Periodically Modulated Thermal Convection

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Many natural and industrial turbulent flows are subjected to time-dependent boundary conditions. Despite being ubiquitous, the influence of temporal modulations (with frequency $f$) on global transport properties has hardly been studied. Here, we perform numerical simulations of Rayleigh-Bénard convection with time periodic modulation in the temperature boundary condition and report how this modulation can lead to a significant heat flux (Nusselt number $Nu$) enhancement. Using the concept of Stokes thermal boundary layer, we can explain the onset frequency of the $Nu$ enhancement and the optimal frequency at which $Nu$ is maximal, and how they depend on the Rayleigh number $Ra$ and Prandtl number $Pr$. From this, we construct a phase diagram in the 3D parameter space ($f$, $Ra$, $Pr$) and identify the following: (i) a regime where the modulation is too fast to affect $Nu$; (ii) a moderate modulation regime, where $Nu$ increases with decreasing $f$, and (iii) slow modulation regime, where $Nu$ decreases with further decreasing $f$. Our findings provide a framework to study other types of turbulent flows with time-dependent forcing.

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Turbulent flows driven by time-dependent forcing are common in nature and industrial applications [1,2]. For example, Earth’s atmosphere circulation is driven by periodical heating from solar radiation, the ocean tidal current by periodical gravitational attractions from both the Moon and the Sun, and the blood circulation by the beating heart.

In periodically driven turbulence in shear flows, a mean-field theory has been used to analyze the resonance maxima of the Reynolds number [3,4]. Periodic forcing in other turbulent systems, for example, in the homogeneous isotropic turbulence [5–8], pipe flow [9–12], channel flow [13,14], Taylor-Couette flow [15,16], and Rayleigh-Bénard (RB) convection [17–19], is also shown to have highly nontrivial response properties.

Here we picked turbulent Rayleigh-Bénard convection as a model system to study how time periodic modulation of temperature boundary condition influences global heat transport. The RB system, consisting of a fluid layer heated from below and cooled from above, has been extensively studied as the paradigmatic and well-defined system for convective thermal turbulence [20–22]. Also, several modulation methods have been studied for that system, such as bottom temperature modulation [17,19,23], rotation modulation [18,24], and gravity modulation [25,26]. Intuitively, one may expect that the modulation effect on time-averaged global quantities is limited because the net force averaged over a cycle vanishes. Indeed, with bottom temperature modulation in experiments, only a small enhancement ($\approx 7\%$) of the heat flux has been observed so far [17,19]. However, in those experiments, the effects of modulation in temperature have not yet been fully explored because of the experimental challenge in having a broad range of modulation frequency due to thermal inertia of the plates. Note that also in numerical simulations thermal inertia can straightforwardly be treated [27,28], but in this study, for conceptional clarity, we keep the problems of thermal transport in the RB cell and in the plates disentangled and assume perfect conductivity of the plates.

In this Letter, we numerically study modulated RB convection within a wide range (more than 4 orders of magnitude) of modulation frequency at the bottom plate temperature and observe a significant ($\approx 25\%$) enhancement in heat transport. To explain our findings, we show the relevance of the Stokes thermal boundary layer (BL), which is analogous to the classical one for an oscillating plate [29], in determining the transitional frequency for the heat transport enhancement and the optimal frequency for the maximal heat transport. In particular, we calculate the transition between the different regimes in phase space and show how they depend on the Rayleigh and Prandtl numbers, which represent the ratios between buoyancy and viscosity and between momentum diffusivity and thermal diffusivity, respectively. Our modulation method...
is complementary to hitherto used concepts of using additional body force or modifying the spatial structure of the system to enhance heat transport, for example, adding surface roughness [30–32], shaking the convection cell [33], including additional stabilizing forces through geometrical modification [34–36], rotation [37], inclination [38,39], or a second stabilizing scalar field [40].

Next to the aspect ratio of the horizontal and vertical extensions of the container, the dimensionless control parameters are the Rayleigh number \( Ra = \alpha g H^3 \Delta / (\nu \kappa) \) and the Prandtl number \( Pr = \nu / \kappa \), with \( \alpha, \nu, \) and \( \kappa \) being, respectively, the thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid, \( g \) the gravitational acceleration, and \( \Delta \) the temperature difference between the bottom and top boundaries. The time, length, and temperature are made dimensionless by the free-fall time \( \tau = \sqrt{H/\alpha g \Delta} \), the height \( H \) of the container, and the temperature difference \( \Delta \), respectively. In the following, all quantities are dimensionless, if not otherwise explicitly stated. In the periodically modulated RB, we give a sinusoidal modulation signal to the bottom temperature as

\[
\theta_{\text{bot}} = 1 + A \cos(2\pi f t). \tag{1}
\]

For modulated RB, two more parameters have to be introduced, namely the modulation frequency \( f \) and its amplitude \( A \), which is kept fixed in this study, \( A = 1 \). The efficiency of the heat transport and flow strength in the system are represented in terms of the Nusselt number \( Nu \) (the dimensionless heat flux) and the Reynolds number \( Re \) (the ratio between inertia and viscous forces). Direct numerical simulation (DNS) for incompressible Oberbeck-Bousinesq flow are employed [41]; the numerical details are provided in the Supplemental Material [42]. The DNS are conducted in a two-dimensional square box with no-slip and impermeable boundary conditions (BCs) for all walls. The explored parameter range spans \( 10^7 \leq Ra \leq 10^9 \), \( 1 \leq Pr \leq 8 \), and \( 10^{-4} \leq f \leq 4 \). We are aware of the limitation of the two-dimensionality of the system on which we focus, but in particular for \( Pr \geq 1 \) two- and three-dimensional RB convections show very close similarities and features [44]. To support that our results are also relevant for 3D RB, we conduct a set of three-dimensional DNS in a cubic box at \( Ra = 10^8 \) and \( Pr = 4.3 \) with various frequencies.

Figure 1(a) shows how the global convective heat flux \( Nu \) depends on the modulation frequency \( f \) at fixed \( Pr = 4.3 \) (corresponding to water). The dependence of \( Nu \) on \( f \) exhibits a universal trend for both, two- and three-dimensional results, which is independent of \( Ra \): When \( f \) is large enough, \( Nu \) is not sensitive to the modulation frequency, and the value is close to the value \( Nu_0 \) for the case without modulation. However, when \( f \) is below a certain onset frequency (denoted as \( f_{\text{onset}} \)), there exists an intermediate regime with significantly enhanced heat flux as compared to \( Nu_0 \). With \( f \) decreasing further, one observes an optimal frequency \( f_{\text{opt}} \) at which \( Nu \) is maximal with an enhancement of approximately 25%. Such a large enhancement of \( Nu \) is highly nontrivial because the time-averaged temperature of the bottom plate is still fixed at 1, and we only have changed the bottom temperature from a steady value to a time periodic signal. In Fig. 1(b), we further examine the \( Nu(f) \) dependence for different \( Pr \), with \( Ra \) fixed at \( 10^8 \). One can see that both \( f_{\text{onset}} \) and \( f_{\text{opt}} \) are much more sensitive to \( Pr \) than to \( Ra \).

We first examine whether the transition is related to the strength of the large-scale circulation (LSC). Figure 1(c) shows the global Reynolds number \( Re \) as function of \( f \) for various \( Ra \), from which we can see that \( Re \) is maximized at a \( Ra \)-dependent frequency \( f_{\text{opt}, Re} \) (see Reynolds resonance in Supplemental Material [42] for further analysis of \( f_{\text{opt}, Re} \)). However, when comparing the \( Nu \) and \( Re \) behavior, one observes that the position of the strongest LSC does not correspond to that of the maximum heat transport.
(\(f_{\text{opt}} \neq f_{\text{opt,Re}}\)). What physics then governs the transitions between the regimes of heat flux?

To gain insight into this problem, we analyze how the flow structure is changed under modulation. Figure 2(a) shows the temperature fields at different phases of modulation at \(f = 10^{-3}\). During the heating phase (\(\theta_{\text{bot}} > 1\)), frequent plume emissions are observed near the bottom plate. On the contrary, during the cooling phase (\(\theta_{\text{bot}} < 1\)), there are no plume emissions from the bottom plate because of the stable stratification near that surface, and the resulting weakening of the circulation. We further calculate the conditional average of the temperature profiles at different phases, and compare these profiles for different modulation frequencies in Figs. 2(b)–2(d). Without modulation, we recover traditional RB with a mean bulk temperature of 0.5 [Fig. 2(b)]. When \(f = 10^{-1}\) as shown in Fig. 2(c), the temperature adjacent to the bottom is significantly affected by modulation, whereas the bulk value is still close to 0.5. However, the overall influence of the modulation is limited because it is too fast to be sensed by the system. With decreasing modulation frequency, the bulk temperature is more and more influenced by the modulation (see Fig. 1 of the Supplemental Material [42]). This suggests that there exists a certain length scale which characterizes how deep the influence of the modulation can penetrate into the convective flow.

To better understand this length scale, we recall the classical Stokes problem. In this flow, a BL is created by an oscillating solid surface with modulating velocity \(U \cos(2\pi ft)\). Likewise, in modulated RB, we can draw the analogy between an oscillating velocity and the oscillating temperature \(\theta\), where \(\theta = \theta - \theta(z)\), with \(\theta(z)\) being the temporally averaged temperature at height \(z\). The governing equation and corresponding BCs are

\[
\frac{\partial \theta'}{\partial t} = (Ra Pr)^{-1/2} \frac{\partial^2 \theta'/\partial z^2},
\]

\[
\theta'(0, t) = A \cos(2\pi ft), \quad \theta'(\infty, t) = 0.
\]

The analytical solution of this PDE is an exponential profile:

\[
\theta'(z, t) = Ae^{-z/\lambda} \cos(2\pi ft - z/\lambda),
\]

with the so-called Stokes thermal BL thickness

\[
\lambda = \pi^{-1/2} f^{-1/2} Ra^{-1/4} Pr^{-1/4},
\]

which is the penetration depth of the disturbance created by the oscillating temperature at the boundary. The distortion [Eq. (3)] travels as a transverse wave through the fluid. From Eq. (4) one can see that the thickness \(\lambda\) of the Stokes thermal BL decreases with increasing modulation frequency. Depending on the relative thicknesses of \(\lambda\), that of the thermal BL \(\lambda_\theta\), and that of the momentum BL \(\lambda_u\), we can obtain three regimes shown in Fig. 2(e). Here we have restricted our discussion to \(1 \leq Pr \leq 8\), where \(\lambda_u \geq \lambda_\theta\).

Regime (i): for \(\lambda < \lambda_\theta < \lambda_u\), the effect of modulation is confined inside the thermal BL, which is also shown by the temperature profiles in Fig. 2(c). In such case, the effect of modulation is negligible and the heat transport is almost unaffected.

Regime (ii): for \(\lambda_\theta < \lambda < \lambda_u\), the plume emission, which occurs at the edge of the thermal BL, can now be influenced by the modulation [Fig. 2(e)], leading to the enhancement of heat transport. We note that in thermal convection with a rough plate, a Nu enhancement can also be observed when the thermal BL is perturbed by roughness [45,46]. Here, we understand the enhancement in Nu by the following mechanism: In the heating phase (\(\theta_{\text{bot}} > 1\)), there is a

\[
\frac{\partial \theta'}{\partial t} = (Ra Pr)^{-1/2} \frac{\partial^2 \theta'/\partial z^2},
\]

\[
\theta'(0, t) = A \cos(2\pi ft), \quad \theta'(\infty, t) = 0.
\]
stronger convective flow and more energetic plumes, as compared to the case without modulation. This can be seen from the value of Nu at the top plate (see Supplementary Material [42]), where it increases to the values above that without the modulation during the heating phase. However, in the cooling phase, Nu starts to decline but still remains at values comparable to that without modulation, due to the remaining convective flow. Therefore, there is a net increase in Nu after one cycle, as compared to the value of Nu without time-dependent modulation.

Regime (iii): for $\lambda_\theta < \lambda_\theta \leq \lambda_S$, the effect of temperature modulation penetrates into the bulk region occupied by the LSC. The role of the bulk flow is to efficiently bring the injected hot/cold fluid near the plates during the heating/cooling phase to the center of the system. Therefore, the center temperature also varies with the phases as seen in Fig. 2(f), in contrast to the situation in regimes (i) and (ii). As a result, at the peak of the heating phase (Fig. 2(f), in contrast to the situation in regimes (i) and (ii). This can be seen in the cooling phase, Nu starts to decline but still remains in the cooling phase becomes weaker for smaller $f$, and the global Nu is expected to decrease for decreasing $f$. When the frequency decreases further and goes to 0, the limiting value of Nu should be higher than without modulation. This is because the asymptotic value of Nu is the integral of the Nu without time-dependent modulation.

According to the physical picture of the three regimes, we compare the relative BL thickness to obtain the boundaries of the regimes, i.e., $f_{\text{onset}}(Ra, Pr)$ and $f_{\text{opt}}(Ra, Pr)$. First, we make use of the relations $\lambda_\theta \sim \text{Nu}^{-1}$ and $\lambda_u \sim \text{Re}^{-1/2}$ for the thermal and momentum BL thicknesses. Then we use the Grossmann-Lohse model for the scaling of Nu(Ra, Pr) and Re(Ra, Pr) in the Lc regime (for large Pr) [47,48]: Nu $\sim$ Pr$^6$Ra$^{1/3}$ and Re $\sim$ Pr$^{-1}$Ra$^{2/3}$. The onset frequency $f_{\text{onset}}$ corresponds to the transition between regime i and regime ii ($\lambda_S \sim \lambda_\theta$), and we obtain

$$f_{\text{onset}} \sim Ra^{1/6} Pr^{-1/2}.$$  

(5)

The optimal frequency $f_{\text{opt}}$ corresponds to the transition between regime ii and regime iii ($\lambda_S \sim \lambda_u$), and we have

$$f_{\text{opt}} \sim Ra^{1/6} Pr^{-3/2}.$$  

(6)

To check these predictions for $f_{\text{onset}}$ and $f_{\text{opt}}$, we replot Nu(f) for various Ra but now versus the rescaled frequency $f/Ra^{1/6}$; see Fig. 3(a) (Pr = 4.3 fixed). Indeed, the figure shows rather good collapses around the onset. Next, we vary Pr for a fixed Ra = 10$^8$ and plot Nu versus the correspondingly rescaled frequencies, namely $f Pr^{1/2}$ for the onset [Fig. 3(b)] and $f Pr^{3/2}$ for the optimum [Fig. 3(c)]. Indeed, one can see the rescaled frequencies (horizontal axis) collapse well, indicating that equations (5) and (6) correctly predict the onset frequency and the optimal frequency for all Pr.

Finally, we present the phase diagram in the $f$ vs Ra and the $f$ vs Pr parameter spaces in Figs. 3(d) and 3(e).

FIG. 3. (a) Normalized Nu as a function of $f Ra^{1/6}$, for different Ra and Pr = 4.3. (b) $f Pr^{1/2}$. (c) $f Pr^{3/2}$ for different Pr and Ra = 10$^8$. Dashed lines show the onset frequency [where Nu(f) starts to be affected, Nu(f)/Nu$_0$ = 1.01] or optimal frequency [where Nu(f) reaches the maximum], averaged for different Ra or Pr. Phase diagram (a) in the $f$ vs Ra and (b) in the $f$ vs Pr parameter spaces. In (a), the lower dashed line shows the optimal frequency $f_{\text{opt}} = 0.65 Ra^{0.22}$ that corresponds to the maximal Nu. The upper dashed line shows the onset frequency $f_{\text{onset}} = 0.015 Ra^{0.14}$ that corresponds to the onset of the heat flux enhancement. In (b), the lower dashed line shows the optimal frequency $f_{\text{opt}} = 0.06 Pr^{-1.35}$, while the upper one shows the onset frequency $f_{\text{onset}} = 0.45 Pr^{-0.65}$.
We classify three regimes: classical RB regime (i), modulation-enhancement regime (ii), and modulation-reduction regime (iii). The boundary between the regimes is found by fitting the numerically obtained \( f_{\text{onset}} \) and \( f_{\text{opt}} \). The fitting scaling relations for onset and optimum (\( f_{\text{onset}} \sim Ra^{0.14} Pr^{-0.65}, \ f_{\text{opt}} \sim Ra^{-0.22} Pr^{-1.35} \)) show a good agreement with the derived ones (\( f_{\text{onset}} \sim Ra^{1/6} Pr^{-1/2}, \ f_{\text{opt}} \sim Ra^{1/6} Pr^{-3/2} \)) except \( f_{\text{opt}} \) vs Ra, corresponding to \( \lambda_s \sim \lambda_u \). We notice that in our model, \( \lambda_s \) is obtained based on a diffusion equation. The neglected advection term can become significant, particularly in regime iii where the Stokes BL may penetrate into the bulk. It, therefore, imposes uncertainty in estimating the weak Ra dependence of \( \lambda_{\text{opt}} \). Our explored parameter range only spans \( 1 < Pr < 8 \) due to extreme costs to explore a wider range. But our model is general for various Pr, as long as the boundary layers exist and follow the given scaling relations. These obviously no longer hold for extreme Pr values (i.e., very large Pr when the flow becomes laminar and very small Pr, when \( \lambda_u < \lambda_0 \)). Moreover, our model indicates the relation of the magnitude of Nu enhancement with Ra and Pr. From Figs. 1(a) and 1(b), the maximal Nu enhancement increases as Pr increases while it is independent of Ra. This is because \( f_{\text{onset}} \) and \( f_{\text{opt}} \) have the same scaling with Ra but different scalings with Pr, as shown in Eqs. (5) and (6). As Pr increases, the gap between \( f_{\text{onset}} \) and \( f_{\text{opt}} \) becomes larger, and Nu keeps increasing in between. Therefore, the maximal Nu increases with increasing Pr.

In conclusion, our results have substantial implications for the investigation of modulated convection systems. For a wide range of parameters in the three-dimensional parameter space (modulation frequency \( f \), Rayleigh number \( Ra \), and Prandtl number \( Pr \)), we have demonstrated how the global heat transport efficiency can be enhanced through temperature modulation in both two- and three-dimensional simulations. The high similarity between 2D and 3D DNS results supports that our results are applicable in both cases and robust. Based on the heat transfer enhancement, we can identify three different regimes: the classical RB regime for fast modulation, an intermediate regime in which the modulation leads to increasing Nu enhancement, and the slow modulation regime in which it leads to decreasing Nu enhancement. The transitions between the regimes are well predicted by the relative thicknesses of thermal, momentum, and Stokes thermal BLs. Our concept of explaining global transport properties in modulated BL flows by the relative thicknesses of the three relevant BLs can also be extended to the angular velocity transfer in modulated turbulent Taylor-Couette flow, or to the kinetic energy transfers in modulated turbulent pipe flow.

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