Quasiparticle Trapping, Excitation, and Tunneling Times in Josephson Junctions with Spatially Inhomogeneous Electrodes

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On the basis of a microscopic model of the proximity effect in an SS'-sandwich we have calculated effective trapping, excitation, and tunneling rates of the reduced gap region in an SS'IS''S junction as a function of temperature, voltage over the junction and strength of the proximity effect. We developed a simple model to describe the effect of magnetic fields on the gap of the superconductor, and calculated the trapping rate as function of the field. The experimentally determined gap reduction and quasiparticle loss times as function of the field were described in terms of this model.

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1. INTRODUCTION

For the development of junction X-ray detectors there is a strong need for a quantitative model of the non-equilibrium superconductivity processes going on in the absorber after the X-ray photon impact. In a detector with a trapping layer, consisting of a proximity sandwich of two superconductors with different bulk energy-gaps, and used to enhance the quasiparticle (qp) collection and tunneling rate,¹ there is an exchange of qps between the bulk of the absorber and the trap. This system may be described with two sets of Rothwarf-Taylor (RT)-equations, for the bulk and the trap respectively, coupled by an effective trapping time τ_{tr} and excitation time τ_{exc} . Additionally an effective time constant τ_{tun} for tunneling out of the trap is needed in the RT-modeling.

The application of a magnetic field to junctions introduces also spatial modulation of the superconducting properties: the gap in the trapping layer will be reduced even more, changing all its time constants. An additional trapping-layer is created at e.g. the substrate-absorber interface, giving rise to qp loss.

In this paper we will give the results of a model from which the effective time constants are calculated as a function of temperature, voltage over the junction, and

strength of the proximity effect. Secondly, the effect of a magnetic field is described in the low temperature limit. The field dependence of experimentally determined gap reductions and loss times are described with this model.

2. THEORY

A microscopic model of the proximity effect in a SS'-sandwich of dirty superconductors was developed for the case of a thin S'-layer ($d \ll \xi_{S'}$) and a thick S-layer ($d_S \gg \xi_S$).² In the limit of good electrical contact between S and S', i.e. opposite to that of McMillan, where a finite potential barrier at the SS'-interface is assumed, the proximity effect can be described with two parameters, $\gamma_m = (\rho_S \xi_S / \rho_{S'} \xi_{S'}) / (d / \xi_{S'})$ and the critical temperature ratio T'_c / T_c . The calculations in this paper are for a Nb/Al-sandwich with $T'_c / T_c = 0.14$, but a variation of the T_c -ratio for other metal combinations can be taken into account by a renormalization of γ_m .²

From this model we can calculate the normalized qp and pair densities of states $\bar{N}$ and $\bar{P}$, as a function of the distance x from the SS'-interface and energy ϵ . These functions enter into the spatial dependent expressions for the time constants for qp scattering with emission (τ_e), respectively absorption of a phonon (τ_a), and qp recombination (τ_r), which are extensions of the well-known Kaplan expressions for the spatially homogeneous superconductor.^{3,4}

2.1. Quasiparticle trapping and excitation in an SS'-sandwich

The trapping time constant τ_{tr} of a single qp (with energy $\epsilon > \Delta_0$, Δ_0 is the bulk gap in S) into the reduced gap region of an SS'-proximity sandwich, due to phonon emission, can now be defined as⁵

$$\frac{1}{\tau_{tr}} = \frac{1}{N_{>}^{tot}} \left| \frac{dN_{>}^{tot}}{dt} \right| = \frac{1}{N_{>}} \left| \frac{dN_{>}}{dt} \right| \left| \frac{1}{1+\alpha} + \frac{1}{N'_{>}} \left| \frac{dN'_{>}}{dt} \right| \frac{\alpha}{1+\alpha} \right|, \quad \alpha = \frac{N'_{>}}{N_{>}} \quad (1)$$

Here $N_{>}^{tot}$, $N_{>}$, and $N'_{>}$ are the total number of qp with energy $\epsilon > \Delta_0$, respectively the qp numbers in S and S'. The trapping rate in S is

$$\left(\frac{1}{\tau_{tr}} \right)_S = \frac{1}{N_{>}} \left| \frac{dN_{>}}{dt} \right| = \int_0^{d_S} dx \int_{\Delta_0}^{\infty} d\epsilon \frac{\bar{N}(\epsilon, x) f(\epsilon)}{\tau_e(\epsilon, x)} / \int_0^{d_S} dx \int_{\Delta_0}^{\infty} d\epsilon \bar{N}(\epsilon, x) f(\epsilon) \quad (2)$$

and similarly for $(\tau_{tr}^{-1})_{S'}$. In the expression for τ_e^{-1} one takes only into account the scatter processes in which a phonon is emitted with energy $\Omega \geq (\epsilon - \Delta_0)$. Introducing an effective length L_{eff} of a trap with zero gap which gives the same total scattering rate as the reduced gap region in S by the relation $(\tau_{tr}^{-1})_S = (L_{eff}(T)/d_S)(\tau_e^{-1}(\Delta_g = 0)) = (L_{eff}(T)/d_S)(1.82/\tau_0)$, and analogously a length L'_{eff} for the S' layer,^{4,5} we obtain finally ($\alpha \ll 1$),

$$\frac{1}{\tau_{tr}} = \frac{1.82}{d_S} \left[\frac{L_{eff}(T, \gamma_m)}{\tau_0} + \beta \frac{L'_{eff}(T, \gamma_m)}{\tau'_0} \right] \quad (3)$$

$\tau_0(\tau'_0)$ are the material dependent time constants of the S (S') metal, as defined by Kaplan.³ The factor $\beta = \alpha(d/d_S)$ is equal to $(v_F/v_{F'})(B'/B)$, with v_F ($v_{F'}$) the Fermi-velocity of S (S'). B'/B is nearly equal 1 for $\gamma_m \ll 1$ and equal

Fig. 1. (a) Effective trapping length (L_{eff}) and (b) excitation rate $(\tau_0/\tau_{exc})_S$ in S as function of temperature for various values of γ_m .

$(2k_B T/\pi\Delta_0)^{1/2}$ for $\gamma_m \geq 1$ and low T . If the area A of S is larger than the area A' of S', τ_{tr}^{-1} is reduced with the factor A'/A , assuming that the fraction of time the qp spend in the reduced gap region is proportional to A'/A . In fig.1 L_{eff} is shown as function of temperature and strength of the proximity effect. For small γ_m the effective length, and thus the trapping rate is very much temperature dependent, reaching a T^2 -dependence at higher T . This reflects the increase of the number of high-energetic particles with higher $\tau_e^{-1}(\epsilon, x)$ when temperature is increased. The trapping rate increases with γ_m , since the trap becomes deeper. L_{eff} shows nearly the same dependence on temperature and γ_m , but is normalized on d instead of ξ_S .

A similar procedure is applied to derive an effective excitation rate out of the trap, resulting in⁵

$$\frac{1}{\tau_{exc}} = \left(\frac{\tau_0}{\tau_{exc}} \right)_S \frac{1}{1+\delta} \frac{1}{\tau_0} + \left(\frac{\tau'_0}{\tau'_{exc}} \right)_{S'} \frac{\delta}{1+\delta} \frac{1}{\tau'_0} \quad (4)$$

with $\delta = N'_</N_< = (v_F/v_{F'}) (E'/E)$ the ratio of the number of qp in the trap in S' and that in S. E'/E is related to the effective tunneling length discussed further on, by $E'/E = d/L_{eff-t}^*(T, \gamma_m)$. In fig.1 the excitation rate $(\tau_0/\tau_{exc})_S$ in S is shown. The rate in S' has a similar dependence on temperature and γ_m . The strong temperature dependence reflects the increasing number of high-energetic phonons that can excite the qp out of the trap.

2.2. Tunneling rate in an SS'IS"-S-junction

The tunneling rate has to be considered separately for electron and hole-like qps, due to the different voltage dependence. Considering first the tunneling time $\tau_{tun,12}^e$ of an electron-like qp out of the trap from electrode 1 to electrode 2 in an (SS')₁I(S'')₂ junction. The rate $(\tau_{tun,12}^e)^{-1} = |dN_{<,1}^e/dt|/N_{<,1}^e$ can be expressed in terms of the tunneling current out of the trap $I_{<,12}^e = e|dN_{<,1}^e/dt|$, which depends on the densities of states at the S'I and IS'' interfaces and the voltage V over the junction. In the low temperature regime one can take the voltage dependent terms out of the tunneling integral and an effective tunneling length L_{eff-t}^* can be defined

Fig. 2. (a) Effective tunneling length L_{eff-t}^* of the reduced gap region in an SS'-sandwich versus temperature for various γ_m -values. (b) $I_{<,12}^h$, $I_{<,12}^e$, and $I_{<,12}^\Sigma$ versus bias voltage, proportional to the tunneling rate $(\tau_{tun,12}^e)^{-1}$. The solid curves give the exact dependence, the dashed curves the approximate dependence through $\bar{N}_2$.

that is only dependent on temperature and γ_m .⁵ With the area of the junction equal to the area A' of S' , the tunneling time becomes

$$\tau_{tun,12}^e(V, T, \gamma_m) = 4e^2 R_N A' \frac{N_1(0) L_{eff-t}^*(T, \gamma_m) + N_1'(0)d}{\bar{N}_2(\Delta_{1g} + eV, x = -d)} \quad (5)$$

Thus the voltage dependence of the tunnel time reflects the density of states $\bar{N}_2$ of the electrode to which the qps tunnel. In the spatially homogeneous case this expression reduces to that of Ivlev⁶ for the tunneling between homogeneous electrodes with different gaps and to the Ginsberg result for similar electrodes in the high voltage regime.⁷ It can be shown straightforwardly that the tunneling time for holes is related to that of the electron-like qps, by the relation $\tau_{tun,12}^h(V, T) = \tau_{tun,12}^e(-V, T)$. The total current of positive charge from electrode 1 to 2 is thus $I_{<,12}^\Sigma = I_{<,12}^h - I_{<,12}^e$. In fig.2 the temperature dependence of L_{eff-t}^* is shown, as well as the voltage dependence of τ_{tun}^{-1} , through the currents $I_{<,12}$, which reduces to the dependence through the term $\bar{N}_2(\Delta_{1g} + eV, x = 0)$ at low T.

2.3. Gap-reduction and quasiparticle loss by magnetic field

For magnetic fields H less than H_{c1} a dirty type II superconductor is in the Meissner state in which screening currents are induced in a surface layer with thickness λ_L . These currents act as a pairbreaking mechanism, giving rise to a reduction of the order parameter in the surface layer. The pairbreaking parameter is given by⁸

$$\alpha_p = \frac{\gamma^*}{2} \Delta_0 (H/H_c)^2 \quad (6)$$

with $\gamma^* \approx 1.78$ the Euler constant. The reduction of the gap in the density of states is then⁹

$$\Delta_g(\zeta) = \Delta(1 - \zeta^{2/3})^{3/2}, \quad \zeta < 1 \quad (7)$$

Fig. 3. (a) Measured energygap as function of Al-layer thickness d_1 at the BE ($\square$), and at the CE with $d_2 = 3$ nm ($\triangle$). The sloped curve gives the fit with the proximity theory; the horizontal line is a guide for the eye for Δ_{g2} . (b) Measured gapvoltages of BE ($\square$) and CE ($\triangle$) as function of applied field. The lines are guides for the eye for Δ_{g1} .

where $\zeta = \alpha_p/\Delta = \zeta_0(\Delta_0/\Delta)$ with $\zeta_0 = \alpha_p/\Delta_0$. The reduction of the order parameter is given by

$$\Delta = \Delta_0 e^{-\pi\zeta/4} \quad (8)$$

For $\zeta \ll 1$ this gives a gap reduction which is nearly linear with ζ_0 and thus quadratic in H . The trapping rate τ_H^{-1} in the surface layer at zero temperature was estimated as⁸

$$\frac{1}{\tau_H} = \frac{1.82}{\tau_0} \frac{L_H}{d_s} (2 - A'/A) \quad (9)$$

where the effective trapping length is approximately $L_H \approx 1.3\lambda_L(H/H_c)^p$ with $p \approx 10/3$.

3. EXPERIMENTS

The gapvoltages Δ_{g1} and Δ_{g2} of the base (BE) and counter (CE) electrodes of a series of $(\text{Nb}/\text{Al})_1/\text{Alox}/(\text{Al}/\text{Nb}/\text{Nb})_2$ tunnel junctions with $d_1 = 5, 10, 15, 20$, and 25 nm (junctions 1 to 5) and $d_2 = 3$ nm were determined, as a function of the applied magnetic field and at low temperatures. In fig.3 Δ_{g1} versus d_1 is shown for $H = 0$, together with the fitted theoretical curve, obtained from the proximity effect model. From the fit we have $\gamma_m/d_1 \approx 0.032$ nm $^{-1}$. Δ_{g2} is smaller and consequently the γ_{m2} -value of the CE is larger than one should expect from the Al $_2$ -layer thickness. This is ascribed to degraded properties of the double Nb $_2$ -layer. Fig.3 shows the field dependence of $\Delta_{g1}(H)$ and $\Delta_{g2}(H)$ for the junctions 1, 2, and 5. It is seen that the gaps decrease with H^2 in accordance with eqs.(6-8). With increasing d_1 the gap Δ_{g1} decreases more with H . This is ascribed to an decrease of H_c with d_1 , hence an increasing pairbreaking parameter α , because the properties of the surface of the SS'-sandwich becomes more like those of Al, which has a much smaller value of H_c (≈ 100 Oe) than Nb (≈ 2000 Oe).

X-ray measurements were performed on junctions for various fields. From the measured gapvoltages γ_m -values were determined from which the time-constants of

Fig. 4. Measured quasiparticle losstime τ_{loss} in the BE of junction 5 ($d_1 = 25$ nm). The solid (dashed) curve is a fit with $p = 4.7$ (3.7).

the trap were estimated.¹⁰ With these values a loss-time constant τ_{loss} of the BE could be determined from the signal output of the junction. In fig.4 τ_{loss} versus H is shown, together with two fits with the equation $\tau_{loss}^{-1} = \tau_H^{-1}(H, p) + \tau_1^{-1}$, with τ_1^{-1} some field independent loss rate, which dominates the loss process at low fields. From these fits we obtain $p \approx 3.7 - 4.7$ and $\tau_1 \approx 150$ nsec. From the field dependence of τ_{loss} we conclude that this loss process is probably due to trapping of qps in reduced gap regions at the surface of the BE, induced by the applied field. τ_1 is ascribed to a degraded surface layer at the interfaces with substrate and insulation layers.

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