A CLOSED-FORM MODEL FOR THE SUPPORT STIFFNESS OF SPATIAL FLEXURE STRIPS WITH LIMITED TWIST

M. Nijenhuis, D.M. Brouwer

Mechanical Automation and Mechatronics, University of Twente, Enschede, The Netherlands
m.nijenhuis@utwente.nl, d.m.brouwer@utwente.nl

Abstract

The stiffness characteristics of flexure strips are important in the design and analysis of mechanisms that they constitute. This paper presents a closed-form parametric nonlinear model that describes the spatial error motions of a fixed-free flexure strip as a function of an arbitrary 3-D end-load.

Method

Euler-Bernoulli beam theory with quadratic strain and curvature measures, and linear material behavior serve as a basis.

We have observed that many flexure mechanisms in practice contain flexure strips that experience only limited torsion deformation. Leveraging this, the general deformation is distinguished into

1. low-stiffness large-deflection motion, occurring in the principal deformation plane (PDP) of Fig. 1.
2. high-stiffness small-deflection motion, perpendicular to the PDP.

By assuming that only the motion in the PDP contributes to load-equilibrium, the deformed configuration admits an explicit solution as a function of the degrees of freedom and the applied loads. Closed-form expressions for the spatial error motions are then obtained as small disturbances with respect to the main deformed shape in the PDP.

For a practical formulation, the equations are formatted in terms of

- degrees of freedom (DOF) $\bar{u}_y$, $\bar{\phi}_z$, $\bar{\phi}_x$,
- applied loads $\bar{F}_x$, $\bar{F}_z$, $\bar{M}_y$, and
- error motions $\bar{u}_x$, $\bar{u}_z$, $\bar{\phi}_y$.

Results

The expressions for the three error motions have the same structure, consisting of

- common linear elastic terms,
- nonlinear “kinematic” terms, quadratic in the DOF, and
- nonlinear “elastokinematic” terms, linear in the applied loads and quadratic in the DOF.

Figure 1: A fixed-free flexure strip with error motions (green), degrees of freedom (blue) and applied loads (black).

These parametric closed-form expressions capture the effects of distributed compliance in flexure strips.

Validation

The error motion expressions have been numerically validated for width-thickness ratios as small as 2, and length-width ratios up to 10, at various load levels, with 95% accuracy.

Conclusion

By considering small spatial deformation with respect to a nominally 2-D deformed shape, accurate expressions are obtained for the geometrically nonlinear aspect of the 3-D behavior of a flexure strip. In practice, this models the performance-limiting reduction of support stiffness that accompanies movement in the intended degrees of freedom of a flexure strip.