Measuring multi-product banks’ market power using the Lerner index

Sherrill Shaffer a,b, Laura Spierdijk c

a University of Wyoming, Department of Economics and Finance, 1000 East University Ave., Laramie, WY 82071, USA
b Centre for Applied Macroeconomic Analysis (CAMA), Australian National University, Australia
c University of Groningen, Faculty of Economics and Business, Department of Economics, Econometrics and Finance, P.O. Box 800, 9700, the Netherlands

ARTICLE INFO

Article history:
Received 8 October 2019
Accepted 13 May 2020
Available online 20 May 2020

JEL classification:
D43
L13
G21

Keywords:
Lerner index
Multi-product banks
Market power
Cost functions

ABSTRACT

The aggregate Lerner index is a popular composite measure of multi-product banks’ market power, based on total assets as the single aggregate output factor. We show that the aggregate Lerner index only qualifies as a consistently aggregated Lerner index if three conditions hold. Under these conditions, the aggregate Lerner index reduces to a weighted-average of the product-specific Lerner indices. We test the three conditions for a sample of U.S. banks covering the years 2011–2017. All three conditions are rejected and we show that they may cause an economically relevant bias to the aggregate Lerner index, depending on the economic context. As a general solution, we propose using the always consistently aggregated weighted-average Lerner index whenever a composite Lerner index is needed.

© 2020 The Author(s). Published by Elsevier B.V.
This is an open access article under the CC BY license. (http://creativecommons.org/licenses/by/4.0/)

1. Introduction

The Lerner index is a widely used measure of market power in the economic literature, whose historical and theoretical foundations have been extensively discussed in the literature (Amoroso, 1933; Lerner, 1934; Amoroso, 1938; 1954; Landes and Posner, 1981; Elzinga and Mills, 2011; Giocoli, 2012; Shaffer and Spierdijk, 2017). A firm’s Lerner index compares the market output price with the firm’s marginal costs of production, where marginal-cost pricing is referred to as the “social optimum that is reached in perfect competition” (Lerner, 1934, p.168). A positive Lerner index is generally associated with the presence of market power and reduced consumer welfare.

The Lerner index was originally derived for a firm producing a single product. The multi-product extension of the Lerner index comprises separate Lerner indices for each product category. This follows from the result that product-specific marginal-cost pricing also characterizes the long-run competitive equilibrium of multi-product firms (Baumol et al., 1982; MacDonald and Slivinski, 1987).

Multi-product measures of market power are relevant for the banking sector, where banks earn a substantial part of their income from investments and off-balance sheet activities, in addition to lending. For instance, for U.S. commercial banks with total assets exceeding $100 million, the sum of securities income and realized capital gains was about 14% of operating income during the 2011–2017 period, on average. For the same group of banks, non-interest income constituted on average about 18% of operating income during this period.1 For an overview of such trends in the European banking sector, see e.g. Lepetit et al. (2008).

Despite the multi-product character of banks, the ‘aggregate’ Lerner index has nevertheless remained popular in the empirical banking literature. This Lerner index is based on total assets as the single aggregate output factor. To obtain this Lerner index, banks’ output price is typically calculated as the average revenue (i.e., total revenue divided by total assets), while the estimate of marginal costs is based on an aggregate cost function with total assets as the single output factor. Product-specific Lerner indices – based on the average revenue per product and a multi-product cost function – have only been used occasionally in banking. Other studies make use of a weighted-average of product-specific Lerner indices (henceforth referred to as ‘the’ weighted-average Lerner index).

Table 1 provides an overview of recent banking studies using the Lerner index. These studies, published between 2013–2020, are grouped into three categories on the basis of the type of Lerner

1 Source: authors’ own calculations using Call Report data for the 2011–2017 period; see Appendix A.
Table 1
Recent Lerner index studies in banking.

<table>
<thead>
<tr>
<th>author(s)</th>
<th>output(s)</th>
<th>sample period</th>
<th>country/region</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Lerner index</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saif-Alyoufi et al. (2020)</td>
<td>total assets</td>
<td>1996–2016</td>
<td>6 G.C.C. countries</td>
</tr>
<tr>
<td>Hirata and Ojima (2020)</td>
<td>total assets</td>
<td>1996–2016</td>
<td>Japan</td>
</tr>
<tr>
<td>Memanova and Mylonidis (2020)*</td>
<td>total earning assets</td>
<td>1997–2010</td>
<td>125 countries</td>
</tr>
<tr>
<td>Wang et al. (2020)</td>
<td>total (earning) assets</td>
<td>2006–2015</td>
<td>19 E.U. countries</td>
</tr>
<tr>
<td>Phan et al. (2020)</td>
<td>total assets</td>
<td>2004–2014</td>
<td>4 Asian countries</td>
</tr>
<tr>
<td>Shamshur and Weill (2019)</td>
<td>total assets</td>
<td>2015</td>
<td>9 E.U. countries</td>
</tr>
<tr>
<td>Deli et al. (2019)*</td>
<td>total earning assets</td>
<td>1997–2014</td>
<td>U.S.</td>
</tr>
<tr>
<td>Clark et al. (2018)</td>
<td>total assets</td>
<td>2005–2013</td>
<td>CIS countries</td>
</tr>
<tr>
<td>Cubillas et al. (2017)</td>
<td>total assets</td>
<td>1989–2007</td>
<td>104 countries</td>
</tr>
<tr>
<td>Fosu et al. (2017)</td>
<td>total assets</td>
<td>1995–2013</td>
<td>U.S.</td>
</tr>
<tr>
<td>Leroy and Lucotte (2017)</td>
<td>total assets</td>
<td>2004–2013</td>
<td>97 large European banks</td>
</tr>
<tr>
<td>Delis et al. (2016)</td>
<td>total earning assets</td>
<td>1997–2009</td>
<td>131 countries</td>
</tr>
<tr>
<td>Carbó-Valverde et al. (2016)</td>
<td>total assets</td>
<td>1994–2010</td>
<td>Spain</td>
</tr>
<tr>
<td>Calderon and Saeckeck (2016)</td>
<td>total assets</td>
<td>1996–2010</td>
<td>124 countries</td>
</tr>
<tr>
<td>Dong et al. (2016)</td>
<td>total assets</td>
<td>2002–2013</td>
<td>China</td>
</tr>
<tr>
<td>Fernández et al. (2016)</td>
<td>total assets</td>
<td>1989–2008</td>
<td>110 countries</td>
</tr>
<tr>
<td>Clerides et al. (2015)*</td>
<td>total earning assets</td>
<td>1997–2010</td>
<td>148 countries</td>
</tr>
<tr>
<td>Anginer et al. (2014)</td>
<td>total assets</td>
<td>1997–2009</td>
<td>63 countries</td>
</tr>
<tr>
<td>Fu et al. (2014)</td>
<td>total assets</td>
<td>2003–2010</td>
<td>14 Asia Pacific countries</td>
</tr>
<tr>
<td>Mirzaei and Moore (2014)</td>
<td>total assets</td>
<td>1999–2011</td>
<td>146 countries</td>
</tr>
<tr>
<td>Beck et al. (2013)</td>
<td>total assets</td>
<td>1994–2009</td>
<td>79 countries</td>
</tr>
<tr>
<td>Hainz et al. (2013)</td>
<td>total assets</td>
<td>2000–2005</td>
<td>70 countries</td>
</tr>
<tr>
<td><strong>Product-specific Lerner indices</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Degl’Innocenti et al. (2017)</td>
<td>loans, customer deposits</td>
<td>1993–2011</td>
<td>Italy</td>
</tr>
<tr>
<td>Huang et al. (2017)</td>
<td>loans, investments, non-interest income</td>
<td>1998–2010</td>
<td>5 E.U. countries</td>
</tr>
<tr>
<td>Tirrito and Ongena (2017)</td>
<td>loans, loan advancements to banks, securities, other earnings assets, derivatives, guarantees (OBS), committed credit lines (OBS)</td>
<td>2000–2014</td>
<td>28 E.U. countries</td>
</tr>
<tr>
<td>Ahamed and Mallick (2017)</td>
<td>loans, securities</td>
<td>1994–2012</td>
<td>India</td>
</tr>
<tr>
<td>Hakenes et al. (2015)</td>
<td>loans, securities and OBS items</td>
<td>1995–2004</td>
<td>Germany</td>
</tr>
<tr>
<td>Inklaur et al. (2015)</td>
<td>interbank loans, commercial loans, securities, OBS items</td>
<td>1996–2006</td>
<td>Germany</td>
</tr>
<tr>
<td>Buch et al. (2013)</td>
<td>interbank loans, customer loans, securities, OBS items</td>
<td>2003–2006</td>
<td>Germany</td>
</tr>
</tbody>
</table>

Notes: This non-exhaustive table lists some recent studies (published since 2013) using aggregate, product-specific and weighted-average Lerner indices. Studies that appear more than once employ different Lerner indices. The abbreviation OBS stands for ‘off-balance sheet’. All studies listed use the translog cost function to estimate the marginal-cost component of the Lerner index, apart from the ones marked with a star. They use a semi-parametric quasi-linear specification. The aggregate Lerner index, used by the studies in the upper panel, is the key focus of our study.

index used: the aggregate Lerner index (upper panel), product-specific Lerner indices (middle panel) and the weighted-average of the product-specific Lerner indices (lower panel).

In the economic literature about ‘consistent aggregation’, functions depending on various disaggregate variables are transformed into functions that depend on a single aggregate variable, or series of disaggregate functions are transformed into a single aggregate function (e.g., Gorman, 1959; Berndt and Christensen, 1974; Chipman, 1974; Vartia, 1976; Brown et al., 1979; Blackorby and Schworm, 1988; Kim, 1986; Blackorby and Primonials, 1980; Blackorby and Russell, 1999). Examples are the aggregation of a firm’s multi-product cost function (depending on multiple outputs) into a single-product cost function (depending on a single aggregate output) and the aggregation of multiple firms’ efficiency indices.
into a single industry-level aggregate efficiency index. In these and other cases, the aggregation must be done ‘consistently’ to ensure that the aggregated form shares the particular economic and mathematical properties associated with the underlying disaggregated forms.

To our best knowledge, the literature has only considered the consistent aggregation of firm-specific measures of market power into industry-wide measures (e.g., Schroeter and Azzam, 1990; Morrison Paul, 1999; Neven and Röller, 1999). In particular, the consistent aggregation of product-specific Lerner indices has not yet been addressed. According to the aforementioned literature, however, consistent aggregation is a necessary property for the aggregate Lerner index to represent a summary measure of a multi-product bank’s market power in different output markets. The continued use in the banking literature of a potentially inconsistently aggregated Lerner index is a key motivation for our study.

Our theoretical contribution to the literature is twofold. First, we define the concept of a consistently aggregated Lerner index in line with the aforementioned consistent-aggregation literature. Second, we derive three conditions under which the aggregate Lerner index is consistently aggregated. If these consistency conditions hold, this Lerner index reduces to the weighted-average Lerner index. If any of these conditions is rejected, however, the aggregate Lerner index is no longer consistently aggregated. This part of our study also provides the missing link among the three different types of Lerner indices used in the literature and included in Table 1.

Although one of the aforementioned conditions has been described as ‘extremely restrictive’ in the literature (Brown et al., 1979), whether the three conditions hold for a given sample is ultimately an empirical matter. We therefore provide an empirical application in addition to our theoretical analysis. Our contribution is that we provide an empirical strategy to test the consistency conditions, applied to a sample of U.S. commercial banks observed during the 2011–2017 period. Here we distinguish among three lines of business of multi-product banks: lending, investments and off-balance-sheet activities. We find that all three conditions are statistically rejected, which means that the aggregate Lerner index is not consistently aggregated for our sample of banks. We show that the statistically rejected conditions may cause economically relevant distortions to the aggregate Lerner index, depending on the economic context.

Our analysis raises the question why the aggregate index should be used in the first place. The user of this index should at least test whether it is consistently aggregated for the particular sample at hand. This already turns out to require the calculation of the components of the weighted-average Lerner index. Furthermore, will show that some cost functions (such as the well-known translog) are never separable in total output. Based on such a cost function, the aggregate Lerner index is a priori known to be inconsistently aggregated. Such a cost function is therefore not suitable for the purpose of estimating the aggregate Lerner index, although it could still provide a good fit to the data.

As an efficient solution to these issues, we propose using the weighted-average Lerner index whenever a composite Lerner index is needed. Because the weighted-average Lerner index is always consistently aggregated regardless of the underlying cost function, this index can simply be based on a cost function that fits the data well without further concerns about this cost function’s separability properties.

The setup of the remainder of this study is as follows. We start with a literature review in Section 2. Section 3 contains the theoretical framework and derives the conditions under which the aggregate Lerner index is consistently aggregated. The setup of our empirical analysis is outlined in Section 4, while the empirical results are presented and discussed in Section 5. Lastly, Section 6 concludes. An online appendix with supplementary material is available.

2. Literature review

From an economic perspective, market power results in higher prices and lower quantities, which reduces consumer and total welfare relative to what can be attained in a hypothetical perfectly competitive outcome. This is the main reason why policymakers care about market power and generally seek to suppress it. Banks’ central role in the provision of credit, the payment system, the transmission of monetary policy and in maintaining financial stability leads to particularly large concerns about their market power.

Driven by these concerns, both the theoretical and the empirical literature have investigated the impact of banking competition on various economic outcomes, including financial stability, bank efficiency, information sharing and economic growth (e.g., Degryse et al., 2018; Coccorese, 2017). The theoretical literature has offered mixed predictions about the effects of banking competition on economic outcomes. For example, the ‘competition-stability’ view predicts a positive impact of banking competition on financial stability, while the ‘competition-fragility’ view conjectures the opposite. A similar ambiguity applies to the competition-growth relation, for which partial equilibrium models tend to predict a negative relation and general equilibrium models a positive one. Also the competition-efficiency relation is subject to such diverging theoretical results: a positive relation is predicted by the ‘efficient-structure’ hypothesis, while the ‘quiet-life hypothesis’ conjectures a negative relation. The last controversy that we mention is the relation between banking competition and relationship lending. While some studies predict that banks operating in a highly competitive environment could be inhibited from forming long-term lending relationships with small and medium-sized enterprises, others conjecture that banking competition boosts relationship lending (e.g., Petersen and Rajan, 1995; Boot and Thakor, 2000).

Because of the widespread ambiguity in the theoretical literature, the impact of banking competition is ultimately an empirical matter. Consequently, there exists an abundant empirical literature that analyzes the effect of banking competition on economic outcomes. We refer to Degryse et al. (2018) and Coccorese (2017) for a recent overview of this literature and a discussion of the reported effects of banking competition on economic outcomes.

The aforementioned empirical banking studies rely on certain measures of market power. Popular measures besides the Lerner index include market shares (such as the four-bank concentration ratio and the Herfindahl-Hirschman index), the Rothschild-Bresnahan conduct index (also known as the conduct parameter), the Panzar-Rosse H-statistic and the Hay-Liu-Boone index (also known as the performance-structure-conduct indicator); see e.g. Shaffer and Spierdijk (2017) and Degryse et al. (2018). Although all of these measures may fail to correctly indicate the absence or presence of market power in specific cases, concentration indices are widely considered to fall short as a reliable measure of market power in general. Also the Panzar-Rosse H-statistic has been shown to be unfit as a measure of market power and has been relegated to the same category as the concentration measures (Hyde and Perloff, 1995; Bikker et al., 2012; Shaffer and Spierdijk, 2015).

Blair and Sokol (2014, p. 325) report that the Lerner index has become “the standard measure of market power (...) among economists”, while Shaffer and Spierdijk (2017) call it a measure that is among the “scant handful of ‘least objectionable’ methods”. The value of the Lerner index is monotonically associated with consumer welfare losses from market power for given costs and demand functions. It has also been shown to represent the slope of a social welfare function (Dansby and Willig, 1979). Like any mea-
sue of market power, also the Lerner index has certain conceptual limitations, related to issues such as inefficiency, economies of scale and a lack of profit maximizing behavior (e.g., Scitovsky, 1955; Cairns, 1995; Koetter et al., 2012; Spierdijk and Zaouaras, 2017; 2018). For a more detailed discussion and comparison of the properties of the popular measures of banking competition, we refer to Shaffer and Spierdijk (2017) and Degryse et al. (2018).

The initial versions of the aforementioned measures of market power have in common that they assume that a bank (or other firm) produces a single output factor. For some of these measures, extensions to the multi-output case have been proposed (e.g., Gelfand and Spiller, 1987; Suominen, 1994; Feenstra and Levinsohn, 1995; Shaffer, 1996; Barbosa et al., 2015). For the Lerner index, originally derived by Lerner (1934) for a firm producing a single product, the multi-product extension is fairly straightforward and relies on the result that product-specific marginal-cost pricing also characterizes the long-run competitive equilibrium of multi-product firms (Baumol et al., 1982; MacDonald and Silovinski, 1987). Baumol et al. (1982) and MacDonald and Silovinski (1987) show this for markets with multi-product firms only and for markets with both single- and multi-product firms, respectively. Their proofs make use of the concept of a perfectly contestable market (PCM). They show that, for both single- and multi-product firms in a PCM market, the first-order conditions imply marginal-cost pricing. This argument then carries over to competitive equilibrium, which is a specific form of a PCM.

From the upper panel of Table 1 it becomes apparent that the aggregate Lerner index has remained popular in the empirical banking literature despite banks’ multi-product character. Commonly used data sources such as BankScope and the U.S. Call Reports provide sufficiently detailed data to obtain product-specific Lerner indices. The popularity of the aggregate Lerner index therefore seems largely driven by the convenience of using a single-output measure of market power (for instance, as an explanatory variable in a regression analysis).

The economic literature has paid only limited attention to the consistent aggregation of measures of market power. For example, the banking studies listed in the lower panel of Table 1 use a weighted-average of product-specific Lerner indices without addressing the topic of consistent aggregation. Gischer et al. (2015) criticize the way the aggregate Lerner index’ average revenue is calculated, but do not refer to consistent aggregation. A few studies consider the consistent aggregation of firm-specific measures of market power into industry-wide measures (e.g., Schroeter and Azzam, 1990; Morrison Paul, 1999; Neven and Röller, 1999), while others aggregate Lerner indices over firms or outputs by means of share weighting without reference to consistent aggregation (e.g., Spiller and Favaro, 1984; Encaoua et al., 1986; Verboven, 1996; Chirinko and Fazzari, 2000). As explained in the introduction, it remains to be seen if the aggregate Lerner index is a consistently aggregated measure of market power.

3. The Lerner index and consistent aggregation

This section will show that the aggregate Lerner index is consistently aggregated only under three conditions. All proofs for this section can be found in Appendix B.

3.1. Definitions

Product-specific and weighted-average Lerner indices We assume a multi-product total cost function $c(y, w)$, where $y = (y_1, \ldots, y_n)$ and $w = (w_1, \ldots, w_K)$. Here $y_j \geq 0$ denotes the level of the $j$th output and $w_k \geq 0$ the value of the $k$th exogenous input price, for $j = 1, \ldots, n$ and $k = 1, \ldots, K$. The marginal costs with respect to each output are denoted $MC_j(y, w) = \partial c(y, w) / \partial y_j > 0$. The observed market output price of the $j$th output is written as $P_j(y)$, with $P_j(y) > 0$ for $y_j > 0$. Although this price will usually also depend on variables other than $y$, we suppress this for simplicity of notation.

The Lerner index for the $j$th output is defined for $y_j > 0$ and captures the relative markup of the market output price over marginal costs. Specifically, the product-specific Lerner indices in the banking literature use the average revenue earned on each output factor, denoted $\tilde{R}_j(y) = R_j(y)/y_j$, as the market output price. This yields

$$L_j(y, w) = \frac{\tilde{R}_j(y) - MC_j(y, w)}{\tilde{R}_j(y)}$$

(1)

The studies in the second panel of Table 1 estimate product-specific Lerner indices and typically use loans, securities and off-balance sheet items as the output factors.

The weighted-average Lerner index based on $n \geq 2$ product-specific Lerner indices is defined as

$$L_{WA}(y, w) = \sum_{j=1}^{n} \omega_j(y) L_j(y, w).$$

(2)

with revenue shares as the weights, as suggested by Encaoua et al. (1986):

$$\omega_j(y) = \frac{\hat{R}_j(y)}{\hat{R}_A(y)}, \quad R_j(y) = \sum_{j=1}^{n} R_j(y).$$

(3)

By rewriting (2) using short-hand notation that leaves out the functions’ arguments for the sake of readability, we find

$$L_{WA} = \sum_{j=1}^{n} \omega_j(y) L_j = \sum_{j=1}^{n} \left[ \frac{R_j \hat{R}_j - MC_j}{R_j} \right] = \sum_{j=1}^{n} \frac{R_j R_j/y_j - MC_j}{R_j}.$$

(4)

where $\omega_j = y_j/\sum_{j=1}^{n} y_j$ denotes the output share of the $i$th output and $\hat{R}_A = \sum_{j=1}^{n} y_j$ the average revenue on total output. From (4), we see that $L_{WA}$ is defined for values of $y$ with $\sum_{j=1}^{n} y_j > 0$ (which we will henceforth denote by $y \neq 0$) and that it can be viewed as a single-output Lerner index with $\hat{R}_A$ as the market output price and weighted-average marginal costs as the marginal costs. The weighted-average Lerner index has recently been used in various banking studies; see the lower panel of Table 1.

Aggregate Lerner index The starting point of the aggregate Lerner index is the existence of an aggregate output factor $y$, with corresponding market output price $P_A(y) > 0$ for $y > 0$, cost function $c_A(y, w)$ and associated marginal cost function $MC_A(y, w) = \partial c_A(y, w)/\partial y > 0$. Specifically, the aggregate Lerner index uses total output $\sum_{j=1}^{n} y_j$ as the aggregate output factor and the average revenue earned on total output ($\hat{R}_A$) as the market price of total output. The index is defined for $y = 0$ and writes as

$$L_A(y, w) = \frac{\hat{R}_A(y) - MC_A(\sum_{j=1}^{n} y_j, w)}{\hat{R}_A(y)}.$$

(5)

---

1 As shown in Shaffer (1983), the average revenue can reflect any two-part tariffs or nonlinear pricing schedules. Average revenue also has the advantage of reflecting actual transaction prices even when they deviate from posted prices (due to errors, idiosyncratic negotiations with selected counterparties, etc.).

2 Because these studies typically write the weighted-average Lerner index in a different way, their index is not directly recognizable as a share-weighted average. Using short-hand notation that leaves out the functions’ arguments for the sake of readability, these studies write $L_{WA} = (\hat{R}_A - AC \sum_{j=1}^{n} e_j)/\hat{R}_A$, where $AC$ denotes average costs and $e_j$ the elasticity of the multi-product cost function $c$ with respect to the $k$th output.
The empirical banking studies listed in the upper panel of Table 1 have recently used the aggregate Lerner index.

**Consistent aggregation** Using the same notation as before, we continue to consider a K-input and n-output bank with input-price vector \( w \), output vector \( y \), multi-product cost function \( c(y, w) \) and market output prices \( P_j(y) \) for \( j = 1, \ldots, n \). In this setting, we consider the product-specific Lerner indices \( L_j(y, w) \) and some Lerner index \( L(y, w) : D \times \mathbb{R}_+^k \to \mathbb{R} \) (with \( D \subset \mathbb{R}_+^k \)) of which we would like to know whether it is consistently aggregated. We will write \( L = (L_1, \ldots, L_k) \) and \( P = (P_1, \ldots, P_n) \) to denote vectors of values of product-specific Lerner indices and output prices, respectively, where \( L \in \mathbb{R}^k \) and \( P \in \mathbb{R}^n \). Furthermore, we will henceforth assume that \( y \in D \) and \( w \in \mathbb{R}_+^k \), even if we do not explicitly mention this.

To define a consistently aggregated Lerner index in line with the literature, we proceed in a way comparable to Blackorby and Russell (1999). From them, we take the requirements of aggregation, monotonicity and (non-)competitive indication, resulting in the following definition:

**Definition 3.1.** A Lerner index \( L(y, w) \) is consistently aggregated if there exists a differentiable function \( F : \mathbb{R}^n \times \mathbb{R}_+^k \to \mathbb{R} \) that satisfies the requirements of aggregation, monotonicity, and (non-)competitive indication:

(i) \[ \text{[Aggregation]} \quad L(y, w) = F(L_1(y, w), \ldots, L_n(y, w), P_1(y), \ldots, P_n(y)) \quad \text{for all } y, w. \]

(ii) \[ \text{[Monotonicity]} \quad \frac{\partial F(L, P, y)}{\partial L_j} > 0 \quad \text{for all } L, P, y. \]

(iii) \[ \text{[(Non-)competitive indication]} \quad F(0, \ldots, 0, P, y) = 0 \quad \text{and} \quad F(1, \ldots, 1, P, y) = 1 \quad \text{for all } P, y. \]

Under the aggregation requirement, \( L \) is a function of the product-specific Lerner indices, ensuring an economic interpretation as a summary measure of a bank’s product-specific Lerner indices. The monotonicity requirement ensures that \( L \) increases following a ceteris paribus increase in one of the product-specific Lerner indices. The requirement of (non-)competitive indication is imposed to ensure that \( L \) shares the particular economic and mathematical properties associated with the underlying product-specific Lerner indices. The first part of requirement (iii) ensures that \( L = 0 \) if each \( L_j = 0 \), which implies that both \( L \) and each \( L_j \) have 0 as the competitive benchmark value. To see this, we recall from the literature review that product-specific marginal cost pricing characterizes long-run competitive equilibrium in markets with only multi-product firms, as well as markets with both single- and multi-product firms (Baumol and Bradford, 1970; MacDonald and Slivinski, 1987). The second part of requirement (ii) ensures that \( L = 1 \) if each \( L_j = 1 \). Hence, if all product-specific marginal costs are zero, then the aggregate index must have the value that corresponds to zero marginal costs at the aggregate level. In Appendix B it is shown that two other natural properties with respect to (non-)competitive indication are automatically satisfied under Definition 3.1.

To see that the class of consistently aggregated Lerner indices is not empty, consider share-weighted Lerner indices of the form

\[
L_{SW}(y, w) = \sum_{j=1}^{n} s_j L_j(y, w), \quad 0 < s_j < 1, \quad \sum_{j=1}^{n} s_j = 1. \tag{6}
\]

Here the shares \( s_j \) are allowed to be functions of the form \( s_j = s_j(P_1(y), \ldots, P_n(y), y) \). Evidently, this class of Lerner indices satisfies the requirements of Definition 3.1 and is thereby consistently aggregated. This also holds for the weighted-average Lerner index as defined in Section 3.1. A further characterization of the class of consistently aggregated Lerner indices is provided in Appendix B.

We need some theory before we can conclude whether or not the popular aggregated Lerner index is consistently aggregated.

**Separability in total output** Several studies about consistent aggregation of functions show that this concept is in some way related to ‘separability’ of the underlying functions (e.g., Berndt and Christensen, 1974). Such a relation also turns out to exist in case of the aggregate Lerner index, where the function of relevance is the multi-product cost function. We provide a definition of the type of separability that is relevant in our case.

Brown et al. (1979) define a separable multi-product cost function as a cost function \( c(y, w) \) for which a single-output cost function \( c_j(y, w) \) and an output aggregation function \( h(y) \) exist such that \( c(y, w) = c_j(h(y), w) \) for all \( y, w \). The output aggregation function aggregates the vector of outputs \( y \) into a scalar measure of aggregate output \( h(y) \). Stated differently, a separable multi-product cost function is equal to a single-output cost function with aggregate output \( h(y) \) as the single output factor (Kim, 1986).

**Definition 3.2.** A multi-product cost function \( c(y, w) \) is separable in total output if there exists a single-output cost function \( c_A(y, w) \) such that \( c(y, w) = c_A(\sum_{j=1}^{n} y_j, w) \) for all \( y, w \).

Hence, a multi-product cost function that is separable in total output reduces to a single-output cost function with total output \( \sum_{j=1}^{n} y_j \) as the aggregate output factor.

**3.2. Consistency conditions**

**Aggregate Lerner index** In line with the literature about consistent aggregation, we find a strong relation between separability in total output of the multi-product cost function and consistent aggregation of the aggregate Lerner index.

**Result 3.1.** Assume that banks’ multi-product cost function is given by \( c(y, w) \), with corresponding \( L_{SW}(y, w) \) as defined in (4). Let \( h(y) = \sum_{j=1}^{n} y_j \). If there exists a single-output cost function \( c_A(y, w) \) such that \( c(y, w) = c_A(h(y), w) \) for all \( y, w \), then \( L_j \) in (5) based on \( c_A \) is consistently aggregated and \( L_A = L_{WA} \). Conversely, if \( c \) is not separable in \( h(y) \), then \( L_A \) based on any single-output cost function \( c_A \) is not consistently aggregated and \( L_A \neq L_{WA} \).

The implications of this result are as follows. If banks’ multi-product cost function is separable in total output, then the aggregate Lerner index \( L_A \) is consistently aggregated and equal to the weighted-average Lerner index \( L_{WA} \). If banks’ multi-product cost function is not separable in total output, it is either separable in a different aggregate output measure or not separable at all. Result 3.1 tells us that, in either case, the aggregate Lerner index based on any single-output cost function we may come up with is not consistently aggregated and not equal to the weighted-average Lerner index.

The practical consequence of Result 3.1 is that we first have to verify whether banks’ multi-product cost function is separable in total output before we can use the aggregate Lerner index in an empirical setting. We note that it does not seem very likely that separability in total output will often exist in practice. Brown et al. (1979, p. 257) call the implications of separability in an aggregate output factor “extremely restrictive”. If separability in some aggregate output factor is already considered extremely restrictive, then separability in a specific aggregate output factor (namely total output) will be even more restrictive. The restrictiveness of separability in total output stems from the implied property that the marginal costs are the same for all outputs. This property follows immediately from the multi-product cost function’s functional form under separability in total output as given in Definition 3.2.

**Empirical aggregate Lerner index** The studies referred to in the upper panel of Table 1 make use of the ‘empirical’ aggregate Lerner index \( L_A^* \), which differs from the ‘theoretical’ aggregate Lerner index \( L_A \) as defined in (5). \( L_A^* \) is based on a different aggregate output
factor, namely total assets instead of total output. It also makes use of a different revenue, namely the sum of interest and non-interest (INI) income instead of the sum of the product-specific revenues. We will use Result 3.1 to derive the conditions under which $I_A^2$ is consistently aggregated.

We start with the following observation. Depending on the chosen banking model, there may be no equivalence between total assets and total output and between INI income and the total revenue. Some components of total assets may not be considered an output, while other components are viewed as an output but are not part of total assets. For example, total assets include fixed assets, which are considered an input instead of an output in the commonly used intermediation model of banking (Klein, 1971; Monti, 1972; Sealey and Lindley, 1977). Furthermore, off-balance sheet activities are not included in total assets, while they are often considered to be an output factor (e.g., DeYoung and Rice, 2004; Wheelock and Wilson, 2012). These two sources of non-equivalence work in opposite directions regarding the mismatch between total assets and total output, so total assets could potentially either overstate or understate total output for individual banks in the intermediation model.

Also the direction of the non-equivalence between INI income and the total revenue is ambiguous. For instance, service fees on deposits are part of INI income, but are not part of the total revenue according to the intermediation model of banking (where deposits are considered an input instead of an output). Furthermore, capital gains on the output factor securities are not part of INI income; they are listed as a separate item on banks’ income statement. Yet securities are included in total assets and part of their revenue stems from these capital gains.

We can thus write total assets as the sum of certain non-output variables and some of the output factors. Similarly, INI income is written as the sum of the income on certain non-output variables and the revenue on some of the output factors. Stated differently, there are included non-output variables and excluded output factors. Informally, we thus write

\[
\text{total assets} = \text{total output} - \text{excluded outputs} + \text{included non-outputs}.
\]

\[
\text{INI income} = \text{total revenue} - \text{revenue on excluded outputs} + \text{income on included non-outputs}.
\]

where we note that the excluded outputs and included non-outputs may differ between total assets and INI income.

We will now formalize the above considerations, while allowing for any banking model and not just the intermediation model as considered above. We assume that there are additional variables $y_1, \ldots, y_m$, which are considered non-output variables according to the chosen banking model. In case of the intermediation model of banking, the variables $y_1, \ldots, y_m$ include fixed assets and deposits. With $z = (y_1, \ldots, y_m, y_1, \ldots, y_n)$, we can now write total assets and INI income as, respectively,

\[
k(z) = h(y) - \sum_{j \in I} y_j + \sum_{k \in K} y_k.
\]

\[
Q_A(z) = R_A(y) - \sum_{i \in I} R_i(y) + \sum_{p \in P} Q_P(z),
\]

for input functions $Q_P$.

Writing $z \neq 0$ for values of $z$ with $k(z) > 0$, we define $\tilde{Q}_A(z) = Q_A(z)/k(z)$ for $z \neq 0$. We can then write the empirical aggregate Lerner index as

\[
I_A^2(z, w) = \frac{\tilde{Q}_A(z) - MC_A(k(z), w)}{\tilde{Q}_A(z)}.
\]

Table 2

<table>
<thead>
<tr>
<th>composite index</th>
<th>conditions for consistent aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_A$</td>
<td>[SEP]</td>
</tr>
<tr>
<td>$I_{WA}$</td>
<td>always consistently aggregated</td>
</tr>
<tr>
<td>$I^2_A$</td>
<td>[SEP], [EQ1], [EQ2]</td>
</tr>
<tr>
<td>$I^2_{WA}$</td>
<td>[EQ2]</td>
</tr>
</tbody>
</table>

Notes: This table lists the condition(s) that must hold for each index to qualify as a consistently aggregated Lerner index. Here $L_4$ is defined in (5), $L_{WA}$ in (4), $I^2_A$ in (8), and $I^2_{WA}$ in (9).

$k(z) = h(y)$ and $Q_A(z) = R_A(y)$ for all $y, z$ (“no excluded outputs and no included non-outputs”), we can use Result 3.1 to determine whether $I_A^2$ is consistently aggregated. This leads to the following result:

**Result 3.2.** Assume that banks’ multi-product cost function is given by $c(y, w)$. Consider $I_A^2$ as defined in (8), based on some single-output cost function $C_k$. If $k(z) \neq h(y)$ or $Q_A(z) \neq R_A(y)$ for some $y, z \neq 0$, then $I_A^2$ is not consistently aggregated. Conversely, if $k(z) = h(y)$ and $Q_A(z) = R_A(y)$ for all $y, z$, then $I_A^2(z, w) = L_A(y, w)$ for all $w$ and $y, z \neq 0$ and Result 3.1 applies.

Result 3.2 leads to the following three necessary and sufficient conditions for $I_A^2$ to be consistently aggregated: separability in total output of the multi-product cost function ([SEP]), equivalence of total output and total assets ([EQ1]), and equivalence of total revenue and INI income ([EQ2]). The intuition behind these consistency conditions is as follows. The first consistency condition ensures that the single-output aggregate cost function used for $I_A^2$ coincides with the underlying multi-product cost function. The second and third conditions ensure that the aggregate output factor and the measured revenue on the aggregate output factor used for $I_A^2$ are both consistent with the choice of outputs in the underlying multi-product cost function. If any of these conditions do not hold, then $I_A^2$ will not be consistently aggregated.

**Empirical weighted-average Lerner index** The studies referred to in the lower panel of Table 1 make use of the ’empirical’ weighted-average Lerner index $L_{WA}^2$. In contrast to the theoretical weighted-average Lerner index $L_{WA}$ as defined in (4), it uses INI income instead of the sum of the product-specific revenues $R_A$. Using the same notation as for $I_A^2$, we can thus write

\[
L_{WA}^2(z, w) = \frac{\tilde{Q}_A(z) - \sum_{i=1}^n \tilde{W}_i(y)MC_i(y, w)}{\tilde{Q}_A(z)},
\]

with $\tilde{Q}_A(z) = Q_A(z)/h(y)$. Following the same line of reasoning as for $I_A^2$, we can show that $I_{WA}^2$ is consistently aggregated if and only if $Q_A(z) = R_A(y)$ for all $y, z$. This result has been relegated to Appendix B.

The upper panel of Table 2 summarizes the conditions under which each of the indices $L_4, L_{WA}, I^2_A$ and $I^2_{WA}$ is consistently aggregated. The statistical testing of these conditions will play a major role in the empirical part of our analysis.

4. Empirical setup

This section describes the data sample, banking model and multi-product cost function that will be used in our empirical application.

4.1. Data and banking model

We use year-end Call Report Data to create an unbalanced sample of U.S. commercial banks covering the 2011–2017 period. We
restrict the sample to commercial banks that are part of a bankholding company, with a physical location in a U.S. state and subject to deposit-related insurance.

The common procedure in the banking literature is to choose a particular banking model in order to define the output and input factors. Subsequently, a specific functional form of the total cost function is chosen. We follow that procedure and base our choice of inputs and outputs on the widely used intermediation model for banking (Klein, 1971; Monti, 1972). More specifically, we assume that banks employ a technology with four inputs and three output factors (Wheelock and Wilson, 2012). The four inputs that we consider are purchased funds, core deposits, labor services, and physical capital. The corresponding input prices are (i) the price of purchased funds of bank \( i \) at year \( t \) \( w_{1,t,i} \), (ii) the core deposit interest rate \( w_{2,t,i} \), (iii) the wage rate \( w_{3,t,i} \), and (iv) the price of physical capital \( w_{4,t,i} \). Total operating costs \( c_{i,t} \) are defined as the sum of expenses on purchased funds, core deposits, personnel, and physical capital.

The three output factors that we consider are total loans and leases \( y_{1,t,i} \), total securities \( y_{2,t,i} \), and off-balance sheet activities \( y_{3,t,i} \). For total loans and leases, we use interest and lease income as the revenue. For total securities (defined as the sum of hold-to-maturity and available-for-sale securities), we use interest and dividend income (also known as securities income) and realized capital gains on securities as the revenue. We define the revenue from off-balance sheet activities as non-interest income minus service fees on deposits (e.g., DeYoung and Rice, 2004; Boyd and Gertler, 1994). Due to a lack of direct output data, the output associated with the off-balance sheet revenue has to be obtained indirectly. We convert the adjusted non-interest income to non-interest income capitalization credit equivalents using the method of Boyd and Gertler (1994). This method measures off-balance sheet activities in units of on-balance sheet assets that would be required to generate the observed level of adjusted non-interest income. The resulting quantity serves as our output measure of off-balance sheet activities. The Boyd-Gertler method assumes that on- and off-balance sheet items are equally profitable at the margin. Clark and Siems (2002) argue that this assumption is reasonable in fairly competitive markets. In such markets, a reallocation of outputs would take place in case of unequal profit margins across different outputs.

The banking literature has emphasized the relevance of bank-specific cost technologies (e.g., Berger and Humphrey, 1997; Kumbhakar and Tsionas, 2008). We therefore use banks’ total output in prices of the year 2017 to stratify our sample and distinguish among four size classes: (i) less than $100 million, (ii) $100–500 million, (iii) $500 million–1 billion, and (iv) more than $1 billion. The next section will present empirical results for each size class.

Having defined the required variables and the size classes, we filter out inconsistent values in the data and use trimming to remove outliers. The exact filtering rules are listed in Appendix A. This appendix also explains how the Call Report series have been used to construct the variables required for our analysis.

Table 3 provides (non-deflated) sample statistics on these variables, including output and revenue shares, output quantities, average revenue, and number of banks and bank-year observations. We highlight a few figures. On average, total loans have larger revenue and output shares than total securities and off-balance sheet activities, regardless of bank size. Banks in the two largest size classes have relatively low average revenue and output shares for loans and securities, but higher average shares for off-balance sheet activities. Regardless of the size class, loans have the highest average revenue, followed by off-balance sheet activities and securities. Banks in the two largest size classes have relatively large average shares of adjusted non-interest income and fiduciary services and they also have a higher average wage rate. The dispersion in output levels is relatively large for banks in the largest size class.

4.2. Cost functions

Multi-product cost functions have a long history in banking (e.g., Benston et al., 1982; Shaffer, 1984). It is well-known that the popular translog cost function – introduced by Christensen et al. (1971, 1973) – requires a relatively homogeneous sample in terms of bank size and product mix to provide an accurate fit (e.g., McAllister and McManus, 1993; Feng and Serletis, 2010; Wheelock and Wilson, 2012). In a multi-product setting, the problem of size heterogeneity is amplified due to the presence of multiple outputs. Even if the translog cost function is estimated separately for relatively homogeneous samples of banks in terms of total output, there can still be substantial variation across banks in terms of one or more individual outputs. This is because the various outputs are not perfectly correlated. For instance, there are banks that are large in terms of loans, but small in terms of securities.

Although non-parametric methods have proven their usefulness in the modeling of cost functions in banking (e.g., Wheelock and Wilson, 2012; 2018), we confine our analysis to parametric cost functions. The main reason for this choice is that our research question requires us to assess whether the cost function is separable in total output.

In the light of the aforementioned problems associated with the translog cost function, we consider another parametric cost function in addition to the translog: the generalized Leontief cost function (Diewert, 1971; Fuss, 1977). Generalized Leontief technologies have been widely used in banking and other fields, both in a single- and a multi-product context (e.g., Thomsen, 2000; Gunning and Sickles, 2011; Martin-Oliver et al., 2013; Miller et al., 2013). Multi-product Leontief cost functions already date back to Hall (1973). Some basic properties of the translog and generalized Leontief cost functions are discussed in Appendix C, where it is shown that both cost functions differ in terms of the possible shapes of the average and marginal cost functions.

4.2.1. Empirical specification: translog

We consider a translog cost function similar to Koetter et al. (2012) and many others. As usual, we impose linear homogeneity in input prices by normalizing total costs and input prices with the price of purchased funds \( w_{1,t,i} \). This normalization will be reflected in our notation by the use of variables with a tilde, indicating that they have been normalized with the price of purchased funds prior to taking the logarithmic transformation. This results in the following four-input and three-output translog cost function for bank \( i \) in year \( t \):

\[
\log (c_{i,t}) = \alpha_i + 4 \sum_{j=2}^{4} \beta_{j,w} \log (\tilde{w}_{j,t,i}) + (1/2) \sum_{j=2}^{4} \beta_{j,ww} \left[ \log (\tilde{w}_{j,t,i}) \right]^2
\]

\[
+ 4 \sum_{j=2}^{4} \sum_{k=2}^{j} \beta_{jk,w} \log (\tilde{w}_{j,t,i}) \log (\tilde{w}_{k,t,i})
\]

\[
+ 4 \sum_{k=2}^{3} \sum_{l=1}^{k-1} \beta_{k,w} \log (\tilde{w}_{k,t,i}) \log (y_{l,t,i})
\]

\[
+ 3 \sum_{l=1}^{3} \beta_{l,y} \log (y_{l,t,i}) + (1/2) \sum_{l=1}^{3} \beta_{ly} \left[ \log (y_{l,t,i}) \right]^2
\]
Table 3

<table>
<thead>
<tr>
<th></th>
<th>ALL</th>
<th>CLASS 1</th>
<th>CLASS 2</th>
<th>CLASS 3</th>
<th>CLASS 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
<td>s.d.</td>
<td>mean</td>
</tr>
<tr>
<td>total loans (TLNS)</td>
<td>1,325,996</td>
<td>20,796,483</td>
<td>35,785</td>
<td>17,721</td>
<td>141,414</td>
</tr>
<tr>
<td>total securities (TESC)</td>
<td>465,813</td>
<td>8,532,902</td>
<td>14,960</td>
<td>11,991</td>
<td>51,502</td>
</tr>
<tr>
<td>off-balance sheet items (OBS, Boyd-Gerrler)</td>
<td>1,263,169</td>
<td>32,892,009</td>
<td>5,337</td>
<td>5,786</td>
<td>31,486</td>
</tr>
<tr>
<td>off-balance sheet items (ANIL, adjusted non-interest income)</td>
<td>28,940</td>
<td>659,572</td>
<td>170</td>
<td>181</td>
<td>990</td>
</tr>
</tbody>
</table>

Notes: The columns captioned ‘mean’ report sample means, while the columns captioned ‘s.d.’ show sample standard deviations. All level variables are in thousands of $. We classify banks on the basis of their total output in 2017 prices. Size classes in prices of the year 2017: 1: less than $100 million, 2: $100–500 million, 3: $500 million–1 billion and 4: $1 billion. Some banks switch from one size class to another over the years if their total output in 2017 prices changes. For this reason, the sum of the number of banks in each size class exceeds the number of banks in the entire sample.
The non-nestedness implies that separability in total output will never hold; the multi-product translog cost function is either separable in a different aggregate output measure, or not separable at all. This leads to the following result:

**Result 4.1.** Assume that banks have a multi-product translog cost function \( c(y, \mathbf{w}) \). Then \( c \) is not separable in total output \( \sum_{j=1}^{J} y_j \) and \( L_\ell \) in (5) is not consistently aggregated regardless of the single-output translog cost function \( c_\ell \) used for \( L_\ell \).

Result 4.1 follows directly from Result 3.1, which states that separability in total output is a necessary condition for the aggregate Lerner index to be consistently aggregated. Result 4.1 tells us that the aggregate Lerner index based on any single-output translog cost function we may come up with is not consistently aggregated.

### 4.2.2. Empirical specification: generalized Leontief

We consider a multi-product cost function similar to the non-homothetic generalized Leontief (NHT-GL) cost function of Fuss (1977). With four inputs and three outputs, the total input-factor costs of bank \( i \) in year \( t \) are given by:

\[
c_{it} = \alpha_i + \sum_{j=1}^{J} \beta_{j,w} w_{jt} + \sum_{j=1}^{J} \sum_{k=j+1}^{J} \beta_{jk,w} w_{jt}^2 w_{kt} + \frac{1}{2} \sum_{j=1}^{J} \sum_{k=j+1}^{J} \beta_{jk,w} w_{jt}^2 w_{kt}^2 + \beta_{\epsilon,c} \epsilon_{it} + \beta_{\epsilon,d} d_{it} + \epsilon_{it}. \tag{13}
\]

Here \( \alpha_i \) denotes a bank-specific effect, \( \epsilon_{it} \) a vector of control factors, \( d_{it} \) a time dummy for year \( s = 2, \ldots, T \) and \( \epsilon_{it} \) a zero-mean error term that is orthogonal to the regressors. The NHT-GL cost function is linearly homogeneous in input prices. The marginal costs corresponding to (13) are given by

\[
MC_{it} = \frac{\partial c_{it}}{\partial y_{j,t}} = \sum_{j=1}^{J} \beta_{j,k,w} w_{jt}^2 w_{kt} + \beta_{\epsilon,c} \epsilon_{it} + \beta_{\epsilon,d} d_{it} + \epsilon_{it}. \tag{14}
\]

*Aggregate cost function* To calculate the aggregate Lerner index, we also estimate the following single-output aggregate NHT-GL cost function in terms of total output or total assets \( y \):

\[
c_{it} = \alpha_i + \sum_{j=1}^{J} \beta_{j,w} w_{jt} + \sum_{j=1}^{J} \sum_{k=j+1}^{J} \beta_{jk,w} w_{jt}^2 w_{kt} + \frac{1}{2} \sum_{j=1}^{J} \sum_{k=j+1}^{J} \beta_{jk,w} w_{jt}^2 w_{kt}^2 + \beta_{\epsilon,c} \epsilon_{it} + \beta_{\epsilon,d} d_{it} + \epsilon_{it}. \tag{15}
\]

The marginal costs corresponding to (15) equal

\[
MC_{it} = \frac{\partial c_{it}}{\partial y_{j,t}} = \sum_{j=1}^{J} \beta_{j,k,w} w_{jt}^2 w_{kt} + \beta_{\epsilon,c} \epsilon_{it} + \beta_{\epsilon,d} d_{it} + \epsilon_{it}. \tag{16}
\]

### Conditions for separability in total output

The aggregate NHT-GL cost function in (15) is a special case of the multi-product NHT-GL cost function in (13). As shown in Appendix C, the necessary parameter restrictions for separability in total output are \( \beta_{j,w} = \beta_{j,m,w} = \beta_{j,oy} \) for some \( j \), \( \beta_{k,w} = \beta_{k,oy} \) for some \( k \), and \( \beta_{j,w} = \beta_{j,oy} \) for some \( j \). Under these 40 linearly independent constraints, the multi-product NHT-GL cost function in (14) reduces to the aggregate NHT-cost function in (15). We can test the parameter constraints using a Wald test.

### 4.3. Cost function estimation

All cost functions are estimated in terms of deflated level variables and by using random-effects (RE) estimation.\(^6\) The random effect \( \alpha_i \) in each specification captures bank-specific heterogeneity, including time-invariant cost inefficiencies, uncorrelated with the cost function’s explanatory variables. Any remaining time-varying cost inefficiencies are contained in the error term and do not have to be specified any further for consistent estimation. In all specifications we include, both linearly and quadratically, bank age as a control factor to allow for different cost behavior of de novo banks (due to e.g. new technologies). We also include the equity ratio as a control factor, with the interpretation of a quasi-fixed input (e.g., Mester, 1996).\(^7\) The cost functions are estimated separately for each of the four size classes.

### 5. Empirical results

Our empirical analysis starts with the estimates of the relevant Lerner indices: (i) \( L_\ell \) and \( L_\ell^* \) (the aggregate Lerner indices), (ii) \( L_{\text{TLNS}} \) and \( L_{\text{LNS}}^* \) (the weighted-average Lerner indices), (iii) \( L_{\text{TLNS}} \) (the Lerner index for total loans and leases), (iv) \( L_{\text{FISC}} \) (the Lerner index for total securities) and (v) \( L_{\text{BOBS}} \) (the Lerner index for off-balance sheet activities). We will verify whether the estimated Lerner indices pass an initial screening based on economic plausibility. Subsequently, we will turn to the empirical aggregate Lerner index, test the three consistency conditions and investigate the economic consequences of using this index anyhow even if the consistency conditions are rejected.

#### 5.1. Estimated Lerner indices

The estimated Lerner indices based on the NHT-GL cost function are reported in Table 4, while the estimated Lerner indices based on the popular translog cost function are reported in Appendix D.

**Non-negativity** Various studies have established some positive values for the estimated Lerner indices (e.g. Fonseca and González, 2010; Jiménez et al., 2013; Coccorese, 2014; Huang et al., 2017). Because prices must weakly exceed marginal costs in equilibrium under profit maximization, negative values may indicate that something is wrong. We therefore start with a negativity check on our Lerner estimates.

The figures in Table 4 make clear that the NHT-GL cost functions hardly ever produce negative estimates of the Lerner indices \( L_{\text{TLNS}} \), \( L_{\text{LNS}}^* \), \( L_{\text{TLNS}}^* \), \( L_{\text{FISC}} \) and \( L_{\text{BOBS}} \). Furthermore, for these Lerner indices the percentage of significantly positive Lerner indices is almost 100% in each size class. To save space, we will therefore only investigate the Lerner index for securities in more detail.

Because negative Lerner indices are only a potential concern if they are **significantly** negative, Table 5 provides detailed information on the sign and significance of the estimated Lerner index for

---

6 We used the All Urban Consumer Price Index for deflation; see Appendix A.

7 Estimation results based on fixed-effect estimation are similar and available upon request.
Table 4
Summary statistics (in %) for the estimated Lerner indices (NHT-GL).

<table>
<thead>
<tr>
<th>CLASS 1</th>
<th>CLASS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_{HSN}$</td>
<td>$\ell_{HS}$</td>
</tr>
<tr>
<td>mean</td>
<td>61.1</td>
</tr>
<tr>
<td>median</td>
<td>62.8</td>
</tr>
<tr>
<td>IQR</td>
<td>14.4</td>
</tr>
<tr>
<td>5% quantile</td>
<td>40.0</td>
</tr>
<tr>
<td>95% quantile</td>
<td>76.5</td>
</tr>
<tr>
<td>mean s.e.</td>
<td>2.0</td>
</tr>
<tr>
<td># bank-years</td>
<td>7207</td>
</tr>
<tr>
<td># banks</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CLASS 3</th>
<th>CLASS 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_{HSN}$</td>
<td>$\ell_{HS}$</td>
</tr>
<tr>
<td>mean</td>
<td>60.0</td>
</tr>
<tr>
<td>median</td>
<td>61.9</td>
</tr>
<tr>
<td>IQR</td>
<td>13.1</td>
</tr>
<tr>
<td>5% quantile</td>
<td>38.1</td>
</tr>
<tr>
<td>95% quantile</td>
<td>75.2</td>
</tr>
<tr>
<td>mean s.e.</td>
<td>3.4</td>
</tr>
<tr>
<td># bank-years</td>
<td>3,208</td>
</tr>
<tr>
<td># banks</td>
<td>861</td>
</tr>
</tbody>
</table>

Notes: For each size class, this table reports sample mean, median, interquartile range (IQR), 5% quantile and 95% quantile of the various estimated Lerner indices. The size classes in prices of the year 2017 are defined as: 1: less than $100 million, 2: $100–500 million, 3: $500 million–1 billion and 4: $1 billion. In each size class, we still use 90%-99% of the sample for that class to calculate the summary statistics. The rows labeled ‘mean s.e.’ report the means of the bootstrap-standard errors of the estimated Lerner indices, providing an indication of the amount of estimation uncertainty. The number of bank-year observations and included banks are reported on the basis of the amount used in the estimations (‘est.’) and in the calculation of the summary statistics (‘stat.’). To obtain the summary statistics, we leave out observations with product-specific output levels less than $100,000 (in prices of 2017) and output prices lower than 1%. We do this because the Lerner indices may become erratic for such small values.

Table 5
Significance of estimated Lerner indices for total securities (NHT-GL).

<table>
<thead>
<tr>
<th>CLASS 1</th>
<th>CLASS 2</th>
<th>CLASS 3</th>
<th>CLASS 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_{HSN}$</td>
<td>$\ell_{HS}$</td>
<td>$\ell_{QRS}$</td>
<td>$\ell_{VAR}$</td>
</tr>
<tr>
<td>L$_{HSN}$ &lt; 0</td>
<td>0.2</td>
<td>1.9</td>
<td>0.4</td>
</tr>
<tr>
<td>L$_{HSN}$ &gt; 0</td>
<td>93.2</td>
<td>88.8</td>
<td>85.4</td>
</tr>
<tr>
<td>L$_{HSN}$ &gt; 1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Notes: For each size class, this table reports the percentage of estimates of $\ell_{HSN}$ that is significantly smaller than 0, significantly larger than 0, and significantly larger than 1, respectively. To determine the significance, we have used a panel wild bootstrap with a chosen significance level of 5%. The summary statistics in this table are based on the same number of bank-year observations as reported in Table 4. Size classes in prices of the year 2017: 1: less than $100 million, 2: $100–500 million, 3: $500 million–1 billion and 4: $1 billion.

The second row of Table 5 nevertheless indicates that the percentage of significantly positive values of $\ell_{HSN}$ varies between 85–93% in the first three size classes, while this percentage is only 51.9% in the largest size class. Hence, in the largest size class a relatively large fraction of the bank-year observations has a value of $\ell_{HSN}$ that does not significantly differ from 0. As part of our sanity check, we provide an explanation for this phenomenon.

The reduced significance of $\ell_{HSN}$ in the largest size class reflects a relatively large amount of estimation uncertainty. This estimation uncertainty is also reflected by the large average standard error of $\ell_{HSN}$ in the largest size class, as reported in the row captioned ‘mean s.e.’ in Table 4. The estimation uncertainty is caused by the modest amount of bank-year observations in this size class and the relatively large amount of output dispersion causing heteroskedasticity.

A related phenomenon is the size effect in the distribution of $\ell_{HSN}$: the mean and median of $\ell_{HSN}$ are substantially lower in the largest size class than in the other size classes. This size effect is a direct consequence of the estimation uncertainty in $\ell_{HSN}$. That is, the size effect in the distribution of $\ell_{HSN}$ disappears if we calculate the sample means, medians and quantiles only over those bank-years for which $\ell_{HSN}$ differs significantly from zero.

As shown in Appendix D, the estimated Lerner indices for securities based on the translog cost function suffer from a substantial negativity issue and much larger estimation uncertainty. This is why we prefer the Lerner estimates based on the NHT-GL cost functions and have relegated the translog estimates to the appendix. In terms of the average values of the estimated Lerner indices, however, the two cost function provide quite similar results.

Prior expectations: As an additional sanity check, we verify whether the estimated values of the product-specific Lerner indices are in line with what we would expect on the basis of the literature. For banks, we expect relatively high Lerner indices due the presence of locally limited borrowers (Petersen and Rajan, 2002; Degryse and Ongena, 2004; Brevoort and Hannan, 2006: Ho and Ishii, 2011) and loan screening and monitoring activities (Ruckes, 2004), among others. Because banks are the main output category for multi-product banks, we expect $\ell_{VAR}$ to be similar to $\ell_{HSN}$.

Superficially, security markets may look highly competitive due to the lack of entry barriers and the high degree of substitutability of well-diversified portfolios. Yet the literature has shown that

---

3 The amount of estimation uncertainty in the largest size class would be even larger if the 53 banks with total output in excess of $30 billion were included. On average, these banks have a total output of $329 billion, while their average total assets equal $241 billion. Although the other results remain about the same, these huge banks act as outliers in the sample and have a negative impact on the model fit. The results reported here for the largest size class do therefore not include these largest banks.
asymmetric information between investors may result in imperfect competition (Grinblatt and Ross, 1985; Kyle, 1989; Holden and Subrahmanyam, 1992; Caballé and Krishnan, 1994; Back et al., 2000; Pasquariello, 2007). Furthermore, in the presence of price uncertainty, we expect positive Lerner indices even in competitive markets (Sandmo, 1971). Because securities are typically subject to liquidity, interest rate and default risk, we therefore expect to find positive Lerner indices for them.

The value of the Lerner index for off-balance sheet activities is more of an empirical matter, because these bank activities tend to be quite diverse across banks. Such heterogeneity suggests product differentiation, which could promote market power and positive Lerner indices. On the other hand, some off-balance sheet activities may be offered primarily as a service or convenience to customers who are already using other banking products. As such, they may sometimes be priced at or below the bank’s cost, which would tend to zero or even negative Lerner indices. In general, we expect that retail and conventional intermediation activity will lead to relatively high market power (De Guevara and Maudos, 2007). Therefore, we expect the Lerner indices for securities and off-balance sheet activities to be lower than for loans, on average.

Several recent studies have established significant economies of scale even for the largest banks and bank-holding companies (e.g., Wheelock and Wilson, 2012; Hughes and Mester, 2013; Wheelock and Wilson, 2018). In the presence of economies of scale, product-specific marginal-cost pricing would imply negative profits for the firm. In the presence of economies of scale, we may therefore expect relatively high Lerner indices for all outputs. Instead of attributing the entire margin to market power, we should realize that positive Lerner indices may simply reflect banks’ need to earn non-negative profits (Lindenberg and Ross, 1981; Elzinga and Mills, 2011; Spierdijk and Zauuras, 2018).

Returning to the estimated Lerner indices in Table 4, we observe fairly high product-specific Lerner indices for loans, securities and off-balance sheet activities, on average. Because we also establish economies of scale for most banks, the relatively high Lerner indices are consistent with our prior expectations. Table 4 shows that \( L_{SEC} \) has the lowest median value among the three product-specific Lerner indices, followed by \( L_{NS} \) and \( L_{BS} \), respectively. The relatively low value of \( L_{SEC} \) is in line with our prior hypotheses. We also confirm our initial expectation that \( L_{BS} \) is largely driven by \( L_{NS} \).

Robustness We performed various robustness checks with respect to the definition of revenue and recalculated the Lerner indices for loans and securities in each case. For loans, we included gains on the sales of loans (a form of non-intermediation income) in the revenue as a robustness check. For securities, we considered two alternative definitions of revenue. First, we excluded realized trading gains (a form of non-intermediation income) from the revenue. Second, we calculated the securities revenue as the sum of securities income, realized trading gains and unrealized holding gains on available-for-sale securities. These changes in the definitions of the revenue for loans and securities did not substantially alter the results. We also varied the definition of inputs and outputs and estimated three alternative cost functions. First, we estimated a two-output cost function in line with Koetter et al. (2012), thus omitting off-balance sheet items and only including loans and securities. Second, we estimated a two-output cost function by excluding securities and only including loans and off-balance sheet items. Third, we estimated a three-input cost function with a single input factor for borrowed funds, similar to Koetter et al. (2012). In the first two cases, we obtained similar Lerner estimates for the remaining output factors as in the three-output case. In the third case, we found similar estimates as in the four-input case. Lastly, we changed the stratification based on size classes by using sample quartiles to form size classes. Also this change did not substantially alter the results.\(^\text{10}\)

5.2. Consistency conditions

Now that the Lerner estimates have passed our initial screening, we turn to the empirical aggregate Lerner index \( L_N^2 \) and the conditions that are required for its consistent aggregation. Statistical tests Table 6 reports the outcomes of the Wald test for separability in total output, applied to the multi-product NHT-GL cost function for each size class. The Wald-tests are based on cluster-robust covariance matrices, making the test robust to heteroskedasticity and serial correlation in the cost functions’ error terms. The adjusted \( R^2 \) for each estimated NHT-GL cost function is also reported in Table 6.\(^\text{11}\) For each size class and at each reasonable significance level, the necessary parameter restrictions for separability in total output are rejected. This finding is in line with our prior expectation based on the existing literature that separability is an “extremely restrictive” requirement.

As explained in Section 3.2, conditions [EQ1] and [EQ2] do not hold in the intermediate model of banking. This can be statistically illustrated by running paired-samples Wilcoxon signed-rank tests. More precisely, to statistically test condition [EQ1] (condition [EQ2]) in a non-parametric way, we use the Wilcoxon test to assess whether the distribution of the difference between total output and total assets (between total revenue and INI income) has median 0. If we reject the null hypothesis of a zero median, we reject the statistical equivalence of the two series. For all size classes, both paired-samples Wilcoxon signed-rank tests reject the null hypothesis that the distribution of the difference has median 0 at each reasonable significance level. Hence, conditions [EQ1] and [EQ1] are both statistically rejected for all size classes.

The non-equivalence between total assets and total output is further illustrated by the summary statistics for the ratio of total output to total assets in Table 3, which ranges on average between 90%–155%. The second largest size class has an average value below 100%, which shows that the effect of including fixed assets

\(^\text{9}\) For a multi-product bank profit under marginal-cost pricing equals \( \pi = c(\sum_{t=1}^{k} e_t - 1) \), where \( e_t \) denotes the cost elasticity with respect to the \( t \)-th output. Hence, profits are negative for \( \sum_{t=1}^{k} e_t < 1 \).

\(^\text{10}\) Measurement error in e.g. input prices is another source of potential misspecification. We leave it as a topic for future research to analyze the extent to which such errors are present in banking data and affect the estimation results.

\(^\text{11}\) The complete estimation results for the multi-product and aggregate cost functions consist of many coefficients and are not reported to save space. They are available upon request.
in total assets outweighs the effect of not including off-balance sheet activities, on average. The largest size class has the largest average value, illustrating the more substantial output share of off-balance sheet activities for the largest banks. Also the dispersion in the ratio of total output to total assets is relatively large in this size class. Similarly, the non-equivalence between the sum of the product-specific revenues and INI income is illustrated by the summary statistics for the ratio of the latter two variables. This ratio is around 95% on average, showing that INI income exceeds the sum of the product-specific revenue on average due to the included service fees on deposits.

On the basis of Result 3.2, we conclude that the empirical aggregate Lerner index \( L^*_A \) is not consistently aggregated; conditions [SEP], [EQ1] and [EQ2] are rejected. Because of Result 3.1, we conclude that also the ‘theoretical’ aggregate Lerner index \( L_A \) is not consistently aggregated. This is important to know since \( L_A \) can be viewed as a ‘repaired’ version of \( L^*_A \) that is subject to the separability condition only. Furthermore, the rejection of consistency condition [EQ2] implies that the empirical weighted-average Lerner index \( L^*_WA \) is not consistently aggregated either. This is also important to know, because the latter Lerner index has been recently used in the empirical banking literature; see the lower panel of Table 1.

Discrepancies between \( L^*_A \) and \( L_B \). Because \( L^*_A \) is not consistently aggregated for our sample of banks, Result 3.2 tells us that \( L^*_A \neq L_B \). We summarize the sample distribution of \( (L^*_A - L_B)/L_B \) in Table 7 to illustrate the discrepancies between \( L^*_A \) and \( L_B \) caused by the inconsistent aggregation of the former index. We observe that the value of the relative difference is between 6–13% on average. The 95% sample quantiles indicate that more extreme differences may also occur; in the largest size class the 95% quantile of the relative difference equals 38.5%.

Fig. 1 visualizes the unsigned relative differences between \( L^*_A \) and \( L_B \) by showing a histogram of \((L^*_A - L_B)/L_B \) for banks in the largest class. We observe that \( L^*_A \) tends to be larger than \( L_B \) for most observations in this sample.

In Appendix D, we show that the discrepancies between \( L^*_A \) and \( L_B \) are mostly due to the rejected consistency conditions [SEP] and [EQ1] rather than the rejected condition [EQ2].

Economic implications \( L_B \) serves as a natural benchmark for \( L^*_A \), because it represents the value \( L^*_A \) would have under consistent aggregation. We therefore study the potential economic implications of using \( L^*_A \) instead of \( L_B \). Fig. 2 displays \( L^*_A \) and \( L_B \) during the 2011–2017 period for one particular bank. For this bank, the difference between the two Lerner indices is substantial. Apart from the difference in value between \( L^*_A \) and \( L_B \), this figure also shows that the two Lerner indices do not always agree on the direction of trend in the price-cost margin over the years. Fig. 3 compares the Lerner indices \( L^*_A \) and \( L_B \) of the same bank (‘bank 1’) to those of another bank (‘bank 2’) from the same size class. On the basis of the inconsistently aggregated \( L^*_A \), we would conclude that bank 1 is more successful in raising prices above marginal costs, while
we tend to conclude the opposite on the basis of the consistently aggregated $L_{\text{WA}}$.

Robustness As mentioned before, our empirical application to U.S. banks is based on specific choices regarding inputs, outputs and the functional form of the cost function. We have already shown that the estimated Lerner values are robust to the specification changes mentioned in Section 5.1. Also the results regarding the rejection of the consistency conditions turn out robust to these changes. Qualitatively speaking, this also holds for the possible economic implications of the rejected consistency conditions.

6. Conclusions

The aggregate Lerner index is widely used as a measure of banks’ market power in the empirical banking literature, on total assets as the single aggregate output factor. We have shown that the consistent aggregation of the aggregate Lerner index depends on
three consistency conditions. If these conditions hold, the aggregate Lerner index reduces to a weighted-average of the product-specific Lerner indices. Otherwise, the aggregate Lerner index is not consistently aggregated and may lead to incorrect economic conclusions.

Although one of the above conditions has been described as “extremely restrictive” in the literature, whether the three conditions hold for a given sample is ultimately an empirical matter. We have therefore provided an empirical application to U.S. multi-product banks observed during the 2011–2017 period. Our empirical strategy to test for consistent aggregation has shown that all three conditions are statistically rejected. Consequently, the aggregate Lerner index is not consistently aggregated for this sample. We have shown that the inconsistent aggregation may cause economically relevant distortions to the aggregate Lerner index, depending on the economic context.

Our analysis raises the question why the aggregate index should be used in the first place. The user of this index should at least verify whether it is consistently aggregated for the particular sample at hand, which requires the calculation of the components of the always consistently aggregated weighted-average Lerner index.

An additional complication is that some cost functions (such as the translog) are never separable in total output. Based on such a cost function, the aggregate Lerner index is a priori known to be inconsistently aggregated. Although such cost functions could still provide a good fit to the data, they are unsuitable for the purpose of estimating the aggregate Lerner index.

How to go from here? The most efficient strategy is to omit the aggregate Lerner index altogether and to work with the always consistently aggregated weighted-average Lerner index whenever a composite Lerner index is required. Because the weighted-average Lerner index is consistently aggregated regardless of the underlying cost function, it can simply be based on a cost function that fits the data well without concerns about this function’s separability properties. We recommend using the product-specific Lerner indices if there is no particular need for a composite Lerner index, because the former indices are by definition more informative about multi-product banks’ market power than a composite index.

Inconsistent aggregation could also occur if single-output non-Lerner measures of market power are used under the assumption that total assets (or another measure of aggregate output) is the single aggregate output factor. Similar issues may arise if a summary measure of banks’ market power in different product markets is created. In such cases, we recommend to extend the concept of a consistently aggregated Lerner index to the chosen measure of market power and to verify whether the summary measure is consistently aggregated. Because the details of such an extension are highly case specific, we leave this as a topic for future research.

Supplementary material

CRediT authorship contribution statement
Sherrill Shaffer: Conceptualization, Methodology, Software, Formal analysis, Data curation, Visualization, Writing - original draft.
Laura Spierdijk: Conceptualization, Methodology, Writing - review & editing.

References