AN ANALYTICAL FORMULATION FOR THE LATERAL SUPPORT STIFFNESS OF A SPATIAL FLEXURE STRIP

M. Nijenhuis[1], J.P. Meijaard[2], J.L. Herder[1], S. Awtar[3], D.M. Brouwer[1]

1) Mechanical Automation and Mechatronics
University of Twente
Enschede, The Netherlands
m.nijenhuis@utwente.nl

2) Olton Engineering Consultancy
Enschede, The Netherlands

3) Precision Systems Design Lab
University of Michigan
Ann Arbor, USA

Abstract

This paper presents a framework for modeling the deformation and stiffness characteristics of static 3-D flexure strips (leaf springs), based on a discrete beam model that is suited for analytical calculations. As a case study, a closed-form parametric expression is derived for the lateral support stiffness of a parallel flexure mechanism.

Continuous model

A spatial Timoshenko beam with Reissner's finite strain measures — capturing shear, bending and torsion deformation — and linear elastic material behavior serves as a model for flexure strips.

Discrete model

A discretized version of the continuous model has been implemented in numeric flexible multibody software as a two-node beam element [1]. It is observed that a single such element captures stiffness characteristics of spatially deforming flexure strips with reasonable accuracy, owing to the inclusion of finite strain measures. As the mathematics of a single element remain comprehensible, the discrete model is well-suited for closed-form analysis. The available software implementation then serves as a calculation aid that facilitates the analytical modeling process.

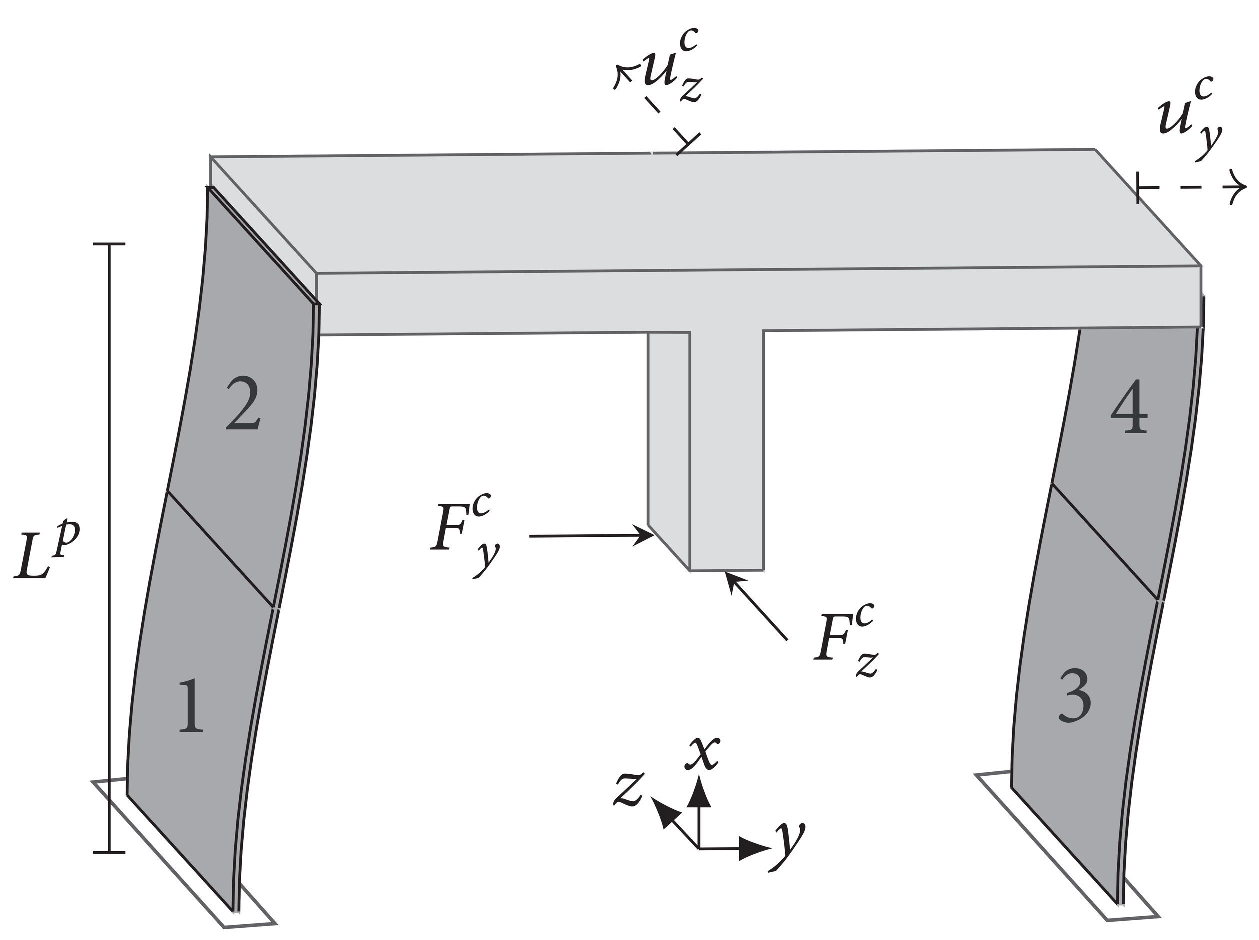

Figure 1: Parallel flexure mechanism modeled by four beam elements (1–4). Forces are applied in the center of compliance. The lateral support stiffness (in z-direction) is investigated.

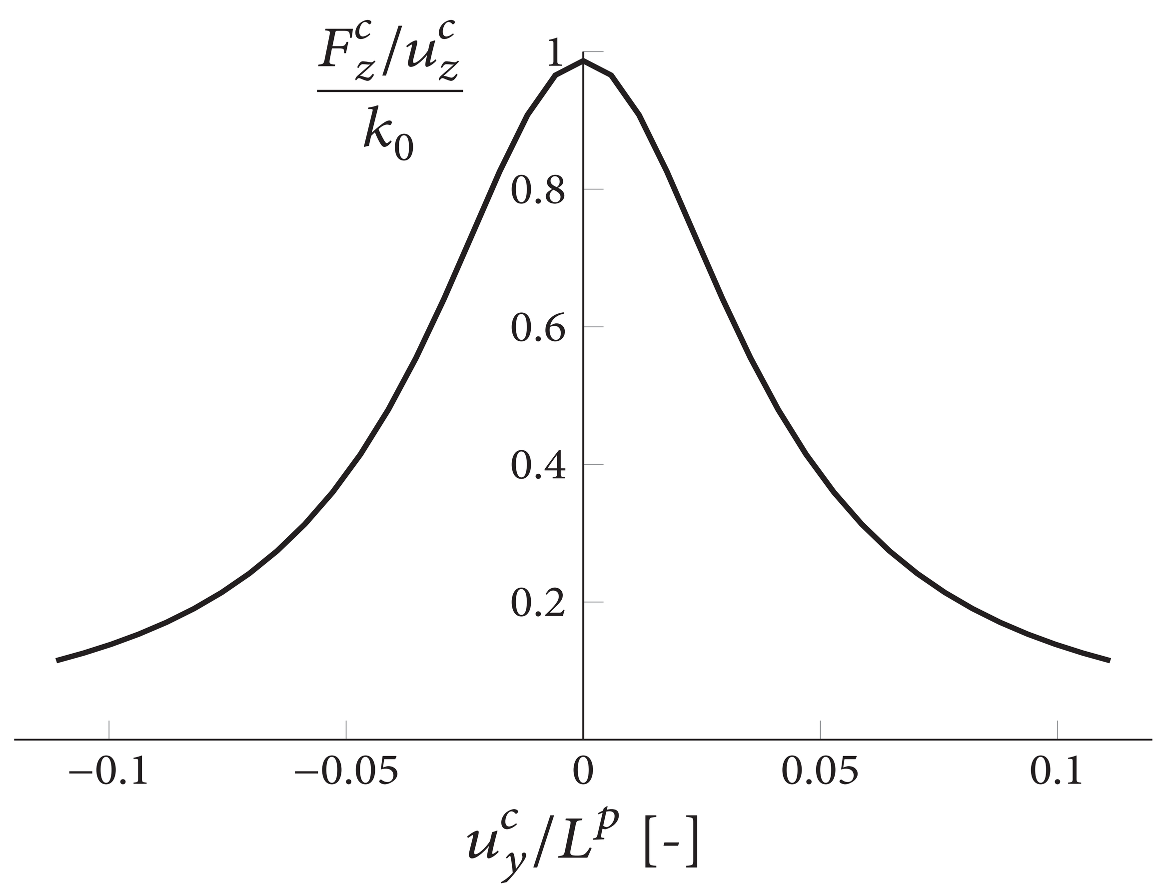

Figure 2: The (normalized) lateral support stiffness decreases significantly with (normalized) DOF displacement.

Case study: parallel flexure mechanism

When a parallel flexure mechanism (figure 1) moves in the degree of freedom, the stiffness characteristics deteriorate: the lateral support stiffness decreases (figure 2). By using four discrete beam elements, a case-specific improvement of the torsion interpolation, and an approximation of the equilibrium configuration, this behavior is captured by the simple closed-form expression

$$\frac{F_z^c}{u_z^c} = \left(\frac{\left(L^p\right)^3}{24EI_y} + \frac{L^p}{2k_z GA} + \frac{L^p}{56GJ}\left(u_y^c\right)^2 \right)^{-1},$$

where F_z^c/u_z^c is the lateral support stiffness, u_y^c the DOF displacement, and the other parameters have conventional definitions.

Conclusion

By incorporating a geometric non-linearity due to torsion and an effective torsional stiffness due to constrained cross-sectional warping, a compact parametric expression is obtained that gives insight into the lateral support stiffness of a parallel flexure mechanism. It is validated against FEA for parameters of practical interest.

References

[1] Jonker et al. *Int. J. Nonlinear Mech.* **2013**, *53*, 63.