



Optimization of maintenance strategies for railway track-bed considering probabilistic degradation models and different reliability levels

Sara Bressi^a, João Santos^{b,*}, Massimo Losa^a

^a Department of Civil and Industrial Engineering (DICE), University of Pisa, Largo Lucio Lazzarino 1, Pisa, Italy

^b Department of Construction Management & Engineering, Faculty of Engineering Technology, University of Twente, Enschede, the Netherlands

ARTICLE INFO

Keywords:

Track management system
Markov chain
Genetic algorithm
Multi-objective optimization
Railway
Maintenance

ABSTRACT

An optimization-based maintenance scheduling framework is an essential tool to plan the necessary investment to maintain the required performance of a railway line. In the present study, a methodology is proposed to minimize the present value of the life cycle maintenance costs and maximize the life cycle quality level of the track-bed considering different levels of reliability. Probabilistic degradation models are developed for predicting the evolution of the railway track condition over time. Afterwards, a Genetic Algorithm based optimization procedure is applied for obtaining a set of optimal solutions taking into account several constraints. The proposed methodology is applied to an Italian railway track-line case study. The results show that it is possible to develop a decision support system to help railway managers to schedule railway track maintenance operations based on the optimal trade-off between maintenance costs and railway track geometry condition for different levels of reliability.

1. Introduction

The strategic planning of Maintenance and Rehabilitation (M&R) is of utmost importance for infrastructure agencies that need to rationalize the scarce available economic resources. Railway track bed materials and geometry, which degrade under traffic loads and environmental conditions, must satisfy specific quality requirements that are very strict, both to avoid speed limitations and to reduce the risk of derailment. Therefore, track geometry is inspected periodically to detect defects before they reach an unacceptable level. The vertical and lateral deformation of the ballast, together with the particle degradation, represents one of the major aspects that govern the maintenance frequency and the durability of the ballasted track [1]. Ballast settlement represents the highest contribution to the total track settlement [2] and occurs in two phases [3], either as a consequence of initial consolidation (after tamping or renewal) or as a further mechanism, such as, for instance, densification, distortion and degradation. Densification and distortion are characterized by the progressive consolidation and the slide and roll of the ballast particles, respectively [4]. In turn, the degradation is caused by the attrition that leads to the breakage and the change of the particles size [4].

In general, two types of maintenance activities can be performed to

ensure the availability and reliability of the railways: corrective or preventive maintenance. Corrective maintenance envisages the intervention as soon as a failure appears. In turn, preventive maintenance is composed of different interventions performed periodically thanks to a pre-determined schedule [5]. In this case, it is possible to apply the maintenance activities before the elements deteriorate to an unacceptable condition. Furthermore, they can be implemented in the intervals between trains operations [6]. Normally, track geometry and components are monitored, and railway authorities decide to perform maintenance or renewal when certain threshold limits are exceeded without regarding the quality of railway investment. The consequence of the "business-as-usual" practice related to the performance of corrective maintenance whenever a failure happens, leads to ignore the quality of the railway investment. The efficiency of the railway network system can be improved through a higher control on the maintenance processes and application timing. It increases the overall quality level of the track-bed, reduces the discomfort experienced by the users, decreases the environmental impacts [7] and promotes a better allocation of the commonly large amount of economic resources needed for maintenance and renewal. For instance, the average cost for maintenance and renewal of 1 km of track of the West European network is around 50.000 € per year considering all the elements [5]. Ballast maintenance plays a

* Corresponding author at: Universiteit Twente, Faculty of Engineering Technology, Horst – Ring, P.O. Box 217, De Horst 2, 7422LW Enschede, the Netherlands.
E-mail address: j.m.oliveiradossantos@utwente.nl (J. Santos).

<https://doi.org/10.1016/j.ress.2020.107359>

Received 12 April 2020; Received in revised form 2 October 2020; Accepted 3 November 2020

Available online 27 November 2020

0951-8320/© 2020 The Author(s). Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

significant role over the entire maintenance activity of a track. Indeed, track geometry is restored by adjusting the ballast under the sleepers by using tamping, which means packing ballast beneath the sleepers in order to correct the alignment of the rails and hence improve the track quality [8].

Reducing the number of M&R activities, thanks to an optimized maintenance strategy, allows reducing the ballast contamination (fouling) and the use of heavy machines and human resources. Furthermore, it also allows reducing the traffic interruptions and thereby the discomfort experienced by the users.

2. Background

2.1. Optimization-based scheduling of track maintenance

To understand when and where the M&R actions are required, the track geometry is assessed at regular intervals of time by using a diagnostic train. The information gathered allows evaluating whether the geometry falls below an acceptable level. If so, maintenance actions are scheduled to restore and bring the geometry to an acceptable range [8]. The complex system of all these activities from inspection, data elaboration to geometry restoration is time and cost consuming. The improvement in the track asset management resulting from performing optimization-based maintenance has the potential to help improving the quality level of the track-bed and the use of the already limited economic resources.

Optimization problems already formulated for scheduling train timetables and setting transit lines as well as for passenger line assignment in general networks [9,10] can be solved by exact or metaheuristic algorithms. Contrary to exact algorithms, which are able to find exact solutions, a metaheuristic is a specific high-level strategy designed to generate a satisfactory (approximate) solution for an optimization problem [11]. The use of this type of algorithms in lieu of the exact algorithms is explained by the fact that for real-world problems the former is often unable to find exact solutions within a human computational time.

Although the design of optimal maintenance strategies for railway assets is relatively recent, there are already in the literature several studies that investigated the problem of planning maintenance from an optimization-based perspective. Budai et al. [12] provided an overview of maintenance (preventive, corrective, and opportunistic) to be performed in the railway track. They proposed a model for the maintenance optimization focused on the medium-term plan. A more complete Decision Support System (DSS) for track management that incorporates new computational optimization models was proposed later [13]. This enables more accurate knowledge on the track state and provides a more accurate decision support rather than a simple schedule optimization [13]. This system was proposed for helping managers to take the most appropriate decision for track asset management considering financial limitations and other constraints [14]. For instance, in Europe, the International Union of Railways (UIC) and European Rail Research Institute proposed and adopted a DSS for railway track maintenance and renewal called ECOTRACK with the purpose of minimizing the track-bed Life Cycle Cost (LCC) [15]. Another advanced management system (RAMSYS) was designed by railway engineers for ballasted track [16].

Peng et al. [17] proposed a mathematical model to systematically solve the Rail Inspection Scheduling Problem (RISP). RISP involves several tasks, inspection team, budget and business constraints. Modeling these problems with an algorithm increases operational efficiency and safety [17]. Other mathematical models were presented to address the Track Maintenance Scheduling Problem (TMSP) where the total travel costs of the maintenance teams were intended to be minimized [18].

Oyama and Miwa [19] built an all-integer linear programming model for optimizing the tamping schedule. The optimization process was developed in two phases: 1) the modeling of the transition process of surface irregularities (degradation and restoration model), and 2) the

Optimal Track Maintenance Schedule (OTMS). Before solving the OTMS model, the authors applied another model for selecting candidate units that possibly lead to higher improvements when maintenance is operated. In simple terms, the authors tried to optimize the performance of maintenance activities by selecting a number of consecutive lots for the maintenance application. More recently, Vale et al. [20] proposed an advanced integer programming model to optimize the scheduling of preventive maintenance activities in a finite time horizon considering track quality and technical aspects related to the operations. Other models have also been proposed for the optimal maintenance of sleepers, electrical railway components and for scheduling rail maintenance [21–23].

Given the computational complexity of (1) the existing nonlinear mixed-integer programming models designed to solve real large-scale problems, (2) the uncertainties characterizing the data gathered from the measurement systems and (3) the track degradation models, in this paper a probabilistic optimization-based DSS is presented to tackle the problem associated with the design of optimal M&R strategies.

2.2. Track defects, deterioration process and tamping recovery

The geometrical parameters that commonly define the track-bed quality are the following: vertical and horizontal alignment (i.e., VA and HA, respectively), gauge, cant and track twist. The existing standards prescribe minimum and maximum acceptable values for these parameters based on the type of railway line [24].

The VA and HA are the main geometric parameters driving the planning of preventive maintenance. They are normally defined as the geometrical deviation of the rail from the design configuration, measured on the vertical or horizontal plane, respectively. VA is the difference in millimetres between the ideal longitudinal profile, as established in the project, and the real point on the top of the rail in the running plane [25]. It is still the fundamental factor for planning maintenance, as stated by UIC [26], despite the trend observed among the infrastructure managers of merging it with other defects measurements to create a unique track quality index (TQI) [27].

VA and HA can be expressed as isolated defects or as the standard deviation (SD) for the short wavelength (3–25 m) of these parameters in a 200-meter long track segment. The Italian standard [24] and CEN [28] prescribes that maintenance has to be planned based on the values assumed by the SD of VA and HA and the limit values of their isolated defects. An extensive literature review [1,29] revealed that field data of track geometry degradation (SD and isolated defects) are best fitted by linear empirical laws depending on the traffic volume, expressed in Million Grosse Tonnes (MGT), typically.

A nonlinear behaviour for certain characteristics determining the track deterioration process has been highlighted [30,31]. However, the linear relationship is widely accepted as a satisfactory approximation by Esveld 2001 [32] and more recently confirmed by real data collected over the years [8,20].

2.3. Uncertainties and reliability in the track geometry degradation

The majority of the models applied for evaluating the track geometry degradation are deterministic. However, the track deterioration process is affected by several factors, such as, for instance, weather conditions, traffic loads, the geometry of the track-bed and the properties of construction materials [27]. These factors are affected by uncertainties and variability [32] and even when the same conditions apply, the track deterioration might be unexpectedly different. Moreover, the degradation rates of the defects have a significant dispersion for different track-bed segments even if they are contiguous [33].

Due to this reason, the uncertainties should be properly considered when dealing with the development of track-bed degradation models, possibly by applying a probabilistic approach. Caetano and Teixeira [25] showed that the accuracy of the estimation of the future behavior of

the track-bed can be improved by increasing the number of records and inspections. A stochastic model responds to the need of integrating into the analysis the uncertainties affecting the different input parameters, which cannot be captured by a deterministic approach.

At each time it is possible to define the value of the defect in a certain segment that corresponds to a certain probability P_f . That means that the track segment at a certain time has the probability P_f of showing a higher value of the defect being considered. Thus, it is possible to develop a degradation curve corresponding to a certain probability of failure and perform preventive maintenance when the probability of failure exceeds a maximum limit established at the beginning, thereby ensuring a certain level of reliability and quality of the track-bed.

Applying a maintenance strategy supported by such a stochastic predictive modeling of the track geometry degradation allows considering the reliability as an indicator that can ensure that the segment does not exceed a certain value of the defect until the next inspection [25]. Eq. (1) defines the reliability or the probability of a track segment being under the maximum geometric thresholds at a certain time:

$$R(t) = 1 - P_f(t) \quad (1)$$

3. Problem statement and research objectives

Uncertainties in the track degradation models have been considered recently by means of probabilistic approaches. Among those, Markov chain models have received particular attention [8,34]. Nevertheless, those models do not take into account a reliability indicator as a measure to support decision makers (DMs) when making decisions regarding the planning of maintenance activities [25]. One of the few exceptions existing in the literature is the work by Caetano and Teixeira [25]. The authors went further in the analysis by using these degradation models to design optimal maintenance strategies. The formulated multi-objective optimization (MOO) problem was solved through a Genetic Algorithm (GA).

Nevertheless, as declared by the authors, selecting a specific solution from the Pareto front is one of the main problem at the end of such analysis and the solution adopted will depend on the preference and/or subjectivity of DMs. This is because there is no one single solution that can be said to be better than the others.

The present paper aims to advance the state-of-the art by combining a probabilistic approach for the definition of railway track condition degradation models for different levels of reliability with a GA for identifying a set of optimal solutions for the design of railway track-bed maintenance strategies. The GA structure has been specifically tailored for the optimization of railway track-bed maintenance strategies by incorporating the uncertainties affecting the degradation of the track-bed. Thus, it can provide more comprehensive and informed results comparatively to those given by conventional deterministic processes, thereby responding to the ultimate need of increasing the stakeholders' capacity in the railway sector to make more strategic and informed decisions regarding the railway management.

Therefore, the objective of this paper is to present an optimization-based railway track maintenance scheduling framework to help designing optimal track maintenance schedules based on probabilistic degradation models developed using historical data and a reliability indicator. For that purpose, a MOO approach is applied for supporting the DMs in scheduling optimization-based preventive maintenance activities according to different level of reliability, considering different objective functions as well as several financial and technical constraints. Finally, a methodology is applied to select a unique optimal solution among those belong to the Pareto front.

The steps required to achieve the aforementioned objectives are the following:

- 1 Analysis of the real data of an Italian railway line to model the degradation process of the track-bed geometric parameters (i.e., VA

and HA) over time by applying the Markov chains approach. The time characterizing each step has a duration of three months instead of one year to increase the control of the entire process and safety level;

- 2 Definition of the probabilistic degradation models related to different levels of reliability;
- 3 Definition of a set of optimal maintenance strategies through the application of a GA;
- 4 Identification of a unique optimal solution using the criterion of the shortest Euclidean distance.

Based on the research steps presented previously, the rest of this paper is organized as follows: the description of the stochastic approach employed to develop the degradation models of the railway track-bed and the characteristics of the GA employed in the optimization process are presented and explained in Section 4. The formulation of the optimization-based railway track-bed maintenance strategy selection problem is given in Section 5. Section 6 introduces the characteristics of the real case study. The results of the application of the developed methodology to the case study are described in Section 7. Finally, the conclusions and avenues for future research are provided in Section 8.

4. Methodology

4.1. Stochastic approach for modeling the ballast degradation: Markov chains

The application of the Markov model to predict the degradation of the railway track-bed is based on the following assumptions [35]:

- The deterioration process has to be conceived in discrete intervals of time. The deterioration of the track geometry is continuous over time; however, it is possible to consider the condition of the track as discrete at a specific time t . This time usually takes the form of one-year duty cycles for road infrastructure [36]. However, in the framework presented in this paper the time characterizing each duty cycle has a duration of three months to increase the control of the entire process and the safety level.
- The track-bed condition is represented by numbered non-overlapping performance intervals (discrete state space). The space of the possible states is defined as a finite number of condition bands represented by the defect under consideration.
- The process has to satisfy the Markov property, i.e. the future state of the system depends on its present state and not on its past states. Thus, the probability at $t+1$ that the state indicator assumes certain values depends exclusively on the values of such indicator at time t , immediately before $t+1$.

Therefore, the deterioration of the track can be modeled by defining transition probabilities p_{ij} , that represent the probability of the portion of the track network in condition i to move to condition j in one interval of time (duty cycle). The transition probabilities can be organized in a *transition probability matrix (TPM)*, that can be written as shown by Eq. (2):

$$TPM = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix} \quad (2)$$

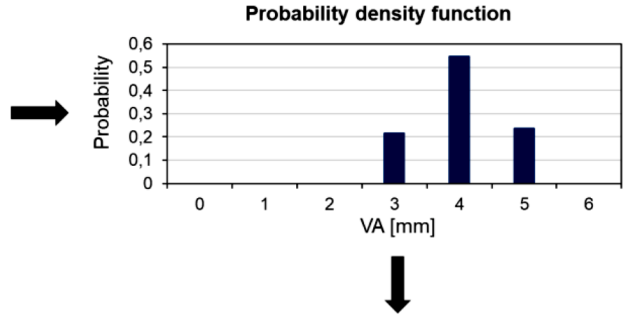
The standard approach to define the p_{ij} in TPM is to observe the historical data, particularly the way in which a track network worsens its state indicators over multiple discrete time intervals. Each element of TPM is the probability that the state indicators VA or HA remain in the same or move to a lower interval.

Moreover, the process can be modeled through a non-stationary approach, i.e. the probability of moving from one track condition to

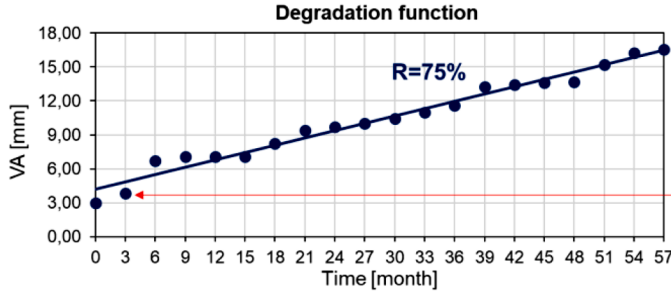
Step 1: State vector definition for each duty cycle

State of the track-bed	VA [mm]	Duty cycle				
		0	5	10	15	19
1	0-3	1,00	0,00	0,00	0,00	0,00
2	3-6	0,00	0,92	0,19	0,00	0,00
3	6-9	0,00	0,08	0,65	0,22	0,07
4	9-12	0,00	0,00	0,16	0,55	0,14
5	12-15	0,00	0,00	0,00	0,24	0,63
6	15-18	0,00	0,00	0,00	0,00	0,16

Step 2: Definition of PDF of the state vector for each duty cycle



Step 4: Definition of the degradation function for each reliability level



Step 3: Definition of CDF of the state vector for each duty cycle

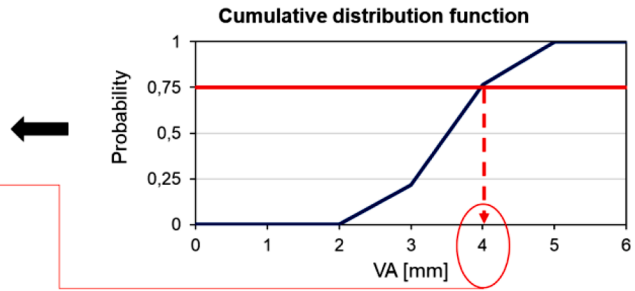


Fig. 1. Schematic representation of the steps to be performed for defining the degradation functions.

another is dependent on the time at which the step is being made. This implies the use of a different transition matrix before and after time t . In this case, since the traffic load gradually increases over time, it affects the probability distribution in every duty cycle. Therefore, the process is described by a non-stationary TPM . Furthermore, following the practice adopted for road infrastructure [37,38], in the framework presented in this paper, it is assumed that the track-bed condition will not drop by more than one state in a single duty cycle, considering that the duty cycle is a short period of time (3 months). Thus, the track-bed will either stay in its current state or move to the immediately lower state during one duty cycle. As such, the TPM will assume a simplified form (semi-Markov process) [39], which is represented by Eq. (3):

$$TPM = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ 0 & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_{nn} \end{bmatrix} \quad (3)$$

The probabilities composing the TPM are given by Eq. (4):

$$p_{ij}^{t,t+1} = \frac{N_{ij}^{t,t+1}}{N_{tot}} \quad (4)$$

Where $p_{ij}^{t,t+1}$ is the probability of dropping from the state i to the state j during a generic duty cycle ($t, t+1$); $N_{ij}^{t,t+1}$ is the number of track sections that drop from state i to state j during a generic duty cycle ($t, t+1$); and N_{tot} is the total number of track-bed sections analyzed.

In an n -state Markov process, the state of the process at any time t is identified by a probability mass function [38] that can be expressed by Eq. (5):

$$a(t) = [a_1^t, a_2^t, \dots, a_n^t] \quad \text{subjectto} \quad \sum_{i=1}^n a_i^t = 1 \quad (5)$$

Where a_i^t is the probability that the track-bed is in state i at time t , and n is the number of states.

Considering that the process starts at time $t=0$, the probability mass function of the process at the generic time t can be derived by multiplying the TPM for each of t transitive steps. Thus, the state vector at a specific time t , can be obtained as shown by Eq. (6):

$$a(t) = a_0 \times TPM^{1,2} \times TPM^{2,3} \times \dots \times TPM^{t-1,t} \quad (6)$$

Where $a(t)$ is the state vector at time t , i.e. the vector of the probability mass function; a_0 is the state vector at time $t=0$; and $TPM^{t-1,t}$ is the transition probability matrix characterizing each duty cycle until time t .

In the framework presented in this paper, the initial state vector (a_0) that expresses the initial state of the track-bed represents the optimal condition just after a maintenance intervention. Knowing that $a(t)$ represents the probability mass function for each duty cycle of the state of the track-bed, it is possible to develop the degradation function of the potential defects for different reliability levels. To do so, the steps listed out below were carried out for both defects (i.e., VA and HA). Fig. 1 illustrates their application for the VA.

1. For each state vector corresponding to a specific duty cycle, a probability density function and the relative cumulative distribution function were associated. The different probability distributions were obtained by performing goodness-of-fit tests to measure the difference between the data and the distribution and to compare the "distance" to a threshold value. If the "distance" is lower than the critical value, the distribution succeeds the test and the fit is considered satisfactory. Three different statistical tests were performed: Kolmogorov-Smirnov, Anderson Darling and Chi-Squared [40,41]. The EasyFit 5.6® software was used to assign a rank to each distribution, indicating with higher position in the ranking the distribution that shows a lower "distance" from the data. Therefore, by comparing the fitted models and the ranking for the three tests it was possible to select the most appropriate probability distribution.
2. Once the cumulative distribution functions were defined for each state, three different levels of reliability were considered (i.e., 75%, 85% and 95%);
3. The values of VA and HA corresponding to the different reliability levels were reported in a graph for each duty cycle, leading to the obtention of three different degradation functions.

The definition of the different degradation functions allows going a

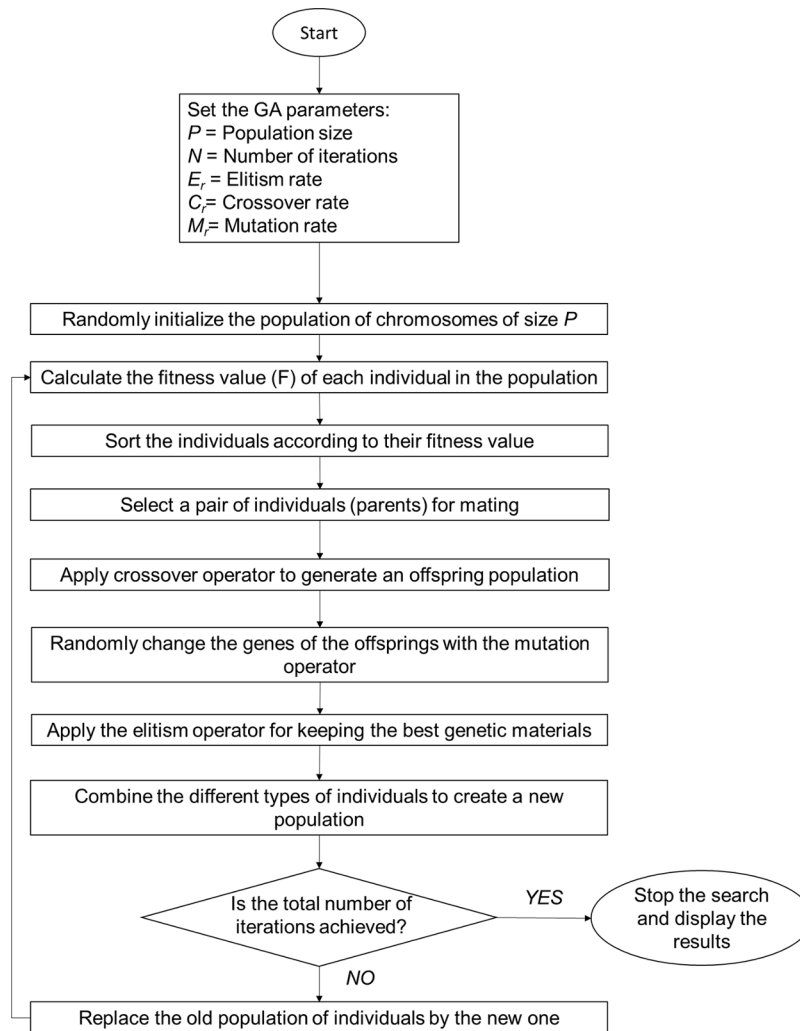


Fig. 2. Schematic representation of the structure of the algorithm.

step further in optimizing the maintenance strategy for each reliability level.

4.2. Optimization model for scheduling maintenance activities: Genetic Algorithms (GA)

Proposed in 1975 by Holland [42], GA are stochastic search engines employed for solving optimization problems. They started to be used in the last decades in transportation applications because they (1) do not require the objective function to be continuous or differentiable; (2) have good robustness for many applications; (3) have outstanding global search capabilities for convex and non-convex problems; (4) have inherent parallel processing capabilities; (5) are relatively easy to implement; and (6) are adaptable and flexible [43–45].

While in pavement management engineering GA have been widely applied, as documented by the literature [36,46–49], less research studies can be found in scheduling optimized M&R activities in railway engineering [25,50]. Therefore, one of the objectives of this research work is to contribute to the knowledge of the application of GA for designing optimal maintenance strategies of railway track-bed, integrating a probabilistic approach to predict the evolution of the parameters representing the quality level of the track-bed.

4.3. Structure of the GA

Similar to other Evolutionary Algorithms, the application of GA

starts by the definition of the size of the population (P), which consists of n chromosomes representing potential solutions of the problem. The elements that form each string of chromosomes are called genes. In this research work each gene represents a different type of maintenance intervention performed during a certain duty cycle. The dimension of the chromosomes is given by the project planning period. For every chromosome it is calculated the *fitness* value, which is an indicator of the appropriateness of a chromosome as a solution for the problem. It enables the selection of the best chromosomes and the generation of a new population through the recombination of the genes. Once the fitness of each chromosome is calculated, a sub-population is selected to generate new solutions. There are different techniques for selecting the set of solutions for the reproduction (mating pool). The most immediate and simple is to order the n chromosomes of the population by sorting their fitness function value from the highest to the lowest and select the first (in a maximization problem) or the last (in a minimization problem) k solutions (with $k < n$).

Next, two chromosomes (parents) are randomly selected from the mating pool and a cut point (crossover point) is randomly selected. The portions of the two chromosomes at the right of the crossover point are exchanged, which generates two new chromosomes called children or offsprings [47]. After the creation of the children through the application of the crossover operator, certain elements of new chromosomes are randomly changed (mutation). The application of the mutation operator is essential because it helps to prevent the potential convergence to a local minimum or maximum.

To preserve the best genetic material, the chromosomes characterized by the highest fitness values are copied into the subsequent generation [50]. Moreover, before any new chromosomes are added to the new population, a given number of old chromosomes are copied from the old population according to the so-called elitism rate (E_r). The main idea of elitism is to preserve the best genetic material, copying the best members from each generation to the subsequent generations.

The process summarily described above is repeated until a termination criterion is met. A flowchart representing the working mechanism of the GA is displayed in Fig. 2.

4.4. Generation of optimal solutions: the Pareto front

To solve the MOO problem, it is necessary to introduce the concept of domination. In light of the Pareto dominance concept extended to solutions, a solution x_1 is said to dominate another solution, x_2 , if both conditions are true (for a maximization problem):

- The solution x_1 is not worse than x_2 for all the objective functions, i. e., $f_i(x_1) \geq f_i(x_2)$ for all $i \in (1, 2, \dots, N)$, where N is the total number of objective functions;
- The solution x_1 is strictly better than x_2 in at least one objective, i. e. $f_i(x_1) > f_i(x_2)$, for at least one $i \in (1, 2, \dots, N)$, where N is the total number of objective functions.

Thus, every solution is directly compared with another to determine which one is the best in relation to the objective i . By extending this comparison procedure to all the objectives N , it is possible to define a set of non-dominated solutions, called *Pareto optimal set*, which represents the solutions of the MOO problem. The objective function values of the *Pareto optimal set* in the objective space is named *Pareto front* [48].

There are several optimisation methods that can be used to generate the *Pareto optimal set*. Among other, they include aggregation methods (e.g., weighted sum method (WSM)), weighted metric methods (e.g., compromise programming methods), goal programming methods, achievement functions methods, goal attainment method, ϵ -constrained method, dominance-based approaches (e.g., NSGA-II, SPEA2, PESA-II, etc.) [48].

According to Marler and Arora [51], there is not a single method that is in general superior to the others. In this research work, the MOO problem is solved by means of the WSM. According to this method the various objective functions are combined into one performance function by assigning weights (preferences) to each objective. This results in a SOO problem (Eq. (7)). By varying uniformly the values of the set of weights (Eq. (8)), the solutions that fall between the objectives' boundaries are obtained.

$$\min \sum_{i=1}^{N_{obj}} w_i \times \frac{OF_i(\vec{X}) - OF_i^{\min}}{OF_i^{\max} - OF_i^{\min}} \quad (7)$$

Subject to

$$w_i \geq 0, \quad i = 1, \dots, N_{obj}, \quad \sum_{i=1}^{N_{obj}} w_i = 1 \quad (8)$$

Where w_i is the weight assigned to the objective i , which varies from 0 to 1 according to a given incremental step; $OF_i(\vec{X})$ is the value of the objective function i for the solution \vec{X} ; OF_i^{\min} is the minimum value of the i th objective function; OF_i^{\max} is the maximum value of the i th objective function, and; N_{obj} is the number of objectives for the MOO problem being considered.

Given that the result of a MOO problem is not a single optimal solution, but instead a set of optimal solutions, there is a need for the DM to select a particular solution from the Pareto front, depending on his/her own judgment or preference. A methodology that can be applied to select a single solution relies on the use of guiding constraints, such as,

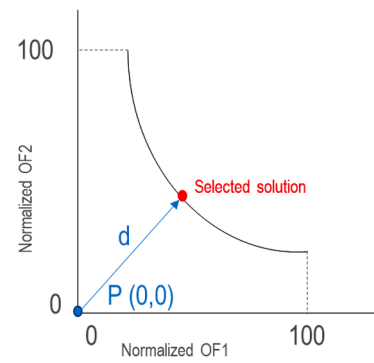


Fig. 3. Schematic representation of the criterion of the shortest Euclidean distance.

for instance, the total maintenance cost nearest to the available budget. If there are no indications about any guiding constraints, the most convenient solution that represents a satisfactory balance of the different objective functions can be found according to the criterion of the shortest Euclidean distance [37]. The main idea of this criterion consists of finding the point in the space (P) that represents the ideal best optimal solution when the best possible values of both objective functions are achieved simultaneously [52]. This solution is often called Utopia point. Therefore, the solution to be selected and implemented is the closest one to the point P in terms of normalized distance. A schematic representation of the criterion of the shortest normalized distance is shown in Fig. 3, considering as example a minimization problem.

Because different objective functions are commonly measured with different measurement units, it is necessary, beforehand, to normalize the obtained optimal Pareto front. The normalized value for each non-dominated solution is calculated based on its objective function values. The normalized value of objective function i of a generic solution x belonging to the Pareto front is scaled to a value ranging from 0 to 100 according to Eq. (9) [37]:

$$NOF_i^x = \frac{OF_i^x - OF_i^{\min}}{OF_i^{\max} - OF_i^{\min}} \times 100, \quad i = 1, \dots, N_{obj} \quad (9)$$

Where NOF_i^x is the normalized value of objective function i of a generic x solution belonging to the Pareto Front; OF_i^x is actual value of objective function i of a generic x solution belonging to the Pareto Front; OF_i^{\max} is the maximum value of objective function i of all solutions belonging to the Pareto Front, and; OF_i^{\min} is the minimum value of objective function i of all solutions belonging to the Pareto Front.

For a n -objective optimization problem the solution that has the shortest Euclidean distance d from the ideal solution is the solution to be selected. d can be calculated according to Eq. (10).

$$d = \sqrt{[NOF_1^x - NOF_1^{\min}]^2 + [NOF_2^x - NOF_2^{\min}]^2 + \dots + [NOF_{N_{obj}}^x - NOF_{N_{obj}}^{\min}]^2} \quad (10)$$

5. Problem formulation: the railway track-bed maintenance strategy selection problem

In the research work presented in this paper, the railway track-bed maintenance strategy selection problem was formulated as an optimization problem where the aim is to optimize different objectives concomitantly, while satisfying several technical quality standards and budgetary requirements. The main set of decision variables of this optimization problem, which are defined by an integer value, is conceived to represent all feasible maintenance operations that can be performed in each duty cycle of the analysis period. Examples of other sets of variables include those describing the railway track-bed condition in each duty cycle of the analysis period. As far as the specification

of the objective functions is concerned, it is as follows:

- Objective 1: Minimize the mean vertical alignment (VA) over the analysis period

This objective function can be expressed as follow:

$$\text{Min. } OF_1 = \frac{\sum_{t=1}^T Y_{tR}}{T} \text{ for } R \in \{75\%, 85\%, 95\%\} \quad (11)$$

Where Y_{tR} is the value of the degradation function (derived from Markov model application) at each time t (duty cycle) for the reliability level R considered (i.e., 75, 85 and 95%); T is the analysis period (time) expressed in duty cycles, and; OF_1 is the objective function value related to the mean VA of the railway track-bed.

- Objective 2: Minimize the present value (PV) of the total M&R costs incurred over the analysis period

This objective function can be expressed by Eq. (12).

$$\text{Min. } OF_2 = \sum_{t=1}^T \sum_{m=1}^M \frac{MC_{mt}}{(1+d)^t} \times X_{mt} \quad (12)$$

Where MC_{mt} is the maintenance cost incurred due to the execution of maintenance intervention m to track-bed section at time t ; X_{mt} is equal to one if maintenance intervention m is applied to track-bed section at time t , otherwise it is equal to zero; d is the discount rate; T is the analysis period (time) expressed in duty cycles, and; OF_2 is the PV of the total M&R costs incurred over the analysis period.

The optimization problem is subject to the following general set of constraints:

$$Y_{tR} = \Phi(Y_{0R}, X_{11}, \dots, X_{1t}, \dots, X_{m1}, \dots, X_{mt}), m = 1, \dots, M; t = 1, \dots, T \quad (13)$$

$$Y_{tR} \leq Y_R^{\max}, t = 1, \dots, T \quad (14)$$

$$X_{mt} \in \Omega(Y_{tR}), m = 1, \dots, M; t = 1, \dots, T \quad (15)$$

$$\sum_{m=1}^M X_{mt} = 1, t = 1, \dots, T \quad (16)$$

$$\sum_{t=1}^T \sum_{m=1}^M \frac{MC_{mt}}{(1+d)^t} \times X_{mt} \leq B_{\max} \quad (17)$$

$$\sum_{t=1}^T C_t = T \quad (18)$$

$$\{MC_{mt}\} = \Psi(Y_{tR}, X_{mt}), m = 1, \dots, M; t = 1, \dots, T \quad (19)$$

Constraints (13) correspond to the railway track-bed conditions functions. In this formulation they express the value of the degradation function of the railway track-bed section in each time t as a set of functions of the initial condition and the maintenance activities previously applied to the section. Constraints (14) are the warning level constraints which define the maximum values (Y_R^{\max}) for the railway track-bed condition variables (Y_{tR}) for a given reliability level R . Constraints (15) represent the feasible maintenance sets, i.e. the M&R activities that can be applied to maintain or rehabilitate the railway track-bed section in relation to its quality condition. Constraint (16) indicates that only one M&R activity can be performed in each duty cycle. Constraint (17) is the budget constraint and specifies the maximum budget (B_{\max}) available to be spent in maintenance activities during the total analysis period T . Constraint (18) controls the number of correct maintenance activities applied at the right time (C_t), i.e., when certain defect thresholds are not exceeded. Constraints (19) represent the maintenance costs which are computed in relation to the condition of

Table 1
Discretization of intervals for VA and HA.

State	VA [mm]	HA [mm]
1]0; 3]]0; 3]
2]3; 6]]3; 6]
3]6; 9]]6; 9]
4]9; 12]]9; 12]
5]12; 15]]12; 15]
6]15; 18]	-

the railway track-bed section and the maintenance activity applied to the section in a given duty cycle.

6. Case study application

6.1. Development of probabilistic condition degradation models

The inputs for the development of the probabilistic condition degradation models (i.e., VA and HA) of the railway track-bed were obtained through an analysis of 5 years (from 2005 to 2010) of historical data from the track geometry inspections of the homogeneous trafficking Italian railway line Pistoia-Lucca. The Pistoia-Lucca line is a 42-km long single track line. The maximum train running speed is 160 km/h. The rails are all long-welded rails type 60 UIC. Prestressed concrete sleepers are positioned at a 60 cm center-distance. The fastening systems are of the type Pandrol.

The track data were collected using the "Archimede" diagnostic train, whereas maintenance interventions were collected at the technical office. Among the years of the data analysis, a time span of "non-intervention" for each section (starting after the implementation of a tamping operation) was considered to determine the degradation function without any maintenance intervention.

The main parameters considered in the development of the models were the following:

- Geometric parameters analysed: Vertical Alignment (VA) and Horizontal Alignment (HA);
- Data acquisition interval: 5 years with no-intervention. Those data were used to calculate the Probability Transition matrix (TPM);
- Number of sections: 25;
- Duty cycle: 3 months.

Furthermore, for the sake of an omogeneous partition, the three-by-three discretization of intervals was selected and set to be equal for VA and HA as shown in Table 1.

At the beginning of the process (year 2005), tamping was carried out in all sections, bringing the VA and HA of each individual section to a value of 1.50 mm. Thus, all the sections start the degradation process in an excellent condition (state 1). Afterwards, the degradation process follows a different evolution for the different sections. Fig. 4 depicts examples of the evolution of the VA defect for different sections.

Nineteen different TPMs (non-homogeneous Markov chains) were obtained for each defect (i.e., VA and HA). The values of the TPMs were obtained by dividing the number of the track-bed sections that remained in their current conditions from one duty cycle to another by the total number of track-bed sections considered in the study (Eq. (4)). Once obtained the TPMs, it is possible to obtain the state vectors for each duty cycle by applying Eq. (6). The results of the state vectors obtained in each duty cycle without applying maintenance interventions are reported for VA and HA in Tables 2 and 3, respectively. Figs. 5 and 6 represent the probability of finding the track-bed sections in a certain state at a certain time (duty cycle).

As can be seen from Figs. 5 and 6, at the beginning of the process the probability of finding the track-bed line in state 1 (excellent state) is very high. As the time evolves, the probability density function representing

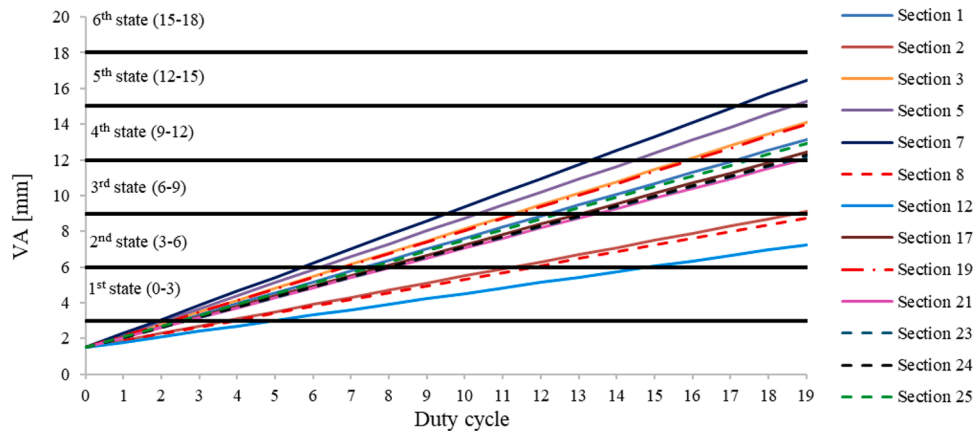


Fig. 4. Evolution of VA values for different sections considered in the case study.

every state shifts and the standard deviation increases. This means that the probability of finding the track-bed in one single state is lower and the dispersion higher. Towards the end of the established interval, there is a high probability (approximately 63% at the duty cycle 19 for VA) of finding the track-bed in state 5. In this state immediate maintenance is required.

Finally, once the distributions of the state vectors were calculated, their average can be determined according to the following expression [49]:

$$\bar{y} = a_i \cdot c^Y \quad (20)$$

Where c^Y is a vector consisting of the midpoints of the condition states for the defect Y (Table 4). The results of \bar{y} for both VA (\overline{VA}) and HA (\overline{HA}) are reported in Fig. 7a and b.

Fig. 7 shows the evolution of the degradation process of VA and HA over time. The maintenance interventions are planned based on the warning levels established by the Italian Railway Standard [24], which are also presented in Fig. 7. The levels within which the railway is normally operated are divided into three levels of quality:

- 1 *First quality level*: within this range it is not necessary to execute any maintenance intervention (0–6 mm for VA and HA);
- 2 *Second quality level*: within this range it is appropriate to plan the possible execution of maintenance interventions (6–10 mm for VA and 6–8 mm for HA);
- 3 *Third quality level*: within this range the execution of maintenance interventions is expected to take place within a limited time frame (10–14.5 mm for VA and 8–10.4 mm for HA).

Looking at Fig. 7, it is possible to observe that VA remains under the limit of the 1st quality level for almost 7 duty cycle (21 months), while for HA this value rises to 9 duty cycles, which corresponds to 27 months. That means that during this time span the track-bed does not require the execution of any corrective action.

Afterwards, the sections exceed the first quality level, remaining under the second quality level approximately until the 14th duty cycle for both VA and HA. The second quality level establishes that the geometric conditions of the track-bed allow the normal operation of the railway without any type of restriction. Nevertheless, for this interval the actions presented below are required to be performed:

- Analysis of the causes of degradation;
- Evaluation of the progress rate of the geometric parameter;
- Plan the possible maintenance work on the geometry according to the evolution of the defect detected locally.

The third quality level still allows the railway operation without any type of restriction provided that maintenance work is carried out shortly, so that the time span that will elapse until the actual execution of the maintenance does not exceed the maximum value allowed by the third quality level (14.5 mm). If the maximum value is exceeded, it is necessary to intervene by imposing speed restrictions.

For the period considered in the present study, the VA and HA were not found to reach the thresholds corresponding to the third level (14.5 mm for VA and 10.5 mm for HA). Comparing the average degradation of VA and HA in Fig. 7, it can be seen that the degradation process is approximately similar for both alignments.

6.2. Railway track condition degradation functions and reliability levels

A statistical analysis was performed to define the reliability level (i. e., 75, 85 and 95%) of the railway track condition degradation functions. Tables 5 and 6 show the results of the selection of the probability distributions for each duty cycle and the values of VA and HA corresponding to the different reliability levels, also for each duty cycle. Table 7 presents the condition degradation functions for VA and HA for the different reliability levels. They were defined according to the methodology schematically represented in Fig. 1. The resulting unconditional probabilities describe a non-homogeneous and non-stationary or invariant process distribution, i. e. the probability distribution does not remain constant as the Markov chain evolves over time, even if a higher level of stationarity is observed for the HA degradation compared to that of the VA. Despite the fact that most agencies that adopt Markov chain degradation models rely on static transition probabilities, the subjective knowledge of the experts and the high variability of all the parameters considered result in the modeling of the degradation process that is not completely adequate [39]. Therefore, the probability distributions identified in this case study for each duty cycle represent a step further towards a model more adherent to the reality of the degradation.

6.3. Optimization algorithm: parameters setup and constraints handling mechanism

To start the implementation of the GA used to solve the optimization model it is necessary to define the value of a set of parameters, such as the population size (P), mutation rate (M_r), crossover rate (C_r) and elitism rate (E_r).

A large population size improves diversity but slows down the convergence process and increases computational time. On the contrary, if the population is too small, the risk of a premature convergence of the algorithm towards a poor local optimal solution is high [53]. A high crossover probability may destroy the structure of important genetic

Table 2
State vectors corresponding to each duty cycle (3 months) for the VA model.

State	Duty cycle																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1.00	0.92	0.23	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.08	0.77	0.92	1.00	0.92	0.57	0.32	0.19	0.19	0.19	0.06	0.06	0.06	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.08	0.43	0.68	0.81	0.73	0.65	0.61	0.46	0.15	0.22	0.22	0.22	0.22	0.14	0.07
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.16	0.32	0.48	0.70	0.63	0.55	0.47	0.31	0.07	0.14
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.16	0.24	0.31	0.39	0.63	0.63
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.16	0.16

Table 3
State vectors corresponding to each duty cycle (3 months) for the HA model.

State	Duty cycle																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1.00	1.00	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.55	1.00	1.00	1.00	1.00	0.82	0.55	0.36	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.45	0.64	0.91	1.00	1.00	0.73	0.55	0.45	0.36	0.18	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.45	0.55	0.64	0.82	0.91
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09

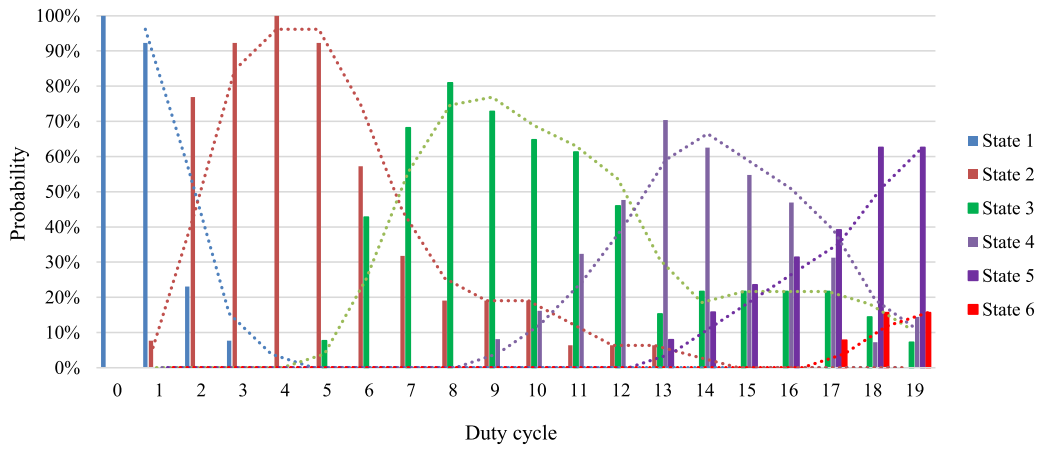


Fig. 5. Probabilities of finding the track-bed sections in a certain VA state at a certain time.

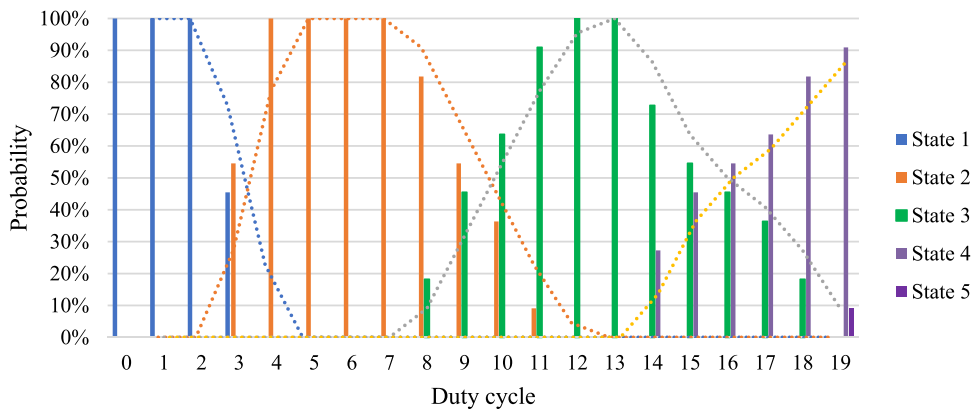
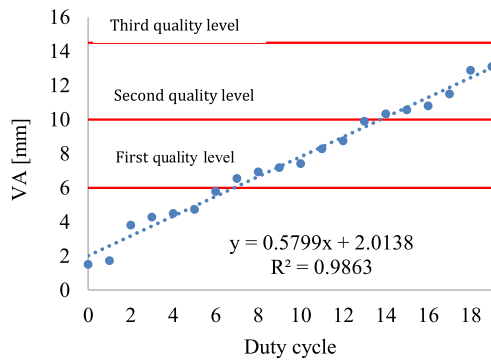


Fig. 6. Probabilities of finding the track-bed sections in a certain HA state at a certain time.

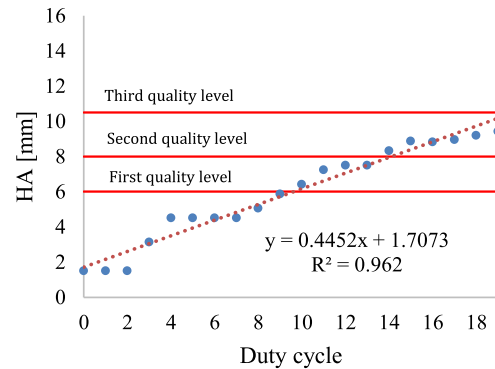
Table 4

Midpoint of the condition bands for VA and HA.

Condition state	VA range [mm]	HA range [mm]	e^{VA} [mm]	e^{HA} [mm]
1]0; 3]]0; 3]	1.5	1.5
2]3; 6]]3; 6]	4.5	4.5
3]6; 9]]6; 9]	7.5	7.5
4]9; 12]]9; 12]	10.5	10.5
5]12; 15]]12; 15]	13.5	13.5
6]15; 18]	-	16.5	-



(a)



(b)

Fig. 7. Average of the state vectors (\bar{y}) for (a) VA (\bar{VA}) and (b) HA (\bar{HA}).

Table 5
Probability distribution identified for the values of the VA at each duty cycle and corresponding values for different reliability levels.

Duty cycle	Month	Probability distribution	VA [mm]		
			75%	85%	95%
1	3	Nakagami	3.86	4.29	5.03
2	6	Weibull	6.70	7.38	8.49
3	9	Log-Logistic	7.04	7.98	10.13
4	12	Log-Logistic	7.04	7.98	10.13
5	15	Frechet	7.04	7.98	10.13
6	18	Frechet	8.22	9.27	11.79
7	21	Weibull	9.39	9.98	11.79
8	24	Weibull	9.68	10.22	11.79
9	27	Uniform	10.00	10.53	11.79
10	30	Uniform	10.45	11.07	11.79
11	33	Log Logistic	11.00	11.89	13.80
12	36	Frechet	11.58	13.01	16.46
13	39	Weibull	13.25	14.07	16.46
14	42	Uniform	13.42	14.06	16.46
15	45	Lognormal	13.60	14.30	16.46
16	48	Lognormal	13.66	14.60	16.46
17	51	Lognormal	15.19	16.13	17.33
18	54	Gumbel Min	16.27	16.92	17.85
19	57	Weibull	16.54	17.37	18.65

Table 6
Probability distribution identified for the values of the HA at each duty cycle and corresponding values for different reliability levels.

Duty cycle	Month	Probability distribution	HA [mm]		
			75%	85%	95%
1	3	Rayleigh	3.00	3.00	3.00
2	6	Rayleigh	3.00	3.00	3.00
3	9	Rayleigh	6.18	7.23	9.08
4	12	Rayleigh	6.18	7.23	9.08
5	15	Rayleigh	6.18	7.23	9.08
6	18	Rayleigh	6.18	7.23	9.08
7	21	Rayleigh	6.18	7.23	9.08
8	24	Frechet	7.21	7.23	9.59
9	27	Frechet	8.30	7.92	11.95
10	30	Weibull	9.28	9.37	11.95
11	33	Gen. Ext. Value	9.28	9.89	11.95
12	36	Gen. Ext. Value	9.28	9.89	11.95
13	39	Gen. Ext. Value	9.28	9.89	11.95
14	42	Frechet	10.62	11.45	13.32
15	45	Frechet	11.33	12.35	14.67
16	48	Weibull	12.03	12.61	14.67
17	51	Weibull	12.27	12.83	14.67
18	54	Weibull	12.66	13.14	14.67
19	57	Gumbel Max	12.72	13.14	14.67

Table 7
Railway track condition degradation functions for VA and HA for different reliability levels.

Reliability level [%]	Alignment type	Degradation function [mm]	R ²
75	VA	$y(x) = 0.21x + 4.19$	0.98
	HA	$y(x) = 0.18x + 3.21$	0.91
85	VA	$y(x) = 0.22x + 4.79$	0.97
	HA	$y(x) = 0.18x + 3.94$	0.92
95	VA	$y(x) = 0.0001x^3 - 0.0116x^2 + 0.57x + 4.12$	0.94
	HA	$y(x) = 0.0001x^3 - 0.0136x^2 + 0.64x + 1.69$	0.96

Table 8
Parameters of the GA adopted in this study.

Parameter	Value
P	500
C _r	0.6
M _r	0.1
E _r	0.1

Table 9
Unit costs of the M&R interventions.

ID	Name	M&R cost [€/km]
1	No intervention	0
2	Inspection	111
3	Tamping	7200

Note: The costs values were taken from the literature [25,54,55].

Table 10
Summary of the parameters considered in the application of the optimization model to the case study.

Name	Value	Unit	Note
Maximum budget available (B_{max})	120 000 (for 75 and 85% reliability levels) and 180 000 (for 95% reliability level)	€/km	Parameter considered in constraint (17) of the optimization model. Its value was defined according to the values found in literature [25].
Maximum admissible VA value ($Y_R^{VA, max}$)	14.5	mm	Parameter considered in constraint (14) of the optimization model and defined according to the Italian standard [26]
Period of analysis	20	years	
Discount rate	3.5	%	[36]

information of some chromosomes, whereas a low value may constraint their diversity. The mutation is meant to allow genetic diversity. Its rate generally varies between 0.001 and 0.1 and when it is too low some useful genes may never be explored [37].

Based on the considerations presented above, the values commonly reported by the literature, and an exploratory analysis, the parameters were set as presented in Table 8.

Finally, due to the stochastic nature of the search process employed by GA, it might happen that the application of the genetic search operators produce infeasible solutions, i.e., solutions that do not meet one or multiple constraints considered. Therefore, those undesirable solutions need to be somehow penalised so that they are not considered as good solutions. An intuitive approach to handle those solutions consists simply of rejecting them. However, this procedure might lead to the GA being stuck in a local optima. In order to prevent this scenario, the penalty-based approach presented by Santos et al. [46] was implemented in this research work. According to this approach, the fitness value of an infeasible solution not only depends on the amount of constraints violation, but also on the population of individuals of each generation.

6.4. Aspects related to the implementation of the optimization model

When integrated in the optimization model, the use of the railway track condition degradation functions allows the design of optimal long-

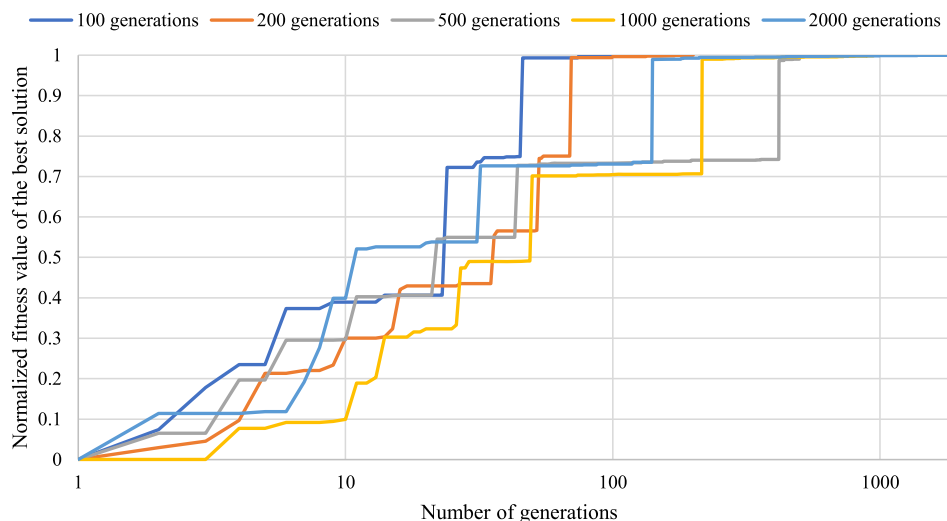


Fig. 8. Normalized fitness value of the best solution when the GA is run with different number of maximum generations.

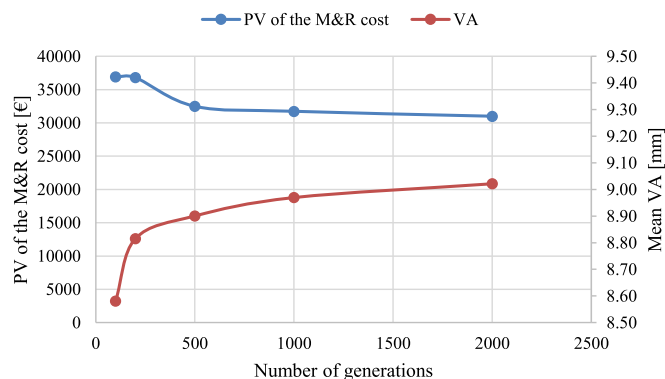


Fig. 9. Objective functions value of the best solution for 100, 200, 500, 1000 and 2000 generations.

term M&R strategies according to the objective functions and constraints considered. On this subject, it matters to mention that the implementation of tamping intervention corrects the defects related to both alignments, i.e., vertical and horizontal. If the intervention to restore the VA is applied, the HA is corrected as well. Therefore, since the degradation trends of VA and HA were found to be approximately similar, only the VA degradation function was used for the design of optimal maintenance strategies.

The applicability of the optimization model was illustrated through a track-bed section of the Lucca- Pistoia railway with a standard length of 1 Km. Moreover, in this case study 3 mutually exclusive maintenance interventions were considered: (1) maintenance 1 "no intervention", (2) maintenance 2 "inspection", and; (3) maintenance 3 "tamping". These interventions were selected based on the warning levels for the defects defined by the Italian standards [24]. Their unit costs are presented in Table 9. The remaining parameters considered in the application of the optimization model to the case study are summarized in Table 10. Finally, it should be mentioned that for the analysis period considered in the case study (i.e., 20 years) the renewal activity was not considered to be available for application based on the criterion that considers the 30% of particles passing the 22.4 mm sieve as a limit for the application of that activity [1,54].

7. Results and discussion

The optimization model was written in MATLAB® programming software (MATLAB, 2015), and run on a computational platform Intel Core i7-6700HQ 2.60 GHz processor with 16.00 GB of RAM, on the Windows 10 Home operating system.

7.1. Evolution of the stability of solutions

In order to evaluate the evolution of the fitness value of the best solution in the search for the optimal solution, the GA was implemented by subjecting the initial population of size 500 to an evolutionary process considering 100, 200, 500, 1000 and 2000 generations. This is a common procedure used in evolutionary computing to decide when enough generations have been run based on the stabilisation of results [56] in order to avoid excessive and unnecessary computing time. Under those circumstances, when no (or residual) changes in the fitness value of the best solution are observed, one can say that the optimal solution has been reached.

Fig. 8 shows the fitness value of the best solution when the GA was run considering 100, 200, 500, 1000 and 2000 generations and equal weights were assigned to both objective functions. Additionally, Fig. 9 presents the M&R costs and mean VA over the entire analysis period (i.e., 20 years) for the best solution found throughout the search process. From the analysis of both figures it can be seen that no considerable improvements in the best fitness value was achieved after approximately 500 generations. As such, 1000 generations have been conservatively adopted as the number of generations for this case study.

The Pareto fronts obtained for the different reliability levels are presented in Fig. 10. This Figure shows the existence of a clear trade-off between the two objective functions, meaning that a reduction in the PV of the life cycle M&R costs can only be achieved at the expense of an increase in mean value of the VA.

After displaying the Pareto fronts, the next and final step of the methodology consists of selecting the final solution among the several Pareto solutions according to the minimum Euclidean distance criterion [37]. For the analyzed problem, the OF_i^{\min} and OF_i^{\max} values represented in Eq. (9) are summarized in Table 11 for the different reliability levels. The normalized Pareto optimal fronts obtained after applying Eq. (16) are illustrated in Fig. 11 for each reliability level. Finally, the optimal solution is identified as the solution that has the smallest Euclidean distance from the point in the Cartesian plane that ideally represents the

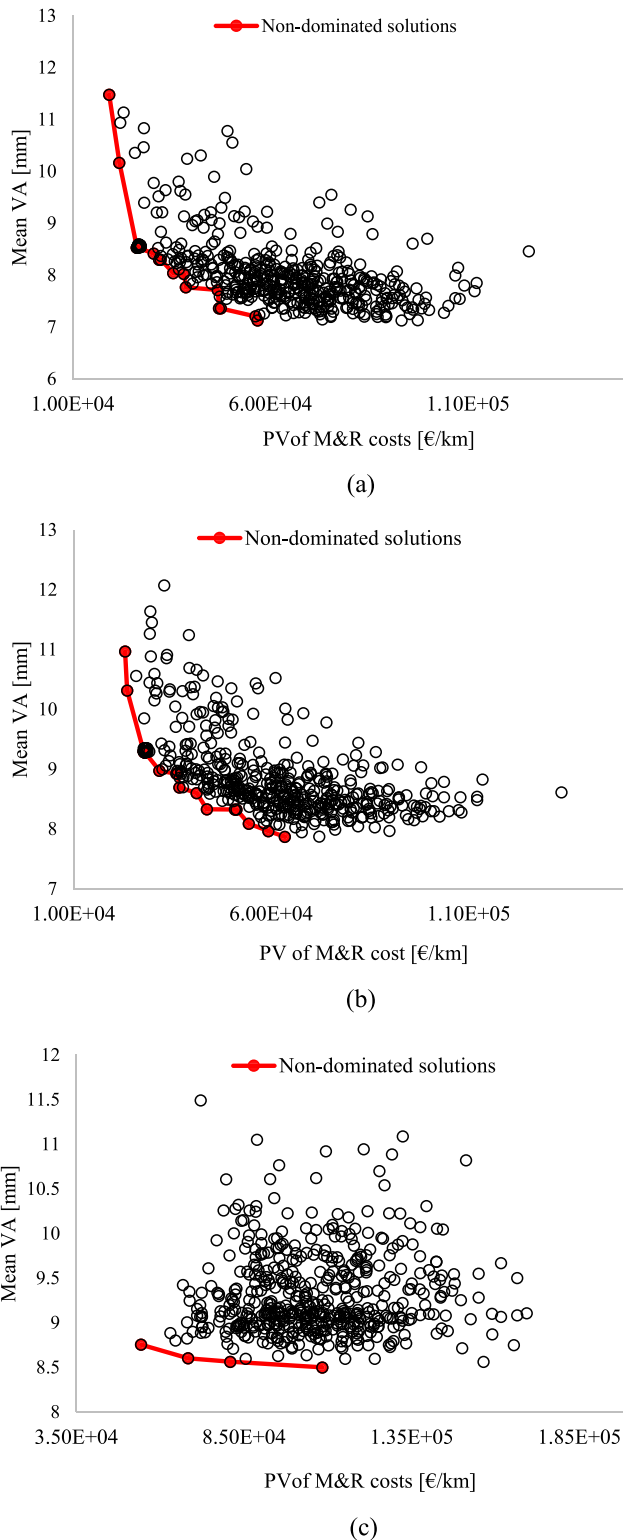


Fig. 10. Pareto optimal fronts obtained when the considered reliability level is equal to (a) 75%, (b) 85% and (c) 95%.

minimum life cycle M&R costs and the minimum mean VA. The normalized Euclidean distance of those solutions as well as their objective function values are summarized in Table 12.

Table 11

Lower and upper bound values of the Pareto optimal set for the different reliability levels.

Objective Function		Reliability level [%]		
Name	Nomenclature	75	85	95
Minimize PV of the life cycle M&R	OF_2^{\min} [€/km]	19 196	23 032	54 339
	OF_2^{\max} [€/km]	56 769	63 484	108 356
Minimize the life cycle mean VA	OF_1^{\min} [mm]	7.13	7.87	8.50
	OF_1^{\max} [mm]	11.48	10.96	8.75

7.2. Optimal M&R schedules

The results presented in Table 12 show that the selected Pareto optimal solution with the lowest level of reliability entails a total M&R cost of € 26191 €/km and a mean VA of 8.53 mm throughout the 20-year analysis period. That means that by employing the corresponding M&R plan (Fig. 12) in 1 km of the railway line, the probability of keeping the mean VA equal to 8.53 mm is equal to 75%. It corresponds to carry out 5 tamping operations and a consistent number of inspections (35) during the 20-year analysis period in order to ensure an adequate level of safety and ride comfort. It should be mentioned that the M&R plan is very detailed because every duty cycle is equal to three months.

For the intermediate level of reliability (i.e., 85%) a total M&R cost of € 31731 €/km and a mean VA value of 8.97 mm are observed if the M&R plan corresponding to the selected Pareto optimal solution is implemented (Fig. 13). It requires to perform 5 tamping operations and 42 inspections during the 20-year analysis period in order to ensure an adequate level of safety and ride comfort. By doing so, the probability of keeping the mean VA equal to 8.97 mm is 85%.

Finally, for the highest level of reliability considered (i.e, 95%) the selected Pareto optimal solution is associated with a total M&R cost of € 68359 €/km and a mean VA value of 8.60 mm. That means that by employing these economic resources, the probability of keeping a mean VA equal to 8.60 mm along 1 km of the railway line is equal to 95%. The corresponding M&R plan envisages to execute 12 tamping operations and 53 inspections throughout 20-year analysis period in order to guarantee an adequate level of safety and driving comfort (Fig. 14).

Fig. 15 depicts the three Pareto optimal fronts obtained for the different reliability levels. From the comparison of the Pareto optimal fronts it is possible to observe that as the reliability increases there is a diagonal shift (towards the right upper corner) and a flattening of the Pareto optimal fronts. In practice that means the worsening of the marginal value of the money, in the sense that for each additional euro spent in M&R the railway agency can expect less expressive reductions in the mean VA, and thereby increase in safety and comfort. The trade-off between the two objectives functions (Mean VA and PV of M&R costs) follows a nonlinear curve as shown in Fig. 16. This result is in line with the results obtained recently by other studies [25].

8. Conclusions and future research

The framework presented in this paper introduces a methodology conceived to support the design of optimal preventive maintenance strategies that allows maintaining a satisfactory quality level of a railway track-bed while minimizing the present value of the life cycle maintenance costs. For solving the optimization-based maintenance planning problem three main steps were carried out: (1) the degradation process over time of the geometric parameters were analyzed according with a probabilistic approach; (2) three degradation functions were conceived considering three different reliability levels; and (3) an optimal maintenance strategy was defined by applying a GA to solve the

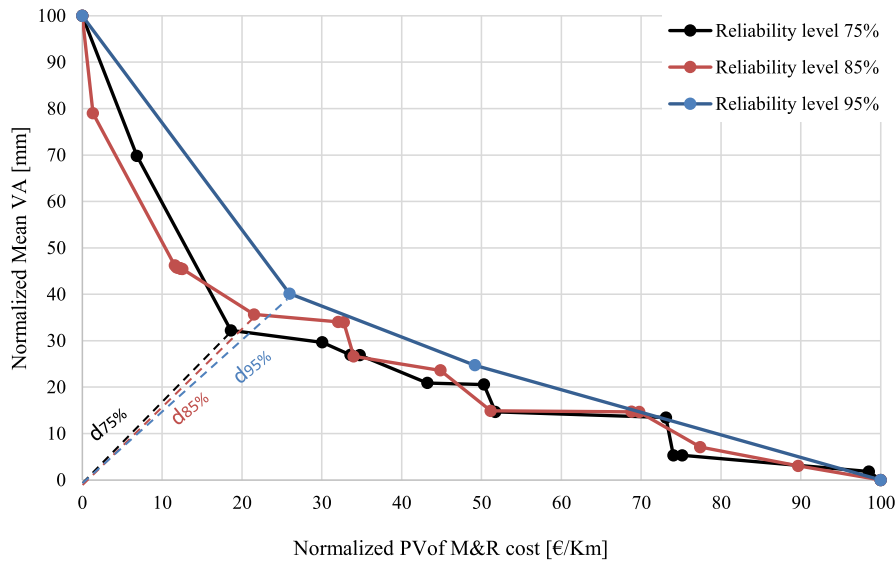


Fig. 11. Normalized Pareto optimal front for the different reliability levels.

Table 12

Characteristics of the selected Pareto optimal solutions.

Item	Reliability level [%]		
	75	85	95
Euclidean distance (d)	37.2	41.63	47.80
Life cycle M&R costs [€/km]	26 191	31 731	68 359
Life cycle mean VA[mm/km]	8.53	8.97	8.60

multi-objective maintenance planning problem. By using real data obtained from inspections on the Italian Lucca-Pistoia railway track-bed, it was possible to determine the level of reliability for the evaluation of the degradation of the geometric parameters considered.

Based on the characteristics of the case study considered and the results obtained, the following main conclusions can be drawn:

- Through the use of a MOO approach, it is possible to develop a decision-making process to support the design of optimal strategies for the planning of inspections and tamping operations with a certain reliability level. Furthermore, a criterion was applied to establish the

balance between the objectives involved (i.e., the concomitant minimization of the PV of the M&R cost and the mean VA over the analysis period). The procedure clearly allowed the identification of a specific maintenance strategy for each reliability level.

- The selected Pareto optimal solution for each reliability level led to mean VA values that varies in the range of approximately 8.5 and 9 mm. To ensure that those values are attained, different economic resources are necessary to be spent per km depending on the reliability level. In particular, approximately 26200, 32000 and 68500 € are necessary when considering 75, 85 and 95% level of reliability. The cost-reliability trade-off indicates that to achieve a higher level of reliability and control for properly scheduling inspections and tamping operations, the necessary investment in maintenance increases according to an exponential relationship. In particular, for an increase of 10% in the reliability level from 75% to 85%, it is necessary to increase the budget allocated to railway track maintenance by 22%, whereas an increase in the reliability level from 85% to 95% requires an increase of the budget equal to 114%.
- The optimal maintenance plans require the execution of different numbers of inspections and tamping operations depending on the reliability level being considered. In particular, 5, 5 and 12 tamping

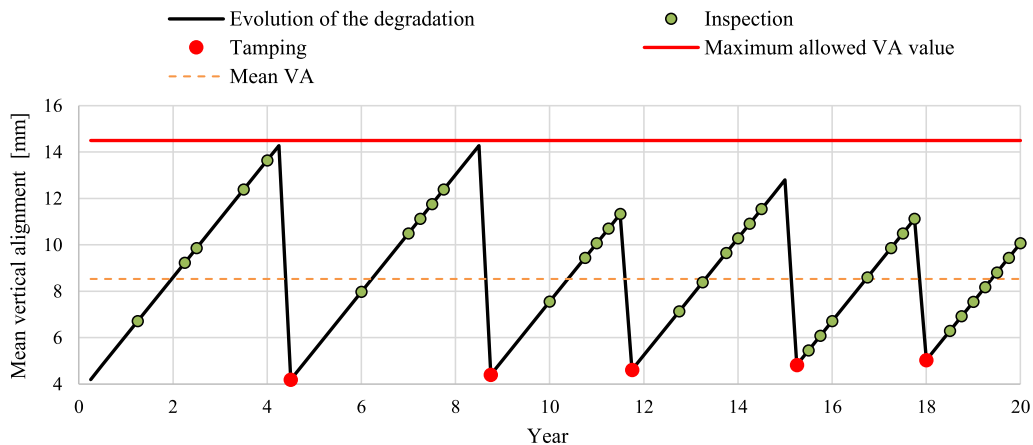


Fig. 12. M&R plan corresponding to the optimal solution with 75% of probability of ensuring an average VA equal to 8.53 mm.

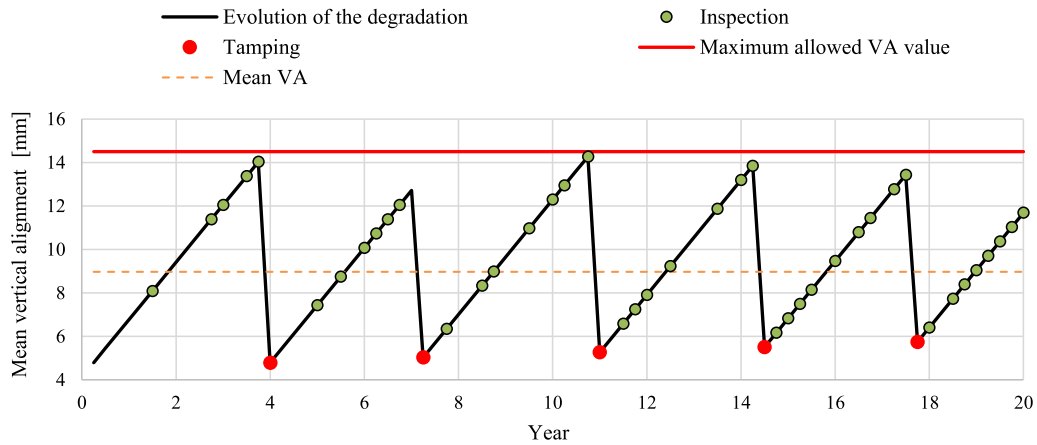


Fig. 13. M&R plan corresponding to the optimal solution with 85% of probability of keeping an average VA equal to 8.97 mm.

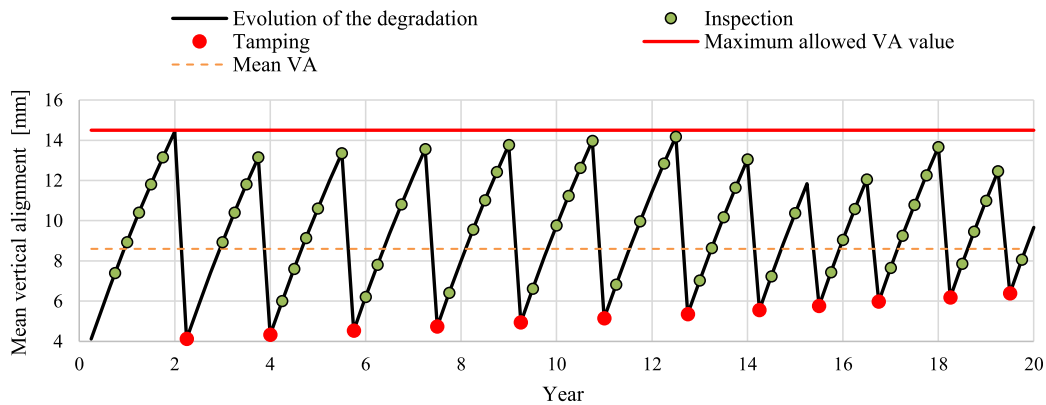


Fig. 14. M&R plan corresponding to the optimal solution with 95% of probability of ensuring an average VA equal to 8.60 mm.

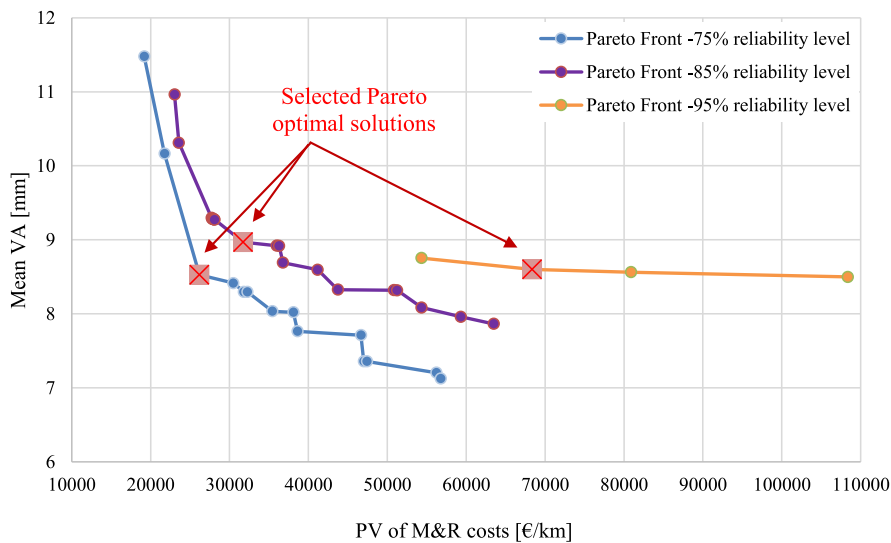


Fig. 15. Comparison of the Pareto optimal fronts of the three different reliability levels.

operations are necessary for the reliability levels of 75, 85 and 95%, respectively. Moreover, increasing the reliability level also implies an increase in the number of inspections.

Although the proposed methodology has proved to be useful for real applications, its applicability can be further extended either by including other geometric parameters or by incorporating additional/different maintenance/renewal operations that have the potential to

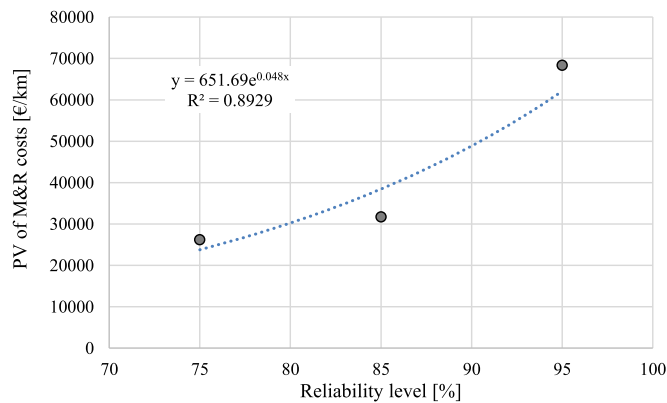


Fig. 16. Relationship between the PV of the M&R costs associated with the selected Pareto optimal solutions and the level of reliability considered.

increase the geometric quality of the track, such as, for example, the cleaning of the railway track-bed ballast or the improvement of the drainage.

CRedit authorship contribution statement

Sara Bressi: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **João Santos:** Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **Massimo Losa:** Supervision.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

References

- D'Angelo G, Bressi S, Giunta M, Lo Presti D, Thom N. Novel performance-based technique for predicting maintenance strategy of bitumen stabilised ballast. *Constr Build Mater* 2018. <https://doi.org/10.1016/j.conbuildmat.2017.11.115>.
- Read D, Li D. Research results digest 79-design of track transitions. *Transp Coop Res Progr* 2006.
- Dahlberg T. Railway track settlements-a literature review. Rep EU Proj SUPERTRACK 2003.
- Sun QD, Indraratna B, Nimbalkar S. Deformation and degradation mechanisms of railway ballast under high frequency cyclic loading. *J Geotech Geoenviron Eng* 2016. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001375](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001375).
- Guler H. Optimisation of railway track maintenance and renewal works by genetic algorithms. *Gradjevinar* 2016. <https://doi.org/10.14256/JCE.1458.2015>.
- Higgins A, Ferreira L, Lake M. Scheduling rail track maintenance to minimise overall delays. *Transp Traffic Theory* 1999.
- Bressi S, D'Angelo G, Santos J, Giunta M. Environmental performance analysis of bitumen stabilized ballast for railway track-bed using life-cycle assessment. *Constr Build Mater* 2018;188. <https://doi.org/10.1016/j.conbuildmat.2018.08.175>.
- Prescott D, Andrews J. Investigating railway track asset management using a Markov analysis. *Proc Inst Mech Eng Part F* 2015. <https://doi.org/10.1177/0954409713511965>.
- Cacchiani V, Caprara A, Toth P. Scheduling extra freight trains on railway networks. *Transp Res Part B* 2010. <https://doi.org/10.1016/j.trb.2009.07.007>.
- Guan JF, Yang H, Wirasinghe SC. Simultaneous optimization of transit line configuration and passenger line assignment. *Transp Res Part B* 2006. <https://doi.org/10.1016/j.trb.2005.12.003>.
- Moghaddam KS, Usher JS. Preventive maintenance and replacement scheduling for repairable and maintainable systems using dynamic programming. *Comput Ind Eng* 2011. <https://doi.org/10.1016/j.cie.2010.12.021>.
- Budai G, Dekker R, Nicolai RP. Maintenance and production: a review of planning models. Maintenance and production: a review of planning models. Springer; 2008. https://doi.org/10.1007/978-1-84800-011-7_13.
- Guler H. Decision support system for railway track maintenance and renewal management. *J Comput Civ Eng* 2013. [https://doi.org/10.1061/\(ASCE\)CP.1943-5487.0000221](https://doi.org/10.1061/(ASCE)CP.1943-5487.0000221).
- Rashidi M, Lemass B, Gibson P. A decision support system for concrete bridge maintenance. In: AIP Conference proceedings; 2010. <https://doi.org/10.1063/1.3452105>.
- Rivier RE. ECOTRACK: A tool for track maintenance and renewal managers. In: *Proc. int. conf. comput. aided des. manuf. oper. railway and other adv. mass transit systems*; 1998.
- Jovanovic S, Guler H. Railway infrastructure assets management by using appropriate systems. In: *7th International congress on advances in civil engineering, Istanbul, Turkey*; 2006.
- Peng F, Ouyang Y, Somani K. Optimal routing and scheduling of periodic inspections in large-scale railroad networks. *J Rail Transp Plan Manag* 2013. <https://doi.org/10.1016/j.jrtpm.2014.02.003>.
- Peng F, Kang S, Li X, Ouyang Y, Somani K, Acharya D. A heuristic approach to the railroad track maintenance scheduling problem. *Comput Civ Infrastruct Eng* 2011. <https://doi.org/10.1111/j.1467-8667.2010.00670.x>.
- Oyama T, Miwa M. Mathematical modeling analyses for obtaining an optimal railway track maintenance schedule. *Jpn J Ind Appl Math* 2006. <https://doi.org/10.1007/BF03167551>.
- Vale C, Ribeiro IM, Calçada R. Integer programming to optimize tamping in railway tracks as preventive maintenance. *J Transp Eng* 2011. [https://doi.org/10.1061/\(ASCE\)TE.1943-5436.0000296](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000296).
- Zhao J, Chan AHC, Burrow MPN. Reliability analysis and maintenance decision for railway sleepers using track condition information. *J Oper Res Soc* 2007. <https://doi.org/10.1057/palgrave.jors.2602251>.
- Zorita AL, Duque O, Fernández MA, García-Escudero LA. Determination and optimization of the maintenance frequencies in the overhead contact line system. *J Transp Eng* 2010. [https://doi.org/10.1061/\(ASCE\)TE.1943-5436.0000166](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000166).
- Gorman MF, Kanet JJ. Formulation and solution approaches to the rail maintenance production gang scheduling problem. *J Transp Eng* 2010. [https://doi.org/10.1061/\(ASCE\)0733-947X\(2010\)136:8\(701\)](https://doi.org/10.1061/(ASCE)0733-947X(2010)136:8(701)).
- Rete Ferroviaria Italiana (RFI). Standard di qualità geometrica del binario con velocità fino a 300km/h 2007.
- Caetano LF, Teixeira PF. Predictive maintenance model for ballast tamping. *J Transp Eng* 2016. [https://doi.org/10.1061/\(ASCE\)TE.1943-5436.0000825](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000825).
- UIC. <http://www.uic.org> 2016.
- Shafahi Y, Hakhamaneshi R. Application of a maintenance management model for Iranian railways based on the Markov chain and probabilistic dynamic programming. *Sci Iran* 2009.
- European Committee for standardization (CEN). Railway applications—track—track geometry quality. Part 5: Geometric quality levels. EN 13848-5. Brussels: 2008.
- Audley M, Andrews JD. The effects of tamping on railway track geometry degradation. *Proc Inst Mech Eng Part F* 2013. <https://doi.org/10.1177/0954409713480439>.
- Ubalde L, López Pita A, Teixeira P, Bachiller A, and Gallego I. Track deterioration in high-speed railways: Influencing of stochastic parameters. 8th International conference on railway engineering, n.d., p. 54–62.
- Veit P. Track auality: luxury or necessity? *Railw Tech Rev Spec Maint Renew* 2007; 8–12.
- Esveld C. *Modern railway*. MRT Prod 2001.
- Maji A, Das A. Reliability considerations of bituminous pavement design by mechanistic-empirical approach. *Int J Pavement Eng* 2008. <https://doi.org/10.1080/10298430600997240>.
- Andrade AR, Teixeira PF. Uncertainty in rail-track geometry degradation: lisbon-orporto line case study. *J Transp Eng* 2010. [https://doi.org/10.1061/\(ASCE\)TE.1943-5436.0000206](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000206).
- Lyngby N, Hokstad P, Vatn J. RAMS Management of railway tracks. *Handb Performability Eng* 2008. https://doi.org/10.1007/978-1-84800-131-2_68.
- Saloff-Coste L, Bremaud P. Markov chains: Gibbs fields, Monte Carlo simulation, and queues. *J Am Stat Assoc* 2000. <https://doi.org/10.2307/2669802>.
- Yang C, Remenyte-Preccott R, Andrews JD. Pavement maintenance scheduling using genetic algorithms. *Int J Performability Eng* 2015.
- Butt AA, Shahin MY, Carpenter SH, Carnahan JV. Application of markov process to pavement management systems at network level. In: 3rd International conference on managing pavements; 1994. <https://doi.org/10.1093/ije/24.1.177>.
- Yang J, Gunaratne M, Lu JJ, Dietrich B. Use of recurrent Markov chains for modeling the crack performance of flexible pavements. *J Transp Eng* 2005. [https://doi.org/10.1061/\(ASCE\)0733-947X\(2005\)131:11\(861\)](https://doi.org/10.1061/(ASCE)0733-947X(2005)131:11(861)).
- Massey FJ. The Kolmogorov-Smirnov test for goodness of fit. *J Am Stat Assoc* 1951. <https://doi.org/10.1080/01621459.1951.10500769>.
- Razali NM, Wah YB. Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. *J Stat Model Anal* 2011. <https://doi.org/10.1515/bile-2015-0008>.
- Holland JH. *Adaptation in natural and artificial systems*. 2019. 10.7551/mitpress/1090.001.0001.
- Artificial Intelligence in transportation: information for application. Washington: 2007. 10.17226/23208.
- Khoshnevisan B, Rafiee S, Pan J, Zhang Y, Liu H. A multi-criteria evolutionary-based algorithm as a regional scale decision support system to optimize nitrogen consumption rate; a case study in North China plain. *J Clean Prod* 2020. <https://doi.org/10.1016/j.jclepro.2020.120213>.
- Costa-Carrapiço I, Raslan R, González JN. A systematic review of genetic algorithm-based multi-objective optimisation for building retrofitting strategies towards energy efficiency. *Energy Build* 2020. <https://doi.org/10.1016/j.enbuild.2019.109690>.
- Morcous G, Lounis Z. Maintenance optimization of infrastructure networks using genetic algorithms. *Autom Constr* 2005. <https://doi.org/10.1016/j.autcon.2004.08.014>.

- [47] Fwa TF, Tan CY, Chan WT. Road-maintenance planning using genetic algorithms. II: analysis. *J Transp Eng* 1994. [https://doi.org/10.1061/\(ASCE\)0733-947X\(1994\)120:5\(710\)](https://doi.org/10.1061/(ASCE)0733-947X(1994)120:5(710)).
- [48] Santos J, Ferreira A, Flintsch G. A multi-objective optimization-based pavement management decision-support system for enhancing pavement sustainability. *J Clean Prod* 2017. <https://doi.org/10.1016/j.jclepro.2017.07.027>.
- [49] Santos J, Ferreira A, Flintsch G. An adaptive hybrid genetic algorithm for pavement management. *Int J Pavement Eng* 2019. <https://doi.org/10.1080/10298436.2017.1293260>.
- [50] Zhang T, Andrews J, Wang R. Optimal scheduling of track maintenance on a railway network. *Qual Reliab Eng Int* 2013. <https://doi.org/10.1002/qre.1381>.
- [51] Marler RT, Arora JS. Survey of multi-objective optimization methods for engineering. *Struct Multidiscip Optim* 2004. <https://doi.org/10.1007/s00158-003-0368-6>.
- [52] Fwa TF, Chan WT, Hoque KZ. Multiobjective optimization for pavement maintenance programming. *J Transp Eng* 2000. [https://doi.org/10.1061/\(ASCE\)0733-947X\(2000\)126:5\(367\)](https://doi.org/10.1061/(ASCE)0733-947X(2000)126:5(367)).
- [53] Deb K. Multi-objective optimisation using evolutionary algorithms: an introduction. *multi-objective evol. Optim Prod Des Manuf* 2011. https://doi.org/10.1007/978-0-85729-652-8_1.
- [54] Giunta M, Bressi S, D'Angelo G. Life cycle cost assessment of bitumen stabilised ballast: a novel maintenance strategy for railway track-bed. *Constr Build Mater* 2018. <https://doi.org/10.1016/j.conbuildmat.2018.04.020>.
- [55] Khouy IA, Larsson-Kräik PO, Nissen A, Juntti U, Schunnesson H. Optimisation of track geometry inspection interval. *Proc Inst Mech Eng Part F* 2014. <https://doi.org/10.1177/0954409713484711>.
- [56] Kielhauser C, Martani C, Adey BT. Development of intervention programs for inland waterway networks using genetic algorithms. *Struct Infrastruct Eng* 2018. <https://doi.org/10.1080/15732479.2017.1373299>.