AN INTERDISCIPLINARY EYE ON MATHEMATICS SERVICE TEACHING

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ABSTRACT

The bachelor programmes at the University of Twente are designed as a series of thematic modules each centred on a project supported by disciplinary units. Ideally, a mathematics course included in a module is linked with other units and is related to the project, thereby encouraging interdisciplinary ways of thinking. Typically, first-year mathematics courses are largely decoupled from the projects as they are taught to many programmes simultaneously. To explicitly link the service mathematics to other fields, they include the option of contextualized and scaffolded exercises which we call “Cases”.

To achieve this explicit link, the design of any Case requires lecturers to communicate to align with both project and mathematical topics, as well as support in the roster. Most Cases were developed in the period 2014-2016. Meanwhile, both the mathematics courses and the modules have evolved, and some programmes dropped Cases altogether. These developments lead to the question of whether their design and use support the intended role and add value.

We evaluate each Case on alignment with the relevant mathematics course, alignment with the module and its design, highlighting Biomedical and Civil Engineering examples. We also determine whether the modules still incorporate it and, if not, the reason for exclusion. Preliminary data suggests that their design and use unevenly support the intended role of the Cases. Preliminary conclusions suggest certain Cases need a redesign, but also cause us to reflect on the existence of the need that the Cases are supposed to be meeting.

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1 INTRODUCTION

1.1 Mathematics in context

Engineers and engineering students use mathematics within their disciplines in many and various forms. Every university engineering programme includes mathematics prominently in the suite of courses offered to the students. Teaching mathematics as an isolated subject to engineering students runs the risk of engineering graduates having had a thorough mathematical education yet struggling to use that mathematics in the workplace [1]. Students need to see clearly the role mathematics plays in their disciplines and to develop not only mathematical competence but disciplinary-specific mathematical understanding. If they simply learn to apply some techniques, they might not see the limitations or necessary adaptations for a particular example.

In their 2013 report for the SEFI Mathematics Working Group, Alpers et el [2] stress that a competence-based approach to mathematics teaching must be integrated into engineering programmes to develop students' ability to apply mathematics in engineering contexts. They argue that students' motivation and engagement are encouraged through seeing mathematics in context and suggest projects or case studies as suitable vehicles for this contextualisation. Such case studies would provide "more demanding learning scenarios" (p. 59) than merely pointing out connections or providing isolated contextualised exercises. For instance, Härterich et al. [3] report on the success of three projects based firmly in engineering context requiring first-year students' skills at calculus and linear algebra, much as our Cases which we discuss here.

1.2 The Twente Educational Model and Cases

The Twente Educational Model (TOM, for Twents Onderwijs Model) structures the bachelor's programmes as a series of thematic modules broadly grounded in project-based learning [4]. In each module, students complete a group project and take part in disciplinary courses that support the project and one another to varying degrees. All technical first-year modules include mathematics courses in calculus and linear algebra; together these courses are termed the "mathematics line" which is taught to many programmes simultaneously. A key feature of TOM is interdisciplinary learning and teaching [5], avoiding the unfortunate silo effect that is prevalent in many higher education institutions, an effect to which mathematics is particularly vulnerable. To assist the linking of mathematics with other disciplinary courses and to distinguish it from the generic mathematics for each programme in a module the courses in the mathematics line can include so-called Cases.

A Case is a single problem or collection of related problems that contextualise mathematics in a context of relevance to the module. An example is the Civil Engineering module "Traffic and Transport" where the Case uses linear algebra to model traffic flow across a road network. The basic premise is that all four first-year modules, each including a course of the mathematics line, will also include a Case as part of the teaching package and that the Case will be well-aligned with the module. Most of the Cases were designed in 2016 at the beginning of the institution-wide roll out of TOM. Meanwhile, modules and courses have evolved, staff has changed, and the use of Cases has been unproblematised and unmonitored.

We aim to answer the question “Are the Cases fulfilling their role?” by asking the subquestions “Does each module have a Case assigned and are they aligned?” and “How is their role perceived by the teachers?”.

2 METHODOLOGY

2.1 Repository review

A central server, accessible to all teaching staff, contains all the documentation related to the Cases. We accessed that server and took note of what Cases are present and looked for omissions. A full review of the Cases is underway. However, for brevity in this paper, we only make brief reference to the entirety of the Cases and instead focus closely on two programmes: Biomedical Engineering (BME) and Civil Engineering (CE).

2.2 Teachers’ evaluation of the Case

Each module has a team of lecturers who discuss educational issues together. For each team, one lecturer acts as module coordinator, typically keeping an overview, leading a course and the project. Each team also has a single responsible lecturer from the mathematics department as a contact person for the team, and sometimes also another leading the Case. We approached both persons for all four modules of BME and CE, module coordinator and Case lecturer. Of the mathematics lecturer leading the Case, we asked whether they had sufficient knowledge regarding the problem and whether they felt part of the module team. Based on their experiences, we wanted to know whether the students could achieve the Case, whether students learned something from the Case, and if the Case brought motivation to the students (the original argument for the Cases). Of the module coordinator, we asked their general impression and appreciation of the Cases as well as about any practical issues, since they were not involved in teaching the Case.

3 RESULTS

3.1 Review of all Cases

The overarching project of which this paper forms a part aims to assess all the Cases in the mathematics line for merit. The first step was to extract from the server mentioned above, all the Cases designed to date. Table 1 lists the Case topics for the departments of Advanced Technology and Electrical Engineering. These two departments have a slightly different first-year mathematics curriculum to the other departments since their curriculum includes vector calculus. In contrast, the others encounter vector calculus in their second year of study. These two programmes are also unique in that they have chosen to abandon the Cases in their educational portfolio as they feel students see mathematics in context sufficiently without it. Table 2 lists the Case topics for the other programmes. Notably, some programmes do not have Cases available for all mathematics courses. A line of inquiry in our project will be to determine why, and whether this gap can be filled.

Table 1. Contexts of Cases by Programme and Course - inactive

	Calculus 1	Calculus 2	Linear Algebra	Vector Calculus
Advanced Technology	Complex pendulum (Mechanics)	Ideal gas laws (Thermodynamics)	Chemical equations (Fundamentals of Materials)	Drug costs (Dynamics)
Electrical Engineering	Complex pendulum (Intro to Elec Eng & Electronics)	(Electric circuits)	Electrical networks (Electronics)	Wave propagation and Maxwell's equations (Fields and Waves)

Module name in parentheses

Table 2. Contexts of Cases by Programme and Course - active

	Calculus 1A	Calculus 1B	Linear Algebra	Calculus 2
Business Information Technology	(Intro to BIT)	Retail inventory model (Software Systems)	Markov chains (Business Intelligence & IT)	(Data and Information)
Biomedical Technology	Branching blood vessels (Molecular Construction of Human Body)	Tumour growth (Microscopic Cancer Detection)	Least squares fit (Biomedical Measurement)	Bone inertial moment (Implant Design)
Civil Engineering	Moments of inertia (Intro to CE)	Error measurements (Water management)	Traffic flow (Traffic and Transport)	Bayes' Rule OR Capital growth (Designing Constructions)
Computer Science	Combinations (context-free) (Pearls of CS)	(Software Systems)	Data transmission (Network Systems)	(Data and Information)
Industrial Design	Piston models (Introduction to IDE)	Branching blood vessels (Ideation)	Force equilibrium in a truss	(Smart Products)

			(Realisation of Products)	
Mechanical Engineering	Force and material properties (Design and Manufacturing)	Ideal gas laws (Energy and Materials)	Force equilibrium in a truss (Energy and Sustainability)	Brayton gas turbine (Design and Mechanics)
Chemical Science and Engineering	Chemical reactions (Chemistry)	Heat exchanger (Process Technology)	Chemical equations (Materials)	Drug costs (Physical and Analytical Chemistry)
Industrial Engineering and Management	Company share price and advertising (Intro to IEM)	Retail inventory model (Operations Management)	Markov chains (Business Intelligence and IT)	Cobb Douglas production function (Supply Chain Management)

In almost all instances, looking at Table 2 above, it is clear that the topic of the Case appears to be aligned with the theme of the corresponding module. Of course, deeper understanding requires digging down into the nature of the Case and the teaching plan of the module. For this paper, we focus on Civil Engineering (CE) and Biomedical Engineering (BME) as two programmes which utilise Cases in all four modules.

3.2 Focus on Biomedical Engineering and Civil Engineering

The first module of BME concerns the design of a biomaterial as an aid for some body function. The courses involve Chemistry, Biochemistry and Anatomy. Initially Calculus 1A treated first-order differential equations too. The Case then dealt with Michaelis-Menten kinetics to study the effect of an enzyme complementing the treatise by Biochemistry. After a reshuffle of the content of the mathematics line, Calculus 1A no longer introduced differential equations but instead discussed the extreme value theorem for a single variable. The current Case discusses the branching of a blood vessel as a one-dimensional branching problem linking to Anatomy.

The second BME Module involves the courses Cell Biology and Microscopy with a project to detect cancerous tissue in a small sample. Calculus 1B always involved integration techniques which are easily related to solving differential equations. The Case discusses various population growth models, e.g. exponential, logistic or Gompertz, and relates the predicted outcome to observations of tumours made by a pathologist.

In the third quartile, BME students get acquainted with medical sensors to measure, for instance, diminished lung function in COPD. The courses teach Physiology, Network Analysis, Measurement Systems and programming with Matlab, while the Mathematics concerns Linear Algebra dealing with solving linear systems. These systems are either underdetermined or square. Within the project, students get lots of data, so the Case considers the least-squares solution for overdetermined systems. The Case shows how to use Matlab for linear regression, and later on, students use the technique to analyse their data.

The first year finishes with a module centred around bones and implants. Students follow courses in Mechanics, Material Science and Imaging. The moment of inertia relates to a bones' strength, and X-rays are used to image fractures. Calculus 2 teaches multivariable calculus, e.g. multi-dimensional integration and different coordinate systems. The connection through the Case comes from computing the moment of inertia for bones with various geometries, e.g. the humerus with a triangular hollow shape for which students get a real X-ray image to estimate real-life parameters. As an example, we outline the content of this case and the classroom experience.

The Case starts with simple rectangular geometries (also discussed in the Mechanics course). We then introduce cylindrical shapes for hollow bones requiring to transform to polar coordinates. A particular bone (humerus) has a triangular shape. This shape turns out to be some 10% stronger than a plain cylindrical one. Students get an X-ray image of the humerus to obtain realistic parameters. Finally, they design an implant for this bone using material properties. Students work in groups of four and can ask support from lecturers and teaching assistants. Students familiarise themselves with the context starting with a few easy questions. Midway, the groups achieve the biggest mathematical hurdle. A few final questions follow for interpretation and application of the results. A typical session lasts 4 hours, sufficient for most groups to finish the Case. After grading (pass/fail and repair), the general solution strategy is discussed in a short feedback session.

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The first module of Civil Engineering (CE) concerns the redesign of the Enschede Station Concourse. The courses consist of Mathematics (Calculus 1A), Matlab, Professional Skills Development and three civil engineering courses: Structural Mechanics 1, Construction Materials and Fundamentals of Civil Engineering. Within the course, the students learn, among other things, how to design a structural system, to determine forces and deformation of beams and to calculate reaction forces and moments. The Case deals with using the method of region slicing to determining moments of inertia as single variable integrals. The Case does not draw on any of the mathematics content of Calculus 1A but instead uses school level mathematics to provide a mathematical context for moments of inertia.

The second module in CE focuses on the physical and policy aspects of water management, including the behaviour of different natural water systems and the effects of human interventions on these systems. The module units are Mathematics (Calculus 1B), Fluid Mechanics, Water, Policy Process and the project, which concerns the design of a dam and management of a reservoir in the Blue Nile in

Ethiopia. The Case concerns error, error propagation and least-squares approximations. The Case is not contextualised in the topic of water management but expands on an important topic related to approximation and data analysis.

Traffic and Transport is the third CE module in which the students learn basic theory from traffic engineering. The courses are Mathematics (Linear Algebra) and Transport Studies, both theory and project. The project concerns a substantial modification towards CO_2-neutral transportation in Enschede, taking into account accessibility and economics. The Case involves modelling traffic flow through a network with varying costs or weights as a system of linear equations.

The final module of the CE first-year programme and its central project relate to designing constructions. The courses within the module are Introduction Construction Project (project management, project organisation and project finance), Systems Engineering, the circular economy, Mechanics, Mathematics (multivariable calculus) and Professional Skills. There are two available Cases for this module. One Case relates to Bayes' rule on conditional probability, and the other Case is financial, looking at capital growth.

3.3 Views of teaching staff and module coordinators

Some of the mathematics lecturers leading the Case had been involved with its development. Others were replaced over the years. The latter expressed that they had sufficient general knowledge to deal with the Case; however, the context available during the development was not easily transferred to a new teacher. As a consequence, specific links to other parts of the module were lost. This disconnect came across in their feeling whether they felt part of the module team. If the teacher would be entirely new, (s)he was not yet embedded in the team, and continuity was lost. Discussions on educational development would be restarted every time.

We then discussed the experience of executing the Case. The Cases for Civil Engineering were certainly doable, i.e. completed within a session of four hours. The Cases for Biomedical Technology, on the other hand, turned out to be too extensive. The lecturers indicated that students mostly understood the idea of the Case, but their formula manipulation skills were overestimated, leaving the problems challenging to finish within the time allocated. While a few Cases taught a nice and relevant point, others did not achieve this link. We learned that one Case was cut short as students would use techniques from a different course to solve the Case, bypassing the pedagogical idea of the Case. Limited time for reflection is another hurdle to achieve a tangible link. In general, the mathematics teachers did not observe that the Cases raised enthusiasm for mathematics, and students mostly just did the Case because it was compulsory.

The module coordinators of Biomedical Technology were all convinced that Cases had added value, as it added to some of their courses or projects. We note that three out of four were involved with the development of the Case. For the fourth, there was a Case that linked well to a course initially, but it was replaced by something else because the contents of the mathematics line changed, and developing a better Case had a lower priority. For Civil Engineering, the module coordinators expressed their support for the aim of Cases and wish to keep the Cases, although they were unsure whether the current Case had added value for their module. For both BME and CE, it matters whether the Case takes place just before or after related practicals, and whether the relevant mathematics has been covered already in class.

A superficial perspective of the fit of the Case to the module was achieved by constructing Table 2. Speaking to the teachers of the Cases as well as the module coordinators provided two far more informative perspectives. The student perspective remains to be sought, both historical (through course evaluations) and current (through questionnaires or interviews)

4 SUMMARY AND CONCLUSION

The Cases were designed as contextualised and scaffolded exercises to explicitly link service mathematics courses with the disciplinary programmes. As time has passed since the current Cases were designed modules and mathematics courses have evolved and teaching staff have changed, prompting concern that the Cases may no longer be meeting the need for which they were designed. Returning to our original questions, we conclude that even though only some Cases fulfil their role, they are appreciated by the non-maths teachers. As with many interdisciplinary subjects, communication between teachers is essential for good Cases. In an observational study, Gast [6] concluded that professional development takes place within teams of lecturers, but this development would not happen automatically. We observed that staff changes in teams were detrimental for the added value of Cases, and that active support would be necessary to have sufficient discussion every year about the Case for each module.

At this early point in our project, we have learned valuable information about the CE and BME Cases, some extendable across all the Cases, some specific. We shall broaden our enquiry to include more departments and also to incorporate the student perspective. Cases have the potential to encourage students' motivation and engagement through seeing mathematics in context [2]. If there is a disconnect between Case and module, for whatever reason, it should be remedied as they provide an excellent educational material to tighten the link between maths and engineering.

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References

[1] Wood, LN (2010), Graduate capabilities: putting mathematics into context, *International Journal of Mathematical Education in Science and Technology*, Vol. *41, No.* 2, pp. 189-198.

[2] Alpers, BA, Demlova, M, Fant, CH, Gustafsson, T, Lawson, D, Mustoe, L, et al. (2013), A framework for mathematics curricula in engineering education: a report of the mathematics working group. European Society for Engineering Education (SEFI), Loughborough University

[3] Härterich, J, Kiss, C, Rooch, A, Mönnigmann, M, Schulze Darup, M and Span, R (2012), MathePraxis–connecting first-year mathematics with engineering applications, *European Journal of Engineering Education*, Vol. *37, No.* 3, pp. 255-266.

[4] Centre of Expertise in Learning and Teaching (CELT) – University of Twente, (2017) *The Twente Educational Model*, Enschede: University of Twente. Retrieved 21 April 2020 from https://www.utwente.nl/en/tom/

[5] Craig, TS (2019), The Twente Educational Model in theory and practice: Two modules as exemplars, *Proceedings of the Twelfth Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics (Swan Delta), Editor Pettigrew, J, Rylands, L, Shearman, D, Yeung, A, Perth, pp. 19-28.*

[6] Gast, I (2018), Team-based professional development: possibilities and challenges of collaborative curriculum design in higher education, PhD thesis University of Twente.