Topology Optimization with Additive Manufacturing Constraints

Grzegorz S. Misiun

TOPOLOGY OPTIMIZATION WITH ADDITIVE MANUFACTURING CONSTRAINTS

Grzegorz S. Misiun

This dissertation has been approved by:

supervisor

prof. dr. ir. A.H. van den Boogaard

supervisor

dr. ir. H.J.M. Geijselaers

Printed by: Gildeprint

ISBN: 978-90-365-5149-6

DOI: 10.3990/1.9789036551496

This research was carried out under project number S12.7.14549b in the framework of the Metals for Additive Manufacturing Partnership Program of the Materials innovation institute M2i (www.m2i.nl) and the Netherlands Organization for Scientific Research (www.nwo.nl).

Copyright ©2020 by G.S. Misiun, Enschede, The Netherlands.

TOPOLOGY OPTIMIZATION WITH ADDITIVE MANUFACTURING CONSTRAINTS

DISSERTATION

to obtain
the degree of doctor at the Universiteit Twente,
on the authority of the rector magnificus,
prof. dr. ir. A. Veldkamp,
on account of the decision of the Doctorate Board
to be publicly defended
on Friday 26 March 2021 at 16:45

by

Grzegorz Stefan Misiun

born on 19th December 1988
in Gdynia, Poland

Graduation Committee:

Chairman / secretary	prof.dr.ir. H.F.J.M. Koopman
Supervisors	prof.dr.ir. A.H. van den Boogaard dr.ir. H.J.M. Geijselaers
Committee Members	prof.dr.ir. A. van Keulen prof.dr.ir. M. Schevenels prof.dr. I. Gibson prof.dr.ir. G.R.B.E. Römer

Summary

Additive manufacturing (AM) is a new production technology that in recent years has gained much attention both from the scientific world and from industry. It is a method of building three-dimensional objects by depositing layer upon layer of material in an incremental manner. Its biggest advantage, when compared with other conventional production methods, is virtually unlimited freedom of design and the fact that AM is often suitable for metals that are hard to produce by other means.

Various metal AM technologies can be used, depending on the form of an input material and the energy source. This work focuses on the powder bed system, selective laser melting (SLM). In SLM a part is built from thin powder layers that are selectively melted by means of a laser beam. The SLM process creates parts in a range of centimetres for which several thousands of layers are needed and the printing process may take up to several dozens of hours.

A problem related to metal AM is frequently occurring production failures due to shape distortions that happen during the production. This is caused by thermal shrinkage of the consecutive material layers, an inherent property of AM. It leads to accumulation of residual stresses and, in consequence, distortions. Since in the event of failure the costly process has to be repeated, it is beneficial to support the design stage with numerical models and to predict possible distortions in advance.

The problem connected with numerical modelling of SLM is that the length-scales are small, the layer thickness and laser spot are in the range of a few dozen micrometres and it takes a long time to produce a single part. It is therefore impossible to perform simulation of SLM on a part scale with commonly known numerical techniques, for example from welding simulations. Therefore, the first goal of this doctoral research is to develop a model that predicts distortions within a reasonable time.

In this thesis the proposed numerical model combines some existing techniques and supplements them with new approaches. It accounts for all steps of the simulations process, starting from mesh creation, and it includes all production stages: printing, heat treatment and release. Moreover, a special element definition is proposed to model support structures.

The second topic investigated in this research regards topology optimization (TO). TO is a structural optimization method that searches for the optimal material distribution for a given part application with some assumed restrictions. It often results in organic geometries that are hard to produce with conventional methods. Therefore, additive manufacturing and topology optimization seem like a perfect match. The only problem that still remains is the distortions, which can be especially severe for the complex geometries. Hence, the idea is to incorporate numerical modelling of

additive manufacturing into a TO loop and to set constraints on distortions.

In this thesis a method is presented to couple additive manufacturing simulations with topology optimization. Two possible causes of failure of powder bed additive manufacturing are accounted for: recoater collision and global distortion of the product. Both are calculated by simulation of the build process and their limits are applied as constraints to a topological optimization based ‘Solid Isotropic Material with Penalization’ method. An adjoint method is used to derive the sensitivities of the additive manufacturing constraints. Two optimization approaches are investigated. In the first the optimizer alternates a part shape in such a way that distortions are compensated. The second approach assumes unalterable shape of a part and it tries to limit the distortions solely by proper placement of auxiliary structures, supports. The methods are applied to the optimization of brackets, both in 2D and 3D and, additionally, supports distribution is optimized for a use case geometry.

In every case the optimization found a solution that maximizes the part performance and preserves constraints. In the case of part shape optimization the obtained designs show features that are aimed at facilitating the printing process rather than improving the load carrying function. These features resemble supports. Optimization of supports results in distributions that are not easy to interpret. It indicates that it may be hard to effectively choose supports location based only on intuition and experience. Moreover, it has been proven that it is possible to partially reduce distortion of a part by tailoring the supports distribution.

Samenvatting

Additive Manufacturing (AM) is een nieuwe productietechnologie die de afgelopen jaren veel aandacht heeft gekregen van zowel de wetenschappelijke wereld als de industrie. Het is een methode om driedimensionale objecten te bouwen door laag-op-laag, op incrementele wijze, materiaal te deponeren. Het grootste voordeel ten opzichte van andere conventionele productiemethoden is de vrijwel onbeperkte ontwerpvrijheid en het feit dat AM vaak geschikt is voor metalen die met andere methoden moeilijk te vervaardigen zijn.

Er zijn verschillende metaal AM technologieën, afhankelijk van de vorm van het invoermateriaal en van de energiebron. Dit werk richt zich op het poederbedsysteem selective laser melting (SLM). Bij SLM wordt een onderdeel opgebouwd uit dunne poederlagen die met behulp van een laser selectief worden gesmolten. Bij het SLM-proces worden onderdelen vervaardigd in een bereik van centimeters, waarvoor enkele duizenden lagen nodig zijn. Het printproces kan tot enkele tientallen uren duren.

Een probleem bij metaal AM zijn de vaak voorkomende productiefouten als gevolg van vervormingen tijdens de productie. Deze worden veroorzaakt door de thermische krimp van de opeenvolgende materiaallagen. Dit is een inherente eigenschap van AM, leidt tot een accumulatie van restspanningen en aldus tot vervormingen. Omdat het kostbare proces in het geval van storingen moet worden herhaald, is het wenselijk om de ontwerpfase te ondersteunen met numerieke modellen om mogelijke vervormingen vooraf te voorspellen.

Het probleem met de numerieke modellering van SLM is dat de lengteschalen klein zijn, dat de laagdikte en de grootte van de laservlek in het bereik van enkele tientallen micrometers liggen en dat de productietijd van één onderdeel lang is. Daarom is het onmogelijk om SLM te simuleren op een deelschaal met algemeen bekende numerieke technieken, bijvoorbeeld uit lassimulaties. Daarom bestaat het eerste doel van dit doctoraatsonderzoek erin een model te ontwikkelen dat vervormingen binnen een redelijke tijd voorspelt.

In dit proefschrift combineert het voorgestelde numerieke model enkele van de reeds bekende technieken en worden deze uitgebreid met nieuwe benaderingen. Het omvat alle stappen van het simulatieproces, te beginnen bij het maken van eindige elementen net, en het omvat alle productiestadia: het printen, de warmtebehandeling en lossnijden van de basisplaat. Bovendien wordt een speciale elementdefinitie voorgesteld om ondersteunende structuren te modelleren.

Het tweede onderwerp dat in dit onderzoek wordt onderzocht heeft betrekking op Topologie-optimalisatie (TO). TO is een structurele optimalisatiemethode die op zoek gaat naar de optimale materiaalverdeling voor een bepaalde onderdeelapplicatie met

enkele veronderstelde beperkingen. Het resulteert vaak in organische geometrieën die moeilijk te vervaardigen zijn met behulp van conventionele methoden. Daarom lijken Additive Manufacturing en Topologie-optimalisatie perfect op elkaar afgestemd. Het enige probleem dat nog overblijft zijn de vervormingen, die met name voor de complexe geometrieën aanmerkelijk kunnen zijn. Daarom is het idee om de numerieke modellering van additive manufacturing in een TO-lus op te nemen en beperkingen op vervormingen in te stellen.

In dit proefschrift wordt een methode gepresenteerd om Additive Manufacturing simulaties te koppelen aan Topologie-optimalisatie. Er wordt rekening gehouden met twee mogelijke oorzaken voor het falen van de vervaardiging van poederbed additieven: een botsing met een recoater en een globale vervorming van het product. Beide worden berekend door simulatie van het bouwproces en hun limieten worden toegepast als beperkingen voor een "Solid Isotropic Material with Penalization" (SIMP)-methode, gebaseerd op topologische optimalisatie. Er wordt een aanvullende methode gehanteerd om de gevoeligheden van de beperkingen van de additieve productie af te leiden. Er worden twee optimalisatiemethoden onderzocht. In de eerste genereert de optimalisator een onderdeelvorm zodanig af dat de vervormingen worden gecompenseerd. De tweede benadering gaat uit van een onveranderlijke vorm van een onderdeel en probeert de vervormingen alleen te beperken door de juiste plaatsing van hulpconstructies en ondersteuning. De methoden worden toegepast op de optimalisatie van een draagconstructie, zowel in 2D als in 3D, en de distributie van de ondersteuning wordt bovendien geoptimaliseerd voor een gebruikssituatie-geometrie.

In alle gevallen heeft de optimalisatie een oplossing gevonden die de prestaties van het onderdeel maximaliseert en die de beperkingen in stand houdt. In het geval van optimalisatie van de onderdeelvorm tonen de verkregen ontwerpen kenmerken die gericht zijn op het vergemakkelijken van het printproces in plaats van het verbeteren van de lastdragende functie. Deze functies gelijken op ondersteuning. Optimalisatie van de ondersteuning resulteert in distributies die niet gemakkelijk te interpreteren zijn. Het geeft aan dat het moeilijk kan zijn om de locatie van de ondersteuning effectief te kiezen op basis van intuïtie en ervaring alleen. Bovendien is het bewezen dat het mogelijk is om de vervorming van een onderdeel te verminderen door de distributie van ondersteuning op maat te maken. Dit is evenwel slechts tot op zekere hoogte mogelijk.

Contents

- 1 Introduction 1**
  - 1.1 Selective laser melting 2
  - 1.2 Numerical modelling of additive manufacturing 4
  - 1.3 Topology optimization for additive manufacturing 6
  - 1.4 Research objective 9
  - 1.5 Outline of the thesis 10
- 2 Basics 11**
  - 2.1 Continuum mechanics 11
    - 2.1.1 Kinematics and strains 11
    - 2.1.2 Stresses and equilibrium 13
    - 2.1.3 Thermodynamics 14
  - 2.2 Finite element method 14
    - 2.2.1 Strong and weak form 15
    - 2.2.2 Spatial discretization 15
    - 2.2.3 Time integration 17
  - 2.3 Constitutive models 18
    - 2.3.1 Plasticity 18
    - 2.3.2 Viscoplasticity 22
  - 2.4 Summary 24
- 3 Numerical Modelling of SLM 25**
  - 3.1 Finite element modelling of additive manufacturing 25
    - 3.1.1 Meshing script 26
    - 3.1.2 Inherent strains 27
    - 3.1.3 Element deposition 28
  - 3.2 Finite element analysis of additive manufacturing 31
    - 3.2.1 Simulation of SLM by finite element method 31
    - 3.2.2 Support elements 33
    - 3.2.3 Distortions after release from the base plate 34
    - 3.2.4 Simulation without supports 37
    - 3.2.5 Stress relaxation 38
  - 3.3 Summary 38
- 4 Numerical Verification of SLM Model 41**
  - 4.1 Lumped approach 41
    - 4.1.1 Bending bar 41

4.1.2	3D use case	45
4.1.3	Conclusions	49
4.2	Element growth method	49
4.2.1	Bending bar	49
4.2.2	3D use case	50
4.2.3	Conclusions	51
4.3	Support elements	51
4.3.1	Bending bar	52
4.3.2	3D use case	53
4.4	Stress relaxation	54
4.4.1	Bending bar	54
4.4.2	3D use case	55
4.5	Summary	56
5	Topology Optimization and Adjoint Sensitivities	57
5.1	Topology optimization	57
5.1.1	Shape optimization with distortion constraint	59
5.1.2	Supports distribution optimization	63
5.2	Sensitivity analysis	65
5.2.1	Sensitivities for shape optimization	65
5.2.2	Sensitivities for support distribution	68
5.3	Summary	71
6	Topology Optimization of Additive Manufacturing with Printing Constraints	73
6.1	2D examples	73
6.1.1	Recoater collision	74
6.1.2	Loaded node distortions	77
6.1.3	Recoater collision and loaded node distortion combined	81
6.2	3D examples	85
6.2.1	Recoater collision	87
6.2.2	Distortion constraint	88
6.2.3	Distortion and recoater collision constraints combined	89
6.3	Summary	90
7	Topology Optimization of Support Structures	93
7.1	2D examples	95
7.1.1	Part distortions	95
7.1.2	Supports distribution vs. part shape optimization	100
7.1.3	Mesh size dependence	104
7.2	3D examples	106
7.2.1	Part distortions	106
7.2.2	Supports distribution vs. part shape optimization	110
7.3	3D use case	114
7.4	Summary	122
8	Conclusions and Recommendations	125
8.1	Conclusions	125

8.1.1	Numerical modelling of selective laser melting	125
8.1.2	Topology optimization with additive manufacturing constraints	126
8.2	Further research recommendations	127
A		129
A.1	Derivation of tangent stiffness matrix for plasticity	129
A.2	Derivation of tangent stiffness matrix for viscoplasticity	131

Abbreviations and Symbols

Abbreviations

AM	additive manufacturing
CAD	computer aided design
FE	finite element
FEM	finite element method
PI	producibility index
PNG	portable network graphics
SLM	selective laser melting
SIMP	Solid Isotropic Method with Penalization
STL	stereolithography
SVG	scalable vector graphics
TO	topology optimization

Scalars

A	creep material constant
c	compliance objective function
d	displacement of a node
E	Young's modulus
f	material fraction
f_c	prescribed load
g	constraint function
G	shear modulus
H	linear hardening coefficient
K	bulk modulus
m	creep material constant
n_{row}	number of rows
N	interpolation function
N	number of
Q	activation energy
p_s	SIMP penalization exponent
p	p-norm parameter
r	scaling parameter
r	heat source
R	Boltzmann constant

S	surface
t	layer thickness
t	time
T	temperature
u	internal energy
V	volume
ε	lower density threshold
ζ	local coordinate
η	local coordinate
λ	consistency parameter
ν	Poisson's ratio
ξ	local coordinate
ρ	density
ρ_p	relative metal powder density
σ	stress
Φ	von Mises yield criterion

Tensors

$\mathbf{A}$	arbitrary matrix
$\mathbf{b}$	arbitrary vector
$\mathbf{B}$	gradient operator
$\mathbf{c}$	arbitrary vector
$\mathbf{C}$	Cauchy-Green tensor
$\mathbf{d}$	node displacement vector
$\mathbf{D}$	rate of deformation tensor
$\mathbf{D}$	fourth order material tensor
$\mathbf{E}$	Green-Lagrange tensor
$\mathbf{f}$	force vector
$\mathbf{F}$	deformation tensor
$\mathbf{F}$	force vector
$\mathbf{h}$	interpolated variables vector
$\mathbf{K}$	stiffness matrix
$\mathbf{L}$	velocity gradient
$\mathbf{L}$	interpolation matrix
$\mathbf{n}$	normal vector
$\mathbf{q}$	heat flux vector
$\mathbf{r}$	reaction force vector
$\mathbf{R}$	residual force vector
$\mathbf{s}$	deviatoric stress tensor
$\mathbf{S}$	selection matrix
$\mathbf{t}$	surface traction vector
$\mathbf{u}$	displacement vector
$\mathbf{w}$	vertical components of displacement vector
$\mathbf{v}$	velocity vector
$\mathbf{x}$	spatial coordinate vector
$\mathbf{X}$	referential coordinates

$\mathbf{Z}$	transformation matrix
α	thermal expansion tensor
ε	true strain
κ	Lagrange multiplier vector
λ	Lagrange multiplier vector
μ	Lagrange multiplier vector
ρ	design variables
φ	nodal densities
$\mathbf{1}$	second order identity tensor
$\mathbb{I}$	fourth order projection tensor

Sub- and superscripts

cr	creep
d	distortion
dev	deviatoric
e	elastic
e	elements
eq	equivalent
f	unsuppressed degrees of freedom
g	constraints
i	interface degrees of freedom
i	layer number
inh	inherent
n	step number
max	maximal
M	final building step
$M + 1$	release step
M	modified elastic material matrix
p	plastic
p	part degrees of freedom
r	remaining degrees of freedom
s	support degrees of freedom
sym	symmetric
t	trial
t	surface nodes
y	yield
v	vertical fraction
vm	von Mises
vol	volumetric
w	wiper blade collision
0	initial
0	base plate degrees of freedom

Chapter 1

Introduction

Before additive manufacturing, the situation in production technology of metals seemed to be stable. Most of the existing production methods, such as forging, casting, welding or machining, were well developed and any novelties were touch-ups rather than game changers. What could be done with metal prefabricates was settled and no one expected any serious breakthroughs. That is why additive manufacturing—the production method that could fill the production gap of customized small-series parts, which are often made from hard-to-produce materials—quickly caught the attention of industry [1]. But to fully master the method and to make it applicable on an industrial scale many problems still have to be tackled, which is an interesting challenge for the scientific community [2].

Additive manufacturing (AM) or 3D printing is a technology that at the beginning was known as *rapid prototyping*. The main goal of rapid prototyping was to create in a short time a visual representation of a product, a prototype, that could be shown to managers or clients in the early phase of product development [3]. The main advantage of rapid prototyping was the ease with which a part could quickly be produced directly from its digital version, for example in a CAD file. But as the development of the process proceeded, the product quality improved and the term *rapid prototyping* became obsolete. It was therefore decided to change it to *additive manufacturing*, which grasps the nature of the process that is so different from conventional subtractive production methods or forming processes [4]. An unquestionable advance in the development of additive manufacturing technologies was the utilization of metal as source material. Setting foot in the metal industry opened new possibilities for various engineering fields and commanded the attention of big companies [5].

Without doubt, one of the advantages of additive manufacturing that distinguishes it from other production methods is its almost unlimited freedom of design and the fact that the cost of the part does not increase as the part complexity increases [6]. At the outset, most of the metal additive manufacturing technologies used material in a form of a powder and a laser source as the energy source to melt it [7]. But across the years many different systems have been developed. In general, metal additive manufacturing systems may be classified into four different categories [8]:

- Powder bed fusion, where material in the form of a powder bed is selectively melted layer-by-layer by using a focused energy source, e.g. a laser or electron

beam.

- Direct energy deposition, where a molten pool is created by using an energy source, either a laser beam, electron beam or welding arc, and the material is directly fed to the meltpool as a stream of a blown powder or a wire.
- Binder jetting, where binder is deposited on a metal powder, then the binder is cured and the powder is joined, for example by sintering.
- Sheet lamination, where stacks of 2D metal sheets are cut and joined in order to create a 3D part. Methods used for joining may be for example laser or resistance welding.

Additive manufacturing technology is still under development and many new approaches, as well as improvements of the existing technologies are constantly being proposed [9]. Recent ones include for example multi-laser systems [10], where more than one laser source is used to enhance productivity, multi-material systems [11], which combine more than one material to enhance the product functionality and implementation of in-situ process monitoring to improve the quality of a product [12]. As mentioned, metal additive manufacturing is a relatively new production method and still has many problems that have to be dealt with. The most common problems are part distortions leading to out-of-spec dimensions, cracking due to the accumulation of residual stresses, local overheating due to the insufficient heat conduction and improper bonding of the material due to wrongly chosen process parameters. Limited control over the process leads to production failures and unnecessary costs. To prevent these problems it is crucial to fully understand the phenomena that cause such behaviour [13].

1.1 Selective laser melting

This work focuses on the powder bed fusion production method that has gained much attention from industry, selective laser melting (SLM). In SLM a part is built from material in the form of metallic powder, in a layer-by-layer manner starting from the base plate (Figure 1.1). The process takes place in a building chamber in the presence of inert gas, either argon or nitrogen. Every building increment starts by lowering the base plate by one layer thickness, i.e. 30–100 μm . Next, a recoater, also known as a roller or a wiper blade, sweeps over the building area. It deposits a layer of powder of the right thickness by removing surplus particles above the threshold height. After that a laser beam, steered by mirrors, selectively scans the area where the solid part should be present, following the path from the machine code, known as G-code. After the scanning has finished the whole process repeats until the full part shape is obtained. When the printing is done, the final step is to remove the auxiliary structures, supports, and to cut the part off the base plate [14].

When the scanning laser beam passes over a given material point, the powder is quickly heated up to the point where it melts and sometimes even evaporates [15]. Next, since the laser has already moved on, the material starts to cool down. It solidifies and then starts to shrink because of the temperature decrease. Due to the local character of the laser beam action, only a few layers at the top are affected,

Figure 1.1: Schematic of the SLM process.

which manifests itself in substantial temperature gradients. The temperature of the layers below is much lower than that of the top layers. This temperature difference leads to differences in shrinkage. As the top layers cool down, they squeeze the layers below. Since all of them are already in the solid state, equilibrium is formed at cool down with the presence of internal stresses in the structure. These residual stresses are unwanted and often lead to shape distortions and production failures [16]. The most common problems connected with occurrence of residual stresses are cracking, out-of-specification dimensions and wiper blade collision due to excessive distortions in the positive printing direction.

There are several methods that deal with SLM-related problems: they range from the product design phase up to post-processing [17]. At the design stage such parameters as minimum feature size, product tolerances and possible ways of material removal have to be taken in account. During the manufacturing planning the product orientation or its placement on the base plate is important. By choosing the right orientation of the product it is possible to minimize the number of overhangs, prevent the part from overheating and lower distortions [18].

Overhangs are regions that are unprintable due to the insufficient amount of material underneath, leading to lack of bonding and problems with heat conduction [19]. The measure connected with overhangs is the overhang angle which limits printable regions. The value of the allowable overhang angle depends on the production process and material that is used. At this stage support structures are often proposed as well. Supports are auxiliary geometrical features that are usually slender pillars or lattice structures that are used to connect a part member with the base plate or with a member that is immediately underneath it. Supports allow for printing of the aforementioned overhangs. A second role of supports is to prevent overheating. The powder itself in SLM is an insufficient conductor, and heat may accumulate in the part instead of being conducted to the base plate, leading to wrongly bonded material, undesired microstructure or even material evaporation. The third application of supports is prevention of part distortions, caused by residual stresses. The supports may be applied to counter distortion by adding additional stiffness to the structure, but at the cost of increasing the amount of used material and problematic removal in

the post-processing stage [20].

In the post-processing phase the part is separated from the base plate and the supports are removed. Before cutting off, heat treatment may be applied. The common approach is to perform Hot Isostatic Pressing with annealing. Heat treatment is done to relieve the accumulated residual stress and to increase microstructure homogeneity [21].

An important method that may be used in order to prevent some of the aforementioned failures regards the part design. In theory, if it would be possible to predict the outcome of additive manufacturing of a given shape, it should also be possible to modify it in such a way that it achieves the desired behaviour after printing. Hence, by tailoring the part shape and accounting for its production constraints during the design stage a part would fulfil its function while ensuring its manufacturability.

1.2 Numerical modelling of additive manufacturing

Supporting the design process with numerical simulations is a commonly adopted approach [22]. Simulations allow for virtual verification of the design and its assumptions. If the total cost of a product is high, a lot of money may be saved by performing such calculations instead of a trial-and-error approach. But to do so and to get reliable results, a sufficient amount of accurate data has to be provided and there is always a trade-off between the accuracy and the calculation time.

Numerical methods for simulation of additive manufacturing originate from welding simulations. This is due to the obvious resemblance between the processes [23–26]. In both a moving heat source is present, usually in the form of a Goldak model [27]. Moreover, as the process proceeds, new material is added which can be simulated in a finite element (FE) setting by element deposition techniques. Michaleris [28] compared two of the available element deposition methods: ‘inactive elements’ and ‘quiet elements’. In the inactive element technique elements are absent from the calculation until their virtual deposition has taken place. The quiet element technique, on the other hand, has all the elements active from the start but elements representing part of the domain that has not yet been deposited have their properties scaled down hence the name ‘quiet’. The solution stage for the quiet elements method will take more time especially at the beginning due to the bigger matrix size consisting of scaled down, unnecessary elements. In the inactive elements technique the solver initialization and equation renumbering needs to be done every time that new elements are introduced which also affects computational time. To handle the aforementioned issues Michaleris proposed a new hybrid approach that combines advantages of both methods, ultimately resulting in reduced computational time [28]. Two other widely used methods are application of one way coupled thermo-mechanical simulation and disregard of the behaviour of the melt pool. In a one way coupled thermo-mechanical simulation temperatures obtained from a thermal model are used to generate loads in a mechanical simulation [29–31]. The behaviour of the pool of molten metal is often ignored and only a change in material properties is applied to imitate transition from powder to solid [32–34].

The heat source travels much faster in SLM than in welding, and the ratio between the total build volume and the heat source size is much bigger. This mismatch of the length scales gives rise to a very fine spatial discretization of a large domain

which leads to a high number of degrees of freedom in an FE setting. Therefore AM simulations are computationally very demanding [28, 35]. The development of numerical models that are capable of predicting the temperature evolution and the associated distortions in a computationally efficient way has been a topic of ongoing research [36].

Initial attempts to predict the distortions during additive manufacturing can be found in [37, 38], where the commercial FE software package Abaqus is used to simulate deposition of several material layers in a simplified 1D-thermal and 2D-mechanical model. Matsumoto *et al.* [39] used their in-house software to simulate deposition of a single layer. Simulations performed in 3D can be found in [40, 41], where the hatching of a single layer was simulated by means of a 3D model based on Abaqus and Ansys software. However, non-standard techniques must be applied to perform simulations on a part scale.

An interesting technique was developed for the 3DSIM package which is now a part of Ansys [42–44]. Within this framework a method for fast mesh refinement has been proposed that allows for simulation speed-up by having a detailed mesh that follows the heat source. Another innovative approach is the work of Hodge *et al.* [45]. Here not only material that is going to be solidified is simulated but also the domain that is filled with powder is considered. As the heat source travels it changes properties of the heated material from powder to solid. Although this is in line with the real metal 3D printing process and is certainly insightful on a small scale, it still seems impractical for application to part-scale simulations due to the enormous mesh size that would be required.

Furthermore, a multiscale approach is often used [46, 47]. It reduces computational demands by performing simulations with different levels of complexity on different scales. Results from lower scales are used as boundary conditions in the scale above [48–52]. Usually on a microscale a single line scan is simulated, while the mesoscale applies to a hatch, i.e. an area covered by several vectors, and the macroscale addresses a group of layers or even a whole part.

Besides this approach, two additional modelling techniques are frequently implemented: ‘inherent strains’ and ‘layer lumping’. When inherent strains are prescribed, the simulation of the thermal history is fully replaced by its mechanical equivalent of shrinkage. This means that the thermal shrinkage that would occur during the cooling down is calculated in advance and is prescribed as mechanical strain, or the associated stress. Usually this is applied to a whole layer at once. Complex thermal calculations on the part scale are then superfluous and information from lower-scale simulations or experiments can be used [29, 51, 53–55]. The idea behind layer lumping is to reduce the size of the FE mesh and the number of the increments by applying an element size that is bigger than a single layer thickness. In this way, in a simulation, several layers of material are deposited simultaneously, which substantially reduces the computational burden [51–53, 56–58].

Another approach that focuses on mesh refinement strategies is an adaptative mesh coarsening proposed by Hajjalizadeh *et al.* [59]. It was implemented into Abaqus as a Python routine that allows for reduction of computational time by a factor 3 while still capturing the dominant stress distribution. Baiges *et al.* [60] recently published an article describing an adaptive octree mesh refinement strategy that was developed with stress and force correction terms that accounted for mesh equilibrium before and

after the refinement. The method was verified for tetragonal and hexahedral meshes for both purely academical and industrial examples and showed high accuracy and scalability.

At this moment many of the aforementioned methods for 3D printing simulations have been implemented and are available as a part of commercial FEM packages. Prediction of global distortion in a reasonable time frame has been made possible, although the accuracy of the results is still an issue especially for complex shapes [61, 62].

1.3 Topology optimization for additive manufacturing

The challenge of finding the best design for a given problem goes as far back as the development of the wheel or even the first tools. In recent decades the process of purely intellectual invention has been supported with and often even replaced by the use of mathematical algorithms. Currently a separate optimization branch, called structural optimization can be distinguished that focuses on, matters such as finding the optimal shape of a given structure. Three different types of structural optimization can be set apart: size optimization, to optimize parameters of predefined geometrical features, shape optimization, to optimize the shape of the structure by means of the either predefined parameters or chosen functions, and topology optimization (TO), which has ability to alter the topology of a design [63].

Topology optimization is a mathematical technique that searches for the optimal part shape (material distribution) for a given use case (objective function) under assumed restrictions (constraints). It typically creates complex designs, which cannot be realized with traditional manufacturing techniques [64]. As mentioned, shape complexity is not a limiting factor in AM as it offers virtually unlimited possibilities to realize a desired geometry. Therefore, additive manufacturing and topology optimization seem like a perfect match [65, 66].

At the beginning of this project there were only few examples of works that couple TO with AM. As the years passed new contributions were published and it was clear that the topic attracts a lot of interest. Based on the research goals and approach that was chosen, the published works may be categorized differently.

Topology optimization of part shape

The first group of research focuses on minimization of part distortions by altering the part shape. Minimization of compliance combined with thermal distortions induced by AM was investigated in [67]. For the purpose of thermal modelling two strategies have been used. In the first strategy cooling has been assigned to all elements at once. This was realized by changing the temperature from an initial to a final value in one step which caused deformations due to thermal contraction. In the second strategy cooling was assigned in the same manner but this time combined with an element birth scheme. Mesh activation was performed incrementally element by element. Results for 2D and 3D problems were presented for different boundary conditions. Although the novelty of this report is undeniable, its application to engineering problems seems

impracticable. Due to the calculation of sensitivities through finite differences, the time complexity and thus the calculation time of the optimization increase greatly with the number of design variables. This will make optimization inefficient, especially for the element birth model, where for a single printing simulation many incremental steps are to be calculated.

In [68] a variational formulation for the adjoint problem and the shape derivatives was used in an optimization based on the level-set method. Two transient thermo-elastic numerical models of 3D printing were used with the design domain consisting of two subdomains: powder and solid. For the calculation of sensitivities an adjoint method has been used. The considered objective functions aim at minimization of thermal stresses and vertical displacement of just deposited layers. The latter is relevant in preventing recoater collisions, where the recoater can hit the not-fully-built part due to its distortions. Often such an event marks the breakdown of the printing process. The optimized designs achieved improvements for the considered objective functions. However, the thermal transient model used for simulation seems to be too complex and may limit application of the method to purely academic examples.

Topology optimization of supports distribution

A separate branch of topology optimization for additive manufacturing may be optimization of supports distribution. To use supports in the right way their shape and placement have to be chosen properly. This has been based mainly on experience and trial and error. As an alternative, topology optimization supported by numerical modelling may be used. In recent years, many articles were published that fall into the scope of this broad topic [19]. One way of dealing with supports is by optimizing the shape of the structure such that supports are no longer needed. Many published research papers focus on providing such solutions by for example addressing the problem of overhangs [69–72]. Another approach, which is also a topic of this work, focuses on the right placement of the supports. The change of part shape is not always possible, because some elements have predefined geometry that is dictated by their application, for example body implants. Therefore, the result may only be influenced by changing of process parameters, altering of the part orientation, and placement of supports.

One of the first works that dealt with supports-free structures was presented by Leary *et al.* [69], where the local density gradient was used to identify overhangs and to prevent them. Gaynor *et al.* [70, 73] proposed a method to impose a minimum length scale and maximal overhang angle by the use of two projection functions. The removal of overhangs was carried out by applying local wedge-shaped projections. Langelhaar [71, 74] proposed a method that measures overhang based on the local density gradient calculated from elements beneath, but it is limited to voxel shaped mesh. Mass *et al.* [75] used a virtual skeleton method to first identify non-overhanging regions, which were then mapped on a mesh for a Solid Isotropic Material with Penalization (SIMP) method for which optimization was performed. Guo *et al.* [76] used methods of Moving Morphable Components and Moving Morphable Voids to take overhangs into account by making use of geometrical features that are characteristic for these techniques. Liu *et al.* [77] adopted TO based on the level-set method to incorporate design anisotropy due to the printing direction by rotation of an anisotropic constitu-

tive matrix originating from laminate theory: they account for overhangs by applying multi-level set modelling. Zhang *et al.* [78] used a level-set method with polygon hole-features projections to assess the occurrence of overhangs. Van de Ven *et al.* [72] proposed an overhang filter based on a front propagation method which is suitable for any type of mesh. Amir *et al.* [79] used an approach where the fabrication process is divided into several stages to assess overhanging regions based on gravitational load. Garaigordobil *et al.* [80] used an edge detection algorithm from image analysis to identify overhangs and to constrain them in a TO routine. The same method was used in [81] for a 3D printed compliant mechanism. A constraint that is evaluated as a single integral over the whole domain for a level-set based optimization has been proposed and successfully verified by Wang *et al.* [82]. In [83] overhangs are identified by means of a logistic segregation function. Pellens *et al.* [84] combined length scale [85] and minimum angle [74] filters. They used them in different sequences to obtain designs that are overhang-free and have a proper feature size as well. Zhang *et al.* [86] also combined two constraints, one for the overhang angle based on density of elements beneath every element in the domain and a second that defines the minimum feature size in the horizontal direction.

The second type of works tries to make use of them in the most optimal way, instead of focusing on reduction of supports in the build job. For this purpose, Mirzendehtel and Suresh [87] used a Pareto approach with a level-set method to choose the right supports volume. Kuo *et al.* [88] focused on optimization of supports stiffness that was combined with constraints on easy removability, by assuring proper sparsity of supports in the contact area between the part and supports and by incorporating of a cost function for supports printing that is applicable for fused deposition modelling. Mezzadri *et al.* [89] obtained supports distribution by prescription of an artificial load on the edge of the domain and then did optimization with compliance minimization for load-carrying supports. Zhou *et al.* [90] did a topology optimization of supports distribution to allow for overhang and heat conduction. For this purpose they simulated printing in an element-by-element fashion. Bartsch *et al.* [91] first performed a numerical simulation of printing for fixed degrees of freedom in place of supports. Then they used calculated maximal loads at these degrees of freedom to do topology optimization of supports distribution for compliance minimization. Cheng *et al.* [92], on the other hand, optimized supports distribution by using graded lattice structures. In this method the domain of optimization is filled with a lattice structure with varying density. The obtained optimization results provide density distributions that reduce residual stresses. The results have been confirmed with experiments. Pellens *et al.* [93] proposed a method of limiting recoater collision by tailoring the support distribution. The optimization's objective was to minimize support volume, while setting a constraint on maximal vertical distortions of just deposited elements. Lattice structures were used as supports and the inherent strain method was used to model material shrinkage. The method was verified on both 2D and 3D examples, considering a beam geometry. Zhang *et al.* [94] also investigated lattice-based supports for distortions minimization. Optimizations were performed with only gravitational load considered or with gravitational load combined with the inherent strains method. The goal was to minimize distortions under these two loading conditions for different values of allowed volume. The method was verified on a 3D beam geometry and results were confirmed experimentally.

There are also more advanced examples that try to optimize not only support structures but also other parameters. Langelaar [95] optimized simultaneously for overhangs, build direction and part shape. In [96] optimization of supports shape with parallel optimization of part shape has been done. Zhao *et al.* [97] used porous structures to obtain support-free design and combined it with part orientation optimization. Wang *et al.* [98], on the other hand, not only optimized the structure shape but also included the sequence of its production, which is applicable for example in wire arc additive manufacturing. As can be seen, optimization of supports may be done in many different ways and so far there is no consensus on which one is the best.

Other approaches

Other researchers that are worth alluding to but that do not fall into any of the above categories, are mentioned in this section. Yang *et al.* [99] optimized lattice structures and produced them with AM. A similar approach - but for optimization of a heat sink - was taken by Dede *et al.* in [100]. In [101, 102] a design with no closed cavities was obtained by means of a virtual temperature method that creates an artificial thermal model based on the existing geometry and uses it to assess the presence of cavities based on the accumulated heat. A design methodology for additive manufacturing that is either based on a Producibility Index (PI) for existing designs or for entirely new designs created by topology optimization was proposed in [103]. In [104] Ranjan *et al.* proposed a method for TO that controls local overheating by performing steady-state simulations on overlapping fragments of analysed geometry. An open source framework that converts results from TO based on the level-set method into a CAD file that can be directly used for printing has been proposed by Vogiatzis *et al.* in [105]. A similar method but for SIMP-based optimization was presented by Liu *et al.* [106] for 2D problems although extension to 3D seems to be possible. Xiao *et al.* [107] optimized lattice structures by means of TO and later on verified their results by using of structures produced by SLM with satisfactory results. Mirzendehtdel *et al.* [108] performed TO whereby instead of compliance the strength of a structure is maximized not by compliance but on the basis of the Tsai–Wu [109] criterion, which takes anisotropy of the material into account.

Other works relating to AM and TO have not been mentioned here because they fall too far from the scope of this work (they focus mainly on optimization of lattice structures). For a detailed overview the reader is referred to [19, 110].

1.4 Research objective

None of the commercially available simulation software couple TO with numerical modelling of distortion due to 3D printing. The state of the art of dealing with part distortion is based on geometry compensation without changing the overall design [50, 111]. This is a crude approach, since dependence of distortion is nonlinear with respect to geometrical changes and it involves manual labour.

In the research, as the previous section clearly suggests, there were only a few attempts at coupling a numerical model of additive manufacturing with topology optimization. The biggest problem is the computational burden of AM simulations. When simulations are incorporated into a TO loop they have to be repeated hundreds of times

which makes most of the approaches intractable.

The goal of this work may therefore be divided into two parts. The first one is to develop and implement suitable techniques for simulation of distortions in full-scale 3D printing that may also be used in topology optimization. The second one is to use the results of this model and to integrate them into a topology optimization loop. By doing that, it will be possible to obtain designs that are not only optimal with respect to their purpose but that can also be manufactured by means of AM.

The considered AM constraints are distortions of a part measured after the separation from the base plate and wiper blade collision. Two possible methods of optimization will be investigated. In the first, the optimizer searches for the shape of a part that optimizes the part performance and preserves constraints. The second one does the same by only looking for the right supports distribution and assuming the unalterable shape of a part.

All examples in this thesis are performed on eight cores of a 12-core machine, Intel(R) Xeon(R) CPU E5-2690 v3 @ 2.60GHz and calculations are parallelized by means of OpenMP framework.

1.5 Outline of the thesis

In Chapter 2 first the theoretical background is considered. The mathematical framework of continuum mechanics that describes movement and deformation of a body, the conditions necessary for a body to be in an equilibrium, and constitutive models that relate to stresses and deformations are derived. Moreover, a method of transferring them into numerical form suitable for computer calculations, the finite element method (FEM) is presented. Chapter 3 concerns numerical modelling of SLM, in particular the methods that were implemented in order to make numerical modelling of AM on the part scale feasible. The presented methods are then verified on numerical examples in Chapter 4. Next, a topology optimization framework for distortion constraints and equations for sensitivity calculations by means of the adjoint method are introduced in Chapter 5. In Chapter 6 and Chapter 7 results of the combination of Topology Optimization with numerical modelling of metal 3D printing are discussed. First of all, in Chapter 6 the possibility of influencing the printing by alteration of part shape is investigated for both 2D and 3D examples. Then, in Chapter 7 the focus is on tailoring of supports distribution. Stand-alone examples are given, as well as comparisons with results from Chapter 6. In the end optimizations of supports distribution for a use case example are carried out. The thesis ends with concluding remarks and recommendations for further research that are set out in Chapter 8.

Chapter 2

Basics

In this chapter basic theoretical components that are the foundation for this work are presented. First, continuum mechanics with its description of body kinematics, strain and stress definitions and conservation laws is introduced. Next, the finite element method is formulated that is used for simulations of additive manufacturing. The approach in which the equilibrium equations are enforced in the weak form is first discussed, followed by the methods for spatial and time discretization. Finally, material constitutive relations for plastic and viscoplastic material behaviour are derived. In this thesis, scalars will be denoted with regular letters (e.g. a) and tensors will be denoted with bold letters (e.g. $\mathbf{a}$ or $\mathbf{A}$). In the case of numerical algorithms, tensorial notation is dropped and Voigt notation is used instead, where one-dimensional arrays will be denoted with bold lower-case letters and two-dimensional arrays with bold capital letters.

2.1 Continuum mechanics

In this section the basic relations between the motion of a body and the forces that act on it are formulated using continuum mechanics. The continuum approach treats a body as a continuous object rather than as discrete particles, therefore microstructural changes are treated as globally averaged in the macroscopic description. The material considered here is homogeneous and isotropic, unless stated otherwise. Homogeneity of material means that it has uniform composition and properties at every infinitesimal point. Isotropy implies the same properties in every direction. The outline of this section is based on the work of Gurtin [112].

2.1.1 Kinematics and strains

Assuming that the initial position of the body, its trajectory and its current position are known, a motion of an arbitrary body may be described using one of two methods: Lagrangian or Eulerian.

In the Lagrangian description, which is going to be used in this work, the current position of a material point $\mathbf{x}$ is described based on its initial position $\mathbf{X}$ and the current time t (Figure 2.1):

Figure 2.1: Motion of a material point.

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t). \quad (2.1)$$

Based on initial and current position the displacement of a material point is defined as

$$\mathbf{u} = \mathbf{x} - \mathbf{X}. \quad (2.2)$$

Time derivative of displacements gives velocity

$$\mathbf{v} = \dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt}. \quad (2.3)$$

For a line element, a map from its initial configuration to the current configuration may be formulated

$$d\mathbf{x} = \mathbf{F} \cdot d\mathbf{X}. \quad (2.4)$$

The tensor $\mathbf{F}$ is called the deformation gradient and is defined as

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \mathbf{x} \bar{\nabla}_0 \quad (2.5)$$

and $\bar{\nabla}_0$ is a post-gradient operator with respect to the initial coordinates. Since the deformation gradient is non-symmetric, a different deformation measure is preferred to be used. For example the right Cauchy-Green tensor

$$\mathbf{C} = \mathbf{F}^T \cdot \mathbf{F} \quad (2.6)$$

It is called a metric tensor because it describes deformed metric of a body and in case of no deformation or rigid body motion it equals the unit tensor. The strain tensor is expected to describe a relative change of length, and in the case of rigid body motion, to be equal to $\mathbf{0}$. Therefore other strain measures are defined. Strain that describes deformation of the body with respect to the undeformed configuration and vanishes in the case of rigid body motions is the Green-Lagrange or Lagrangian strain tensor

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{1}), \quad (2.7)$$

where $\mathbf{1}$ is a second order identity tensor.

In terms of displacements it may be written as

$$\mathbf{E} = \frac{1}{2}[\mathbf{u}\vec{\nabla} + \vec{\nabla}\mathbf{u} + (\vec{\nabla}\mathbf{u}) \cdot (\mathbf{u}\vec{\nabla})]. \quad (2.8)$$

The presented strain is geometrically nonlinear, as the third term indicates, but the influence of nonlinearity may be neglected for small displacements. In that way the strain definition for the classical linear theory is obtained

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\mathbf{u}\vec{\nabla} + \vec{\nabla}\mathbf{u}]. \quad (2.9)$$

Additionally, a velocity gradient is formulated

$$\mathbf{L} = \mathbf{v}\vec{\nabla} \quad (2.10)$$

and a rate of deformation tensor which is defined as

$$\mathbf{D} = \frac{1}{2}(\mathbf{v}\vec{\nabla} + \vec{\nabla}\mathbf{v}) = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T). \quad (2.11)$$

2.1.2 Stresses and equilibrium

Stress is the response of a body to the deformation that is expressed by internal body forces. It is defined as a force per unit area.

Figure 2.2: Infinitesimal stress tetrahedron.

Assuming an infinitesimal tetrahedron with force vector $\mathbf{t}$ acting on one of its surfaces it may be defined

$$\mathbf{t} = \boldsymbol{\sigma} \cdot \mathbf{n} \quad (2.12)$$

and $\mathbf{n}$ is the unit normal to the surface and $\boldsymbol{\sigma}$ is the stress tensor (Figure 2.2). If the stress is defined in the current configuration it is called the Cauchy stress or true stress $\boldsymbol{\sigma}$. From equilibrium of moments the symmetry of the Cauchy tensor results

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T, \quad (2.13)$$

which is also known as the second law of Cauchy.

Based on the above definitions conservation laws may be formed that apply to every material point. They concern preservation of mass, momentum and heat.

The conservation of mass requires that

$$\dot{\rho} + \rho(\vec{\nabla} \cdot \mathbf{v}) = 0. \quad (2.14)$$

Moreover, balance of momentum or the first Cauchy law states that for an arbitrary body

$$\rho \dot{\mathbf{v}} = \boldsymbol{\sigma} \cdot \vec{\nabla} + \rho \mathbf{f}, \quad (2.15)$$

where $\mathbf{f}$ are the body forces, measured per unit of mass, and $\rho \dot{\mathbf{v}}$ are inertia terms. For many forming processes the inertia term may be ignored leaving the static equilibrium equation

$$\boldsymbol{\sigma} \cdot \vec{\nabla} + \rho \mathbf{f} = \mathbf{0}. \quad (2.16)$$

Finally, the conservation of energy requires a body to remain in thermal equilibrium and it is governed by the first law of thermodynamics

$$\rho \dot{u} = \boldsymbol{\sigma} : \mathbf{D} - \vec{\nabla} \cdot \mathbf{q} + \rho r, \quad (2.17)$$

here u is the specific internal energy, $\mathbf{q}$ is the heat flux vector, r is the specific heat source and $\boldsymbol{\sigma} : \mathbf{D}$ represents the mechanical energy dissipation.

2.1.3 Thermodynamics

Although heat plays an important role in additive manufacturing, including thermodynamics and imposing Eq. (2.17) increases the complexity substantially. Since, emphasis is put on the calculation time in this work, it is desirable to reduce the problem to mechanical only. Therefore, instead of applying heat flux explicitly and calculating thermal balance its assumed influence on stress $\boldsymbol{\sigma}$ is predetermined and applied.

2.2 Finite element method

In this work numerical modelling of the additive manufacturing process is done by means of the finite element method. The starting point for this method is equilibrium Eq. (2.16). Finding a solution to the equation in that form is possible only for simple problems. To handle more advanced examples approximations have to be done and therefore FEM is used. To obtain a solution the equilibrium equations with boundary conditions have to be rewritten in a weak form (Section 2.2.1), the domain of the problem has to be discretized (Section 2.2.2) and time integration has to be applied in the case of transient problems (Section 2.2.3). The literature used for this section is [113, 114].

2.2.1 Strong and weak form

In structural mechanics a problem to be solved reads as follows: *For a body volume V , find a solution $\mathbf{u}$, that is in accordance with geometrical boundary conditions and that fulfils mechanical equilibrium.* In the mathematical form, the equilibrium Eq. (2.16) combined with boundary conditions is used:

$$\begin{aligned}\boldsymbol{\sigma} \cdot \tilde{\nabla} + \rho \mathbf{f} &= \mathbf{0} && \text{in } V \\ \mathbf{u} &= \mathbf{u}_0 && \text{in } S_u \\ \boldsymbol{\sigma} \cdot \mathbf{n} &= \mathbf{t} && \text{in } S_t.\end{aligned}\tag{2.18}$$

Here $\mathbf{t}$ is the force acting on boundary S_t and $\mathbf{n}$ is the normal to this boundary. This is the natural boundary condition. Whereas, $\mathbf{u}_0$ are prescribed displacements on boundary S_u , which are the essential boundary conditions.

The equations presented above are a strong form. To solve this system numerically, by means of the finite element method, a weak form has to be formulated. The conservation of momentum and the natural boundary conditions are therefore weighted and integrated

$$\int_V \delta \mathbf{v} \cdot (\boldsymbol{\sigma} \cdot \tilde{\nabla} + \rho \mathbf{f}) dV - \int_{S_t} \delta \mathbf{v} \cdot (\boldsymbol{\sigma} \cdot \mathbf{n} - \mathbf{t}) dS_t = 0 \quad \forall \delta \mathbf{v}.\tag{2.19}$$

Here $\delta \mathbf{v}$ are virtual velocities that have to be zero at S_u and that are sufficiently differentiable. The weak form in Eq. (2.19) is rewritten in order to use it in FEM. Use is made of the chain rule

$$(\delta \mathbf{v} \cdot \boldsymbol{\sigma}) \cdot \tilde{\nabla} = \delta \mathbf{v} \cdot (\boldsymbol{\sigma} \cdot \tilde{\nabla}) + (\delta \mathbf{v} \tilde{\nabla}) : \boldsymbol{\sigma}\tag{2.20}$$

and the Gauss' divergence theorem to form the equation

$$\int_V (\delta \mathbf{v} \tilde{\nabla}) : \boldsymbol{\sigma} dV = \int_V \rho \delta \mathbf{v} \cdot \mathbf{f} dV + \int_{S_t} \delta \mathbf{v} \cdot \mathbf{t} dS_t \quad \forall \delta \mathbf{v}\tag{2.21}$$

and since $\boldsymbol{\sigma}$ is symmetric it can be rewritten as

$$\int_V \delta \mathbf{D} : \boldsymbol{\sigma} dV = \int_V \rho \delta \mathbf{v} \cdot \mathbf{f} dV + \int_{S_t} \delta \mathbf{v} \cdot \mathbf{t} dS_t \quad \forall \delta \mathbf{v}\tag{2.22}$$

in which $\delta \mathbf{D}$ is virtual rate of deformation.

In this way the strong equilibrium is changed to be valid in the integral form only over a defined volume instead of in every material point, and thus it is enforced in the weak form.

2.2.2 Spatial discretization

In the finite element method spatial discretization of a domain is required to obtain the solution of the weak equilibrium equations. The FEM gives a lot of freedom and flexibility with respect to the size and type of spatial discretization. In principle, a domain is divided into a finite number of subdomains in the form of elements and the interpolation functions are chosen. They interpolate displacements and primary variables. Based on the interpolation of the displacements other variables - stresses

and strains - are calculated. In this work two types of elements will be used: bilinear quadrilateral elements for 2D problems and trilinear hexahedral for 3D, depicted in Figure 2.3.

Figure 2.3: Finite elements used for the numerical analysis.

The position of every element is described by spatial points called nodes and the displacements are interpolated within an element by means of shape functions that depend on local coordinates. Isoparametric elements will be used, which means that the same interpolation functions describe both displacements and geometry. Relation of primary variables within an element by means of shape function is described as

$$\mathbf{x} = \sum_M N^M \mathbf{x}^M, \text{ with } N^M = N(\xi, \zeta, \eta)^M \quad (2.23)$$

where $\mathbf{x}$ is a primary variable, N are the interpolation functions, the index M refers to the nodal points and ξ, ζ, η are the local coordinates, see Figure 2.3. From now on the summation convention will be used and the summation operator will be omitted. Starting from the weak form in Eq. (2.22) all primary field variables and their gradients can be described by the proposed interpolation functions

$$\begin{aligned} \mathbf{v} &= N^M \mathbf{v}^M \\ \delta \mathbf{v} &= N^M \delta \mathbf{v}^M. \end{aligned} \quad (2.24)$$

The rate of deformation can be now written using index notation as

$$\begin{aligned} D_{ij} &= \frac{1}{2} \left(\frac{\partial N^M}{\partial x_j} \delta_{ik} + \frac{\partial N^M}{\partial x_i} \delta_{jk} \right) v_k^M = B_{ijk}^M v_k^M, \\ \delta \mathbf{D} &= \mathbf{B}^M \delta \mathbf{v}^M, \end{aligned} \quad (2.25)$$

where δ_{ik} is the Kronecker delta and $\mathbf{B}^M$ is a third order tensor, the gradient operator, which maps nodal velocities to the rate of deformation.

Using Eqs. (2.24) and (2.25) the discretized weak form may be obtained

$$\delta \mathbf{v}^M \cdot \int_V \mathbf{B}^M : \boldsymbol{\sigma} \, dV = \delta \mathbf{v}^M \cdot \left(\int_V N^M \mathbf{f} \, dV + \int_{S_t} N^M \mathbf{t} \, dS_t \right). \quad (2.26)$$

The left-hand side of the equation describes the work done by internal forces $\mathbf{F}_{\text{int}}^M$, whereas the right-hand side is for the work done by the external forces $\mathbf{F}_{\text{ext}}^M$, so that

$$\delta \mathbf{v}^M \cdot \mathbf{F}_{\text{int}}^M = \delta \mathbf{v}^M \cdot \mathbf{F}_{\text{ext}}^M, \quad \forall \delta \mathbf{v}^M. \quad (2.27)$$

Eq. (2.27) has to be true for all kinematically admissible virtual velocity fields at every node in the domain. These velocity fields are interpolated locally, being non-zero only in the elements connected to a given node, creating a pyramid-like field with a node in the centre. But since it holds true for all nodes in the domain the superscript M may be omitted, leaving the global internal and external force vectors:

$$\mathbf{F}_{\text{int}} = \mathbf{F}_{\text{ext}}. \quad (2.28)$$

The internal force vector $\mathbf{F}_{\text{int}}$ is a nonlinear function of displacement, so a proper procedure has to be applied in order to obtain the equilibrium.

2.2.3 Time integration

To solve a transient in a time system a time discretization has to be applied. The weak equilibrium equations are therefore evaluated at fixed times. The change of time between individual time points is called the time increment, thus it is often called an incremental procedure. The state of the system is known only at these fixed time points and in the incremental steps the change between them is calculated, based on the data from the beginning of the increment. If there is a nonlinear dependence of some state variable an accurate solution in a single calculation step may not be possible and an iterative procedure has to be applied, where the process is repeatedly solved until a required convergence is achieved.

Since not only velocities are important in the solution but also other history dependent variables such as stresses or equivalent plastic strain it is convenient to assume a constant value of the velocity throughout a given time step and to calculate a displacement increment instead

$$\Delta \mathbf{u} = \mathbf{v} \Delta t \quad (2.29)$$

and to do the same with strains

$$\Delta \boldsymbol{\epsilon} = \mathbf{D} \Delta t = \frac{1}{2} [(\Delta \mathbf{u}) \bar{\nabla} + \bar{\nabla}(\Delta \mathbf{u})]. \quad (2.30)$$

The iterative Newton–Raphson procedure is applied to provide a solution. Knowing that Eq. (2.28) has to be fulfilled and with assumption of already known, from the iteration $((k))$, displacement vector $\Delta \mathbf{u}^{((k))}$ a solution scheme can be built. The change of the displacement vector due to the iteration $((k+1))$ may be then written as

$$\Delta \mathbf{u}^{((k+1))} = \Delta \mathbf{u}^{((k))} + \Delta \Delta \mathbf{u}^{((k+1))}. \quad (2.31)$$

In this section increments are described with superscript in single brackets, $()$; superscript in double brackets, $(( ))$, is used for iterations.

Substituting Eq. (2.31) into Eq. (2.28) gives

$$\mathbf{F}_{\text{int}}(\Delta \mathbf{u}^{((k+1))}) = \mathbf{F}_{\text{int}}(\Delta \mathbf{u}^{((k))} + \Delta \Delta \mathbf{u}^{((k+1))}) = \mathbf{F}_{\text{ext}}. \quad (2.32)$$

After the Taylor series expansions and eliminating of the higher order terms the equation to be solved is

$$\mathbf{K}^{((k))} \Delta \Delta \mathbf{u}^{((k+1))} = \mathbf{R}^{((k))}, \quad (2.33)$$

with

$$\begin{aligned}\mathbf{R}^{((k))} &= \mathbf{F}_{\text{ext}} - \mathbf{F}_{\text{int}}(\Delta \mathbf{u}^{((k))}) \\ \mathbf{K}^{((k))} &= \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} \big|_{\Delta \mathbf{u}^{((k))}},\end{aligned}\tag{2.34}$$

where $\mathbf{R}^{((k))}$ is the residual force vector and $\mathbf{K}^{((k))}$ is the tangent stiffness matrix. The internal, reaction forces are calculated from stresses and compared with the applied force vector, the difference being the residual. The whole process is iteratively repeated until the norm of the residual is sufficiently small.

2.3 Constitutive models

Constitutive equations describe the response of the material by defining the relation between stresses and strains. For metals, two main regimes may be usually distinguished: elastic and plastic deformations. In the elastic range metals in general show a linear dependence between applied force and the resulting deformation. In the case of material plasticity, or other complex nonlinear material behaviour, the situation is not that simple and advanced experimental and modelling techniques have to be applied.

In this subsection derivation of two such models is presented. First, a plastic model with linear isotropic hardening is introduced which will be used in the 3D printing process simulation to quantify the residual stresses. A second model, which considers viscoplastic material response, describes the creep behaviour of the material and will be used to simulate the stress relief that occurs during the heat treatment after the printing stage. The content of this section is based mostly on Simo and Hughes [115] and articles that derive from it, such as [116], [117] and [118].

2.3.1 Plasticity

The material yields when its elastic limit is exceeded. This means that irreversible microstructural changes occur, which cause permanent plastic deformation. Different yielding conditions can be defined. In metal plasticity it is often observed that yielding behaviour does not depend on hydrostatic stress $\frac{1}{3}\boldsymbol{\sigma} : \mathbf{1}$ but rather on the deviatoric part only $\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3}\boldsymbol{\sigma} : \mathbf{1} \otimes \mathbf{1}$, where $\mathbf{1}$ is a second order identity tensor. Based on the norm of the deviatoric stress the von Mises yield criterion measure Φ may be written:

$$\Phi = \sqrt{\mathbf{s} : \mathbf{s}} - \sqrt{\frac{2}{3}}\sigma_y = 0,\tag{2.35}$$

which may be thought of as the description of a surface in a stress space.

In this work plasticity is limited to linear isotropic hardening. This means a linear relation H between the accumulated deformation measure, equivalent plastic strain ε_{eq} , and the yield limit σ_y

$$\sigma_y = \sigma_0 + H\varepsilon_{\text{eq}}.\tag{2.36}$$

Hence, the yield function is

$$\Phi = \|\mathbf{s}\| - \sqrt{\frac{2}{3}}(\sigma_0 + H\varepsilon_{\text{eq}}), \quad (2.37)$$

and the equivalent plastic strain relates to the plastic strain $\boldsymbol{\varepsilon}_p$ as

$$\dot{\varepsilon}_{\text{eq}} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}_p : \dot{\boldsymbol{\varepsilon}}_p}, \quad (2.38)$$

Moreover, for small deformations the assumption is made that the elastic and plastic strains decompose additively

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_p \quad (2.39)$$

and Hooke's law of elasticity is in the form

$$\boldsymbol{\sigma} = \mathbf{D}_{\text{el}} : \boldsymbol{\varepsilon}_e. \quad (2.40)$$

Here $\mathbf{D}_{\text{el}}$ is a fourth order elasticity tensor

$$\mathbf{D}_{\text{el}} = 3K\mathbf{I}^{\text{vol}} + 2G\mathbf{I}^{\text{dev}}, \quad (2.41)$$

where G and K are shear and bulk modulus respectively and $\mathbf{I}^{\text{vol}} = \frac{1}{3}\mathbf{1} \otimes \mathbf{1}$ is a fourth order volumetric projection tensor, $\mathbf{I}^{\text{dev}}$ is the deviatoric part of the fourth order identity tensor $\mathbf{I}^{\text{dev}} = [\mathbf{I}^{\text{sym}} - \mathbf{I}^{\text{vol}}]$ and $\mathbf{I}^{\text{sym}}$ is the symmetric fourth order identity tensor.

Setting Eq. (2.40) in a rate form and substitution of Eq. (2.39) gives

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}_{\text{el}} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p). \quad (2.42)$$

Under the assumption of classical associative plasticity, the direction of the plastic strain is normal to the yield surface. It is written as

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda} \frac{\partial \Phi}{\partial \boldsymbol{\sigma}}, \quad (2.43)$$

where λ is the consistency parameter and the yield function derivative may be rewritten

$$\frac{\partial \Phi}{\partial \boldsymbol{\sigma}} = \frac{\partial \|\mathbf{s}\|}{\partial \boldsymbol{\sigma}} = \frac{\mathbf{s}}{\|\mathbf{s}\|} = \mathbf{n}. \quad (2.44)$$

Moreover, from Eq. (2.38)

$$\dot{\lambda} = \sqrt{\frac{3}{2}}\dot{\varepsilon}_{\text{eq}}. \quad (2.45)$$

Substituting Eq. (2.43) in Eq. (2.42) gives

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}_{\text{el}} : (\dot{\boldsymbol{\varepsilon}} - \dot{\lambda}\mathbf{n}). \quad (2.46)$$

The magnitude of the plastic strain is determined by the consistency condition, which states that when the plastic flow occurs the stress state is always located on the yield surface

$$\dot{\Phi}(\boldsymbol{\sigma}, \lambda) = 0. \quad (2.47)$$

The dependence of Φ on λ is determined by Eqs. (2.37) and (2.45). In order to eliminate $\dot{\lambda}$ from Eq. (2.46) the consistency condition is expanded

$$\mathbf{n} : \dot{\boldsymbol{\sigma}} + \frac{\partial \Phi}{\partial \lambda} \dot{\lambda} = 0 \quad (2.48)$$

and then combined with Eq. (2.46)

$$\dot{\lambda} = \frac{\mathbf{n} : \mathbf{D}_{\text{el}} : \dot{\boldsymbol{\epsilon}}}{\mathbf{n} : \mathbf{D}_{\text{el}} : \mathbf{n} - \frac{\partial \Phi}{\partial \lambda}}. \quad (2.49)$$

Finally, using it to rewrite Eq. (2.42), with $\frac{\partial \Phi}{\partial \lambda} = -\frac{2}{3}H$, since only linear hardening is considered here and after rearranging of terms

$$\dot{\boldsymbol{\sigma}} = \left(\mathbf{D}_{\text{el}} - \frac{\mathbf{D}_{\text{el}} : \mathbf{n} \otimes \mathbf{n} : \mathbf{D}_{\text{el}}}{\mathbf{n} : \mathbf{D}_{\text{el}} : \mathbf{n} + \frac{2}{3}H} \right) : \dot{\boldsymbol{\epsilon}}. \quad (2.50)$$

In this work the stress and strain integration is done by means of the radial return algorithm [119]. The main idea is to first assume fully elastic total strain increment, the so called elastic predictor, and if the equivalent stress exceeds the yield limit to project it back to the nearest point on the yield surface, the so called plastic corrector. The method is implicit because, although direction of stress is determined at the beginning of the increment, the final value of yield stress is found with the implicit approach.

The value of the stress at the end of the next increment is equal to

$$\boldsymbol{\sigma}^{(n+1)} = \boldsymbol{\sigma}^{(n)} + \Delta \boldsymbol{\sigma}, \quad (2.51)$$

whereas the elastic trial stress due to the elastic predictor is

$$\boldsymbol{\sigma}_{\text{t}} = \boldsymbol{\sigma}^{(n)} + \mathbf{D}_{\text{el}} : \Delta \boldsymbol{\epsilon}, \quad (2.52)$$

and application of the plastic corrector gives

$$\boldsymbol{\sigma}^{(n+1)} = \boldsymbol{\sigma}_{\text{t}} - 2G\Delta\epsilon_{\text{p}}^{(n+1)}. \quad (2.53)$$

As mentioned, the yield stress is obtained based on the final step values

$$\Delta\epsilon_{\text{p}}^{(n+1)} = \Delta\lambda \mathbf{n}^{(n+1)}. \quad (2.54)$$

Since plastic deformations depends only on deviatoric stress Eq. (2.53) may be rewritten in terms of $\mathbf{s}$

$$\mathbf{s}^{(n+1)} = \mathbf{s}_{\text{t}} - 2G\Delta\lambda \mathbf{n}^{(n+1)} \quad (2.55)$$

and contracted with $\mathbf{n}^{(n+1)}$

$$\left\| \mathbf{s}^{(n+1)} \right\| = \left\| \mathbf{s}_{\text{t}} \right\| - 2G\Delta\lambda \quad (2.56)$$

Now, the yield function Eq. (2.37) is put in the iterative form

$$\Phi^{(n+1)} = \|\mathbf{s}^{(n+1)}\| - \sqrt{\frac{2}{3}}\sigma_0 - \sqrt{\frac{2}{3}}H\varepsilon_{\text{eq}}^{(n+1)}, \quad (2.57)$$

and substituting Eq. (2.56) into it gives equation that can be solved iteratively in order to obtain the plastic strain increment

$$\|\mathbf{s}_t\| - 2G\Delta\lambda - \sqrt{\frac{2}{3}}\sigma_0 - \sqrt{\frac{2}{3}}H\varepsilon_{\text{eq}}^{(n+1)} = 0. \quad (2.58)$$

Although in general an iterative solution for Eq. (2.58) is required, in the case of linear hardening with the von Mises yield criterion the solution can be obtained in one explicit step. The exception is when for a given material point in the most recent increment only elastic strain increment occurred and now plasticity deformation develops or the opposite happens, so plasticity occurred in the previous increment and only elastic strain develops. This may be also seen as a change of position of a stress state from lying on the inside of the ellipsoid drawn by Φ from Eq. (2.35) to its surface or the other way around.

The numerical integration of state variables requires the material tangent matrix to follow the numerical approach as well. Otherwise, if the continuum tangent matrix is applied a poor convergence ratio in the iterative procedure from Section 2.2.3 is expected. The consistent material tangent (algorithmic version of Eq. (2.50)) takes into account the numerical integration procedure and an asymptotic quadratic rate of convergence is expected for it.

To obtain the consistent material tangent Eq. (2.42) has to be linearised:

$$d\boldsymbol{\sigma}^{(n+1)} = \mathbf{D}_{\text{el}} : d\boldsymbol{\varepsilon}^{(n+1)} - 2G d\Delta\boldsymbol{\varepsilon}_p^{(n+1)}, \quad (2.59)$$

Additionally, a differentiation of Eq. (2.54) at increment (n+1) gives

$$d\Delta\boldsymbol{\varepsilon}_p^{(n+1)} = d\Delta\lambda \mathbf{n}^{(n+1)} + \Delta\lambda d\mathbf{n}^{(n+1)}. \quad (2.60)$$

Writing Eq. (2.59) in terms of total strains

$$d\boldsymbol{\sigma}^{(n+1)} = \left[\mathbf{D}_{\text{el}} - 2G \frac{\partial \Delta\boldsymbol{\varepsilon}_p^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} \right] : d\boldsymbol{\varepsilon}^{(n+1)}, \quad (2.61)$$

shows that derivative of the second term in brackets is needed

$$\frac{\partial \Delta\boldsymbol{\varepsilon}_p^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} = \mathbf{n}^{(n+1)} \otimes \frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}^{(n+1)}} + \Delta\lambda \frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}}. \quad (2.62)$$

Substituting Eq. (2.62) into Eq. (2.61) gives

$$d\boldsymbol{\sigma}^{(n+1)} = \left[\mathbf{D}_{\text{el}} - 2G\mathbf{n}^{(n+1)} \otimes \frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}^{(n+1)}} - 2G\Delta\lambda \frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} \right] : d\boldsymbol{\varepsilon}^{(n+1)}. \quad (2.63)$$

To obtain the relation between the stress and strain two components are yet to be determined: $\frac{\partial \Delta\lambda}{\partial \boldsymbol{\varepsilon}^{(n+1)}}$ and $\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}}$. Since their derivation requires complex and lengthy calculation, an interested reader is referred to Appendix A.1. Otherwise please accept the final form of Eq. (2.63), which after substitution of missing terms and some rearranging gives

$$d\boldsymbol{\sigma}^{(n+1)} = \left[\mathbf{D}_{\text{el}} - \frac{(2G)^2 \Delta\lambda}{\|\mathbf{s}_t^{(n+1)}\|} \mathbf{I}^{\text{dev}} - \left(\frac{2G}{1 + \frac{H}{3G}} - \frac{(2G)^2 \Delta\lambda}{\|\mathbf{s}_t^{(n+1)}\|} \right) \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right] : d\boldsymbol{\epsilon}^{(n+1)}, \quad (2.64)$$

for which the consistent tangent matrix is

$$\mathbf{D}_{\text{pl}} = \mathbf{D}_{\text{el}} - \frac{(2G)^2 \Delta\lambda}{\|\mathbf{s}_t^{(n+1)}\|} \mathbf{I}^{\text{dev}} - \left(\frac{2G}{1 + \frac{H}{3G}} - \frac{(2G)^2 \Delta\lambda}{\|\mathbf{s}_t^{(n+1)}\|} \right) \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)}. \quad (2.65)$$

Here, $\mathbf{D}_{\text{pl}}$ is the consistent tangent matrix for elasto-plastic deformations, which in case of no plastic deformations, $\Delta\lambda = 0$, reduces to the elastic tangent matrix.

2.3.2 Viscoplasticity

In powder bed additive manufacturing when the printing has finished, but before the separation from the base plate and the removal of the supports, heat treatment may be applied. The purpose of heat treatment is to reduce the accumulated residual stresses and therefore lower distortions after the cut-off, to increase the homogeneity of the microstructure and to remove voids. The decrease in stress is caused by transformation from elastic strain to plastic strain, i.e. relaxation.

Relaxation is the time dependent decrease of the stress under a constant strain. It resembles creep, which is time dependent deformation due to a constant stress. Both phenomena have the same underlying mechanisms, only the stimuli and response are switched, and it is common to assume that they are complementary and that their parameters can often be used interchangeable [120]. Therefore relaxation is usually simulated using a creep model [121, 122].

Creep is both temperature and stress dependent. It is often assumed that the influence of time, temperature and stress are separable. A typical creep curve is determined under a constant temperature and a constant load. In this work a widely adopted power law creep model [123] is used:

$$\dot{\epsilon}_{\text{cr}} = A \sigma_{\text{vm}}^m e^{\frac{-Q}{RT}}, \quad (2.66)$$

where A and m are material parameters, which may depend on temperature. Q is the activation energy of the creep mechanism, R is Boltzmann's constant ($8.31 \text{ J mol}^{-1} \text{ K}^{-1}$) and T is the absolute temperature.

Assuming that during the stress relaxation no additional plastic deformation occurs the stress rate is

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}_{\text{el}} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}_{\text{cr}}), \quad (2.67)$$

Eq. (2.67) in the rate form may be integrated with assumption of the fully implicit, backwards Euler integration scheme and may be written in a form of the elastic trial step and the inelastic correction step

$$\boldsymbol{\sigma}^{(n+1)} = \mathbf{D}_{\text{el}} : \boldsymbol{\epsilon}^{(n+1)} - 2G \Delta \boldsymbol{\epsilon}_{\text{cr}}^{(n+1)}, \quad (2.68)$$

Using Eq. (2.66) a creep strain rate tensor may be defined as

$$\dot{\boldsymbol{\epsilon}}_{cr} = A\sigma_{vm}^m e^{\frac{-Q}{RT}} \sqrt{\frac{3}{2}} \mathbf{n}, \quad (2.69)$$

where $\mathbf{n}$ in this case is the direction of the deviatoric stress.

Eq. (2.69) may be put in the incremental form

$$\Delta \boldsymbol{\epsilon}_{cr} = \Delta t A (\sigma_{vm}^{(n+1)})^m e^{\frac{-Q}{RT}} \sqrt{\frac{3}{2}} \mathbf{n}^{(n+1)}. \quad (2.70)$$

Creep strain depends fully on the deviatoric stress $\mathbf{s}$, with $\sigma_{vm}^{(n+1)} = \sqrt{\frac{3}{2}} \|\mathbf{s}^{(n+1)}\|$. Using Eqs. (2.68) and (2.70) the creep strain increment may be found using an iterative approach

$$\|\mathbf{s}^{(n+1)}\| = \|\mathbf{s}_t^{(n+1)}\| - 2G \sqrt{\frac{3}{2}} \Delta t A \left(\sqrt{\frac{3}{2}} \|\mathbf{s}^{(n+1)}\| \right)^m e^{\frac{-Q}{RT}}. \quad (2.71)$$

The equation above has to be iteratively solved for each material point separately.

Now, in order to obtain proper convergence rate of the numerical algorithm a consistent material tangent matrix has to be derived. To obtain a consistent tangent matrix Eq. (2.68) has to be linearised

$$d\boldsymbol{\sigma}^{(n+1)} = \mathbf{D}_{el} : d\boldsymbol{\epsilon}^{(n+1)} - 2G d\Delta \boldsymbol{\epsilon}_{cr}^{(n+1)}. \quad (2.72)$$

Please note that in this instance linearisation is done for a numerical algorithm at the fixed time step $(n+1)$, otherwise the strain would also depend on time. For the second term differentiation of Eq. (2.70) is required

$$d\Delta \boldsymbol{\epsilon}_{cr} = \Delta t A d(\sigma_{vm}^{(n+1)})^m e^{\frac{-Q}{RT}} \sqrt{\frac{3}{2}} \mathbf{n}^{(n+1)} + \Delta t A (\sigma_{vm}^{(n+1)})^m e^{\frac{-Q}{RT}} \sqrt{\frac{3}{2}} d\mathbf{n}^{(n+1)}. \quad (2.73)$$

After substitution

$$\begin{aligned} d\boldsymbol{\sigma}^{(n+1)} = \mathbf{D}_{el} : d\boldsymbol{\epsilon}^{(n+1)} - 2G \left(\Delta t A d(\sigma_{vm}^{(n+1)})^m e^{\frac{-Q}{RT}} \sqrt{\frac{3}{2}} \mathbf{n}^{(n+1)} \right. \\ \left. + \Delta t A (\sigma_{vm}^{(n+1)})^m e^{\frac{-Q}{RT}} \sqrt{\frac{3}{2}} d\mathbf{n}^{(n+1)} \right). \end{aligned} \quad (2.74)$$

Following the same steps as for derivation of the consistent tangent matrix for plasticity, Eq. (2.74) is written in terms of total strains

$$\begin{aligned} d\boldsymbol{\sigma}^{(n+1)} = \left(\mathbf{D}_{el} - 2G \left(\sqrt{\frac{3}{2}} \right)^{m+1} \Delta t A e^{\frac{-Q}{RT}} \mathbf{n}^{(n+1)} \frac{\partial \|\mathbf{s}^{(n+1)}\|^m}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right. \\ \left. - 2G \left(\sqrt{\frac{3}{2}} \right)^{m+1} \Delta t A \|\mathbf{s}^{(n+1)}\|^m e^{\frac{-Q}{RT}} \frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right) : d\boldsymbol{\epsilon}^{(n+1)}. \end{aligned} \quad (2.75)$$

The terms that need further attention in Eq. (2.75) are $\frac{\partial \|\mathbf{s}^{(n+1)}\|^m}{\partial \boldsymbol{\epsilon}^{(n+1)}}$ and $\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}$. Again, the reader is referred to Appendix A.2 for full derivation of the missing terms. Otherwise please accept that the final form of the consistent material tangent matrix for viscoplastic calculation is

$$\mathbf{D}_{cr} = \left[3K\mathbf{I}^{\text{vol}} + 2G \frac{1}{1 + \Delta t \gamma} \mathbf{I}^{\text{dev}} + 2G \frac{\Delta t \gamma (m-1)}{1 + m + \Delta t \gamma m + \Delta t \gamma^2 m} \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right] \quad (2.76)$$

with

$$\gamma = 2G \left(\sqrt{\frac{3}{2}} \right)^{m+1} A e^{\frac{-Q}{RT}} \left\| \mathbf{s}^{(n+1)} \right\|^{m-1}, \quad (2.77)$$

$\mathbf{D}_{cr}$ is the consistent material tangent for viscoplasticity, which in the case of zero deviatoric stress, $\mathbf{s} = \mathbf{0}$, reduces to the elastic tangent.

2.4 Summary

In this chapter the theoretical background for numerical modelling of continuum mechanics that is the basis for the rest of this work has been presented. Different components of continuum mechanics, thermodynamics, and numerical schemes can be used to achieve this. The choice depends on the application and the level of fidelity that is sought. This work puts the emphasis on calculation time, so only small displacements theory with purely mechanical simulation will be used. Additionally, two separate constitutive models for the printing and for the heat treatment stage have been introduced along with derivation of the constitutive material tangent matrix. The AM oriented techniques will be dealt with in detail in the following chapter.

Chapter 3

Numerical Modelling of SLM

Numerical modelling of SLM is a complex task and may be performed on different scales depending on the type of results that are sought. In research, various techniques have been used ranging from microstructural evolution [124, 125] and melt pool simulation [126, 127] to the coarse assessment for overall part behaviour [53, 57, 128, 129]. Including everything into a single simulation is impossible because there is always a trade-off between accuracy and the computational time. In this work, a numerical model will be used to calculate distortions on the part scale level. Therefore, it is important to use methods that will allow for its accurate prediction.

Fidelity of numerical models depends greatly on the amount and accuracy of the input data and the level of sophistication of the model. But the increase in accuracy almost always comes at a cost to the increase in the time that is required for the calculation of the solution often non-proportional with the number of degrees of freedom [130].

In this work, a numerical model will be used for topology optimization. In topology optimization an iterative algorithm is utilized where a new solution is found based on the current and most recent increments. Often many iterations are required to obtain a satisfactory solution. It means that even hundreds of simulations may be needed. Consequently, it is of great importance to use numerical techniques that will help to reduce the computational time, while maintaining sufficient accuracy.

Work relating to the numerical modelling of additive manufacturing is divided into two chapters. In this chapter numerical techniques used to model SLM will be presented, whereas Chapter 4 is dedicated to numerical verification of the presented model and the proposed techniques. This chapter has two main sections. The first one deals with various modelling components that are necessary in order to perform additive manufacturing simulations. The second focuses on the numerical model itself and consists of derivation of essential FEM equations that are specific to this work.

3.1 Finite element modelling of additive manufacturing

In this section numerical modelling techniques that are necessary for additive manufacturing simulations are presented. First, a meshing script that transforms the

Figure 3.1: Meshing script flowchart.

geometry or machine G-code into a FEM mesh is introduced. It is followed by inherent strains, a modelling method about calculation and prescription of thermal shrinkage as a purely mechanical load. Finally, two element deposition techniques are presented. The widely adopted lumped approach is first described and as an alternative, an element growth technique is proposed.

3.1.1 Meshing script

Additive manufacturing provides a freedom of design that was previously unknown. Not only it is possible to produce complex designs but also the overall part production cost does not depend on its complexity. It is a great opportunity from the designer's point of view but also a great challenge for FEM analysts to create proper discretization of the designed shape.

In simulation of additive manufacturing there is also an additional limitation coming from the printing direction and the sequential process character. It organizes mesh parts into domains, the element rows, that are going to be deposited in the specified order. To achieve that a complex, shape-preserving algorithm that follows thoroughly the surface of an object could be used. On the other hand, there are more basic methods that perform voxelization of a described shape but at the cost of generalization of small features. Since some assumptions regarding numerical modelling have already been made and creation of a complex meshing algorithm is out of the scope of this thesis the latter option is chosen.

The mesh used in this work will be only in the form of pixels for 2D and voxels for 3D problems. To create a mesh the part's geometry is virtually sliced by planes parallel to the base plate, the same as for the areas that are scanned by the laser beam. The distance between the planes imposes the element height in the building direction. Slicing is done by either cutting the geometry on the chosen heights provided in the STL format or by using a G-code that describes a laser beam path for each layer. It gives information about the areas that are scanned by the laser beam path, and thus where powder is turned into a solid part.

The flowchart of the script is presented in Figure 3.1. The whole routine is based on Python and as mentioned it takes an input file in the form of STL or G-code. In the case of an STL file, a part is cut by planes on heights corresponding to the bottom

Figure 3.2: Comparison of STL geometry (left) and obtained mesh (right) for mesh size 1 mm [133].

of every row of elements. This results in vector plots of the cross sections in a form of SVG files.

For G-code a different procedure is applied. The machine code is searched, so as to identify fragments of code that describe laser beam paths for heights corresponding to the bottom of every row of elements. Then code used to describe these laser beam paths is then used to create their graphical representation in the form of the SVG file. These SVG files are then transformed into bitmaps, the PNG files, by means of Inkscape software [131]. Every bitmap has the same reference point, so the bitmaps can be stacked. Depending on the bitmap resolution, there can be a different number of pixels that describe a given cross section, and thus an element size may be different in the horizontal direction. The elements height, on the other hand, are determined by the distance between the cutting planes, or searched layers in case of G-code. After the transformation with use of Inkscape each PNG file is treated as a map for consecutive rows of elements. If a pixel is black or sufficiently grey the presence of an element is assumed, while the absence of elements is indicated in white. To distinguish supports from the part geometry either they have to be provided in a separate STL file which will be joined during this stage or the G-code has to be properly commented that a given laser beam path belongs to support structures, as for example in the case of Slic3r software [132]. Finally, the 2D-element maps are joined into a 3D array, which is used to create the mesh.

An example of application of the meshing script is shown in Figure 3.2. The presented mesh grasps the main features of the investigated part. Although the surface results, will certainly have to be treated with some caution, achieving meaningful results still should be possible, especially when the mesh resolution is increased.

3.1.2 Inherent strains

Complex thermo-mechanical phenomena that occur at the melt pool require application of high-fidelity numerical models that are computationally expensive. Physics of the melt pool, interaction of the laser beam with material and change of material

state are the most elemental phenomena that occur at this scale but many more may be included. It is impossible to take them all into account and still be able to simulate the part behaviour on the macro scale. Therefore, in this work, to reduce the nonlinearity of the problem it was decided to change the problem from a thermo-mechanical into a mechanical-only one. It is expected that the simplification method should take into account the resulting mechanical effects of all of the relevant phenomena. The most widely used method is the inherent strains technique [51, 53]. In this approach thermal contraction due to the cooling of the material is calculated beforehand and its mechanical equivalent is applied in accordance with

$$\boldsymbol{\varepsilon}_{\text{inh}} = \boldsymbol{\alpha}\Delta T, \quad (3.1)$$

where $\boldsymbol{\alpha}$ is the thermal expansion tensor. Consequently, thermo-mechanical multi-incremental calculations may be replaced by a single mechanical increment. The method itself originates from welding simulations [134]. The value of inherent strains may either be assumed from Eq. (3.1), calculated from thermo-mechanical simulations performed on the lower scale [54] or obtained experimentally [55], depending on the desired accuracy and available resources.

3.1.3 Element deposition

A constant increase in the amount of material is one of the aspects that distinguishes additive manufacturing from standard production methods. For numerical modelling it means that the number of elements will constantly rise. Two main approaches may be distinguished that handle this in different ways: the quiet element technique and the active/inactive element technique, which is also known as the element ‘birth and death’ method. In this work a whole row of elements is activated at once with use of the birth approach only. Moreover, the calculations are purely mechanical and the inherent strains are prescribed to the whole row at once. This subsection focuses on two different approaches of handling multiple layers of material within a single element: a lumped approach and an element growth technique.

Lumped approach

One of the difficulties that comes with simulations of additive manufacturing is the difference in length scales. A single part, the size of which may be up to several centimetres, is built from layers that have a thickness of 30 to 100 micrometres. In order to describe the full part it is impossible to have a mesh that treats every material layer as a separate detail. Therefore, methods that describe multiple material layers in a single element row are used. In one of the approaches the layers are gathered in bundles that are treated as a single entity that is deposited at once as a row, see Figure 3.3. The standard element integration is used and the properties of the layers are averaged at the integration points. This method is called here the *lumped approach*. The method is now common and its biggest advantage is reduction of the number of elements which makes part scale calculations achievable.

One of the inevitable costs of such a mesh size coarsening is loss of accuracy. The gradient that is normally introduced between consecutive layers within an element is completely ignored and the same value of inherent strains is prescribed to all the

Figure 3.3: Typical element used in lumped approach that describes deposition of multiple material layers. ‘x’ marks the location of integration points.

integration points [53]. In reality, every layer is usually filled with a different laser beam path. Depending on the direction of the scanning laser beam path for a given material point differently oriented shrinkage occurs, and thus a different value of the inherent strains should be used. If the lumped approach is implemented, this issue is usually ignored; if not, it can be addressed only in an average sense over the element layer thickness.

Element growth technique

To handle the problem of a different shrinkage direction an alternative approach is proposed. In order to keep the mesh size reasonable while making it possible to simulate deposition of separate material layers an element growth technique is introduced. The main idea in this method is to incorporate deposition of a single layer into simulations while keeping the mesh size similar to that of the lumped approach. To do that, multiple rows of integration points are introduced into the element’s definition. Each row corresponds to a physical material layer within an element. Moreover, the size of the element gradually increases with deposition of every new layer.

Figure 3.4: Different stages of layer deposition, each describing deposition of a new layer (left) and shrinkage due to inherent strains (right).

The method may be divided into two phases: growth and regularization. Figure 3.4

shows the growth. In the first phase a row of elements is filled with the material layers. A single element row is going to incorporate multiple material layers. The bottom nodes are used to describe the bottom position of the lowest layer, whereas the top nodes represent the top of the currently highest layer. Deposition starts from a single material layer, which means that both top and bottom nodes describe its position. A row of integration points is added to provide description of each layer's state variables such as stresses and strains. The row means that for elements with quadrilateral elements there are two integration points that have a standard location for the Gaussian quadrature rule in horizontal directions; for the hexahedral there are 2×2 integration points, so four. The local coordinate for the building direction is then determined for each row by:

$$\zeta^J = \frac{2(J-1) + 1 - n_{\text{row}}}{n_{\text{row}}}, \quad (3.2)$$

where ζ^J is the local coordinate of the mid plane of layer J and n_{row} is the number of layers that is deposited in the element. Next, the inherent strains of the last deposited layer are prescribed to its integration points and displacement increments of the nodes are calculated. Then, the simulation proceeds and the deposition of the next layer takes place. The position of only the top element nodes is updated in such a way that they correspond to the nominal position of newly deposited material layer and their total displacement vector is set to 0. The new row of integration points is added and the position of the ones that have already been deposited is changed, so now there are two layers with $\zeta^1 = -0.5$ and $\zeta^2 = 0.5$. Again inherent strains are prescribed, but now to the layer $J = 2$ only, and displacements are calculated. The whole process is repeated until the number of layers that fit in the element are deposited.

The second phase, element regularization, takes place after the element has been fully filled with material layers. Within an element the number of integration points in the building direction is equal to the number of layers. In the regularization process the number of integration points is simply reduced to correspond to the regular 2×2 Gaussian points integration scheme, as presented in Figure 3.5 for 2D.

Figure 3.5: Schematic showing number of integration point before (left) and after (right) the regularization stage.

The (yet unknown) state variables in the 2×2 integration points 'n' are related to those in the $n_{\text{row}} \times 2$ integration points 'o' by $\mathbf{h}_o = \mathbf{L}\mathbf{h}_n$, where $\mathbf{h}_o$ and $\mathbf{h}_n$ are values of interpolated variables at the old and new integration points localization respectively and $\mathbf{L}$ is a $2n_{\text{row}} \times 4$ interpolation matrix made of values of the shape functions at the new integration points locations $\mathbf{h}_n$. Then the new ones can be evaluated from a least squares fit

$$\mathbf{L}^T \mathbf{L} \mathbf{h}_n = \mathbf{L}^T \mathbf{h}_o. \quad (3.3)$$

For stresses interpolation should hold that the equilibrium is not disturbed. This can be written formally as $\mathbf{B}_n^T \boldsymbol{\sigma}_n = \mathbf{B}_o^T \boldsymbol{\sigma}_o$. It is realized by the following minimization problem: $\min : \frac{1}{2}(\mathbf{L}\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_o)^T(\mathbf{L}\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_o) + \boldsymbol{\kappa}^T(\mathbf{B}_n^T \boldsymbol{\sigma}_n - \mathbf{B}_o^T \boldsymbol{\sigma}_o)$, which gives the equation

$$\begin{bmatrix} \mathbf{L}^T \mathbf{L} & \mathbf{B}_n \\ \mathbf{B}_n^T & \mathbf{o} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\sigma}_n \\ \boldsymbol{\kappa} \end{Bmatrix} = \begin{Bmatrix} \mathbf{L}^T \boldsymbol{\sigma}_o \\ \mathbf{B}_o^T \boldsymbol{\sigma}_o \end{Bmatrix}, \quad (3.4)$$

where $\mathbf{L}$ is made of values of the shape functions at the new integration points locations separately for each $\boldsymbol{\sigma}$ component and $\boldsymbol{\kappa}$ is a Lagrange multipliers vector.

3.2 Finite element analysis of additive manufacturing

For the purpose of numerical modelling a finite element method is used with the Lagrangian description of motion as described in Chapter 2. Moreover, geometrical linearity is used, assuming that the deformations after solidification are small. Distortions are calculated not by updating the nodal position but by storing cumulative displacements in a separate vector. This simplifies calculations, reduces calculation time and gives possibilities for solution parallelization as proven in [128]. Finally, to reduce computational time, modelling techniques presented in the previous section are used.

This section focuses on derivation of finite element equations for the aim of SLM process simulation. The simulation may be divided into three steps: the printing, the optional heat treatment and the release stage. The purpose of this section is not only to introduce the necessary equations but also to create a base for adjoint sensitivities calculation in topology optimization. In topology optimization only the lumped method is used. Therefore the description below is mainly restricted to this approach.

3.2.1 Simulation of SLM by finite element method

At the start the domain consists only of inactive elements and with every new step one element row is activated. In every step of the printing stage a row that may consist of the elements of a part and support is activated. Assuming that M element layers are needed to fully print the part, the printing stage will consist of M incremental steps. The final release stage increment will be therefore increment $M + 1$. The deposition of a bundle of layers is simulated by activation of a new row of elements. In our notation a vector $\mathbf{x}^{(i)}$ refers to a solution valid for the i^{th} building step and a vector $\mathbf{x}_j$ is a selection of a field quantity referring only to the j^{th} layer of finite element nodes. Consequently $\mathbf{x}_m^{(n)}$ is a vector referring to the m^{th} layer of nodes at the n^{th} building step. This means that always $m \leq n$. A schematic illustration of the printing process simulation and of the notation is shown in Figure 3.6.

Printing always starts from the base plate. The displacement degrees of freedom of nodes connected to the base plate $\mathbf{u}_0$ are completely suppressed. After deposition of

Figure 3.6: Schematic of numerical modelling.

$j - 1$ layers, i.e. at the $(j - 1)^{\text{th}}$ step, the total set of degrees of freedom is $\mathbf{u}^{(j-1)} = [\mathbf{u}_0^T, \mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_{j-1}^T]^T$, where $\mathbf{u}$ are displacement degrees of freedom. Deposition of a new set of layers starts with activation of a new row of elements. The degrees of freedom of the newly activated row $\mathbf{u}_j$ are added to the current set of degrees of freedom

$$\mathbf{u}^{(j)} = [\mathbf{u}^{(j-1)T}, \mathbf{u}_j^T]^T = [\mathbf{u}_0^T, \mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_{j-1}^T, \mathbf{u}_j^T]^T. \quad (3.5)$$

The global stiffness matrix is assembled from element stiffness matrices

$$\mathbf{K}^{(j)} = \int_{V^{(j)}} \mathbf{B}^T \mathbf{D} \mathbf{B} dV, \quad (3.6)$$

where $V^{(j)}$ is the volume of the build including step j . The forces on the right hand side come from the inherent strains in the just deposited elements

$$\mathbf{f}^{(j)} = - \int_{V_j} \mathbf{B}^T \mathbf{D} \boldsymbol{\epsilon}_{\text{inh}} dV, \quad (3.7)$$

where V_j is the volume of the last deposited layer j . The system of equations in that form is solved

$$\mathbf{K}^{(j)} \Delta \mathbf{u}^{(j)} = \mathbf{f}^{(j)} \quad (3.8)$$

and as mentioned above, the nodal position is not updated, but instead the solution is stored in a separate vector so the total displacements of nodes in row i until deposition j ($j \geq i$) layers can be calculated as

$$\mathbf{u}_i^{(j)} = \sum_{k=i}^j \Delta \mathbf{u}_i^{(k)}. \quad (3.9)$$

When the printing stage has finished the part is released from the base plate. The release stage is a single increment within which the part is separated from the base

plate and the supports are removed. The nodes at the bottom that were suppressed are freed and all elements that describe supports are being deactivated. This modification of boundary conditions leads to unbalance in the internal forces at the nodes for which the change occurred. Therefore in the release stage we build the force vector from unbalanced forces at these nodes

$$\mathbf{K}_{ff}^{(M+1)} \Delta \mathbf{u}^{(M+1)} = -\mathbf{S}_f \mathbf{r}^{(M)}, \quad (3.10)$$

where $\mathbf{S}_f$ is a selection matrix which chooses only unsuppressed degrees of freedom and $\mathbf{r}^{(M)}$ are the reaction forces at the released part's nodes and part's nodes that were connected to support elements. The global stiffness matrix and the displacement vector are described only by degrees of freedom that are unsuppressed in the release stage and belong to the part, which is denoted with the subscript f

$$\mathbf{K}_{ff}^{(M+1)} = \int_{V_f^{(M+1)}} \mathbf{B}^T \mathbf{D} \mathbf{B} dV, \quad (3.11)$$

where $V_f^{(M+1)}$ is the volume of the part only, so with supports removed.

The system in that form is solved and the calculated distortions are stored. As mentioned, the main goal of the numerical model is to provide distortions in a fast way while still being reliable. Our main limitation here is computational time which should be as short as possible when aiming for dozens or even hundreds of simulations. We therefore introduce these simplifications intentionally. The equations above are for linear material only because their main aim is to provide backbone for topology optimization that will be presented in Chapters 6 and 7. Of course their extension to nonlinear material is possible and for this purpose the iterative approach from Section 2.2.3 can be used. The numerical model results presented in this chapter consists of both a linear and a nonlinear material model. Although the model seems to be crude and neglects many factors, it has been shown by Munro *et al.* [128] that valuable results can be obtained.

3.2.2 Support elements

Despite the fact that there is much freedom in the forms and shapes of supports, in the most common and basic form, they are slender vertical structures that are used to connect the part with the base plate or with a member that is right underneath it. Of course, different support shapes can be implemented, but they are not a standard in current AM software. For example, if supports need to have stiffness variation in the building direction lattice structures can be used [110, 135, 136]. The disadvantage of a lattice-based approach is that a proper lattice structure has to be chosen and its stiffness variation over the domain has to be defined. Then, its digital form has to be transferred to a printer and it still has to be produced. Since lattices may have small members, especially for less stiff cells, they may be difficult to produce, and may be prone to manufacturing errors for not fully controlled processes such as SLM. The whole process is rather complex and, as mentioned, it is not a common feature of AM-related programs, so special algorithms for this purpose would have to be created. Therefore, the common approach proposed by most of the additive man-

ufacturing design supporting software is based on structures that are either columns or honeycombs that only provide a connection in the vertical, building direction. Depending on the cross sections of a pillar-like structure a different stiffness between the vertical and horizontal direction may be obtained. If thin pillars are to be simulated a fine mesh would have to be used. In the case of a crude mesh resulting from voxel-based methods there is no such option, so a different approach has to be taken. In this work, a two-fraction material model is used to model this behaviour within a FEM framework. The material is divided into two volume fractions, one with only stiffness in the vertical, building direction f_v and a second, much smaller with isotropic stiffness.

The choice of the size of the fractions is here arbitrary and for non-numerical applications it should be validated by experiments. In this work, the first fraction has only stiffness in the building direction (vertical) and describes 0.99 of volume ($f_v = 0.99$), whereas the second fraction is isotropic and has only 0.01 of volume ($1 - f_v$). Therefore the stiffness matrix of support elements takes the form

$$\mathbf{K}_{ss} = \int_{V_s} \mathbf{B}^T (f_v \mathbf{D}_M + (1 - f_v) \mathbf{D}) \mathbf{B} dV. \quad (3.12)$$

In which the modified $\mathbf{D}_M$ is simply a matrix with all entries equal to zero, except for the printing direction which is set equal to the value that would correspond to the one dimensional problem, so for the elastic material response it is equal to Young's modulus E ,

$$\mathbf{D}_M = E \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3.13)$$

In the case of material plasticity, only the vertical fraction f_v is assumed to be plastic. It therefore changes the consistent tangent from Eq. (2.65) into

$$\mathbf{D}_M = \frac{EH}{E + H} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3.14)$$

It is assumed that by changing the values of fractions it could be also possible to obtain a description of supports with different embodiment.

3.2.3 Distortions after release from the base plate

During the release stage two separate phenomena occur at the same time. First of all, the part is separated from the base plate which means that reaction forces that are acting on the nodes that were connected to the bottom are released (Figure 3.7c).

Figure 3.7: Origin of forces acting on the part in the release stage.

Secondly, support structures are removed. This means that elements that describe supports are deactivated and that there are unbalance forces acting on the nodes that connect the part to the supports (Figure 3.7e). Finally, to avoid rigid body movement new fixed degrees of freedom are defined. These suppressed degrees of freedom do not necessarily have to belong to the previously suppressed set but they have to belong to the part's elements since supports are deactivated (Figure 3.7f). In the end $\mathbf{r}^{(M)}$ in Eq. (3.10) may be divided into two separate components

$$\mathbf{r}^{(M)} = \mathbf{r}_0^{(M)} + \mathbf{r}_i^{(M)}, \quad (3.15)$$

where $\mathbf{r}_0^{(M)}$ are reaction forces at the base plate and $\mathbf{r}_i^{(M)}$ are reaction forces at the part-support interface coming from the supports removal.

The reaction forces in the release stage coming from suppressed and supports nodes may be obtained in two ways: keeping track either of stresses at the elements connected to these nodes or forces at their nodes. Both methods are equivalent in the sense of calculation time and complexity as well as accuracy. Here the force tracking

option is chosen to use it later also for adjoint sensitivities calculations. Therefore, in every building step the system of equations may be divided into suppressed degrees of freedom $\mathbf{u}_0$ and the rest $\Delta \mathbf{u}_f^{(j)} = [\Delta \mathbf{u}_1^T, \dots, \Delta \mathbf{u}_j^T]^T$

$$\begin{bmatrix} \mathbf{K}_{00} & \mathbf{K}_{0f} \\ \mathbf{K}_{f0} & \mathbf{K}_{ff} \end{bmatrix}^{(j)} \begin{Bmatrix} \mathbf{0} \\ \Delta \mathbf{u}_f \end{Bmatrix}^{(j)} = \begin{Bmatrix} \Delta \mathbf{r}_0 \\ \mathbf{f}_p \end{Bmatrix}^{(j)}. \quad (3.16)$$

Usually only $\mathbf{K}_{ff}$ is assembled since displacements for fixed degrees of freedom are known beforehand.

In this work there is a distinction between the part and support elements thus Eq. (3.16) may be further rewritten as

$$\begin{bmatrix} \mathbf{K}_{00}^p & \mathbf{K}_{0r}^p & \mathbf{K}_{0r}^p & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{f0}^p & \mathbf{K}_{rr}^p & \mathbf{K}_{ri}^p & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{i0}^p & \mathbf{K}_{ir}^p & \mathbf{K}_{ii}^p + \mathbf{K}_{ii}^s & \mathbf{K}_{i0}^s & \mathbf{K}_{ir}^s \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{0i}^s & \mathbf{K}_{00}^s & \mathbf{K}_{0r}^s \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{ri}^s & \mathbf{K}_{r0}^s & \mathbf{K}_{rr}^s \end{bmatrix}^{(j)} \begin{Bmatrix} \mathbf{0} \\ \Delta \mathbf{u}_r^p \\ \Delta \mathbf{u}_i^p \\ \mathbf{0} \\ \Delta \mathbf{u}_r^s \end{Bmatrix}^{(j)} = \begin{Bmatrix} \Delta \mathbf{r}_0^p \\ \mathbf{f}_r^p \\ \mathbf{f}_i^p + \mathbf{f}_i^s \\ \Delta \mathbf{r}_0^s \\ \mathbf{f}_r^s \end{Bmatrix}^{(j)}, \quad (3.17)$$

where superscripts p are the entries of the degrees of freedom belonging to a part and s are for supports. The subscripts i denote the interface between a part and supports, 0 are the fixed nodes at the base plate and r is the remaining degrees of freedom. In this work the stiffness of supports in the horizontal direction is much smaller than in the build direction and thus from here on it is assumed that there is no shrinkage induced stress in support elements, so $\mathbf{f}^s = \mathbf{0}$ and forces acting on the interface come only from the part, $\mathbf{f}_i^s = \mathbf{0}$.

Since in this approach support elements are also removed in the release stage, reaction forces on the interface i have to be tracked. For this purpose the term $\mathbf{K}_{ii}^p + \mathbf{K}_{ii}^s$ can be taken apart with use of a Lagrange multiplier vector

$$\begin{bmatrix} \mathbf{K}_{00}^p & \mathbf{K}_{0r}^p & \mathbf{K}_{0i}^p & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{r0}^p & \mathbf{K}_{rr}^p & \mathbf{K}_{ri}^p & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{i0}^p & \mathbf{K}_{ir}^p & \mathbf{K}_{ii}^p & \mathbf{0} & \mathbf{0} & \mathbf{0} & -1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{00}^s & \mathbf{K}_{0r}^s & \mathbf{K}_{0i}^s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{r0}^s & \mathbf{K}_{rr}^s & \mathbf{K}_{ri}^s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{i0}^s & \mathbf{K}_{ir}^s & \mathbf{K}_{ii}^s & 1 \\ \mathbf{0} & \mathbf{0} & -1 & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \end{bmatrix}^{(j)} \begin{Bmatrix} \mathbf{0} \\ \Delta \mathbf{u}_r^p \\ \Delta \mathbf{u}_i^p \\ \mathbf{0} \\ \Delta \mathbf{u}_r^s \\ \Delta \mathbf{u}_i^s \\ \Delta \mathbf{r}_i \end{Bmatrix}^{(j)} = \begin{Bmatrix} \Delta \mathbf{r}_0^p \\ \mathbf{f}_r^p \\ \mathbf{f}_i^p \\ \Delta \mathbf{r}_0^s \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}^{(j)}, \quad (3.18)$$

where the Lagrange multiplier $\Delta \mathbf{r}_i$ is the interface force to enforce that $\Delta \mathbf{u}_i^p - \Delta \mathbf{u}_i^s = \mathbf{0}$. Now, in order to keep track of the value of reaction forces $\mathbf{r}$ two components of Eq. (3.15) have to be found

$$\mathbf{r}_0^{p(j)} = \mathbf{r}_0^{p(j-1)} + \mathbf{K}_{0r}^{p(j)} \Delta \mathbf{u}_r^{p(j)} + \mathbf{K}_{0i}^{p(j)} \Delta \mathbf{u}_i^{p(j)}; \quad j = 1..M, \quad (3.19)$$

where $\mathbf{r}_0^{p(0)} = \mathbf{f}_0^{p(1)}$,

and

$$\begin{aligned} \text{either } \mathbf{r}_i^{(j)} &= \mathbf{S}_{ii}^{(j)T} \mathbf{r}_i^{(j-1)} + \mathbf{K}_{ir}^{p(j)} \Delta \mathbf{u}_r^{p(j)} + \mathbf{K}_{ii}^{p(j)} \Delta \mathbf{u}_i^{p(j)} + \mathbf{f}_i^{p(j)} \\ \text{or } \mathbf{r}_i^{(j)} &= \mathbf{S}_{ii}^{(j)T} \mathbf{r}_i^{(j-1)} - \mathbf{K}_{ir}^{s(j)} \Delta \mathbf{u}_r^{s(j)} - \mathbf{K}_{ii}^{s(j)} \Delta \mathbf{u}_i^{s(j)}; \quad j = 1..M. \end{aligned} \quad (3.20)$$

The latter version is used in this thesis.

Rewriting the first equation in terms of $\Delta \mathbf{u}_f$ gives

$$\mathbf{r}_0^{(j)} = \mathbf{r}_0^{(j-1)} + \mathbf{K}_{0r}^{(j)} \mathbf{S}_{rf}^{(j)} \Delta \mathbf{u}_f^{(j)} + \mathbf{K}_{0i}^{(j)} \mathbf{S}_{if}^{(j)} \Delta \mathbf{u}_f^{(j)}; \quad j = 1..M, \quad (3.21)$$

where $\mathbf{r}_0^{(0)} = \mathbf{f}_0^{(1)}$,

and the second results in

$$\mathbf{r}_i^{(j)} = \mathbf{S}_{ii}^{(j)T} \mathbf{r}_i^{(j-1)} - \mathbf{K}_{ir}^{(j)} \mathbf{S}_{rf}^{(j)} \Delta \mathbf{u}_f^{(j)} - \mathbf{K}_{ii}^{(j)} \mathbf{S}_{if}^{(j)} \Delta \mathbf{u}_f^{(j)}; \quad j = 1..M. \quad (3.22)$$

$\mathbf{S}_{rf}^{(j)}$ and $\mathbf{S}_{if}^{(j)}$ are selection matrices which choose from the set of free degrees of freedom only the ones belonging to the rest and interface for the part degrees of freedom for increment j and $\mathbf{S}_{rf}^{(j)}$ and $\mathbf{S}_{if}^{(j)}$ do the same but they select from the support elements degrees of freedom. Moreover, as optimization proceeds new interface degrees of freedom are added and the size of $\mathbf{r}_i$ increases. Hence, to prescribe reaction forces from $\mathbf{r}_i^{(j-1)}$ to $\mathbf{r}_i^{(j)}$ a selection matrix $\mathbf{S}_{ii}$ has to be used. $\mathbf{S}_{ii}$ chooses from $\mathbf{r}_i^{(j)}$ only degrees of freedom that belong to $\mathbf{r}_i^{(j-1)}$; $\mathbf{S}_{ii}^{(j)T}$ does the opposite. The distortions upon release can be simulated by

$$\begin{bmatrix} \mathbf{K}_{00}^p & \mathbf{K}_{0r}^p & \mathbf{K}_{0i}^p \\ \mathbf{K}_{r0}^p & \mathbf{K}_{rr}^p & \mathbf{K}_{ri}^p \\ \mathbf{K}_{i0}^p & \mathbf{K}_{ir}^p & \mathbf{K}_{ii}^p \end{bmatrix}^{(M+1)} \begin{Bmatrix} \Delta \mathbf{u}_0^p \\ \Delta \mathbf{u}_r^p \\ \Delta \mathbf{u}_i^p \end{Bmatrix}^{(M+1)} = \begin{Bmatrix} -\mathbf{r}_0^p \\ \mathbf{0} \\ -\mathbf{r}_i \end{Bmatrix}^{(M)} \quad (3.23)$$

or in terms of f degrees of freedom Eq. (3.10) may be rewritten

$$\mathbf{K}_{ff}^{(M+1)} \Delta \mathbf{u}^{(M+1)} = -\mathbf{S}_{f0}^{(M)} \mathbf{r}_0^{(M)} - \mathbf{S}_{fi}^{(M)} \mathbf{r}_i^{(M)} \quad (3.24)$$

where $\mathbf{S}_{f0}^{(M)}$ and $\mathbf{S}_{fi}^{(M)}$ are selection matrices that choose unsuppressed degrees of freedom belonging to the base plate and the interface respectively.

Note that for calculation of distortion after release from the base plate it is not the internal stresses are monitored but the manifestation of internal stresses by reaction forces in the suppressed base plate nodes. Both approaches are equivalent in the amount of stored data and calculation. The force vector option has been chosen because it provides more intuitive description of the system for the adjoint sensitivities calculations.

3.2.4 Simulation without supports

In Chapter 5 the optimization will be done without supports structures and a numerical model that does not incorporate supports elements will therefore be used. For that case, a simulation without supports, Eq. (3.10) reduces to

$$\mathbf{K}_{ff}^{(M+1)} \Delta \mathbf{u}^{(M+1)} = -\mathbf{S}_{f0}^{(M)} \mathbf{r}_0^{(M)}, \quad (3.25)$$

and the reaction force tracking is based on Eq. (3.16) only

$$\mathbf{r}_0^{(j)} = \mathbf{r}_0^{(j-1)} + \mathbf{K}_{0f}^{(j)} \Delta \mathbf{u}_f^{(j)}; \quad j = 1..M, \quad \text{where } \mathbf{r}_0^{(0)} = \mathbf{f}_0^{(1)}. \quad (3.26)$$

This equation may be seen as a simpler form of Eq. (3.21): as there are no supports structures there is also no distinction between degrees of freedom belonging either to the base plate, supports, interface or the rest, so superscript and most of selection matrices $\mathbf{S}$ are gone.

3.2.5 Stress relaxation

Heat treatment may be applied after the printing stage and before the separation from the base plate. For Ti6Al4V this is usually Hot Isostatic Pressing with two hours of annealing at 788°C. The goal of heat treatment is to get rid of residual stresses, to increase the homogeneity of the microstructure and to remove any possible voids. After the heat treatment, separation from the base plate and removal of supports are carried out. In that way it is possible to reduce the distortions after the separation. The heat treatment is simulated by means of the viscoplastic material model of Section 2.3.2. The creep simulation will be performed for the support elements as well. Viscoplasticity is assumed here for both the vertical and the isotropic fraction. The isotropic fraction is solved with Eq. (2.71) and has the consistent material tangent defined in Eq. (2.76). For the vertical fraction Eq. (2.71) is reduced to a single stress component and the consistent material tangent matrix has to be one dimensional, which takes the form

$$\mathbf{D}_M = \frac{E}{1 + EmA \exp^{-Q/RT} \Delta t \left| \boldsymbol{\sigma}_{vm}^{(n+1)} \right|^{m-1} \text{sgn}(\boldsymbol{\sigma}_{vm}^{(n+1)})} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3.27)$$

For the creep in Eq. (2.66) the parameters A and m have to be determined. In literature, most creep test for Ti6Al4V are performed at a lower temperature range than the one at which the heat treatment takes place. Therefore in this model only values based on Badea *et al.* [137] are used. Since their work considers only the temperature range from 450°C to 600°C the values are extrapolated and should be therefore treated as indicative values used for purely numerical studies. For a temperature of 788° the extrapolated value of A is 2.16×10^{-15} , $m = 5.72$ and $Q = 354 \text{ kJ/mol}$.

3.3 Summary

In this section various components that can be used for simulation of additive manufacturing were introduced. They can be used to perform full numerical modelling of the SLM process that consists of printing, heat treatment, removal of supports and cutting from the base plate. Various existing modelling techniques have been presented, such as a meshing script, element deposition, layers lumping and inherent strains. Additionally, the element growth technique has been proposed as an alternative to the lumped approach. Moreover, a separate model to describe support structures based on material fractions has been developed. Finally, the FEM proce-

dure with all necessary equations has been derived to model 3D printing process and to create a basis for topology optimization.

Chapter 4

Numerical Verification of SLM Model

This chapter presents numerical verification of methods proposed in Chapter 3 is presented. Separate sections are dedicated to different modelling techniques. First, simulations using only the lumped approach are performed to check convergence of results with mesh refinement. The simulation with an element row for each material layer is taken as a reference. Then, the result from the lumped approach are compared with those from the element growth method. Next, implementation of support elements is verified and compared with results obtained by using either fixed degrees of freedom or regular material instead of supports. Finally, stress relaxation simulations are performed for the heat treatment stage.

4.1 Lumped approach

In this section accuracy of the lumped approach will be considered. The goal of this section is to investigate the influence of the mesh size on the accuracy of results, both displacements and stresses, and on the calculation time. For this purpose two numerical examples will be used: bending of a bar and a use case.

4.1.1 Bending bar

First a simple beam printing case will be simulated. Usually the beam is printed with supports underneath. The role of the supports is to allow the printing of overhangs and to prevent distortions of the part. After printing, the beam and supports are removed from the base plate and the characteristic beam warp occurs due to the presence of residual stresses. The beam curvature may be measured as a quantification of the shape distortions. Based on the accuracy of the numerical model different values for the curvature may be obtained and they may consequently be used for the assessment of the accuracy of the simulation. The curvatures will be compared with the curvature obtained from a reference simulation where the element height corresponds to the layer thickness. The beam used in this work with its dimensions is presented in Figure 4.1.

Figure 4.1: Dimensions of the bending bar in mm.

In this example as in the rest of this work Ti6Al4V is used as material, with Young's modulus $E = 110$ GPa and Poisson's ratio $\nu = 0.342$. For models with plasticity the yield strength is set to 910 MPa and a linear isotropic hardening model is assumed with $H = 10$ MPa. The inherent strain of $\epsilon_{\text{inh}} = [0.00598, 0.0, 0.0, 0.0, 0.0, 0.0]$ is assumed, which imitates a unidirectional scanning strategy.

The inherent strain value corresponds to the shrinkage of Ti6Al4V from the temperature 800°C to 150°C [138] with thermal expansion coefficient $9.2 \times 10^{-6} \text{ K}^{-1}$ and is similar to inherent strains assumed by Munro *et al.* [128].

During printing the bottom of both pillars is fully fixed. Moreover, supports underneath the beam are simulated by suppressing the displacements of the bottom nodes of the beam part in the z -direction. After the printing stage, the separation from the baseplate is simulated by releasing the bottom nodes of the right pillar and removal of supports by freeing the bar's bottom nodes.

The simulation is performed for different mesh sizes and the results are compared. Mesh sizes of 1 mm, 0.5 mm, 0.25 mm and of the material layer height 0.05 mm are used. Since the element height 0.05 mm is equal to the layer height it is treated as a reference case, with which other simulation results will be compared. The number of elements and of degrees of freedom for a given mesh size are compared in Table 4.1.

Table 4.1: Mesh data of bending bars.

	1 mm	0.5 mm	0.25 mm	0.05 mm
Number of nodes	3597	23688	170355	803723
Number of elements	2390	19060	152240	761200

The material layer thickness is $50\mu\text{m}$ which means that 20, 10, 5 or 1 layers per element will be deposited. The number of calculation increments is equal to 8, 16, 32 and 160 respectively for the printing stage and a single increment for the release stage for all mesh sizes.

To fully investigate the performance of the methods, simulations for both elastic and plastic material behaviour will be done. Moreover, two types of scanning will

Figure 4.2: Distortions obtained after the cutting from the base plate.

be considered: longitudinal, where the same inherent strain ϵ_{inh} is applied in every deposition, corresponding to scanning every layer in the same direction; and scanning with 30° rotation of the orientation of each consecutive layer, which means rotation of the inherent strains tensor in each growth step and applying a direction averaged inherent strain in the case of the lumped method. First unidirectional scanning is investigated.

The curvatures obtained after separation of the beam from the base plate are presented in Figure 4.2. The curvatures from rotational scanning strategy are smaller than those from the unidirectional strategy which is due to the lower stresses in the x -direction. Surprisingly, bigger curvatures are obtained for simulations with plastic material behaviour. Curvature depends mostly on the gradient of σ_{xx} in the building direction. Two pillars on the bar edges are stiff supports that prevent contractions of bar layers, so little strain relaxation due to layer shrinkage occurs. In the case of plasticity yield stress works as a threshold that determines maximal values of stress and the gradient depends also on the hardening coefficient H . In the case of linear elastic material properties putting the same amount of inherent strains with no room for its relaxation leads to a smaller gradient. Consequently, in the release stage the bar contracts in the x -direction instead of bending up.

The stress plots after the printing stage for the elastic and plastic material properties are presented in Figure 4.3. The plot shows that for the elastic material there are still high stresses at the bottom of the beam, contrary to the results from the material with plasticity. These high tensile stresses at the bottom act in the opposite direction

Figure 4.3: Stress in the x -direction for elastic material (top) and plastic material (bottom) before the release from the base plate, for element size 0.05 mm.

to the stresses that produce the upward curvature and therefore they reduce it.

Table 4.2 shows the calculation times required for every simulation. The increase in calculation time with the increase of the number of elements is clear but the calculation time is reasonable even for the element height 0.05 mm.

The comparison of equivalent stress before and after the release stage for different mesh sizes for material with plasticity is presented in Figure 4.4. As the element size increases the distributions clearly become less detailed. This can be observed in particular for the horizontal bar, where for the element size equal to 0.5 mm and 1.0 mm the equivalent stress distribution is almost uniform and is much lower than the stresses from the reference case. Nevertheless, when comparing stress distributions with the curvature plots from Figure 4.2, in case of elastic material properties the element size 0.5 mm still yields reasonable results.

Nodal position update due to the distortions

The above examples were performed without the mesh position update and with storage of the total displacements in the separate vector as Eq. (3.9) indicates. To prove the correctness of this approach and to investigate how it influences the obtained results, simulations with and without mesh movement were performed and compared. The lumped approach is considered for both the elastic and plastic material model. The curvatures obtained in that way are presented in Figure 4.5. No clear difference was observed in curvatures and the same applies also for stress and plastic strain distribution, using mesh update is therefore clearly not necessary.

Table 4.2: Time required for each simulation (h:mm:ss).

	1 mm	0.5 mm	0.25 mm	0.05 mm
elastic	0:00:05	0:00:32	0:07:36	3:33:59
plastic	0:00:08	0:00:57	0:12:43	3:52:10

Figure 4.4: Equivalent stress for plastic material properties before (left) and after (right) the release stage for unidirectional scanning strategy.

Figure 4.5: Comparison of curvatures for methods with and without mesh update.

4.1.2 3D use case

To further investigate the possibilities of the method a simulation for a use case geometry was performed. It is a typical part that can be used in the aerospace industry for connection of hoses. The part geometry with some of its dimensions is presented in Figure 4.6.

The part has been rotated by an angle of 45° to get rid of most of the overhanging surfaces. Moreover, supports for the part were created by means of Slic3r software and its box feature. The support structures have been generated for every downwards pointing surface that is located inside the box regions. The geometry with support-forcing boxes and the obtained final geometry are shown in Figure 4.7.

The material and inherent strains used are the same as in the bending bar example. Here, two mesh sizes will be compared, 1 mm and 0.5 mm, with again a layer thickness of 0.05 mm. The total height of the part in the build job is 49 mm. This means that

Figure 4.6: Geometry of a part that serves as a use case.

the approach for size 1 mm, 49 printing steps and single release step are needed, and for size 0.5 mm, 98 printing steps and a single release stage step. In the release stage the part is separated from the base plate, supports are deactivated, and sufficient degrees of freedom at the bottom are suppressed to fix the part in space, avoiding rigid body movements. In total 21591 nodes connected by 15269 elements are needed to describe the part for a mesh size of 1 mm and 101463 elements connected by 125934 nodes for a mesh size of 0.5 mm. Here only plastic material behaviour is considered and a rotation of the scanning vector by 30° for each consecutive layer is assumed. Elements that describe the supports have the same properties as the solid part.

Figure 4.7: Geometry of the part with bounding boxes that enforces supports (left) and visualization of the final geometry (right) in Slic3r software.

The simulations have been performed in that form. In total the calculation time was over 1 minute for the element size 1 mm and almost 12 minutes for the element size 0.5 mm. The calculation times are short but it has to be stated that in a SLM chamber parts of bigger size could be printed for which longer calculation times are expected.

Figure 4.8: Equivalent stress before (top) and after (bottom) the release stage for the element size 1 mm (left) and 0.5 mm (right).

Figure 4.8 shows a comparison of equivalent stress between the two mesh sizes before and after the release stage. The difference in plots is substantial and it seems that element size 1 mm greatly underestimates the stress level.

The plots of equivalent plastic strain in Figure 4.9 indicate the same level of underestimation in case of the element size 1 mm. For the element size 0.5 mm the areas where there is a concentration of plastic deformation can be clearly seen. It occurs mostly in the corners and in places where supports are connected to the part and the base plate.

Displacements in the z -direction before (Figure 4.10) and after release from the base plate (Figure 4.11) are also compared. Before release, the maximal distortions are around 0.2 mm for the element size 1 mm and 0.3 mm for the element size 0.5 mm. The values increase significantly after the release stage up to 0.55 mm and 0.75 mm. The highest distortions after the release stage are located on the sides of the part, where long horizontal layers occur.

Figure 4.9: Equivalent plastic strain before the release stage for the element size 1 mm (left) and 0.5 mm (right)

Figure 4.10: Displacement in the z -direction before the release stage for the element size 1 mm (left) and 0.5 mm (right).

Figure 4.11: Displacement in the z -direction after the release stage for the element size 1 mm (left) and 0.5 mm (right).

4.1.3 Conclusions

Based on the results it can be stated that the accuracy of the presented approach depends greatly on the mesh size. The element size 1 mm leads to excessive mesh coarsening and in the end underestimates both distortions and stresses. Therefore it has been decided from here on to use an element size of 0.5 mm as a maximal possible mesh size that still yields reasonable results.

4.2 Element growth method

In this section results of the lumped and the element growth method will be compared. Again the results will be evaluated for two geometries: the bending bar and the use case. This time only mesh size 0.5 mm is shown to reduce the amount of presented results.

4.2.1 Bending bar

For the comparison the same geometry as in Section 4.1.1 is used with the same parameters. In this section the bending bar simulation with element layer thickness equal to layer thickness is used as a reference example. When the height of an element row is equal to the layer thickness the lumped approach is the same as the growth method. As mentioned, results are considered only for the element size 0.5 mm and two scanning strategies are again compared, the unidirectional and the rotational strategy.

The simulation using the element growth method consists of 160 increments for the printing stage and 1 increment for the release stage. In total, calculations for the unidirectional scanning strategy took 2 minutes and 45 seconds in the case of elastic material and 4 minutes and 5 seconds for plastic material. Time is more than 5 and 4 times longer, respectively than the same calculation for the lumped approach. The expected calculation time extension was 10 times, since for every element row ten times more increments are needed, but in the end, the calculations take less time than expected. Probably reading of an input file, memory allocation and other programming related issues are important for time consumption. The calculation times for unidirectional and rotational scanning strategy are similar.

In Figure 4.12 comparison of curvatures obtained after the separation from the base plate are presented. The plots show that the growth method yields more accurate distortion predictions in every case except for the rotational scanning strategy with elastic material properties. In the case of elastic material properties, most of the deformation happens at $x = 12$ mm, where the beam connects to the left pillar and it seems that the growth method has difficulty with correctly describing these deformations.

The equivalent stress distributions in Figure 4.13 clearly show that the accuracy of stress predictions substantially improves in the case of the growth method. The results obtained from the growth method are on a similar level to those from the reference example and reflect the distribution in detail.

Figure 4.12: Curvatures obtained after the cutting off from the base plate.

Figure 4.13: Equivalent stress for unidirectional scanning strategy with plastic material properties before (left) and after (right) the release stage for reference (top), growth (middle) and lumped (bottom) method.

4.2.2 3D use case

The same comparison has been made for the use case for mesh size 0.5 mm. Since this time it is impracticable to print the use case with an element height equal to a layer thickness the comparison is made between the lumped and growth method with the assumption that the growth method yields more accurate results. This can be supported by multiple examples performed for the bending bar with different mesh sizes, the results of which are not presented in this thesis for the sake of brevity. The calculations for the growth method consist of 980 increments for the printing stage and a single increment for the release, which took 3 hours and 26 minutes. This is almost 18 times longer than the lumped approach, with the number of increments

Figure 4.14: Results of the element growth method: equivalent stress (left), plastic strain (middle) before the release stage and displacement in the z -direction after the release stage.

increased by a factor of 10. This is probably due to the material nonlinearity, and much higher stresses and plastic deformations than the lumped approach.

Figure 4.14 shows different results for the simulation performed using the growth technique. When compared with the results from Figure 4.8, stresses are on a slightly higher level than those of the lumped method. The comparison of the equivalent plastic strains before the release stage to Figures 4.9 clearly shows that the growth method predicts much more plastic deformation, which confirms the reasoning behind the extended calculation time. Finally, the plot of displacement in the z -direction shows lower displacements after the release stage in comparison with Figure 4.11. This is in contrast to the bigger distortion in the case of the bending bar and indicates that the dependence of distortions on residual stresses is not always straightforward and relates to the geometry of a part.

4.2.3 Conclusions

After the investigation of the use case results it may be concluded that the application of the element growth method rather than the lumped approach is not always beneficial. For the higher parts, it makes more sense to run the lumped method with a finer mesh size because for many material layers the required calculation time increase of the growth method is too big. It seems that the growth approach should be used mainly in certain cases where the geometry is complex and application of inherent strains for bundles of layers may lead to too much averaging and to the underestimation of results. Consequently, it is decided to use in the rest of this work the lumped approach.

4.3 Support elements

This section covers numerical verification of support elements implementation proposed in Section 3.2.2. Simulation will be here performed using only the lumped approach since the interest is focused on the effect of the application of supports alone. The element size is set to 0.5 mm and again both the bending bar and the use case will be considered. The obtained results will be compared with the results from

Figure 4.15: Geometry of the bending bar with bounding boxes that enforces supports (left) and visualization of the final geometry (right) in Slic3r software.

Section 4.1.

4.3.1 Bending bar

The same parameters as previously are utilized. In the material model of the supports (Eq. (3.12)), the size of the vertical fraction is set to 0.99 and the isotropic fraction to 0.01. Both elastic and plastic material responses are investigated. For the bending bar, support elements are introduced by means of the bounding box feature of Slic3r software, which enforces support structures under the bending bar. Bar geometry with the bounding box is depicted in Figure 4.15.

During the simulation of the printing stage the bottom of the part is fixed in every direction. In the release stage only the left pillar is left attached to the base plate and the supports elements are deactivated, which mimics their virtual removal. The simulations are compared with the results of the previous simulation that used the lumped approach, where supports were imitated by suppression of degrees of freedom.

Figure 4.16: Curvatures obtained after the release stage for simulation with and without support elements.

The obtained curvatures for the elastic and plastic material properties with unidirectional scanning strategy are presented in Figure 4.16. The addition of support elements clearly leads to less beam curvature.

The plot of equivalent stress before and after the release stage for the plastic material properties is presented in Figure 4.17. In the case of support elements definition, for the equivalent stress the absolute value of stress for fraction f_v is plotted since this

Figure 4.17: Equivalent stress for plastic material properties, before (top) and after (bottom) the release stage for simulations with (left) and without (right) support elements.

is the quantity that determines plastic deformation. The obtained distributions are similar. What can be observed from the plots is that after the printing stage there is less stress at the bottom of the beam for the case with support elements. In the case of the release stage there is a lot of similarity between the plots and hardly any difference can be spotted by visual inspection.

4.3.2 3D use case

To further check the proposed implementation of the support elements the use case geometry is again used. Since in a previous Section (4.1.2) there were elements that described supports geometry, the simulations are performed this time for the same part shape but with the changed material properties for supports.

Inspection of the equivalent stress in Figure 4.18 and its comparison with Figure 4.8 does not show much difference in stress distribution in the part. In the supports, on the other hand, stresses have changed significantly. The accumulated stress in the left pillar is much lower due to the assumption that there is no inherent strain in

Figure 4.18: Equivalent stress before (left) and after (right) the release stage for simulation with support elements.

Figure 4.19: Displacement in the z -direction before (left) and after (right) the release stage for simulation with support elements.

the supports. Moreover, there is a stress concentration in the bottom right corner of the supports that connect the wider member of the part to the base plate. Wide dimensions of the part there means that long, continuous layers of material have been deposited, which curl up, leading to the stress concentration. In the case of the stress distribution after the release hardly any differences can be spotted between the case with and without the support elements.

Distortions in the z -direction before and after the release stage are shown in Figure 4.19. When compared with simulations with the support elements not having the special material properties in Figures 4.10 and 4.11 it can be noticed that distortions before the release stage simulation with support elements are slightly lower and after the release the trend is reversed and that distortions are slightly higher, especially on the laterally extending sides of the part.

4.4 Stress relaxation

In this section simulation results of stress relaxation due to heat treatment after printing are presented. The viscoplastic material model of Section 2.3.2 is applied for this purpose. The heat treatment lasts for two hours and takes place at a temperature of 788°C . After the stress relief stage the release stage takes place, as in the previous examples.

Again, the same two geometries with identical material properties and boundary conditions are used with mesh size 0.5 mm . The support elements model is applied in both cases.

4.4.1 Bending bar

In the case of the bending bar the simulation took over 52 minutes for elastic material properties and over 1 hour and 6 minutes for material with plasticity. Compared to the previous simulations the calculation time increased largely. Mostly it is due to the fact that simulations with the extrapolated creep parameters require fine time stepping in the range of $10^{-7} - 10^{-10}\text{ s}$, especially at the beginning, depending on the mesh size. With too big step size, high creep rate values lead to problems with

Figure 4.20: Curvatures obtained after the cutting from the base plate for bending bar with and without heat treatment.

convergence. In the end hundreds of creep calculation increments are required for two hours of annealing.

The obtained curvatures are compared with the simulation without stress relief in Figure 4.20. The beams remain almost perfectly flat, which means that most of residual stresses are gone.

The stresses just after the creep stage for material with plasticity are presented in Figure 4.21. The plot confirms that most of the residual stresses are gone due to the annealing. What is left from residual stress is not greater than 30 MPa which explains the flat lines obtained in curvature plots. Unfortunately, there is no experimental validation.

4.4.2 3D use case

The same use case geometry with mesh size 0.5 mm has been used but this time a stress relief occurred before the release stage. The simulations for 0.5 mm took 10:02:20, so calculation time again increased considerably.

The equivalent stress, creep strain and displacement in the z -direction are presented in Figure 4.22. The same trend as for the bending bar can be observed. Almost all stresses are gone and they do not exceed 30 MPa. Moreover, most of the equivalent creep strain can be observed on the edges and in the corner of the part where the

Figure 4.21: Equivalent stress distribution for simulation with plasticity after the stress relief stage.

Figure 4.22: Equivalent stress (left), creep (middle) after the heat treatment and displacement in z -direction after the release stage.

stress concentration occurred. Comparison with displacements presented in Figure 4.11 shows that the biggest displacement in the positive z -direction now occurs in the middle of the part, which indicates that the high tangential stresses that were causing the curl are gone.

4.5 Summary

In this section various components that can be used for simulation of additive manufacturing were verified. Based on multiple simulations it has been concluded that for geometries whose dimensions do not exceed 50 mm the minimum voxel size should be at least 0.5 mm. Moreover, the lumped approach has been compared with the element growth method. The element growth technique gives more accurate and detailed results than the lumped approach. Nevertheless, it seems impracticable to use it on the part scale for the whole simulation time but it may still be advantageous to apply it for some selected parts of the mesh that may require more detailed treatment of inherent strains. Introducing support elements with direction-dependent stiffness allowed for inclusion of these auxiliary elements without changing the mesh size. Finally, a creep model has been used for the stress relief simulation to perform heat treatment. All of the methods have been subjected to preliminary verification but, unfortunately, only numerically. An obvious next step should be to use experimental validation in order to tune the properties of each of the models.

Chapter 5

Topology Optimization and Adjoint Sensitivities

In this chapter equations that are required to apply topology optimization to additive manufacturing are developed. For this purpose the numerical techniques for simulation of 3D printing presented in Chapter 3 are utilized. With their use, information about the part performance is gathered by means of selected distortion measure functions. Moreover, sensitivities of the shape change with respect to these functions have to be defined in order to use a gradient-based optimization scheme. For calculation of sensitivities the adjoint method is utilized which is suitable for application in optimization problems with many design variables and few evaluation functions. Although, the optimizations will be performed for a numerical model consisting of the layer lumping technique only, the presented formulation is also valid for the element growth method.

The first investigated problem concerns maximization of the part stiffness, or minimization of compliance, with constraints on shape distortions that are characteristic for selective laser melting. The second problem assumes a fixed shape of the part and focuses on minimization of the printing distortions by changing the supports distribution. Both are solved with the assumption of elastic material behaviour only. When heat treatment is applied, it is assumed that the residual stresses are gone after heat treatment and no distortions occur after the release stage. Therefore, if the optimized process incorporates heat treatment the final distortions are only the sum of the printing stage steps; other stages are not required.

5.1 Topology optimization

Topology optimization by the SIMP method is used [64] in this work. The design space is divided into a regular mesh containing N_e identically sized finite elements, each with volume V_0 . The goal is to find the optimal material distribution ρ . In general its elements could have one of two states: $\rho = 0$ for void and $\rho = 1$ for presence of material. Using ρ in a zero-one state results in a discrete optimization problem. Discrete problems are difficult to solve when the number of design variables

is in the range of thousands or millions, which is typical for topology optimization. Therefore it is beneficial to relax the discrete and hard-to-solve problem and change it into a feasible, continuous form. This is done by allowing ρ elements to be in a continuous range: $0 \leq \rho_e \leq 1$.

One of the commonly adopted problems solved by topology optimization is minimization of the structure compliance, thus maximization of the stiffness with a constraint on the allowable volume. The generic mathematical form of this problem may be defined as

$$\begin{aligned}
 \min_{\rho} : \quad & c(\rho) , \\
 \text{s.t.} : \quad & \frac{1}{N_e} \sum_{e=1}^{N_e} \rho_e V_e \leq V, \\
 & g_i \leq 0 \quad i = 1, \dots, N_g, \\
 & 0 < \varepsilon \leq \rho_e \leq 1 \quad e = 1, \dots, N_e, \\
 \text{with:} \quad & \mathbf{K}(\rho)\mathbf{u} = \mathbf{f},
 \end{aligned} \tag{5.1}$$

where $c(\rho)$ is an objective function, V_e is the volume of an element, V is the maximum allowable relative volume (%), N_e is the number of design variables, g_i are inequality constraints other than volume constraint (for example maximum allowable distortion during printing) and N_g is the number of the constraints ε is the lower density threshold, introduced to prevent singularity of the solution matrix, $\mathbf{K}(\rho)$ is the structural stiffness matrix, $\mathbf{f}$ and $\mathbf{u}$ are the load vector and the displacement vector.

In this work the nested approach is used. This means that instead of enforcing a structural equilibrium equation explicitly in the optimization and solving it simultaneously for ρ and $\mathbf{u}$, the structural equilibrium, known as the analysis problem, is evaluated separately. In that way optimization reduces to searching only for the optimal density distribution for which a finite element analysis is performed in order to evaluate displacements, the objective and constraint functions value. Finally, sensitivities of these functions with respect to the design variables are calculated and the process repeats until the convergence criterion is fulfilled.

The continuous optimization problem results in design variables that are not only 0 or 1 but can also have intermediate values. The main issue with the intermediate values is that they are often hard to justify physically, especially for complex multi-physical problems. Moreover, they create difficulties when it comes to manufacturing. In the case of mechanical problems the use of lattice structures is possible, but obtaining a full map between the density of an element and its stiffness is often difficult. For that reason it is preferable to have a density distribution that consists of elements that belong either to void or solid material.

To get rid of the intermediate states the densities are mapped to properties of elements by scaling of the Young's modulus E

$$E_e = \rho_e^{p_s} E. \tag{5.2}$$

The penalization exponent p_s , which is used to enforce a black-and-white solution, was proposed by Bendsoe [139] as a part of the SIMP approach and it is applied in this work. It makes intermediate densities expensive in the sense of 'material cost', because the stiffness of elements is scaled in a progressive way with density (see

Figure 5.1: Relative Young's modulus plotted for several p_s values.

Figure 5.1). It is usually set to 3 to force optimization into producing designs with densities that are either close to 0 or 1.

Eq. (5.1) is usually not solved directly but filtering is applied in order to get rid of mesh-dependence and checkerboard patterns [140]. Mesh dependence originates from the fact that the continuum problem is ill-posed, i.e. there is no optimal solution and with a decrease of the mesh size there is always a refined solution that is closer to optimum. A checkerboard pattern is a numerical error that introduces artificial stiffness to the structure. A common way of dealing with these problems is by applying mesh filtering. Here density based filtering is implemented [141].

5.1.1 Shape optimization with distortion constraint

In this work, the shape optimization of a structure is pursued, that minimizes compliance (maximizes stiffness) under a prescribed load with constraints on structure volume and on distortions during additive manufacturing. Therefore, Eq. (5.1) takes the form

$$\begin{aligned}
 \min_{\boldsymbol{\rho}} : \quad & c(\boldsymbol{\rho}) = \mathbf{f}^T \mathbf{u} , \\
 \text{s.t.} : \quad & \frac{V_0}{N_e} \sum_{e=1}^{N_e} \rho_e \leq V, \\
 & g_w \leq 0, \\
 & g_d \leq 0, \\
 & 0 < \varepsilon \leq \rho_e \leq 1 \quad e = 1, \dots, N_e, \\
 \text{with:} \quad & \mathbf{K}(\boldsymbol{\rho}) \mathbf{u} = \mathbf{f},
 \end{aligned} \tag{5.3}$$

where $c(\boldsymbol{\rho})$ represents compliance. Two distortion constraints are investigated. The first one, g_w , is distortion during build, which could result in a catastrophic collision of the recoater with the distorted product. The second, g_d , is the final distortion of the work-piece once released from the build plate, which could result in a product with out-of-spec dimensions.

Application of distortion constraints requires solving a numerical model in addition to the one which assesses the compliance for a given material distribution. The

Figure 5.2: Schematic of optimization for compliance minimization only (left) and with additive manufacturing constraints (right).

second model considers the printing of the part and it follows the numerical scheme as presented in Section 3.2.1. The flowchart of the optimization is presented in Figure 5.2.

Recoater collision

As the printing process proceeds, with every new layer deposition the build plate is lowered by a distance equal to the layer thickness and a new layer of powder is evenly spread over the working area with help of a recoater or a wiper blade. The recoater scans over the area and sweeps off the surplus powder, assuring a uniform powder layer height. In the case of excessive shape distortions in the positive build direction the recoater may collide with the part, see Figure 5.3.

The danger of recoater collision is determined after the deposition of each layer, by the positive z -distortion of the top nodes of that layer. The constraint equations are of the form

$$\max(\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \circ \Delta \mathbf{w}^{(i)}) - d_w \leq 0, \quad (5.4)$$

where $\Delta \mathbf{w}^{(i)}$ are the vertical (z -)components of the displacement increments $\Delta \mathbf{u}_i^{(i)}$ of the i^{th} layer top nodes after it has been deposited. The symbol ' $\circ$ ' denotes the Hadamard product: element-by-element multiplication of two arrays, resulting in an array of the same dimension. A zero density means that no material is associated with the node of interest and no danger of collision exists. The array $\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho})$ contains nodal densities that are defined as the average of the densities of the currently active

Figure 5.3: Schematic recoater collision scenario.

elements connected to a given node

$$\varphi_k^{(i)}(\boldsymbol{\rho}) = \frac{\mathbf{S}_k^{(i)\top} \boldsymbol{\rho}^{(i)}}{N_k^{(i)}}, \quad (5.5)$$

where $\mathbf{S}_k^{(i)}$ selects the currently active elements, connected to node k and $N_k^{(i)}$ is the number of active elements connected to node k . The allowable distortion d_w depends on the deposited layer thickness t and the relative density of the metal powder ρ_p

$$d_w = \frac{t}{\rho_p}. \quad (5.6)$$

To make Eq. (5.4) differentiable, an envelope function [142] is used to approximate the max function

$$g_w^{(i)} = \frac{1}{r} \log \left(\mathbf{1}^\top \exp \left(r \frac{\varphi^{(i)}(\boldsymbol{\rho}) \mathbf{w}^{(i)}}{d_w} \right) \right) - 1 \leq 0, \quad (5.7)$$

where, r is a scaling parameter. $\exp()$ of the array is assumed to give an array of the exponentials of the array components. Its product with $\mathbf{1}^\top$ is a summation of its elements resulting in a scalar. The constraints of Eq. (5.7) may either be applied to every individual layer i or aggregated by the smooth max function over all layers into a single constraint g_w . In this way the number of constraints is reduced but the convergence of optimization may be affected.

Shape distortion

Distortion of a node consists of a series of displacement increments from each separate calculation step starting from the step in which the node got activated up to the final distortion from the release stage. If the distortion of one node is to be measured and constrained relative to other nodes, that also distort during the building, a transformation has to be applied. Assume that between a number of nodes, of which the first has been deposited as part of layer j , a final distortion vector is calculated in the form

Figure 5.4: Schematic of global distortion calculation. The theoretical shape is indicated in grey.

of

$$\mathbf{d} = \sum_{i=j}^{M+1} \mathbf{Z}^{(i)} \Delta \mathbf{u}_f^{(i)}, \quad (5.8)$$

where $\mathbf{Z}^{(i)}$ is a transformation matrix, which picks only the considered nodes and applies a transformation to the intended assembly position, see Figure 5.4. If it is assumed that heat treatment took place right after the printing stage, a simulation is performed without the release stage, i.e. up to step M .

Figure 5.4 depicts distortion during consecutive manufacturing stages. Due to the mesh distortions the theoretical assembly points (bright yellow dots) move away from their assumed positions (faded yellow dots), so after the release from the bottom stage the whole structure has to be transformed in such a way that it puts the assembly nodes back in their intended position. The displacement of a node, on which a constraint is applied (bright green dot) is then measured with respect to its designed coordinates (faded green dot) after the transformation of the part.

Figure 5.5: Design domain and design variables for supports distribution optimization.

The actual constraint g_d is a scalar function and is specified as:

$$g_d = \frac{1}{2} \mathbf{d}^T \mathbf{d} - \frac{1}{2} d_{\max}^2 \leq 0, \quad (5.9)$$

which signifies that the displacement of the considered node is constrained to a spherical tolerance region.

5.1.2 Supports distribution optimization

In supports distribution optimization it is assumed that the shape of the part is already defined and cannot be altered. This is the case for example for biomedical implants, where the shape of the body defines the geometry which has to be exactly as designed. If the geometry and position of a part are fixed, only the position and the amount of supports can be optimized. To that end the printed part is treated as a collection of passive elements with predefined densities equal to 1, see Figure 5.5. As explained in Section 3.2.2, supports are assumed to be slender structures that create connections in the vertical direction. The mesh is in the form of voxels. Therefore several elements are grouped in a column to create a single support element, i.e. a column of elements with the same density ρ_e . It is assumed that supports not only connect the part with the base plate but may also interconnect part members in the vertical direction. Every region that is below an element that belongs to the part is considered as belonging to the potential support domain, see e.g. Figure 3.7.

In SIMP intermediate densities are unwanted and a crisp design is favoured. In the case of supports there is a desire to create supports with varying densities. The design will not be fully pushed toward a black-and-white solution; instead, intermediate densities will be allowed. The resulting density can be translated to the area density of vertical supports. Moreover, mesh filtering is turned off for the same reasons.

Shape distortion and recoater collision

The constraints are again shape distortion and recoater collision. But this time distortions of all outer nodes of the part are considered instead of the chosen region.

The distortions of all the nodes that are on the outside of the part are gathered into a single aggregation p -norm function as

$$g_p = \sqrt[p]{\mathbf{1}^T \left| \sum_{i=1}^{M+1} \Delta \mathbf{u}_t^{(i)} \right|^p}. \quad (5.10)$$

Here $\mathbf{u}_t$ is a vector of the degrees of freedom that belong to the part and lie on its surface. It does not have a constant length and may increase with newly deposited elements. To choose nodes that lie on the surface a selection matrix $\mathbf{S}_{tf}^{(i)}$ is applied

$$\Delta \mathbf{u}_t^{(i)} = \mathbf{S}_{tf}^{(i)} \Delta \mathbf{u}_f^{(i)}. \quad (5.11)$$

Note here, that first summation is done over increments for the same node, so distortion in the opposite direction between different increments may cancel out. When stress relaxation heat treatment is applied before release from the base plate, summation is done up to step M .

In supports optimization a part shape is predefined and compliance minimization cannot be an objective. The objectives considered in this work are either minimization of shape distortions or supports volume. If in the optimization routine the objective is to minimize the distortion with a constraint on the supports volume the optimization problem reads as follows

$$\begin{aligned} \min_{\boldsymbol{\rho}} : \quad & g_p(\boldsymbol{\rho}) \\ \text{s.t.:} \quad & \frac{1}{N_e} \sum_{e=1}^{N_e} \rho_e V_e \leq V, \\ & g_w \leq 0, \\ & \rho_j^e = \rho_e \quad j = 1, \dots, N_s^e \quad e = 1, \dots, N_e, \\ & 0 < \varepsilon \leq \rho_e \leq 1 \quad e = 1, \dots, N_e, \\ \text{with:} \quad & \mathbf{K}(\boldsymbol{\rho}) \mathbf{u} = \mathbf{f}. \end{aligned} \quad (5.12)$$

Here, volume is calculated only through summing the volume of elements belonging to supports elements. The densities of elements ρ_j^e belonging to a design variable e have the same density equal to ρ_e . There are N_e design variables, thus columns. Every column has a different number of elements N_s^e . V is the maximal allowable volume and v_e is the total volume belonging to design variable e : $V_e = N_s^e V_0$.

Since the densities of each column of elements are gathered into a single design variable, the number of variables is substantially less than in the standard topology optimization approach. Consequently, it is expected that optimization may require fewer iterations than a similar problem where every element is assigned its own density. Moreover, referring to Figure 5.2, the shape of the part is predefined and not optimized. Therefore, there is no need to do the numerical calculations that assess its performance in the compliance simulation, and only the additive manufacturing simulation is performed.

5.2 Sensitivity analysis

Sensitivities are a must for optimization algorithms calculation. They provide information about a function descent direction, thus the path that an algorithm should follow. Sensitivities may be obtained in different ways. The most popular approaches are finite difference, discrete direct and adjoint method. The finite difference is a crude method in which response of the system is measured after variation of a single design variable at a time. The system is then treated as a black box and the number of required calculation steps is at least equal to the number of design variables. The discrete direct method, on the other hand, is obtained through differentiation of governing equations in the discrete form. The main disadvantage of this approach is that it still requires a separate calculation for each design variable. Finally, the adjoint method is best suited to problems where there are many design variables but few constraints. In the adjoint approach objective or constraints equations are augmented by products of Lagrange multipliers and governing equations. After some transformations sensitivities are solved for the same system matrix as in the original problem but in an adjoint state, hence the name [143].

In topology optimization the number of design variables is usually much bigger than the number of constraints; in that case it is profitable to use the adjoint method for the sensitivity analysis. Moreover, application of continuous density instead of discrete variables allows for the use of a gradient-based optimization method. For this purpose mostly first-order nonlinear programming optimization methods are usually utilized. Here, the Method of Moving Asymptotes [144] is used, which is a reliable method that is widely adopted for topology optimization problems [145].

5.2.1 Sensitivities for shape optimization

In this subsection adjoint sensitivities of the two constraint functions presented in Section 5.1.1 are derived following a standard approach for this method.

Recoater collision

To obtain the sensitivity of the collision constraint $g_w^{(i)}$ Eq. (5.7) is augmented by governing Eq. (3.8)

$$\tilde{g}_w^{(i)} = \frac{1}{r} \log \left(\mathbf{1}^T \exp \left(r \frac{\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \circ \Delta \mathbf{w}^{(i)}}{d_w} \right) \right) - 1 + \boldsymbol{\kappa}^{(i)T} \left(\mathbf{K}_{ff}^{(i)} \Delta \mathbf{u}_f^{(i)} - \mathbf{f}_f^{(i)} \right), \quad (5.13)$$

where $\boldsymbol{\kappa}^{(i)}$ is a Lagrange multiplier vector. Note that the term in brackets which is multiplied by $\boldsymbol{\kappa}^{(i)}$ is equal to $\mathbf{0}$ and therefore the product is $\mathbf{0}$ as well. This means that the final outcome of Eq. (5.13) has not been changed.

The design derivative of $\tilde{g}_i$ is

$$\begin{aligned} \frac{\partial \tilde{g}_w^{(i)}}{\partial \boldsymbol{\rho}} &= \mathbf{h}_i \frac{\partial (\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} + \mathbf{q}_i \frac{\partial \boldsymbol{\varphi}^{(i)}}{\partial \boldsymbol{\rho}} \\ &+ \boldsymbol{\kappa}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} + \mathbf{K}_{ff}^{(i)} \frac{\partial (\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right), \end{aligned} \quad (5.14)$$

where

$$\mathbf{h}_i = \frac{1}{d_w \left(\mathbf{1}^T \exp \left(r \frac{\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \circ \Delta \mathbf{w}^{(i)}}{d_w} \right) \right)} \left(\exp \left(r \frac{\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \circ \Delta \mathbf{w}^{(i)}}{d_w} \right) \circ \boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \right)$$

$$\mathbf{q}_i = \frac{1}{d_w \left(\mathbf{1}^T \exp \left(r \frac{\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \circ \Delta \mathbf{w}^{(i)}}{d_w} \right) \right)} \left(\exp \left(r \frac{\boldsymbol{\varphi}^{(i)}(\boldsymbol{\rho}) \circ \Delta \mathbf{w}^{(i)}}{d_w} \right) \circ \Delta \mathbf{w}^{(i)} \right)$$

$\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)}$ is an ambiguous term that involves operation on a 3D matrix with a vector. In this work such a product means

$$\frac{\partial \mathbf{A}}{\partial \mathbf{b}} \mathbf{c} = \frac{\partial A_{ij}}{\partial b_k} c_j = A_{ij,k} c_j \quad (5.15)$$

In Eq. (5.14) the derivative $\partial(\Delta \mathbf{u}_f^{(i)})/\partial \boldsymbol{\rho}$ is difficult to obtain and is often referred to as an implicit term, whereas the derivatives $\frac{\partial \boldsymbol{\varphi}^{(i)}}{\partial \boldsymbol{\rho}}$ and $\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}}$ are straightforward and are referred to as explicit terms [146].

The main idea of the adjoint method is to rearrange the terms in Eq. (5.14) in such a way that calculation of the implicit term will not be necessary. For that, the terms with $\partial(\Delta \mathbf{u}_f^{(i)})/\partial \boldsymbol{\rho}$ are sorted

$$\frac{\partial \tilde{g}_w^{(i)}}{\partial \boldsymbol{\rho}} = \left(\mathbf{h}_i + \mathbf{K}_{ff}^{(i)T} \boldsymbol{\kappa}^{(i)} \right) \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} + \mathbf{q}_i \frac{\partial \boldsymbol{\varphi}^{(i)}}{\partial \boldsymbol{\rho}} + \boldsymbol{\kappa}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right). \quad (5.16)$$

Now, to omit the calculation of the implicit derivative the terms in brackets has to be set to $\mathbf{0}$, so $\boldsymbol{\kappa}^{(i)}$ is solved from

$$\mathbf{K}_{ff}^{(i)T} \boldsymbol{\kappa}^{(i)} = -\mathbf{h}_i, \quad (5.17)$$

where in general $\mathbf{K}_{ff}^{(i)T}$ is adjoint—the conjugate transpose—of the matrix $\mathbf{K}_{ff}^{(i)}$. In the investigated case $\mathbf{K}_{ff}^{(i)}$ is symmetric so $\mathbf{K}_{ff}^{(i)} = \mathbf{K}_{ff}^{(i)T}$.

Finally, knowing the value of $\boldsymbol{\kappa}^{(i)}$, the sensitivities of $g_w^{(i)}$ are calculated using only the explicit terms

$$\frac{\partial g_w^{(i)}}{\partial \boldsymbol{\rho}} = \mathbf{q}_i \frac{\partial \boldsymbol{\varphi}^{(i)}}{\partial \boldsymbol{\rho}} + \boldsymbol{\kappa}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right). \quad (5.18)$$

In this approach the collision constraint at each deposition increment is treated as a separate constraint. If one max-norm were to be put on constraints at all increments, Eq. (5.13) would be augmented by governing equations from all deposition steps but a separate $\boldsymbol{\kappa}^{(i)}$ for each increment would still need to be calculated.

Shape distortion

In the recoater collision constraint each step can be treated separately. In the case of shape distortions the release stage depend on all printing stage steps so the system of equations is transient in time, which is an additional difficulty.

To obtain the sensitivity of the distortion constraint g_d by means of the adjoint method, Eq. (5.9) is augmented by the equilibrium Eqs. (3.25) and (3.8) and the equation for evolution of the reaction forces Eq. (3.26)

$$\begin{aligned} \tilde{g}_d = & \frac{1}{2} \mathbf{d}^T \mathbf{d} - \frac{1}{2} d_{\max}^2 + \boldsymbol{\lambda}^{(M+1)T} \left(\mathbf{K}_{ff}^{(M+1)} \Delta \mathbf{u}_f^{(M+1)} + \mathbf{S}_{f0} \mathbf{r}_0^{(M)} \right) \\ & + \sum_{i=1}^M \boldsymbol{\lambda}^{(i)T} \left(\mathbf{K}_{ff}^{(i)} \Delta \mathbf{u}_f^{(i)} - \mathbf{f}_f^{(i)} \right) + \sum_{i=1}^M \boldsymbol{\mu}^{(i)T} \left(\mathbf{r}_0^{(i)} - \mathbf{r}_0^{(i-1)} - \mathbf{K}_{0f} \Delta \mathbf{u}_f^{(i)} \right), \end{aligned} \quad (5.19)$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are Lagrange multiplier vectors. The derivative with respect to the design variables of $\tilde{g}_d$ is

$$\begin{aligned} \frac{\partial \tilde{g}_d}{\partial \boldsymbol{\rho}} = & \mathbf{d}^T \left(\sum_{i=j}^{M+1} \mathbf{Z}^{(i)} \frac{\partial (\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \right) \\ & + \boldsymbol{\lambda}^{(M+1)T} \left(\frac{\partial \mathbf{K}_{ff}^{(M+1)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(M+1)} + \mathbf{K}_{ff}^{(M+1)} \frac{\partial (\Delta \mathbf{u}_f^{(M+1)})}{\partial \boldsymbol{\rho}} + \mathbf{S}_{f0} \frac{\partial \mathbf{r}_0^{(M)}}{\partial \boldsymbol{\rho}} \right) \\ & + \sum_{i=1}^M \boldsymbol{\lambda}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} + \mathbf{K}_{ff}^{(i)} \frac{\partial (\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right) \\ & + \sum_{i=1}^M \boldsymbol{\mu}^{(i)T} \left(\frac{\partial \mathbf{r}_0^{(i)}}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{r}_0^{(i-1)}}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{K}_{0f}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} - \mathbf{K}_{0f}^{(i)} \frac{\partial (\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \right). \end{aligned} \quad (5.20)$$

In Eq. (5.20) there are two sets of implicit terms, $\partial(\Delta \mathbf{u})/\partial \boldsymbol{\rho}$ and $\partial \mathbf{r}/\partial \boldsymbol{\rho}$, that have to be eliminated in order to calculate the sensitivities explicitly. Moreover, the equation consists of governing equations from all simulation increments, which also have to be taken into account. Therefore, the terms with $\partial(\Delta \mathbf{u})/\partial \boldsymbol{\rho}$ and $\partial \mathbf{r}/\partial \boldsymbol{\rho}$ are sorted per step

$$\begin{aligned} \frac{\partial \tilde{g}_d}{\partial \boldsymbol{\rho}} = & \left(\boldsymbol{\lambda}^{(M+1)T} \mathbf{K}_{ff}^{(M+1)} + \mathbf{d}^T \mathbf{Z}^{(M+1)} \right) \frac{\partial \mathbf{u}_f^{(M+1)}}{\partial \boldsymbol{\rho}} \\ & + \left(\boldsymbol{\lambda}^{(M+1)T} \mathbf{S}_{f0} + \boldsymbol{\mu}^{(M)T} \right) \frac{\partial \mathbf{r}_0^{(M)}}{\partial \boldsymbol{\rho}} \\ & + \sum_{i=j}^M \left(\mathbf{d}^T \mathbf{Z}^{(i)} + \boldsymbol{\lambda}^{(i)T} \mathbf{K}_{ff}^{(i)} - \boldsymbol{\mu}^{(i)T} \mathbf{K}_{0f}^{(i)} \right) \frac{\partial \mathbf{u}_f^{(i)}}{\partial \boldsymbol{\rho}} \\ & + \sum_{i=1}^{j-1} \left(\boldsymbol{\lambda}^{(i)T} \mathbf{K}_{ff}^{(i)} - \boldsymbol{\mu}^{(i)T} \mathbf{K}_{0f}^{(i)} \right) \frac{\partial \mathbf{u}_f^{(i)}}{\partial \boldsymbol{\rho}} \\ & + \sum_{i=1}^M \left(\boldsymbol{\mu}^{(i)T} - \boldsymbol{\mu}^{(i-1)T} \right) \frac{\partial \mathbf{r}_0^{(i)}}{\partial \boldsymbol{\rho}} + \boldsymbol{\lambda}^{(M+1)T} \frac{\partial \mathbf{K}_{ff}^{(M+1)}}{\partial \boldsymbol{\rho}} \mathbf{u}_f^{(M+1)} \\ & + \sum_{i=1}^M \boldsymbol{\lambda}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right) - \sum_{i=1}^M \boldsymbol{\mu}^{(i)T} \frac{\partial \mathbf{K}_{0f}^{(i)}}{\partial \boldsymbol{\rho}} \mathbf{u}_f^{(i)}. \end{aligned} \quad (5.21)$$

To set the explicit terms from Eq. (5.21) to $\mathbf{0}$ a series of equations is obtained for the multiplier vectors

$$\mathbf{K}_{ff}^{(M+1)\text{T}} \boldsymbol{\lambda}^{(M+1)} = -\mathbf{Z}^{(M+1)\text{T}} \mathbf{d} , \quad (5.22)$$

$$\boldsymbol{\mu}^{(M)} = -\mathbf{S}_{f0}^{\text{T}} \boldsymbol{\lambda}^{(M+1)} , \quad (5.23)$$

$$\boldsymbol{\mu}^{(i-1)} = \boldsymbol{\mu}^{(i)} \quad 1 \leq i \leq M , \quad (5.24)$$

$$\mathbf{K}_{ff}^{(i)\text{T}} \boldsymbol{\lambda}^{(i)} = \mathbf{K}_{f0}^{(i)} \boldsymbol{\mu}^{(i)} - \mathbf{Z}^{(i)\text{T}} \mathbf{d} \quad j \leq i \leq M , \quad (5.25)$$

$$\mathbf{K}_{ff}^{(i)\text{T}} \boldsymbol{\lambda}^{(i)} = \mathbf{K}_{f0}^{(i)} \boldsymbol{\mu}^{(i)} \quad 1 \leq i \leq j-1. \quad (5.26)$$

Note, that the first equation solved in this set is for increment $(M+1)$ whereafter comes the rest, so they are solved in the reversed order, which is typical for an adjoint sensitivity calculation of a transient processes [147, 148].

Finally, the sensitivity of g_d is calculated from explicit terms

$$\begin{aligned} \frac{\partial g_d}{\partial \boldsymbol{\rho}} = & \left(\frac{\partial \mathbf{K}_{ff}^{(M+1)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(M+1)} \right)^{\text{T}} \boldsymbol{\lambda}^{(M+1)} + \sum_{i=1}^M \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right)^{\text{T}} \boldsymbol{\lambda}^{(i)} \\ & - \sum_{i=1}^M \left(\frac{\partial \mathbf{K}_{0f}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} \right)^{\text{T}} \boldsymbol{\mu}^{(i)}. \end{aligned} \quad (5.27)$$

The length of $\boldsymbol{\mu}$ equals the number of degrees of freedom, which are released to simulate release from the base plate. Inspection of Eqs. (5.24) and (5.26) suggests that the sensitivity $\partial g_d / \partial \boldsymbol{\rho}$ can be calculated without resorting to reversed order evaluation by calculating the product of $\mathbf{K}_{ff}^{(i)-1} \mathbf{K}_{f0}^{(i)}$ beforehand and in the final step only multiplying them by $\boldsymbol{\mu}^{(i)}$. This is true but involves storing of full matrices. Therefore reverse order evaluation has been implemented.

5.2.2 Sensitivities for support distribution

This subsection focuses on derivation of adjoint sensitivities of the shape distortion function for optimization of the support distribution that was presented in Section 5.1.2.

Shape distortion

To obtain the sensitivity of the part distortion function g_p Eq. (5.10) is augmented by the equilibrium Eqs. (3.24) and (3.8) and equations that describe evolution of reaction

forces: Eqs. (3.21) and (3.22)

$$\begin{aligned}
 \tilde{g}_p = & \sqrt[p]{\mathbf{1}^T \left| \sum_{i=j}^{M+1} \Delta \mathbf{u}_f^{(i)} \right|^p} + \boldsymbol{\kappa}^{(M+1)T} \left(\mathbf{K}_{ff}^{(M+1)} \Delta \mathbf{u}_f^{(M+1)} + \mathbf{S}_{f0}^{p(M)} \mathbf{r}_0^{(M)} + \mathbf{S}_{fi}^{s(M)} \mathbf{r}_i^{(M)} \right) \\
 & + \sum_{i=1}^M \boldsymbol{\kappa}^{(i)T} \left(\mathbf{K}_{ff}^{(i)} \Delta \mathbf{u}_f^{(i)} - \mathbf{f}_f^{(i)} \right) \\
 & + \sum_{i=1}^M \boldsymbol{\lambda}^{(i)T} \left(\mathbf{r}_0^{p(i)} - \mathbf{r}_0^{p(i-1)} - \mathbf{K}_{0r}^{p(i)} \mathbf{S}_{rf}^{p(i)} \Delta \mathbf{u}_f^{(i)} - \mathbf{K}_{0i}^{p(i)} \mathbf{S}_{if}^{p(i)} \Delta \mathbf{u}_f^{(i)} \right) \\
 & + \sum_{i=1}^M \boldsymbol{\mu}^{(i)T} \left(\mathbf{r}_i^{(i)} - \mathbf{S}_{ii}^{(i)T} \mathbf{r}_i^{(i-1)} + \mathbf{K}_{ir}^{s(i)} \mathbf{S}_{rf}^{s(i)} \Delta \mathbf{u}_f^{(i)} + \mathbf{K}_{ii}^{s(i)} \mathbf{S}_{if}^{s(i)} \Delta \mathbf{u}_f^{(i)} \right),
 \end{aligned} \tag{5.28}$$

where $\boldsymbol{\kappa}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are Lagrange multipliers. The size of $\mathbf{r}_i$ and $\boldsymbol{\mu}$ in every increment changes. The derivative with respect to design variables is taken

$$\begin{aligned}
 \frac{\partial \tilde{g}_p}{\partial \boldsymbol{\rho}} = & \mathbf{b} \sum_{i=j}^{M+1} \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} + \boldsymbol{\kappa}^{(M+1)T} \left(\frac{\partial \mathbf{K}_{ff}^{(M+1)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(M+1)} + \mathbf{K}_{ff}^{(M+1)} \frac{\partial(\Delta \mathbf{u}_f^{(M+1)})}{\partial \boldsymbol{\rho}} + \right. \\
 & \left. \mathbf{S}_{f0}^p \frac{\partial \mathbf{r}_0^{p(M)}}{\partial \boldsymbol{\rho}} + \mathbf{S}_{fi}^s \frac{\partial \mathbf{r}_i^{(M)}}{\partial \boldsymbol{\rho}} \right) + \sum_{i=1}^M \boldsymbol{\kappa}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} + \mathbf{K}_{ff}^{(i)} \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right) \\
 & + \sum_{i=1}^M \boldsymbol{\lambda}^{(i)T} \left(\frac{\partial \mathbf{r}_0^{p(i)}}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{r}_0^{p(i-1)}}{\partial \boldsymbol{\rho}} - \frac{\partial \mathbf{K}_{0r}^{p(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{rf}^{p(i)} \Delta \mathbf{u}_f^{(i)} - \mathbf{K}_{0r}^{p(i)} \mathbf{S}_{rf}^{p(i)} \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \right. \\
 & \quad \left. - \frac{\partial \mathbf{K}_{0i}^{p(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{if}^{p(i)} \Delta \mathbf{u}_f^{(i)} - \mathbf{K}_{0i}^{p(i)} \mathbf{S}_{if}^{p(i)} \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \right) \\
 & + \sum_{i=1}^M \boldsymbol{\mu}^{(i)T} \left(\frac{\partial \mathbf{r}_i^{(i)}}{\partial \boldsymbol{\rho}} - \mathbf{S}_{ii}^{(i)T} \frac{\partial \mathbf{r}_i^{(i-1)}}{\partial \boldsymbol{\rho}} + \frac{\partial \mathbf{K}_{ir}^{s(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{rf}^{s(i)} \Delta \mathbf{u}_f^{(i)} + \mathbf{K}_{ir}^{s(i)} \mathbf{S}_{rf}^{s(i)} \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \right. \\
 & \quad \left. + \frac{\partial \mathbf{K}_{ii}^{s(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{if}^{s(i)} \Delta \mathbf{u}_f^{(i)} + \mathbf{K}_{ii}^{s(i)} \mathbf{S}_{if}^{s(i)} \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \right),
 \end{aligned} \tag{5.29}$$

where

$$\mathbf{b} = \left(\mathbf{1}^T \left| \sum_{i=j}^{M+1} \mathbf{S}_{tf}^{(i)} \Delta \mathbf{u}_f^{(i)} \right|^p \right)^{\frac{1-p}{p}} \left| \sum_{i=j}^{M+1} \mathbf{S}_{tf}^{(i)} \Delta \mathbf{u}_f^{(i)} \right|^{p-1} \text{sgn} \left(\sum_{i=j}^{M+1} \mathbf{S}_{tf}^{(i)} \Delta \mathbf{u}_f^{(i)} \right). \tag{5.30}$$

To build an adjoint system of equations, the terms in Eq. (5.29) belonging to implicit terms $\partial(\Delta \mathbf{u})/\partial \boldsymbol{\rho}$ and $\partial \mathbf{r}/\partial \boldsymbol{\rho}$ are now grouped and sorted per increment

$$\begin{aligned}
\frac{\partial \tilde{g}_p}{\partial \boldsymbol{\rho}} = & \left(\boldsymbol{\kappa}^{(M+1)T} \mathbf{K}_{ff}^{(M+1)} + \mathbf{b} \right) \frac{\partial(\Delta \mathbf{u}_f^{(M+1)})}{\partial \boldsymbol{\rho}} + \left(\boldsymbol{\lambda}^{(M)T} + \boldsymbol{\kappa}^{(M+1)T} \mathbf{S}_{f0}^{p(M)} \right) \frac{\partial \mathbf{r}_0^{p(M)}}{\partial \boldsymbol{\rho}} \\
& \left(\boldsymbol{\mu}^{(M)T} + \boldsymbol{\kappa}^{(M+1)T} \mathbf{S}_{fi}^{s(M)} \right) \frac{\partial \mathbf{r}_i^{(M)}}{\partial \boldsymbol{\rho}} + \sum_{i=1}^M \left(\boldsymbol{\lambda}^{(i-1)T} - \boldsymbol{\lambda}^{(i)T} \right) \frac{\partial \mathbf{r}_0^{p(i-1)}}{\partial \boldsymbol{\rho}} \\
& + \sum_{i=1}^M \left(\boldsymbol{\mu}^{(i-1)T} - \boldsymbol{\mu}^{(i)T} \mathbf{S}_{ii}^{(i)T} \right) \frac{\partial \mathbf{r}_i^{(i-1)}}{\partial \boldsymbol{\rho}} \\
& + \sum_{i=j}^M \left(\boldsymbol{\kappa}^{(i)T} \mathbf{K}_{ff}^{(i)} - \boldsymbol{\lambda}^{(i)T} \left(\mathbf{K}_{0i}^{p(i)} \mathbf{S}_{if}^{p(i)} + \mathbf{K}_{0r}^{p(i)} \mathbf{S}_{rf}^{p(i)} \right) \right. \\
& \left. + \boldsymbol{\mu}^{(i)T} \left(\mathbf{K}_{ii}^{s(i)} \mathbf{S}_{if}^{s(i)} + \mathbf{K}_{ir}^{s(i)} \mathbf{S}_{rf}^{s(i)} \right) + \mathbf{b} \right) \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \\
& + \sum_{i=1}^j \left(\boldsymbol{\kappa}^{(i)T} \mathbf{K}_{ff}^{(i)} - \boldsymbol{\lambda}^{(i)T} \left(\mathbf{K}_{0i}^{p(i)} \mathbf{S}_{if}^{p(i)} + \mathbf{K}_{0r}^{p(i)} \mathbf{S}_{rf}^{p(i)} \right) \right. \\
& \left. + \boldsymbol{\mu}^{(i)T} \left(\mathbf{K}_{ii}^{s(i)} \mathbf{S}_{if}^{s(i)} + \mathbf{K}_{ir}^{s(i)} \mathbf{S}_{rf}^{s(i)} \right) \right) \frac{\partial(\Delta \mathbf{u}_f^{(i)})}{\partial \boldsymbol{\rho}} \\
& + \boldsymbol{\kappa}^{(M+1)T} \frac{\partial \mathbf{K}_{ff}^{(M+1)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(M+1)} + \sum_{i=1}^M \boldsymbol{\kappa}^{(i)T} \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \boldsymbol{\rho}} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \boldsymbol{\rho}} \right) \\
& - \sum_{i=1}^M \boldsymbol{\lambda}^{(i)T} \left(\frac{\partial \mathbf{K}_{0i}^{p(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{ir}^{p(i)} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{K}_{0r}^{p(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{rf}^{p(i)} \Delta \mathbf{u}_f^{(i)} \right) \\
& + \sum_{i=1}^M \boldsymbol{\mu}^{(i)T} \left(\mathbf{S}_{ii}^T \frac{\partial \mathbf{K}_{ii}^{s(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{if}^{s(i)} \Delta \mathbf{u}_f^{(i)} + \mathbf{S}_{ii}^T \frac{\partial \mathbf{K}_{ir}^{s(i)}}{\partial \boldsymbol{\rho}} \mathbf{S}_{rf}^{s(i)} \Delta \mathbf{u}_f^{(i)} \right).
\end{aligned} \tag{5.31}$$

Again, the terms in brackets should be set to $\mathbf{0}$ to eliminate implicit terms. This results in a series of equations

$$\begin{aligned}
\mathbf{K}_{ff}^{(M+1)T} \boldsymbol{\kappa}^{(M+1)} &= -\mathbf{b}, \\
\boldsymbol{\lambda}^{(M)} &= -\mathbf{S}_{0f}^{p(M)} \boldsymbol{\kappa}^{(M+1)}, \\
\boldsymbol{\mu}^{(M)} &= -\mathbf{S}_{if}^{s(M)} \boldsymbol{\kappa}^{(M+1)}, \\
\boldsymbol{\lambda}^{(i-1)} &= \boldsymbol{\lambda}^{(i)} & 1 \leq i \leq M, \\
\boldsymbol{\mu}^{(i-1)} &= \mathbf{S}_{ii}^{(i)} \boldsymbol{\mu}^{(i)} & 1 \leq i \leq M, \\
\mathbf{K}_{ff}^{(i)T} \boldsymbol{\kappa}^{(i)} &= \left(\mathbf{K}_{0i}^{p(i)} \mathbf{S}_{if}^{p(i)} + \mathbf{K}_{0r}^{p(i)} \mathbf{S}_{rf}^{p(i)} \right)^T \boldsymbol{\lambda}^{(i)} \\
&\quad - \left(\mathbf{K}_{ii}^{s(i)} \mathbf{S}_{if}^{s(i)} + \mathbf{K}_{ir}^{s(i)} \mathbf{S}_{rf}^{s(i)} \right)^T \boldsymbol{\mu}^{(i)} - \mathbf{b} & j \leq i \leq M, \\
\mathbf{K}_{ff}^{(i)T} \boldsymbol{\kappa}^{(i)} &= \left(\mathbf{K}_{0i}^{p(i)} \mathbf{S}_{if}^{p(i)} + \mathbf{K}_{0r}^{p(i)} \mathbf{S}_{rf}^{p(i)} \right)^T \boldsymbol{\lambda}^{(i)} \\
&\quad - \left(\mathbf{K}_{ii}^{s(i)} \mathbf{S}_{if}^{s(i)} + \mathbf{K}_{ir}^{s(i)} \mathbf{S}_{rf}^{s(i)} \right)^T \boldsymbol{\mu}^{(i)} & 1 \leq i \leq j.
\end{aligned} \tag{5.32}$$

These equations are again solved in reverse order starting from the release stage. When the values of Lagrange multipliers have been calculated, the sensitivities may be obtained with the explicit terms only

$$\begin{aligned}
\frac{\partial g_p}{\partial \rho} = & \left(\frac{\partial \mathbf{K}_{ff}^{(M+1)}}{\partial \rho} \Delta \mathbf{u}_f^{(M+1)} \right)^T \boldsymbol{\kappa}^{(M+1)} + \sum_{i=1}^M \left(\frac{\partial \mathbf{K}_{ff}^{(i)}}{\partial \rho} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{f}_f^{(i)}}{\partial \rho} \right)^T \boldsymbol{\kappa}^{(i)} \\
& - \sum_{i=1}^M \left(\frac{\partial \mathbf{K}_{0i}^p}{\partial \rho} \mathbf{S}_{ir}^{p(i)} \Delta \mathbf{u}_f^{(i)} - \frac{\partial \mathbf{K}_{0r}^p}{\partial \rho} \mathbf{S}_{rf}^{p(i)} \Delta \mathbf{u}_f^{(i)} \right)^T \boldsymbol{\lambda}^{(i)} \\
& + \sum_{i=1}^M \left(\mathbf{S}_{ii}^T \frac{\partial \mathbf{K}_{ii}^s}{\partial \rho} \mathbf{S}_{if}^s \Delta \mathbf{u}_f^{(i)} + \mathbf{S}_{ii}^T \frac{\partial \mathbf{K}_{ir}^s}{\partial \rho} \mathbf{S}_{rf}^s \Delta \mathbf{u}_f^{(i)} \right)^T \boldsymbol{\mu}^{(i)}.
\end{aligned} \tag{5.33}$$

Similarly to the calculations of sensitivities for shape optimization the calculations for sensitivities of the distortions constraint in Eq. (5.32) could be performed without a reverse evaluation, but this would require operations on matrices instead of vectors. Since this would cost more time and memory the solution method using substitutions in reverse order is preferred.

5.3 Summary

The topology optimization schemes presented in this chapter are based on the SIMP method for structural topology optimization and extend it by additional components that are vital for the application of SIMP to the optimization of the additive manufacturing processes. Of main importance is here the definition of the constraint functions and the approach for calculation of sensitivities. Although computation of the sensitivities by means of the adjoint method involves derivation of additional equations, in the end it comes down to solving in backwards increment-by-increment the same stiffness matrices as in the process simulation but for a different right-hand side for each of the constraints. Similarly to the FEM calculation, many routines of the adjoint scheme can be solved in parallel by using multiprocessing programming. The sensitivities obtained by the adjointed method have been verified by means of central differences. The central difference perturbations were applied to all design variables in a small 2D model and to a small number of randomly chosen design variables in a larger 3D model.

Chapter 6

Topology Optimization of Additive Manufacturing with Printing Constraints

The scheme proposed in Sections 5.1.1 for topology optimization and adjoint sensitivities calculation is applied in this chapter to optimizations with different combinations of additive manufacturing related constraints on both 2D and 3D examples. The optimization case investigated is a bracket that is fixed on one side and loaded on the other side. The numerical algorithms from Sections 3.2.1 and 3.2.3 are utilized here.

6.1 2D examples

In 2D examples a rectangular design domain is considered that is described by 120×80 bilinear quadrilateral elements of 1×1 mm with a standard Gaussian integration scheme. Compliance is minimized for the structure when it is supported on the left side and a vertical load is applied in the centre of the right edge, see Figure 6.1. The material is Ti6Al4V with Young's modulus $E = 110$ GPa and Poisson's ratio $\nu = 0.342$. The downward prescribed load f_c has a magnitude of 1000 N. A plane strain configuration is chosen in the simulation. The maximum volume fraction is set to 40%.

The prescribed inherent strains $\boldsymbol{\varepsilon}_{\text{inh}} = [0.00598, 0, 0]$ in Eq. (3.7), following the Voigt notation. The value is chosen to correspond to the x -component of thermal contraction of Ti6Al4V when cooled down from 800°C to 150°C . Convergence for the optimization is reached when the relative change in compliance averaged over the last three iterations is less than 10^{-5} and all constraints are satisfied.

As a reference case, a compliance minimization without any printing constraint is performed. This results in the symmetric shape shown in Figure 6.1b. The obtained compliance value is $c = 345$ mJ. The optimization requires 133 iterations to converge which took 7 minutes and 5 seconds. The printing simulation of this configuration reveals extensive recoater collision, which is shown in Figure 6.2. The distortion of the reference point amounts to 0.61 mm after the separation from the base plate.

Figure 6.1: Depiction of a) the design domain with boundary conditions and b) obtained part layout for compliance minimization without printing constraints.

To show the optimization capabilities, 5 types of optimization with 17 different optimization cases are performed. An overview of all optimization cases is presented in Table 6.1. First an optimization with the recoater collision constraint aggregated over all the printing steps is performed. Then, four optimizations for different values of $d_{\max}$ with the distortion constraint prescribed to the node, at which the load is applied, are performed for simulations without the release stage. When heat treatment is carried out after the printing stage, it relaxes most of the stresses, leading to no distortions after the separation. In that case the release stage can be disregarded. After that, optimization is performed for the same values of $d_{\max}$, but this time for simulations that include the release stage. Finally, all of the simulations for different $d_{\max}$ with and without the release stage are combined with the recoater collision constraint.

6.1.1 Recoater collision

Recoater collision occurs when the top nodal row of a newly deposited layer distorts upwards in the printing direction, above a critical value. In the investigated case a layer thickness t of $50 \mu\text{m}$ and powder density ρ_p equal to 50% are assumed, consequently the collision threshold d_w according to Eq. (5.6) has been set to $100 \mu\text{m}$. The regularization parameter r in Eq. (5.7) is set to 25 and the constraint has been aggregated into a single value for all the layers.

Given an element height of 1 mm and a layer thickness of $50 \mu\text{m}$, 20 layers are lumped in a single row of elements. For the deposition of every row of elements a separate solution step is required. Therefore, for the considered bracket each AM printing simulation takes 80 steps in total.

It is observed that printing of the reference design obtained from compliance minimization only, would result in recoater collision for nodes depicted in Figure 6.2a. It is worth recalling that in the case of the recoater collision, distortions are multiplied by the nodal densities (see Eq. (5.4)), so only those nodes that belong to the part might violate the constraint.

There are several regions where the recoater collides with the work piece. The first is located in the bottom right corner, where the thick member starts ascending. This is caused by the horizontal length of the structure for these layers and their significant

Table 6.1: Constraints applied in different optimization cases of the 2D example.

Stage	$d_{\max}$ (mm)	d_w (mm)	Section
-	-	-	6.1
-	-	0.1	6.1.1
printing only	0.5	-	6.1.2
printing only	0.2	-	6.1.2
printing only	0.1	-	6.1.2
printing only	0.05	-	6.1.2
printing & release	0.5	-	6.1.2
printing & release	0.2	-	6.1.2
printing & release	0.1	-	6.1.2
printing & release	0.05	-	6.1.2
printing only	0.5	0.1	6.1.3
printing only	0.2	0.1	6.1.3
printing only	0.1	0.1	6.1.3
printing only	0.05	0.1	6.1.3
printing & release	0.5	0.1	6.1.3
printing & release	0.2	0.1	6.1.3
printing & release	0.1	0.1	6.1.3
printing & release	0.05	0.1	6.1.3

distance to the base plate connection. Moreover, at the end of the same member a collision can be observed near the tip where the load is to be applied and on the sides of some of the inside members. The largest area, where collision occurs, is located in the top left corner where long material layers exist with no support to the base plate and with a void beneath the member.

Two plots of the distortion at intermediate build stages after 9 and 72 printing steps are depicted in Figures 6.2b and 6.2c, where the nodal displacements have been amplified 3 times for clarity. The red colour indicates the presence of material, $\rho = 1.0$, and blue indicates the void region, $\rho = 0$. In step 9 collision occurs on the right side of the part due to presence of the long horizontal layers that curl up at the ends. In step 72 the top horizontal layer warps upwards due to its small thickness. It is a thin cantilever that is sensitive to distortions caused by the residual stresses.

The result of TO with the recoater collision constraint as developed in Section 5.1.1 is presented in Figure 6.3. For this structure no nodes violate the constraint. It can be noticed that the bottom member of the structure, for which collision occurred, becomes thinner where the part starts to ascend (marked with A). The reduction of horizontal length for layers where collision happened could prevent it and therefore the material has been removed. Moreover, an additional small column is introduced which connects it to the base plate. Since it has no connection with the left edge, its only function is to limit distortion during printing by adding mechanical support. Its small cross-section resembles the supports used in actual powder bed AM processes. All the external members have a slightly reduced thickness and additional internal members have been introduced. The horizontal length of layers on the top member has been reduced by a change of the inclination angle which results in a shorter top

Figure 6.2: Recoater collision in the reference design: a) nodes for which collision will occur (marked in red). Deformed configuration after b) step number 9 and c) step number 72, red indicates material and blue indicates void.

edge and a void region (marked with B) that cuts into it from below. Some internal supporting material can also be observed in this region. Moreover, the symmetry of the part has been broken. Inner supporting branches have moved, to compensate the influence of changes introduced by the collision constraint application.

The evolution of the compliance and the constraint function is plotted against optimization iterations in Figure 6.4. Convergence has been obtained after 180 iterations and the final compliance is equal to 363 mJ which is 5.2% higher than that of the reference case. The computational time is 41 minutes and 6 seconds which is almost 6 times longer than that of the reference case. This is due to the fact that a single optimization iteration now requires 80 steps of printing simulation and adjoint sensitivity calculation, in addition to the compliance simulation. The constraint plot indicates collision during some iterations at the start of the optimization but almost no violation of the constraint afterwards. The jump after 100 iterations, indicates that the design violated the constraint and that the solution was impracticable for a

Figure 6.3: Design obtained from compliance minimization with the recoater collision constraint.

Figure 6.4: Compliance (left) and constraint (right) history as a function of iteration for the optimization with recoater collision constraint.

few iterations. This is not uncommon behaviour for optimization problems with non-linear constraints, when the constraint is active. The volume constraint is preserved for all of the optimization iterations.

6.1.2 Loaded node distortions

As a second optimization case a constraint on nodal distortion, Eq. (5.9), has been imposed on the node where the load is to be applied. Printing distortion is due to thermal contraction of the layer containing that node and the thermal contraction of all subsequent layers during the printing stage. Thermal contractions that are constrained by clamping the work-piece to the base plate give rise to residual stresses. When the part is released from the base plate additional distortion occurs due to the relaxation of these residual stresses. Two cases are considered here. In the first case, the final distortion is only the sum of the part building steps and the second case includes also the separation from the base plate step. The first case may be seen as a production process where right after the printing a part has been subjected to heat treatment. The effect of the heat treatment is that most of residual stresses are released, such that after the separation from the base plate no further distortions occur.

Optimization without the release stage

First the results of optimizations without the release stage are considered. The constraint is set on the final position of the loaded node, measured with respect to its theoretical assembly position (see Figure 5.4). Optimization has been performed for four values of allowable distortion: $d_{\max}$ equal to 0.5, 0.2, 0.1 and 0.05 mm. In the reference design (Figure 6.1b), a distortion $d \approx 0.3$ mm is observed for the node under consideration.

Figure 6.5 shows obtained shapes. Two types of designs can be observed here. The first one resembles the original compliance optimization without the distortion constraint and results from mild values of $d_{\max}$, equal to 0.5 and 0.2 mm. In the case of $d_{\max} = 0.5$ mm the shape is almost identical to that of the reference case and only

Figure 6.5: Designs obtained for compliance minimization for the printing stage only with different values of $d_{\max}$.

Table 6.2: Results for optimization with the distortion constraint and without the release stage.

$d_{\max}$ (mm)	Compliance (mJ)	Iterations	Time (hh:mm:ss)
-	345	133	0:07:05
0.5	345	353	2:00:07
0.2	350	245	0:55:32
0.1	369	1174	4:26:06
0.05	371	912	3:31:28

minor changes in shape, which manifest themselves by breaking the symmetry, may be noticed. For more strict values, $d_{\max} = 0.1$ and 0.05 mm, a complete change of design occurs. The top members become straight with the strictness of the constraint and point in the direction of the constrained node. This is done to limit the amount of material above the constrained node. The material layer above any given point leads to its increased distortions due to the thermal shrinkage that is introduced with its deposition, so the optimizer tends to avoid it.

Optimization data are gathered and compared with the reference case in Table 6.2. A trend of increase of the compliance value with increase of the strictness of the constraint can be observed. Moreover, there is a substantial difference between the number of iterations for $d_{\max}$ equal to 0.5 and 0.2 mm, which represent the first type of obtained shape and the $d_{\max}$ equal to 0.1 and 0.05 mm which represents the second one. The longest calculation time was for $d_{\max} = 0.1$ mm, which required 1174 iterations and the simulation took in total almost four and a half hours.

Plots of the evolution of objective and constraint functions are presented in Figure 6.6. The compliance plots are smooth, with the exception of that for $d_{\max} = 0.05$ mm where at the start the objective function increases in the first step probably due to the severe violation of the distortion constraint. However, it is hard to observe this on the plot. The distortion function plots are not smooth and numerous jumps can be observed. For $d_{\max} = 0.5$, 0.2 and 0.05 mm convergence is obtained with an inactive constraint, i.e. for distortions being below the allowed threshold, which indicates that even lower values of allowable constraint could be used. Moreover, in the case of $d_{\max} = 0.5$ mm the constraint is not activated at all, because the constraint is too loose and $d_{\max} = 0.2$ mm is activated for only a few increments.

Plots of nodal displacements for $d_{\max} = 0.05$ mm are presented in Figure 6.7. In the case of displacements in the x -direction the values are within a reasonable range, starting from -0.7 mm for the top member and up to 0.6 mm for the bottom one. The

Figure 6.6: Compliance (left) and distortion (right) history as a function of iteration for optimization with a distortion constraint for the printing stage only.

most concerning are positive displacements in the y -direction of the bottom member, which are above 8 mm. This is caused by the long horizontal layers which have no connection to the base plate. In Section 6.1.3 it will be shown that simultaneous application of distortion constraint and recoater collision constraint automatically solves this issue. Moreover, an alternative solution could be application of support structures to connect the troublesome region with the base plate.

Optimization with the release stage

Optimizations with the same boundary conditions and identical $d_{\max}$ values have been performed but this time accounting for distortions in the release stage. In the reference design (Figure 6.1b), a distortion $d \approx 0.61$ mm is observed for the node under consideration.

The obtained shapes are gathered in Figure 6.8. It can be again observed that with the decrease of the allowed distortion one of the horizontal members of the design becomes more straight and the number and complexity of internal members increases. When compared with the designs for optimization without the release stage, the obtained shapes resemble the second type of the designs, i.e. the stricter ones, but mirrored in the y -direction. The designs are mirrored because the release stage has

Figure 6.7: Displacement plots (in mm) of the shape obtained for $d_{\max} = 0.05$ mm for simulation with the printing stage only in the x -direction (left) and the y -direction (right).

Figure 6.8: Designs obtained for compliance minimization for printing and release stage with different values of $d_{\max}$.

Table 6.3: Results for optimization with distortion constraint including the release stage.

$d_{\max}$ (mm)	Compliance (mJ)	Iterations	Time (hh:mm:ss)
-	345	133	0:07:05
0.5	347	137	0:30:02
0.2	354	286	1:03:37
0.1	362	310	1:08:43
0.05	369	309	1:07:59

been introduced and long horizontal members that have a connection to the bottom would accumulate a lot of residual stresses that would then cause severe distortion in the final stage. That is why designs with long horizontal member at the top, where there is no connection to the base plate are preferred. Moreover, in comparison with the outcome of pure compliance, the designs feature more complex topologies and the vertical symmetry is again broken.

The results of the performed optimizations, the achieved compliance value, the number of required optimization iterations, and the required computational time are summarized in Table 6.3. The compliance value increases as the constraint becomes stricter. The same happens for the number of iterations required for convergence except for $d_{\max} = 0.05$ mm, which requires one iteration fewer than $d_{\max} = 0.1$ mm. The calculation time increases substantially but still remains within a reasonable range.

Figure 6.9: Compliance (left) and distortion (right) history as a function of iteration for optimization with a distortion constraint.

Figure 6.10: Displacement plots (in mm) of shape obtained for $d_{\max} = 0.05$ mm for simulation with the release stage in the x -direction (left) and the y -direction (right).

A comparison of evolution of compliance and constraint functions for four different distortion constraints is presented in Figure 6.9. Compliance plots are almost on top of each other, indicating similar convergence behaviour. For $d_{\max} = 0.1$ and 0.05 mm a sudden increase of the compliance occurs at the start of the optimization, due to a significant violation of the allowed distortion constraint, but it does not influence the convergence later on (the peak is not visible). In the case of the distortion constraint plot characteristic jumps may be observed at the earlier iterations for all of the plots. This means that for the optimizer it is harder to remain in the feasible space but after 50 iterations the behaviour is quite smooth.

Investigation of the displacement plots for $d_{\max} = 0.05$ mm depicted in Figure 6.10 shows that the constrained upper left corner node as well as the node to which the force is applied stay in place. The positive displacements in the x -directions are this time more severe and go up to 1.1 mm for the thin member in the top left corner. The positive displacements in the y -direction have been reduced due to the reduction of the layer lengths and now the highest value is over 4.6 mm, whereas for the rest of the part displacements in the y -direction are fairly slight.

Relevance of the release stage

In the case of the optimizations that incorporate both production stages there is a possibility that distortions that accumulate in the printing stage may be compensated during the separation. To show this, the distortions of the constrained node as a function of printing steps for $d_{\max} = 0.05$ mm are plotted in Figure 6.11. It can clearly be seen that for the simulation that includes the release stage in the final step the distortions drop from over 0.15 mm to 0.05 mm. In the case of the simulation without the release stage the constraint is preserved from step 70 without any sudden jumps.

6.1.3 Recoater collision and loaded node distortion combined

In the final 2D optimization cases both the recoater collision and the loaded node distortion constraints have been applied simultaneously. Two types of simulation have been performed, without and with the release stage, for $d_{\max}$ equal to 0.05 and 0.1 , 0.2 and 0.5 mm. All are combined with the recoater collision threshold d_w set to $100 \mu\text{m}$.

Figure 6.11: Relevance of the release stage: comparison of distortion of the constrained node for the optimized structures as a function of build step. The orange line indicates the allowed distortion.

Table 6.4: Results for optimization with distortion and recoater collision constraint.

Stage	$d_{\max}$ (mm)	d_w (mm)	Compliance (mJ)	Iteration	Time (hh:mm:ss)
-	-	-	345	133	0:07:05
pr. only	0.5	0.1	362	161	0:48:30
pr. only	0.2	0.1	362	374	1:48:14
pr. only	0.1	0.1	434	340	1:43:55
pr. only	0.05	0.1	444	239	1:10:16
pr. & rel.	0.5	0.1	371	397	2:07:52
pr. & rel.	0.2	0.1	379	639	3:13:33
pr. & rel.	0.1	0.1	387	329	1:42:27
pr. & rel.	0.05	0.1	410	202	1:01:41

The obtained compliance values and other optimization data are set out in Table 6.4. The trend of increase of the objective function value with the increase of the strictness of the distortion constraint can still be observed, but the number of required iterations no longer follows this trend which may be due to the increased nonlinearity of the problem. The longest calculation time occurs for the case with $d_{\max} = 0.2\text{ mm}$ and $d_w = 0.1\text{ mm}$, which is over 3 hours. This is caused by the large number of iterations (639) that were required for convergence. This behaviour is rather random, because the optimizer oscillates above the converge criterion and many times almost fulfills it. Optimizations without the distortions at the release stage do not require a large number of iterations for the stricter constraint values but substantial increase of the objective function values occurs. For $d_{\max}$ equal 0.1 and 0.05 mm without the release stage the value is 25.8% and 28.7% respectively greater than for the reference case. There is some similarity between designs obtained with the same $d_{\max}$ for optimizations with and without the recoater collision constraints (Figure 6.5 and Figure 6.8 vs. Figure 6.12), but overall the shapes which include both constraints are more complex.

Figure 6.12: Designs obtained for compliance minimization for different values of $d_{\max}$ combined with d_w .

For optimizations without the release stage (Figure 6.5 vs. top row of Figure 6.12) two types of design may be again noted, depending on the strictness of the constraint. Moreover, for cases with the release stage (Figure 6.8 vs. bottom row of Figure 6.12) a similar straightening of the lower ascending member can be observed. Comparable patterns can be seen with respect to similarity with the design obtained for the recoater collision constraint only (Figure 6.3). Firstly, the optimizer avoids material in the top left corner for which collision occurs. Furthermore, the irregularity of member thickness is much more pronounced than before. Finally, many thin structures that connect either the part with the base plate or the members themselves are introduced. These structures are reminiscent of the support structures that are introduced in 3D printing practice. Many of the supports are still greyish, which indicates that more optimization iterations may be still required, with either additional filtering or the better convergence criteria that takes into account whether most of the elements are either black or white. Only the change of the objective function is evaluated here. Since the main role of the introduced supports is to help preserve the constraints the optimization is recognized as converged even for supports that are not yet fully black. Additionally, their limited thickness even complicates the problem further and slows down obtaining of the crisp design because the application of density filtering always smears such thin shapes with a void region surrounding it. On the other hand, support structures are mostly thin elements with limited stiffness, so in their case intermediate densities can be justified. That is why it has been decided that no further developments with respect to the obtained shape convergence will be made in this work and that the results are satisfactory. Their main function is to support the production process, since they have been introduced by the printing constraints, and they do not participate in the part performance.

Visual inspection of displacement plots in Figure 6.13 again confirms that the constrained nodes stay in place (within the allowed tolerance). For the case without the release stage the highest distortions in the y -direction occur for the supports region

Figure 6.13: Displacement plots (in mm) of shape obtained for $d_{\max} = 0.05$ mm for simulation with distortion and recoater collision constraint in the x -direction (left) and the y -direction (right).

in the top left corner. In the case of the simulation with the release stage the highest distortions occur for the thin support in the bottom right corner. These displacements are caused by the release stage where the bottom of the member is freed. Note that this support distortion does not affect the loaded node distortion constraint, since that is aimed at controlling the loaded node position in the assembled configuration. Moreover, the overall part distortions are smaller than in the cases without the recoater collision constraint (Figures 6.7 and 6.10). The reason for this is that suppression of recoater collision is a constraint that affects all layers, therefore the distortions in the whole structure are reduced.

The objective and constraint history plots in Figure 6.14 again show that at the beginning the optimizer has problems with remaining in a feasible space, spikes in plots can be observed, but after 200 iterations the plots are rather smooth. The biggest drop in the objective function occurs in first couple of dozen iterations. Moreover, the optimization always converges to a feasible solution.

Figure 6.14: Compliance (left), distortion (middle) and recoater collision (right) as a function of iteration for optimization with a distortion and recoater constraint for the printing stage only.

6.2 3D examples

To demonstrate the application potential of the proposed TO formulation for 3D design cases, optimizations in 3D have been performed. The design domain used for optimization is composed of $80 \times 40 \times 40$ elements of $1 \times 1 \times 1$ mm standard bilinear hexahedral elements with Gaussian integration scheme. Compliance is minimized for the structure that is supported on the left side and the load is applied vertically along the line going through the middle of the right side (Figure 6.15a). During the printing stage the bottom is fully constrained (Figure 6.15b) and in the release stage the bottom left edge is constrained in the x and z -direction, the top edge is constrained in the x -direction and the middle node of the bottom left edge is fully suppressed (Figure 6.15c).

The material used is Ti6Al4V with the same properties as for the 2D cases and the inherent strain applied in simulations is $\epsilon_{\text{inh}} = [0.00299, 0.00299, 0, 0, 0, 0]$ which would be an average value for the lumped method with the rotational scanning strategy. For every case the maximum volume fraction is set to 40%. Note that in this 3D example, in contrast to the 2D case, many nodes are loaded and will be considered for distortion. The convergence criterion is again set to the relative change of objective function averaged over last 3 iterations lower than 10^{-5} . The compliance minimization

Figure 6.15: Boundary conditions for different stages of simulation.

Figure 6.16: Reference shape obtained from compliance minimization.

Table 6.5: 3D Topological Optimization results with different Additive Manufacturing constraints.

Stage	$d_{\max}$ (mm)	d_w (mm)	Compliance (J)	Iteration	Time (hh:mm:ss)
-	-	-	29.9	182	9:24:12
-	-	0.1	30.3	683	276:30:54
pr. only	0.2	-	30.9	281	92:51:04
pr. only	0.2	0.1	31.0	373	222:33:44
pr. & rel.	0.1	-	32.6	327	110:20:07
pr. & rel.	0.1	0.1	33.1	308	184:11:02

with only the volume constraint is performed as a reference case. Then the objective function value is 29.9 J and is reached after 182 iterations. In total the optimization took over 9 hours. The obtained shape is presented in Figure 6.16, only the elements that have density above or equal to 0.9 are plotted.

Five 3D optimizations will be presented. The first concentrates on the recoater collision constraint applied. Next, two optimizations for distortion after the printing stage only are carried out for $d_{\max}$ set to 0.2 mm and again with and without a recoater collision constraint with $d_w = 0.1$ mm. Finally, optimizations that include the release stage with $d_{\max}$ set to 0.1 mm, with and without the recoater collision constraint are performed. The $d_{\max}$ is this time set to all of the loaded nodes and gathered into a single p -norm function following Eq. (5.10). The max-norm is approximated by setting $p = 7$. The results of the performed optimizations are summarized in Table 6.5.

The trend of increasing objective value with the strictness of the constraints is again noticed and the same goes for the increase of the required number of iterations. The number of required iterations is on a similar level to 2D. Nevertheless, the calculation time increased significantly. The longest calculation time is 11 and a half days, which is a substantial amount of time.

Iteration history plots for all 3D optimization cases are presented in Figure 6.17. Most of the objective function reduction occurs in the first 50 increments, and the same goes for jumps in distortion and collision constraints. The constraints are preserved for most of the iterations and their evolution resembles the convergence plots from optimization in 2D.

Figure 6.17: Compliance (left), distortion (right) and recoater collision (bottom) as a function of optimization iteration.

6.2.1 Recoater collision

The same conditions as for 2D are needed if recoater collision is to happen. Therefore, d_w is set to $100 \mu\text{m}$. Again the parameter r in Eq. (5.7) is set to 25 and the constraint has been aggregated for all of the layers into a single function. Since the domain height is 40 elements, 40 separate deposition steps are gathered. In the case of the printing of the obtained reference design, collision occurs for nodes marked by red dots in Figure 6.18.

In total 2153 nodes violate the constraint, and as can be observed, they are all gathered in the top side of the structure, similarly to the main area for which most of the recoater collision occurred in the case of 2D. The highest distortion in the z -direction is 0.33 mm which is well above the threshold.

The design obtained after the application of the recoater collision constraint is presented in Figure 6.19. The change of the shape in relation to the reference case mostly affects the internal members. The shape and position of the inner members are more complex and the symmetry of the part in the z -direction is broken. An additional member has been added on the left side (marked with A) supporting the top of the structure where the collision occurred. The side of the structure where the load is applied did not change much. Only the width and thickness of the outer members now vary.

Figure 6.18: Recoater collision for the reference shape.

Figure 6.19: Shape obtained from compliance minimization with recoater collision constraint. Full shape (left) and shape cut in half by y -plane for $y = 20$ mm (right).

6.2.2 Distortion constraint

The maximal distortion of constrained nodes for the reference shape is found to be 0.9 and 0.66 mm for simulations without and with the release stage respectively. Therefore $d_{\max}$ is set to 0.2 and 0.1 mm for simulations following that order. Designs obtained for these parameters are presented in Figure 6.20. In the plots the elements that have a density above or equal to 90% are plotted as having full density, while the transparent elements have a density that is below 90% but more than 20%. This is done in order to allow for visualization of possible low-density support structures, as observed in the 2D case.

In the case of simulation with the release stage, $d_{\max} = 0.2$ mm, it can be seen that the optimization proposes a structure that is hollow. Moreover, the importance of internal members has decreased. Their thickness is much less and they have shifted to the left side. In the case of the printing stage only and $d_{\max} = 0.1$ mm the structure is something in between the reference case and the case mentioned above. The hollow part is only in the front and the internal members on the left are still important. Regarding the supports in the case of $d_{\max} = 0.2$ mm there are still supports that are not fully dense. On the other hand $d_{\max} = 0.1$ mm creates two fully dense pillars in the corners of the design domain. The reason for this difference is the fact that the constraint for the first structure is stricter and it may be harder to find the supports that will help to enforce it. Compared with the reference design (Figure 6.16) the symmetry is broken and some straightening of the bottom with the increased strictness of the constraint as for the 2D examples occurs.

Figure 6.20: Full shape (left) and shape cut in half by y -plane for $y = 20$ mm (right) obtained for minimization of compliance with distortion constraints.

6.2.3 Distortion and recoater collision constraints combined

The constraints studied separately in the previous subsection are now combined in a single optimization involving both distortion and recoater constraints. The shapes obtained from the optimization are presented in Figure 6.21.

Again for the simulation with the printing and release stage a shell-like structure with thinner inner members that were moved to the left side can be observed. In the case of the printing stage only, similarly to the previous solution a structure is obtained whose shape is in between the reference example and the shell-like. Regarding supports for simulation with the release stage an additional smaller pillar is introduced closer to where the structure starts to ascend from the base plate. In the case of the printing stage only, the supports are thicker than in the case without the recoater collision constraint.

Figure 6.21: Full shape (left) and shape cut in half by y -plane for $y = 20$ mm (right) obtained for minimization of compliance with distortions constraints combined with the recoater collision constraint.

6.3 Summary

In this chapter topology optimization of a structure with constraints on distortions that may occur during additive manufacturing has been performed for examples both in 2D and 3D. Two types of constraints have been investigated: the distortion of the nodes that are loaded during the part performance assessment and recoater collision. In the case of a constraint on the distortion of the loaded node the optimization was found to compensate distortions by changing its shape and by adding supports. In the case of optimizations that includes both simulations of the printing and release stage, the proposed design compensates some of distortions that are accumulated during the build stage by additional distortion during release.

Optimization in 3D of a bracket example including the proposed distortion constraints is characterized by the creation of box-like structures, which distribute the mass concentrated in the middle of the domain in the case of the reference design to the sides. When the recoater collision constraint is applied, many additional support structures result from the optimization, which strongly resemble supports proposed by designers in powder bed based additive manufacturing. The total optimization time for 3D was in the order of days which is still in a reasonable range considering the fact that it involves a transient numerical model.

The addition of additive manufacturing constraints to SIMP-based topology optimization expectedly results in an increase of calculation time and a modest increase of the

compliance objective by 4 to 29%, depending on the strictness of the constraints. However, given the high cost of failure at a late stage of the design or printing process, it is highly recommended to include these crucial constraints in the design for additive manufacturing.

Chapter 7

Topology Optimization of Support Structures

Tailoring of the printing distortions by optimization of supports distribution is the second optimization case for additive manufacturing investigated in this thesis. To demonstrate its capabilities a series of examples will be presented. First, 2D examples will be considered with the aim of minimizing general part printing distortions, measured for all nodes belonging to a part geometry, in a bracket example. Next, optimizations for the same shape as the reference example in Chapter 6 will be investigated but this time only by tailoring the distribution of supports. This means that examples where a chosen node was constrained with respect to its theoretical assembly or the recoater collision constraint will again be optimized by using support structures distribution only. The purpose of this is to compare the two optimization approaches and to check whether it is possible to achieve the same results as when the part shape alters. Additionally, the influence of the mesh size on the obtained solution, the mesh dependence, will be studied. Next, examples will be presented in 3D for general part distortions and the cases will be compared with the results of previously introduced 3D optimizations. Finally, to show the full capabilities of the method the optimization will be used to find the optimal distribution of the supports for the use case example that was presented in Chapter 4.

In supports optimization a part shape is predefined and compliance minimization cannot be an objective. The objectives considered in this work are either minimization of shape distortions or supports volume. If in the optimization routine the objective is to minimize the distortion with a constraint on the supports volume the optimization

problem reads as follows

$$\begin{aligned}
& \min_{\boldsymbol{\rho}} : g_p(\boldsymbol{\rho}) \\
& \text{s.t.:} \quad \frac{1}{N_e} \sum_{e=1}^{N_e} \rho_e v_e \leq V, \\
& \quad g_w \leq 0, \\
& \quad \rho_j^e = \rho_e \quad j = 1, \dots, N_s^e \quad e = 1, \dots, N_e, \\
& \quad 0 < \varepsilon \leq \rho_e \leq 1 \quad e = 1, \dots, N_e, \\
& \text{with: } \mathbf{K}(\boldsymbol{\rho})\mathbf{u} = \mathbf{f},
\end{aligned} \tag{7.1}$$

where g_p and g_w are p -norm and recoater collision functions respectively. $\boldsymbol{\rho}$ is a vector of design variables ρ_e , ρ_j^e is the density of element j that belongs to design variable e so all elements that belong to a given design variable have the same density ρ_e . This is due to the fact that a single design variable may refer to the density of a column of multiple elements. V is the maximal allowable volume, N_e is a number of design variables, and N_s^e is a number of support elements belonging to a given design variable e .

All examples use a material model for Ti6Al4V with Young's modulus $E = 110$ GPa and Poisson's ratio $\nu = 0.342$. As before, bilinear quadrilateral elements are used for 2D and trilinear hexahedral elements for 3D, both with the edge length equal to 1 mm are used, unless otherwise stated. Their integration is carried out with 2×2 and $2 \times 2 \times 2$ Gaussian's schemes. The prescribed inherent strain is equal to $\boldsymbol{\varepsilon}_{\text{inh}} = [0.00598, 0, 0]$ and $\boldsymbol{\varepsilon}_{\text{inh}} = [0.00299, 0.00299, 0, 0, 0, 0]$ for 2D and 3D respectively, following the Voigt notation.

Optimization of supports distribution to prevent overhangs is not the aim of this work but algorithms that are developed for it [72, 86] may be incorporated if needed. Consequently, overhanging regions are allowed in the optimizations.

For better readability the performed optimizations are summarized in a list:

1. 2D results.

(a) Part distortions:

- minimization of part distortions with either $p = 1$ or $p = 7$;
- constraint set on maximal allowable support volume;
- for the printing only stage or the printing & release stage.

(b) Supports distribution vs. part shape optimization:

- minimization of volume;
- two constraints, either the recoater collision constraint or the distortion constraint on the loaded node;
- used for comparison with results from Chapter 6.

(c) Mesh size dependence:

- minimization of overall part distortions for $p = 1$;
- constraint set on maximal allowable support volume;

2. 3D results.

(a) Part distortions:

- optimization for two bracket geometries;
- minimization of part distortions with either $p = 1$ or $p = 7$;
- constraint set on maximal allowable support volume;
- for the printing stage only and the printing & release stage.

(b) Supports distribution vs. part shape optimization:

- minimization of volume;
- two constraints either the recoater collision constraint or the distortion constraint on loaded nodes;
- used for comparison with results from Chapter 6.

3. 3D use case:

- support distribution of the use case optimized for two mesh sizes: 1 mm and 0.5 mm;
- minimization of part distortions with either $p = 1$ or $p = 7$;
- constraint set on maximal allowable support volume;
- for the printing only stage or the printing & release stage.

7.1 2D examples

The examples in 2D are investigated first, since it is easier to interpret results and calculation times are much shorter than for the 3D examples.

7.1.1 Part distortions

The reference bracket geometry of Section 6.1 will be used to demonstrate optimization of support distribution in 2D. It is the outcome of a compliance minimization when the structure is fixed on the left edge while a force is applied in the middle of the right edge and with volume constraint set to 0.4. The domain dimension is 120×80 mm, and the shape after 133 topology optimization iterations is used retaining only elements whose density is above 0.8. During the optimization of the supports these are passive elements, whose densities do not change.

The geometry of the part and the domain used for supports optimization are presented in Figure 7.1. For optimization of the supports distribution the structure is shifted in the building direction by 5 mm to make room for supports. All void region in the y -direction that is below passive elements is chosen as the optimization domain. Passive elements are used to describe the part. They do not enter in the optimization but are still present in the numerical modelling of the production process. During the printing stage the bottom is fully fixed to mimic the base plate. For the release stage the structure is fixed by suppressing degrees of freedom marked by blue symbols. The purpose of optimization in this section is to use supports to minimize either overall, $p = 1$, or maximal, $p = 7$, distortions that occur in the production process. The part

Figure 7.1: Depiction of bracket obtained from minimization of compliance (left) and optimization domain (right), grey light colour, with passive part, grey dark colour with black contour, and boundary conditions, blue colour, during the release stage.

performance and its theoretical assembly are therefore not taken into account; after the release stage fixed degrees of freedom are chosen, purely to prevent rigid body movements.

Four different optimization cases are going to be considered that can either optimize overall distortions, $p = 1$, or maximal distortions, $p = 7$, and either for simulation with the printing stage only or the printing and release stage combined. The goal is to minimize the distortions of the passive part with max support volume constraint set to 30%. Convergence is reached when the relative change in objective function averaged over the last 3 iterations is less than 10^{-5} and all constraints are satisfied. The displacements before optimization of supports distribution for supports volume set to be equal to its initial value of 30% are presented in Figure 7.2. This will be used as a reference example. Clearly for both cases the top left corner distorts the most and there is a big similarity in distribution of the x -direction plots but in case of release stage the part shifts in positive direction. In general, addition of the release stage increases overall distortion of the part which can especially be observed for distortions in the y -direction.

Supports distributions obtained from optimizations are presented in Figure 7.3. The average time per iteration was over 1 minute. The number of iterations needed for every optimization were 70, 106, 183 and 227 for $p = 1$ printing only, $p = 1$ printing and release, $p = 7$ printing only, and $p = 7$ printing and release respectively. Following the same order the objective function values— p -norm of the part nodal displacements—have been reduced to 62%, 59%, 52% and 39% of the initial value, i.e. for the supports equally distributed over the domain. This confirms that supports are correctly placed and are effective even with the limit on maximal stiffness of 1% of the solid material in the in-plane direction.

Despite having the SIMP interpolation parameter set to 3 there are still many greyish elements. Since it is possible to produce supports with a varying cross-section it was decided to not push designs towards the black-and-white solution and to assume that supports are printable in that form. Generally, it is hard to visually interpret obtained distributions and find reasoning behind the given supports distribution by looking at the supports position only.

The distortion plots presented in Figure 7.5 for all of the cases show improvements. Mostly it can be noticed by observation of maximal distortions which, for both $p = 1$ and $p = 7$, are smaller than in the reference case (Figure 7.2). In the case of $p = 1$

Figure 7.2: Displacement components of bracket with initial support distribution for a) x -direction and b) y -direction printing stage only; c) x -direction and d) y -direction printing and release.

there is much more area in the colour having small displacements, around zero, for both simulations with and without the release stage. For the minimization of maximal distortions, $p = 7$, the distortions in the y -direction of the top left corner have been largely reduced for simulations with and without the release stage and it seems that distortions in the x -direction in the bottom left corner of the part are now on the same level.

By looking at convergence plots in Figure 7.4 it can clearly be seen that the algorithm works properly and is able to reduce the objective functions. In the case of $p = 1$ the objective function is divided by a number of degrees of freedom, so averaged distortions are plotted. As can be observed for $p = 1$ fewer increments are needed than when $p = 7$. This is probably due to higher nonlinearity of the problem when the max-norm is approximated by raising the p -value. The same trend can be observed between optimization for printing only and for printing and release. The volume constraint is active in every example. At the start a drop in used volume can be observed, which is dictated by removing material from design variables that actually increase the objective function. But after fewer than 50 iterations in every case the constraint becomes active again.

Figure 7.3: Support distributions for bracket geometry obtained for $p = 1$ a) printing only, b) printing and release; $p = 7$ c) printing only, d) printing and release.

Figure 7.4: Objective and constraint function plots for $p = 1$ a) printing only, b) printing and release; $p = 7$ c) printing only, d) printing and release.

Figure 7.5: Displacement components of bracket for different p -values with optimized supports distribution for a) x -direction and b) y -direction, printing only; c) x -direction and d) y -direction, printing and release.

7.1.2 Supports distribution vs. part shape optimization

The bracket geometry presented in the previous section is the same shape that was used as the reference example in Chapter 6, being the result of the minimization of compliance with a constraint on the maximal allowable volume. It was decided to investigate whether it is possible to achieve the same goals without changing the part shape but by proper placement of supports. In that way the part would be printable and the part performance would not be affected.

Recoater collision

The first investigated case is putting the constraint on recoater collision. The geometry with nodes for which collision would occur if printed in that form is presented in Figure 7.6. As can be noticed, the bracket has again been shifted by 5 mm in the y -direction to make room for supports. Moreover, it has been decided to force supports under the bottom member and to connect it with the base plate. It was done to increase the credibility of the example. The purpose of this in actual 3D printing process is to allow for cutting off of a part from the base plate. Moreover, it also helps to avoid purely numerical solutions that propose no supports at all. In this case the goal of the optimization is to minimize the volume of the supports with the constraint on occurrence of recoater collision. The recoater collision margin has been again set to $100\ \mu\text{m}$ for positive distortion in the y -direction for every top nodal layer of newly deposited layer of elements.

The design domain with the obtained supports distribution is presented in Figure 7.7. The number of increments required for convergence was 232. Clearly little extra supports are needed if the bottom member is supported due to the overhangs and in the end their volume was only 9% of the available domain. A noteworthy point is that supports are quite evenly spread around a big part of the domain, indicating the global character of the constraint.

The objective and constraint function plots are presented in Figure 7.8. The objective function plot is staggered at the start which is the result of constraint violation in the first 100 increments. When the recoater collision constraint is fulfilled the objective function decrease to the point where convergence is reached.

Figure 7.6: The nodes (red) for which collision would occur if only the bottom of the domain were to be supported.

Figure 7.7: The optimization domain at the start of optimization (left) and final supports distribution (right).

Figure 7.8: Volume (left) and recoater collision (right) as a function of iteration for optimization with a recoater collision constraint.

Global nodal distortions

The second example considers distortions of the loaded node with respect to its theoretical assembly position. It is hard to assume how much of distortions it will be possible to reduce because stiffness of support in the in-plane direction is low. Therefore the objective function is to minimize the node distortions with a constraint on allowable supports volume. Three support volume constraints will be considered: 20%, 30% and 40%.

Boundary conditions for two production stages are presented in Figure 7.9. During the printing stage the domain is fully fixed at the bottom to imitate printing on the base plate. In the release stage these supports are removed. The left bottom

Figure 7.9: The bracket with design domain (left) and part during the release stage with fixed degrees of freedom (right).

Figure 7.10: Depiction of a bracket obtained from minimization of nodal distortions for V equal to a) 20%, b) 30% and c) 40%.

constrained node is translated back to its initial position and the whole structure is rotated to set the x -coordinate of the top constrained node to its intended value. Then, the distortions of the considered node are measured with respect to its original position. In the same way as for the recoater collision constraint the presence of support structures is enforced under the bottom member after vertically shifting the structure.

Results obtained for the three volume constraint values are presented in Figure 7.10. The optimization required 81, 40 and 51 iterations for volume constraint set to 20%, 30% and 40% respectively. The final distortion of the node is 0.15, 0.25 and 0.25 mm, following the same order. In the obtained distribution there is almost no difference between volume constraint set to 30% and 40%. The biggest difference occurs in the case of constraint set to 20%, manifested by the absence of material to support the considered node on the right side of the domain. Despite this, distribution obtained for supports volume constrained to 20% has the lowest distortions. This indicates that the problem is non-convex.

In all of the distortion plots shown in Figure 7.11 the constrained nodes have almost no distortions. Moreover, there is almost no difference in distortion between volume constraints set to 30% and 40%. In the case of 20% a slight shift of the contour with zero displacements in both the x and y -direction towards the constrained node can be observed.

The plots of objective and constraint functions presented in Figure 7.12 indicate that none of the volume constraints are active. The final results for $V_c = 30\%$ and 40% seem to be the same but they have been obtained in a slightly different way due to the difference in the initial conditions of the optimization, the starting domain volume.

Figure 7.11: Displacement of the bracket for optimized supports distribution for a) x -direction and b) y -direction, for volume constraint set to 20% (top), 30% (middle) and 40% (bottom).

Figure 7.12: Distortion (left) and volume (right) as a function of iteration for optimization with a distortion minimization.

7.1.3 Mesh size dependence

In this section the influence of the mesh size on the obtained results will be investigated. The same geometry will be printed with different mesh resolution and therefore a different number of layers per single starting pixel. The dimensions of the starting domain are 60×40 mm and 1×1 mm elements are used. The shape used for optimization is an outcome of compliance minimization with volume constraint. Optimization is performed for $p = 1$, printing and release stage with volume constraints set to 30%. The part is shifted by 5 mm in the building direction, resulting in the domain of 60×45 mm. Two more optimizations will be performed. Second mesh resolution will be doubled so 120×90 elements will be used and, the third one will be quadrupled which means 240×180 elements in the domain.

The initial domain and obtained supports distribution for different mesh sizes are presented in Figure 7.13. As can be seen there is a big similarity between the obtained results, especially for the two designs with finer mesh, indicating the mesh size independence of the proposed approach. For the coarsest mesh size 126 optimization iterations were required, whereas it was 116 and 156 for doubled and quadrupled mesh respectively.

Inspection of the plots of the evolutions of objective function and constraint (Figure 7.14) clearly shows that the plots are almost on top of each other. Usually, even for mesh size independent results, it would be expected that a refined mesh size is connected with slower convergence but in this case the algorithm follows almost the same path. This indicates that the algorithm not only yields similar results but also that it obtains them in a similar fashion.

Figure 7.13: Depiction of a) bracket obtained from minimization of compliance with domain build based on it and boundary conditions during the release stage, results for mesh size b) 60×45 , c) 120×90 , and d) 240×180 .

Figure 7.14: Evolution of objective function (left) and volume constraint (right) plots.

7.2 3D examples

The proposed scheme has been fully transferred into 3D. To show the applicability of the method two examples will be considered for overall part distortions and four examples will be used as a comparison with the optimization of part shape. One of that four will be used to constrain recoater collision and three will be used for global nodal distortions.

7.2.1 Part distortions

In this section two bracket examples are considered. Both are a result of compliance minimization with fully constrained left surface and force equally distributed either on the edge going through the middle of the right surface, for case 1, which is the same as the reference case for part shape optimization in Section 6.2, or on the top right edge, for case 2. The domain size was $80 \times 40 \times 40$ mm. The volume constraint in both examples is again set to 40% and the results for which relative change of objective function in the last three increments is less than 10^{-5} are chosen as converged.

The obtained shape for case 1, with domain for supports distribution underneath its members can be seen in Figure 7.15. Once again, the domain is built by shifting the whole part 5 mm in the positive printing direction to make room for supports. During the release stage the bottom left edge is constrained in the x and z -direction. Additionally, a single node in the middle of this edge is constrained in the y -direction and the top left edge is fully constrained in the x -direction. The second case with the same boundary conditions during the release stage is presented in Figure 7.16.

Optimizations are performed for the same parameter variation as previously but for the sake of brevity only some of the obtained supports distributions will be shown in plots and other optimization related parameters will still be reported in tables. The volume constraint is set to 30% and so is the initial supports volume. The domains after the vertical shifting could be written into a block with dimensions $80 \times 40 \times 45$ mm. The total number of elements for the first shape is 134024 and the number of design variables is 6168 and for the second shape the numbers are 143742 and 7440 respectively. In both cases 45 printing increments followed by a single release step are required and the average calculation time is 35 minutes for the first shape and 40 minutes for the second shape, including adjoint sensitivities calculation. The number of iterations required for different parameters is presented in Table 7.1.

As can be seen, the number of iterations is similar to that required for the 2D examples, even though the number of elements and design variables increased. The optimizer had a problem with convergence for case 1, $p = 7$ for printing only simulation and case 2, $p = 1$ for printing and release, this seem quite random and is manifested by non-monotonic convergence behaviour.

Tables 7.2 and 7.3 give the information about the final objective function value and

Table 7.1: Number of optimization iterations required per optimization

	$p = 1$, pr.	$p = 1$, pr. & rel.	$p = 7$, pr.	$p = 7$, pr. & rel.
Case 1	85	76	208	86
Case 2	98	159	76	126

Figure 7.15: Depiction of case 1: a) bracket obtained from minimization of compliance with domain for supports optimization b) with boundary conditions during the release stage.

Figure 7.16: Depiction of case 2: a) bracket obtained from minimization of compliance with domain for supports optimization b) with boundary conditions during the release stage.

volume used in the converged solution. In all optimizations a significant decrease of the objective function can be observed. Special attention is given to case 1: $p = 1$ for printing and release, and case 2: $p = 1$ and $p = 7$ for printing and release, where the optimal solutions do not use all of the available volume. This suggests that addition of the release stage increases the nonlinearity of the problem and that the optimal solutions not always have the volume constraint active, indicating that addition of supports material increases the objective function, at least for this local minimum. The results that will be plotted in detail are case 1 for $p = 1$ and case 2 for $p = 7$, both of the cases for printing only and for printing and release stage. In Figure 7.17 two results of case 1 optimization are compared, for $p = 1$, printing only stage and

Table 7.2: Final value of the objective function (part distortions).

	$p = 1$, pr.	$p = 1$, pr. & rel.	$p = 7$, pr.	$p = 7$, pr. & rel.
Case 1	58%	77%	51%	62%
Case 2	60%	69%	51%	50%

Table 7.3: Volume of the supports for the converged solution.

	$p = 1$ pr.	$p = 1$ pr. & rel.	$p = 7$ pr.	$p = 7$ pr. & rel.
Case 1	30%	29.5%	30%	30%
Case 2	30%	27.5%	30%	24%

Figure 7.17: Depiction of supports distribution obtained for case 1 for the full domain (top) and the domain cut by xy -plane on height $z=25$ mm (bottom).

printing and release stage. In the case of printing only, there is a clear tendency to add supports on the bounds of the domain. Moreover, at the bottom a separate island has formed in the centre. Most of the elements for the printing only case are either fully dense or void whereas in the case of printing and release on the left side many supports with intermediate densities can be observed.

Figure 7.18 compares distributions obtained for case 2 for $p = 7$ between printing only and printing and release simulations. In the case of printing only, supports are more scattered at the bottom with many elements that are not fully dense. The supports for simulation with printing and release create a distinct shape at the bottom with most of the elements being fully dense. Moreover, the fact that 5% less volume has been used is noticeable. Regarding the supports in the middle of the domain, that connect the top and the bottom member of the part, most are located on the boundaries.

The plots of evolution of objective functions are shown in Figure 7.19. Case 1 and

Figure 7.18: Depiction of supports distribution obtained for case 2 for the full domain (top) and the domain cut by xy -plane on height $z=25$ mm (bottom).

case 2 show a similar convergence behaviour: the fastest decay happens in the first 20 iterations; the remainder are mainly for tuning the values of design variables. In both cases the whole allowable constraint function volume is used for the printing only simulation. In the case of printing and release the volume used is less than the allowable value. A quick drop in the volume functions at the start indicates that some of the design variables have a negative effect on the objective function and their presence increases the distortion instead of limiting it. But after the quick initial decrease, the printing only case finds an optimum that uses all allowable volume, whereas the volume for the printing and release case stays below 30%.

Figure 7.19: Distortions functions plots for case 1 (left), case 2 (right) and volume constraint for both (bottom) as a function of optimization iteration.

7.2.2 Supports distribution vs. part shape optimization

In the same way as for 2D an investigation will be carried out into the possibility of reducing unwanted outcome of the printing process by means of proper supports distribution in place of altering a part shape. The geometry that is going to be used is the same as case 1 in the previous section which is the same as the reference example for 3D from Chapter 6. To create a design domain the whole shape is shifted 5 mm in the positive z -direction and again, so as to increase credibility and prevent purely numerical examples where there is almost no support volume used, supports are introduced below the bottom of the part as passive elements. As before, two constraints are investigated: recoater collision and global nodal distortions. The domain with enforced supports is presented in Figure 7.20.

Recoater collision

The goal of the first optimization is to prevent recoater collision, using as little supports volume as possible. As previously collision is assumed when a threshold of

Figure 7.20: Depiction of design domain with passive supports (colour green).

Figure 7.21: Depiction of nodes for which collision would occur if the support domain volume were to be set to 0.01.

100 μm in the positive printing direction is exceeded. In the case of part printing only with the supports enforced below the bottom member and with the density of all of the rest of the support domain elements set to 0.01, the collision would occur for the red nodes in Figure 7.21.

The results for optimization of supports distribution with the recoater collision constraint are presented in Figure 7.22. The starting volume of domain was 30%. Clearly the same trend of separate supports scattered over the domain can be observed but this time concentrated at the boundary of the domain. Supports on the edge counteract the distortions in the most efficient way, since the top member is connected with the bottom in the middle. Moreover, uniform scatter of the supports again confirms the global character of the constraint.

Figure 7.22: Depiction of supports distribution obtained for volume minimization with the recoater collision constraint.

Global nodal distortions

The second optimization case considers minimization of deformation of the same loaded nodes as in Figure 6.15a but this time only by changing distribution of the supports. The p -parameter is set to 7 to reduce the maximal distortions of the nodes. Moreover, after the release from the base plate and the removal of the supports the shape is again transferred to its theoretical assembly position. Since it is hard to assess how little distortion is feasible, it was decided to minimize distortion with a constraint on available supports volume. Three optimizations with different constraint values will be considered: 10%, 20% and 30%.

The distributions obtained for each of them are presented in Figure 7.23. The number of iterations until convergence for constraint 10%, 20% and 30% was equal 70, 54 and 25, which is surprisingly low. Clearly, in every case supports are deposited in the same spots and with the increase of allowable volume their number increases.

As can be seen in Figure 7.24 the capabilities of supports with respect to minimization of nodal distortions are limited and are always above 0.4 mm, whereas with tailoring of part shape 0.2 mm was easily achievable. There are similarities between the distribution plots in Figure 7.24 and all of the simulations use all of the allowable constraint volume.

Figure 7.23: Depiction of supports distribution obtained for distortion minimization for different values of V .

Figure 7.24: Distortions (left) and volume (right) as a function of iteration for optimization with a distortion minimization.

7.3 3D use case

The use case from Chapter 4 is taken as a final example for supports distribution optimization. First, optimizations are performed for the element size $1 \times 1 \times 1$ mm to get an overview of the general performance of the method in a reasonable time frame. The results are then compared with optimization for mesh size $0.5 \times 0.5 \times 0.5$ mm to check whether the obtained distributions are comparable. The supports that are necessary for the printing of geometry due to downward pointing edges have been identified and are applied as passive elements. The optimization domain consists only of design variables that connect part geometry to the base plate to prevent supports in locations that make their removal difficult. During printing, the bottom nodes are suppressed in order to mimic printing on the base plate. In the release stage supports are removed and only a sufficient number of bottom nodes are retained to keep the part fixed in space.

Mesh size 1 mm

The part geometry with a design domain and passive supports marked in green is presented in Figure 7.25. In the left picture the bottom of the domain is indicated in grey. In total the model consists of 25488 elements and 29067 nodes and requires 49 simulation increments for the printing stage and a single increment for the release stage. The design domain consists of 1195 design variables. For a simulation of that size a single optimization iteration requires around 8 minutes. The objective of the optimization is to minimize distortions of the part with a constraint on maximal allowable supports volume set to 30% and the results were taken for which relative change of objective function in the last three increments was less than 10^{-5} . Four optimization cases are considered. Optimizations are done for two p -values either equal to 1 or 7 and for printing only or for printing and release combined.

The number of optimization iterations required for each of the cases is presented in Table 7.4. In general fewer iterations are required than for the 3D examples for overall part distortions in the previous section. In all cases it is possible to reduce the objective function to as little as 53%. The volume constraint is active in every case. The obtained distributions are presented in Figures 7.26 and 7.27. In all of the examples most of the supports are located at the front of the part where the mounting plates extend. This is caused by the existence in that location of long material layers, which will warp upwards if left unsupported, leading to significant distortions. A similarity in pattern for printing only and for printing and release between different p -values can be observed but it is hard to find a reasoning for a given pattern.

The evolutions of objective and constraint functions is presented in Figure 7.28. The objective function plots show smooth convergence for both $p = 1$ and $p = 7$ with

Table 7.4: Number of optimization iterations required per optimization and final relative objective function value

	$p = 1$, pr.	$p = 1$, pr. & rel.	$p = 7$, pr.	$p = 7$, pr. & rel.
iterations	80	77	40	58
objective	81%	69%	53%	76%

Figure 7.25: The design domain for support optimization of the use case. Passive supports are shown in green. The gray area indicates where supports are allowed.

almost no jumps. For the volume constraint a characteristic drop at the start can be observed.

Figure 7.26: Depiction of supports distribution obtained for $p = 1$ for printing only (top) and printing & release (bottom).

Figure 7.27: Depiction of supports distribution obtained for $p = 7$ for printing only (top) and printing & release (bottom).

Figure 7.28: Compliance plots for $p = 1$ (left), $p = 7$ (right) and volume constraint for both (bottom) as a function of optimization iteration.

Mesh size 0.5 mm

As mentioned, to check whether results will change with a more refined mesh size two optimizations for an element size $0.5 \times 0.5 \times 0.5$ mm are performed and compared. The chosen optimizations are $p = 1$, printing stage only and $p = 7$, printing and release stage. The same boundary conditions and simulation parameters were used as previously. Moreover, passive supports have been again forced under the downward pointing edges. In total the model consists of 250844 elements and 268714 nodes and the simulation requires 98 steps for the printing stage and a single increment for the release stage. The domain consists of 4399 design variables. The part geometry with design domain and passive supports marked in green is shown in Figure 7.29. A single optimization iteration of that size takes around 5 hours and 25 minutes. Once again, the objective function was to minimize part distortions measured using a p -norm function with a constraint on maximal allowable supports volume set to 30%; convergence is assumed when a relative change of objective function in the last three increments is less than 10^{-5} .

The number of required optimization iterations and relative change of objective func-

Figure 7.29: The domain for distortion minimization of the use case with passive supports for element size 0.5 mm.

Table 7.5: Number of optimization iterations required per optimization and final relative objective function value

	$p = 1$, printing	$p = 7$, printing & release
iterations	95	131
objective	74%	66%

tion are presented in Table 7.5. Mesh size 0.5 mm required more iterations than mesh size 1 mm (Table 7.4) and also required much more time per iterations. In the end the simulation for $p = 7$ took around 30 days. In both cases the more detailed simulation results in lower relative local minima. The support distributions obtained for the two mesh sizes are compared in Figures 7.30 and 7.31. There is a big similarity between the obtained supports distributions but, as expected, more detailed solutions are provided by mesh size 0.5 mm. Finally, objective functions and constraint evolution are compared in Figure 7.32. The convergence plots in every case are smooth. For $p = 1$ the initial value of objective function is on the same level but the final obtained value is lower. In case of $p = 7$ the maximal distortions are different because mesh size 1 mm underestimates distortions, as concluded in Section 4.1.2. Nevertheless, as Figure 7.31 indicates, it leads to similar distributions.

Figure 7.30: Depiction of supports distribution obtained for $p = 1$ for printing only for element size 1 mm (top) and 0.5 mm (bottom).

Figure 7.31: Depiction of supports distribution obtained for $p = 7$ for printing & release for element size 1 mm (top) and 0.5 mm (bottom).

Figure 7.32: Compliance plots for $p = 1$ (left), $p = 7$ (right) and volume constraint for both (bottom) as a function of optimization iteration for two mesh sizes.

7.4 Summary

A method of optimizing the supports distribution for minimal distortion during and after production of AM products has been proposed and verified. For this purpose, a numerical model of 3D printing presented in Chapter 3 and adjoint sensitivities presented in Chapter 5 have been implemented in a topology optimization scheme. Examples for both 2D and 3D cases have been investigated. A relatively small number of iterations is required for the optimizer to converge. This is due to the supports definition, which gathers densities of each column of elements into a single design variable and therefore reduces the number of design variables considerably. Additionally, the volume constraint is not always active, which means that the amount of volume used in the optimal result is less than the value allowed in the constraint. For every case of 2D and 3D optimization the value of objective function has been reduced significantly. Finally, it has been shown that the results are not mesh dependent.

A separate problem has been formulated for the reference shapes in Chapter 6 with the aim of limiting the same printing constraints by tailoring supports distribution instead of changing the structure geometry. In the case of recoater collision the goal

of optimization was to minimize the volume of supports with a constraint on collision. In both 2D and 3D cases little extra volume was needed and a characteristic scatter of supports over the domain was observed in case of 2D, or over the boundaries of the domain in the case of 3D. In the case of minimization of distortion of chosen nodes for different values of volume constraints both 2D and 3D optimization led to similar results. Surprisingly, the best results were achieved in the case of 2D for the smallest volume constraint equal to 20% and for 3D for the second smallest volume constraint also equal to 20%. The minimization of distortion was only possible to certain extent, limited mostly by the stiffness of the used supports. In order to further minimize distortions, different supports or optimization of part shape have to be used. Nevertheless, satisfactory results have been presented without affecting the part performance.

Chapter 8

Conclusions and Recommendations

The main aim of the research that is described in this thesis was to develop topology optimization techniques that incorporate numerical modelling of additive manufacturing (AM) to take into account distortion constraints. The thesis may be divided into two main parts: firstly, the development of a numerical model, where various numerical techniques for modelling printing distortions on the part scale have been implemented and tested, and secondly, the development of topology optimization routines, in which the numerical model has been integrated. Two different optimization types can be distinguished here: optimization of a part shape and optimization of supports distribution. The optimizations incorporated two AM constraints: recoater collision and distortions of a part measured after the separation from the base plate. Separate conclusions for each subject will be presented first. Recommendations for further research will be suggested based on those conclusions.

8.1 Conclusions

The goal of this work, as defined in Chapter 1, has been realized, namely to provide a numerical model of additive manufacturing to simulate distortions on a part scale and to incorporate it into topology optimization to reduce distortions. A numerical model that applies several simplifications has been implemented and verified numerically. Next, a topology optimization code that includes different distortion constraints has been developed and integrated with it. Adjoint sensitivity equations have been derived for the constraint functions. Two types of optimization have been investigated: optimization of the part shape and optimization of support distributions. The optimizations have been performed on examples both in 2D and 3D.

8.1.1 Numerical modelling of selective laser melting

- Development of a numerical model that predicts distortions on a part scale in a reasonable time (couple of hours) is possible when proper numerical techniques are applied. Such a model may incorporate all of the production stages.

- Voxel-based meshes with inherent strains have to be used with care, as does the lumped method. The correct element size must be chosen depending on the part dimensions, and a representative value of the inherent strains is of the utmost importance.
- The proposed element growth technique allows for more detailed calculations but at the cost of increased calculation time. Using it to calculate every single layer is rather impractical.
- To incorporate support structures into voxel-based simulations either a special material or a special element model has to be implemented. Such a model has to take into account the stiffness anisotropy that comes from supports geometry.
- Simulation of stress relief is possible using a creep model but proper parameters for the creep model have to be obtained from experiments. Values that were acquired by extrapolation of the parameters from literature definitely overestimate the true coefficients and in consequence lead to excessively high stress rates. The results from stress relaxation calculations indicate that practically all residual stress disappears during heat treatment.

8.1.2 Topology optimization with additive manufacturing constraints

Optimization of part shape

- Tailoring a part's final distortion through redesign of the part shape is possible but comes at a cost of the part performance. As the constraint gets stricter, the part performance deteriorates. With respect to the required number of iterations there is no strong trend of the number of iterations increasing as the strictness of the constraint increases. This is due to the non-monotonic convergence behaviour.
- In the case of stricter values of the global distortions and the recoater collision constraints, the appearance of auxiliary members can be observed, the only role of which is to prevent part distortions and which do not participate in load carrying during the compliance simulation. Such structures resemble supports proposed in real-life 3D printing.
- Obtained shapes differ a lot, depending on whether or not the release stage has been incorporated into the optimization. Simulations that are performed without the release stage represent the assumption that after the printing stage heat treatment is applied to relieve all the stresses. Consequently, distortions after separation may be ignored. When optimization is done including the release stage it is possible to tailor residual stresses in such a way that distortions of the part that accumulated during the printing stage are compensated by distortions in the release stage.
- For 3D structures, as the strictness of the distortion constraint increases, a trend can be observed of designs that start to take a shell-like form with a cavity inside.

Optimization of supports distribution

- Reduction of part distortions through design of supports distribution is possible only to a limited extent. This is caused by limits set on stiffness of the assumed supports. Of course it is possible to apply supports with the same stiffness as the printed material of the part but removing them is cumbersome.
- In principle, support distributions obtained from topology optimization are often hard to explain a priori. Their placement is not intuitive, which indicates that it would be hard to find the optimized distribution without application of such an optimization algorithm.
- With optimization of supports distribution, elements with intermediate densities are often obtained. It has been decided to not push design fully towards a black-and-white solution and to allow for grey areas on the assumption that supports with varying cross-sections can be printed.
- Application of the recoater collision constraint leads to a uniform distribution of supports over most of the domain in the case of 2D optimization and on the edges of the domain in the case of 3D optimization.
- It has been shown that optimization results are mesh size independent and similar support distributions are obtained in the case of different resolutions of the mesh. This has been proved both on 2D and 3D examples.

8.2 Further research recommendations

Although the main research objective has been achieved, it could be beneficial to deal with some of the topics in more detail and many new questions arose as the research proceeded. The most important open issue is the lack of experimental validation of the proposed numerical model. Purely numerical verification allows the identification of some key numerical parameters, such as the minimal element size. To make the proposed model applicable for industrial problems, it is necessary to determine material parameters and assess them by experiments. The most important parameters that require determination by experiments are inherent strains, parameters for the supports and creep parameters for the material model. Moreover, to make the method applicable for industrial-scale 3D optimization further developments with respect to reduction of calculation time are needed. Furthermore, even if application of the element growth technique for every layer is impractical, turning on the method might be justified for particular chosen regions for which application of layer lumping may be too crude, so it is worth checking.

In the case of topology optimization the main goal has been fully achieved but improvements are possible. One of the topics that could increase the fidelity of the obtained results is the incorporation of a nonlinear material model to predict plastic deformation. This would have big impact on the adjoint method and would require the history of each integration point to be stored. Moreover, the heat treatment has been modelled by the simple assumption that it relieves all the stresses and therefore the influence of the release stage may be ignored. To increase accuracy the heat treatment stage should also be incorporated into the optimization.

The element growth technique has been proposed and compared with the lumped method but it has not been shown how application of this technique influences shapes obtained from the topology optimization. So the natural next step would be to perform optimization with the growth technique to compare the results with the lumped method. Additionally, other 3D printing-related restrictions could be incorporated into the optimization. For example, a stress constraint could be implemented to inspect how limits on this quantity influence the optimization results and to prevent material failure by cracking. Finally, other AM-related subjects could be incorporated into the topic of this thesis for example the overhang constraint. It could be implemented in order to obtain designs that have constraint distortions and are fully overhang-free.

Appendix A

A.1 Derivation of tangent stiffness matrix for plasticity

The starting point for derivation is Eq. (2.63):

$$d\boldsymbol{\sigma}^{(n+1)} = \left[\mathbf{D}_{\text{el}} - 2G\mathbf{n}^{(n+1)} \otimes \frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}} - 2G\Delta\lambda \frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right] : d\boldsymbol{\epsilon}^{(n+1)}. \quad (\text{A.1})$$

Here, three components are yet to be determined: $\mathbf{n}^{(n+1)}$, $\frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}}$ and $\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}$. For the $\mathbf{n}^{(n+1)}$ the direction in $(n+1)$ is the same as the direction of the trial stress

$$\mathbf{n}^{(n+1)} = \frac{\mathbf{s}^{(n+1)}}{\|\mathbf{s}^{(n+1)}\|} = \frac{\mathbf{s}_t^{(n+1)}}{\|\mathbf{s}_t^{(n+1)}\|}. \quad (\text{A.2})$$

To obtain the second term $\frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}}$ a derivative of Eq. (2.58) with respect to $\boldsymbol{\epsilon}^{(n+1)}$ is taken

$$\frac{\partial \|\mathbf{s}_t^{(n+1)}\|}{\partial \boldsymbol{\epsilon}^{(n+1)}} - 2G \frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}} - \sqrt{\frac{2}{3}} H \frac{\partial \varepsilon_{\text{eq}}^{(n+1)}}{\partial \Delta\lambda} \frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}} = 0 \quad (\text{A.3})$$

$$2G \frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}} + \frac{2}{3} H \frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}} = 2G\mathbf{n}^{(n+1)} \quad (\text{A.4})$$

$$\frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}} = \frac{\mathbf{n}^{(n+1)}}{1 + \frac{H}{3G}} \quad (\text{A.5})$$

The last term, $\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}$, is obtained by differentiation of Eq. (A.2)

$$\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} = \frac{1}{\|\mathbf{s}_t^{(n+1)}\|} \frac{\partial \mathbf{s}_t^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} - \frac{\mathbf{s}_t^{(n+1)}}{\|\mathbf{s}_t^{(n+1)}\|^2} \frac{\partial \|\mathbf{s}_t^{(n+1)}\|}{\partial \varepsilon_{\text{eq}}^{(n+1)}} \frac{\partial \varepsilon_{\text{eq}}^{(n+1)}}{\partial \Delta\lambda} \otimes \frac{\partial \Delta\lambda}{\partial \boldsymbol{\epsilon}^{(n+1)}}, \quad (\text{A.6})$$

where

$$\frac{\partial \mathbf{s}_t^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} = 2G \mathbf{I}^{\text{dev}}, \quad (\text{A.7})$$

because

$$\begin{aligned} \mathbf{s}_t^{(n+1)} &= \mathbf{s}^{(n)} + 2G \Delta \mathbf{e} \\ \Delta \mathbf{e} &= \mathbf{e}^{(n+1)} - \mathbf{e}^{(n)} \\ \mathbf{e}^{(n+1)} &= \mathbf{I}^{\text{dev}} \boldsymbol{\varepsilon}^{(n+1)}, \end{aligned} \quad (\text{A.8})$$

where $\mathbf{e}$ is a deviatoric strain component. Finally the expression for $\frac{\partial \|\mathbf{s}_t^{(n+1)}\|}{\partial \varepsilon_{\text{eq}}^{(n+1)}}$ is derived from Eq. (A.3)

$$\frac{\partial \|\mathbf{s}_t^{(n+1)}\|}{\partial \varepsilon_{\text{eq}}^{(n+1)}} = 2G \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}} H. \quad (\text{A.9})$$

Putting Eqs. (A.5), (A.7) and (A.9) into (A.6) and by knowing that since $\frac{\partial \Delta \lambda}{\partial \varepsilon_{\text{eq}}^{(n+1)}} = \sqrt{\frac{3}{2}}$ then $\frac{\partial \varepsilon_{\text{eq}}^{(n+1)}}{\partial \Delta \lambda} = \sqrt{\frac{2}{3}}$ yields

$$\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} = \frac{2G}{\|\mathbf{s}_t^{(n+1)}\|} (\mathbf{I}^{\text{dev}} - \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)}). \quad (\text{A.10})$$

Substituting missing terms into Eq. (A.1) and making some rearranging gives

$$d\boldsymbol{\sigma}^{(n+1)} = \left[\mathbf{D}_{\text{el}} - \frac{(2G)^2 \Delta \lambda}{\|\mathbf{s}_t^{(n+1)}\|} \mathbf{I}^{\text{dev}} + \left(\frac{(2G)^2 \Delta \lambda}{\|\mathbf{s}_t^{(n+1)}\|} - \frac{2G}{1 + \frac{H}{3G}} \right) \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right] : d\boldsymbol{\varepsilon}^{(n+1)}, \quad (\text{A.11})$$

for which the consistent tangent matrix is:

$$\mathbf{D}_{\text{pl}} = \mathbf{D}_{\text{el}} - \frac{(2G)^2 \Delta \lambda}{\|\mathbf{s}_t^{(n+1)}\|} \mathbf{I}^{\text{dev}} + \left(\frac{(2G)^2 \Delta \lambda}{\|\mathbf{s}_t^{(n+1)}\|} - \frac{2G}{1 + \frac{H}{3G}} \right) \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)}. \quad (\text{A.12})$$

A.2 Derivation of tangent stiffness matrix for viscoplasticity

The starting point for this derivation is Eq. (2.74) written in terms of total strains

$$\begin{aligned} d\boldsymbol{\sigma}^{(n+1)} = & \left(\mathbf{D}_{\text{el}} - 2G \left(\sqrt{\frac{3}{2}} \right)^{m+1} \Delta t A e^{\frac{-Q}{RT}} \mathbf{n}^{(n+1)} \frac{\partial \|\mathbf{s}^{(n+1)}\|^m}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right. \\ & \left. - 2G \left(\sqrt{\frac{3}{2}} \right)^{m+1} \Delta t A \left\| \mathbf{s}^{(n+1)} \right\|^m e^{\frac{-Q}{RT}} \frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right) : d\boldsymbol{\epsilon}^{(n+1)}. \end{aligned} \quad (\text{A.13})$$

Two terms have to be obtained in order to get the consistent tangent matrix $\frac{\partial \|\mathbf{s}^{(n+1)}\|^m}{\partial \boldsymbol{\epsilon}^{(n+1)}}$ and $\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}$.

The former can be written as

$$\frac{\partial \|\mathbf{s}^{(n+1)}\|^m}{\partial \boldsymbol{\epsilon}^{(n+1)}} = \frac{\partial \|\mathbf{s}^{(n+1)}\|^m}{\partial \|\mathbf{s}^{(n+1)}\|} \frac{\partial \|\mathbf{s}^{(n+1)}\|}{\partial \boldsymbol{\sigma}^{(n+1)}} \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} = m \left\| \mathbf{s}^{(n+1)} \right\|^{m-1} \mathbf{n}^{(n+1)} \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}. \quad (\text{A.14})$$

Similarly for the latter

$$\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} = \frac{\partial \mathbf{n}^{(n+1)}}{\partial \mathbf{s}^{(n+1)}} \frac{\partial \mathbf{s}^{(n+1)}}{\partial \boldsymbol{\sigma}^{(n+1)}} \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}. \quad (\text{A.15})$$

Following [115] it can be proven that

$$\frac{\partial \mathbf{n}^{(n+1)}}{\partial \mathbf{s}^{(n+1)}} = \frac{1}{\|\mathbf{s}^{(n+1)}\|} \left(\mathbf{I}^{\text{sym}} - \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right) \quad (\text{A.16})$$

and

$$\frac{\partial \mathbf{s}^{(n+1)}}{\partial \boldsymbol{\sigma}^{(n+1)}} = \mathbf{I}^{\text{dev}}. \quad (\text{A.17})$$

Substituting Eqs. (A.16) and (A.17) in Eq. (A.15) gives

$$\frac{\partial \mathbf{n}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} = \frac{1}{\|\mathbf{s}^{(n+1)}\|} \left(\mathbf{I}^{\text{dev}} - \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right) \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}}. \quad (\text{A.18})$$

Setting $\gamma = 2G \left(\sqrt{\frac{3}{2}} \right)^{m+1} A e^{\frac{-Q}{RT}} \left\| \mathbf{s}^{(n+1)} \right\|^{m-1}$ and after substituting (A.14) and (A.18) into (A.13)

$$\begin{aligned} d\boldsymbol{\sigma}^{(n+1)} = & \left(\mathbf{D}_{\text{el}} - m \Delta t \gamma \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right. \\ & \left. - \Delta t \gamma \left(\mathbf{I}^{\text{dev}} - \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right) \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\epsilon}^{(n+1)}} \right) : d\boldsymbol{\epsilon}^{(n+1)}. \end{aligned} \quad (\text{A.19})$$

While after rearrangement this is

$$\begin{aligned} d\boldsymbol{\sigma}^{(n+1)} = & \left(\mathbf{D}_{\text{el}} - \Delta t \gamma \mathbf{I}^{\text{dev}} \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} + (\Delta t \gamma \right. \\ & \left. - m \Delta t \gamma) \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} \right) : d\boldsymbol{\varepsilon}^{(n+1)}, \end{aligned} \quad (\text{A.20})$$

using $\mathbf{I}^{\text{sym}} = \mathbf{I}^{\text{vol}} + \mathbf{I}^{\text{dev}}$ and grouping terms by $d\boldsymbol{\sigma}^{(n+1)}$ gives

$$\left(\mathbf{I}^{\text{vol}} + (1 + \Delta t \gamma) \mathbf{I}^{\text{dev}} + (m \Delta t \gamma - \Delta t \gamma) \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right) : d\boldsymbol{\sigma}^{(n+1)} = \mathbf{D}_{\text{el}} : d\boldsymbol{\varepsilon}^{(n+1)}. \quad (\text{A.21})$$

Here, use has been made of dependence $\frac{\partial \boldsymbol{\sigma}^{(n+1)}}{\partial \boldsymbol{\varepsilon}^{(n+1)}} : d\boldsymbol{\varepsilon}^{(n+1)} = d\boldsymbol{\sigma}^{(n+1)}$, because for any arbitrary tensors $\mathbf{A}$ and $\mathbf{B}$

$$\frac{\partial \mathbf{A}}{\partial \mathbf{B}} : d\mathbf{B} = \frac{\partial A_{ij}}{\partial B_{kl}} (\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k \otimes \mathbf{a}_l) : dB_{kl}(\mathbf{a}_k \otimes \mathbf{a}_l) = dA_{ij}(\mathbf{a}_i \otimes \mathbf{a}_j) = d\mathbf{A}, \quad (\text{A.22})$$

where $\mathbf{a}$ denotes a unit base vector.

Now in order to obtain $\mathbf{D}_{cr}$ we may rewrite the terms in the bracket as

$$\mathbf{A} : d\boldsymbol{\sigma}^{(n+1)} = \mathbf{D}_{\text{el}} : d\boldsymbol{\varepsilon}^{(n+1)}. \quad (\text{A.23})$$

and perform inversion

$$d\boldsymbol{\sigma}^{(n+1)} = \mathbf{A}^{-1} : \mathbf{D}_{\text{el}} : d\boldsymbol{\varepsilon}^{(n+1)} = \mathbf{D}_{cr} : d\boldsymbol{\varepsilon}^{(n+1)}. \quad (\text{A.24})$$

where

$$\mathbf{A}^{-1} = \left[\mathbf{I}^{\text{vol}} + \frac{1}{1 + \Delta t \gamma} \mathbf{I}^{\text{dev}} + \frac{\Delta t \gamma (m - 1)}{1 + m + \Delta t \gamma m + \Delta t \gamma^2 m} \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right] \quad (\text{A.25})$$

with $\mathbf{D}_{\text{el}} = 3K\mathbf{I}^{\text{vol}} + 2G\mathbf{I}^{\text{dev}}$ the final form of consistent tangent matrix for viscoplastic creep calculation is:

$$\mathbf{D}_{cr} = \left[3K\mathbf{I}^{\text{vol}} + 2G \frac{1}{1 + \Delta t \gamma} \mathbf{I}^{\text{dev}} + 2G \frac{\Delta t \gamma (m - 1)}{1 + m + \Delta t \gamma m + \Delta t \gamma^2 m} \mathbf{n}^{(n+1)} \otimes \mathbf{n}^{(n+1)} \right]. \quad (\text{A.26})$$

Bibliography

- [1] F. Thiesse, M. Wirth, H.-G. Kemper, M. Moisa, D. Morar, H. Lasi, F. Piller, P. Buxmann, L. Mortara, S. Ford, et al. Economic implications of additive manufacturing and the contribution of MIS. *Business & Information Systems Engineering*, 57(2):139–148, 2015.
- [2] W. Gao, Y. Zhang, D. Ramanujan, K. Ramani, Y. Chen, C. B. Williams, C. C.L. Wang, Y. C. Shin, S. Zhang, and P. D. Zavattieri. The status, challenges, and future of additive manufacturing in engineering. *Computer-Aided Design*, 69: 65–89, 2015.
- [3] I. Gibson, D. W. Rosen, and B. Stucker. *Additive manufacturing technologies*, volume 17. Springer, 2014.
- [4] I. Campbell, D. Bourell, and I. Gibson. Additive manufacturing: rapid prototyping comes of age. *Rapid prototyping journal*, 2012.
- [5] S. A. Tofail, E. P. Koumoulos, A. Bandyopadhyay, S. Bose, L. O’Donoghue, and C. Charitidis. Additive manufacturing: scientific and technological challenges, market uptake and opportunities. *Materials today*, 21(1):22–37, 2018.
- [6] T. Pereira, J. V. Kennedy, and J. Potgieter. A comparison of traditional manufacturing vs additive manufacturing, the best method for the job. *Procedia Manufacturing*, 30:11–18, 2019.
- [7] T. Wohlers and T. Gornet. History of additive manufacturing. *Wohlers Report*, 24(2014):118, 2014.
- [8] W. J. Sames, F. A. List, S. Pannala, R. R. Dehoff, and S. S. Babu. The metallurgy and processing science of metal additive manufacturing. *International Materials Reviews*, 61(5):315–360, 2016.
- [9] O. Abdulhameed, A. Al-Ahmari, W. Ameen, and S. H. Mian. Additive manufacturing: Challenges, trends, and applications. *Advances in Mechanical Engineering*, 11(2), 2019.
- [10] EOS metal systems. <https://www.eos.info>. Accessed: 2020-06-30.
- [11] A. Hinojos, J. Mireles, A. Reichardt, P. Frigola, P. Hosemann, L. E. Murr, and R. B. Wicker. Joining of inconel 718 and 316 stainless steel using electron beam melting additive manufacturing technology. *Materials & Design*, 94:17–27, 2016.

- [12] S. K. Everton, M. Hirsch, P. Stravroulakis, R. K. Leach, and A. T. Clare. Review of in-situ process monitoring and in-situ metrology for metal additive manufacturing. *Materials & Design*, 95:431–445, 2016.
- [13] B. Zhang, Y. Li, and Q. Bai. Defect formation mechanisms in selective laser melting: a review. *Chinese Journal of Mechanical Engineering*, 30(3):515–527, 2017.
- [14] C. Y. Yap, C. K. Chua, Z. L. Dong, Z. H. Liu, D. Q. Zhang, L. E. Loh, and S. L. Sing. Review of selective laser melting: Materials and applications. *Applied physics reviews*, 2(4), 2015.
- [15] F. Verhaeghe, T. Craeghs, J. Heulens, and L. Pandelaers. A pragmatic model for selective laser melting with evaporation. *Acta Materialia*, 57(20):6006–6012, 2009.
- [16] C. Li, Z.Y. Liu, X.Y. Fang, and Y.B. Guo. Residual stress in metal additive manufacturing. *Procedia CIRP*, 71:348–353, 2018.
- [17] M. Schmidt, M. Merklein, D. Bourell, D. Dimitrov, T. Hausotte, K. Wegener, L. Overmeyer, F. Vollertsen, and G. N. Levy. Laser based additive manufacturing in industry and academia. *CIRP Annals*, 66(2):561–583, 2017.
- [18] V. Griffiths, J. P. Scanlan, M. H. Eres, A. Martinez-Sykora, and P. Chinchapatnam. Cost-driven build orientation and bin packing of parts in selective laser melting (SLM). *European Journal of Operational Research*, 273(1):334–352, 2019.
- [19] J. Jiang, X. Xu, and J. Stringer. Support structures for additive manufacturing: a review. *Journal of Manufacturing and Materials Processing*, 2(4):64, 2018.
- [20] J.-P. Järvinen, V. Matilainen, X. Li, H. Piili, A. Salminen, I. Mäkelä, and O. Nyrhilä. Characterization of effect of support structures in laser additive manufacturing of stainless steel. *Physics Procedia*, 56:72–81, 2014.
- [21] K. Chadha, Y. Tian, J. G. Spray, and C. Aranas Jr. Effect of annealing heat treatment on the microstructural evolution and mechanical properties of hot isostatic pressed 316l stainless steel fabricated by laser powder bed fusion. *Metals*, 10(6):753, 2020.
- [22] Z. Gronostajski, Z. Pater, L. Madej, A. Gontarz, L. Lisiecki, A. Lukaszek-Solek, J. Luksza, S. Mróz, Z. Muskalski, W. Muzykiewicz, et al. Recent development trends in metal forming. *Archives of Civil and Mechanical Engineering*, 19: 898–941, 2019.
- [23] W. E. Frazier. Metal additive manufacturing: a review. *Journal of Materials Engineering and Performance*, 23(6):1917–1928, 2014.
- [24] R. J. Silva, G. F. Barbosa, and J. Carvalho. Additive manufacturing of metal parts by welding. *IFAC-PapersOnLine*, 48(3):2318–2322, 2015.

-
- [25] Z. Luo and Y. Zhao. A survey of finite element analysis of temperature and thermal stress fields in powder bed fusion additive manufacturing. *Additive Manufacturing*, 21:318–332, 2018.
- [26] A. Bandyopadhyay and K. D. Traxel. Invited review article: Metal-additive manufacturing—modeling strategies for application-optimized designs. *Additive Manufacturing*, 22:758–774, 2018.
- [27] J. Goldak, A. Chakravarti, and M. Bibby. A new finite element model for welding heat sources. *Metallurgical Transactions B*, 15(2):299–305, 1984.
- [28] P. Michaleris. Modeling metal deposition in heat transfer analyses of additive manufacturing processes. *Finite Elements in Analysis and Design*, 86(86):51–60, 2014.
- [29] D. Deng, H. Murakawa, and W. Liang. Numerical simulation of welding distortion in large structures. *Computer Methods in Applied Mechanics and Engineering*, 196(45-48):4613–4627, 2007.
- [30] P. Michaleris and A. DeBiccari. Prediction of welding distortion. *Welding Journal-Including Welding Research Supplement*, 76(4):172s, 1997.
- [31] P.-L. Mollicone, D. Camilleri, T. G. F. Gray, and T. Comlekci. Simple thermo-elastic-plastic models for welding distortion simulation. *Journal of Materials Processing Technology*, 176(1-3):77–86, 2006.
- [32] N. S. Prasad and T. K. S. Narayanan. Finite element analysis of temperature distribution during arc welding using adaptive grid technique. *Welding Journal (Miami, Fla)*, 75(4):123s–128s, 1996.
- [33] H. Runnemalm and S. Hyun. Three-dimensional welding analysis using an adaptive mesh scheme. *Computer Methods in Applied Mechanics and Engineering*, 189(2):515–523, 2000.
- [34] X. K. Zhu and Y. J. Chao. Effects of temperature-dependent material properties on welding simulation. *Computers & Structures*, 80(11):967–976, 2002.
- [35] E. R. Denlinger, M. Gouge, J. Irwin, and P. Michaleris. Thermomechanical model development and in situ experimental validation of the laser powder-bed fusion process. *Additive Manufacturing*, 16:73–80, 2017.
- [36] B. Schoinochoritis, D. Chantzis, and K. Salonitis. Simulation of metallic powder bed additive manufacturing processes with the finite element method: A critical review. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, 231(1):96–117, 2017.
- [37] R. K. Chin, J. L. Beuth, and C. H. Amon. Thermomechanical modeling of successive material deposition in layered manufacturing. In *Solid Freeform Fabrication Proceedings*. Proc. 1996 Solid Freeform Fabrication Symposium, Austin, 1996.

- [38] N. W. Klingbeil, J. L. Beuth, R. K. Chin, and C. H. Amon. Measurement and modeling of residual stress-induced warping in direct metal deposition processes. In *Solid Freeform Fabrication Proceedings*, pages 367–374. Proc. 1998 Solid Freeform Fabrication Symposium, Austin, 1998.
- [39] M. Matsumoto, M. Shiomi, K. Osakada, and F. Abe. Finite element analysis of single layer forming on metallic powder bed in rapid prototyping by selective laser processing. *International Journal of Machine Tools & Manufacture*, 42(1): 61–67, 2002.
- [40] A. H. Nickel, D. M. Barnett, and F. B. Prinz. Thermal stresses and deposition patterns in layered manufacturing. *Materials Science and Engineering a-Structural Materials Properties Microstructure and Processing*, 317(1-2):59–64, 2001.
- [41] K. Dai and L. Shaw. Distortion minimization of laser-processed components through control of laser scanning patterns. *Rapid Prototyping Journal*, 8(5): 270–276, 2002.
- [42] M. Nikoukar, N. Patil, D. Pal, and B. E. Stucker. Methods for enhancing the speed of numerical calculations for the prediction of the mechanical behavior of parts made using additive manufacturing. In *Solid Freeform Fabrication Proceedings*. Proc. 2013 Solid Freeform Fabrication Symposium, Austin, 2013.
- [43] N. Patil, D. Pal, R. H. Khalid, K. Zeng, A. Moreland, A. Hicks, D. Beeler, and B. E. Stucker. A generalized feed forward dynamic adaptive mesh refinement and derefinement finite element framework for metal laser sintering - Part I: Formulation and algorithm development. *Journal of Manufacturing Science and Engineering*, 137(4):041001, 2015.
- [44] K. Zeng, D. Pal, N. Patil, and B. E. Stucker. A new dynamic mesh method applied to the simulation of selective laser melting. In *Proceedings of the Solid Freeform Fabrication Symposium, Austin, TX, August 2013*, pages 12–14, 2013.
- [45] N. E. Hodge, R. M. Ferencz, and J. M. Solberg. Implementation of a thermo-mechanical model for the simulation of selective laser melting. *Computational Mechanics*, 54(1):33–51, 2014.
- [46] M. Markl and C. Körner. Multiscale modeling of powder bed-based additive manufacturing. *Annual Review of Materials Research*, 46:93–123, 2016.
- [47] M. Megahed, H.-W. Mindt, N. N’Dri, H. Duan, and O. Desmaison. Metal additive-manufacturing process and residual stress modeling. *Integrating Materials and Manufacturing Innovation*, 5(1):61–93, 2016.
- [48] C. Li, C. H. Fu, Y. B. Guo, and F. Z. Fang. A multiscale modeling approach for fast prediction of part distortion in selective laser melting. *Journal of Materials Processing Technology*, 229:703–712, 2016.
- [49] C. Li, J.F. Liu, X.Y. Fang, and Y.B. Guo. Efficient predictive model of part distortion and residual stress in selective laser melting. *Additive Manufacturing*, 17:157–168, 2017.

-
- [50] N. Keller and V. Ploshikhin. New method for fast predictions of residual stress and distortion of AM parts. In *Solid Freeform Fabrication*, 2014.
  - [51] F. Neugebauer, N. Keller, V. Ploshikhin, F. Feuerhahn, and H. Köhler. Multi scale FEM simulation for distortion calculation in additive manufacturing of hardening stainless steel. In *Proc. Int. Workshop on ‘Thermal Forming and Welding Distortion’*, 2014.
  - [52] J. Schilp, C. Seidel, H. Krauss, and J. Weirather. Investigations on temperature fields during laser beam melting by means of process monitoring and multiscale process modelling. *Advances in Mechanical Engineering*, 6, 2014.
  - [53] P. Alvarez, J. Ecenarro, I. Setien, M.S. Sebastian, A. Echeverria, and L. Eciolaza. Computationally efficient distortion prediction in powder bed fusion additive manufacturing. *International Journal of Engineering Research & Science*, 2(10):39–46, 2016.
  - [54] Q. Chen, X. Liang, D. Hayduke, J. Liu, L. Cheng, J. Oskin, R. Whitmore, and A. C. To. An inherent strain based multiscale modeling framework for simulating part-scale residual deformation for direct metal laser sintering. *Additive Manufacturing*, 28:406–418, 2019.
  - [55] I. Setien, M. Chiumenti, S. van der Veen, M. San Sebastian, F. Garciandía, and A. Echeverría. Empirical methodology to determine inherent strains in additive manufacturing. *Computers and Mathematics with Applications*, 78:2282–2295, 2018.
  - [56] T. A. Krol, S. Westhäuser, M. F. Zaeh, J. Schilp, and C. Groth. Development of a simulation-based process chain-strategy for different levels of detail for the preprocessing definitions. *Simulation Notes Europe*, 21:135–140, 2011.
  - [57] C. Seidel, M. F. Zaeh, M. Wunderer, J. Weirather, T. A. Krol, and M. Ott. Simulation of the laser beam melting process – approaches for an efficient modelling of the beam-material interaction. *Procedia CIRP*, 25:146–153, 2014.
  - [58] M. F. Zäh and G. Branner. Investigations on residual stresses and deformations in selective laser melting. *Production Engineering*, 4(1):35–45, 2009.
  - [59] F. Hajializadeh and A. Ince. Finite element–based numerical modeling framework for additive manufacturing process. *Material Design & Processing Communications*, 1(1):e28, 2019.
  - [60] J. Baiges, M. Chiumenti, C. A. Moreira, M. Cervera, and R. Codina. An adaptive finite element strategy for the numerical simulation of additive manufacturing processes. *Additive Manufacturing*, 2020, in press.
  - [61] V. D. Savane, R. Fu, P. Gaborik, and C. Chin. Distortion prediction of Ti6Al4V parts in selective laser melting: An industrial case study. In *AIAA Scitech 2020 Forum*, page 1475, 2020.

- [62] M. Seabra, J. Azevedo, A. Araújo, L. Reis, E. Pinto, N. Alves, R. Santos, and J. P. Mortágua. Selective laser melting (SLM) and topology optimization for lighter aerospace components. *Procedia Structural Integrity*, 1:289–296, 2016.
- [63] S. Zargham, T. A. Ward, R. Ramli, and I. A. Badruddin. Topology optimization: a review for structural designs under vibration problems. *Structural and Multidisciplinary Optimization*, 53(6):1157–1177, 2016.
- [64] M. P. Bendsøe and O. Sigmund. *Topology Optimization. Theory, Methods, and Applications*. New York: Springer Verlag, 2003.
- [65] J. Liu and Y. Ma. A survey of manufacturing oriented topology optimization methods. *Advances in Engineering Software*, 100:161–175, 2016.
- [66] L. Meng, W. Zhang, D. Quan, G. Shi, L. Tang, Y. Hou, P. Breitkopf, J. Zhu, and T. Gao. From topology optimization design to additive manufacturing: Today’s success and tomorrow’s roadmap. *Archives of Computational Methods in Engineering*, pages 1–26, 2019.
- [67] R. A. Wildman and A. T. Gaynor. Topology optimization for reducing additive manufacturing processing distortions. Report ARL-TR-8242, Weapons and Materials Research Directorate, US Army Research Laboratory, Aberdeen Proving Ground, United States, 2017.
- [68] G. Allaire and L. Jakabčín. Taking into account thermal residual stresses in topology optimization of structures built by additive manufacturing. *Mathematical Models and Methods in Applied Sciences*, 28(12):2313–2366, 2018.
- [69] M. Leary, L. Merli, F. Torti, M. Mazur, and M. Brandt. Optimal topology for additive manufacture: A method for enabling additive manufacture of support-free optimal structures. *Materials & Design*, 63:678–690, 2014.
- [70] A. T. Gaynor and J. K. Guest. Topology optimization considering overhang constraints: Eliminating sacrificial support material in additive manufacturing through design. *Structural and Multidisciplinary Optimization*, 54(5):1157–1172, 2016.
- [71] M. Langelaar. Topology optimization of 3D self-supporting structures for additive manufacturing. *Additive Manufacturing*, 12, Part A:60–70, 2016.
- [72] E. van de Ven, R. Maas, C. Ayas, M. Langelaar, and F. van Keulen. Continuous front propagation-based overhang control for topology optimization with additive manufacturing. *Structural and Multidisciplinary Optimization*, pages 1–17, 2018.
- [73] A. T. Gaynor, N. A. Meisel, C. B. Williams, and J. K. Guest. Topology optimization for additive manufacturing: considering maximum overhang constraint. In *15th AIAA/ISSMO multidisciplinary analysis and optimization conference*, page 2036, 2014.

-
- [74] M. Langelaar. An additive manufacturing filter for topology optimization of print-ready designs. *Structural and multidisciplinary optimization*, 55(3):871–883, 2017.
  - [75] Y. Mass and O. Amir. Topology optimization for additive manufacturing: Accounting for overhang limitations using a virtual skeleton. *Additive Manufacturing*, 18:58–73, 2017.
  - [76] X. Guo, J. Zhou, W. Zhang, Z. Du, C. Liu, and Y. Liu. Self-supporting structure design in additive manufacturing through explicit topology optimization. *Computer Methods in Applied Mechanics and Engineering*, 323:27–63, 2017.
  - [77] J. Liu and A. C. To. Deposition path planning-integrated structural topology optimization for 3D additive manufacturing subject to self-support constraint. *Computer-Aided Design*, 91:27–45, 2017.
  - [78] W. Zhang and L. Zhou. Topology optimization of self-supporting structures with polygon features for additive manufacturing. *Computer Methods in Applied Mechanics and Engineering*, 334:56–78, 2018.
  - [79] O. Amir and Y. Mass. Topology optimization for staged construction. *Structural and Multidisciplinary Optimization*, 57(4):1679–1694, 2018.
  - [80] A. Garaigordobil, R. Ansola, J. Santamaría, and I. F. de Bustos. A new overhang constraint for topology optimization of self-supporting structures in additive manufacturing. *Structural and Multidisciplinary Optimization*, 58(5):2003–2017, 2018.
  - [81] A. Garaigordobil, R. Ansola, E. Veguería, and I. Fernandez. Overhang constraint for topology optimization of self-supported compliant mechanisms considering additive manufacturing. *Computer-Aided Design*, 109:33–48, 2019.
  - [82] Y. Wang, J. Gao, and Z. Kang. Level set-based topology optimization with overhang constraint: towards support-free additive manufacturing. *Computer Methods in Applied Mechanics and Engineering*, 339:591–614, 2018.
  - [83] Y. Kuo and C. Cheng. Self-supporting structure design for additive manufacturing by using a logistic aggregate function. *Structural and Multidisciplinary Optimization*, pages 1–13, 2019.
  - [84] J. Pellens, G. Lombaert, B. Lazarov, and M. Schevenels. Combined length scale and overhang angle control in minimum compliance topology optimization for additive manufacturing. *Structural and Multidisciplinary Optimization*, 59(6):2005–2022, 2019.
  - [85] L. Hägg and E. Wadbro. Nonlinear filters in topology optimization: existence of solutions and efficient implementation for minimum compliance problems. *Structural and Multidisciplinary Optimization*, 55(3):1017–1028, 2017.
  - [86] K. Zhang, G. Cheng, and L. Xu. Topology optimization considering overhang constraint in additive manufacturing. *Computers and Structures*, 212:86–100, 2019.

- [87] A. M. Mirzendehtdel and K. Suresh. Support structure constrained topology optimization for additive manufacturing. *Computer-Aided Design*, 81:1–13, 2016.
- [88] Y. Kuo, C. Cheng, Y. Lin, and C. San. Support structure design in additive manufacturing based on topology optimization. *Structural and Multidisciplinary Optimization*, 57(1):183–195, 2018.
- [89] F. Mezzadri, V. Bouriakov, and X. Qian. Topology optimization of self-supporting support structures for additive manufacturing. *Additive Manufacturing*, 21:666–682, 2018.
- [90] M. Zhou, Y. Liu, and Z. Lin. Topology optimization of thermal conductive support structures for laser additive manufacturing. *Computer Methods in Applied Mechanics and Engineering*, 353:24–43, 2019.
- [91] K. Bartsch, F. Lange, M. Gralow, and C. Emmelmann. Novel approach to optimized support structures in laser beam melting by combining process simulation with topology optimization. *Journal of Laser Applications*, 31(2):022302, 2019.
- [92] L. Cheng, X. L., J. Bai, Q. Chen, J. Lemon, and A. To. On utilizing topology optimization to design support structure to prevent residual stress induced build failure in laser powder bed metal additive manufacturing. *Additive Manufacturing*, 27:290–304, 2019.
- [93] J. Pellens, G. Lombaert, M. Michiels, T. Craeghs, and M. Schevenels. Topology optimization of support structure layout in metal-based additive manufacturing accounting for thermal deformations. *Structural and Multidisciplinary Optimization*, 61(6):2291–2303, 2020.
- [94] Z.-D. Zhang, O. Ibhado, U. Ali, C. F. Dibia, P. Rahnama, A. Bonakdar, and E. Toyserkani. Topology optimization parallel-computing framework based on the inherent strain method for support structure design in laser powder-bed fusion additive manufacturing. *International Journal of Mechanics and Materials in Design*, 2020.
- [95] M. Langelaar. Combined optimization of part topology, support structure layout and build orientation for additive manufacturing. *Structural and Multidisciplinary Optimization*, 57(5):1985–2004, 2018.
- [96] G. Allaire and B. Bogosel. Optimizing supports for additive manufacturing. *Structural and Multidisciplinary Optimization*, 58(6):2493–2515, 2018.
- [97] J. Zhao, M. Zhang, Y. Zhu, X. Li, L. Wang, and J. Hu. A novel optimization design method of additive manufacturing oriented porous structures and experimental validation. *Materials & Design*, 163:107550, 2019.
- [98] W. G. Wang, D. Munro, C. CL. Wang, F. van Keulen, and J. Wu. Space-time topology optimization for additive manufacturing. *Structural and Multidisciplinary Optimization*, pages 1–18, 2019.

-
- [99] L. Yang, O. Harrysson, D. Cormier, H. West, H. Gong, and B. Stucker. Additive manufacturing of metal cellular structures: design and fabrication. *The Journal of The Minerals, Metals & Materials Society (TMS)*, 67(3):608–615, 2015.
  - [100] E. M. Dede, S. N. Joshi, and F. Zhou. Topology optimization, additive layer manufacturing, and experimental testing of an air-cooled heat sink. *Journal of Mechanical Design*, 137(11), 2015.
  - [101] S. Liu, Q. Li, W. Chen, L. Tong, and G. Cheng. An identification method for enclosed voids restriction in manufacturability design for additive manufacturing structures. *Frontiers of Mechanical Engineering*, 10(2):126–137, 2015.
  - [102] Q. Li, W. Chen, S. Liu, and L. Tong. Structural topology optimization considering connectivity constraint. *Structural and Multidisciplinary Optimization*, 54(4):971–984, 2016.
  - [103] R. Ranjan, R. Samant, and S. Anand. Integration of design for manufacturing methods with topology optimization in additive manufacturing. *Journal of Manufacturing Science and Engineering*, 139(6), 2017.
  - [104] R. Ranjan, Y. Yang, C. Ayas, M. Langelaar, and F. van Keulen. Controlling local overheating in topology optimization for additive manufacturing. In *Proceedings of euspen special interest group meeting: additive manufacturing, Leuven, Belgium*, 2017.
  - [105] P. Vogiatzis, S. Chen, and C. Zhou. An open source framework for integrated additive manufacturing and level-set-based topology optimization. *Journal of Computing and Information Science in Engineering*, 17(4), 2017.
  - [106] S. Liu, Q. Li, J. Liu, W. Chen, and Y. Zhang. A realization method for transforming a topology optimization design into additive manufacturing structures. *Engineering*, 4(2):277–285, 2018.
  - [107] Z. Xiao, Y. Yang, R. Xiao, Y. Bai, C. Song, and D. Wang. Evaluation of topology-optimized lattice structures manufactured via selective laser melting. *Materials & Design*, 143:27–37, 2018.
  - [108] A. M. Mirzendehtdel, B. Rankouhi, and K. Suresh. Strength-based topology optimization for anisotropic parts. *Additive Manufacturing*, 19:104–113, 2018.
  - [109] S. W. Tsai and E. M. Wu. A general theory of strength for anisotropic materials. *Journal of Composite Materials*, 5(1):58–80, 1971.
  - [110] J. Liu, A. T. Gaynor, S. Chen, Z. Kang, K. Suresh, A. Takezawa, L. Li, J. Kato, J. Tang, C. Wang, et al. Current and future trends in topology optimization for additive manufacturing. *Structural and Multidisciplinary Optimization*, 57(6):2457–2483, 2018.
  - [111] S. Afazov, W. Denmark, B. L. Toralles, A. Holloway, and A. Yaghi. Distortion prediction and compensation in selective laser melting. *Additive Manufacturing*, 17:15–22, 2017.

- [112] M. E. Gurtin. *An introduction to Continuum Mechanics*. Academic press, 1982.
- [113] J. Huétink. *On the Simulation of Thermo-Mechanical Forming Processes*. PhD thesis, University of Twente, 1986.
- [114] O. C. Zienkiewicz, R. L. Taylor, and J. Z. Zhu. *The Finite Element Method: Its Basis and Fundamentals, Sixth Edition*. Butterworth-Heinemann, 2005.
- [115] J. C. Simo and T. J. R. Hughes. *Computational Inelasticity*, volume 7. Springer Science & Business Media, 2006.
- [116] M. Kojić and K.-J. Bathe. The 'effective-stress-function' algorithm for thermo-elasto-plasticity and creep. *International Journal for Numerical Methods in Engineering*, 24(8):1509–1532, 1987.
- [117] N. Prakash, C. Matthews, and D. Versino. Stress and tangent update equations for combined time-hardening creep and J2 plasticity in an implicit hypo-elastic formulation. Technical Report LA-UR-18-23619, Los Alamos National Lab.(LANL), Los Alamos, NM (United States), 2018.
- [118] N. Prakash, C. Matthews, D. Versino, and C. Unal. A general constitutive framework for the combined creep, plasticity and swelling behavior of nuclear fuels in an implicit hypoelastic formulation. Technical Report LA-UR-19-20166, Los Alamos National Lab.(LANL), Los Alamos, NM (United States), 2019.
- [119] J. C. Simo and R. L. Taylor. Consistent tangent operators for rate-independent elastoplasticity. *Computer Methods in Applied Mechanics and Engineering*, 48(1):101–118, 1985.
- [120] S. K. Reif, K. J. Amberge, and D. A. Woodford. Creep design analysis for a thermoplastic from stress relaxation measurements. *Materials & Design*, 16(1): 15–21, 1995.
- [121] K.-I. Ohguchi and K. Sasaki. Elastic-plastic-creep simulation of Pb/Sn solder alloys by separation of plastic and creep. *JSME International Journal Series A Solid Mechanics and Material Engineering*, 46(4):559–566, 2003.
- [122] P. Liu, Y. Zong, D. Shan, and B. Guo. Relationship between constant-load creep, decreasing-load creep and stress relaxation of titanium alloy. *Materials Science and Engineering: A*, 638:106–113, 2015.
- [123] J. Betten. *Creep Mechanics*. Springer Science & Business Media, 2008.
- [124] P. Nie, O. A. Ojo, and Z. Li. Numerical modeling of microstructure evolution during laser additive manufacturing of a nickel-based superalloy. *Acta Materialia*, 77:85–95, 2014.
- [125] Y. S. Lee and W. Zhang. Modeling of heat transfer, fluid flow and solidification microstructure of nickel-base superalloy fabricated by laser powder bed fusion. *Additive Manufacturing*, 12:178–188, 2016.

-
- [126] S. A. Khairallah, A. T. Anderson, A. Rubenchik, and W. E. King. Laser powder-bed fusion additive manufacturing: Physics of complex melt flow and formation mechanisms of pores, spatter, and denudation zones. *Acta Materialia*, 108(108): 36–45, 2016.
  - [127] Al. J. Wolfer, J. Aires, K. Wheeler, J.-P. Delplanque, A. Rubenchik, A. Anderson, and S. Khairallah. Fast solution strategy for transient heat conduction for arbitrary scan paths in additive manufacturing. *Additive Manufacturing*, 30: 100898, 2019.
  - [128] D. Munro, C. Ayas, M. Langelaar, and A. van Keulen. On process-step parallel computability and linear superposition of mechanical responses in additive manufacturing process simulation. *Additive Manufacturing*, 28:738–749, 2019.
  - [129] P. Prabhakar, W. J. Sames, R. Dehoff, and S. S. Babu. Computational modeling of residual stress formation during the electron beam melting process for Inconel 718. *Additive Manufacturing*, 7:83–91, 2015.
  - [130] A. Krishnamoorthy and D. Menon. Matrix inversion using Cholesky decomposition. *Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA)*, pages 70–72, 2013.
  - [131] Inkscape Project. Inkscape. URL <https://inkscape.org>.
  - [132] Slic3r open source 3D printing toolbox. <https://slic3r.org>. Accessed: 2020-06-30.
  - [133] Flowalistik A. Low-poly pikachu. <https://www.thingiverse.com/thing:376601>. Accessed: 2020-06-20.
  - [134] Y. Ueda and M. G. Yuan. Prediction of residual stresses in butt welded plates using inherent strains. *Journal of Engineering Materials and Technology*, 115 (4):417–423, 1993.
  - [135] A. Panesar, M. Abdi, D. Hickman, and I. Ashcroft. Strategies for functionally graded lattice structures derived using topology optimisation for additive manufacturing. *Additive Manufacturing*, 19:81–94, 2018.
  - [136] B. Vaissier, J.-P. Pernot, L. Chougrani, and P. Véron. Genetic-algorithm based framework for lattice support structure optimization in additive manufacturing. *Computer-Aided Design*, 110:11–23, 2019.
  - [137] L. Badea, M. Surand, J. Ruau, and B. Viguier. Creep behavior of Ti-6Al-4V from 450° C to 600° C. *University Polytechnica of Bucharest Scientific Bulletin*, 76, 2014.
  - [138] H. Ali, L. Ma, H. Ghadbeigi, and K. Mumtaz. In-situ residual stress reduction, martensitic decomposition and mechanical properties enhancement through high temperature powder bed pre-heating of selective laser melted Ti6Al4V. *Materials Science and Engineering: A*, 695:211–220, 2017.

- [139] M. P. Bendsøe. Optimal shape design as a material distribution problem. *Structural Optimization*, 1(4):193–202, 1989.
- [140] M. Zhou, Y.K. Shyy, and H.L. Thomas. Checkerboard and minimum member size control in topology optimization. *Structural and Multidisciplinary Optimization*, 21(2):152–158, 2001.
- [141] O. Sigmund. Morphology-based black and white filters for topology optimization. *Structural and Multidisciplinary Optimization*, 33(4-5):401–424, 2007.
- [142] G. Kreisselmeier and R. Steinhauser. Systematic control design by optimizing a vector performance index. In *IFAC Symposium on Computer-Aided Design of Control Systems*, Zurich, 1979.
- [143] R. T. Haftka and Z. Gürdal. *Elements of structural optimization*, volume 11. Springer Science & Business Media, 2012.
- [144] K. Svanberg. The method of moving asymptotes - a new method for structural optimization. *International Journal for Numerical Methods in Engineering*, 24(2):359–373, 1987.
- [145] K. Svanberg. The method of moving asymptotes (MMA) with some extensions. In *Optimization of large structural systems*, pages 555–566. Springer, 1993.
- [146] P. Michaleris, D. Tortorelli, and C.A. Vidal. Tangent operators and design sensitivity formulations for transient non-linear coupled problems with applications to elastoplasticity. *International Journal for Numerical Methods in Engineering*, 37:2471–2499, 1994.
- [147] F. van Keulen, R.T. Haftka, and N.H. Kim. Review of options for structural design sensitivity analysis. part 1: Linear systems. *Computer Methods in Applied Mechanics and Engineering*, 194(30-33):3213–3243, 2005.
- [148] S. H. Chung, L. Fourment, J. L. Chenot, and S. M. Hwang. Adjoint state method for shape sensitivity analysis in non-steady forming applications. *International Journal for Numerical Methods in Engineering*, 57:1431–1444, 2003.