the origin of large scale turbulent vortices as the result of small scale unmixed plumes.

In Fig. 2.14, we show the PDFs of $Nu_{\omega}^{c,net}$ for various different radii r for the case of $Re_S = 2.14 \times 10^5$ at $\mu = -0.36$. The negative tails of the PDFs are largely identical for all investigated radial positions, while the right tails strongly depends on $\tilde{r}$. From the inner to the outer cylinder wall, the width of the positive tail and therefore the asymmetry decreases. As was shown in Fig. 2.11b, the maximum of the outflow contribution to the overall momentum transport is close to $\tilde{r} = 0.3$. At this location, the right tail of the PDF reflects extreme positive events due to the emission of plumes and is nearly covered by the PDF of the outflow (see Fig. 2.14b). However, in the outer gap region at $\tilde{r} = 0.7$, the asymmetric right tail of the PDF is comparably formed by both the in- and outflow region.

2.7 Azimuthal energy co-spectra and correlations

2.7.1 Azimuthal energy co-spectra

As shown in the previous section, the net convective Nusselt numbers suggest the presence of small scale plumes concentrated in the in- and outflow regions of the turbulent Taylor vortices, which dominate the transport. Hence, we analyse the spatial energy co-spectra to detect the presence of small scale

Figure 2.14: (a) Probability density functions of the local net convective angular momentum transport calculated over space (cylindrical surfaces) and time for $\text{Re}_S = 2.14 \times 10^5$ at $\mu = -0.36$ at different radial positions ($\text{C7}_{\text{UR,M}}$). (b,c) Same PDFs at $\tilde{r} = 0.3$ and $\tilde{r} = 0.7$, respectively with the corresponding PDFs related to the vortex inflow, vortex center, and vortex outflow.

structures. We assume velocity fluctuations at a constant radial r_c and axial z_c position in the homogeneous φ -direction with a total number of azimuthal points of $n_\varphi = 0, 1, \dots, N-1$ equidistantly spaced by $\Delta s = \Delta\varphi r_c$ i.e.

$$u_r^*(r_c, \varphi, z_c, t) = u_r(r_c, \varphi, z_c, t) - \langle u_r(r_c, \varphi, z_c, t) \rangle_t, \quad (2.8)$$

$$u_\varphi^*(r_c, \varphi, z_c, t) = u_\varphi(r_c, \varphi, z_c, t) - \langle u_\varphi(r_c, \varphi, z_c, t) \rangle_t. \quad (2.9)$$

The discrete spatial Fourier transform $U_{r,\varphi}$ of both fluctuation components $u_{r,\varphi}^*$ is given by

$$U_{r,\varphi}(n_\varphi) = \sum_{k=0}^{N-1} u_{r,\varphi}^*(k) \exp\left(-\frac{2\pi i k n_\varphi}{N}\right), \quad (2.10)$$

where for simplicity we have not written out the dependences on r_c , z_c and t . Thus, the spatial energy co-spectrum $E_{r\varphi}$ can be calculated as

Figure 2.15: (a) Temporally and azimuthally averaged azimuthal kinetic energy co-spectra evaluated at $\tilde{r} = 0.5$. Spectra are normalized to cover an area of 1 under their respective curves and by the gap width d . (b) Local scaling exponent γ of the co-spectra for $E_{r,\varphi} \sim k_\varphi^\gamma$ calculated with a bin size of $\log_{10}(k_\varphi) = 0.5$. Colors are chosen according to table 2.1.

$$E_{r\varphi}(k_\varphi^{n_\varphi}) = \begin{cases} \frac{1}{N^2} |U_r^{n_\varphi} \cdot U_\varphi^{n_\varphi}|, & \text{for } n_\varphi = [0, \frac{N}{2}] \\ \frac{1}{N^2} (|U_r^{n_\varphi} \cdot U_\varphi^{n_\varphi}| + |U_r^{N-n_\varphi} \cdot U_\varphi^{N-n_\varphi}|), & \text{for } n_\varphi = [1, \dots, \frac{N}{2} - 1], \end{cases} \quad (2.11)$$

with the wavenumber vector $k_\varphi^{n_\varphi} = (\Delta s)^{-1} n_\varphi / N$. The co-spectra are determined for each time step t and afterwards ensemble-averaged over 1500 snapshots for every case. To enable a comparison of the co-spectra for different Re_S and radial positions, we normalize all co-spectra with the area under its graph $A_E \approx (2\Delta s)^{-1} \sum_{n_\varphi=0}^{N/2-1} [E_{r\varphi}(k_\varphi^{n_\varphi}) + E_{r\varphi}(k_\varphi^{n_\varphi+1})]$ based on the trapezoidal integration method.

In Fig. 2.15a, we show the temporally and axially averaged azimuthal energy co-spectra at $\tilde{r} = 0.5$. The kinetic energy co-spectra show that most of the energy lies within the large scales, corresponding to small wavenumbers. The co-spectra depict a noticeable drop for $k_\varphi d \approx 20$. It is worth mentioning that we do not see any peak in the large scale regime, because of the limited range of the azimuthal coordinate in the experiments. Moreover, we cannot resolve the energy content of axisymmetric Taylor rolls, as they correspond to an azimuthal wavenumber of $k_\varphi = 0$.

In the case of Re_S being in the classical regime, the spectra show, compared to the other flow states, a stronger decrease of the spectral energy at mid-scales and a kink in the region of $10 \leq k_\varphi d \leq 20$. When Re_S is increased into the ultimate regime (with μ fixed at $\mu = \mu_{max}$), this kink continuously diminishes and energy is redistributed from the large to the small scales. Based on the power-law ansatz $E_{r,\varphi} \sim k_\varphi^\gamma$, the local scaling exponent γ corresponding to the co-spectra is shown in Fig. 2.15b. γ is calculated with a bin size of $\log_{10}(k_\varphi) = 0.5$. We observe that the spectra neither show -1 nor $-5/3$ scaling, which is consistent with the results of Ref. [55], Ref. [80] and Ref. [54]. In the ultimate regime (with μ fixed at $\mu = \mu_{max}$), the exponent decreases with increasing k_φ before it strongly drops down in the viscous regime beyond $k_\varphi d \approx 20$. This decrease becomes smaller with increasing Re_S . However, in the case of the highest investigated shear Reynolds number at $Re_S = 3.51 \times 10^5$ and $\mu = 0$, the local exponent is nearly constant for $k_\varphi d \leq 10$ with a value of $\gamma \approx -1.52$, which is slightly above $-5/3$.

Next, we focus on the local pre-multiplied energy co-spectra at the axial height of the vortex inflow, vortex center, and vortex outflow respectively. *Pre-multiplied* means that the co-spectra are multiplied with the wavenumber vector k_φ , such that the area under its graph corresponds to the kinetic energy [98]. In Fig. 2.16, we show the pre-multiplied energy co-spectra in the classical regime for $Re_S = 9.32 \times 10^3$ and μ_{max} at the three vortex positions. Firstly, we address the peak that corresponds to the large scales. For the vortex inflow in Fig. 2.16a, the large scale peak shifts its location from $k_\varphi d \approx 1.1$ (close to the OC at $\tilde{r} = 0.9$) to $k_\varphi d \approx 1.1$, independent of $\tilde{r}$ within the bulk. Close to the IC wall however, the peak shifts to $k_\varphi d \approx 2.2$ at $\tilde{r} = 0.1$. In Fig. 2.16c, at the location of the vortex outflow, the opposite behavior is observed. Here, the large scale peak shifts to smaller wavenumbers when the radial position increases from the IC to the OC. At the vortex center in Fig. 2.16b, the large scale peak is shifted from smaller wavenumbers in the center of the gap ($\tilde{r} = 0.5$) to larger ones at both cylinder walls, in an almost symmetric manner. This suggests that the formation of this large scale peak is connected to the mean radial velocity field, which is in itself caused by the large scale turbulent Taylor rolls. When fluid impacts on the cylinder walls due to these rolls, structures of the size $k_\varphi d \approx 2.2$ are formed. Since the axisymmetric flow has a wavenumber of $k_\varphi = 0$, the existence of a large scale peak may be an indication of modified turbulent Taylor vortices. This point will be discussed in more detail in section 2.8.

With respect to the small scales peak, we observe that it is located at wavenum-

Figure 2.16: Temporally averaged pre-multiplied azimuthal kinetic energy co-spectra in the classical regime for $\mu_{\max}$ (C2_{CR,M}) at different radial positions at the axial height of (a) vortex inflow, (b) vortex center and (c) vortex outflow. The region of $k_{\phi} d \in [10, 20]$ with enhanced plume emission is marked in light blue. Sketches above the co-spectra for the different vortex regions are added for clarity.

bers around $k_\varphi d \in [10, 20]$ for all three depicted heights, independent of the radial coordinate. However, its amplitude strongly varies with $\tilde{r}$ and the height z . In the region of the vortex inflow shown in Fig. 2.16a, the small scale peak is most pronounced near the OC and its amplitude decreases monotonically with decreasing $\tilde{r}$. On the contrary, at the height of the vortex outflow in Fig. 2.16c, the small scale peak amplitude is largest close to the IC and decreases in amplitude towards the OC. In the region of the vortex center in Fig. 2.16c, we find the highest amplitude of the small scale peak in the center of the gap. This is due to the emission of coherent plumes from the cylinder walls which give rise to the formation of Taylor rolls, as was already mentioned before. Thus, this peak should indeed be most pronounced in the ejecting regions, i.e. in the outflow region at the IC and in the inflow region at the OC, respectively. At the height of the vortex center —where no predominant flow direction concerning the radial velocity component is present— the behavior is different: Plumes rise from both cylinder walls and travel towards the bulk flow which results in the highest peak amplitude at $\tilde{r} = 0.5$.

The pre-multiplied energy co-spectra in the ultimate regime at $\text{Re}_S = 2.1 \times 10^5$ for the three vortex locations are depicted in Fig. 2.17 for $\mu = 0$ (a-c) and μ_{max} (d-f). For $\mu = 0$, no large scale peak can be seen in any of the co-spectra at the investigated heights. This is consistent with the finding shown in Fig. 2.3c, where we observed that the Taylor rolls have faded away. Accordingly, the shape of the co-spectra is less dependent on both the axial coordinate and on $\tilde{r}$. However, the co-spectra show a prominent change in the slope around $k_\varphi d \approx 20$: In Fig. 2.17a (vortex inflow), a small scale peak is formed around $k_\varphi d \in [10, 20]$ close to the OC at $\tilde{r} = 0.9$. This is comparable to the previous case in the classical regime. Also in the region of the vortex outflow (Fig. 2.17c), we observe a peak within the same range of scales at $\tilde{r} = 0.1$. For the vortex center however (Fig. 2.17b), no peak is visible.

Also when μ is changed to μ_{max} (see Figs. 2.17d,e), we identify a similar behavior as in the classical regime, although the energy is more homogeneously distributed over all scales. At the height of the vortex inflow as seen in Fig. 2.17d, a large scale peak is present which shifts to $k_\varphi d \approx 3.9$ at $\tilde{r} = 0.1$. At the vortex outflow in Fig. 2.17f, this shift appears close to the OC wall at $\tilde{r} = 0.9$ for the same wavenumber. In the region of the vortex center (Fig. 2.17e), the large scale peak is most pronounced at $\tilde{r} = 0.5$ at a smaller wavenumber. For all the heights explored, the co-spectra do not reveal a peak in the small scale regime. However, at the vortex inflow the energy is redistributed continuously from large to small scales when $\tilde{r}$ is varied from 0.1 to 0.9, as it is

Figure 2.17: Upper row: Temporally averaged pre-multiplied azimuthal kinetic energy co-spectra for C6_{UR,0} at different radial positions at the axial height of (a) vortex inflow, (b) vortex center and (c) vortex outflow. Region of $k_{\varphi} d \in [10, 20]$ is marked in light blue. Lower row: Same spectra for C7_{UR,M} at the axial height of (d) vortex inflow, (e) vortex center and (f) vortex outflow.

clearly visible in the marked regime of $k_\varphi d \in [10, 20]$. We also observe that the energy contained in the small scales increases in the vortex outflow region with decreasing radial position.

In summary, should prominent turbulent Taylor rolls exist in the flow, a peak at large scales exists and small scale structures are present throughout the gap, but most prominently in the vortex ejecting regions. However, when the Taylor rolls have faded, the peak at large scales vanishes and small scale structures are only detectable close to the cylinder walls. These findings reveal yet another evidence of the existence of turbulent non-axisymmetric Taylor rolls and support the idea that the large scale rolls are consisted of small scale unmixed plumes.

2.7.2 Spatial correlation coefficients

As it was shown in the previous section, the pre-multiplied energy co-spectra demonstrate the existence of small scale structures (coherent plumes) and large scale azimuthal structures, which are both connected thanks to the presence of turbulent Taylor vortices. For the sake of clarity, the investigation was confined to three specific cases. Within this section, however, we extend the analysis of the large scale peak based on the azimuthal two-point autocorrelation coefficient. The spatial two-point autocorrelation coefficient of the velocity fluctuations between two points separated by $\Delta\varphi$ in the azimuthal direction is given by

$$R_{rr}(r, z, \Delta\varphi) = \frac{\langle u_r^*(r, \varphi, z, t) u_r^*(r, \varphi + \Delta\varphi, z, t) \rangle_{\varphi, t}}{\langle u_r^{*2}(r, \varphi, z, t) \rangle_{\varphi, t}}, \quad (2.12)$$

$$R_{\varphi\varphi}(r, z, \Delta\varphi) = \frac{\langle u_\varphi^*(r, \varphi, z, t) u_\varphi^*(r, \varphi + \Delta\varphi, z, t) \rangle_{\varphi, t}}{\langle u_\varphi^{*2}(r, \varphi, z, t) \rangle_{\varphi, t}}. \quad (2.13)$$

In Fig. 2.18a, we show the spatial two-point autocorrelation function of u_r at $\tilde{r} = 0.1$ in the ultimate regime for three different flow cases at μ_{max} , and for both the vortex inflow and outflow. We observe that independently of the flow case, the autocorrelation function at the vortex inflow shows a minimum at $r\varphi/d \approx 0.13$, which indicates the existence of azimuthal structures of that size. In contrast, for the outflow, we observe a monotonically decrease of the autocorrelation function. Close to the outer cylinder wall (see Fig. 2.18b), the opposite behavior is observed with a pronounced minimum of the autocorrelation function for the vortex outflow at $r\varphi/d \approx 0.15$. This suggests that when

Figure 2.18: Upper row: Azimuthal two-point autocorrelation function of the fluctuation radial velocity component at the axial height of vortex in- and outflow at (a) $\tilde{r} = 0.1$ and (b) $\tilde{r} = 0.9$. Lower row: Azimuthal two-point autocorrelation function of the fluctuation (c) azimuthal and (d) radial velocity component at the axial height of vortex center at $\tilde{r} = 0.5$.

the large scale Taylor rolls transport fluid against the cylinder walls, azimuthal structures seem to be stimulated close to these walls in good agreement with the findings of the spectral analysis shown in section 2.7.1.

Next, in Figs. 2.18c,d, we show both the azimuthal and radial two-point autocorrelation function for the center of the vortex evaluated at $\tilde{r} = 0.5$. At this position, the large scale peak in the co-spectra is most pronounced as it was shown in Figs. 2.16 and 2.17. Here, we compare three flow states in the classical and ultimate regime where turbulent Taylor vortices are present and a case where they are absent ($C6_{UR,0}$). These figures reveal that in all vortex dominated cases, a large scale oscillating behavior is developed, while in absence of rolls, the auto correlation simply decreases towards zero over the whole azimuthal measurement length. We thus conclude, that the large scale peak in the spectra apparently results from a wavy azimuthal pattern connected to the Taylor rolls.

2.8 Complex Proper Orthogonal Decomposition (CPOD)

In order to reveal the flow structure connected to the large scale oscillation within the flow, we perform a Proper Orthogonal Decomposition (POD). The POD, also known as Empirical Orthogonal Function analysis (EOF), is a technique to extract modes like coherent structures from a flow field that contribute most to the energy of the flow [99]. It can be further used to identify the most relevant degrees of freedom in a dynamical system. For additional information we refer the reader to the review of Ref. [100]. As a classical POD mode cannot capture propagating structures, we perform a complex POD using the Hilbert transform (see Ref. [101]), where a mode is split into two patterns namely its real and its imaginary part with a phase difference of $\pi/2$, based on the algorithm described by Ref. [102]. We further use a combined analysis of both velocity components u_r and u_φ for the POD analysis. The applied algorithm is based on three steps [103]. At first we use the Hilbert transform, to make the velocity components complex. Secondly, the imaginary, transformed velocity fluctuations are arranged in a data matrix $\mathbf{D}$, where columns and rows are assigned to the spatial grid points and the time signals, respectively. At the end, a Singular Value Decomposition (SVD) of the data matrix is performed. Therefore, the velocity fluctuations at a specific axial location z can be reconstructed by

$$\mathbf{u}^*(r, \varphi, t) = \sum_{i=1}^N a_i(t) \mathbf{CPOD}_i(r, \varphi), \quad (2.14)$$

where N represents the number of snapshots and therefore the number of modes. The complex modes $\mathbf{CPOD}_i(r, \varphi)$ are the singular vectors of the data matrix and the time dependent coefficients $a_i(t)$ result from a projection of the complex modes onto the data matrix, i.e. $a_i = \mathbf{D}^T \mathbf{CPOD}_i$. $\mathbf{D}^T$ represents the transpose of the data matrix. The square of the singular values is a measure of the kinetic energy captured by the individual modes. Note that the eigenvalues are sorted in descending order together with the complex modes, meaning that the first mode represents a larger fraction of kinetic fluctuation energy of the full field than the second one and so forth. In the following we will present the CPOD results for two flow states at the height of the vortex center, where the azimuthal two-point-correlation showed a large scale oscillating behavior. The first case is in the classical regime at $\mu = 0$ and the second one in the ultimate regime at μ_{max} .

2.8.1 CPOD in the classical regime

The first CPOD mode for $Re_S = 9.32 \times 10^3$ and $\mu = 0$ is depicted in Fig. 2.19 for both velocity components. With the first temporal coefficient $a_1(t)$ and the first mode $\mathbf{CPOD}_1$, the corresponding velocity fluctuation field is reconstructed by $\mathbf{u}_1^*(r, \varphi, t) = a_1(t) \mathbf{CPOD}_1(r, \varphi)$. In Fig. 2.19a, the real part of $\mathbf{CPOD}_1$ representing the radial velocity component, shows nearly circular regions of positive and negative velocity, alternating in the azimuthal coordinate direction. This azimuthal wave pattern becomes azimuthally shifted in the corresponding imaginary part in Fig. 2.19c, indicating an azimuthally traveling wave. The real part of the azimuthal velocity of $\mathbf{CPOD}_1$ in Fig. 2.19b, depicts diagonal bands with pointy edges close to the cylinder walls. These regions again show alternating positive and negative velocities. The corresponding imaginary part in Fig. 2.19d is also azimuthally shifted, showing the same pattern. As a result, it is revealed that the velocity field consists of counter-rotating vortices in the horizontal plane, whose vorticity axes are co-axial to the rotation axis of the system. The pattern appears similar to that of the well known wavy Taylor vortices (WTV) at lower Reynolds number TC flow. Thus, here we show that in the classical turbulent regime, turbulent Taylor vortices can also feature azimuthal waves.

Figure 2.19: **CPOD**₁ for $\text{Re}_S = 9.32 \times 10^3$ and $\mu = 0$ at the axial height of the vortex center for both velocity components. (a) Real and (c) imaginary part of **CPOD**₁ for the radial velocity component. (b) Real and (d) imaginary part of **CPOD**₁ for the azimuthal velocity component. Black arrows represent the resulting velocity field of **CPOD**₁.

Figure 2.20: (a) Turbulent energy fraction captured by the CPOD modes for the case $Re_S = 9.32 \times 10^3$ and $\mu = 0$ at the axial height of the vortex center. Only the first 20 of the total 1500 modes are plotted. (b) Temporal power spectrum of the real part of the temporal coefficient $a_1(t)$ of **CPOD**₁ and of the radial velocity component of the full field at $\tilde{r} = 0.5$, averaged over φ . The frequency f is normalized by the difference of the cylinder frequencies $f_1 - f_2$.

In Fig. 2.20a, we show the energy fraction captured by the CPOD modes for $Re_S = 9.32 \times 10^3$ and $\mu = 0$. The first mode captures approximately 17% of the total energy fluctuation and we observe a strong drop to the second mode down to 6.7%. Accordingly, the first CPOD mode is dominant and is therefore the only one discussed. The temporal energy spectra of the real part of the first mode's temporal coefficient $a_1(t)$ is pictured in Fig. 2.20b, together with the temporal energy spectra of the full field radial velocity component, calculated at $\tilde{r} = 0.5$ and averaged over φ . The dominant frequency of **CPOD**₁ is $f / (f_1 - f_2) = 3.03$, identical to the one of the radial velocity component. Thus we can assume that the first mode captures the temporal behavior of the full field quite accurately. The frequencies f_1 and f_2 represent the rotation frequencies of the inner and outer cylinder, respectively.

To illustrate the spatio-temporal character of the first mode, we depict in Fig. 2.21a its reconstructed radial velocity component as well as the corresponding full field at $\tilde{r} = 0.5$ as a function of the azimuthal coordinate φ and time t . The space-time plot of **CPOD**₁ depicts diagonal bands of alternating positive and negative velocity, representing azimuthally propagating waves into the direction of the mean flow. Furthermore, the same diagonal

Figure 2.21: Space-time diagram of (a) the reconstructed radial velocity component based on the first CPOD mode and (b) the full field radial velocity component for $\text{Re}_S = 9.32 \times 10^3$ and $\mu = 0$ at the axial height of the vortex center as function of φ and t . The radial coordinate is fixed at $\tilde{r} = 0.5$.

bands -superimposed by turbulent fluctuations- are observed in the full field. A reappearance of azimuthal waves for pure inner cylinder rotation at a similar radius ratio of $\eta = 0.733$ has already been found by [88] in the Reynolds number regime of $20 \leq \text{Re}/\text{Re}_C \leq 38$. In our study in the classical regime for $\mu = 0$ at $\eta = 0.714$, we find $\text{Re}_S/\text{Re}_{S,C} \approx 99$, where the critical shear Reynolds number is located around $\text{Re}_{S,C} \approx 94.5$ [10]. This provides evidence of the reappearance of an azimuthal wave in the classical turbulent regime for a much larger turbulence intensity. Moreover, we can extract the corresponding wave frequency f_w , wavelength λ_w and phase speed c_w of **CPOD**₁. The temporal phase function ϕ_i is defined as

$$\phi_i(t) = \arctan \left(\frac{\Im(a_i(t))}{\Re(a_i(t))} \right). \quad (2.15)$$

Its temporal derivative is equal to the angular frequency of the CPOD mode [104], leading to a wave frequency of $f_{w,1} = \partial_t \phi_1(t)/(2\pi) = 1.80$ Hz. In addition, the azimuthal derivative of the spatial phase function $\Phi_i(x)$ is defined as

$$\Phi_i(r, \varphi) = \arctan \left(\frac{\Im(\text{CPOD}_i(r, \varphi))}{\Re(\text{CPOD}_i(r, \varphi))} \right), \quad (2.16)$$

which is then a measure for the local wavenumber $k_{\varphi,w}$. Note that the CPOD analysis yields only one temporal coefficient $a_i(t)$, but two spatial modes concerning the radial and azimuthal velocity components. Therefore we can extract two wavelengths, which are however nearly identical. The averaged wavelength, evaluated for $\tilde{r} = 0.5$, is given by $\lambda_{w,1} = 2\pi/(\partial_\varphi \Phi_1(\tilde{r} = 0.5, \varphi)) = 0.68$ rad, leading to an azimuthal wavenumber of around $k_{\varphi,w,1} = 2\pi/\lambda_{w,1} \approx 9$. In this way, the phase speed becomes $c_{w,1} = \lambda_{w,1} f_{w,1} = 1.22$ rad/s = $0.35\omega_1$. As the detected wave pattern reminds us of wavy Taylor vortices, we compare our result with the wave speeds for WTV measured by Ref. [105], although one should keep in mind the large Reynolds number difference. Ref. [105] find for the case of pure inner cylinder rotation at $\eta = 0.73$, $R/R_c \approx 14$, and an average axial wavelength of $\bar{\lambda}_w/d = 2.4$ a wave speed of $c_w \approx 0.2\omega_1$, while the corresponding pattern features $k_{\varphi,w} = 2$. Additionally, they show for a slightly larger radius ratio of $\eta = 0.84$, that the wave speed increases when $\text{Re}/\text{Re}_c > 18$. Thus, we can conclude that the wave speeds superimposed to the Taylor vortices in the wavy Taylor vortex regime and in the turbulent Taylor vortex regime are in the same order. Moreover, the wave speed seems to increase with the forcing Re_S , as the wave speed in our case is noticeable higher than the one reported by Ref. [105].

Figure 2.22: $\mathbf{CPOD}_1$ for $\text{Re}_S = 6.68 \times 10^4$ and $\mu = -0.36$ at the axial height of the vortex center for both velocity components. (a) Real and (c) imaginary part of $\mathbf{CPOD}_1$ for the radial velocity component. (b) Real and (d) imaginary part of $\mathbf{CPOD}_1$ for the azimuthal velocity component. Black arrows represent the resulting velocity field of $\mathbf{CPOD}_1$.

2.8.2 CPOD in the ultimate regime

We now turn our attention to the ultimate regime at the torque maximum rotation rate. In Figs. 2.22a-d, the first CPOD mode, representing the real and imaginary part of the radial and azimuthal velocity component, for $\text{Re}_S = 6.68 \times 10^4$ and $\mu = -0.36$ at the axial height of the vortex center is depicted. The flow in the ultimate regime features nearly the same pattern as already shown in 2.19 with alternating regions of positive and negative velocity. In case of the radial and azimuthal velocity component, these regions form circular patches and diagonal bands with pointy edges, respectively. In addition, the

Figure 2.23: (a) Turbulent energy fraction captured by the CPOD modes for the case $Re_S = 6.68 \times 10^4$ and $\mu = -0.36$ at the axial height of the vortex center. Only the first 20 of the total 1500 modes are plotted. (b) Temporal power spectrum of the real part of the temporal coefficient $a_1(t)$ of **CPOD**₁ and of the radial velocity component of the full field at $\tilde{r} = 0.5$, averaged over φ . The frequency f is normalized by the difference of the cylinder frequencies $f_1 - f_2$

patterns of the imaginary part are azimuthally shifted relative to the real parts.

The turbulent energy fraction in Fig. 2.23a shows again a strong drop between the first and second mode from approximately 12% to 5%. Further, the temporal power spectrum of the temporal coefficient $\Re(a_1(t))$ and of the full field in terms of u_r at $\tilde{r} = 0.5$ depict the same prominent peak at $f/(f_1 - f_2) = 0.92$ (see Fig. 2.23b). It is worth mentioning, that this peak is much more prominent than the one seen in the classical regime without the emergence of any secondary peak or higher harmonics.

The space-time diagram of the reconstructed radial velocity component based on **CPOD**₁ in Fig. 2.24a depicts the before seen diagonal bands of alternating velocity as result of azimuthally traveling waves. In comparison to **C1**_{CR,0}, the frequency of the wave has strongly increased, as visible by the higher number of bands within the same time interval. We find a wave frequency of $f_{w,1} = 3.67$ Hz, a wavelength of $\lambda_{w,1} = 0.74$ rad and a phase speed of $c_{w,1} = 2.72$ rad/s = $0.11\Delta\omega$. Remarkably, the turbulent Taylor vortices feature azimuthal waves also in the ultimate turbulent regime at μ_{max} . To the

Figure 2.24: Space-time diagram of the reconstructed radial velocity component based on the first CPOD mode for (a) $\text{Re}_S = 6.69 \times 10^4$ and $\mu = -0.36$ and (b) $\text{Re}_S = 2.15 \times 10^5$ and $\mu = 0$ at the axial height of the vortex center as function of φ and t . The radial coordinate is fixed at $\tilde{r} = 0.5$.

	η	Re_S	μ	$k_{\varphi,w,1}$	$c_{w,1}$ [rad/s]
Classical regime	0.714	9.32×10^3	0	≈ 9	$0.35\Delta\omega$
Ultimate regime	0.714	6.68×10^4	-0.36	≈ 9	$0.11\Delta\omega$
Ref. [105]	0.730	1.05×10^3	0	2	$0.21\Delta\omega$

Table 2.2: Overview of detected azimuthally traveling waves in the classical and ultimate regime in comparison to the findings of Ref. [105].

best of our knowledge, this is the first study reporting such a wave pattern in that highly turbulent regime. As additional test, we performed a CPOD analysis in the ultimate regime for $Re_S = 2.15 \times 10^5$ and $\mu = 0$, where the large Taylor rolls disappear in the mean field. The space-time diagram for the reconstructed radial velocity component connected to the first CPOD mode is depicted in Fig. 2.24b. The diagram shows horizontal bands with random size. Therefore, no traveling waves can be detected, when large scale Taylor rolls are absent in the very high ultimate regime at $\mu = 0$. As a comparison tool, we show in table 2.2 the results concerning the azimuthally traveling waves found in this study and the findings of Ref. [105] for $\eta = 0.73$.

To what extent the detection of azimuthal traveling waves is dependent on the turbulence level (Re_S) and the geometry of the system (η, Γ) is not known. We note that further experimental as well as numerical investigations would be required to address this issue and reveal whether it is a intrinsic property of the flow or not.

2.9 Summary & conclusions

By means of planar PIV measurements performed in horizontal planes at different axial heights, we showed the dependence of the small-scale statistics and flow organization on the presence of large scale Taylor rolls in high-Reynolds number TC flow. The ratio of angular velocities μ is the appropriate parameter to control whether or not the Taylor rolls within the gap are prominent and stable in a statistical sense. While in the classical turbulent regime Taylor rolls are observed for both $\mu = 0$ and $\mu < 0$, they fade out when the ultimate regime is reached for $\mu = 0$, but can be formed again when sufficient counter-rotation ($\mu < 0$) is introduced, in particular at μ_{max} where the flux of angular

momentum is maximum. This turbulent state can then be used to compare the flow—for the same Re_S —with and without the presence of these large scale rolls.

We showed that the PDFs of the radial and azimuthal velocity component are close to Gaussian, when evaluated at a fixed height, in accordance with the results of Ref. [54]. However, when all heights covering one vortex pair are included, the Gaussian shape is preserved for $\mu = 0$ for both velocity components, but changes drastically for μ_{max} . There, the PDFs of u_φ feature a cusp-like shape with exponential tails. It is also seen that the skewness and kurtosis varies strongly with the radial coordinate with a nearly identical behavior as for the temperature PDFs in RB flow [76, 92].

In order to better reveal the physical mechanism of the momentum transport in TC flow, we introduced the net convective Nusselt number (which is based on the correlation of the velocity fluctuations), which captures the main fraction of the transport in the bulk flow. We showed that the transport is dominated by the vortex inflow and especially outflow regions, where the radial and azimuthal velocity component are strongly correlated. While the contribution of the vortex inflow reaches a nearly fixed value in the ultimate regime of around $\approx 30\%$ at μ_{max} in the outer gap region, the contribution of the outflow increases monotonically with Re_S . Here we find a large value of the angular momentum transport of $\approx 60\%$ in the inner gap region at $Re_S = 2.14 \times 10^5$. We showed also that the corresponding PDFs of $Nu_\omega^{c,net}$ fluctuate around zero and not the averaged value of the Nusselt number and can be described (for $\mu = 0$) by a jointly Gaussian shape. At μ_{max} however, the right hand tail broadens with increasing Re_S , which can be attributed to the vortex in- and outflow dominance. We conclude that the resulting rare and strong events of momentum flux originate from the ejection of coherent plumes from the cylinder walls into the gap, which highlights the strong physical analogies between TC and RB flow [76, 82, 83].

A footprint of these coherent structures could be detected by studying the azimuthal energy co-spectra. We showed the appearance of a peak at small scales within the region $k_\varphi \in [10, 20]$, whose amplitude is most pronounced in regions where the large scale rolls transport fluid away from the wall, i.e. the so-called ejection regions [10]. In addition, the energy spectra show a peak in the region of large scales at a wavenumber larger than the one for axisymmetric Taylor rolls. This peak shifts to smaller scales, where the fluid transported by the rolls impacts on the cylinder walls; a process which seems to stimulate mid-scale azimuthal structures. Furthermore, the spatial correlation

function of the velocities clearly reveals that the peak at large scales features an oscillating behavior.

In order to finally capture the underlying flow structure, we performed a complex POD analysis at the height of the vortex center. Here we showed that large scale azimuthally traveling waves superimposed on the turbulent Taylor rolls are present in the flow, not only in the classical regime at $\mu = 0$, but also in the ultimate regime at μ_{max} . These waves propagate into the direction of the mean flow and are similar to the well-known wavy Taylor vortices. While the wave speeds of $0.35\Delta\omega$ in the classical, and $0.11\Delta\omega$ in the ultimate regime are in the same order as the one for the wavy Taylor vortices measured by Ref. [105], the azimuthal wave number found of $k_{\varphi,w,1} = 9$ is much higher than in the laminar regime.

These findings reveal the intrinsic statistical relation between structures of different scales: large-scale Taylor rolls and small-scale plumes in high-Reynolds number TC flow. These relations strongly rely on the specific locations along the vortex (inflow, center or outflow) and since they play a prominent role for the flow organization and the momentum transport, our study underlines the importance of an axial exploration when studying the statistics of turbulent TC flows.

We finally note that we believe that our results are much more general than only holding for turbulent TC flow. Clearly, they will generalize to turbulent RB flow with its organization on very large scales [106, 107], but also to pipe and channel flow with highly organized structures in spanwise direction [98, 108, 109]. What mechanism however sets the lengthscale of the organisation of such superstructures in spanwise direction remains unclear.

3

Controlling the secondary flows in turbulent Taylor-Couette flow using spanwise varying roughness: experiments and simulations[°]

Highly turbulent Taylor-Couette flow with spanwise-varying roughness is investigated experimentally and numerically to determine the effects of the normalized axial size $\tilde{s}$ of roughness patches on the total drag on the inner cylinder and the local flow structures. We apply sandgrain roughness, in the form of alternating bands to the inner cylinder. Numerically, the Taylor number is $\mathcal{O}(10^9)$ and $\tilde{s}$ is varied such that $0.47 \leq \tilde{s} \leq 1.24$ and is simulated using a direct numerical simulation in conjunction with an immersed boundary method. Experimentally, we explore $Ta = \mathcal{O}(10^{12})$ and $0.61 \leq \tilde{s} \leq 3.74$ in the Twente Turbulent Taylor-Couette facility (T³C). For both approaches the radius ratio is fixed at $\eta = 0.716$. We present how the flow scales with Ta and how it depends on the boundary conditions set by $\tilde{s}$. Both numerically and experimentally, we find a maximum in the angular momentum transport when $\tilde{s}$ is varied. We attribute this to the re-arrangement of the large-scale structures triggered by the presence of the rough patches, which yields effectively, tuned turbulent vortices with different axial wavelengths. We describe how the local flow rearranges for varying $\tilde{s}$ and how these local effects are reflected in the global response of the system.

[°]In preparation: Dennis Bakhuis, **Rodrigo Ezeta**, Pieter Berghout, Pim. A. Bullee, Dominic Tai, Daniel Chung, Sander G. Huisman, Roberto Verzicco, Detlef Lohse and Chao Sun. Experiments are done by Bakhuis, Ezeta and Bullee. Numerical simulations are done by Berghout. Analysis of PIV and LDA is done by Ezeta. Analysis of the torque and LDA is done by Bakhuis. Writing is done by Bakhuis, Ezeta, Berghout, Bullee and Huisman. Supervision by Chung, Huisman, Verzicco, Lohse and Sun. Proofread by everyone.

3.1 Introduction

Many turbulent flows are bounded by irregular, rough, boundaries. These flows are extensively studied, under the approximation that the roughness is homogeneous [110]. Homogeneously rough surfaces have a characteristic length scale k that is much smaller than the largest wall normal length scale δ . The effects of the roughness in these flows is believed to be confined to the immediate vicinity of the wall (i.e. the roughness sublayer), whereas in the outer, inertial, layer, the flow only experiences the effective shear stress of the surface, i.e. Townsend's outer layer similarity [111]. As such, the focus of many studies was to find functional relationships between the parameters that describe both the roughness geometry and the skin friction coefficient C_f [112]. In practice, however, flows are bounded by rough boundaries that not only vary on the scale of k , but also on a much larger scale s , where $s = O(\delta)$. Whereas these variations can be either laterally (spanwise) or longitudinally (streamwise), we focus here on the former. Examples of these flows are found in shipping (i.e. the formation of patches of biofouling on ship hulls [113]) and geophysical flows (e.g. the atmospheric flows over spanwise-varying terrain [114]).

Hitherto, the research is focused on the effects of spanwise-varying rough surfaces on canonical systems of wall-bounded turbulence research, i.e. pipe flow [115], boundary layer flow [116] and channel flow [117]. The hallmark of flows over these surfaces is the presence of spanwise wall-normal secondary flows of the size $O(\delta)$, with mean streamwise vorticity. Examples of studies where this has been observed are the works of Ref. [115] on the effects of convergent and divergent grooves (reminiscent of shark skin) and the work by [118] on spanwise-varying riverbeds. We note that earlier research dates back to the works of Ref. [119, 120] on the field of surface stress variations in duct flows.

Following up on the work of [115], [121] set out to perform a parametric study of the converging-diverging riblets surface in a zero pressure gradient BL. They find a thickening of the BL height above the converging regions, and a thinning of the BL height above the diverging regions. Furthermore, the energy spectra shows an increased energy content of the larger scales. [122] performed stereo particle image velocimetry (PIV) in the spanwise wall-normal plane of the flow over a turbine blade replica and found spanwise variations of the order of δ in the mean velocity field. With the same configuration, Ref. [123] identified

regions of low momentum pathways (LMPs) and high momentum pathways (HMPs) in the instantaneous fields. Here, LMPs coincide with regions of enhanced turbulent kinetic energy (TKE) and Reynolds shear stress (RSS), and rather remarkably, these regions do seem to occur at recessed roughness heights. [124] found very similar behavior of the secondary flows for a much more regular surface geometry. [125] found that only when $s/\delta \gtrapprox 0.5$, where s is the spacing between the streamwise aligned Lego[®] blocks, secondary flow formation is observed. However, for $s/\delta \lesssim 0.5$ the secondary flows are confined to the roughness sublayer. Interestingly, contrary to the findings of Ref. [123], they find LMPs on top of their elevated blocks, and HMPs in between the roughness strips. [126], however, found $s/H \gtrapprox 0.2$, with H the channel half height, as the threshold for heterogeneous behavior of the streamwise aligned pyramid elements. By carefully assessing the terms in the transport equation of TKE, [116] found that spanwise variations of roughness leads to a local imbalance of production and dissipation of TKE, as already proposed by [119]. Since the secondary flows are driven by a spatial gradient in the RSS, they find that the mean secondary flows are Prandtl's secondary flow of the second kind [127]. [128] observed a breakdown of outer layer similarity in the local profiles of the mean flow, turbulent intensity, and the energy spectra, evidently induced by the presence of the secondary vortices. Finally, [117] studied the influence of the spacing of idealized (i.e. no geometric induced disturbances to the flow) regions of low shear stress and high shear stress. They find that for $s/\delta \lesssim 0.39$ the notion of outer layer similarity is retained. Interestingly, for $s/\delta \gtrapprox 6.28$, they find a sign reversal of the isovels (stream velocity contour lines), with respect to the orientation of the secondary flows, that remain upwelling over low shear stress regions.

The aforementioned studies were all carried out in systems that lack two characteristics which are intrinsic to many applications, namely the curvature in the streamwise direction (as in turbine blades), and the presence of strong secondary motions (as in the atmospheric boundary layer). A canonical system in which these two properties can be simultaneously observed is the Taylor–Couette (TC) flow. TC flow is the flow in between two coaxially, independently rotating cylinders. Its geometry is characterized by the inner cylinder radius r_i , outer cylinder radius r_o , and the height of the cylinders L , captured by two dimensionless parameters; the radius ratio $\eta = r_i/r_o$ and the aspect ratio $\Gamma = L/d$, where $d = r_o - r_i$ is the gap in between the cylinders. Since TC is a closed system, one can directly relate global and local quantities through exact mathematical relations [14]. The driving in TC flow is expressed in

dimensionless form by the Taylor number:

$$\text{Ta} = \frac{1}{4} \sigma d^2 \frac{(r_i + r_o)^2 (\omega_i - \omega_o)^2}{\nu^2}, \quad (3.1)$$

where $\omega_{i,o}$ are the inner and outer angular velocity of the cylinders respectively, ν is the kinematic viscosity, and $\sigma = ((1 + \eta) / (2\sqrt{\eta}))^4$ is the so-called geometric Prandtl number, in analogy to the Prandtl number in Rayleigh-Bé convection [14]. Alternatively, when the outer cylinder is at rest ($\omega_o = 0$), the driving can also be expressed with a Reynolds number based on the outer scales $\text{Re}_i = r_i \omega_i d / \nu$. This Reynolds number and Ta (for $\omega_o = 0$), are related by $\text{Re}_i = (8\eta^2 / (1 + \eta)^3) \sqrt{\text{Ta}}$. In TC flow, the angular velocity flux J^ω is radially conserved. Here, $J^\omega = r^3 (\langle u_r \omega \rangle_{A,t} - \nu \frac{\partial}{\partial r} \langle \omega \rangle_{A,t})$, where the brackets $\langle \cdot \rangle_{A,t}$ denote averaging over a cylindrical surface and time. The angular momentum flux for the case of laminar flow is $J_{lam}^\omega = 2\nu r_i^2 r_o^2 (\omega_i - \omega_o) / (r_o^2 - r_i^2)$. In this way the response of the flow is quantified with the dimensionless Nusselt number (Nu_ω), which is also directly related to the torque $\mathcal{T}$ that is required to drive the cylinders at constant speed, i.e.

$$\text{Nu}_\omega = \frac{J^\omega}{J_{lam}^\omega} = \frac{\mathcal{T}}{2\pi L \rho J_{lam}^\omega}. \quad (3.2)$$

Alternatively, the torque of the system can be non-dimensionalized to form the friction coefficient $C_f = \mathcal{T} / (\rho L \nu^2 \text{Re}_i^2)$, which is directly related to the Nusselt number:

$$\text{Nu}_\omega = C_f \omega_i (r_o - r_i)^2 (r_o^2 - r_i^2) / (4\pi \nu r_o^2). \quad (3.3)$$

The inner friction velocity $u_{\tau,i}$ is also related to the torque by $u_{\tau,i} = \sqrt{\mathcal{T} / (2\pi r_i^2 \rho L)}$, which is used to non-dimensionalize quantities in the inner layer. Lastly, a frictional Reynolds number based on the inner scales can be defined as $\text{Re}_\tau = u_{\tau,i} d / (2\nu)$.

The scaling of the response of the flow with the driving has been extensively studied in the literature [11, 18, 19, 22, 55, 74, 77]. In the so-called ultimate regime of turbulence [15, 16], where the boundary layers are fully turbulent ($\text{Ta} > \mathcal{O}(10^8)$), it was shown that effectively $\text{Nu}_\omega \propto \text{Ta}^{0.4}$, independently of both η and the rotation ratio of the cylinders $a = -\omega_o / \omega_i$ [17, 20].

Secondary flows are featured in TC flow, in the form of large scale vortices with a mean streamwise vorticity component, the so-called turbulent Taylor Vortices (TTV). These structures are reminiscent of laminar Taylor vortices,

which transition through a series of instabilities into turbulence once the flow becomes predominantly unstable after a critical value of the Reynolds number [12]. As noted by Ref. [31], the axial wavelength λ of the TTVs, *i.e.* the distance between two rolls, is primarily a function of η and Re . When Re is large ($\mathcal{O}(10^6)$), the rolls are observed to persist in the system [21]. Here, multiple states for $\eta = 0.716$ can be observed in a certain regime of counter-rotating cylinders, namely $a \in [0.17, 0.51]$, where $a = -\omega_o/\omega_i$ is the rotation ratio of the cylinders. These multiple states are characterized by a change in the number of rolls present in the system, and as a consequence, in their averaged axial wavelength ($\lambda/d = 1.46$ or $\lambda/d = 1.96$). These states—being strongly hysteretic—result in different torques for the same rotation rates, which reflects the importance of the large scale structures (TTV) in transporting angular momentum. At pure inner cylinder rotation however ($a = 0$), no multiple states are detected and the rolls are observed to be less coherent and stable. Finally, we note that the effect of the curvature of the cylinders is quantified with the radius ratio η , and it has a tremendous impact on the flow organization as it was thoroughly reported by Ref. [10, 90]. For a more detailed review we refer the reader to Ref. [9, 11].

It is not the first time that roughness is studied in a TC geometry. Ref. [129, 130] used obstacle roughness, in the form of axial riblets, to study the scaling of the angular momentum transport with the driving. Ref. [131] investigated the influence of grooves for large Ta ($\mathcal{O}(10^{10})$), and find that at the tips of the grooves, plumes are preferentially ejected. In a more recent work, Ref. [132] find that by using a similar configuration of rough walls as Ref. [130], the scaling that corresponds to the ultimate regime, originally predicted by Kraichnan *i.e.* $\text{Nu}_\omega \propto \text{Ta}^{1/2}$ [15], can be achieved. They attribute this to a dominance of the pressure drag over the viscous drag on the cylinders. Structure in the form of grooves in the streamwise direction were studied by Very recently, [133] studied the influence of sandgrain roughness in TC flow, and found similarity of the roughness function with the same type of roughness in pipe flow [134]. We highlight that none of the works described above, reported an influence of the roughness over the axial wavelength of the rolls.

In this chapter we study the effects of spanwise-varying roughness in highly turbulent TC flow $\mathcal{O}(10^{12})$, where the effect of curvature is present due to the cylinders, and for the case of pure inner cylinder rotation $a = 0$, where secondary flows are present in the form of TTVs. In particular, we focus on the effect of spanwise-varying roughness on the TTVs and thus, on the

global and local response of the flow. We introduce the roughness through a series of patches which extend along the entire circumference of the inner cylinder (IC). This gives rise to a spanwise (axial) arrangement of roughness which we characterize with the width of the rough patch s . We conduct both experiments and direct numerical simulations (DNS) for various $\tilde{s} = s/d$, i.e. the width of the rough patch normalized with the gap width.

The structure of the chapter is as follows. In Sec. 3.2 we introduce the experimental and numerical methods. In Sec. 3.3.1 we show the local response of the flow due to the varying roughness width. In Sec. 3.3.2, we study its effect on the global quantities. In Sec. 3.3.3 we link the global and local observations and provide the physical mechanism between the interaction of the rolls and the roughness. We finalize the chapter in Sec. 3.4 with some conclusions and future work.

3.2 Methods

3.2.1 Experimental apparatus with spanwise roughness

The experiments were performed in the Twente Turbulent Taylor-Couette (T³C) facility as shown in Fig. 3.1a (details of the experimental facility can be found in Ref. [18]). The inner cylinder has a radius $r_i = 200$ mm and the outer cylinder has a radius $r_o = 279.4$ mm, such that the gap size is $d = r_o - r_i = 79.4$ mm, and the radius ratio $\eta = 0.7146$. The length of the cylinders is $L = 927$ mm, which leads to an aspect ratio $\Gamma = L/d = 11.7$. The outer cylinder (OC), is made from transparent acrylic which allows for optical access to the flow. The working fluid is demineralised water. We apply axially varying roughness to the inner cylinder (IC), which leads to patterns of uniformly rough and hydrodynamically smooth bands in the spanwise direction (see Fig. 3.1a). The rough patches are made of P36 ceramic industrial grade sandpaper and are fixed to the IC using double-sided adhesive tape. In Fig. 3.2, we show the height scan of a roughness element using confocal microscopy. The scan revealed that the height (h_r) of the roughness is mostly within $\pm 2\sigma(h_r)$ of the mean, giving a characteristic length scale $k \equiv 4\sigma(h_r) = 695 \mu\text{m}$, see Fig. 3.2b. More statistics of the roughness is shown in table 3.1. We fix the roughness coverage at 56% such that $0.56A_i$ of the cylinder is rough, where

Figure 3.1: a) Schematic of the Twente Turbulent Taylor–Couette showing the sandpaper on the inner cylinder in red. PIV measurements in the r – θ plane are illuminated from the side using a high-power laser creating a horizontal sheet. The sheet is imaged through a window in the bottom. Using LDA the azimuthal velocity is measured along the axial direction. The torque is measured in the middle section of the IC, which has a length of $L_{mid} = 536$ mm. b) Numerical domain for the case of $\tilde{s} \equiv s/d = 0.47$, sandpaper roughness taken from the scan of the material used in the experiment, see Fig. 3.2.

$A_i = 2\pi r_i L$ is the area of the entire IC. In addition, the torque is measured only in the middle section of the IC with a coverage of also 56%.

Global measurements: Torque

We measured the torque, $\mathcal{T}$, that is required to drive the inner cylinder at constant angular velocity (the outer cylinder is kept at rest). For this we use a hollow flange reaction torque transducer connecting the driving shaft and the inner cylinder. We continuously measure the torque while quasi-statically ramping the frequency of the inner cylinder, f_i , from 5 Hz to 18 Hz. This corresponds to $Ta \approx 4 \times 10^{11}$ and $Ta \approx 6 \times 10^{12}$. All the experiments are performed at 21 ± 1 °C and all quantities are calculated using the actual measured temperature. Table 3.2 shows additional experimental parameters.

Figure 3.2: a) Height scan captured using confocal microscopy of a roughness patch of $20\text{ mm} \times 20\text{ mm}$ with a resolution of $2.5\text{ }\mu\text{m}$. The typical size of the grains is given by $k \equiv 4\sigma(h_r) = 695\text{ }\mu\text{m}$ where h_r is the height and σ the standard deviation. The normalized typical grain size is then $k/d \approx 0.01$. b) Probability density function (PDF) of the measured height of the roughness patch, with subtracted mean $h'_r = h_r - \langle h_r \rangle$.

Metric	Value
$\sigma(h_r) = \sqrt{\langle h_r'^2 \rangle}$	$174\text{ }\mu\text{m}$
$\langle h'_r \rangle$	$134\text{ }\mu\text{m}$
$\min(h'_r)$	$-527\text{ }\mu\text{m}$
$\max(h'_r)$	$738\text{ }\mu\text{m}$
$\text{median}(h'_r)$	$-19.6\text{ }\mu\text{m}$
$\text{mode}(h'_r)$	$-27\text{ }\mu\text{m}$
$\text{IQR} = Q3 - Q1 = \text{CDF}^{-1}(0.75) - \text{CDF}^{-1}(0.25)$	$215\text{ }\mu\text{m}$
$\langle h_r'^3 \rangle / \langle h_r'^2 \rangle^{3/2}$	0.928
$\langle h_r'^4 \rangle / \langle h_r'^2 \rangle^2$	4.361
wetted area/flat area	≈ 1.6

Table 3.1: Various statistics of the roughness $h'_r = h_r - \langle h_r \rangle$ based on the data obtained from confocal microscopy, see also figure 3.2.

Local measurements: LDA and PIV

We performed an axial scan of the azimuthal velocity with laser Doppler anemometry (LDA). The scan was performed at the middle of the gap, $\tilde{r} = (r - r_i)/d = 0.5$, at a fixed $Ta = 9.5 \times 10^{11}$. The flow was seeded using $5\text{ }\mu\text{m}$ -diameter polyamide particles with density of 1030 kg m^{-3} that act as tracers [8]. The laser beam goes through the outer cylinder and is focused in the middle of the gap. We correct for curvature effects by numerically ray tracing the LDA beams as it was shown in Ref. [20]. The axial extent of the LDA scans is $0 \leq z/L \leq 0.5$. Particle image velocimetry (PIV) measurements were performed at $Ta = 9.5 \times 10^{11}$ (same as LDA) in the radial-azimuthal plane. The scan is done for different heights and for different $\tilde{s}$. The working fluid is seeded with PPMA fluorescent particles (**Dantec FPP-RhB-10**) with diameters of $1\text{--}20\text{ }\mu\text{m}$ with a seeding density of ≈ 0.01 particles/pixel. These particles have an emission peak at $\approx 565\text{ nm}$. We illuminate the particles with a **Quantel Evergreen 145 532 nm**, double pulsed laser. A cylindrical lens is used to create a light sheet of $\approx 1\text{ mm}$ thickness. The images are captured with an **Imager SCMOS** (2560×2160 pixel) 16 bit camera with a **Carl Zeiss 2.0/100** lens. The camera is operated in double frame mode with a frame rate f which is always smaller than the interframe time $1/\Delta t$, i.e. $\Delta t \ll 1/f$. In order to enhance the particle contrast in the images, we add a **Edmund High-Performance Longpass 550 nm** filter to the camera lens. For every $\tilde{s}$, the axial extent of the experiments is different. This is done because—as will be shown later—the aspect ratio of the rolls change depending on $\tilde{s}$. For the smallest $\tilde{s} = 0.63$ however, the axial resolution is $\delta z/L \approx 0.011$ while for the largest value $\tilde{s} = 3.74$, $\delta z/L \approx 0.022$. Since we scan in the axial direction, the focus of the camera is changed accordingly. The fields are resolved with a commercial PIV software (**Davis 8.0**) based on a multi-step method. The initial window size is set to 64×64 pixels and it decreases to 32×32 pixels for the last iteration. The fields are calculated in cartesian coordinates, which we transform to polar coordinates. The final result is the fields in the form $\vec{u} = u_r(r, \theta, t)\hat{e}_r + u_\theta(r, \theta, t)\hat{e}_\theta$, where u_r and u_θ are the radial and azimuthal velocity component which depend on the radius r , the azimuthal (streamwise) direction θ and time t .

3.2.2 Numerical methods

The Navier-Stokes (NS) equations are spatially discretized by using a central second-order finite-difference scheme and solved in cylindrical coordinates by means of a semi-implicit procedure [135, 136]. The staggered grid is homogeneous in both the spanwise and streamwise directions (the axial and azimuthal directions, respectively). The wall-normal grid consists of double cosine (Chebychev-type) grid stretching. Below the maximum roughness height, we employ a cosine stretching such that the maximum grid spacing is always smaller than 0.5 times the viscous length scale $\delta_\nu = \nu/u_\tau$. In the bulk of the fluid, we employ a second stretching, such that the maximum radial grid spacing in the bulk is $\approx 1.7\delta_\nu$. The minimum radial grid spacing is $\approx 0.33\delta_\nu$, and thus is located at the position of the maximum roughness height, where we expect the highest shear stress. In table 3.2, we show a summary of the relevant parameters in the simulations. Time advancement is performed by using a fractional-step third-order Runge–Kutta scheme in combination with a Crank–Nicolson scheme for the implicit terms. The Courant–Friedrichs–Lewy (CFL) $(u\Delta t)/(\Delta x) < 0.8$ time-step constraint for the non-linear terms is enforced to ensure stability. We scale the roughness patch such that the maximum roughness height, and thus the maximum blockage ratio, is $\max(h_r) = 0.1d$. Depending on $\tilde{s}$, we cut out a portion of roughness from the scanned surface. The roughness is then mirrored and pasted together to obtain a smooth, streamwise periodic, stripe. Note that we do not mirror the surface in the spanwise direction. The streamwise and spanwise lengths of the computational domain are set to match the minimum computational domain size as studied in [17]. A moving average over 10×10 points is employed to smooth the scan from measurement noise. Finally, we set the resolution based on the demands $(\Delta z^+, r_i \Delta \theta^+ < 3)$, which is small enough to recover the smallest geometrical features of the surface. The sandpaper roughness is implemented in the code by an immersed boundary method (IBM) [137]. In the IBM, the boundary conditions are enforced by adding a body force $\mathbf{f}$ to the NS equations. A regular, non-body fitting, mesh can thus be used, even though the rough boundary has a very complex geometry. We perform interpolation in the spatial direction preferential to the normal surface vector to transfer the boundary conditions to the momentum equations. The IBM has been validated previously [65, 131, 132, 137–139].

$\tilde{s}$	$N_\theta \times N_z \times N_r$	Ta $\times 10^9$	Γ	Re $_\tau$	C_f	Nu $_\omega$	$\Delta(\omega)^+$	$\Delta r_{\min}^+$	$\Delta r_{\max}^+$
Simulations									
smooth	$758 \times 600 \times 840$	2.39	2.08	697	0.049	30.1	—	0.28	2.44
uniformly rough	$1324 \times 1012 \times 1200$	1.19	2.00	689	0.095	41.5	8.11	0.33	1.76
0.47	$1324 \times 1275 \times 1200$	1.33	2.52	686	0.084	38.9	6.36	0.33	1.76
0.62	$1324 \times 1121 \times 1200$	1.33	2.22	690	0.085	39.4	6.78	0.33	1.77
0.93	$1324 \times 1682 \times 1200$	1.45	3.32	692	0.079	38.1	6.21	0.33	1.77
1.24	$1324 \times 1121 \times 1200$	1.37	2.22	685	0.082	38.3	6.20	0.33	1.75
Experiments									
smooth		$\times 10^{12}$		$\times 10^3$					
0.61		1.00	11.7	10.1	0.024	307			
0.93		1.00	11.7	13.1	0.041	509			
1.23		1.00	11.7	13.5	0.043	534			
1.87		1.00	11.7	13.3	0.041	508			
3.74		1.00	11.7	12.9	0.039	491			
		1.00	11.7	14.4	0.049	517			

Table 3.2: List of parameters involved in both the simulations and the experiments. $\tilde{s} = s/d$ is the normalized roughness width. $N_\theta \times N_z \times N_r$ is the numerical resolution in the azimuthal, axial, and radial direction, respectively. $\Delta(\omega)^+$ is the downward shift of the angular velocity profile ω^+ . $\Delta r_{\min}^+$ is the minimum spacing in the wall normal direction at the location of the maximum roughness height. $\Delta r_{\max}^+$ is the maximum spacing in the wall normal direction. $r_i^+ \Delta\theta = \Delta z^+ \approx 2.7$ ($r_o^+ \Delta\theta \approx 3.8$) is the grid spacing in the streamwise and spanwise directions. In the DNS, the roughness height $k^+ = 4\sigma(h_r)^+ = 130 \pm 1$ for all rough cases. The roughness is glued to the surface and thus protruding, such that $r_k > r_i$, with r_k the radial coordinate of the rough surface patches and r_i the radial position of the smooth patches.

Figure 3.3: Standard deviation of the azimuthal velocity $\sigma(u_\theta)$, normalized by the inner cylinder azimuthal velocity u_i , as a function of $\tilde{z} = z/d$ for various $\tilde{s}$. $Ta = 1 \times 10^{12}$ for all experiments. The enforced roughness pattern is indicated in red and a light blue shade. The signature of the roughness pattern is clearly visible at mid-gap in the bulk flow. For $\tilde{s} = 0.61$, the roughness pattern shows does not leave a distinct imprint of its topology in the midgap flow statistics.

3.3 Results

3.3.1 Response of the Turbulent Taylor Vortices

In order to get a first insight on the effect of the roughness on the flow, we performed axial scans of the azimuthal flow velocity at midgap using LDA. Subsequently, we calculated the standard deviation of the azimuthal velocity. In Fig. 3.3, we show the standard deviation of the azimuthal velocity $\sigma(u_\theta)$ normalized with the inner cylinder velocity u_i , as a function of the height, for various $\tilde{s}$. Here, the axial coordinate is normalized using the height of the cylinders such that $\tilde{z} = z/d$. The standard deviation is a measure of the magnitude of the velocity fluctuations and therefore, we can now quantify the effect of the applied roughness. Fig. 3.3 reveals that for the case of the largest patch size ($\tilde{s} = 3.74$), the smooth section has, on average, a value of $\sigma(u_\theta)/u_i \approx 0.03$, slightly larger than $\sigma(u_\theta)/u_i \approx 0.023$ that is found for the smooth case (for $Ta = 1.5 \times 10^{12}$) in the same apparatus [8]. Above the rough

section, towards the center of the setup (i.e. for large $\tilde{z}$), $\sigma(u_\theta)$ gradually increases to a value of approximately $\sigma(u_\theta)/u_i \approx 0.04$. A similar, but not so clear trend can be seen at the lower roughness section ($\tilde{z} \approx 0.93$) of this case. However, this might be influenced by the lower bottom plate of the system. When looking at the $\tilde{s} = 1.87$ case, we see very similar, however more pronounced dynamics. Azimuthal velocity fluctuations are promoted in regions where the roughness is present, as suggested by the appearance of local peaks centered at the position of the rough patches. This effect is further seen for the cases of $\tilde{s} = 1.23$ and $\tilde{s} = 0.93$, where we clearly observe similar profiles. At their smooth areas however, we observe plateaus for $\sigma(u_\theta)$, similar to the ones for the $\tilde{s} = 3.74$ case, although with a slightly higher value. For the final case with $\tilde{s} = 0.61$ this trend seems to fade away and we see that $\sigma(u_\theta)$ becomes more axially independent, i.e. the peaks are less pronounced, and do not seem to follow the topology of the roughness patches. The results from Fig. 3.3 hints that the presence of the roughness might have an effect on the morphology of the flow, far away from the roughness sublayer region [133], on the order of the gap width d , and reminiscent to what is found in studies of pipe and channel flow [115, 117]. To gain more insight into how the roughness alters the flow, we set out to measure the velocity field in the meridional plane using PIV at multiple heights.

In figure 3.4, we show the temporal and azimuthally averaged radial velocity component u_r , normalized with u_i , in the spanwise wall-normal plane ($\tilde{z}-\tilde{r}$), where the radial coordinate is normalized such that $\tilde{r} = (r - r_i)/d$. Fig. 3.4 shows that for the case of $\tilde{s} = 3.74$, a very large structure can be seen, which consists of a large outflow region (positive u_r) around $\tilde{z} = 5.84$, while a large inflow region (negative u_r) is detected around $\tilde{z} = 2.10$. The situation is more pronounced for the cases of $\tilde{s} = 1.87$, $\tilde{s} = 1.23$, and $\tilde{s} = 0.93$, where a clear roll-like structure (i.e. the TTV) can be observed. Note that the radial component in the flow changes sign along the axial direction as it should in the presence of a TTV. What is rather remarkable, is that the wavelength of the rolls λ changes for different values of $\tilde{s}$. For the large structure at $\tilde{s} = 3.74$, the normalized wavelength is $\tilde{\lambda} = \lambda/d \approx 4.01$. As $\tilde{s}$ decreases to $\tilde{s} = 1.87$, $\tilde{\lambda} \approx 1.49$. At $\tilde{s} = 1.23$, the wavelength decreases to a value of $\tilde{\lambda} \approx 1.42$. At $\tilde{s} = 0.93$, $\tilde{\lambda} \approx 0.94$, and finally for the smallest value of $s = 0.61$, the wavelength increases slightly to $\tilde{\lambda} = 1.09$. We remind the reader that the work of Ref. [21], revealed that for counter-rotation ($a \approx 0.4$), the average wavelength of the rolls could be either $\tilde{\lambda} = 1.46$ or $\tilde{\lambda} = 1.96$ depending on the *state* the system is in. The current work shows that by an appropriate choice of $\tilde{s}$, the wavelength

Figure 3.4: Temporal and azimuthal average of the radial velocity u_r , normalized with the inner cylinder azimuthal velocity u_i , obtained from PIV for varying roughness patch sizes $\tilde{s}$. A positive value of u_r denotes outflow, while a negative value denotes inflow, with respect to the inner cylinder. It can be seen that the rolls are pinned by the roughness and their wavelength changes with $\tilde{s}$. The red and gray areas at the left side of each plot indicate the positions of the rough and smooth areas, respectively. Note that the typical grain size is $k/d \approx 0.01$. The gray shaded areas in the gap represent unexplored heights.

of the rolls can firstly, abandon its natural state; and secondly, it can be tuned within the range $\tilde{\lambda} \in [0.94, 4.01]$ by an appropriate choice of $\tilde{s}$. The wavelengths described above were calculated by measuring the locations of two consecutive maximum and minimum values of $\langle u_r \rangle_{t,\theta,r_{bulk}}$ along z which are closest to midheight. Here, the symbol $\langle \cdot \rangle_{t,\theta,r_{bulk}}$ denotes average over time, the streamwise direction and the bulk region, i.e. $(r_{bulk} - r_i)/d \in [0.3, 0.7]$.

In addition, we observe that outflow regions are created in axial regions where the roughness is located; and conversely, inflow regions are created in the smooth areas. Note that this orientation of the secondary flows is opposite to what is found in other canonical systems (e.g. pipe flow and channel flow [117, 124, 126]), where one finds inflow regions above the rough patches and outflow region above the smooth patches. Another interesting observation is that because the driving is now from the BL rather than the bulk, the strength of the rolls change depending on the value of $\tilde{s}$, as evidenced by the magnitude of $|u_r|$. In order to explore this feature in more detail, we quantify the strength of the rolls with $\tilde{u}_r' \equiv \sqrt{\langle (u_r/u_i)^2 \rangle_{t,\theta,r_{bulk},z_\lambda}}$ as a function of $\tilde{s}$. Here, the symbol $\langle \cdot \rangle_{t,\theta,r_{bulk},z_\lambda}$ denotes an average over time, the streamwise direction, the bulk region, and the axial region that defines the wavelength of a single roll z_λ . In Fig. 3.7(c), we show $\tilde{u}_r'$ as a function of $\tilde{s}$, where we observe that the strength of the rolls increases with decreasing $\tilde{s}$ for $\tilde{s} \in [0.93, 3.74]$. However at $\tilde{s} = 0.61$ the trend is broken, where we observe that $\tilde{u}_r'$ decreases with respect to the case of $\tilde{s} = 0.93$.

In order to obtain more insight into the mechanism(s) that lead to the varying $\tilde{\lambda}$ for varying $\tilde{s}$, we turn to DNS, albeit at a much lower Ta ($\approx 1.0 \times 10^9$), and much higher roughness height ($k/d \approx 0.1$). Since very large $\tilde{s}$ cases are not feasible for DNS, we focus on matching the exact $\tilde{s}$ in the lower range. We will show that, despite the $O(10^3)$ difference in Ta , the same observations found in the numerics are also found in the experiments.

First, we look at the azimuthal velocity component. In Fig. 3.5, we plot the difference of the temporal and azimuthal average of the angular velocity $\langle \omega^+ \rangle_{t,\theta}$ with respect to the temporal, azimuthal and, axial average of the angular velocity $\langle \omega^+ \rangle_{t,\theta,z}$ in wall units. This is done to emphasize the underlying organization of the TTVs. Here, we clearly observe that for all $\tilde{s}$, ejecting regions of angular velocity are originated in the rough patches, similar to the preferential plume ejection sides at the tips of grooves in Ref. [131]. These ejecting regions advect fluid from the roughness patch on or at the inner cylinder towards the outer cylinder. As a consequence, an array of plume-like structures are formed along the axial direction. In TC flow (without

Figure 3.5: Deviation of the temporal and azimuthally averaged angular velocity $\langle \omega \rangle_{t,\theta}$ with respect to the temporal, azimuthal, and axial averaged angular velocity $\langle \omega \rangle_{t,\theta,z}$ obtained from DNS at $Ta \approx 1.0 \times 10^9$, and for all $\tilde{s}$ explored in DNS. The fields are normalized with the inner angular velocity $\omega_i = u_i/r_i$. Positive values represent velocities that are closer to the IC velocity. The leftmost panel corresponds to the case of no roughness while the rightmost panel is the case where the entire IC is uniformly rough. Hatched regions are copied from the actual numerical domains—which are periodic in the axial direction—to allow for straightforward comparison. Ejecting regions can be seen in axial locations where the roughness is present. Notice the similarity of the structures with those found in the experiments shown in figure 6.

roughness), plume-like structures are clear signatures of the presence of TTVs [10,17]. A closer inspection of Fig. 3.5 reveals that for the largest value of $\tilde{s} = 1.24$, the plumes have enough separation such as not to interact between them. When $\tilde{s}$ is lowered to $\tilde{s} = 0.93$, we observe that the plumes come closer, and can, in fact, begin to interact with each other. At the lower $\tilde{s} = 0.62$ however, the situation is rather different. Here, one rough patch does not create a single plume as for the previous cases; a plume is created from the interaction of two ejecting regions. For the $\tilde{s} = 0.47$ case, we observe finally that a plume-like structure is originated from three different rough patches. The behavior of the plumes for $\tilde{s} = 0.62$, and $\tilde{s} = 0.47$ is the result of the merging of the plumes that arise from the roughness patches. These observations help us to rationalize the change in the wavelength and strength of the rolls shown in Fig. 3.4. If $\tilde{s}$ decreases, the plumes are effectively forced to come closer to each other; and as a result, the roll changes its wavelength and becomes stronger due to the added interaction of the plumes. In Fig. 3.6 we show $(\langle\omega\rangle_{t,\theta} - \langle\omega\rangle_{t,\theta,z})/\omega_i$, the same quantity discussed previously, albeit now for the experiments. Here, we can clearly see that a similar mechanism takes place. Plume-like structures are originated at the centers of the roughness elements and interact with each other if the spacing (small $\tilde{s}$) is reduced.

The LDA, PIV and DNS explored in this section reveal that there is a mean effect of the spanwise-varying roughness on the large scale secondary flows that exist in turbulent TC flow. We have seen thus far that the roughness pins the rolls, and that their wavelength and strength can be tuned depending of the choice of $\tilde{s}$ over a wide range of Ta , and a wide range of roughness heights h . However, how does the flow respond globally, i.e. the angular momentum transport, to this change in morphology? This will be addressed in the following section.

3.3.2 Global response

The global response of the TC system can be expressed with the Nu_ω (3.2). In Fig. 3.7(a), we show the compensated Nusselt number as a function of the driving, where a scaling of $Nu_\omega \propto Ta^\alpha$, with $\alpha = 0.45$ is revealed for all the $\tilde{s}$ explored; except for $\tilde{s} = 1.87$, where the scaling is closer to $\alpha = 0.44$. In the absence of roughness and within the same range of Ta , the scaling is found to be effectively $Nu_\omega \propto Ta^{0.40}$ [18,19,21]. In contrast, when both of the solid walls are made uniformly rough (i.e. pressure drag dominates), the scaling

Figure 3.6: Deviation of the temporal, and azimuthal averaged angular velocity $\langle \omega \rangle_{t,\theta}$ with respect to the temporal, azimuthal, and axial averaged angular velocity $\langle \omega \rangle_{t,\theta,z}$ obtained from the experiments at $Ta = 9.5 \times 10^{11}$, and for all $\tilde{s}$ explored in experiments. The fields are normalized with the inner angular velocity $\omega_i = u_i/r_i$. Ejecting regions can be seen in axial locations where the roughness is present. Notice the similarity of the structures with those found in the numerics shown in figure 5.

Figure 3.7: (a) Compensated Nusselt number $Nu_\omega Ta^{-0.45}$ as a function of Ta for varying $\tilde{s}$. The shaded area indicates the error of the measurements, which can be seen to decrease with increasing driving. (b) Compensated Nusselt number $Nu_\omega Ta^{-0.45}$ as a function of $\tilde{s}$ for 3 selected Ta . Here, an optimum value in the transport of angular momentum is observed close to $s/d \approx 1$. The inset in (b) shows the results obtained with the DNS, where the maximum can be observed at a lower $\tilde{s}$, namely $\tilde{s} \approx 0.6$. (c) Normalized RMS of the radial velocity u'_r as a function of $\tilde{s}$, obtained from the PIV experiments. (d) Friction coefficient C_f as a function of the driving, expressed with the Reynolds number Re_i , for various $\tilde{s}$.

asymptotes to the ultimate regime scaling famously predicted by Kraichnan, *i.e.* $Nu_\omega \propto Ta^{0.5}$ [15,132]. In [132], the closest configuration to our study is the case of rough IC and smooth OC, for which the exponent $\alpha = 0.43$ is found. We note that this exponent is somewhat smaller than the ones observed in the current study. The reason behind this, is currently unknown. We notice, however, that the roughness type in our study is rather different. In this study we use spanwise-varying sandgrain roughness, while the roughness in Ref. [132] is made of rib obstacles and is oriented along the streamwise direction.

In order to connect the observed dynamics of the TTVs with the global response, we plot in Fig. 3.7(b), the compensated Nusselt number $Nu_\omega Ta^{-0.45}$ as a function of $\tilde{s}$ for both the experiments and the numerics. We note that the exponent found for $\tilde{s} = 1.87$ ($\alpha = 0.44$) is nearly the same as $\alpha = 0.45$. We see rather remarkably, the appearance of a maximum around $\tilde{s} \approx 0.93$ for the experiments, and $\tilde{s} = 0.61$ for the DNS. We attribute the appearance of this peak to the strengthening of the TTVs, which is caused by the variation of $\tilde{s}$, and thus of $\tilde{\lambda}$. Explicitly, by lowering $\tilde{s}$, we can lower the wavelength of the rolls and bring them closer together (see Sec. 3.3.1). As a consequence, the rolls are strengthened which leads to an enhancement of the angular momentum transport; and thus, the peak around $\tilde{s} = 0.93$. This occurrence is also observed by Ref. [21], although the mechanism observed there is quite different. While the rolls in their study are enhanced by counter-rotating the OC; in our case, the rolls are strengthened by forcing $\tilde{s}$ below their natural wavelength due to the right choice of the spanwise varying roughness. This is also supported by the observation that the magnitude of the radial velocity shows a maximum around $\tilde{s} = 0.93$, as shown in Fig. 3.7(c). We note, however, that the torque is not measured throughout the entire axial length of the cylinders $L = 927$ mm, but in a smaller section of length $L_{\text{mid}} = 536$ mm. As a result, the large structure identified previously for the case of $\tilde{s} = 3.74$ ($\tilde{\lambda} = 4.01$), does not fit entirely in the measurement section (see the first panel of Fig. 3.4). As a result, the Nusselt number that corresponds to this case, could be under or overestimated.

We also note that in the case of the numerics, the position of the maximum is different than in the experiments. We attribute this to a combination of two effects. On the one hand the DNS is performed at a lower Ta , which has an effect on the natural wavelength of the rolls as it was shown by Ref. [31], who show that for similar values of η , the wavelength of the rolls can decrease with decreasing Ta . On the other hand, the axial domain of the DNS is bounded by $\Gamma \in [2.08, 3.32]$, which gives rise to limited box-sizes. Thus, when $\tilde{s}$ is varied,

Figure 3.8: (a) Angular velocity ω^+ profiles versus the wall normal distance $y^+ - h_m^+$ for various $\tilde{s}$, where h_m^+ is the virtual origin and equals the melt-down (i.e. mean) height of the rough surface. The solid black line represents the uniformly rough case. (b) Angular velocity shift $\Delta\omega^+$ as a function of $y^+ - h^+$ for varying $\tilde{s}$. In the inset of (b), we show the angular velocity shift $\Delta\omega^+$ versus the wall normal distance $y^+ - h_m^+$. Here, we observe a maximum downwards shift of the angular velocity profile for the simulation where we cover the entire inner cylinder with sandpaper roughness (i.e. uniformly rough).

the rolls could suffer from an additional *constraint* due to the limited axial domain. In addition to this discrepancy, we also note that the scaling in the range of Ta at which the DNS is done ($\approx 1.0 \times 10^9$), is not known a priori. Since no other exponent is presently available to us, we chose to compensate the numerical data with the same exponent found in the experiments (Fig. 3.7(b)). However, we note that this exponent might be different due to the 2 decades of separation in Ta between the numerics and experiments, as was also shown by Ref. [132]. We would like to emphasize, however, that in spite of these discrepancies, a maximum in angular momentum transport is observed for a given $\tilde{s}$ in both the experiments and the numerics, which is solely a consequence of the varying axial wavelength of the TTV, dictated by the spanwise-varying roughness.

3.3.3 Velocity profiles

Having discussed the dynamics of the TTVs and the corresponding global response, in terms of the dimensionless torque, we now set out to study the streamwise, angular, velocity profiles (rather than the azimuthal profiles, as discussed in [140] and [133]). To allow for straightforward comparison between

the respective velocity profiles, we run the DNS at constant friction Reynolds number $Re_\tau = 690 \pm 10$. The profiles are then temporally, azimuthally, and axially averaged $\omega^+ = \langle \omega \rangle_{t,\theta,z} / \omega_\tau$. The profiles still exhibit a logarithmic region when averaged over the entire axial coordinate. Fig. 3.5 shows however that the TTVs in the flow, following the spanwise-varying roughness, do not exhibit any outer similarity. Deviations of the azimuthal and temporal averages from the mean logarithmic profiles are found up to $\Delta\omega^+ \approx 2$.

For turbulent flows over rough walls, the streamwise velocity profiles retains its logarithmic form. However, the hallmark effect of rough walls is a downwards shift of this region (for any drag increasing surface), which can also be understood as an increase of the skin friction factor C_f [141]. Fig. 3.8(a) shows the angular velocity profiles ω^+ as a function of $(y^+ - h_m^+)$, where h_m^+ is the virtual origin and equals the melt-down (i.e. mean) height of the rough surface. We choose the melt-down height of the roughness over the full inner cylinder as the virtual origin. In Fig. 3.8(b) we show the velocity shift versus the wall normal distance. The inset gives a vertical cut at $y^+ = Re_\tau$. It is evident that also in this representation, an optimum in the velocity shift, and thus in C_f can be observed. The position of this maximum ($\tilde{s} = 0.61$) is the same as the one obtained from the angular momentum transport (see Sec. 3.3.2).

3.4 Conclusions and outlook

In this study, we investigate, both numerically and experimentally, large Taylor number Taylor–Couette flow in the presence of spanwise-varying roughness, which consists of an arrangement of patches of width $\tilde{s} = s/d$, with d the gap width, that covers the entire circumference of the inner cylinder. In the experiments, the patches are made from sandpaper, while in the numerics a confocal microscopy scan of the surface is implemented by means of the immersed boundary method (IBM).

Remarkably, we find that by varying $\tilde{s}$ in the domain $\tilde{s} = [0.61, 3.74]$ we can alter the axial wavelength of the turbulent Taylor vortices within the range $\tilde{\lambda} \in [0.94, 4.01]$, even if the roughness height is very low ($k/d \approx 0.01$). This manipulation is observed to hold in a range of 3 decades in Ta ($\mathcal{O}(10^9)$ – $\mathcal{O}(10^{12})$).

In the experiments, the scaling of the Nusselt number with the driving is found to be effectively $Nu_\omega \propto Ta^{0.45}$ for $Ta \in [5 \times 10^{11}, 5 \times 10^{12}]$; except for $\tilde{s} = 1.87$,

where a very similar exponent is found ($\alpha = 0.44$). The experiments also revealed that inflow regions ($u_r < 0$) originate between the rough patches, where the inner cylinder is hydrodynamically smooth (in contrast to secondary flows induced by spanwise-varying roughness in channel flow, where the orientation of the vortices is reversed [117]). Conversely, at the center of the rough patches, we observe the creation of outflow regions ($u_r > 0$) which are accompanied by the promotion of azimuthal velocity fluctuations $\sigma(u_\theta)$ at midgap. At these axial locations (center of rough patches), we observe, in both the numerics and experiments, the emission of plume-like structures, which are responsible for the creation and pinning of the rolls. Since the coverage of the roughness is fixed, we show that by reducing $\tilde{s}$, we can effectively bring these structures closer, and enhance the interaction of the rolls, as evidenced by the increment in $|u_r|$. As a consequence of this interaction, the flow responds globally by inducing a maximum of angular momentum transport at $\tilde{s} = 0.93$ in the experiments, and $\tilde{s} = 0.62$ in the numerics.

We highlight that in this study, the change in the morphology of the large-scale structures is only due to the spanwise-varying roughness (of very low height) and not by a change of Γ or η , which opens the possibility of exploring different configurations in which the rolls can be tuned at such large turbulence levels. Many questions arise from the aforementioned observations. Understanding the mechanisms leading to the merging of plume ejection regions, and accompanied parameter boundaries at which this occurs, would lead to a further insight into the dynamics of the TTVs. Furthermore, it would be intriguing, in the spirit of Ref. [28], to study the influence of spanwise-varying regions of idealized high and low wall shear stress, without geometrical induced disturbances. It is an open question whether one could also alter λ , without the interaction of the plumes.

Local maxima of angular momentum transport in small gap $\eta = 0.91$ Taylor-Couette turbulence^o

We study Taylor-Couette flow for the case of low curvature $\eta = 0.91$, in the vicinity of the ultimate regime with both experiments and direct numerical simulations. We probe the phase space of TC flow, namely in the regimes of co and counter-rotation as a function of the driving which is controlled by the Taylor number Ta . In the range of $\mathcal{O}(Ta) \in [10^8, 10^{10}]$, we observe the appearance of two local angular momentum transport peaks; a broad and a narrow one. We confirm, experimentally, that the broad peak is accompanied by the strengthening of the large-scale structures in the flow, and that the narrow peak only appears with sufficient driving. Furthermore, we explore the near wall-region with the numerics, and find that the origin of the narrow peak is related to instabilities from the BLs. This supports the idea that the broad peak is caused by centrifugal instabilities and that the narrow peak is a consequence of shear instabilities as firstly evidenced numerically by [142]. In addition, we find that the location of the narrow peak changes with driving. We attribute this to a series of changes in the flow structure at $Ta \approx 10^{10}$, where quiescent wind regions disappear and the whole boundary layers can emit plumes. Finally, close to the transition to the ultimate regime, we experimentally observe the appearance of two stable *disconnected* rolls. We numerically explore this *unusual* state and attribute it to changes in the underlying roll characteristics during the transition to the ultimate regime.

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4.1 Introduction

Taylor–Couette (TC) flow, the flow in between two coaxial, independently-rotating cylinders has been used successfully as a model for shear flows to study turbulence, flow patterns, instabilities, nonlinear dynamics, and transitions [9–12, 19, 48, 142]. The TC geometry is characterized by two parameters. The first one is the radius ratio $\eta = r_i/r_o$, where r_i and r_o are the inner and outer radii, respectively. The second one is the aspect ratio $\Gamma = L/d$, where L is the height of the cylinders and $d = r_o - r_i$ is the width of the gap. The driving of the flow is expressed in dimensionless form with the Taylor number [14]

$$\text{Ta} = \frac{(1 + \eta)^4}{64\eta^2} \frac{d^2(r_o + r_i)^2(\omega_i - \omega_o)^2}{\nu^2}, \quad (4.1)$$

where $\omega_{i,o}$ are the inner and outer angular velocities respectively, and ν is the kinematic viscosity. In statistically stationary TC flow, the flux of angular momentum J^ω is conserved radially, i.e. $dJ^\omega/dr = 0$. Here, $J^\omega = r^3(\langle u_r \omega \rangle_{A,t} - \nu \partial_r \langle \omega \rangle_{A,t})$, where u_r is the radial velocity, ω is the angular velocity, r is the radius, and the symbol $\langle \cdot \rangle_{A,t}$ denotes a time average on a cylindrical surface coaxial with the cylinders. The response of the system can then be characterized by normalizing J^ω with its laminar value $J_{\text{lam}}^\omega = 2\nu r_i^2 r_o^2 (\omega_i - \omega_o)/(r_o^2 - r_i^2)$, which gives rise to the Nusselt number in TC flow

$$\text{Nu}_\omega = \frac{J^\omega}{J_{\text{lam}}^\omega}. \quad (4.2)$$

Distinct differential rotations of the cylinders can produce the same shear, so it is necessary to introduce a second control parameter, the rotation ratio

$$a = -\omega_o/\omega_i. \quad (4.3)$$

In this manner, $a < 0$ denotes corotation of the cylinders while the converse $a > 0$, denotes counter-rotating cylinders. The value of $a = 0$ corresponds to the case of pure inner cylinder rotation. One of the key questions is to describe accurately the transport of the flow throughout the parameter space, i.e. $\text{Nu}_\omega = \text{Nu}_\omega(\text{Ta}, \eta, a, \Gamma)$.

The response of the flow with the driving can be better understood in the context of Rayleigh–Bénard (RB) convection. RB flow, the convective flow in between two plates, one heated from below and the other one cooled from above,

together with TC flow has been used to study the transition to the so-called ultimate regime. As first predicted by Ref. [15], the flow is expected to undergo a transition where the viscosity is no longer important after some critical value of the driving. The driving in RB flow is due to the thermal difference between the plates and it is quantified with the Rayleigh number $Ra = \alpha_p g H^3 \Delta T / (\kappa \nu)$, where α_p is the thermal expansion coefficient, g the gravitational acceleration, H the height of the cell, ΔT the temperature difference, and κ the thermal diffusivity. In the original prediction of Kraichnan, the scaling of the Nusselt number Nu in RB flow is $Nu \propto Ra^{1/2} \times \log$ corrections, and using the analogy with TC flow [14], this implies $Nu_\omega \propto Ta^{1/2} \times \log$ corrections. Both experimental and numerical studies have revealed that the transition to the ultimate regime in TC flow occurs at $Ta \approx 3.0 \times 10^8$ for $\eta = 0.714$ [17, 20], and at this radius ratio it is independent of the rotation ratio a . Here the boundary layers (BLs)—which were originally laminar at lower values of Ta —undergo a shear transition and become turbulent. As a consequence, the scaling is *contaminated* by the interplay between BLs and bulk and the scaling changes to effectively $Nu_\omega \propto Ta^{0.4}$. In the ultimate regime, this scaling is also found to be independent of the radius ratio η and the rotation ratio a [10, 19, 142]. The difference between the predicted exponent $\alpha = 1/2$ and $\alpha = 0.4$ is due to logarithmic corrections that arise from the BLs [16]. Below the transition, however, the BLs have not undergone the transition and they remain laminar; and as a consequence $Nu_\omega \propto Ta^{0.3}$. Another relevant response parameter of the flow is the wind Reynolds number Re_w , which quantifies the strength of the secondary flows. Below the transition to the ultimate regime, it is predicted to scale as $Re_w \propto Ta^{3/7}$. However, in the ultimate regime, the logarithmic corrections from both BLs remarkably cancel each other out which yields $Re_w \propto Ta^{1/2}$, i.e. the original prediction of Kraichnan.

The effect of the rotation of the cylinders a has been thoroughly investigated in the literature, and is intimately linked to that of the radius ratio η . In particular, it has been shown that the rotation ratio a influences tremendously the organization of the flow. When the flow is driven at a constant driving (fixed Ta), a maximum in angular momentum transport (Nu_ω) can be found for a certain a [19, 142]. For the cases of $\eta < 0.9$, the origin of the maximum has been associated to the strengthening of the large-scale wind and vortical structures [8, 21, 78], and the presence of turbulent intermittent bursts originated from the BLs [8]. At optimal momentum transport, the gradient of the angular velocity profiles is nearly zero, which leads to a flat profile throughout the bulk [8, 77]. Past the point of optimal transport $a > a_{opt}$, when the

counter-rotation is strong, the stabilizing effect of the outer cylinder leads to the detachment of mean vortices from the outer layer which leads to radial intermittency and decreases the overall transport [78].

The influence of the curvature of the cylinders to the flow is quantified with the radius ratio η . At $\eta = 0.5$, the position of maximum angular momentum transport is $a_{\text{opt}} \approx 0.2$. In contrast, at $\eta = 0.714$, $a_{\text{opt}} \approx 0.4$ [21]. An extensive study of the influence of η in TC flow has been performed by Ref. [90]. Here it is shown that as η increases from 0.5, a_{opt} moves towards higher counter-rotation values. However, as η increases towards the vanishing curvature limit $\eta \rightarrow 1$, a_{opt} moves back towards the case of pure inner cylinder rotation. In addition, as η increases the peak becomes broader, which makes a_{opt} difficult to precisely identify. This reveals overall a non-monotonic influence of η over the optimal momentum transport. Ref. [90] studied also the influence of a_{opt} with the driving, and it is found that as the driving increases, a_{opt} moves towards the case of $a = 0$. The effect of η has also an influence also on the dynamics of the boundary layers as it has been shown by Ref. [79] and Ref. [17]. Only when the driving is large enough, the influence of the curvature is suppressed and a logarithmic profile for both the angular and azimuthal velocity can be attained.

In a more recent work, Ref. [143] studied numerically the transition from TC flow to rotating plane Couette flow (RPCF), as $\eta \rightarrow 1$. Strikingly, they find that for $\eta > 0.9$ (low curvature), not one maximum of angular momentum transport is present, but two. The first one, they denoted as the broad peak and is located in the corotating regime. The second one, denoted as the narrow peak, is found for counter-rotating cylinders. Only when the driving is sufficiently large, the narrow peak appears and it is speculated that it will surpass the broad peak with sufficient driving. The broad peak is associated with strong vortical motions, as evidenced by the radial velocity fluctuations which show a maximum at optimal transport [143]. Ref. [143] linked the counter-rotating peak of $\eta = 0.5$ and $\eta = 0.714$ to the broad peak for large η found for co-rotation, as they both contained strong ordered motions. The intermittency originated from the stabilizing effect of the OC does not fully explain the origin of the broad peak, as it appears for corotating cylinders as curvature effects vanish. The appearance of two peaks for small gaps means that several competing mechanisms for optimum formation must exist, and that these become blurred for large gaps as the stabilizing effects due to curvature add a third factor.

Ref. [144] attempted to explain the origin of the two optimal transport maxima

in small gaps. In the spirit of Ref. [145], they divided the TC system into three sub-systems, with the boundary layers representing marginally stable TC systems. Using this, they were able to explain and predict the location of the broad peak which arose from centrifugal considerations, finding good agreement for the prediction at moderate Ta. As the shear increased, Ref. [144] also found that as a function of the rotation of the system, the BLs transition into turbulence earlier for a fixed driving. This was linked to the appearance of the narrow peak as the turbulent BLs enhance the angular momentum transport. Using this argument, they argued that the narrow peak will dominate the broad peak once the centrifugal instabilities are superseded by shear instabilities. Once the BLs become turbulent for the values of a in the broad peak, it will vanish, and only the narrow peak will exist. [144] predicted this to happen at $Ta > 4.95 \times 10^9$.

At the same time, the shear instabilities supersede the centrifugal instabilities, the dependence of Nu_ω on the roll wavelength, and thus on Γ vanishes [146, 147]. This was first seen as a crossing of the different $Nu_\omega(\Gamma)$ curves around the transition to the ultimate regime [10, 147], and later as a complete vanishing of the Nu_ω - Γ dependence [10, 148] at high Ta. Recently, [149] showed that the behaviour of the rolls for small gaps is affected around the transition to the ultimate regime. They also showed that pinned structures exist for anti-cyclonic rotation in plane Couette indicating that other mechanisms aside from centrifugal instabilities dominate the formation of pinned large-scale structures.

Finally, we note that in addition to describing the control parameters of TC flow with Ta, η and a , one could also describe the parameter space as was done by [72]. There, the driving is the shear Reynolds number $Re_S = 2r_i r_o d |\omega_i - \omega_o| / ((r_i + r_o)\nu)$ ($Re_S \propto \sqrt{Ta}$), the effect of the curvature is quantified with the curvature number $R_C = (1 - \eta) / \sqrt{\eta}$, and the rotation of the cylinders is quantified with the rotation number $R_\Omega = (1 - \eta)(1 - a/\eta) / (1 + a)$. While the choice of one set of parameters might seem arbitrary at first, we note that in particular, R_Ω is the quantity that controls the magnitude of the Coriolis force when the equations of motion are transformed to the rotating reference frame [143]. As we will show later, the choice of either a or R_Ω becomes especially relevant to elucidate certain effects.

In this study, we probe globally and locally the angular momentum transport close to the transition to the ultimate regime for the case of low curvature $\eta = 0.91$, where the appearance of two peaks for angular momentum transport were observed in the numerics. This will be performed in an experimental

setup with very large aspect ratios, which allows for the flow to switch between states, i.e. different roll wavelengths. By doing this, not only can we confirm the appearance of multiple angular momentum optima experimentally in TC flow, which has not been reported, but we will study the transition between regimes dominated by narrow and broad peaks. We will also rule out that they are an effect of artificially constraining the flow to small periodic aspect ratios. Switching between two and three-roll states as the driving was changed was already reported in [90], and this could have an effect on the two peaks. Secondly, we will test the predictions of Ref. [143,144] regarding the underlying mechanisms for both peaks. Is the broad peak related to vortical motions, which are strengthened by centrifugal forces? Is the narrow peak a consequence of shear? And if so, will it surpass the broad peak and at what turbulence level? By carefully examining the regime where the boundary layers transition, we can better explore the mixed dynamics arising when centrifugal effects and shear are competing side-by-side, and further understand what is happening at the transition to the ultimate regime.

To address these questions we conducted both experiments (torque measurements and Particle Image Velocimetry (PIV)) and Direct Numerical Simulations (DNS). The structure of the paper is as follows. In Sec. 4.2, we introduce the experimental methods. In Sec. 4.3, we introduce the numerical details of the simulations. In Sec. 4.4, we study experimentally the global response of the flow through the parameter space of Ta and a . In Sec. 4.4, we study the transitions and local maxima of the angular momentum transport with the experimental data. In Sec. 4.5.2, we look at the numerics and discuss in detail the effect of the rotation of the system over the Taylor rolls at this value of $\eta = 0.91$. We finalize our study by introducing some concluding remarks and some future work.

4.2 Experimental setup

The experiments were carried out in the Twente Turbulent Taylor-Couette facility (T³C) [142]. In this apparatus, the ratio η and aspect ratio Γ can be manipulated by installing outer cylinders of different dimensions. In this study, the radius of the inner cylinder (IC) is $r_i = 200$ mm and the radius of the outer radius (OC) is set to $r_o = 220$ mm. As a consequence, the radius ratio is $\eta = r_i/r_o = 0.91$ and the aspect ratio is $\Gamma = L/d = 46.35$, where $d = r_o - r_i$ is the gap in between the cylinders and $L = 927$ mm is the height

Working Fluid	(%) Glycerol	ν/ν_w	ρ/ρ_w
Mixture 1	58.72	18.20	1.18
Mixture 2	55.60	13.11	1.17
Mixture 3	45.39	5.67	1.13
Mixture 4	40.18	4.09	1.11
Water	0	1.0	1.0

Table 4.1: Properties of the different mixtures used in the experiments. The percentage of glycerol is based on volume. Both the density and kinematic viscosity ratios are calculated with respect to the density ρ_w and the kinematic viscosity ν_w of water at 20 °C. Data taken from Ref. [150].

of the cylinders. Two acrylic windows located at the bottom cylinder, which cover the entire gap d , allow for the capture of PIV fields in the $r - \theta$ plane at a fixed height.

4.2.1 Global measurements: Torque

We measure the torque $\mathcal{T}$ that is required to drive the cylinders at constant speed with a hollow flanged reaction torque transducer connecting the driving shaft and the inner cylinder. From the torque measurements, the Nusselt number can be calculated as follows

$$\text{Nu}_\omega = \left(\frac{r_o^2 - r_i^2}{2\nu r_i^2 r_o^2 \Delta\omega} \right) \left(\frac{\mathcal{T}}{2\pi\ell_{\text{eff}}\rho} \right), \quad (4.4)$$

where $\ell_{\text{eff}} = 536$ mm is the effective length along the cylinder where the torque is measured, the difference of angular frequencies is $\Delta\omega = 2\pi\Delta f = 2\pi(f_o - f_i)$ with $f_{i,o}$ the driving frequency of the inner and outer cylinder respectively, and ρ is the density. Typically, the (T³C) operates in the ultimate regime of turbulence, where both boundary layers (inner and outer) are turbulent i.e. $\text{Ta} > \mathcal{O}(10^8)$. Thus, in order to capture the transitional regime ($\mathcal{O}(10^7) < \text{Ta} < \mathcal{O}(10^8)$) we use working fluids with different values of the kinematic viscosity ν to control the driving. The working fluid—depending on the desired range of Ta to be resolved—is a mixture of water and pure glycerol. The percentage of glycerol in the mixtures, along with its corresponding kinematic viscosity and density can be found in Tab. 4.1. The liquid temperature is kept constant at 21 °C during all the experiments.

We probe the phase space of Nu_ω in two different ways by conducting two types of torque experiments. The first one is an a-ramp, where the angular frequency difference $\Delta f = (f_o - f_i)$ is kept constant and the rotation ratio $a = -f_o/f_i$ is varied. In this way, we can measure different states in the co and counter-rotation regime while the driving (Ta) is fixed. The second type of experiments is the converse: a Ta-ramp, where a is fixed and Δf is slowly increased which effectively increases Ta.

4.2.2 Local measurements: PIV

We seed the flow with polyamide fluorescent particles with diameters up to $\approx 20\mu\text{m}$ with a seeding density of ≈ 0.01 particles/pixel. The emission peak of these particles is centered at $\approx 565\text{nm}$. We image the particles in the flow with an **Imager SCMOS** (2560×2160) 16 bit camera using a **Carl Zeiss Milvus 2.0/100** objective. The illumination of the particles is provided by a **Quantel Evergreen 145**, 532 nm double pulsed laser. A cylindrical lens is positioned at the laser output to create a thin light sheet of $\approx 1\text{mm}$. A set of mirrors is placed along with a traverse system which makes the laser sheet move with the frame of the T^3C (see Fig. 5.1). Explicitly, the laser sheet (at the laser output) hits a first mirror which is tilted 45° . Light will then be redirect upwards towards a second mirror (also tilted at 45°) which redirects it finally towards the T^3C , perpendicular to both cylinders. The second mirror is attached to the traverse system which can move freely in the axial direction. Since the traverse is fixed to the frame of the T^3C , the result of this arrangement is no relative motion between the camera and the laser sheet due to mechanical vibrations.

The experiments require the OC to move freely; thus, a special trigger for the camera is used for the acquisition of the images. This triggering is done via two hall switch transducers located on top of the OC and a detector mounted onto the frame of the T^3C which outputs a voltage signal every time the transducer has been detected. Using this signal as a trigger, we are able to capture two fields (each one corresponding to one window in the bottom plate) per revolution of the OC. The camera is operated in double frame mode with a framerate f that depends on the rotation rates of the outer cylinder f_o . In all cases however $\Delta t \leq 1/f$, where Δt is the interframe time. In order to increase the contrast between the emission of the light from the particles and the background, we use a **Edmund High-Performance Longpass filter**

550 nm in front of the camera lens.

We scan axially 7 different flow states which are characterized by the magnitude of the rotation ratio of the cylinders i.e. $a = -f_o/f_i$, and the driving of the system i.e. the Taylor number Ta . These states correspond—as will be shown later, to local maxima of the angular momentum transport flux for both co and counter-rotation. In total 10 different heights were explored for each state, and 500 fields were recorded for each height. The axial span for all experiments is in the range $0.402 < z/L < 0.5$, with a resolution of $\Delta z/L \approx 0.1$, where Δz is the length of the axial scan (see Fig. 5.1). The movement of the laser sheet in the axial direction results in defocusing of the images, therefore the focus is adjusted accordingly for all the heights explored.

The velocity fields are measured in the $r - \theta$ plane and are resolved using a multi-pass algorithm from a commercial software (Davis 8.0). The first pass is set to 64×64 pixels and the last one is set to 24×24 pixels with 50% overlap of the windows. The fields obtained are then expressed in cylindrical coordinates of the form $\vec{u} = u_r(r, \theta, t)\hat{e}_r + u_\theta(r, \theta, t)\hat{e}_\theta$, where u_r and u_θ are the radial and azimuthal velocities respectively, which depend on the radius r , the angular (streamwise) direction θ and time t . Here, $\vec{e}_r$ and $\vec{e}_\theta$ are the unit vectors in the radial and azimuthal direction respectively.

4.3 Numerical details

We perform direct numerical simulation (DNS) using an energy-conserving second-order centered finite-difference code for the spatial discretization, while a fractional time advancement is computed using a low-storage third order Runge-Kutta method. The complete description of the algorithm can be found in [135, 136]. This code has been extensively used and validated for TC flow [10].

We perform the simulations in a convective reference frame [72] such that the cylinders rotate with opposite velocities $\pm U/2$, and any combination of differential rotations of the cylinders is reflected as a Coriolis force. In this frame the two control parameters become the shear Reynolds number Re_S and Rotation number R_Ω of [72]. We remind the reader that Re_S is roughly equal to Ta^2 , and that R_Ω is equivalent to a , which are linked by

$$a = -\frac{\omega_o}{\omega_i} = \frac{\eta(1 - \eta) - \eta R_\Omega}{(1 - \eta) + \eta R_\Omega}.$$

Figure 4.1: Sketch of the experimental setup. An arrange of mirrors is set such that the light sheet and camera are mounted on the frame of the T³C.

In order to compare the experimental results with the DNS, the radius ratio $\eta = 0.91$ is chosen, which is the same as in Ref. [10,90]. We perform two sets of simulations with fixed Reynolds numbers, $\text{Re}_S = 2.25 \times 10^4$ and $\text{Re}_S = 3.4 \times 10^4$ (or $\text{Ta} = 5.10 \times 10^8$ and $\text{Ta} = 1.17 \times 10^9$) while varying R_Ω (or equivalently a). Axially periodic boundary conditions are taken with a periodicity length corresponding to the height of the cylinder L , and are expressed by the aspect ratio Γ . In the azimuthal direction, the system is naturally periodic; however, an imposed artificial rotational symmetry of order n_{sym} is chosen in order to reduce computational costs.

For both values of Re_S , we take $\Gamma = 2.33$ and $n_{sym} = 20$, which leads to a small computational box, similar to the one used by [151], where it is found that it does not affect the mean statistics of the flow [148]. For the case of $\text{Re}_S = 2.25 \times 10^4$, we also run a medium-sized boxed with an aspect ratio of $\Gamma = 12.56$, and a rotational symmetry of $n_{sym} = 3$ is chosen. This allows the flow some freedom to switch between different roll states, as in [152]. A uniform grid is used in the azimuthal and axial directions, while a Chebychev-type clustering near the cylinders is used in the radial direction. The spatial resolution for the small domain cases at $\text{Re}_S = 2.25 \times 10^4$ and $\text{Re}_S = 3.4 \times 10^4$ was then

chosen as $n_\theta \times n_r \times n_z = 384 \times 512 \times 768$ in the azimuthal, radial and axial directions, which in wall units for the more restrictive case of $\text{Re}_S = 3.4 \times 10^4$ is a resolution of $\Delta z^+ \approx 5$, $\Delta x^+ = r\Delta\theta^+ \approx 9$ and $\Delta r^+ \in (0.5, 5)$. For the medium-size box at $\text{Re}_S = 2.25 \times 10^4$, a grid of $n_\theta \times n_r \times n_z = 1728 \times 384 \times 1728$ was chosen, which yields a resolution of $\Delta z^+ \approx 5$, $\Delta x^+ = r\Delta\theta^+ \approx 9$ and $\Delta r^+ \in (0.4, 2.5)$. In order to achieve temporal convergence, the simulations are run until the difference between the time-averaged torque of the inner and the outer cylinders is less than 1%. The torque is then taken as the average between these two values. The simulations are then run for at least 40 large eddy turnover times (tU/d).

4.4 Transitions and local maxima in $\text{Nu}_\omega(\text{Ta}, a)$

4.4.1 Transitions in the $\text{Nu}_\omega(\text{Ta})$ scaling

First, we show that the transition to the ultimate regime ($\text{Ta} \propto \text{Nu}_\omega^\alpha$, with $\alpha \approx 0.4$) at this η coincides for experiments and numerics. We show in Fig. 4.2 the compensated Nusselt number with the scaling in the classical regime, i.e. $\text{Nu}_\omega \text{Ta}^{-0.33}$ as a function of the driving for pure inner cylinder rotation only ($a = 0$). Here, we also included the DNS of Ref. [10], where we see an excellent agreement between the numerics and the experiments. As mentioned before, for values of the driving $\text{Ta} < 10^7$, the flow is still in the classical regime, where both BLs are still laminar and $\alpha = 0.3$. When the driving is increased and $\text{Ta} \in [\mathcal{O}(10^7), \mathcal{O}(10^8)]$, the flow enters a transitional regime wherein both turbulent and laminar BLS can coexist. The exponent that corresponds to this transition is $\alpha \approx 0.2$. However, past the minimum value of the compensated Nusselt number, at which $\text{Ta} \approx 3.0 \times 10^8$, a clear change in the scaling exponent to $\alpha = 0.4$ can be seen. This signals that indeed, the transition to the ultimate regime takes place—just as it is found in the numerics of Ref. [90]—at $\text{Ta} \approx 3.0 \times 10^8$.

Figure 4.2 reveals also a second phenomenon, which was previously unreported in experiments. In [90], the local scaling-law was found to be $\text{Nu}_\omega \sim \text{Ta}^{0.4}$ for $\text{Ta} > 10^{10}$. Indeed, provided $\text{Ta} > 3 \times 10^8$ the local scaling-law appears to be the same, with one caveat: a *jump* in the curve around $\text{Ta} \approx 10^{10}$ can be seen, where the Nusselt number suddenly increases. The region where

Figure 4.2: Compensated Nusselt number as a function of the driving for the case of pure inner cylinder rotation ($\eta = 0.91$) in log-log scale. The gray data points ($Ta < 10^8$) corresponds to DNS from [90]. The gray data points for $Ta > 10^8$ are also DNS simulations but from a different study [10]. In addition, each colored marker at fixed Ta corresponds to the driving variation as shown in the legend of Fig. 4.3c. The open circle in light blue corresponds to the DNS data of the current study for $\Gamma = 2.33$. Here the transition to the ultimate regime is observed at $Ta \approx 3 \times 10^8$. The green shaded area corresponds to the region where a *jump* can be seen due to the disappearance of quiescent wind shearing regions. The black solid lines serve as a reference to indicate the corresponding scaling.

this occurs is highlighted in Fig. 4.2. For DNS at $a = 0$, this *jump* was attributed by Ref. [10] to the disappearance of quiescent wind shearing regions, and the whole boundary layer emitting plumes. After this transition, the dependence of Nu_ω on the roll wavelength was completely lost [148]. Here, we find evidence that experiments see a similar, sharp *jump* as observed in figure 4.2. This will be further investigated in Sec. 4.5.1, where we will explore how this behaviour is seen across the a -range, and its effect on the local maxima of angular momentum transport.

4.4.2 Appearance and shifting of the local maxima

In Fig. 4.3, we quantify the angular momentum transport as a function of both the rotation ratio a and the driving Ta from both the experiments and DNS. This figure reveals, that for $Ta < \mathcal{O}(10^8)$, a clear peak which corresponds to maximum transport is located for co-rotation ($a < 0$). The same observation was seen in the simulations of Ref. [90] for the same η . As the driving increases ($Ta > \mathcal{O}(10^8)$), the prominence of this local maximum in the compensated Nusselt $Nu_\omega(a_{co})Ta^{-0.33}$ decreases and a second local maximum in the counter-rotating regime arises. This second local maximum ($Nu_\omega(a_{counter})$) can be fully appreciated around $Ta \approx 10^8$ and seems to increase with Ta , consistent with the DNS of Ref. [143, 144] for $\eta = 0.91$. As we increase the driving beyond $Ta > 10^{10}$, we observe that the first local maximum that corresponds to the corotating regime completely vanishes and only one peak remains in the counter-rotating regime. We note that Fig. 4.3 contains both a - and Ta -ramps, and that both measurements are mutually consistency throughout 3 decades of Ta . Furthermore, we emphasize again the excellent agreement between the numerical and the experimental data. An animated version of Fig. 4.3 can be found in the supplementary material.

These global results shows that at this $\eta = 0.91$, the appearance of a second peak in the angular momentum transport is only a function of the driving. If the driving is not enough ($Ta < \mathcal{O}(10^8)$), only one peak is present and it is located in the corotating regime. However, if the driving is too large ($Ta > \mathcal{O}(10^{10})$), again only peak will be visible but this time in the counter-rotation regime. For a narrow band of values of Ta , both peaks can be observed. Indeed, the original prediction of Ref. [143] is correct: the peak for counter rotation (narrow) will eventually surpass the peak for co-rotation (broad) if driving is large enough. Interestingly, the value of Nu_ω at both local maxima is the same, and close to the expected value of the transition to the ultimate regime, i.e. $Ta \approx 3 \times 10^8$. In addition, for the cases of $Ta = 2.23 \times 10^9$ and $Ta = 3.31 \times 10^9$, and for $a < 0$, jumps in the $Nu_\omega(a)$ can be observed. These will be revisited in Sec. 4.5.2.

Another interesting property to investigate is the relative magnitude of the peaks. Thus, we define

$$\zeta \equiv \frac{Nu_\omega Ta^{-0.33}(a = a_{co})}{Nu_\omega Ta^{-0.33}(a = a_{counter})}, \quad (4.5)$$

Figure 4.3: Compensated Nusselt number $Nu_\omega Ta^{-0.33}$ as a function of the rotation ratio a and the driving Ta . The projection in (a) allows a better visualization of the peak for counter-rotation for $Ta > 10^{10}$ while the projection in (b) highlights the peak for corotation at lower Ta . The projection in (c) shows the compensated Nusselt number as function of a only. The projection in (d) shows the compensated Nusselt number as function of Ta only. The solid colored lines in all figures represent the experiments done with a -ramps, i.e. Ta fixed and variable a . The solid black lines in all figures represent the experiments done with Ta -ramps, i.e. a fixed and variable Ta . The gray dashed line in all figures corresponds to DNS from Ref. [90]. The light blue dashed line is the present DNS data for $Re_S = 3.4 \times 10^4$ ($Ta = 1.17 \times 10^9$) and $\Gamma = 2.33$. The dashed vertical black line in (c) separates the co- and counter regimes. The colors in all figures represent the Ta -variation as shown in the legend of (c). The numbers shown in (d) represent the value of a for the corresponding a -ramp.

Figure 4.4: (a) Location in the phase space of the local maxima of angular momentum transport. The blue triangles for $Ta < 2.5 \times 10^7$ are the DNS of Ref. [90]. The blue triangles located at $Ta = 1.17 \times 10^9$ are from DNS of the current study. The stars represent experimental data for the maxima in the corotating regime while the open circles represent experimental data for the maxima in the counter-rotation regime. The markers in red represent turbulent states where we perform PIV experiments as described in Sec. 4.4.3. (b) Ratio of the magnitude of the angular momentum transport peaks. The colored points represent the experimental data. The open circle represents the DNS simulation shown in (a) for $Ta \approx 10^9$. The pink shaded areas represent the turbulence at which only one peak in angular momentum is observed. The dashed red line represents the prediction of [144] for the disappearance of the (broad) peak found in corotation, namely at $Ta > 4.95 \times 10^9$.

where a_{co} and $a_{counter}$ denotes the a -value that corresponds to the peak for co and counter-rotation respectively. In Fig. 4.4b, we show ζ as a function of the driving where we observe that before the transition to the ultimate regime $\zeta < 1$, and past the transition value of $Ta \approx 3 \times 10^8$, $\zeta > 1$. This yields an alternative representation of what was originally shown in Fig. 4.3: with sufficient driving the peak for counter-rotation will surpass the peak for co-rotation. As mentioned previously, the magnitude of both peaks seems to be nearly the same ($\zeta \approx 1$) close to the transition to the ultimate regime. Finally, we note that the broad peak is predicted to disappear at $Ta = 4.95 \times 10^9$ [143]. Figure 4.4b, shows also an excellent agreement with this prediction: past $Ta = 4.95 \times 10^9$ only one peak can be detected.

Next, we look at the location of the observed local maxima shown firstly in Fig. 4.3. In Fig. 4.4a, we show their location throughout the parameter space (a, Ta) . Here, we also include the DNS data of Ref. [90] for the same radius ratio $\eta = 0.91$ albeit for much lower values of Ta . We note that as the driving increases from $Ta = \mathcal{O}(10^4)$ towards the transition to the ultimate regime, the peak for corotation moves towards $a = 0$. Past the transition, the location of this peak remains relatively unchanged at $a \approx -0.2$. Regarding the peak for counter-rotation, we see that the peak only appears when $Ta > 10^8$ and moves as the driving increases. When $Ta \in [1.33 \times 10^8, 1.17 \times 10^9]$, the peak moves in a towards higher values of counter-rotation. However, for $Ta > \mathcal{O}(10^{10})$, the peak seems to move back towards $a = 0$. We note, however, that at this driving, $Nu_\omega Ta^{-0.33}$ becomes less a -dependent which could over- or underestimate the precise location of the maximum. The shifting of the narrow peak happens around the same Ta the jumps in the $Nu_\omega(Ta)$ relation appeared, and this will be revisited in §4.5.1.

4.4.3 Local flow structure and its relation to the local Nu_ω maxima

In the previous sections, it was shown that one of the peaks will surpass its counterpart for a given value of the driving. Now, we look at the velocity fields obtained with PIV and investigate the local properties of the two peaks for 4 different Ta . In order to do so, we probe the peaks at their corresponding a -values before, close to, and after the transition to the ultimate regime. In Fig. 4.5, we show the compensated Nusselt number (same as in Fig. 4.3) but only for the Ta we explore with PIV. As a reference, we highlight explicitly the turbulent states to be probed for both co and counter-rotation.

Figure 4.5: Compensated Nusselt number as a function of the rotation ratio a for 4 selected Ta . The red markers represent turbulent states where PIV experiments are performed: (stars) for corotation and (open circles) for counter-rotation. The radial velocity $u_r(r, z)$ that correspond to this states is shown in figure 6.

We first investigate the strength of the radial flow by looking at u_r in the $(r - z)$ plane. In Fig. 4.6, we show $u_r(r, z)$ for both peaks (broad and narrow), which are located in the corotating and counter-rotating regime respectively, and as a function of Ta . Here, we can clearly identify regions of negative and positive radial velocity along the axial direction which signals the presence of Taylor rolls. Strikingly, we find that for all Ta explored, $|u_r|$ is much larger for the corotation than for counter-rotation. This hints that, indeed, the broad peak is accompanied by strong and coherent rolls as predicted by Ref. [143]. In addition, it also coincides with what was analyzed in [149], where rolls were shown to be pinned for values of $R_\Omega \approx 0.1$.

In addition, we note that for the case of the corotating peak, the magnitude of the radial velocity decreases with increasing Ta , while the converse occurs for the counter-rotating peak. This supports the global results presented in the previous sections where we showed that the peak in the counter-rotating becomes more dominant with driving as compared to the other peak which eventually disappears at a given Ta . The former holds except for the largest Ta explored ($Ta = 1.40 \times 10^{10}$). Here, the peak is probed at $a = 0.125$ and the magnitude of the radial velocity is smaller than for the case of $Ta = 3.31 \times 10^9$. We remind the reader that at this Ta , the maximum becomes more a -dependent. The reason for the decrease in the magnitude of the radial velocity is presumably because there is simply not enough counter-rotation to stabilize the roll as it is also observed for the case of $\eta = 0.7$ [21].

In order to give a more quantitative picture of the strength of the vortex as a function of the driving, we look at

$$\text{RMS}(\tilde{u}_r) \equiv \sqrt{\langle (\langle u_r/u_S \rangle_{t, \theta, r_{\text{bulk}}})^2 \rangle_z}, \quad (4.6)$$

which is the root mean square (RMS) of the normalized radial velocity profile along the z -direction. Here, r_{bulk} denotes an average over the radial domain that defines the bulk region of the flow, namely $(r - r_i)/d \in [0.3, 0.7]$. Due to the different vortical structures found for every state, the axial average has to be carefully chosen. For the cases of $a = -0.27$ and $a = -0.15$, the axial average is done such that a roll pair is considered; explicitly, we take the average from the maximum outflow of the first roll to the minimum inflow of the second roll. The same average is done for their corresponding counter-rotation counterparts at the same Ta ($a = 0.46, 0.44$). In contrast, the axial averages for the cases $a = -0.21, 0.49$, and 0.125 are done throughout the entire axial domain. The reason is that no coherent structures are detected for $a = 0.49, 0.125$, and for the case of $a = 0.125$, two symmetric rolls are

Figure 4.6: Azimuthally and time averaged normalized radial velocity obtained from PIV experiments as described in Fig. 4.5. The upper row represents measurements of the peak in the corotating regime while the bottom row are measurements of the peak for counter-rotation. Along a single column, the Ta is fixed for both co- and counter rotating states. The dashed line is added to emphasize the difference between the Ta values. The legend on top of each figure represents the value of (a, R_Ω) .

contained within the full axial domain. In Fig. 4.7a, we show the $\text{RMS}(\tilde{u}_r)$ as a function of the driving, where we observe that indeed, the $\text{RMS}(\tilde{u}_r)$ for the peak in corotation decreases with driving while the converse occurs for the peak in the counter-rotating regime. For the highest Ta however, we observe that the RMS of the radial velocity decreases with increasing driving. The reason is, as it was mentioned before, there is not enough counter-rotation at this state ($a = 0.125$) to stabilize the roll.

In addition to the strength of the radial velocity, we can also look at what happens to their fluctuations as a function of the driving, i.e. the wind. The standard deviation of the radial velocity is calculated as

$$\sigma_{\text{bulk}}(u_r) \equiv \langle \sigma_{t,\theta}(u_r) \rangle_{r_{\text{bulk}}, z}, \quad (4.7)$$

where $\sigma_{t,\theta}(u_r)$ is the standard deviation of the radial velocity in the azimuthal direction and over time. Here, the averages are done in the same manner as for $\text{RMS}(\tilde{u}_r)$. From this characteristic velocity we can construct the so-called wind Reynolds number $\text{Re}_w = (d/\nu)\sigma_{\text{bulk}}(u_r)$. In the classical regime of turbulence, the unifying theory of Ref. [16] predicts a scaling of the Reynolds wind $\text{Re}_w \propto Ta^{3/7}$. When the driving is increased, within the ultimate regime, the logarithmic corrections remarkably cancel each other out which yields the effective scaling predicted by Ref. [15], namely $\text{Re}_w \propto Ta^{1/2}$. In Fig. 4.7b, we plot the compensated Reynolds wind with the scaling of the ultimate regime $\text{Re}_w Ta^{-1/2}$ as a function of Ta . Here we see that indeed, past the transition into the ultimate regime, Re_w slowly asymptotes into a $Ta^{-1/2}$ scaling for both peaks.

We focus now on the case of $Ta = 5.10 \times 10^8$; which is very close to transitional value and where both angular momentum peaks have the same magnitude (see Fig. 4.4). As shown in Fig. 4.6, for corotation at $a = -0.21$ (second panel of first row in Fig. 4.6), a peculiar pattern of the radial velocity can be appreciated in the axial direction. Here, a region of positive velocity, i.e. an outflow region is observed at midheight $z/L = 0.5$. As we move towards lower heights, we encounter a region of negative radial velocity, i.e. an inflow region. The peculiarity arises when we notice that as we move further down to $z/L \approx 0.44$, another inflow region can be found which would correspond to another roll located in between $z/L \in [0.4, 0.44]$ which rotates in the opposite direction with respect to the one above. This pattern is rather puzzling since one would expect consecutive inflow and outflow regions which denote a consecutive arrangement of rolls in the axial direction. Here however, it seems that two Taylor rolls coexist, although slightly disconnected. One possible

Figure 4.7: (a) RMS along the axial direction of the azimuthally-time-bulk averaged radial velocity normalized with the shear velocity u_s as a function of the driving. (b) Reynolds-wind (based on the radial velocity) as a function of the driving. In both figures, the stars represent states that correspond to the peak in the corotating regime, while the open circles represent measurements for the peak for counter-rotation.

explanation to this unusual state is roll-splitting, i.e. one roll is locally split into two more rolls which would result in a similar configuration, and as a consequence change also the number of rolls in the system. This observation will be revisited in Sec. 4.5.3, with more data coming from the simulations.

4.5 Boundary layer transitions and state switching

In this section, we use simulations to elucidate some of the questions that cannot be answered in experiments by closely examining the velocities, especially in the boundary layers, and the changes in roll states.

4.5.1 Disappearance of the broad peak

In order to explain the first type of jumps, i.e. those in the $Nu_w(Ta)$ curves, and the movement of the counter-rotating peak at $Ta \sim 10^{10}$, we show in figure

Figure 4.8: Compensated Nusselt number as a function of driving for several rotation ratios. A reordering of the curves around $Ta \sim 10^{10}$ can be seen. The green shaded area corresponds to the region where a *jump* can be seen due to the disappearance of quiescent wind shearing regions. The black dashed line represents the scaling of $Nu_w \propto Ta^{0.4}$.

4.8 the behaviour of the $Ta(Nu_w)$ curves for 3 selected values of a . These states correspond to states before and past the narrow peak for fixed Ta . We previously mentioned how for $a = 0$, a second phenomena, consisting of jumps in the curves was seen at $Ta \approx 10^{10}$, which coincided with the disappearance of the torque on the roll-wavelength. In figure 4.8, we see how during this transition, a reordering of the curves occur, as for low values of a , a discrete jump increases the torque, while for high values of a the opposite process seems to happen, and the torque decreases.

By relating this to the transition observed in Ref. [10], where the quiescent regions near the boundary layer disappeared, we can explain why the counter-rotating maximum shifts. Indeed, in figure 4.9 we show the structure of the near-wall region for several values of R_Ω (and a). As R_Ω is increased, the near-wall structures become sharper, but at the same time the turbulent Taylor-roll is stabilized and quiescent regions appear. This is also consistent with the idea that the creation of the narrow peak is due to shear instabilities that arise from the BLs [144]. As mentioned previously, it is known that at high Ta , the quiescent regions disappear, providing the large jump in the torque, where the $R_\Omega = 0.09$ ($a = 0$) curve surpasses the $R_\Omega = 0.03$ ($a = 0.47$) curve. As there

Figure 4.9: Instantaneous azimuthal velocity at $r^+ = 15$ for $Ta = 5 \times 10^8$ for (a) $R_\Omega = 0.02$ ($a = 0.59$), (b) $R_\Omega = 0.03$ ($a = 0.47$), and (c) $R_\Omega = 0.09$ ($a = 0$). The DNS corresponds to the case of $\Gamma = 2.33$.

is no quiescent area to eliminate for low values of R_Ω , then these branches of the $Nu_\omega(Ta)$ curve do not jump and remain at a lower value.

4.5.2 Roll state switches

We note that previous simulations, both by [144] and [90] used a small Γ domain which essentially fixed the roll size. In order to explain the second type of jumps, i.e. those in the $Nu_\omega(a)$ curve, we inspect the results given by DNS, using both a small computational box which accommodates a single roll pair, as well as medium-sized boxes which can sustain changes in roll number. In this way we can capture how the rolls appear as the system goes from pure counter rotation to co-rotation, and if state switching takes place as a is varied. Indications of state switching are already present in the experimental $Nu_\omega(a)$ curve at $Ta = 2.23 \times 10^9$ and $Ta = 3.31 \times 10^9$ for $a < 0$ (see Fig. 4.3), where jumps in the $Nu_\omega(a)$ curve were seen. In Figure 4.10, we show the compensated Nusselt number for all values of Ta simulated, for both the small and the medium boxes, and for an experimental value. We can see that while the results see a degree of collapse for $a < -0.5$, for $a \geq -0.5$ there are significant discrepancies between the datasets.

In figure 4.11 we show the azimuthally-time averaged radial velocity for the medium-sized domain, varying a from the negative value of the broad peak to positive values. Or, in terms of R_Ω , varying the Coriolis forces from anti-

Figure 4.10: Compensated Nusselt number as a function of a obtained from the DNS of the current study. The green solid line is the experimental data performed at $Ta = 5.1 \times 10^8$.

cyclonic to zero. We can see that the flow self-organizes in Taylor rolls as the Coriolis force start to play a role, i.e., when $R_\Omega \neq 0$ [149], and the intensity of the structures increases as the peak in Nu_ω is approached. As R_Ω increases, we first find that the rolls become sharper and more prominent, as it also observed in the experiments (see Fig. 4.6). Notice how also $|u_r|$ increases monotonically with R_Ω . As we approach what is the “broad peak”, the number of roll pairs switch, first from four to five, then from five to six, sharply reducing the roll wavelength. These reflect on the torque and appear as discrete jumps, as seen in Fig. 4.10. A switch between roll states will affect the torque, the change in the roll wavelength changes the proportion between plume ejection and impact regions, and can cause the appearance of larger quiescent regions in the boundary layers. As Ta increases, these discrete jumps in the $Nu_\omega(a)$ curve disappear, because roll state switching does not modify the fraction of boundary layer region which emit plumes. In a way, they are analogous to the jumps in the $Nu_\omega(Ta)$ relationships.

The $\Gamma = 12.56$ simulation and the experiment coincide in the region $0.3 < a < 1$, where the Coriolis force is small and the rolls are only starting to become organized, while in the region between $-0.5 < a < 0.3$ there are significant discrepancies. These are probably due to the state switching, as in the medium size box, where state switching can take place, the values the roll wavelength

can take are more restricted. In the experiment, Γ is much larger, and thus a state switch might or might not have a large footprint on the torque.

4.5.3 Roll splitting and fracture

During the transition, the rolls occupy the axial domain not in an axially homogeneous manner (see Fig. 4.11), as they have different sizes and also in an asymmetric manner: the distance between the maximum outflow and inflow is not always the same. This results could be a possible explanation for the pattern depicted at the end of last section, represented in the second panel of figure 4.6, i.e., the presence of two consecutive rolls that rotate in opposite direction. We first note that while DNS results in an average over all azimuths, experimental results examine a single azimuth and assume that the flow is azimuthally homogeneous. As we have seen repeatedly, the Taylor rolls break the assumption of axial homogeneity, so it is possible that azimuthal homogeneity is also broken. As R_Ω increases, the rolls become sharper, until at a certain point, one of them must split up locally, such that transiently, outflow, or inflow regions with the same sign coexist next to each other at certain values of the azimuth. As R_Ω is further increased, the roll splits up completely to form a new roll pair. However, the speed at which this roll “dislocation” exists and propagates to “fracture” the roll is a priori unknown. If we now consider the rotation number of that case, indeed, we have $a = -0.21$, or $R_\Omega = 0.142$, that is an intermediate case between the first two panels of figure 4.11, where we see the signature of two outflow regions that an extra couple of rolls has taken a place in $\Gamma \in [6, 12.56]$. So, what is seen in experiments could be just a part of this process of duplication of the rolls.

Indeed, a similar process was seen at $Ta = 5.10 \times 10^8$ for rotating plane Couette flow (RPCF) at $R_\Omega = 0.1$ ($a = -0.046$), as shown in the instantaneous azimuthal velocity in figure 4.12. The roll “fracture” propagates at a characteristic velocity of the order of the frictional velocity u_τ , and persists for the length of the simulation. We expect the same phenomena to be seen in small-gap Taylor-Couette flow, as the dynamics of the rolls are the same at high Reynolds numbers [149]. It thus seems that close to the transition to the ultimate regime, there is a change in the roll dynamics which occasionally leads to persistent transient effects seen both in simulations and experiments which are hard to capture as they occur only at certain azimuths, which means experiments could miss them, and are washed out when taking streamwise

Figure 4.11: Azimuthally-time averaged radial velocity normalized with the shear velocity u_S obtained from DNS for $Ta = 5.10 \times 10^8$. Each plot represents a different rotation state, quantified by either a or R_Ω . The legend on top of each figure represents the value of (a, R_Ω) .

Figure 4.12: Instantaneous azimuthal velocity at the mid-gap from DNS normalized with the shear velocity u_S for $Ta = 5.10 \times 10^8$ and pure inner cylinder rotation. The roll splitting procedure generating consecutive inflow and outflow regions can clearly be seen.

averages, the usual representation in simulations.

4.6 Conclusions

In this study, we probe the angular momentum transport with both experiments and direct numerical simulations in $\eta = 0.91$ TC flow in the transition to the ultimate regime. We show that in the experiments, it is also possible to detect multiple local angular momentum maxima for a constant driving in the range of $Ta \in [1.33 \times 10^8, 3.31 \times 10^9]$ as it was first seen in the numerics of Ref. [143, 144], and that this effect is not just an artifact of the axial domain in the numerics. We confirm experimentally that indeed, the broad peak is related to the strengthening of the large-scale due to centrifugal forcing quantified by the magnitude of R_Ω ; and numerically, that the narrow peak is only a consequence of shear instabilities that arise from the BLs. With sufficient driving, the shear instabilities dominate over centrifugal instabilities and the broad peak disappears at around $Ta = 4.95 \times 10^9$, which we also observe in

the experiments. Past this value only the narrow peak can be detected. We show also that the narrow peak moves as the driving increases. We attribute this to an unexplored phenomena at $Ta \approx 10^{10}$, where the quiescent wind sheared regions at the cylinder walls disappear, and the whole BLs can emit plumes. In the experiments, a peculiar state of the Taylor rolls is observed around $Ta = 5 \times 10^8$, which is close to the transition to the ultimate regime, and where both the narrow and broad peaks are present. At this state, two neighboring Taylor rolls are observed to rotate opposite to each other. We explore the possibility that this *unusual* state is produced by roll splitting, which is controlled by the amount of the Coriolis force in the system (R_Ω). This peculiarity is also observed in the limit of $\eta = 1$ (RPCF), which suggests that the dynamics of the rolls, for very small TC flow ($\eta \rightarrow 1$), are subject to transient effects close to the transition to the ultimate regime. Further experimental and numerical work is needed in order to explore thoroughly this new feature of the secondary flows in TC flow.

5

Drag reduction in boiling Taylor-Couette turbulence[°]

The nucleation of vapor bubbles (“boiling”) is an omnipresent phenomenon in daily life, in nature, and in technology. In energy conversion processes and in industrial applications, this includes turbulent flow boiling, such as in riser tubes of steam generators, in boiler tubes of power plants and in coolant channels of boiling water nuclear reactors. The physics governing the pressure drop and, therefore, the wall shear stress in such turbulent boiling flows is not understood, also due to the lack of controlled experiments. Here we create a highly controlled lab environment—accessible to both global and local monitoring—to analyse turbulent boiling flows and in particular their shear stress in a statistically stationary state. Namely, by precisely monitoring the drag of strongly turbulent Taylor-Couette flow (the flow in between two co-axially rotating cylinders, Reynolds number $Re \approx 10^6$) during its transition from non-boiling to boiling, we show that the intuitive expectation, namely that a few volume percent of vapor bubbles would correspondingly change the global drag by a few percent, is wrong. Rather, we find that for these conditions a dramatic global drag reduction of up to 50% occurs. We connect this global result to our local observations, showing that for major drag reduction the vapor bubble deformability is crucial, corresponding to Weber numbers larger than one. We compare our findings with those for turbulent flows with gas bubbles, which obey very different physics than vapor bubbles. Nonetheless, we find remarkable similarities and explain these.

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5.1 Introduction

When the temperature of a liquid increases, the corresponding vapor pressure P_v also increases. Once the vapor pressure equals the surrounding pressure P_{atm} , all of the sudden *boiling* starts and vapor bubbles can grow [35, 39–45] (preferentially starting on nucleation sites such as on immersed microparticles), dramatically changing the characteristics of the flow [153–156]. Similarly, *cavitation* requires that the pressure is locally lowered so that it matches the vapor pressure at the given temperature, and again vapor bubbles emerge, again dramatically changing the flow characteristics [39, 157]. Both boiling and cavitation lie at the basis of a vast range of different phenomena in daily life, nature, technology, and industrial processes. For boiling, many of them are connected with energy conversion such as handling of LNGs (e.g., pumping through pipes or pipelines), liquified CO₂, riser tubes of steam generators, boiler tubes of power plants, or cooling channels of boiling water nuclear reactors. The sudden change of the global flow characteristics at the boiling point can lead to catastrophic events [153–156]. In nature, examples for such catastrophic events in boiling turbulent flows include volcano or geyser eruptions [158, 159]. For cavitation, one example is the cavitation behind rapidly rotating ship propellers, drastically reducing the propulsion [160, 161] and another one is the occurrence of cavitating bubbles in turbomachinery, where the corresponding huge pressure fluctuations can cause major damage [162]. In spite of the relevance and the omnipresence of boiling and cavitation in turbulent flows, the physics governing the corresponding pressure drop and thus the reduced wall shear stress or propulsion is not fully understood, also due to the lack of controlled experiments.

In this paper, we want to contribute to the understanding of boiling turbulent flows, by performing and analysing controlled boiling experiments in turbulent Taylor-Couette (TC) flow [11], i.e. the flow in between two coaxial co- or counterrotating cylinders. This flow has the unique advantage of being closed, with global balances, and no spatial transients. Also the underlying equations and boundary conditions are well-known and well-defined so that the turbulent flow is mathematically and numerically accessible. More concretely, the experiments are performed in the Boiling Twente Taylor-Couette (BTTC) facility [36], which allows us to control and access at the same time both global and local flow quantities, namely to measure the torque $\mathcal{T}$ required to drive the cylinders at constant speed, the liquid temperature T_{TC} in the cell, the

Figure 5.1: Vapor bubble generation and migration close to the surface of the IC in the boiling regime. Each image corresponds to a different stage during the experiment. Time moves from left to right: the numbers on top of each image corresponds to the time steps shown in Fig. 5.3d. The nucleation of vapor bubbles starts on top of the cell because the hydrostatic pressure there is smaller. The vapor bubble front then travels downwards by bubble dispersion until the surface of the IC is fully covered. Note that the volume fraction is increasing with time.

pressure P of the system, the volume fraction α of the vapor, and in the beginning of the vapor bubble nucleation process even the positions and sizes of individual vapor bubbles, all as function of time, so that we can study the dynamics and evolution of the boiling process. The focus of the work is on the onset and evolution of turbulent drag reduction induced by the nucleating vapor bubbles in the turbulent flow. We will compare it with the case of air bubbles in turbulent flow with similar turbulence level and fixed gas volume fraction. For air bubbles, van Gils *et al.* [30] have shown that with a small volume fraction of only $\alpha \approx 4\%$, very large drag reduction of $\approx 40\%$ can be achieved, provided the turbulence level is high, i.e. $\mathcal{O}(\text{Re}) \approx 10^6$. We will not only compare the drag reduction effect of vapor and gas bubbles, but also their deformabilities, which have turned out to be essential for bubbly drag reduction [30, 34, 163–165].

5.2 Experimental setup and procedure

In order to properly measure the drag reduction and the vapor volume fraction α in the flow as function of time, we have extended the BTTC facility [36] for boiling TC flow, see Fig. 5.1 for examples of vapor bubbles in the BTTC at different stages in time and Fig. 5.2 for a detailed sketch of the experimental setup. Boiling water requires liquid temperatures of $\approx 100^\circ\text{C}$, which turns out to be impractical in the BTTC facility: the glass transition temperature of PMMA, from which the outer cylinder is made of, can be as low as $\approx 82^\circ\text{C}$. Therefore, instead of water, we use the commercially available low boiling point Novec Engineered Fluid 7000 ($\text{C}_3\text{F}_7\text{OCH}_3$) liquid instead. This liquid boils at roughly 34°C at atmospheric pressure. The density of the liquid ρ_ℓ , its kinematic viscosity ν_ℓ , and its surface tension (air/liquid) σ depend on temperature. These dependences can be found in Rausch *et al.* [166]. In order to allow for liquid expansion due to changes in temperature, the cell of the BTTC is connected to a transparent closed cylindrical container, which we refer to as the *upper vessel* (see Fig. 5.2). The cell of the BTTC, the upper vessel, and the tubing that connects them form a closed reservoir; neither liquid nor vapor can leak out of the system. The pressure of the system is fully controlled and monitored as shown in Fig. 5.2.

The experimental procedure is as follows: we fill the system with the low-boiling point liquid until an initial liquid height can be seen in the upper vessel. Next, we rotate the inner cylinder (IC) at a fixed rotation frequency $f_i =$

Figure 5.2: Diagram of the experimental apparatus. The control volume defined by the blue dashed lines corresponds to the volume of the cell V_{TC} . Correspondingly, in green dashed lines we highlight the volume of the tubing V_{tube} . Finally, in red dashed lines we highlight the volume of the liquid height in the upper vessel $V_L(t)$.

20 Hz while maintaining an initial liquid temperature of 20 °C. At this stage, the entire fluid is still liquid. Once the system reaches thermal equilibrium, we strongly increase the liquid temperature T_{TC} in the cell within the range $T_{TC} \in [20^\circ\text{C}, 55^\circ\text{C}]$. Eventually, the boiling point of the liquid is reached and we observe the nucleation of small vapor bubbles on top of the cylinder as shown in Fig. 5.1①. The nucleation of vapor bubbles starts on top of the cell because the hydrostatic pressure there is smaller. Since the nucleated vapor bubbles are small, they are carried away by the flow and are able to migrate downwards close to the surface of the IC as shown in Fig. 5.1②. The bubble migration continues until the bubble front reaches the bottom plate of the BTTC (Fig. 5.1③). Figs. 5.1①–④ shows a typical boiling experiment where we highlight four different stages of the experiment in which the bubble migration can be fully appreciated.

In Fig. 5.3a we show the temperature ramp of the liquid phase for the experiment shown in Fig. 5.1, where a heating rate of $dT_{TC}/dt \approx 0.022$ K/s was used. This value is calculated by applying a linear fit to the data in the range $60\text{ s} < t < t_{\text{boil}}$, where 60 s is the time at which the liquid temperature is observed to increase linearly with time, and t_{boil} is the time at which the liquid starts boiling. Since the system is closed, the pressure P of the system increases monotonically with temperature as can be seen in Fig. 5.3b. The boiling temperature T_{boil} is a function of P ; thus, we calculate the boiling point by plotting the vapor pressure of the liquid P_v along with P as shown in Fig. 5.3b. The boiling point occurs at $t = t_{\text{boil}}$, i.e. the instant of time where $P = P_v$, and as a consequence $T_{\text{boil}} \equiv T_{TC}(t_{\text{boil}})$. In this manner, we define the *non-boiling* regime as the time interval during the experiment where the liquid experiences pure thermal expansion and no vapor is present, i.e. $t < t_{\text{boil}}$ (or equivalently $T_{TC} < T_{\text{boil}}$). This region is shaded in gray in Fig. 5.3. Conversely, we define a *boiling* regime, where $t \geq t_{\text{boil}}$ (or equivalently $T_{TC} \geq T_{\text{boil}}$).

As the temperature ramp continues beyond the boiling point, we observe that the amount of vapor in the system increases. This effect can be seen throughout the experiment as shown in Figs. 5.1③–④, where one observes more and more bubbles as time goes by. From the conservation of mass, based on the measurements of the liquid temperature T_{vessel} and the liquid height h_L in the upper vessel (see Fig. 5.2), we can compute the instantaneous volume fraction $\alpha(t)$. The results are shown in Fig. 5.3c, where we observe, indeed, that the instantaneous volume fraction $\alpha(t)$ increases monotonically until a maximum value of $\approx 8\%$ is reached at the end of the temperature ramp. This calculation

Figure 5.3: (a) Liquid temperature T_{TC} , (b) pressure P , volume fraction α , (d) the compensated Nusselt number $Nu_w Ta^{-0.4}$, and (e) the drag reduction (DR) as a function of time. The gray shaded area and the blue lines correspond to data in the non-boiling regime, i.e. $t < t_{\text{boil}}$. The boiling point is defined using the intersection $P = P_v$ at a certain time as it is shown in (b). The time steps in (d) correspond to the photographs shown in Fig. 5.1.

is self-consistent: within the non-boiling regime $\alpha \approx 0$ and $\alpha > 0$ in the boiling regime. We would like to point out that no free parameters are involved in the calculation of $\alpha(t)$. It is rather remarkable that our boiling experiments yield such large and well-controlled values of the vapor volume fraction.

In order to quantify the turbulence level of the experiments, we use the Taylor number which is defined as [11]

$$\text{Ta} = \left(\frac{(1 + \eta)^4 d^2 (r_i + r_o)^2 (2\pi f_i)^2}{64\eta^2} \right) \frac{1}{\nu(\alpha)^2}, \quad (5.1)$$

with $\nu(\alpha) = \nu_\ell(1 + 5\alpha/2)$, i.e. including the so-called Einstein correction for the viscosity due to the presence of the dispersed phase [167], r_i and r_o the radii of the inner and outer cylinder of the BTTC, respectively, the gap width is $d = r_o - r_i$, and $\eta = r_i/r_o$ the radius ratio. The Taylor number and the Reynolds number $\text{Re}_i = 2\pi f_i r_i d / \nu$ are related by $\text{Ta} = \frac{(1+\eta)^6}{64\eta^4} \text{Re}_i^2$. The response of the system, the torque or equivalently the angular velocity transfer, can—in analogy to the heat transfer in Rayleigh-Bénard flow—best be expressed in dimensionless form as the so-called generalized Nusselt number [14]

$$\text{Nu}_\omega = \left(\frac{r_o^2 - r_i^2}{8\pi^2 \ell_{\text{eff}} r_i^2 r_o^2 f_i} \right) \frac{\mathcal{T}}{\rho_\ell \nu(\alpha)}, \quad (5.2)$$

where $\ell_{\text{eff}} = 489 \text{ mm}$ is the effective length along the cylinder where the torque is measured. In the ultimate regime of TC turbulence [11], where both the bulk and the boundary layers are turbulent ($\text{Ta} > 3 \times 10^8$), it was found that $\text{Nu}_\omega \propto \text{Ta}^{0.4}$ [18–21]. In our experiments, Ta is of the order $\mathcal{O}(10^{12})$ and thus the compensated quantity $\text{Nu}_\omega \text{Ta}^{-0.4}$ can be used as a measure to characterize the amount of drag reduction. In Fig. 5.3d, we show that indeed $\text{Nu}_\omega \propto \text{Ta}^{0.4}$ during the experiment, up to a dramatic drop some time after the boiling point is reached. In the absence of a dispersed phase (non-boiling regime), $\text{Nu}_\omega \text{Ta}^{-0.4}$ should remain constant because the driving strength (i.e., the angular velocity f_i of the inner cylinder) is fixed and all the temperature-dependent quantities are contained in both Nu_ω and Ta (see Fig. 5.3d). Then, in the boiling regime, the occurrence of the vapor bubbles modifies the local angular velocity flow near the IC, causing a dramatic drop in the transport of momentum (Nusselt number) [30], which we directly observe as a drop in the global torque. This is the signature of drag reduction in the flow, shown in Fig. 5.3e. Notice how the start of the drop in the compensated Nusselt number and the corresponding drag reduction occur at a time after the boiling point

is reached. This is simply because a certain amount of time (determined by turbulent diffusion) is needed for the vapor bubbles to migrate downwards and cover the entire surface of the IC.

5.3 Quantifying the drag reduction

We quantify the level of drag reduction DR through

$$DR = 1 - \frac{Nu_\omega Ta^{-0.4}}{Nu_\omega Ta^{-0.4}(t = t_{\text{boil}})}, \quad (5.3)$$

where $Nu_\omega Ta^{-0.4}(t = t_{\text{boil}})$ is the compensated Nusselt number evaluated at the boiling point, where the system is still in a single phase state. In this way, $DR > 0$ corresponds to a finite amount of drag reduction (boiling regime), while $DR = 0$ corresponds to zero drag reduction (non-boiling regime). In Fig. 5.4a we show DR as function of Ta and α for the experiment described in Figs. 5.1 and 5.3 along with five other experiments we performed, which confirm the reproducibility of our controlled experiments. Figs. 5.4ab unambiguously reveal that drag reductions approaching 50% can be achieved in the boiling regime, with vapor bubbles as dispersed phase, with a volume fraction only up to 6–8%.

Note that Ta is a non-monotonic function in the boiling regime due to the vapor fraction correction of the liquid viscosity. The reason is that when the boiling regime is reached, α increases at a faster rate than the rate at which ν_ℓ decreases; and as a consequence the corrected viscosity ν increases which leads to a decrease of Ta (see Eq. (5.1)). However, this effect is not significant. The converse takes place at the final stage of the experiment (see Fig. 5.3c), when α saturates to a certain value and as a consequence the corrected viscosity ν decreases which leads to an increment of Ta (see Fig. 5.4a). A 3D representation of Fig. 5.4 which shows the instantaneous drag reduction as a function of both Ta and α can be found in the Supplementary Material.

In Fig. 5.4b we show the DR as a function of the volume fraction α , for all the corresponding experiments shown in Fig. 5.4a. This figure reveals the amount of instantaneous drag reduction that can be achieved for given vapor fraction α at that instant. Again, it is remarkable that with only 6 - 8% vapor bubble fraction drag reduction of nearly 50% can be achieved. This resembles the

Figure 5.4: (a) Drag reduction DR as a function of Ta and α for different experiments. Note that neither α , nor DR, nor Ta are control parameters, but responses of the system to the temperature change. The horizontal axis on top of the figure represents the Reynolds number Re_i . The color bar represents the volume fraction α . The gray data points correspond to data in the non-boiling regime ($DR \approx 0, \alpha \approx 0$). The open circles represent the drag reduction obtained with air bubbles at a fixed $Ta = 1.51 \times 10^{12}$ [30]. A 5% error bar is shown that applies to all experiments. (b) DR as a function of the volume fraction α . The colored lines represent the different experiments as shown in (a). The open circles correspond to the data of ref. [30] for drag reduction using air bubbles. The error bars for both quantities DR (5%) and α (0.5%) are included. A 3D animation of the data is included as Suppl. Material. Note the degree of reproducibility of our controlled experiments, which is remarkable for the boiling process, which is considered to be random and irregular.

large drag reduction of up to 40% achieved by the injection of only 4% volume fraction of *air bubbles* into the TC system [30].

5.4 Comparison to drag reduction with gas bubbles

Therefore we will now, compare in more detail, the drag reduction achieved in boiling turbulent flow (vapor bubbles) with the well-known effect of drag reduction in turbulent flow with gas bubbles [23, 26, 30, 163–165, 168–177]. Vapor and gas bubbles display fundamental differences [35]. While the creation, growth, and stability of gas bubbles is entirely controlled by mass diffusion, vapor bubbles are controlled by the much faster (100 times) heat diffusion and by phase transitions. Since the surface tension in a vapor bubble is temperature-dependent, thermal Marangoni flows can further affect the vapor bubble dynamics. Given these major differences between vapor and gas bubbles, one wonders whether these major differences also reflect in different bubbly drag reduction behavior.

The answer can be read off Figs. 5.4ab, in which we have also included the drag reduction data of van Gils *et al.* [30], which correspond to the case of air bubbles at a fixed $Ta = 1.51 \times 10^{12}$. The inspection of this figure reveals, strikingly, that both vapor and air bubbles produce a comparable amount of drag reduction when α and Ta are approximately the same for the two cases. This is even more remarkable as in the gas bubble injection experiments, the gas volume fraction α is a control parameter, whereas in the boiling experiments, the vapor volume fraction α is a response of the system to the temperature increase. Indeed, for an equivalent Ta , the same amount of very large drag reduction can be obtained by either vapor or gas bubbles, given that their volume fraction is the same.

5.5 Bubble deformability

As relevant mechanism for achieving large drag reduction in high-Re flows with relative small gas bubble volume fraction, the gas bubble deformability has been identified as crucial ingredient. This is supported by experimental and numerical studies in high Reynolds number Taylor-Couette flow [23, 30, 34, 163–165, 170, 174, 175, 178–180] and in other canonical flows. Whether a

bubble is deformable or not is determined by the corresponding Weber number, comparing inertial and capillary forces. It is defined by

$$\text{We} = \frac{\rho_\ell u_\theta'^2 d_b}{\sigma}, \quad (5.4)$$

where d_b is the typical bubble size, $u_\theta'^2$ is the variance of the azimuthal velocity, σ is the surface tension of the liquid/air interface. A large Weber number $\text{We} > 1$ implies that the bubble is deformable. Indeed, Verschoof *et al.* [165] showed that for fixed gas volume fraction $\alpha \approx 4\%$ and fixed large Reynolds number, the large drag reduction ($\approx 40\%$) could basically be “turned off” by adding a surfactant, which hinders bubble coalescence and leads to much smaller bubbles with $\text{We} < 1$ in the strongly turbulent flow.

To obtain the Weber numbers of the vapor bubbles in our boiling experiments, we performed high-speed image recordings to measure the bubble size and shape. With this information at hand, we calculate the distribution of the Weber number for different volume fractions during the experiment. The details of the bubble sizing experiments can be found in the Materials and Methods section. The velocity fluctuations u_θ' (required to calculate We) as function of Ta are given by $u_\theta' = 11.3 \times 10^{-2} (\nu_\ell/d) \text{Ta}^{0.44}$, as measured earlier in the same BTTC facility [181]. The temperature variation during a high-speed measurement is $< 0.3 \text{ K}$. Therefore, all the values of the quantities in Eq. (5.4) (except for d_b) are taken as the temperature-dependent value at the beginning of each recording.

In Fig. 5.5, we show the probability density function (PDF) and the mean value of the Weber number for different α during a typical experiment in the boiling regime. These distributions reveal that $\text{We} > 1$, independent of the volume fraction, i.e. the vapor bubbles in our experiments are deformable, which supports the idea that also for vapor bubbles deformability is key for achieving large drag reduction, just as shown for bubbly drag reduction with gas bubbles [30, 165]. Moreover, we find that the maximum of all distributions lies at $\text{We} \approx 2.5$, which corresponds to bubbles of size $\approx 0.27 \text{ mm}$. The tail of every PDF extends up to $\text{We} \approx 8$, which indicates that some of the bubbles are highly deformable. Furthermore, the inset in Fig. 5.5b reveals that in the range of the volume fraction explored with the high-speed image experiments ($\alpha \in [3.1\%, 6.2\%]$), the mean Weber number $\langle \text{We} \rangle$ slightly increases, namely from a value of $\langle \text{We} \rangle \approx 3.1$ to $\langle \text{We} \rangle \approx 3.7$. Notice also that as the volume fraction increases, the probability to find a larger value of We is also larger for $\text{We} > 3.5$, which indicates that the deformability of the vapor bubbles increases when the number of bubbles is increased. Since the variation of

Figure 5.5: (a) Probability density function of the bubble Weber number during the boiling experiment for different volume fractions in linear scale. The colors represent the variation of α as shown in the legend. N_b is the number of detected bubbles in every measurement. (b) Same as in (a) but in log-log scale. The inset in (b) represents the mean Weber number as function of the volume fraction. The error bars in (b) correspond to $\pm\sigma(We)$, where $\sigma(We)$ is the standard deviation of the Weber number for a given α .

$\rho \ell u_0'^2 / \sigma$ is very small throughout the boiling regime ($\approx 7\%$), this shows that with increasing volume fraction it is more likely to find bigger bubbles. This can also be seen at the other tail of the distribution for values of $We < 3.5$, where the probability decreases with increasing volume fraction. So when advancing the boiling process, the emerging extra vapor manifests itself in larger vapor bubbles, be it by growth or coalescence, and not in more freshly nucleated small bubbles.

5.6 Conclusions and Outlook

We have investigated the transport properties (i.e., the drag) of strongly turbulent boiling flow with increasing temperature in the highly controlled BTTC setup and correlated them with the vapor bubble fraction and the vapor bubble characteristics. Our highly reproducible and controlled findings reveal a sudden and dramatic drag reduction at the onset of boiling and that the emerging vapor bubbles are similarly efficient in drag reduction as injected air bubbles: Nearly 50% drag reduction can be achieved with a volume fraction of about $\approx 6\%$. In both cases the main reason for the drag reduction lies in the bubble deformability, reflected in large $We > 1$. Furthermore, at later stages of the boiling process when the vapor fraction is higher, on average the bubbles are also bigger. Though this seems to be in line with our daily life experience when watching boiling tea water, it is not evident at all, as the enhanced vapor fraction could also manifest itself in more small, freshly nucleated bubbles.

In our experiments (Fig. 5.3a), the temperature increase is relatively modest, both in absolute numbers beyond the boiling temperature and in rate, due to experimental limitations. Furthermore, the drag reduction effect is spatially smeared out, as we measure the global drag of the whole cylinder. Nonetheless, within minutes the overall drag of the system reduces by a factor of 2. Within industrial devices, such as riser tubes of steam generators, boiler tubes of power plants, coolant channels of boiling water nuclear reactors, or when handling liquefied natural gases (LNGs) and liquefied CO_2 , such sudden and large drag change can have dramatic consequences. In our experiments, the time scale of the sudden drag is determined by the heating rate and by the turbulent mixing of the emerging bubbles over the whole measurement volume. For larger heating rate and smaller volume, the rate of the drag change will be

even more dramatic. Our experiments give guidelines on how to explore such events in a controlled and reproducible way.

5.7 The Boiling Twente Taylor-Couette (BTTC) facility

The experiments were performed in the BTTC facility. Fig. 5.2 shows a sketch of the experimental setup. The radius of the inner and outer cylinder of the BTTC are $r_i = 75$ mm and $r_o = 105$ mm, respectively. The gap is then $d = r_o - r_i = 30$ mm and the radius ratio is $\eta = r_i/r_o = 5/7 = 0.714$, which is very close to $\eta = 0.716$ of the Twente Turbulent Taylor-Couette facility [142]. The height of the cylinders is $\ell_{\text{eff}} = 549$ mm, which gives an aspect ratio of $\Gamma = \ell/d = 18.3$. In order to allow for liquid expansion due to changes in temperature, the cell is connected to a transparent closed cylindrical container, i.e. the *upper vessel*. The cell and the upper vessel are connected via plastic tubing and an aluminum heat exchanger which is in direct contact with a water liquid bath. The coil and the liquid bath forms a very efficient condenser that condenses liquid back into the cell. A circulator Julabo FP50-HL unit controls the temperature of the water bath for the condenser. A PT 100 temperature sensor in the upper vessel monitors the liquid temperature $T_{\text{vessel}}(t)$. Since the system is closed, the increment in temperature translates into an increment in relative pressure which we monitor with an Omega PXM409-002BGI pressure sensor. The pressure signal is then calculated as $P(t) = P_{\text{atm}} + p(t)$, where $p(t)$ is the measured relative pressure. In order to avoid a possible overpressure of the system, a 1 bar pressure release valve is connected to the upper vessel. Finally, we use a Nikon D300 camera to record the liquid height $h_L(t)$ during the experiment. From this measurement, the liquid volume in the upper vessel can be calculated as $V_{\text{vessel}}(t) = \pi(D/2)^2 h_L(t)$, where $D = 100$ mm is the inner diameter of the upper vessel. The heating of the liquid phase is done via the surface of the IC which is itself heated through channels, where hot water can flow due to a second circulator Julabo FP50-HL. The IC is made out of stainless steel. Three temperature sensors distributed along the vertical axis of the IC measure the liquid temperature during the experiment. We take the average of the three sensors to calculate the liquid temperature inside the cell $T_{TC}(t)$. Finally, a Althen 01167-051 hollow flanged reaction torque transducer (located inside the IC) measures the torque $\mathcal{T}$ required to drive the IC at constant speed. For a more detailed overview of the experimental setup, see Huisman *et al.* [36].

5.8 Calculation of the volume fraction α

We calculate dynamically the volume fraction $\alpha(t)$ using the following conservation of mass argument. At the beginning of the experiment, i.e. the start of the temperature ramp, the temperature is $T(t_0) = T_0$. At this stage, the initial mass m_0 is composed only of the liquid mass which is known a priori. This is $m_0 = \rho_\ell(T_0)(V_{TC} + V_{\text{tube}} + V_{\text{vessel}}(t = t_0))$, where ρ_ℓ is the liquid density of the $\text{C}_3\text{F}_7\text{OCH}_3$, V_{TC} is the volume of the BTTC cell, V_{tube} is the volume that corresponds to the tubing that connects the cell to the upper vessel and $V_{\text{vessel}}(t = t_0)$ is the liquid volume inside the upper vessel. Once the temperature ramp starts, the liquid experiences thermal expansion. This leads to a redistribution of the initial mass into V_{TC} , V_{tube} and V_{vessel} (see Fig. 5.2):

$$\begin{aligned} m_0 &= m_{TC} \\ &+ \rho_\ell(T_{\text{vessel}}(t))V_{\text{tube}} + \rho_\ell(T_{\text{vessel}}(t))V_{\text{vessel}}(t), \end{aligned} \quad (5.5)$$

where m_{TC} is the mass inside the cell and $T_{\text{vessel}}(t)$ the temperature measured in the upper vessel, which we also assume is the temperature that corresponds to the tubing. Note that the only time dependent volume is $V_{\text{vessel}}(t)$. As the temperature T_{TC} increases, the boiling point T_{boil} is eventually reached and vapor bubbles start nucleating. At this stage, the mass inside the cell m_{TC} is a mixture of both the liquid and the vapor phase i.e.

$$m_{TC} = \rho_\ell(T_{TC}(t))V_\ell(t) + \rho_v(T_{TC}(t), P(t))V_v(t), \quad (5.6)$$

where T_{TC} is the measured temperature inside the cell, $V_\ell(t)$ is the volume occupied by the liquid inside the cell and $V_v(t)$ is the volume occupied by the vapor inside inside the cell, i.e. $V_\ell(t) + V_v(t) = V_{TC}$. The vapor density denoted by ρ_v in Eq. (5.6) is dependent on both temperature and pressure due to the natural compressibility of the vapor phase. The vapor density is calculated by using tabulated values [166] of $\rho_v(T, P_{\text{atm}})$ and assuming that the vapor experiences adiabatic expansion such that $\rho_v(T, P) = (P/P_{\text{atm}})\rho_v(T, P_{\text{atm}})$. In the absence of vapor, i.e. $\alpha = 0$, $V_v(t < t_{\text{boil}}) = 0$. Using Eqs. 5.5 and 5.6, along with the definition of the volume fraction $\alpha = V_v/V_{TC}$ and using that $1 - \alpha = V_\ell/V_{TC}$, we find that

$$\alpha(t) = \frac{1 - \frac{m_0}{\rho_\ell(T_{TC}(t))V_{TC}} + \frac{\rho_\ell(T_{\text{vessel}}(t))}{\rho_\ell(T_{TC}(t))} \left(\frac{V_{\text{tube}} + V_{\text{vessel}}(t)}{V_{TC}} \right)}{1 - \frac{\rho_v(T_{TC}(t), P(t))}{\rho_\ell(T_{TC}(t))}}. \quad (5.7)$$

5.9 High-speed recordings

The high-speed recordings were done with a **Photron SA-X** high-speed camera. The frame rate was set to 13,500 fps. The framing of the camera results in a viewing area of $43.5 \text{ mm} \times 43.5 \text{ mm}$ and it was recorded at mid-height. The focus plane of the camera is located at $(r - r_i)/d \approx 0.9$, thus the imaged bubbles are very close to the OC. The higher density of vapor bubbles near the IC makes the detection of these bubbles less reliable than for bubbles close to the outer cylinder.

Conclusions

In this thesis, we make use of experiments and direct numerical simulations to study highly turbulent Taylor—Couette flow, the flow in between two coaxial and independently rotating cylinders. The end-goal of this thesis was to study multiphase turbulence in TC flow, which is created by a flow carrier and a second phase which consists of vapor bubbles. We started our study however, by looking at fundamental properties of highly-turbulent TC flow in the absence of a dispersed phase.

In Chapter 1, we use particle image velocimetry to investigate the flow locally for the case of pure inner cylinder rotation, in the $r - z$ plane and at a fixed height. By doing so, we are able to calculate structure functions D in the streamwise direction s for both the radial and azimuthal components of the velocity, as a function of the radial coordinate r . From the bulk of the flow, and using the scaling of the second order of the structure functions $D \propto (\epsilon s)^{2/3}$, we extracted from the inertial range of turbulence, a local dissipation rate ϵ_{bulk} which we used to construct the Taylor-Reynolds number. This quantity is defined as $\text{Re}_\lambda = (\sigma_{\text{bulk}}(u_\theta))^2 (15/(\nu \epsilon_{\text{bulk}}))^{1/2}$, where $\sigma_{\text{bulk}}(u_\theta)$ is the also measured standard deviation of the azimuthal velocity in the bulk. With this dimensionless group at hand, we are able now to compare the local turbulence generated in the bulk of TC flow to the turbulence generated in other canonical flows (pipe, channel, etc), where Re_λ is often used to describe the turbulence response. We find that the scaling of this turbulence strength with the driving of the flow, the Taylor number Ta , can be characterized by the power-law $\text{Re}_\lambda \propto \text{Ta}^{0.18}$.

In chapter 2, we use particle image velocimetry to probe the flow in the meridional plane, albeit this time also in the axial direction and for different rotation ratios of the cylinders. By taking the velocity fluctuations of the velocity, we constructed the local Nusselt number (convective term only) as a function of the axial coordinate z and r . We showed that the angular momentum transport is mostly confined in the regions of inflow and outflow, where the averaged

radial velocity is positive and negative respectively. In this regions, we looked at the statistics of the flow by means of the probability density functions and found intermittent bursts at the tails of the distributions. We attributed these bursts to the presence of small-scale plumes, which are also responsible for intermittent bursts in RB flow. Revealing yet again, the powerful analogy between RB and TC flow. Furthermore, we detected the presence of superimposed azimuthally travelling waves by performing proper orthogonal decomposition. We showed that these large-scale structure are most pronounced at the center of a Taylor roll and that are very similar to the more well-known pattern of *wavy* Taylor vortices, although with larger azimuthal wavenumbers.

In chapter 3, we studied the effect of spanwise allocated roughness in TC flow with both experiments and direct numerical simulations. Here, we showed that depending on the width of the rough patches s —which are placed along the inner cylinder—we were able to firstly pin the rolls; and secondly by varying s , we were able to change the wavelength, and number of the rolls in the system. We showed that this type of turbulence control induces a local and global response of the flow. Locally, we showed that ejecting plumes are originated at the centers of the rough patches. Since the total coverage of the area is fixed, a decrease in s translates into bringing these structures closer together. As a result of this interaction between plumes of different rolls, the flow responds globally by exhibiting a maximum in angular momentum transport when the width of the patch is roughly the same size as the gap in between the cylinders. We concluded that by using this type of spanwise roughness the Taylor rolls can be tailored, as evidenced by both the experiments and the numerics.

In chapter 4, we revisited TC flow in the ultimate regime for the case of small curvature, where the radius ratio is fixed at $\eta = 0.9$. Here, again we make use of both experiments and direct numerical simulations to probe the flow globally and locally in the phase space of $(Ta - a)$ close to the transition to the ultimate regime. We showed that at this radius ratio, two maximums for angular momentum transport can be detected at two different rotation ratios of the cylinder and for the same Ta . As it was previously found in numerical studies, one of the peaks (termed the broad peak) is located for corotating cylinders, while the second peak (termed the narrow peak) is found when the cylinders are counter-rotating. We showed firstly that the narrow peak only appears with sufficient driving, leading to the detection of both peaks within the range of $Ta \in [1.33 \times 10^8, 3.31 \times 10^9]$. Close to the transitional value to the ultimate regime, i.e. $Ta \approx 3 \times 10^8$, both the narrow and broad peak

seem to have the same magnitude. In addition, we showed that the broad peak disappears when $Ta > 4.95 \times 10^9$, leaving only the narrow peak in the counter-rotating regime. We showed that the original prediction of [143] is correct: the broad peak is created due to centrifugal instabilities, as indicated by a strong enhancement of the rolls, and that the narrow peak is created due to shear instabilities that are originated from transitions of the boundary layers. The reason why the broad peak disappears is that with sufficient driving, these shear instabilities become more dominant as compared to the centrifugal ones. We also showed that the narrow peak moves non-monotonically in a as the driving increases. We attributed this to a second transition in the scaling around $Ta = \times 10^{10}$, in which a sharp *jump* in the $Nu_\omega - Ta$ curve is observed. The reason behind this transition is that wind-sheared regions in the boundary walls disappear, allowing for the ejecting of plumes throughout the whole boundary layers. Furthermore, at $Ta = 5.10 \times 10^8$, which is very close to the transitional value to the ultimate regime, the velocimetry revealed two neighboring rolls which rotate opposite to each other. By describing the magnitude of the Coriolis force with the rotation number R_Ω , we found firstly that the rolls increase their strength with increasing R_Ω ; and secondly, that the existence of the *unusual* state of the rolls mentioned previously could be due to roll-splitting, which seems to occur when the value of R_Ω is large enough so as to dislocate a roll and divide it in two new ones.

In the final chapter of this thesis, we studied bubbly TC turbulence. The bubbles however are created by boiling a liquid within a temperature-controlled experiment. Here, we showed that in the presence of a small amount of vapor bubbles ($\alpha \approx 6\%$), and when the turbulence level is large ($Ta \approx \mathcal{O}(10^{12})$), the drag in the flow can be reduced by almost half. We showed in this way, that vapor bubbles are just as efficient as air bubbles in reducing the drag of a highly turbulent flow. We extended the investigation and found that the vapor bubbles in the system are deformable, as indicated by finding a Weber number We larger than unity. This sustains the idea that the deformability of the bubbles is the key ingredient in achieving large amounts of drag reduction, whether the bubbles are made of air or vapor. Mastering the phenomenon of boiling is no easy task, it is a rather dynamical physical state. However, in our experiments we were able to control the creation of vapor and monitor it as a time-dependent volume fraction $\alpha(t)$, which was calculated from first principles and measured experimentally. The result of this, was a monotonic (and repeatable for that matter) increment of vapor volume fraction in the flow. What is the consequence of such a dynamical volume fraction to the

morphology of the bubbles? We showed from the probability density function of the Weber number, that with increasing volume fraction (and thus, increasing drag reduction), the probability to detect smaller bubbles decreases, while the probability to detect larger bubbles increases. From this we can conclude that at such large turbulence levels ($Ta \approx \mathcal{O}(10^{12})$), and for increasing α , the mechanism of bubble coalescence will be dominated over the break-up of the bubbles.

Outline

As the title of this thesis suggests, we are in the direction of fully unveiling the rich physics between the presence of the vapor bubbles and turbulent TC flow. However, a lot of work still needs to be done, and here we name what could be the possible avenues that this research demands. Firstly, the Ta -dependence of the DR. In the last chapter of this thesis, we fix the speed of the inner cylinders to $f_i = 20$ Hz and conducted the boiling experiments. As soon as nucleation is observed, the bubbles are observed to migrate downwards towards the bottom plate, eventually covering the entire inner cylinder. However, if Ta is lowered, the mean flow in the system will be lowered as well. As a result, one would have to start considering the effect of buoyancy. Put in other words, if the velocity of the mean flow (say in the streamwise direction) is very low, it could very well be that buoyancy takes over and thus, the bubbles will not migrate downwards, but stay on top of the setup, where they originally nucleate from. This is of course not beneficial, since the entire inner cylinder, (where the torque is measured) would not be entirely covered, leading to discrepancies in the quantification of the drag reduction.

The second effect to be addressed in the future is probably the most obvious one. What happens when we conduct the experiments at $a \neq 0$? That is, what would happen with the drag reduction when the outer cylinder co or counter-rotates with respect to the inner cylinder. For the case of $\eta = 0.714$, it has been previously shown [21], that by introducing counter-rotation into the flow, the Taylor rolls can be strengthened at large Ta . If we were to conduct boiling experiments in this configuration, we would expect the vapor bubbles to be *trapped* by the rolls because of the lower pressure there. However, as it was also pointed out before [23, 34, 182], the interaction of the bubbles with the boundary layers is required to achieve drag reduction, so it is not clear a priori what would be the effect of this reallocation of the bubbles. Further

experimental work would shed more light into this effect; and finally, what would happen to the morphology of the bubbles? Would they become bigger, or smaller than the case for $a = 0$?

For now, I would like to conclude and thank you, the reader, for the time-spent while reading this work. Hopefully, a *take-away* message has been laid out. The problem of turbulence is far from being solved, but hopefully works like the present could contribute in the future to achieve a full understanding of fluid mechanics as a whole; and that in my very humble opinion, should be the goal of all *fluid-mechanicians* out there.

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Summary

Taylor-Couette (TC) flow is the flow in-between two concentric cylinders which rotate independently. It is together with Rayleigh-Bénard (RB) convection, the flow in-between two plates, heated from below and cooled from above, one of the canonical flows where turbulence can be probed and put to the test. As compared to RB flow, where the flow is thermally-driven, the driving in TC flow is purely mechanical which is far more efficient when designing experiments. In this thesis, we focus our investigation towards a better understanding on the effect of vapor bubbles in high-Reynolds TC flow. These bubbles are the product of boiling a low-boiling point liquid which we use as the working fluid. We have designed an experiment, where the boiling can be well-controlled and monitored in order to quantify the physical response of the flow due to the bubbles, in particular its drag. We find that vapor bubbles are just as efficient in reducing the drag in the flow as air bubbles. Almost 50% drag reduction can be achieved when the volume fraction of the bubbles is $\approx 8\%$, the Taylor number is $Ta \approx \mathcal{O}(10^{12})$ and the Weber number $We > 1$. In addition to the study on vapor bubbles, in this thesis we study fundamental properties of turbulent flows in the context of TC flow. In chapter 1, we measure locally the turbulence level in the bulk of the flow described by a Taylor-Reynolds number Re_λ and find the scaling $Re_\lambda \propto Ta^{0.18}$, where $Ta \in [10^8, 10^{10}]$ is the Taylor number which describes the driving of the flow. By describing the turbulence level in this manner, one can compare the local turbulence generated in TC flow with respect to other canonical flows like pipe flow, channel flow, etc.

In chapter 2, we measure the velocity field in the azimuthal and radial plane and in the axial direction and explore the underlying mechanism of the angular momentum transport. The measurements are carried out before and after the transition to the so-called ultimate regime of turbulence, where the Taylor number spans $\mathcal{O}(10^6 - 10^{10})$. We find that the transport is mostly confined in outflow and inflow regions, where small-scale plumes are responsible for in-

termittent events that appear in the probability density functions of the local angular velocity transport. This is similar to what it is seen in RB flow, revealing yet again the powerful analogy between TC and RB. By performing a proper orthogonal decomposition, we also find superimposed azimuthal travelling waves which are similar to the well known *wavy* Taylor vortices present at lower Taylor numbers.

In chapter 3, we look at the effect of having sand grain roughness applied to the inner cylinder. We perform both experiments and numerical simulations of the flow, where the roughness is placed onto the inner cylinder in the form of stripes of width s . In the experiments, the width of the stripes is $s/d \in [0.61, 3.74]$, where d is the gap width, and the Taylor number is $Ta \approx 10^{12}$. By adequately manipulating s , we find that the wavelength of the Taylor rolls λ can be modified within the range $\lambda/d \in [0.94, 4.01]$. The numerical results reveal, albeit at a lower Ta ($\approx \mathcal{O}(10^9)$), that the flow responds locally by emitting plumes from the rough stripes into the bulk, which is the mechanism responsible for the formation and pinning of the rolls. Globally, the system responds with a maximum of angular momentum transport when the roughness stripe width is $s/d = 0.93$; a result which is obtained from both experiments and numerics.

In chapter 4, we look at TC flow with a narrow gap width and its effect on the global and local responses. We perform both experiments and direct numerical simulations for the radius ratio of $\eta = r_i/r_o = 0.91$. Here $r_{i,o}$ are the inner and outer radii respectively, and the Taylor number spans $Ta \in [10^8, 10^{12}]$. The experiments reveal that it is possible to encounter not only one maximum angular transport, as it is commonly seen for $\eta < 0.9$, but also two angular momentum maxima whose magnitude and location in phase space depend on both Ta and the rotation ratio of the cylinders $a = -\omega_o/\omega_i$, where $\omega_{i,o}$ are the inner and outer angular velocities respectively. One of the peaks, termed the broad peak is located for corotating cylinders ($a < 0$), while the second peak is found for counter-rotation ($a > 0$) and is termed the narrow peak. We find that with sufficient driving, the narrow peak will surpass the broad peak, and eventually only one peak will be left for $Ta > 10^{10}$. The experiments reveal also that roll-splitting can occur due to the large value of the centrifugal forcing, which is also observed in the numerical simulations.

Summary (Spanish)

El flujo de Taylor-Couette (TC) es el flujo entre dos cilindros concéntricos que giran independientemente entre sí. Es junto a convección de Rayleigh Bénard, el flujo entre dos platos, uno calentado por debajo y el otro enfriado por arriba, uno de los flujos canónicos en donde la turbulencia puede ser investigada y puesta a prueba. En comparación con flujo de RB, en donde el flujo es impulsado por diferencias de temperatura, el flujo en TC es impulsado mecánicamente, lo que resulta más eficiente cuando se diseñan experimentos. En esta tesis, presentamos una investigación que tiene como objetivo un mejor entendimiento del efecto de burbujas de vapor en un flujo de TC con un número de Reynolds muy grande. Estas burbujas son el producto de hervir un líquido con un punto de ebullición muy bajo. Hemos diseñado un experimento, en donde el fenómeno de ebullición puede ser eficazmente controlado y monitoreado para cuantificar la respuesta física del flujo debido a las burbujas, en particular su resistencia al movimiento o fricción. Encontramos que las burbujas de vapor son igual de eficientes que las burbujas de aire para reducir la fricción en el flujo. Aproximadamente, 50% de reducción de fricción puede ser alcanzada cuando la fracción volumétrica de las burbujas es $\approx 8\%$, el número de Taylor es $Ta \approx \mathcal{O}(10^{12})$ y el número de Weber es $We > 1$. Adicionalmente al estudio de burbujas de vapor, en estas tesis estudiamos propiedades fundamentales de flujos turbulentos en el contexto de flujo de TC.

En el capítulo 1 medimos localmente el nivel de turbulencia del flujo en una región lejana a las paredes, que está descrito por un número de Taylor-Reynolds Re_λ y encontramos el escalamiento $Re_\lambda \propto Ta^{0.18}$, en donde $Ta \in [10^8, 10^{10}]$ es el número de Taylor que describe el impulso del flujo. Al describir el nivel de turbulencia de esta manera, es posible comparar la turbulencia local generada en flujo de TC con respecto a la producida por otros flujos canónicos como lo son el flujo en un tubo, en un canal, etc.

En el capítulo 2 medimos el campo de velocidad en el plano azimutal y radial, así como en la dirección axial y exploramos el mecanismo fundamental detrás

del transporte de momento angular. Las mediciones son llevadas a cabo antes y después de la llamada transición al régimen último de turbulencia. El número de Taylor cubre el intervalo $\mathcal{O}(10^6 - 10^{10})$. Hallamos que el transporte se lleva a cabo predominantemente en regiones de flujo exterior e interior, en donde estructuras de tamaño pequeño en forma de hongo son responsables de eventos intermitentes que aparecen en las funciones de densidad de probabilidad de momento angular. Esto es similar a lo que se observa en un flujo de RB. Al realizar una adecuada descomposición ortogonal, encontramos también ondas viajeras superpuestas que son parecidas a las presentes en el bien conocido régimen de vórtices de Taylor ondulados, que están presentes a números más bajos de Taylor.

En el capítulo 3, estudiamos el efecto de tener rugosidad de grano de arena aplicada al cilindro interior. Efectuamos ambos experimentos y simulaciones directo-numéricas del flujo, en donde la rugosidad es colocada en el cilindro interior en forma de bandas de ancho s . En los experimentos, el ancho de las bandas es $s/d \in [0.61, 3.74]$, donde d es el ancho de la brecha entre los cilindros, y el número de Taylor es $Ta \approx 10^{12}$. Al manipular adecuadamente el valor de s , encontramos que la longitud de onda que corresponde a los roles de Taylor λ puede ser modificada dentro del intervalo $\lambda/d \in [0.94, 4.01]$. Los resultados numéricos revelan, aunque a un número menor de Taylor ($\approx \mathcal{O}(10^9)$), que el flujo responde localmente al emitir estructuras en forma de hongo desde las bandas de rugosidad en dirección hacia la región del flujo que se encuentra lejos de las paredes, que es el mecanismo responsable de la formación y la fijación de los roles. Globalmente, el sistema responde con un máximo de transporte de momento angular cuando el ancho de las bandas de rugosidad es $s/d = 0.93$; un resultado que es corroborado por ambos experimentos y simulaciones numéricas.

En el capítulo 4, investigamos un flujo de TC con un ancho de brecha entre los cilindros pequeño y su efecto en las respuestas locales y globales. Efectuamos ambos experimentos y simulaciones directo-numéricas para el cociente de radios $\eta = r_i/r_o = 0.91$. En la ecuación anterior, $r_{i,o}$ representa el radio interior y exterior respectivamente. El número de Taylor cubre el intervalo $Ta \in [10^8, 10^{10}]$. Los experimentos revelan que es posible encontrar no sólo un máximo de transporte de momento angular, como es comúnmente observado para el caso de $\eta < 0.9$, sino dos máximos de transporte de momento angular cuyas magnitudes y posición en espacio fase dependen de ambos, el número de Taylor y el cociente de rotación de los cilindros $a = -\omega_o/\omega_i$. En la ecuación anterior $\omega_{i,o}$ representa las velocidades angulares del cilindro interior y exterior

respectivamente. Uno de los picos, denominado el pico ancho se encuentra cuando los cilindros giran en la misma dirección ($a < 0$), mientras que el segundo pico se genera cuando los cilindros giran en sentido opuesto ($a > 0$) y es denominado el pico estrecho. Encontramos que con suficiente impulso, el pico estrecho sobrepasa al pico ancho, y eventualmente sólo un pico puede ser encontrado para $Ta > 10^{10}$. Los experimentos revelan también que los roles pueden sufrir un rompimiento debido al valor tan grande del forzamiento centrífugo, que es también observado por las simulaciones numéricas.

Summary (Dutch)

Taylor-Couette stroming (TC) is de stroming tussen twee concentrische cilinders die onafhankelijk van elkaar kunnen draaien. Samen met Rayleigh-Bénard (RB) convectie, de stroming tussen twee platen waarvan de onderste verhit en de bovenste gekoeld wordt, zijn het twee canonieke stromingen die gebruikt wordt om metingen aan turbulente stromingen te doen en theorieën op de proef te stellen. In vergelijking met RB convectie, waar de stromingen thermisch wordt aangedreven, is de aandrijving van een TC stroming puur mechanisch. Deze manier is veel efficiënter en vergemakkelijkt het ontwerp van de experimenten. In dit proefschrift richten we ons onderzoek op het verkrijgen van een beter begrip over het effect van dampbellen op een TC stroming met hoge Reynoldsgetallen. Deze bellen worden gevormd door het koken van een vloeistof met een laag kookpunt. We hebben een experiment ontworpen, waarbij we gecontroleerd kunnen koken terwijl we de stromingrespons, met name de weerstand, als gevolg van de bellen, accuraat kunnen meten. We vinden dat dampbellen net zo efficiënt zijn in het verminderen van de stromingsweerstand als luchtbelletjes. We bereiken bijna 50% weerstandsvermindering bij een luchtvolumefractie van ongeveer 8% terwijl het Taylor-getal $Ta \approx \mathcal{O}(10^{12})$ en het Weber-getal $We > 1$.

Naast het onderzoek naar dampbellen, bestuderen we in dit proefschrift ook de fundamentele eigenschappen van turbulente TC stromingen. In hoofdstuk 1 meten we lokaal het turbulentieniveau in het midden van de stromingen en beschrijven dat door middel van een Taylor-Reynolds-getal Re_λ . We vinden een schaling $Re_\lambda \propto Ta^{0.18}$, waarbij het Taylor-getal $Ta \in [10^8, 10^{12}]$ de aandrijving van de stroming beschrijft. Door het turbulentieniveau op deze manier te beschrijven kan men de lokale turbulentie die wordt gegenereerd in TC stromingen vergelijken met andere canonieke stromingen, zoals de stroming door leidingen, kanaalstromingen, enz.

In hoofdstuk 2 meten we het snelheidsveld in het azimutale-radiale-vlak en in de axiale richting en onderzoeken we het onderliggende mechanisme van het

impulsmomenttransport. De metingen worden uitgevoerd voor en na de overgang naar het zogenaamde ultieme regiem van turbulentie, waarbij het Taylor-getal in het bereik $\mathcal{O}(10^6\text{--}10^{10})$ zit. We vinden dat het transport meestal beperkt is tot de uitstroom- en instroomgebieden, waar kleinschalige pluimen verantwoordelijk zijn voor intermitterende gebeurtenissen die verschijnen in de lokale-warmtetransportwaarschijnlijkheidsdichtheidsfunctie. Dit is vergelijkbaar met wat in RB stromingen wordt gezien, en benadrukt nogmaals de krachtige analogie tussen TC en RB stromingen. Door een hoofdcomponentenanalyse uit te voeren, vinden we ook gesuperponeerde azimuthale reizende golven die vergelijkbaar zijn met de welbekende *golvende* Taylor-draaikolken die aanwezig zijn bij lagere Taylor-getallen.

In hoofdstuk 3 brengen wij zandkorrelruwheid aan op de binnencilinder en onderzoeken wij het effect op de structuur en weerstand van de stroming. We voeren zowel experimenten als numerieke simulaties uit, waarbij ruwheid op de binnencilinder wordt geplaatst in de vorm van strepen met een breedte s . In de experimenten is de breedte van de strepen $s/d \in [0.61, 3.74]$, waarbij d de spleetbreedte is en het Taylor-getal $Ta \approx 10^{12}$. Door het adequaat manipuleren van s vinden we dat de golflengte van de Taylor-rollen λ kan worden gewijzigd binnen het bereik $\lambda/d \in [0.94, 4.01]$. De resultaten van de numerieke simulaties onthullen, zij het bij een lager Ta ($\approx \mathcal{O}(10^9)$), dat de stroming plaatselijk reageert door pluimen van de ruwe strepen uit te stoten. Dit is het mechanisme die verantwoordelijk is voor de vorming en het vastspelden van de rolstructuren. Globaal reageert het systeem met een maximum aan impulsmomenttransport wanneer de breedte van de ruwheidsstreep $s/d = 0.93$ is; een resultaat dat zowel uit experimenten als numerieke simulaties is verkregen.

In hoofdstuk 4 kijken we naar een TC-stroming met een smalle spleetbreedte en het effect ervan op de globale en lokale respons. Zowel experimenten als numerieke simulaties zijn uitgevoerd voor een radiusverhouding van $\eta = r_i/r_o = 0.91$. Hier zijn $r_{i,o}$ respectievelijk de radius van de binnen- en buitencilinder, en het Taylor-getal is in het bereik van $Ta \in [10^8, 10^{12}]$. De experimenten laten zien dat het mogelijk is om niet een enkel maximum in het impulsmomenttransport te hebben, zoals vaak wordt gezien voor $\eta < 0.9$, maar dat een tweede maximum ook mogelijk is. Bij dit tweede maximum is de grootte en locatie in de faseruimte afhankelijk van zowel Ta als de cilinderrotatieverhouding $a = \omega_i/\omega_o$, waarbij $\omega_{i,o}$ respectievelijk de hoeksnelheden van de binnenste en buitenste cilinder zijn. Één van deze pieken, aangeduid als de brede piek, bevindt zich voor meedraaiende cylinders ($a < 0$), terwijl de tweede piek zich bevindt voor tegendraaiende ($a > 0$). Deze laatste wordt de smalle piek ge-

noemd. We vinden dat met voldoende aandrijving, de smalle piek de brede piek zal overtreffen, en uiteindelijk zal er slechts één enkele piek overblijven voor $Ta > 10^{10}$. Uit de experimenten blijkt ook dat rolsplitsing kan optreden als gevolg van de grote waarde van de centrifugale aandrijving, welke ook wordt waargenomen in de numerieke simulaties.

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You know, almost five years have passed since I joined the Physics of Fluids group; and during that time, I had the opportunity (one would say privilege) to attend quite a few PhD defenses of people in the group. At first, I remember being completely overwhelmed by the sheer amount of work done by these people. So many experiments, so many simulations, so many plots and so many other things I had very little idea about. As a very naive master student, I kept asking to myself -“How can someone produce this amount of work and in such great detail?” Five years later, I am in the position of standing in front of you and defend my very own thesis. To be quite honest with you, sometimes I have the feeling of not knowing how I got here, but there is something, however, I am quite sure of. And that is I would not have made it without the help of so many people. A small disclaimer though. Those who know me, should be well aware by now, that my memory skills are really not the best. As a consequence, the following is bound to have some forgotten names for which I apologize in advance.

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The next person in line is C. Sun. Dear Chao, you have been with me (and the rest of the TC team for that matter) since day one, and there was virtually no day where you were not available for helping us out; even though you were on the other side of the planet. Your optimism while conducting research is worth admiring and enviable. I truly enjoyed working by your side all these

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project. It was really nice working together with you, and I am pretty sure, our results will turn into a very nice publication.

As a PhD student you get to attend many conferences worldwide, and by doing so, you get to talk to many people from different research groups. Over the years I have come to realize, that truly, there is no other place like PoF. The amazing *science* that seems to be always taking place, is only surpassed by the amazing people that spend their days (and sometimes nights) under the roofs of the first and second floor of the Meander building in the UT. Regardless of the situation, or any other kind of problem someone may be facing with, there is always someone in PoF willing to give a hand. We should all be proud to be part of something like that, and in fact, we should celebrate it more often. Let me start by thanking J. Leferink. Dear Joanita, there are very little words I can say to you that haven't been said before. Nevertheless, I will make my best and thank you for all your amazing support over the years. Whether it was something rather insignificant, or something terribly complicated you were always there to help out. Hidden in plain sight, dear Joanita, you are truly, a fundamental member of this big family. Next, I would like to thank the amazing people in office ME257. Let me start with the man, the legend, the guy who makes everything possible and prevents everything from failing. Dear Gert-Wim, I think everyone would agree with me, when I say, that there could be an entire book dedicated only to acknowledgments for you. We fought hard with boiling in the BTTC; and even when everything seemed lost, you would always come up with a great solution along with a big smile on your face. Without you, there would be no experimental PoF, simple as that. I want to thank you for putting up with all my annoying questions over the years, and for your patience and willingness to help me out with all the setups. I am in debt to you, and wish you a very speedy recovery. Dear Bas, on the subject of annoying requests, I would also like to thank you for all the -"Hey Bas, do we have this cable?" over the years. You were always very kind and helpful. I too, really enjoyed all the fun and jokes we had; Carola should remember very well for example. It was great to play on that awesome amplifier of yours too. Finally, I would like to thank M. Bos for all his great technical support to both of the Taylor-Couette setups.

Now, to the matter of my partners in crime. Let me start with D. Bakhuis. -"Hi Dennis, If you're reading this while I am responding to the committee questions, I just wanted to let you know that, it's not so bad as it looks. You'll do just fine, don't worry about it. We got this." Looking back, I think it's pretty fair to say that we have gone through worse. Not only we shared

an office for over 5 years now, but we shared joy, pain, despair, happiness and all those little caveats that only a PhD brings. Thanks for all the amazing trips, beers, plots, and so many things that I probably already forgot. I am very sorry (sort of) I never made it to Crossfit though. You are the best non-french, non-iranian office mate that anyone could ask for...well except maybe when you bring a new keyboard to the office. Seriously, why don't you just get a normal one? Next in line is R. Verschoof. Although we never really collaborated together, I want to thank you, Ruben, for all your help while assembling and disassembling the setups (which almost never happened, right?). I really admire your overall calmness and *chill* mode of doing things. It was great to have you around in the TC team. Thanks also to P. Berghout. Dear Pieter, It was great working together with you on the roughness project. I really enjoyed all the fun and the actual scientific discussions we had. Here, I would also like to thank Roeland van der Veen for helping me out with the setup at the very beginning of my PhD, and for keeping such a fun environment in the lab. And last but not least, is our very own P. Bullee. Dear Pim, not only you are an amazing engineer, scientist, musician, chef, and who knows what else, but you are also a great *life connoisseur*. I really enjoyed working with you, and I wish you all the best for the rest of your PhD. I know you'll do great!

We move now to the most awesome office in the entire building: ME 246B. If you don't agree with me, that is simply because you never had the chance to spend 4 years of your life with these people. Dear Elise, even though you stole my spot for two years (and not because I totally overslept the day we moved offices, so you went and picked the best one without asking me first), I could not imagine a better non-dutch, non-iranian person as an office mate. I really do miss you around in the office. Thanks for all the french lessons, all the complaining seminars on dutch life, and for all the nice drinks, parties, hard times, good times we had together. You are an amazing friend and I wish you and Paul a lifetime of happiness. Dear Mazi, to be quite honest, I had a really bad time writing this down....I know right? I couldn't really come up with something that would make you justice. So bear with me for a second and take the compliment, will you? I am very happy you came to PoF. I guess It doesn't happen very often, your office mate becomes your best bro. So, let me thank you for always being there for me, whether it was for science or any other silly thing. Salma is indeed a very lucky girl, and I wish you both an entire life full of *perritos*. And finally, hey Martin! Thanks for keeping such a nice and easy-going environment in the office. Thanks for all the nice talks

about science and movies....ahem, I mean *film*. What an *uncultured* swine I must be.

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With love,
Rodrigo
January 2019

Bio

Rodrigo Ezeta was born on November the 20th, 1987 in Mexico City, Mexico. He completed his basic education in the year of 2006 and moved on to continue his undergraduate studies on Physics at the Faculty of Sciences from the National Autonomous University of Mexico (UNAM). At UNAM, his first approach to the field of fluid mechanics was when he followed a course on Acoustics, which was lectured by Prof. dr. Pablo Luis Rendón Garrido. As a music *aficionado* himself, he was tremendously fascinated by the physical mechanisms behind musical instruments and the mathematical elegance of musical theory. In 2012, and under the guidance of Prof. Rendón, he went on to obtain a degree in Physics with the thesis entitled “Nonlinear sound propagation in trumpets”. In order to further pursue his affinity for fluid mechanics, he moved in 2014 to the Netherlands to begin his master studies in Applied Physics at the Physics of Fluids group of the University of Twente (UT). He graduated in 2014 with the thesis entitled “Explosive boiling in open capillaries” under the supervision of Dr. Sander Wildeman, Prof. Chao Sun and Prof. Detlef Lohse. In the winter of 2014, he started his doctoral studies at Twente on the topic of boiling in high-Reynolds Taylor-Couette flow under the supervision of Prof. Chao Sun and Prof. Detlef Lohse.

Scientific output

Publications

- **2018:** *Turbulence strength in ultimate Taylor-Couette flow*
Rodrigo Ezeta, Sander G. Huisman, Chao Sun, Detlef Lohse
J. Fluid Mech 836, 397-412
- **Submitted:** *Small-scale statistics of momentum transport and underlying flow structures in turbulent Taylor-Couette flow*
Andreas, Froitzheim, **Rodrigo Ezeta**, Sebastian Merbold, Sander G. Huisman, Chao Sun, and Detlef Lohse
J. Fluid Mech
- **Submitted:** *Vapor bubble drag reduction in boiling Taylor-Couette turbulence*
Rodrigo Ezeta, Sander G. Huisman, Dennis Bakhuis, Chao Sun, and Detlef Lohse
Proc. Natl. Acad. Sci. USA
- **Submitted:** *Statistics of rigid fibers in strongly sheared turbulence*
Dennis Bakhuis, Varghese Mathai, Ruben A. Verschoof, **Rodrigo Ezeta**, Sander G. Huisman, Detlef Lohse, and Chao Sun
- **In preparation:** *Controlling the secondary flows in turbulent Taylor-Couette flow using spanwise varying roughness: experiments and simulations*
Dennis Bakhuis, **Rodrigo Ezeta**, Pieter Berghout, Pim. A. Bullee, Dominic Tai, Daniel Chung, Sander G. Huisman, Roberto Verzicco, Detlef Lohse and Chao Sun
- **In preparation:** *Local maxima of angular momentum transport in small gap $\eta = 0.91$ Taylor-Couette turbulence*
Rodrigo Ezeta, Dennis Bakhuis, Francesco Sacco, Sander G. Huisman, Rodolfo Ostilla-Mónico, Roberto Verzicco, Chao Sun, and Detlef Lohse

- **In preparation:** *Bubbly drag reduction with sandgrain roughness boundaries*
Pim A. Bullee, Dennis Bakhuis, **Rodrigo Ezeta**, Sander G. Huisman, Chao Sun, Rob G. H. Lammertink and Detlef Lohse
- **In preparation:** *Catastrophic Phase Inversion in Turbulent Taylor–Couette flow*
Dennis Bakhuis, **Rodrigo Ezeta**, Pim A. Bullee, Alvaro Marin, Detlef Lohse, Sander G. Huisman, and Chao Sun