Tribology International 44 (2011) 1997–2001

Contents lists available at SciVerse ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

Amplitude reduction in EHL line contacts under rolling sliding conditions

J. Wang [a], C.H. Venner [b], A.A. Lubrecht [c,*]

[a] Qingdao Technological University, China
[b] University of Twente, Faculty Engineering Technology, Enschede, The Netherlands
[c] Université de Lyon, INSA-Lyon, LaMCoS, CNRS UMR 5259, Villeurbanne F69621, France

ARTICLE INFO

Article history:
Received 10 May 2011
Received in revised form
4 July 2011
Accepted 17 August 2011
Available online 26 August 2011

Keywords:
Elasto hydrydynamic lubrication
Surface roughness
Amplitude reduction

ABSTRACT

Surface roughness plays an important role in the performance of highly loaded elastohydrodynamically lubricated contacts. As the pressures are very high, each of the surface roughness components deforms differently, and as a result the roughness inside the highly loaded contact is different from the measured roughness. Under pure rolling conditions the amplitude reduction theory describes the waviness deformation as a function of wavelength and operating conditions. The current work suggests that similar predictions are possible under rolling sliding conditions, provided that the wavy surface velocity u_2 exceeds the smooth surface velocity u_1. For $u_2 < u_1$ the maximum value of A_d/A_i depends on the slide to roll ratio and may be significantly less than 1.0.

1. Introduction

Surface roughness has an important influence on the performance of concentrated lubricated contacts. The standard way of accounting for the surface roughness is through the film thickness to roughness parameter: h/σ. However, under highly loaded Elasto Hydrodynamically Lubricated (EHL) conditions, the surfaces and the roughness deform. Hence it is important to be able to predict this deformation in order to predict contact performance. The roughness deformation has been studied using semi-analytical methods [1–6] or using numerical techniques [7]. A parallel research effort has used numerical techniques to predict lubrication of real measured roughness profiles [8,9]. Furthermore, experimental techniques have been used to test the numerical predictions, studying the deformation of artificial features [10,11] or the deformation of real roughness [12].

A consensus exists concerning the deformation of sinusoidal waviness under pure rolling conditions, the so called Amplitude Reduction Theory (ART), which predicts the deformation as a function of a single dimensionless parameter ∇, containing the wavelength and the operating conditions. Physically this parameter represents the ratio of the waviness and the length of the inlet pressure sweep [3]. For large values of this parameter the waviness is completely deformed, for short wavelengths the deformation tends to zero. This prediction has been confirmed experimentally for pure rolling [12].

However, for rolling sliding conditions, the situation is more complicated and the semi-analytical models predict a deformation of the waviness for both high and low wavelength values. A first attempt to study the rolling sliding contact numerically was made by Lubrecht et al. [13]. However, this work covered a much smaller parameter range, than the current work. Furthermore, the identification of the mechanics of the amplitude reduction theory (wavelength to inlet length ∇) allows a more comfortable presentation of the results. Recently, Hooke and co-workers [5,6] predicted the film thickness perturbations in EHL contacts under rolling–sliding conditions, using a perturbation analysis and an Eyring lubricant model.

The current work originated from a discussion on the experimental validation under rolling–sliding conditions which proved more complicated than for pure rolling [14]. This incited the authors to revisit the problem of waviness deformation in EHL under rolling/sliding using a numerical analysis of the complete transient equations. For clarity, i.e. to see the dominant first order effects of rolling sliding on waviness deformation, a line contact analysis was chosen, and even though taking into account Eyring rheological behaviour is not significantly more complicated, e.g. see [15], a simple Newtonian model was used. The objective is to see if an overall trend emerges that can be validated experimentally.

2. Theory

The dimensionless Reynolds equation for the transient line contact problem reads:

$$\frac{\partial}{\partial X}\left(\varepsilon\frac{\partial P}{\partial X}\right)-\frac{\partial(\overline{\rho}H)}{\partial X}-\frac{\partial(\overline{\rho}H)}{\partial T}=0 \tag{1}$$

The boundary conditions are $P(X_a,T)=P(X_b,T)=0, \forall T$ where X_a, X_b denote the boundaries of the domain. Furthermore, the

* Corresponding author.
E-mail address: Ton.Lubrecht@insa-lyon.fr (A.A. Lubrecht).

0301-679X/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.triboint.2011.08.009

Nomenclature

A_d	dimensionless deformed amplitude, $A_d = [\max_T(H) - \min_T(H)]/2$
a_i	initial amplitude (m)
A_i	dimensionless initial amplitude $A_i = a_i R_x/b^2$
b	contact half-width (m)
E'	reduced modulus of elasticity, $2/E' = (1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2$ (Pa)
G	dimensionless material parameter $G = \alpha E'$
h	film thickness (m)
H	dimensionless film thickness, $H = hR_x/b^2$
h_0	integration constant
H_0	dimensionless integration constant, $H_0 = h_0 R_x/b^2$
L	dimensionless lubricant parameter, $L = G(2U)^{0.25}$
M	1d dimensionless load parameter $M = W(2U)^{-0.5}$
p	pressure (Pa)
P	dimensionless pressure $P = p/p_h$
p_h	maximum Hertzian pressure
R_x	reduced radius of curvature in x (m)
$\mathcal{R}$	surface waviness
T	dimensionless time $T = \overline{u}t/b$
u_1, u_2	smooth (1) and rough (2) surface velocity (m/s)
$\overline{u}$	mean surface velocity $\overline{u} = (u_1+u_2)/2$
U	dimensionless velocity parameter, $U = \eta_0 \overline{u}/(E'R_x)$
w	1d load per unit length (N/m)
W	1d dimensionless load parameter, $W = w/(E'R_x)$
x	coordinate in direction of rolling (m)
X	dimensionless coordinate $X = x/b$
X_0	dimensionless scaling parameter
α	pressure viscosity index (Pa^{-1})
$\overline{\alpha}$	dimensionless parameter, $\overline{\alpha} = \alpha p_h$
∇	1d dimensionless wavelength parameter, $\nabla = (\lambda/b)(M^{3/4}/L^{1/2})$
$\overline{\nabla}$	1d dimensionless wavelength parameter, $\overline{\nabla} = (\lambda/b)(M^{3/4}/L^{1/2})\sqrt{\overline{u}/u_2}$
$\overline{\lambda}$	dimensionless speed parameter, $\overline{\lambda} = 12(\eta_0 \overline{u} R_x^2)/(b^3 p_h)$
λ	roughness wavelength (m)
λ_h	film thickness modulation wavelength (m)
η	viscosity (Pa s)
$\overline{\eta}$	dimensionless viscosity, $\overline{\eta} = \eta/\eta_0$
ν	Poisson ratio
ρ	density (kg/m^3)
$\overline{\rho}$	dimensionless density, $\overline{\rho} = \rho/\rho_0$

cavitation condition $P(X,T) \geq 0, \forall X,T$ must be satisfied. ε and $\overline{\lambda}$ are defined according to:

$$\varepsilon = \frac{\overline{\rho} H^3}{\overline{\eta}\overline{\lambda}}, \quad \overline{\lambda} = \frac{12\eta_0 \overline{u} R_x^2}{b^3 p_h}$$

The density ρ is assumed to depend on the pressure according to the Dowson and Higginson relation [16] and the Roelands viscosity pressure relation [17] is used.

The film thickness equation is made dimensionless using the same Hertzian parameters and accounting for a moving surface waviness $\mathcal{R}$ reads:

$$H(X,T) = H_0(T) + \frac{X^2}{2} - \mathcal{R}(X,T) - \frac{2}{\pi}\int_\Omega P(X',T)\ln\left(\frac{X-X'}{X_0}\right)dX' \tag{2}$$

where $\mathcal{R}(X,T)$ denotes the undeformed geometry of the surface waviness at dimensionless time T and $H_0(T)$ is an integration constant. X_0 is a dimensionless scaling parameter:

$$R(X,T) = A_i \cos\left(2\pi\left(X - \frac{u_2}{\overline{u}}T\right)\Big/(\lambda/b)\right) \tag{3}$$

The waviness equation is damped using an exponential, to start from a smooth contact stationary solution, and to add the roughness in a physically correct manner. This avoids unphysical start up effects. The deformed amplitude was "measured" when a periodic response of film thickness as a function of time was reached. The A_i value is chosen as 0.1 H_c to remain in the linear range, the non-linear extension is described by Biboulet et al. [18,19].

At all times the force balance condition is imposed, i.e. the integral over the pressure must balance the externally applied contact load. This condition determines the value of the integration constant $H_0(T)$ in Eq. (2). Expressed in the dimensionless variables it reads:

$$\int_\Omega P(X,T)\,dX - \frac{\pi}{2} = 0, \quad \forall T \tag{4}$$

In physical terms this equation means that the acceleration forces of the contacting bodies are neglected. The equations are discretised to second order precision, using narrow upstream discretization of the wedge and squeeze terms in the Reynolds equation, see [21] and solved using MultiLevel techniques [20,22].

The deformed amplitude of the waviness is defined as a function of the operating conditions through the following relation:

$$2A_d = \max_T H(0,T) - \min_T H(0,T) \tag{5}$$

The time T is chosen large enough to avoid start-up effects.

3. Results

Fig. 1 shows the pressure and film thickness distribution as a function of X for a periodic time distribution. From the film thickness distribution the deformed amplitude A_d can be obtained. In order to study the influence of the different operating parameters on the deformed amplitude, the parameters M, L, λ/b and the slide to roll ratio $u_2/\overline{u}$ have been varied and the deformed amplitude was recorded.

Fig. 1b shows a detail of the film thickness distributions as a function of the slide to roll ratio. The deformed waviness changes wavelength and a small amplitude variation can also be observed.

Fig. 2 shows the amplitude reduction A_d/A_i as a function of the standard waviness parameter ∇. The results show that indeed the deformed amplitude tends to zero for large values of ∇. However, the scatter obtained is large and increases towards the low ∇ zone, where the results obtained vary between 0 and 1. For these low ∇ values the scatter renders the curve useless. However, a more detailed analysis shows that the scatter is not random at all!

When plotting the amplitude reduction as a function of the dimensionless wavelength ∇, for a single slide to roll ratio, rather smooth curves are obtained, see Figs. 3 and 4. However, two distinct types of behaviour are observed, for $u_2 < u_1$ (Fig. 3) the maximum value of A_d/A_i is significantly less than 1. Furthermore, this maximum value depends on the slide to roll ratio. For $u_2 > u_1$ (Fig. 4) however, the maximum value of A_d/A_i is very close to 1 and is independent of the slide to roll ratio. Hence it was decided to analyse the results of the cases $u_2 < u_1$ and $u_2 > u_1$ separately.

Fig. 5 shows the amplitude reduction A_d/A_i as a function of the standard waviness parameter ∇ for $u_2 > u_1$ only. The results show that indeed the deformed amplitude tends to zero for small and large values of ∇. However, the scatter on the curve obtained is

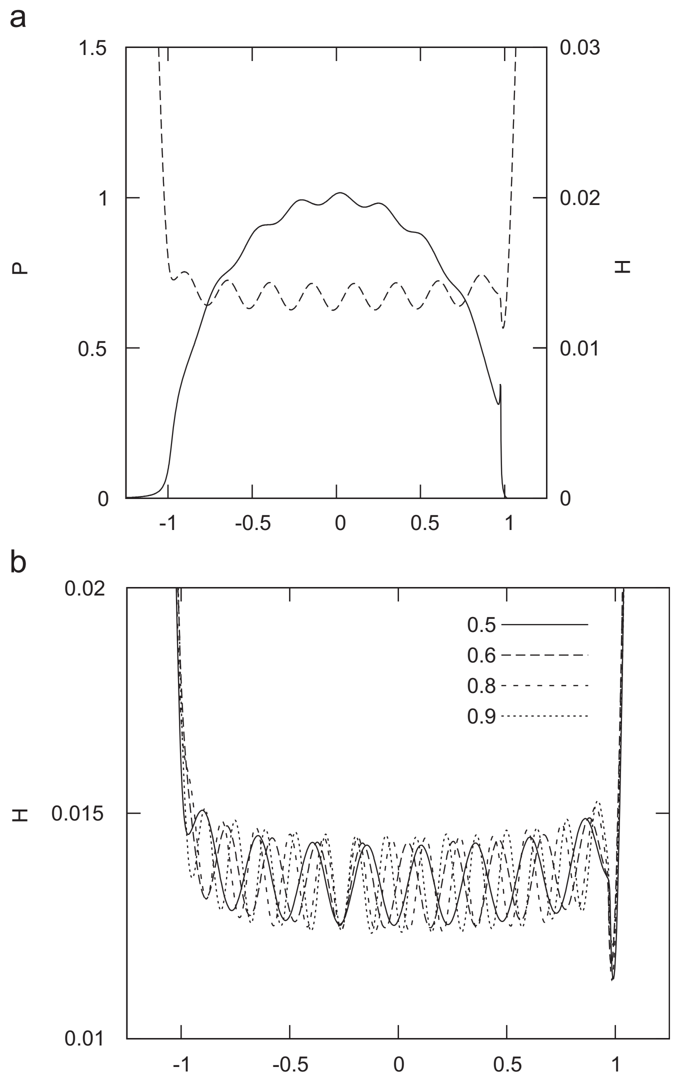

Fig. 1. (a) Pressure and film thickness distribution for $M=100$, $L=10$, $\lambda/b=0.25$ and $u_2/\overline{u}=1.0$. (b) Detail of film thickness distribution for $M=100$, $L=10$, $\lambda/b=0.25$ and $u_2/\overline{u}=1.0\ldots1.8$.

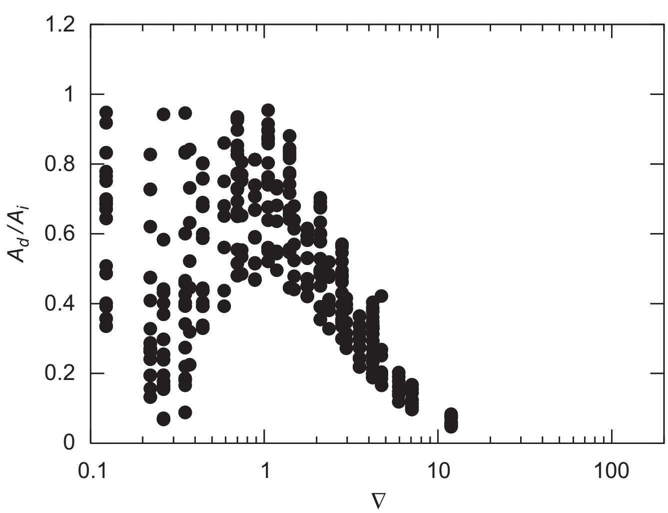

Fig. 2. Amplitude reduction as a function of ∇ for rolling sliding $u_2/\overline{u}=0.20\ldots0.9, 1.1,\ldots1.9$.

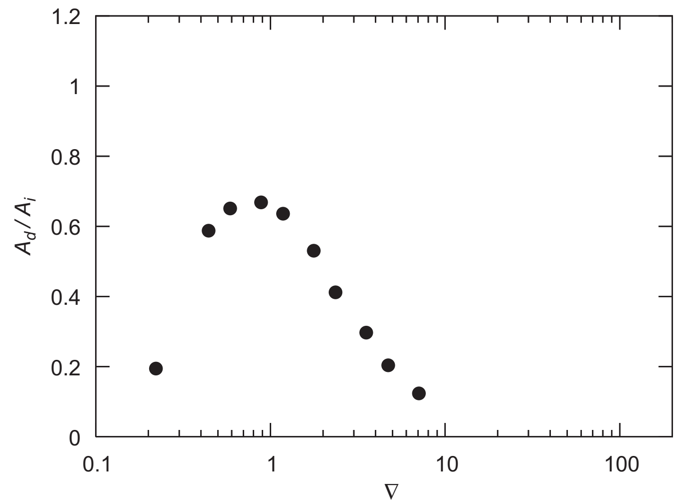

Fig. 3. Amplitude reduction as a function of ∇ for rolling sliding $u_2/\overline{u}=0.5$.

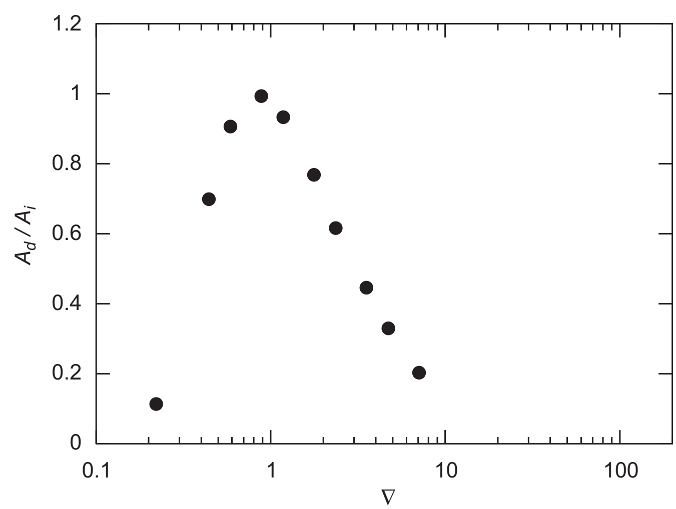

Fig. 4. Amplitude reduction as a function of ∇ for rolling sliding $u_2/\overline{u}=1.5$.

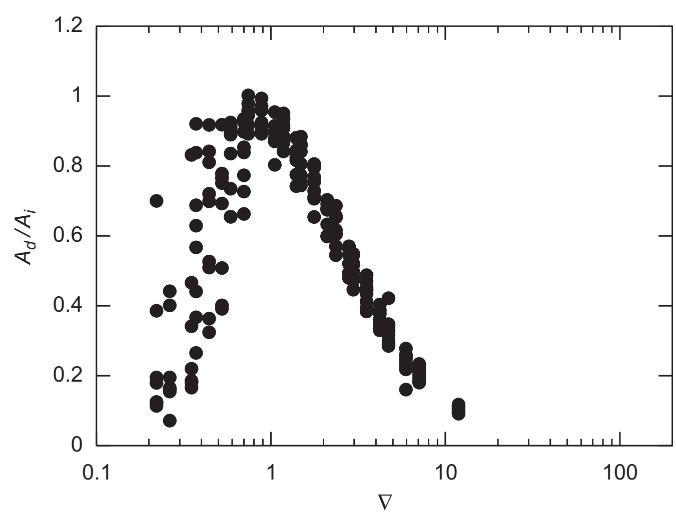

Fig. 5. Amplitude reduction as a function of ∇ for rolling sliding $u_2/\overline{u}=1.1,\ldots1.9$.

rather large, particularly in the low ∇ zone. When analysing the results for low ∇ values, it was found that the order of the points was determined by the slide to roll ratio $u_2/\overline{u}$.

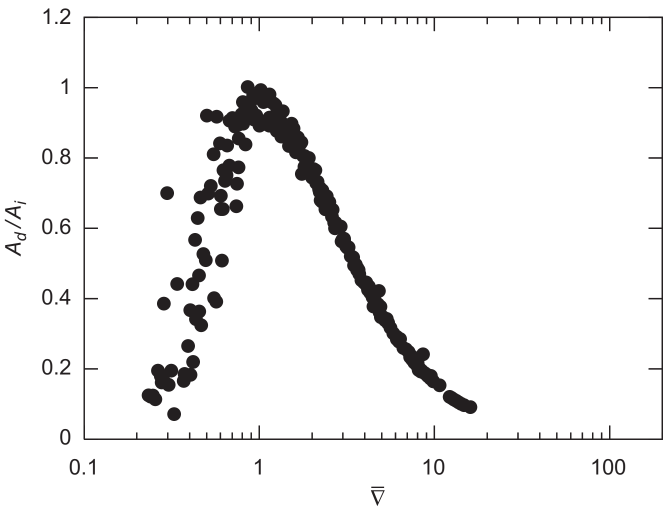

Fig. 6. Amplitude reduction as a function of $\overline{\nabla}$ for rolling sliding $u_2/\overline{u} = 1.1, 1.2, 1.3 \ldots 1.9$.

As such a modified ∇ parameter was introduced. The basic idea from the influence of the slide to roll ratio on the ordering of the points was to replace the mean velocity $\overline{u}$ by the wavy surface velocity u_2. The mean velocity appears in both the M and L parameter. Because of the powers on M and L, $\overline{u}$ appears with power of -0.5 in the ∇ parameter. As such a modified ∇ parameter was introduced, defined as $\overline{\nabla} = \nabla\sqrt{\overline{u}/u_2}$. This dimensionless wavelength parameter uses the velocity of the wavy surface u_2 as the reference velocity, instead of the mean velocity $\overline{u}$ used in the standard ∇ parameter.

Fig. 6 displays the amplitude reduction A_d/A_i as a function of the modified waviness parameter $\overline{\nabla}$. The figure shows that indeed the scatter is much reduced, especially in the low $\overline{\nabla}$ zone and a single master curve is indeed obtained.

4. Discussion

As was the case for the numerically obtained master curve under pure rolling conditions, the results do not exactly fall onto a single line. Numerical tests using finer grids show that the error in the deformed amplitude A_d is generally less than 5%. Furthermore, the linearity was checked using smaller amplitudes A_i. The scatter observed in Fig. 5 does not seem to stem from either of these two numerical sources.

A possible explanation for the scatter might be the influence of secondary order effects, such as the rigid body motion, which is influenced by the force balance over the contact area. Hence, the force balance perturbations would be influenced by the wavelength.

Compared with the results reported in [5], the overall trend is very similar; the dominant parameter is the dimensionless wavelength, compared to the inlet length ∇. The curve shows an amplitude that goes to zero for small and large values of the dimensionless wavelength. Furthermore, a maximum amplitude is found to be less than 1 for the cases where $u_2 < u_1$. Finally, a similar scatter around the master curve is observed by Hooke et al. [5, Fig. 12]. However, some differences are also observed, the constant maximum amplitude for $u_2 > u_1$ and the slightly better fit using the modified wavelength parameter. Experiments will have to determine if the Eyring model is essential in predicting the roughness deformation correctly.

5. Conclusion

The current paper analyses the amplitude reduction of waviness in EHL line contacts for slide to roll ratios $u_2/\overline{u}$ varying from 0.2 to 0.9 and from 1.1 to 1.9. The work suggests that a single master curve exists, describing the amplitude reduction under rolling–sliding conditions, when the wavy surface velocity u_2 exceeds the smooth surface velocity u_1. The amplitude reduction is maximum for both short and long wavelengths, i.e. $A_d/A_i(\overline{\nabla} = 0) = 0$ and $A_d/A_i(\overline{\nabla} = \infty) = 0$. In between these two extremes the waviness amplitude is not reduced, i.e. $A_d/A_i = 1$. This maximum value occurs for all slide to roll ratios from 1.1 to 1.9. The amplitude reduction reduces to a single curve when accounting for the slide to roll ratio $u_2/\overline{u}$, hence the use of the modified waviness parameter $\overline{\nabla}$. It seems that for slide to roll ratios $u_2 < u_1$, the maximum value attained is different, as predicted by Hooke et al. and therefore the quest for a single master curve will involve a more complex parameter on the vertical axis. Finally, the current work requires an extension to circular contacts and two dimensional contact waviness.

Acknowledgement

Dr. J. Wang kindly acknowledges the French CNRS support as a visiting professor at INSA-Lyon.

Appendix A. Numerical example

To illustrate the dependence of amplitude reduction of waviness under sliding conditions, a quantitative example is given. Consider the case with the following parameters: $\alpha = 2.0 \times 10^{-8}$ Pa^{-1}, $E' = 2.10^{11}$ Pa, $\eta_0 = 0{,}04$ Pa s, $\overline{u} = 1.0$ m/s, $R = 0.01$ m, $w = 1.3 \times 10^6$ N/m. The values of the Hertzian parameters are $p_h = 2$ GPa and $b = 4 \times 10^{-4}$ m.

The values of the Dowson and Higginson Parameters are $W = 6.5 \times 10^{-4}$, $U = 2 \times 10^{-11}$, and $G = 4.0 \times 10^3$. The values of the Moes parameters are $M = 100$ and $L = 10$. The central film thickness for this case is 0.22 μm. The value of ∇ for this case $\nabla = 10(\lambda/b)$.

Let the undeformed amplitude and wavelength of three components be given by $a_i(\lambda = 4 \times 10^{-4}) = 0.2$ μm, $a_i(\lambda = 2 \times 10^{-4}) =$ 0.1 μm, and $a_i(\lambda = 1 \times 10^{-4}) = 0.05$ μm. For $(u_2/\overline{u}) = 1$ (pure rolling) the computed values of a_d/a_i for these components are: 0.17, 0.38, and 0.66 (the values obtained from the curve-fit formula given in [7] are 0.16, 0.38, and 0.64), giving values of the deformed amplitudes of 0.035, 0.038 and 0.033 μm, illustrating the wavelength dependent deformation.

For the case of sliding the situation is more complex, as the wavelength of the film thickness oscillation in the contact, say λ_h, differs from the wavelength of the surface waviness λ according to $\lambda_h = \lambda/(u_2/\overline{u})$. For $(u_2/\overline{u}) = 0.5$ (the waviness is on the slower surface) the values of λ_h are 8×10^{-4}, 4×10^{-4} and 2×10^{-4} m, and the values of a_d/a_i are 0.12, 0.30, and 0.53. So, assuming the same initial amplitudes of 0.2, 0.1 and 0.05 μm the amplitudes of the associated film disturbances will be 0.025, 0.030 and 0.027 μm respectively. Note that, due to the wavelength elongation, the film thickness oscillations induced by the waviness significantly differ from those that would be produced by the undeformed waviness passing through the conjunction.

For $(u_2/\overline{u}) = 1.5$ (the waviness is on the faster surface) the apparent wavelength is shorter. For the three components the values are $\lambda_h = 2.67 \times 10^{-4}$, 1.34×10^{-4} and 0.67×10^{-4}. The computed values of a_d/a_i are 0.20, 0.45, and 0.77, so, the amplitudes of the associated film thickness oscillations are 0.041, 0.045, and 0.038 μm.

These examples clearly indicate that a pattern consisting of multiple waves will cause a film oscillation in the contact region that significantly differs from the undeformed waviness and strongly depends on the slide to roll ratio.

References

[1] Greenwood JA, Johnson KL. The behaviour of transverse roughness in sliding elastohydrodynamically lubrication contacts. Wear 1992;153:107–17.
[2] Greenwood JA, Morales-Espejel GE. The behaviour of small-amplitude waviness in transient rough contacts. Proceedings of the Institution of Mechanical Engineers 1994;208:121–32.
[3] Hooke CJ, Venner CH. Surface roughness attenuation in line and point contacts. Proceedings of the Institution of Mechanical Engineers Part J 2000;214:434–44.
[4] Hooke CJ, Li KY. Rapid calculation of the pressures and clearances in rough EHL contacts, part 1. Proceedings of the Institution of Mechanical Engineers Part C 2006;220:901–14.
[5] Hooke CJ, Li KY, Morales-Espejel G. Rapid calculation of the pressures and clearances in rough EHL rolling–sliding elastohydrodynamically lubricated contacts, part 1, low amplitude roughness. Proceedings of the Institution of Mechanical Engineers C 2007;221:534–50.
[6] Hooke CJ, Li KY, Morales-Espejel G. Rapid calculation of the pressures and clearances in rough EHL rolling–sliding elastohydrodynamically lubricated catacts, part 2, general non-sinusoidal roughness. Proceedings of the Institution of Mechanical Engineers Part C 2007;221:534–50.
[7] Lubrecht AA, Venner CH. Elastohydrodynamic Lubrication of rough surfaces. Proceedings of the Institution of Mechanical Engineers Part J 1999;213: 397–404.
[8] Elcoate CD, Evans HP, Snidle RW. Thin film, time dependent, micro-EHL solutions with real surface roughness. Proceedings of the Leeds Lyon symposium on tribology, Tribology Series 1999;36:163–74.
[9] Zhu D, Hu YZ. A computer program package for the prediction of EHL and mixed lubrication characteristics, friction, surface stresses and flash temperatures based on measured 3D surface roughness. Tribology Transactions 2001;44:383–90.
[10] Kaneta M, Sakai T, Nishikawa H. Effect of surface roughness on point contact EHL. Tribology Transactions 1993;36:605–12.
[11] Choo JW, Olver AV, Spikes HA. The influence of transverse roughness in thin film mixed elastohydrodynamic lubrication. Tribology International 2007;40:220–32.
[12] Sperka P, Krupka I, Hartl M. Experimental study of real roughness attenuation in concentrated contacts. Tribology International 2010;43:1893–901.
[13] Lubrecht AA, Graille D, Venner CH, Greenwood JA. Waviness amplitude reduction in EHL line contacts under rolling sliding. ASME Journal of Tribology 1998;120:705–9.
[14] Sperka P, Krupka I, Hartl M. Experimental study of real roughness attenuation in rolling/sliding concentrated contacts, Tribology International, in press, doi:10.1016/j.triboint.2011.06.006.
[15] Jacod B, Lugt PM, Dumont ML, Tripp JH, Venner CH. Amplitude reduction of waviness in elastohydrodynamic lubrication using an Eyring fluid model, Proceedings of the IMechE, Part J: Journal of Engineering Tribology 214: 343–50.
[16] Dowson D, Higginson GR. Elastohydrodynamical lubrication, the fundamentals of roller and gear lubrication contacts. Oxford, Great Britain: Pergamon Press; 1966.
[17] Roelands CJA. Correlational aspects of the viscosity–temperature–pressure relationship of lubricating oils, PhD thesis, Technical University Delft, the Netherlands; 1966.
[18] Biboulet N, Lubrecht AA, Houpert L. Contact pressure in indented elastohydrodynamic contacts. Proceedings of the Institution of Mechanical Engineers Part J 2008;222:415–21.
[19] Biboulet N, Lubrecht AA, Houpert L. Contact pressure of indented wide elliptical contacts: dry and lubricated cases. ASME Journal of Tribology 2008;140:1–7.
[20] Venner CH, Ten Napel WE, Bosma R. Advanced multilevel solution of the EHL line contact problem. ASME Journal of Tribology 1990;112:426–32, doi:10.1115/1.2920277.
[21] Venner CH, Morales-Espejel GE. Amplitude reduction of small-amplitude waviness in transient elastohydrodynamically lubricated line contacts. Proceedings of the IMechE, Part J: Journal of Engineering Tribology 1999;213:487–504.
[22] Venner CH, Lubrecht AA. Multilevel methods in lubrication. In: Dowson D, editor. Elsevier tribology series, vol. 37. Amsterdam: Elsevier; 2000.