Super Coupling Currents in Rutherford Type of Cables due to Longitudinal Non-homogeneities of dB/dt

A.P. Verweij, and H.H.J. ten Kate

University of Twente, Applied Superconductivity Centre, PO Box 217, 7500 AE Enschede, The Netherlands

Abstract - In this paper it is shown that non-homogeneities in the field sweep rate dB/dt along the length of a Rutherford cable provoke a non-homogeneous current distribution during a field sweep. This process can be described by means of Super Coupling Currents (SCC's) flowing through the strands over lengths far larger than the cable pitch. These SCC's can be characterised by a characteristic length, a characteristic time, and a propagation velocity. The dependence of these three parameters on the strand resistance and the contact resistance between strands will be illustrated. Two longitudinal non-homogeneities in dB/dt are considered which are present in accelerator magnets. Firstly, an increase in dB/dt from 0 to a certain value simulating that part of the cable where the cable enters the magnet field. Secondly, a longitudinal decrease in dB/dt which occurs mainly in the heads of the magnet. It is shown that in accelerator magnets a non-homogeneous current distribution induced by the field sweep can not be avoided. However, it seems to be very difficult to estimate the amplitude of the effect.

I. INTRODUCTION

In DC conditions, magnetic measurements on dipole accelerator magnets have shown that several harmonic components vary along the length of the magnet [1],[2]. As these variations have a sinusoidal shape, with a period equal to the cable pitch, it is likely that they are caused by a non-uniform distribution of the transport current among the strands. The currents are assumed to distribute in such a way that the voltage over each strand is the same. Several authors [3],[4] have presented measurements and calculations of this effect for several types of cables showing that mainly differences in the end resistances (between the strands and the current lead) cause the non-homogeneous distribution. A similar effect will also be present in Rutherford type of cables but this aspect is not discussed in this paper.

Recent magnetic measurements on dipole magnets [2],[5] show that under AC conditions additional field distortions are created. These distortions have a sinusoidal shape with an average value and an amplitude which are both linear with dB/dt and a period equal to the cable pitch.

The average field errors (where average indicates the mean value along one cable pitch in axial direction) are caused by the Inter-Strand Coupling Currents (ISCC's). This current distribution can be regarded as a sinusoidal variation of the current (with a minimum on one edge of the cable and a maximum on the other edge), superimposed on the transport current. The variation is due to currents which flow between different strands through cross contact resistances R_c ,

generating the so called inter-strand coupling loss. The ISCC's and the generated power are therefore inversely proportional to R_c . The integral value of the ISCC along an integer times the cable pitch is about 0. The average time constant of the ISCC's for practical Rutherford cables (with non-isolated strands) is typically in the range of 0.1-10 s.

The additional sinusoidally varying field distortions (in axial direction) under AC conditions can be explained by means of so-called Super Coupling Currents (SCC's). These currents are generated due to longitudinal variations of dB/dt . Contrary to the ISCC's, the SCC's have characteristic lengths ξ far larger than the cable pitch L_p so that the integral value of the SCC in a strand differs from 0. In other words: the ISCC's and the SCC's differ mainly in the fact that the SCC's follow the strand over long distances while the ISCC's do not. The fact that the SCC's stay in the strands implies that they generate almost no heat compared to the inter-strand coupling power and that their characteristic time τ_{sc} is far larger than the average time constant of the ISCC's.

In section II the model that is used to calculate the SCC's is briefly described. Results of numerical simulations are presented in section III in which mainly the qualitative dependencies of ξ and τ_{sc} on the cross contact resistance R_c and the strand resistivity ρ_s are stressed.

II. MODEL

All the calculations are based on numerical simulations in which the Rutherford cable is modelled by a 3D-network of nodes interconnected by strands or by contact resistances (see for a detailed description [6]).

The following assumptions are made:

- All the strands in the cable have the same length. Effects due to different self- or mutual inductances between the strands are therefore not considered.
- All the strands have the same end resistance, i.e. the resistance between the strand and the current lead to the power supply.
- All the strands have a similar resistivity ρ_s , which can be correlated to saturation effects or a dynamic resistance.
- A non-homogeneous current distribution within the strand (due to persistent currents and inter-filament coupling currents) is not taken into account.
- Only the field change perpendicular to the wide side of the cable is considered; the field components parallel to the wide side and parallel to the cable axis turn out to have only a minor effect.
- The contact resistance between adjacent strands is larger than the contact resistance R_c between crossing strands.

Manuscript received October 17, 1994.

These assumptions imply that in DC condition the transport current tends to be homogeneously distributed.

Boundary conditions are required for the current distribution in the ends of the cable. The calculations are made assuming that in the ends the current distribution is given by the solution for a cable without longitudinal non-homogeneities which is subjected to the local magnetic field change. Other boundary conditions however, do not influence the results from a qualitative point of view.

III. RESULTS

The simulations are performed on a flat Rutherford type of cable without keystone angle having perfectly round strands (see Fig. 1 for the numbering of the strands). The results differ only slightly from compressed cables or cables with a small keystone angle.

Fig. 1. Numbering of the strand positions in the cross section of a 16-strand Rutherford type of cable.

The effect is considered for two different dB/dt distributions varying along the length of the cable. Field variations along the width of the cable are not considered as they change only slightly the distribution of the ISCC's [6] but they do not generate SCC's. The following distributions are considered:

- dB/dt which is small over a certain length of the magnet, simulating the ends of a dipole magnet, where the cable bends over the beam pipe,
- dB/dt increasing from 0 to a certain value, simulating that part of a magnet where the cable enters the magnet.

The longitudinal position in the cable is denoted by z which is scaled to the cable pitch L_p . First of all a 16-strand cable is considered (with $R_c=1\ \mu\Omega$, $L_p=100\text{ mm}$ and a strand diameter $d_s=1.3\text{ mm}$) which is subjected to a field change of 0.01 Ts^{-1} with a cosine shape 'dip' of length l_{dip} in the middle, see Fig. 2. Fig. 3 pictures the current in the strand at the edge of the cable (position 1, see Fig. 1) for stationary conditions i.e. dB/dt is applied for a time far larger than the time constants of the SCC's. Two curves are shown for two different strand resistivities. (N.B.: In all figures in this paper the discrete points are interpolated to form a continuous line.) Fig. 3 shows first of all that the average strand current at the edge is about -7 A which corresponds to the ISCC for a cable without longitudinal variations. The SCC's can be seen as a current with an amplitude that is maximum close to the dB/dt non-homogeneity. A characteristic length ξ of the SCC's can be defined as the length over which the SCC's decay to $1/e$ of their initial value (for a cable length larger than a few times ξ so that the boundaries do not influence the decay). The decay of the SCC's is strongly affected by ρ_s and R_c ; a small ρ_s value or a large R_c value result in a large characteristic length. If the length of the cable is smaller than a few times ξ the

Fig. 2. Applied transverse field change along the length of the cable. Only a small part of the whole cable is shown. The length of the ' dB/dt -dip' is $1.5L_p$.

Fig. 3. The strand current $I_{s,1}$ at the edge (position 1) of a 16-strand cable as a function of the position z along the length of the cable. $\rho_s=1.4\cdot 10^{-14}$ (small amplitude) and $3.4\cdot 10^{-15}\ \Omega\text{m}$ (large amplitude).

boundaries can also influence the decay. In this case, different boundary conditions will give a qualitatively similar behaviour but quantitatively different results.

A regular pattern exists in the amplitude of the SCC's: in each cross section of the cable opposite strands (for example 3 and 11 or 7 and 15, see Fig. 1) carry an opposite SCC. In addition, strands laying next to each other have only slightly different SCC's (see Fig. 4). This rather regular pattern is typical for SCC's and causes them to generate more pronounced field errors than in the case of a random current distribution between the strands. The amplitude of the SCC's increases and decreases mainly due to cross-over currents I_c flowing between layer 1 (position 1 to 8) and layer 2 (position 9 to 16) through the contact resistances R_c . Therefore, the currents I_c can be regarded as the slope of the I_s vs. z curve. In other words: SCC contributions are created in two opposite strands while their amplitudes change along the length as at each R_c some current flows from one layer to the other. This results also in a periodic behaviour of I_c (see also [7]) and so of the coupling power P_c along the length (see Fig. 5). N.B.: For constant dB/dt and R_c along the length of the magnet these cross-over currents are z -independent, resulting in a constant power P_c of about 0.0022 Wm^{-1} .

Fig. 4. Illustration of the SCC's in each strand at a certain z -position. The legends indicate the strand position as given in Fig. 1.

Fig. 5. The coupling power P_c as a function of the z -position along the length of the cable which is subjected to a field change distribution as shown in Fig. 2.

Fig. 5 shows clearly that a variation of dB/dt does not influence the average coupling power in the whole cable but does affect the distribution of the local power. Similar simulations show that the amplitude of these fluctuations depends on the length of the ' dB/dt -dip'; the amplitude becomes small if the length equals $k \cdot L_p$ with $k=0, 1, 2, \dots$

In Fig. 6 the result is given of the current distribution for a cosine shaped step increase in dB/dt from 0 to 0.01 Ts^{-1} over a length of $1.5L_p$. A similar distribution is generated as for a ' dB/dt -dip'. The step in the average of the current corresponds to the ISCC which is 0 A for $dB/dt=0$ and about -7 A for $dB/dt=0.01 \text{ Ts}^{-1}$. The amplitude of these fluctuations depends on the length of the dB/dt transition and becomes small if the length equals $k \cdot L_p$ with $k=1, 2, 3, \dots$. A step in dB/dt (so the transition length is 0) always induces large SCC's.

By introducing self- and mutual inductances between the strands, time dependent effects of the SCC's can be investigated. As an example the decay of the SCC's in an 8-strand cable (with $d_s=1 \text{ mm}$ and $L_p=100 \text{ mm}$) is calculated after a step in dB/dt from the distribution as shown in Fig. 2 to

Fig. 6. The strand current $I_{s,1}$ at the edge of the cable (position 1) as a function of the z -position along the length of the cable. The field change distribution is shown in the same figure.

0. Fig. 7 shows the decay of the SCC's (scaled to the value at $t=0 \text{ s}$) at several distances z_1 from the centre of the ' dB/dt -dip' (located at $z=18.7L_p$). It is clear that the currents at

Fig. 7. The decay of the SCC's as a function of the time at several distances z_1 from the dB/dt non-homogeneity ($z_1=L_p, 6L_p, 18L_p$ and $28L_p$ respectively). The SCC's are scaled to their initial value at $t=0 \text{ s}$.

large z_1 start to decay later than those at small z_1 . It turns out that the SCC's can be well represented by the following formula, that analytically describes the outcomes of the numerical simulations:

$$I_{SCC}(t, z_1) = A e^{-z_1/\xi} \left(1 + \left(\frac{t}{t_p} \right)^2 \right)^{-n} \quad [\text{A}], \quad (1)$$

where A , ξ , t_p and n depend on the cable geometry, R_a , R_c and ρ_s . In addition, t_p and n are also a function of the z_1 -position. A propagation velocity can now be defined as:

$$v_p = \left(\frac{dt_p}{dz} \right)^{-1} \quad [\text{ms}^{-1}], \quad (2)$$

which is determined to be linearly proportional to R_c . Also a characteristic time τ_{sc} can be defined as the time for which the SCC's close to the dB/dt non-uniformity (so $t_p \rightarrow 0$)

decay to $1/e$ of their initial value. The currents tend to stay in the strands over longer distances if ρ_s is small or R_c is large, which leads to a large ξ -value as illustrated in Figs. 8 and 9 (the calculations are based on an 8-strand cable with $d_s=1$ mm and $L_P=100$ mm). It is clear that ξ strongly depends on

Fig. 8. The characteristic length ξ and the time constant τ_{scc} as a function of the contact resistance R_c .

Fig. 9. The characteristic length ξ and the time constant τ_{scc} as a function of the strand resistivity ρ_s .

ρ_s and tends to large values for practical superconductors especially at low transport current. In first approximation the characteristic time τ_{scc} is linear with ξ/R_c . The coefficient of linearity depends strongly on the cable geometry. It has to be stressed that the simulations presented in this paper are performed for relatively small cables with large ρ_s -values and/or small R_c -values in order to cut down the calculation times. For practical Rutherford cables both ξ and τ_{scc} will be far larger (typically of the order of 10-1000 m and 10^2 - 10^5 s respectively). ξ is therefore often larger than the length of the cable. In this case τ_{scc} is not linear with ξ but linear with the length between the end of the cable and the dB/dt non-homogeneity. More details about the propagation velocity are given in [8].

IV. CABLE WITH LONGITUDINALLY VARYING R_c

In accelerator magnets the R_c -value is likely to change along the wide side of the cable due to the keystone angle. This variation however does not generate SCC's as long as the variation is constant along the cable length. In addition, a longitudinal R_c -variation can easily occur as R_c depends on the transverse prestress onto the cable which varies considerably over the cross section of the magnet and especially in the heads of the magnet and in the soldered connections between different cables in the magnet. Each longitudinal variation in R_c generates SCC's whose amplitude is in general inversely proportional to the local R_c . A detailed analysis of this effect is described in [7],[8].

VI. CONCLUSIONS

So called Super Coupling Currents (SCC's) are generated in (Rutherford type of) cables which are subjected to a changing magnetic field that varies along the length of the cable. These SCC's differ from the 'normal' inter-strand coupling currents in the fact that they stay in the strands over long distances and therefore do not exhibit a significant loss. The decay of the SCC's along the length can be characterised by a length ξ which is of the order of 10^2 - 10^4 times the cable pitch for practical cables. The amplitude depends besides geometrical parameters also on the shape and length of the dB/dt non-homogeneity. The SCC's propagate through the cable with a certain velocity which is in first approximation inversely proportional to R_c . The characteristic time τ_{scc} of the SCC's in practical cables can attain large values (10^2 - 10^5 s) and is therefore several orders of magnitude larger than the time constant of the inter-strand coupling currents. In dipole magnets these currents result in sinusoidally varying field distortions along the magnet axis with a large characteristic time, an amplitude linear to dB/dt , and a period equal to the cable pitch.

REFERENCES

- [1] H. Brück e.a., "Observation of a periodic pattern in the persistent-current fields of the superconducting HERA dipole magnets", *DESY 91-11*, 1991.
- [2] A.K. Ghosh, K.E. Robins, and W.B. Sampson, "The ramp rate dependence of the sextupole field in superconducting dipoles", *IEEE Trans. on Magn.*, vol. 30, pp. 1718-1721 (1994).
- [3] R.I. Schermer, and B.P. Turck, "Current sharing between insulated strands in a SC cable", *Adv. Cryog. Eng.*, vol. 26, p. 599, 1979.
- [4] H.G. Knoopers, H.H.J. ten Kate, and L.J.M. van de Klundert, "Distribution of currents in a 6-strand superconducting cable", *Proceedings MT-9*, C. Marinucci, and P. Weymuth, Eds., p. 539, 1985.
- [5] A.P. Verweij, and R. Wolf, "Field errors due to eddy currents in the cable of the LHC pinkbook dipole", *CERN AT-MA 92-52*, 1992.
- [6] A.P. Verweij, and H.H.J. ten Kate, "Coupling currents in Rutherford cables under time-varying conditions", *IEEE Trans. on Appl. SC*, vol. 3, p. 146, 1993.
- [7] A.A. Akhmetov, A. Devred, and T. Ogitsu, "Periodicity of crossover currents in a Rutherford-type cable subjected to a time-dependent magnetic field", *J. Appl. Phys.*, vol. 75 (6), pp. 3176-3183, 1994.
- [8] A.P. Verweij, PhD thesis University of Twente, the Netherlands, to be published (1995).