Elastohydrodynamics in counterformal and conformal contacts

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Abstract. Since Peppler [1] in 1936 showed that elastic deformation of the mating surfaces in lubricated concentrated contacts plays a dominant role in the pressure built up and film formation, elastohydrodynamic lubrication (E.H.L.) has become a major field of interest in Tribology (Lubrication, Friction and Wear), all over the world. This, of course, is no surprise, since many contacts in machine elements are counterformal and the life of the machine concerned is mainly determined by the life of its most critical parts i.e. the mating surfaces. Examples are rolling element bearings, gears, seals, cam and tappets, etc. The most widely known example of a conformal contact is a journal bearing.

In this paper a review will be given with regard to models and calculational methods concerning film thickness, pressure distribution and traction in E.H.L. and research results, recently obtained at the University of Twente, will be presented. Advantages of application of multilevel techniques will be demonstrated.

Additionally some experimental results will be shown and some applications will be discussed.

Nomenclature

b = Half-width Hertzian dry contact region

$$b = \sqrt{\frac{8wR}{\pi E'}}$$

D = Deborah number

E' = Reduced modulus of elasticity

$$\frac{2}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

E = Elasticity modulus (Young's modulus)

f = Coefficient of friction

$$f = \frac{F_s \cdot b}{w}$$

F_s = Sliding traction force

G = Fluid elastic modulus

H = Dimensionless film thickness

$$H = \frac{hR}{b^2}$$

H_0 = Integration constant in dimensionless film thickness equation

$H_{\min}$ = Dimensionless minimum film thickness (Moes)

$$H_{\min} = \frac{h_{\min}}{R} (U)^{-1/2}$$

$h_{\min}$ = Minimum film thickness

h = Film thickness

L = Dimensionless material parameter (Moes)

$$L = \alpha E'(U)^{1/4}$$

M = Dimensionless load parameter (Moes)

$$M = W(U)^{-1/2}$$

P = Dimensionless pressure

$$P = \frac{p}{p_h}$$

p = Pressure

p_h = Maximum Hertzian pressure

$$p_h = \frac{2w}{\pi b}$$

p_0 = Constant in Roelands' relation

$$P_0 = \frac{p_0}{p_h}$$

R = Reduced radius of curvature

T = Temperature

u_h, u_0 = Velocity of upper and lower surface

u_s = Sum velocity

U = Dimensionless velocity parameter

$$U = \frac{\eta_0 u_s}{E' R}$$

W = Dimensionless load

$$W = \frac{w}{E'R}$$

w = Load per unit width

X = Dimensionless coordinate

$$X = \frac{x}{b}$$

x = Coordinate

z = Pressure viscosity index (Roelands)

α = Pressure viscosity coefficient

ν = Poissons ratio

η = Viscosity

η_0 = Viscosity at atmospheric pressure

$\dot{\gamma}$ = shear rate

$$\dot{\gamma} = \frac{(u_h - u_0)}{h}$$

ρ_0 = Density at atmospheric pressure

ρ = Density

τ = Shear stress.

1. Introduction

In 1916 Martin [2] published his historical paper, entitled: 'Lubrication of Gear Teeth', and evidently his aim was to calculate the minimum thickness of the oil film between two interacting gear surfaces. He calculated the pressure distribution between two rigid gear teeth by solving Reynolds equation for a given geometry. The resulting minimum film thicknesses were of the order of a few hundredths of a micrometer, that is much smaller than the average surface roughness of gear teeth. Full film lubrication, also, seemed out of the question.

The shortcomings of his theoretical model, i.e. the assumption that the surfaces were rigid and that the lubricant behaved like an isoviscous fluid, have been improved by many others later.

Several investigators studied the influence of these effects separately and showed considerable improvements but it was Grubin [3] who, in 1949, combined these two effects and who presented an approximate film thickness equation for highly loaded contacts. His equation predicted film thicknesses being orders of magnitude larger than those predicted by Martin.

Dowson and Higginson [4] firstly presented results of numerical calculations regarding film thickness and pressure distribution and they derived an improved film thickness equation. Much experimental work has been carried out in the sixties and the seventies and, within certain limits, the above mentioned equation has proved its value. For an extensive survey with regard to this subject the reader is referred to Dowson and Higginson [5].

It is now generally accepted that lubricated concentrated contacts, like gear teeth, are almost always separated by an appreciable oil film, provided the surfaces are running at sufficiently high speed. Convincing experimental evidence for this statement has been given by several authors. Not only average film thicknesses have been measured by means of, for instance, capacitive methods but also the film shape and pressure distribution have been measured by application of thin film transducers [6], [7].

From the preceding introduction it might be concluded that the subject regarding film thickness in E.H.L. can be considered as closed. Nothing, however, is less true,

and since about 1980 a renewed interest in this topic can be observed from literature and an appreciable number of papers has been presented. This is the reason why in this paper attention will be drawn to this subject.

With regard to traction calculations the situation is quite different. When traction would be calculated, based on the same physical model as applied to film thickness calculations the results would deviate from experimental values up to some orders of magnitude. The main reason for this is the fact that the film thickness in an E.H.L. contact is mainly determined by the conditions in the region preceding the contact, i.e. the so-called conjunction zone, while the traction in an E.H.L. contact is principally produced in the contact zone itself. The conditions in these two zones with regard to pressure, temperature and shear rate are quite different, i.e. in the contact the influence of the temperature and shear rate on the viscosity can no longer be neglected. Moreover, the viscosity – pressure relationship according to Barus, as very often applied in film thickness calculations can no longer be applied, since this relationship is unreliable for higher pressures (see e.g. [12]). Several theories of non-Newtonian fluid behaviour have, in the past decades, been put forward in trying to explain actual traction behaviour. In this paper the so-called thermal Roelands–Eyring–Maxwell model will be explained and discussed.

Since the models, serving as a basis for numerical calculations of film thickness and traction respectively are quite different; these subjects will be treated separately below.

2. Film shape and pressure distribution

As mentioned before, a renewed interest in the topic of pressure and film thickness calculations in E.H.L. can be observed from literature. Modern and very fast computers are, without any doubt, the first reason, making it possible to investigate, in detail, the influence of several effects on E.H.L. A second reason is the development of numerical techniques, which will be dealt with below.

First the configuration and the equations will be shown.

2.1. Configuration

In this section we will confine ourselves to the two-dimensional case, i.e. the line contact situation. Any concentrated contact, i.e. a contact with two radii which are either largely different (outer contact in a rolling bearing) or located on different sides of the contact (inner contact), can be represented by the contact between a cylinder and a plane. The radii are related according to

$$R^{-1} = R_1^{-1} + R_2^{-1},$$

where R stands for reduced radius and R_1 and R_2 are the radii of the surfaces 1 and 2 respectively.

2.2. Equations

The equations governing the E.H.L. line contact problem are:

Reynolds equation

The Reynolds equation describing the relation between pressure and film shape reads

$$\frac{d}{dx} \left(\rho h^3 \frac{dp}{dx} \right) = 6\eta u_s \frac{d(\rho h)}{dx} \quad (1)$$

with the cavitation condition: $p \geq 0$, and boundary conditions, $p(x_a) = 0$ and $p(x_b) = 0$. The entry zone has to be large enough to avoid so-called starved lubrication, i.e. $x_a \leq -4b$ (see Fig. 1), depending on the load. The end of the pressure region x_b is determined by the cavitation condition.

Elastic deformation

The equation describing the elastic deformation of the surfaces can be written as

$$v(x) = \frac{-2}{\pi E'} \int_{x_a}^{x_b} p(s) \ln(x-s)^2 ds + C, \quad (2)$$

Fig. 1. Pressure distribution and film thickness in an E.H.L. line contact.

Force balance and film thickness equation

The integral over the pressure distribution has to be equal to the externally applied load.

$$w = \int_{x_a}^{x_b} p(s) ds. \quad (3)$$

Once the pressure is determined, the film thickness can be calculated according to

$$h(x) = h_0 + \frac{x^2}{2R} + v(x), \quad (4)$$

with $v(x)$ according to equation 2.

The constant h_0 in this equation is determined by the force balance (equation 3)

Viscosity pressure relationships

The viscosity is dependant on the pressure. Both Barus and Roelands relations have been applied:

$$\eta = \eta_0 \exp(\alpha p), \quad (\text{Barus})$$

$$\eta = \eta_0 \exp \left[\frac{\alpha p_0}{z} \left[\left(1 + \frac{p}{p_0} \right)^z - 1 \right] \right], \quad (\text{Roelands})$$

where $p_0 = 1.98 \cdot 10^8$ [Pa].

The parameters α , p_0 and z in the Roelands equation are mutually dependant:

$$\alpha = \frac{\ln(\eta_0) + 9.67}{p_0} z$$

(parameters in SI-units).

When compressibility is taken into account the lubricant density ρ is assumed to depend on the pressure according to the Dowson and Higginson relation [5].

3. Solution process

3.1. Review

The Newton–Raphson method has become quite popular for solving the above described problem (Okamura [8], Kostreva [9] and many others). Convergence is, provided the starting conditions are close to the solution, fast.

Also calculations based on the Gauss–Seidel iterative method with under relaxation have been presented (Hamrock and Dowson [10] and Chittenden et al. [11]). These calculations, however, required very large computing times, in particular for the highly loaded cases where very large under relaxation factors had to be applied.

The aforementioned Newton–Raphson method also has two severe drawbacks. The first one is the fact that the cavitation boundary condition cannot be treated easily, in particular for circular contact situations. The second disadvantage is a numerical one. Whenever the grid size is halved, so the number of unknowns is doubled, the required computer memory increases by a factor of four and the c.p.u. time increases by a factor of eight. This means that the limits of a computer can quite easily be reached even when working with a supercomputer.

In 1989 Verstappen [13] presented a method based on a variational approach which turned out to be very fast but unfortunately is not suitable for a large number of grid points, which is an absolute requirement when, for instance, effects of surface roughness have to be investigated.

3.2. *Application of multigrid techniques*

At the University of Twente, in 1984, calculations have been carried out, based on the Newton–Raphson method and the above mentioned drawbacks were soon discovered. Results were found for not too highly loaded contacts and a few nodal points. For higher loads the procedure turned out to be unstable, since the Jacobian matrix becomes almost singular, and for more grid points the c.p.u. time, even on mainframe computers, became intolerably high. As expected, the complexity of the method

Table 1. Computing times for three different methods

Computer times in h:m:s			
Number of grid points	Newton–Raphson	Multigrid A	MLMI B
225	01:30:00	:02:00	:12
449	11:40:00	:08:00	:24
897	*91:00:00	:30:00	:48
1793	*750:00:00	02:10:00	:01:38
3585			:03:17
7169			:06:38
14337			:13:20
28673			:26:20
57345			01:03:00
114689			02:08:00
	$O(n^3)$	$O(n^2)$	$O(n)$
	*extrapo.		

turned out to be of $O(n^3)$. In 1986 Lubrecht et al. [14] presented the first of a series of papers on the application of multigrid techniques to E.H.L. problems. They employed a Gauss–Seidel relaxation process to solve the equations using multigrid techniques to accelerate convergence. The reduction in computing time, was quite impressing as can be seen from Table 1. Solutions have been presented for line as well as point contact up to reasonably high pressures and for a large number of nodes. However, the complexity of the algorithm was $O(n^2)$ mainly caused by the calculation of the elastic deformation integrals. Besides, with increasing load, under relaxation and local relaxation with film thickness updates were required to obtain a stable process.

Venner et al. [15] succeeded in 1989 in bringing back the complexity of the algorithm to $O(n \ln n)$, so for large n to almost $O(n)$, by application of multilevel multi-integration to the calculation of the elastic deformation integrals and by the use of an alternative relaxation process. Also very highly loaded situations could be dealt with without problems. The reduction in computing time (see Table 1) makes calculations on a P.C. possible.

4. Results

4.1. Pressure distribution

In Fig. 1 a characteristic pressure distribution P and film shape H are given for an E.H.L. line contact. An interesting feature is the so-called pressure spike at the end of the film just before the restriction in the film. The existence of the spike, its position and height are, amongst others, dependant on:

- the load on the contact
- the pressure viscosity coefficient
- the compressibility of the lubricant
- the non-Newtonian behaviour of the lubricant

Since the spike is very steep and small it is very difficult to calculate its height. By application of the above described method it is possible to carry out these calculations very accurately. In Fig. 2 a detail of the pressure distribution in the spike region is presented. More details about this subject can be found in ref. [16].

4.2. Film thickness

For technical applications the minimum film thickness is, of course, of much interest. For example, the ratio λ of the minimum film thickness to a combined surface roughness parameter is, a dominant factor for the determination of rolling bearing life. See for instance, with regard to this subject, ref. [17]. Very often the results of film thickness calculations are presented in dimensionless parameters. When using the

Fig. 2. Detail of the spike in Fig. 1.

three well known parameters

$$H_{\min} = \frac{h_{\min}}{R} U_{\Sigma}^{-1/2},$$

$$M = \frac{w}{E'R} U_{\Sigma}^{-1/2}$$

and

$$L = (\alpha E') U_{\Sigma}^{1/4},$$

where

$$U_{\Sigma} = \frac{\eta_0 u_s}{E'R},$$

it is possible to represent the minimum film thickness results in one graph (see Fig. 3).

With the exception of the formula proposed by Lubrecht et al. [18] all film thickness formula presented so far describe the curve for L is constant as a straight line, implying that the predicted values for low load conditions are too low. Lubrechts

Fig. 3. Dimensionless film thickness $H_{\min}$ as a function of the load parameter M and material parameter L .

formula follows the curved line, also for low loads, and reads

$$H_m = \left[\left(\left(0.98 \frac{L^{3/4}}{M^{1/8}} \right)^r + \left(\frac{2.04}{M^{1/5}} \right)^r \right)^{s/r} + \left(\frac{2.45}{M} \right)^s \right]^{1/s},$$

where

$$s = 4 - \exp(-L/2) - \exp(-2/M), \quad r = \exp\left(1 - \frac{4}{L+5}\right).$$

4.3. Surface irregularities

Because of the reduction in computing time it is possible now to carry out calculations with regard to surface irregularities. Lubrecht et al. [18] investigated the influence of surface roughness on a stationary surface on the film shape and pressure distribution. In this case the surface roughness had to be applied on the stationary surface since a moving rough surface would introduce time dependant effects in the Reynolds equation.

The problem of a dent or a hole moving through an E.H.L. contact has recently been solved by Venner and will soon be published. He managed to calculate the film thickness and the pressure distribution as a function of time.

This situation is of interest for life determination of concentrated contacts but can also be used in calculating the noise spectrum caused by a defect in a bearing, which can be useful for condition monitoring investigations.

5. Traction

From Fig. 1 it can be seen that the pressure distribution in an E.H.L. contact closely resembles that of the Hertzian contact pressure for dry contact. This is not surprising, since the elastic deformation in comparison with the film thickness is very large. Many investigators (for instance Grubin, ref. [3]) have gratefully taken advantage of this, by assuming that the pressure distribution is known, so the film shape could be calculated by solving the Reynolds equation (1) for the film thickness (the so-called inverse method).

Numerical calculations, carried out later, proved that these assumptions were not bad, especially for high loads or for soft materials (rubber seals for instance). From this it can easily be understood that all kind of phenomena which influence the viscosity, like pressure, temperature and others, do not have a large influence on the pressure and the film shape.

Calculations based on 'Roelands' or 'Barus' (see chapter 2) hardly showed any difference, at least not the differences which could be expected on bases of the different viscosities they predict at higher pressures. Also the influence of a temperature rise in the film is of minor importance.

How different is the situation with regard to traction. Here all these effects play a dominant role. When considering a line contact with a constant film thickness and a Hertzian pressure distribution, the viscous sliding friction force (rolling friction will be neglected) can be expressed as

$$F_s = \int_{-b}^{+b} \tau(x) dx = (u_2 - u_1) \int_{-b}^{+b} \frac{1}{\int_0^h dz/\eta} dx, \quad (5)$$

where z is the coordinate perpendicular to the surfaces.

When the viscosity η is assumed to be constant across the film, which is allowed when no heat is developed, so for low slip, this equation can be written as

$$F_s = \frac{(u_2 - u_1)}{h} \int_{-b}^{+b} \eta dx. \quad (6)$$

It can also be concluded that the frictional force for the isothermal situation strongly depends on the real viscosity in the contact, so on the applied viscosity pressure relationship. It has been shown by ten Napel et al. [12] that application of 'Roelands' instead of 'Barus' provided much better results in comparison with experimental data.

However, when slip increases, thermal effects can no longer be neglected and the temperature development in the contact will have to be incorporated. Zhang Yingjie [20] showed that the most predominant effects are heat dissipation, conduction perpendicular to the surfaces and convection. The energy equation then reads

$$\lambda \frac{\partial^2 T}{\partial z^2} - \rho C_p u \frac{\partial T}{\partial x} + \tau \dot{\gamma} = 0, \quad (7)$$

where C_p is the specific heat of the lubricant and $\dot{\gamma}$ the shear rate. Note that in this equation the shear stress τ occurs; this implies that an iterative procedure has to be applied since in equation 5 the viscosity appears, which is dependant on T . The results of these iterative numerical calculations are presented in Fig. 4, where the slip percentage is defined as $200(u_1 - u_2)/u_s$.

From this result it can be concluded that the Barus viscosity pressure relationship, which has proved its utility in film thickness calculations is useless for calculating traction. From Fig. 4 it can also be seen that, though the application of Roelands relationship improves the results considerable, the agreement with experimental values is still bad. Non-Newtonian fluid behaviour has to be incorporated for the not too highly loaded situations, and for higher pressures even elastic effects have to be taken into account. Only the results of the investigations will be presented here.

The constitutive equation for the fluid, describing both these effects, as applied by ten Napel et al. [21] is the so-called Maxwell-Eyring equation and reads

$$\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_v = \frac{1}{G} \frac{d\tau}{dt} + \frac{\tau_0}{\eta} \sinh\left(\frac{\tau}{\tau_0}\right). \quad (8)$$

Fig. 4. Comparison between theoretical and measured coefficients of friction.

This equation can also be written as

$$\frac{\eta \dot{\gamma}}{\tau_0} = D \frac{d(\tau/\tau_0)}{dX} + \sinh\left(\frac{\tau}{\tau_0}\right), \tag{9}$$

where

$$D = \left(\frac{\eta U}{Gb}\right)$$

represents the so-called Deborah number and G the fluid elastic modulus. τ_0 is the representative Eyring shear stress, below which the fluid behaves Newtonian and above which the apparent viscosity decreases with shear rate.

Fig. 5. Comparison between experimental and theoretical coefficients of friction for a thermal Roelands-Eyring-Maxwell model.

Based on equations 7 and 9 and Roelands equation the traction in an E.H.L. contact has been calculated. The results were quite reasonable. One example of the results is given in Fig. 5.

For more information on this subject the reader is referred to [22].

6. Journal bearings

The theory and results presented so far all concerned so-called concentrated contacts and indeed most of the hydrodynamically lubricated situations where elastic deformation of the surfaces is relevant are concentrated contacts. There are, however, also conformal contact situations where, despite the fact that the pressure is much lower than in counterformal contacts, the elastic deformation cannot be neglected. An example is the connecting rod bearing in combustion engines.

Van der Tempel et al. [23] and Moes et al. [24] solved this problem for flexible short bearings. Since the bearing is dynamically loaded the pressure distribution has to be calculated as a function of time. To reduce the computing time the Ocvirk 'short bearing solution' has been applied. In order to calculate the elastic deformation of the bearing its stiffness matrix is determined once by means of a finite element method. The resulting equations have been solved by a Newton–Raphson method, implying quite considerable c.p.u. times. Van der Tempel also included starvation effects and Moes calculated the influence of temperature development in the bearing on minimum film thickness and other relevant bearing parameters.

Since their methods are based on Newton–Raphson techniques an extension to two dimensional calculations will be difficult. Therefore it is decided that multigrid methods, to solve the Reynolds equation, will have to be applied first, after which more accurate 2-D calculations can be carried out.

7. Applications

As mentioned before life determination of a concentrated contact is a subject which is closely related to the pressure distribution on the surface. Surface roughness and surface damage plays an important role.

At the University of Twente a model has been developed for the calculation of traction in E.H.L. contacts. For an accurate calculation of this traction force it is important to know the film thickness and its shape. This was the main reason why the film thickness project was started.

In co-operation with the Solid Mechanics group recently a research project is started with regard to Condition Monitoring. Under full film conditions, and probably in the mixed lubrication region as well, calculations can be carried out to predict the noise signal that will be produced by a defect in a bearing. Experiments have been carried out on a two disc machine for different defects.

More than a year ago a project is started in co-operation with Alcoa, Pittsburg and Hoogovens, IJmuiden with regard to the cold rolling process. The aim is to calculate film thickness, pressure distribution, traction and deformation as a function of the operational parameters. The problem becomes more complicated since the 'von Karman' equation, describing the plastic deformation, has to be solved simultaneously with the equations mentioned in chapter 2.

Film thickness, pressure distribution and more relevant parameters in journal bearings in reciprocating engines, including thermal, elastic and non-Newtonian effects can be calculated more accurately than has been done until now. Plans have been developed to carry out these calculations.

8. Future research

On the short term the following subjects will be carried out at the University of Twente:

- Influence of non-Newtonian behaviour on film thickness and pressure distribution.
- Influence of inlet shear heating and temperature development in the contact on film thickness.
- Circular and elliptical film thickness calculations.
- Experimental work regarding film shape and traction.
- Surface roughness influence on minimum film thickness and pressure distribution.
- Possibly tackling of mixed lubrication problems.
- Combustion engine bearing calculations.

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