Zhu *et al.* Reply: In their Comment [1] Doering *et al.* question our numerically found [2] onset of a transition to the ultimate regime of 2D Rayleigh-Bénard (RB) convection. We disagree with their reasoning.

To irrefutably settle the issue, we have extended our numerical simulations of Ref. [2] to even larger Ra, namely, now up to $Ra = 4.64 \times 10^{14}$, sticking to the same strict numerical resolution criteria of both boundary layer (BL) and bulk. The simulation at the highest Ra was performed with a grid resolution of $31\,200 \times 25\,600$ with 28 points in the boundary layer. The evidence for the transition to the ultimate regime remains overwhelming:

(1) On the global heat transfer: Nu (Ra), compensated with $Ra^{0.357}$, is shown in Fig. 1(a). An objective least squares fit of an effective power law $Nu \sim Ra^\gamma$ to the last 6 data points gives a scaling exponent $\gamma = 0.345$; the last 5 data points give $\gamma = 0.345$, the last four data points give $\gamma = 0.352$, the last 3 data points give $\gamma = 0.357$, and the last 2 points give $\gamma = 0.358$; i.e., no matter how the data are interpreted, the scaling exponent is *always* larger than $1/3$ and monotonically increasing with Ra.

(2) The key part of Ref. [2] deals with *local* properties of the flow; see Figs. 2–4 of that Letter. For the local heat flux in the plume ejecting regime, beyond 10^{13} the effective

scaling exponents are close to $\gamma = 0.38$, see Fig. 1. In contrast, it remains $\leq 1/3$ in the plume impacting regime, which therefore with increasing Ra loses more and more relevance for the overall heat transfer.

(3) Beyond 10^{13} , the horizontal velocity profiles $u^+(y^+)$ in the BLs become logarithmic (see Fig. 2 of Ref. [2]), signaling a turbulent BL, which is characteristic for the ultimate regime (as a presumption to derive the ultimate regime scaling in Refs. [3,4]), rather than one of laminar type as in the classical regime.

(4) Finally, the transition to ultimate RB turbulence in the numerical data of Ref. [2] has also been confirmed through an extended self-similarity (ESS) analysis of the temperature structure functions; see Ref. [5]. In that paper we find no ESS scaling before the transition. However, beyond the transition and for large enough wall distance $y^+ > 100$, we find clear ESS behavior, as expected for a scalar in a turbulent boundary layer. Therefore, also that analysis provides strong evidence that the observed transition in the global Nusselt number around $Ra = 10^{13}$ indeed is the transition from a laminar type BL to a turbulent type BL.

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FIG. 1. (a) Nusselt number compensated by $\gamma = 0.357$, i.e., a scaling exponent larger than the $\gamma = 1/3$ necessary to prove the transition. We took $\gamma = 0.357$ for the compensated plot as with that value the last three data points show a plateau. The error bars for the data are smaller than the symbols. The inset shows the effective scaling exponent γ , obtained from a power law, fits $Nu \sim Ra^\gamma$ to the last k data points in the main figure. It is always larger than $1/3$, no matter how one interprets the data. (b) The local heat transfer in the plume emitting region (with effective slope 0.38) and in the plume impacting region.

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