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ORIGINAL ARTICLE

# Responsibility and sharing the cost of cleaning a polluted river

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## Abstract

Consider $n$ firms (agents) located at a river, indexed by $1, \ldots, n$ from upstream to downstream. The pollution generated by these firms induce cleaning costs $c_1, \ldots, c_n$, where $c_i$ is the cost for cleaning the water in region $i$ (according to the local environmental standards). The corresponding cost allocation problem is highly interesting both in theory and practice. Among the most prominent allocation schemes are the so-called Local Responsibility and Upstream Equal Sharing. The first one allocates simply each local cost $c_i$ to the corresponding firm $i$. The second distributes each $c_i$ equally among firms $1, \ldots, i$. We propose and characterize a dynamic scheme which, given a particular order of arrival, allocates the current total cost among the firms that have arrived so far. The corresponding expected allocation (*w.r.t.* a random arrival order) turns out to be a convex combination of the two schemes above.

**Keywords** Cost allocation · Local Responsibility Sharing · Upstream Equal Sharing · Axiomatization

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## 1 Introduction

More than 200 rivers around the world flow across different countries (see Ambec and Sprumont (2002) and Barrett (1994)) and many more rivers flow through different regions. On one hand, inhabitants or firms along the river can utilize the water resources. On the other hand, they may also discharge household or industrial waste into the rivers. Thus sharing a river from the aspects of both rights and responsibility is always a highly interesting issue in society.

On the beneficial side, the main aim is to find a fair allocation of the welfare resulting from distributing the water flow. Ambec and Sprumont (2002) first model such situation as a cooperative coalitional game, then they propose the downstream incremental method as a compromise between two precepts: Absolute Territorial Sovereignty (Godana 1985) and Unlimited Territorial Integrity (Kilgour and Dinar 1996). Ambec and Ehlers (2008) consider a more generalised model with satiable agents. Khmelnitskaya (2010) investigates the situation where river basins possess multiple springs.van den Brink et al. (2012) also study such river system by taking account of externalises.

In this paper, we concentrate on the responsibility side of sharing a river. Agents situated along a river benefit from the water resources, but like every coin has two sides, their production processes are accompanied by pollution. There is no doubt that waste can destroy the ecosystem of the river and sometimes even cause natural disasters. Many countries around the world seek to govern the problem of river pollution. For instance, the government of China has established a chief river regulation system coordinating the various involved departments to improve the quality of China's waterways. An important issue is the question how the cost of cleaning the water should be shared among the agents involved.

Ni and Wang (2007) first analysed the river responsibility sharing problem among different segments from a theoretical point of view. In their model, agents (polluters) are located along a river from upstream to downstream and there is an authority that determines how the total cleaning cost should be allocated to the agents. Ni and Wang (2007) introduce two allocation methods, Local Responsibility Sharing (LRS for short) method and Upstream Equal Sharing (UES for short) method. These two methods are proposed under the principles of Local Responsibility and Downstream Responsibility respectively. The LRS method prescribes that each agent should take full responsibility of the cost in its area. In contrast, the UES method forces upstream agents to take some responsibility for its downstream cleaning costs—after all, it is often difficult to determine exactly who is responsible for which part of the pollution. More precisely, the UES method allocates the cleaning cost in a segment equally among the corresponding firm and all its upstream companies.

Alcalde-Unzu et al. (2015) argue that neither LRS nor UES make sense for all scenarios and propose an alternative method based on estimating the transfer rate of waste from one segment to its downstream neighbor. In some cases, this transfer rate may be difficult to estimate, however. In this paper, we therefore propose a somewhat different approach: Instead of estimating the transfer rate, we assume that the firms agree on a minimal responsibility level $\alpha$, meaning that each firm would be happy to pay for an $\alpha$ fraction of the cleaning cost of its segment. (Of course, the value of $\alpha$ would also reflect the general opinion on a lower bound for the transfer rate). Such an agreement may be determined from empirical data, by bargaining or simply government intervention.

Inspired by the work of Malawski (2013) and (Sun et al. 2017a, b), we investigate the allocation method from a dynamic perspective. More specifically, suppose that the firms arrive at the river one by one. When an agent joins the river system, it is ready to pay for its minimal responsibility. Apart from this minimal responsibility, the remaining cost should also be allocated properly. As we have discussed, the upstream agents that have appeared should take the responsibility (Note that downstream agents have no obligation to contribute towards the cleaning cost at upstream locations in our model). To avoid potential uncertainty, every new entrant may want to allocate the cost in its area as long as it joins the procedure, but not wait until its upstream agents join. That is the reason why we assume that an agent shares in the cost of a downstream agent that enters after it but not when it enters after this downstream agent. Proceeding this way, the total cost is shared among the agents given their order of arrival. Of course, the final outcome will depend on the minimal responsibility level $\alpha$.

In case all firms are already present, we define a corresponding expected cost allocation by randomizing over all possible arrival orderings. This defines a new sharing method which we call $\alpha$-responsibility method. Interestingly, the $\alpha$-responsibility method coincides with a convex combination of the LRS and UES method. So the $\alpha$-responsibility in some sense reconciles the two doctrines, Local Responsibility and Downstream Responsibility.

Ni and Wang (2007) characterize the LRS and UES method with Additivity, while van den Brink and van der Laan (2008) introduce axiomatizations of these two methods without Additivity. We show that the $\alpha$-responsibility method can also be fully characterized with or without Additivity. Several new properties, including Weakly Blind Cost, Surplus Monotonicity, Essential Player Property, $\alpha$-Blind Cost, and Upstream (Strong) Surplus Symmetry are proposed to axiomatically characterize the $\alpha$-responsibility method.

The paper is organized as follows. The basic model and concepts are introduced in Sect. 2. In Sect. 3, we study the dynamic implementation of the sharing method and also the definition of the $\alpha$-responsibility method. Axiomatizations of the $\alpha$ -responsibility method are exhibited in Sect. 4.

## 2 Preliminaries

Consider $n$ firms (agents) located along a river, indexed by $N = \{1, 2, \cdots, n\}$ from upstream to downstream. Each of the firms generates some pollutants, which will influence the quality of the waterbody. In order to clean the pollutants according to (local) environmental standards, firms along the river have to pay for the cleaning costs. To this end, the environmental authority fixes a corresponding cost vector $c = (c_1, c_2, \cdots, c_n)$, where $c_i$ denotes the cost incurred to clean the river in segment $i$.

Formally, denote such river sharing system as a pair ($N$, $c$) and the class of all sharing systems with $n$ firms as $\Delta_n$.

Given a river sharing system ($N$, $c$), the authority has to allocate the total cost along the whole river to all firms in a reasonable way. A sharing method is a map $x : (N, c) \to R^n$, and $x_i$ denotes the cost firm $i$ has to pay. Researchers have proposed various allocation methods to allocate the total cost $c(N) = \sum_{i \in N} c_i$. Among these methods, the so-called Local Responsibility Sharing (LRS for short) method and Upstream Equal Sharing (UES for short) method are the most prominent ones. The LRS method states that any firm in segment $i$ should take full responsibility to the cost of cleaning in that segment, i.e.,

**Definition 1** For any river sharing system $(N, c) \in \Delta_n$, the Local Responsibility Sharing method is defined by

$$x_i^{LRS}(c) = c_i, \quad \forall i \in N. \tag{1}$$

The Upstream Equal Sharing method allocates each $c_i$ equally among firms $1, \ldots, i$:

**Definition 2** For any river sharing system $(N, c) \in \Delta_n$, the Upstream Equal Sharing method is defined by

$$x_i^{UES}(c) = \frac{1}{i}c_i + \frac{1}{i+1}c_{i+1} + \cdots + \frac{1}{n}c_n, \quad \forall i \in N. \tag{2}$$

## 3 Dynamic implementation of the sharing method and the $\alpha$-responsibility method

When the transfer of the pollutants is involved, no firm is willing to pay all the cleaning cost in its area, since firms from upstream also contribute to the cost in its region. But in general, it's difficult to obtain the exact transfer rate technically. Instead of estimating the transfer rate as Alcalde-Unzu et al. (2015) did, we investigate an allocation based on *minimal responsibility* of the firms. To measure the minimal responsibility, we introduce a parameter $\alpha \in [0, 1]$, which determines the minimal fraction of the local cost a firm is willing to pay. This fraction can be determined by bargaining and negotiation among firms based on empirical data or some other information. As a consensus, all the firms will agree on this bargaining outcome.

Inspired by the concept of procedural value proposed by Malawski (2013), also studied by Sun et al. (2017a, b), we aim to study the sharing method by analysing the formation of the grand coalition, i.e., all firms along the river. We assume that firms join the system one by one in a given formation order of $N$. Based on above discussion, every new entrant will merely pay for its minimal cost. Next we introduce the explicit dynamic procedure and reveal the relationship between the final allocation and the responsibility level $\alpha$.

– All firms join the system under the formation order $\pi \in \Pi(N)$;

**Table 1** Costs after 3 joins

| Firms | Firm 1 | Firm 2 | Firm 3 |
|---|---|---|---|
| Cost | 0 | 0 | $c_3$ |

- Given a new entrant $i \in N$, it will take position $\pi(i)$ and pay for its minimal responsibility i.e., $\alpha c_i$ in the first place. The surplus, $c_i - \alpha c_i$ is allocated evenly among the firms from its upstream that have already arrived. (Thus, in case there is no firm upstream yet, then $i$ has to take all the cost involved in this area.);
- Proceeding in this way, the total cost $c(N)$ is distributed among all the firms under permutation $\pi$;

We will illustrate this dynamic procedure by discussing a 3-firm example with the formation order $\pi = (3, 1, 2)$. At the beginning, firm 3 comes. It will be charged $c_3$ since there is no other firm, while 1 and 2 pay nothing at the moment (see Table 1).

Then firm 1 joins, and it has to pay $c_1$ since the existing firm 3 is not its upstream. Firm 2 still pays nothing in this stage (Table 2).

Finally firm 2 joins. When firm 2 enters, it is willing to pay $\alpha$ part of the local cost $\alpha c_2$, since the cost $c_2$ also includes the cost of pollution transferred from the upstream. The remaining part $(1 - \alpha)c_2$ will be allocated to firm 1, since it is the unique upstream of firm 2 (Table 3).

Summating the cost in every stage, we obtain the firms' final allocations, which are $c_1 + (1 - \alpha)c_2, \alpha c_2$ and $c_3$, under the permutation $\pi = (3, 1, 2)$.

Generally, the cost of firm $i$ under the formation order $\pi$ consists of two parts. The first is from the cost in its own region, which is

$$B_i^{\alpha\pi}(c) := \begin{cases} c_i, & \text{if } \; i < j, \;\; \forall j \in S_i^{\pi}; \\ \alpha c_i, & \text{otherwise}, \end{cases} \tag{3}$$

where $S_i^{\pi} := \{j \in N : \pi(j) \leq \pi(i)\}$ denotes the set of firm $i$ and all the firms that precede $i$.

Another part of the cost is generated by segments from $i$'s downstream. Denote $G_{ik}^{\alpha\pi}(c)$ as the cost transferred from firm $k$ to $i$ under the formation order $\pi$. Then $G_{ik}^{\alpha\pi}(c)$ has the following form.

$$G_{ik}^{\alpha\pi}(c) := \begin{cases} \frac{c_k - B_k^{\alpha\pi}}{|U_k^{\pi}|}, & \text{if } \; i \in U_k^{\pi}; \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

**Table 2** Costs after 1 joins

| Firms | Firm 1 | Firm 2 | Firm 3 |
|---|---|---|---|
| Cost | $c_1$ | 0 | $c_3$ |

**Table 3** Costs after 2 joins

| Firms | Firm 1 | Firm 2 | Firm 3 |
|---|---|---|---|
| Cost | $c_1 + (1 - \alpha)c_2$ | $\alpha c_2$ | $c_3$ |

where $U_k^\pi$ is the set of all firms from upstream of $k$ in $S_k^\pi$, i.e., $U_k^\pi = \{j \in S_k^\pi | j < k\}$.

Thus we denote $\xi_i^{\alpha\pi}$ as the cost of firm $i$ under the formation order $\pi$ with minimal responsibility level $\alpha$.

$$\begin{aligned}\xi_i^{\alpha\pi} &= B_i^{\alpha\pi}(c) + \sum_{k=1}^{n} G_{ik}^{\alpha\pi}(c)\\ &= B_i^{\alpha\pi}(c) + \sum_{k>i,\pi(k)>\pi(i)} \frac{c_k - \alpha c_k}{|U_k^\pi|}.\end{aligned}$$

In case all firms are present already, it is common to allocate the cost according to their expected share (with the expectation taken with respect to random arrival orderings):

**Definition 3** For any river sharing system $(N, c) \in \Delta_n$, the *α-responsibility method* $\xi^\alpha$ allocates to each firm $i \in N$ the average of its cost over all permutations, i.e.,

$$\xi_i^\alpha(c) = \sum_{\pi \in \Pi(N)} \frac{1}{n!} \xi_i^{\alpha\pi}. \tag{5}$$

**Theorem 1** *For any river sharing system* $(N, c) \in \Delta_n$, *the* $\alpha$ *the convex combination of the Local Responsibility Sharing method and the Upstream Equal Sharing method with coefficient-responsibility method coincides with* $\alpha$, i.e.,

$$\xi^\alpha(c) = \alpha x^{LRS}(c) + (1 - \alpha) x^{UES}(c). \tag{6}$$

***Proof*** If there is no confusion, we omit the item $c$ in the formulas such as $\xi^\alpha$ instead of $\xi^\alpha(c)$ throughout the paper. For any $i \in N$, we have

$$\xi_i^\alpha = \sum_{\pi \in \Pi(N)} \frac{1}{n!} \xi_i^{\alpha\pi} = \sum_{\pi \in \Pi(N)} \frac{1}{n!} B_i^{\alpha\pi} + \sum_{\pi \in \Pi(N)} \sum_{k>i,\pi(k)>\pi(i)} \frac{1}{n!} \frac{c_k - \alpha c_k}{|U_k^\pi|}.$$

Denote the first item on the right hand side by $A$ and the second item by $B$. If agent $i$ enters before all $j < i$ then its contribution in its own cost is $c_i$, and its contribution is $\alpha c_i$ otherwise. Since the probability of permutations where $i$ enter before all $j < i$ is equal to $\frac{(i-1)!}{i!} = \frac{1}{i}$, its expected contribution in its own cost A is equal to $\frac{1}{i} c_i + (1 - \frac{1}{i}) \alpha c_i = \alpha c_i + \frac{1-\alpha}{i} c_i$.

As to item B, we group the coalitions in the second sum according to the size of $U_k^\pi$:

$$\begin{aligned}B =& \frac{1}{n!} \sum_{k=i+1}^{n} \sum_{\pi:\pi(k)>\pi(i)} \frac{c_k - \alpha c_k}{|U_k^\pi|}\\ =& \frac{1}{n!} \sum_{k=i+1}^{n} \sum_{m=1}^{k-1} C_n^{n-k} (n-k)! C_{k-2}^{m-1} m!(k-m-1)! \frac{(1-\alpha)c_k}{m}\\ =& \frac{1}{n!} \sum_{k=i+1}^{n} \sum_{m=1}^{k-1} \frac{n!}{k(k-1)} (1-\alpha) c_k\\ =& \sum_{k=i+1}^{n} \frac{1}{k} (1-\alpha) c_k.\end{aligned}$$

Based on above results, we conclude that for any $i \in N$,

$$\xi_i^\alpha = A + B = \alpha x_i^{LRS} + (1-\alpha) x_i^{UES}. \tag{7}$$

□

For $\alpha = 0$, we obtain the UES method, while $\alpha = 1$ results in the LRS method. Thus $\alpha$ indeed reflects the extent to which the firms take responsibility for the cleaning cost in their segment. Hence, $\alpha$-responsibility method seems to be a proper choice for the name of this allocation concept.

***Remark 1*** Actually, we could also consider a more general model, in which players have different responsibility level. Suppose that the responsibility level for player $i$ is $\alpha_i$, then the final payoff under the above procedure is $\alpha_i c_i + \sum_{k=i}^{n} 1/k(1-\alpha_k)c_k$. The proof is similar to the proof of Theorem 1.

***Remark 2*** In the above model we mainly consider the situation where downstream agents have no obligation to share the costs of their upstream agents. But Dong et al. (2012) proposed another interpretation of the Unlimited Territorial Integrity, which is the opposite of the downstream responsibility. In their model, the agents also have to undertake part of the cost from its upstream agents. Actually, we could also consider the upstream responsibility in our model as follows:

- All firms join the system under the formation order $\pi \in \Pi(N)$;
- Given a new entrant $i \in N$, it will take position $\pi(i)$ and pay for its minimal responsibility i.e., $\alpha c_i$ in the first place. The surplus, $c_i - \alpha c_i$ is allocated evenly among the firms from its *downstream* that have already arrived. (Thus, in case there is no firm *downstream* yet, then $i$ has to take all the cost involved in this area.);
- Proceeding in this way, the total cost $c(N)$ is distributed among all the firms under permutation $\pi$.

Interestingly, under such a procedure, the expected cost allocation is exactly the convex combination of the Local Responsibility Sharing method and the Downstream Equal Sharing method, proposed by Dong et al. (2012).

## 4 Axiomatizations of the $\alpha$-responsibility method

In this section, we introduce several axiomatizations of the $\alpha$-responsibility method with or without Additivity, which is a classical property in cooperative game theory. Recall that Ni and Wang (2007) characterize the LRS method and UES method with the following axioms.

- *Efficiency* $\sum_{i\in N} x_i = \sum_{i\in N} c_i$ for any $(N, c) \in \Delta_n$.
- *Additivity* for any $(N, c^1)$ and $(N, c^2) \in \Delta_n$, it holds that $x_i(c^1) + x_i(c^2) = x_i(c^1 + c^2)$.
- *No Blind Cost* $x_i(c) = 0$ for any $(N, c) \in \Delta_n$ such that $c_i = 0$.
- *Independence of Upstream Costs* if $(N, c^1)$, $(N, c^2) \in \Delta_n$ and $i \in N$ with $c^1_k = c^2_k$, $\forall k > i$, then $x_j(c^1) = x_j(c^2)$ for all $j > i$.
- *Upstream Symmetry* if $c_j = 0$ for all $j \neq i$, then $x_j(c) = x_i(c)$ for all $j < i$.

Ni and Wang (2007) showed that the LRS method is the only sharing method that satisfies Efficiency, Additivity and No Blind Cost. The UES method is the unique sharing method that satisfies Efficiency, Additivity, Upstream Symmetry and Independence of Upstream Costs. We also propose some new axiomatizations by modifying these properties.

### 4.1 Axiomatizations based on No Blind Cost

No Blind Cost is a kind of "null player" axiom stating that a firm with no cleaning cost in its local area does not have to contribute anything. Note, however that cleaning cost $c_i = 0$ does not imply that firm $i$ produces no waste. For example, environmental standards in a downstream country $i + 1$ may well imply nonzero cleaning costs for the waste that firm $i$ produces. This motivates a somewhat weaker Blind Cost axiom as follows.

- *Weakly Blind Cost* $x_i(c) = 0$ for any $(N, c) \in \Delta_n$ with $c_j = 0, \forall j \geq i$.

For any potential sharing method $x$, the difference between $x_i$ and $\alpha c_i$ can be viewed as the extra "social responsibility" of firm $i$. The following axiom states that upstream firms are to take more social responsibilities than downstream firms.

- *Surplus Monotonicity* for any $c \in R^n_+$ and $i \in N$, it holds that $x_i(c) - \alpha c_i \geq \max_{j\geq i}\{x_j(c) - \alpha c_j\}$.

Consider a river sharing system in which only one segment incurs pollution, then regardless of the position of the firm, it seems reasonable that such an essential firm should bear no less responsibility than the other firms.

- *Essential Player Property* for any $j \in N$, $x_i(0, \cdots, 0, c_i, 0, \cdots, 0) - \alpha c_i \geq x_j(0, \cdots, 0, c_i, 0, \cdots, 0) - \alpha c_j$.

Together with Efficiency and Additivity, these three axioms fully characterize the $\alpha$ -responsibility method.

**Theorem 2** For any $(N, c) \in \Delta_n$, *the* $\alpha$ *-responsibility method is the only sharing method that satisfies Efficiency, Additivity, Weakly Blind Cost, Surplus Monotonicity and Essential Player Property.*

***Proof*** It is easy to verify that the $\alpha$-responsibility method satisfies the five properties. In order to show the uniqueness, let $x$ be a method with the mentioned properties and consider $c^k$ given as $c_i^k = 0$ for $i \neq k$ and $c_k^k = 1$. By Weakly Blind Cost, $x_i(c^k) = 0$ for all $i > k$. Surplus Monotonicity implies that for all $j < k$,

$$\begin{aligned} x_j(c^k) - \alpha c_j^k &\geq x_k(c^k) - \alpha c_k^k \\ \Rightarrow x_j(c^k) &\geq x_k(c^k) - \alpha, \quad \forall j < k. \end{aligned}$$

By Essential Player Property, it follows that

$$\begin{aligned} x_k(c^k) - \alpha c_k^k &\geq x_j(c^k) - \alpha c_j^k \\ \Rightarrow x_k(c^k) - \alpha &\geq x_j(c^k), \quad \forall j < k. \end{aligned}$$

Thus we conclude that $x_j(c^k) = x_k(c^k) - \alpha$ for all $j < k$. Together with Efficiency, we have

$$\sum_{i=1}^{n} x_i(c^k) = \sum_{i<k} x_i(c^k) + x_k(c^k) = \sum_{i<k} [x_k(c^k) - \alpha] + x_k(c^k) = 1, \tag{8}$$

which gives that $x_k(c^k) = \alpha + \frac{1-\alpha}{k}$ and $x_i(c^k) = \frac{1-\alpha}{k}$ for $i < k$. Notice that the set $\{c^k\}_{k=1}^n$ forms a basis of $R^n$. For any given $c \in R^n$, $c = \sum_{i \in N} c_i c^i$, together with Additivity, it holds that

$$\begin{aligned} x_i(c) &= \sum_{k \in N} c_k x_i(c^k) \\ &= c_i x_i(c^i) + \sum_{k \neq i} c_k x_i(c^k) \\ &= \alpha c_i + \frac{1-\alpha}{i} c_i + \sum_{k > i} (1-\alpha) \frac{1}{k} c_k \\ &= \alpha c_i + (1-\alpha) \sum_{k=i}^{n} \frac{1}{k} c_k \end{aligned}$$

□

It's worth noting that the axioms of Theorem 2 are quite close to those of van den Brink et al. (2018), but we do propose these axioms independently. Surplus monotonicity and the essential player property are variations of structural monotonicity and the necessary agent property, which correspond to $\alpha = 0$. In this way, Theorem 2 generalizes one of the axiomatizations of the UES method given by van den Brink et al. (2018).

We have argued that a firm with no waste in its segment may also bear some cost generated from its downstream, but the problem is how to determine the amount of the responsibility. Recall that we have introduced the minimal responsibility level $\alpha$ to measure the amount of cost that the firms are willing to bear, then the remaining should be shared among the firms from their upstream. Therefore, compared to Weakly Blind Cost, we propose a stronger property, which states that any firm with a zero cost in its segment assumes some of the cost from its downstream i.e.,

– *$\alpha$-Blind Cost* $x_i(c) = (1-\alpha)\sum_{j>i}\frac{1}{j}c_j$ for any $(N,c) \in \Delta_n$ with $c_i = 0$.

The strengthening of the Weakly Blind Cost property allows us to abandon Surplus Monotonicity and the Essential Player Property in Theorem 2. As a result, $\alpha$ -Blind Cost can fully characterize the $\alpha$-responsibility method together with Efficiency and Additivity.

**Theorem 3** For any $(N,c) \in \Delta_n$, *the $\alpha$ -responsibility method is the only method that satisfies Efficiency*, *$\alpha$-Blind Cost and Additivity*.

***Proof*** It is easy to verify that the $\alpha$-responsibility method satisfies the three properties. Let $x \in R^n$ be a method with the mentioned properties, then we show that $x$ is uniquely determined.

Let $c^k$ be defined as $c_i^k = 0$ for $i \neq k$ and $c_k^k = 1$. By $\alpha$-Blind Cost, it follows that

$$x_i(c^k) = \begin{cases} (1-\alpha)\frac{1}{k}, & \textit{if} \ \ i < k \\ 0, & i > k \end{cases} \tag{9}$$

Together with Efficiency, we have $x_k(c^k) = 1 - (1-\alpha)\frac{k-1}{k}$. Similar to the last part of the proof of Theorem 2, by applying Additivity, it holds that

$$x_i(c) = \alpha c_i + (1-\alpha)\sum_{k=i}^{n}\frac{1}{k}c_k.$$

□

Young (1985) provides an axiomatization of the Shapley value (Shapley 1953) for the class of cooperative games without using Additivity but Strong Monotonicity. van den Brink and van der Laan (2008) show that a sharing method satisfies Independence of Upstream Costs when the corresponding solution concepts obeys Strong Monotonicity, then they introduce a new characterization of the UES method. Following the approach of van den Brink, we show that the $\alpha$ -responsibility method can also be fully axiomatized by replacing Additivity in Theorem 3 with Independence of Upstream Costs.

**Theorem 4** For any $(N,c) \in \Delta_n$, *the $\alpha$ -responsibility method is the only method that satisfies Efficiency*, *$\alpha$-Blind Cost and Independence of Upstream Costs*.

***Proof*** First of all, it is not difficult to verify that the method satisfies the three properties. It remains to show the uniqueness. Let $x \in R^n$ be a method with the mentioned properties. We are to prove that $x$ is uniquely determined.

Let $c^n \in R^n_+$ be defined by $c^n_i = 0$ for $i < n$ and $c^n_n = c_n$. $\alpha$-Blind Cost implies that $x_i(c^n) = \frac{(1-\alpha)}{n} c^n_n = \frac{(1-\alpha)}{n} c_n$ for all $i < n$. Efficiency implies that $x_n(c^n) = c_n - (n-1)\frac{(1-\alpha)}{n} c_n = \alpha c_n + (1-\alpha)\frac{1}{n} c_n$. Together with Independence of Upstream Costs, it holds that $x_n(c) = x_n(c^n) = \alpha c_n + (1-\alpha)\frac{1}{n} c_n$, which means that the cost for $n$ is uniquely determined. Similarly, define $c^j$ as $c^j_i = c_i$ for all $i \geq j$ and $c^j_i = 0$ for all $i < j$. According to $\alpha$-Blind Cost, we have that $x_i(c^j) = (1-\alpha)\sum_{k=j}^{n} \frac{1}{k} c^j_k$ for all $i < j$ are uniquely determined. Together with Independence of Upstream Costs, we then deprive $x_j(c) = x_j(c^j)$, which indicates that the allocation method with the three properties can be uniquely determined. □

***Remark 3*** As a matter of fact, combined with $\alpha$-Blind Cost, Additivity implies the Independence of Upstream Costs. For any $(N, c^1), (N, c^2) \in \Delta_n$ and $i \in N$ such that $c^1_k = c^2_k, \forall k > i$, it holds that $x_j(c^1) - x_j(c^2) = x_j(c^1 - c^2)$ by Additivity. With $\alpha$-Blind Cost, we have $x_j(c^1 - c^2) = (1-\alpha)\sum_{k>j} \frac{1}{k}(c^1_k - c^2_k) = 0$. Thus we conclude that $x_j(c^1) - x_j(c^2) = 0$ for all $j > i$, which is the Independence of Upstream Costs property. However, the Independence of Upstream Costs is not equivalent to Additivity. The sharing method $x_i(c) = [c_i]^2$ satisfies the Independence of Upstream Costs but not Additivity, since Efficiency plays a significant part in this situation.

### 4.2 Axiomatizations based on Upstream Symmetry

Ni and Wang (2007) propose the Upstream Symmetry to capture the idea that any given downstream costs are shared evenly among the upstream firms. In our model, as we have seen in Theorem 4, only a $(1-\alpha)$-fraction of the local cost is distributed to the upstream firms. Correspondingly, we have the following modified symmetry axiom.

– *Upstream Surplus Symmetry* $x_j(c) - \alpha c_j = x_k(c) - \alpha c_k$ for all $j, k \leq i$, where $c_j = 0$ for all $j \neq i$.

**Theorem 5** For any $(N, c) \in \Delta_n$, *the* $\alpha$ *-responsibility method is the only method that satisfies Efficiency, Additivity, Upstream Surplus Symmetry and Independence of Upstream Costs.*

***Proof*** First, the $\alpha$-responsibility method satisfies the four properties. In order to show the uniqueness, let $x$ be a solution with the mentioned properties and consider $c^k \in R^n$ defined by $c^k_i = 0$ for all $i \neq k$ and $c^k_k = 1$. By Independence of Upstream Costs, $x_j(c^k) = x_j(0, 0, \cdots, 0)$ for all $j > k$. Upstream Surplus Symmetry implies that $x_i(0,0,\cdots,0) = x_j(0,0,\cdots,0)$ for all $i,j \in N$. Together with Efficiency, we then have $x_i(0,0,\cdots,0) = 0$ for all $i \in N$. Therefore $x_j(c^k) = x_j(0,0,\cdots,0) = 0$ for all $j > k$. By Upstream Surplus Symmetry, there must exist $\beta, \gamma \in R$ such that $x_k(c^k) = \gamma = x_j(c^k) + \alpha = \beta + \alpha$ for all $j < k$. Moreover, Efficiency implies that

$$(k-1)\beta + \gamma = (k-1)\beta + \beta + \alpha = 1, \tag{10}$$

which gives that

$$x_i(c^k) = \begin{cases} \frac{1-\alpha}{k}, & if \;\; i < k; \\ \frac{1-\alpha}{k} + \alpha, & if \;\; i = k; \\ 0, & i > k. \end{cases} \tag{11}$$

Based on above consideration and Additivity, for any $c \in R^n$, One could easily obtain that ( the details are left to the reader )

$$x_i(c) = \alpha c_i + (1-\alpha) \sum_{k=i}^{n} \frac{1}{k} c_k.$$

□

Finally, we present a strengthening of Upstream Symmetry which allows us to get rid of Additivity:

– *Strong Upstream Surplus Symmetry* $x_j(c^i) - \alpha c^i_j = x_k(c^i) - \alpha c^i_k$ for all $j,k \leq i$, where $c^i_j = 0$ for all $j < i$.

It is not difficult to show that Upstream Surplus Symmetry and Additivity imply Strong Upstream Surplus Symmetry. In fact, compared with Theorem 5, this property can characterize the $\alpha$-responsibility method without Additivity.

**Theorem 6** *For any $(N,c) \in \Delta_n$, the $\alpha$ -responsibility method is the only method that satisfies Efficiency, Strong Upstream Surplus Symmetry and Independence of Upstream Costs.*

***Proof*** One can easily verify that the $\alpha$-responsibility method obeys the properties. It remains to prove the uniqueness. Suppose $x$ is a method with the three properties. Consider $c^n \in R^n$ defined by $c^n_i = 0$ for $i < n$ and $c^n_n = c_n$. By Upstream Strong Surplus Symmetry, there exist $\beta, \gamma \in R$ such that $x_i(c^n) = \beta = x_n(c^n) - \alpha c^n_n = \gamma - \alpha c^n_n$. Together with Efficiency, it holds that

$$\sum_{i \in N} x_i(c^n) = (n-1)\beta + \gamma = c^n_n, \tag{12}$$

which gives that $\beta = \frac{1-\alpha}{n}$

$$x_i(c^n) = \begin{cases} \frac{1-\alpha}{n} c_n, & \text{if } \ i < n; \\ \frac{1-\alpha}{n} c_n + \alpha c_n, & \text{if } \ i = k; \end{cases} \tag{13}$$

Moreover, Independence of Upstream Costs implies that $x_n(c) = x_n(c^n) = \frac{1-\alpha}{n} c_n + \alpha c_n$.

By inducing on $i$, we assume that $x_i(c) = \alpha c_i + (1-\alpha) \sum_{j=i}^{n} \frac{1}{j} c_j$ for all $i \geq j+1$. Then define $c^j$ as $c_i^j = 0$ for $i < j$ and $c_i^j = c_i$ for $i \geq j$. The assumption and Independence of Upstream Costs imply that $x_i(c) = x_i(c^j)$ for $i \geq j+1$. And with Upstream Strong Surplus Symmetry, we have $x_i(c^j) = x_j(c^j) - \alpha c_j$ for $i < j$. Together with Efficiency, it holds that

$$\begin{aligned} x_j(c^j) &= \sum_{i=1}^{n} c_i - \sum_{i=1}^{j-1} x_i(c^j) - \sum_{i=j+1}^{n} x_i(c^j) \\ &= \sum_{i=j}^{n} c_i - \sum_{i=1}^{j-1} \left[ x_j(c^j) - \alpha c_j \right] - \sum_{i=j+1}^{n} \left[ \alpha c_i + (1-\alpha) \sum_{j=i}^{n} \frac{1}{j} c_j \right] \end{aligned}$$

It turns out that $x_j(c^j) = \alpha c_j + (1-\alpha) \sum_{i=j}^{n} \frac{1}{i} c_i$. Again by Independence of Upstream Costs, $x_j(c) = x_j(c^j)$ is uniquely determined. □

## 5 Conclusion

We study the cost sharing method for cleaning a polluted river by investigating the dynamic formation of the river system. It turns out that the new sharing method is exactly the convex combination of the Local Responsibility Sharing and Upstream Equal Sharing method, therefore it reconciles both local responsibility and downstream responsibility.

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