Proceedings of the 40th Chinese Control Conference
July 26-28, 2021, Shanghai, China

Scale-free Design for Delayed Regulated Synchronization of Discrete-time Heterogeneous Multi-agent Systems subject to Unknown Non-uniform and Arbitrarily Large Communication Delays

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Abstract: In this paper, we study delayed regulated output synchronization for discrete-time heterogeneous multi-agent systems (MAS) with introspective agents subject to unknown, non-uniform and arbitrarily large communication delays. A delay transformation is utilized to transform the original MAS to a new system without delayed states. The proposed *scale-free* dynamic protocols are developed solely based on agent models and localized information exchange with neighbors such that we do not need any information about the communication networks and the number of agents.

Key Words: Heterogeneous multi-agent systems, Delayed regulated output Synchronization, Scale-free protocols

1 Introduction

Cooperative control of multi-agent systems (MAS) such as synchronization, consensus, swarming, flocking, has become a hot topic among researchers because of its broad application in various areas such as biological systems, sensor networks, automotive vehicle control, robotic cooperation teams, and so on. See, for example, the books [26, 31, 13, 2] and references therein.

State synchronization inherently requires homogeneous networks. On the other hand, for heterogeneous network it is more reasonable to consider output synchronization since the dimensions of states and their physical interpretation may be different. For heterogeneous MAS with non-introspective agents , it is well-known that one needs to regulate outputs of the agents to *a priori* given trajectory generated by a so-called exosystem (see [30, 9]). Other works on synchronization of MAS with non-introspective agents can be found in the literature as [11, 10]. Most of the literature for heterogeneous MAS with introspective agents are based on modifying the agent dynamics via local feedback to achieve some form of homogeneity. There have been many results for synchronization of heterogeneous networks with introspective agents, see for instance [12, 34, 14, 19, 25, 4].

In real applications, networks may be subject to delays. Time delays may affect system performance and can even lead to instability. As discussed in [3], two kinds of delays have been considered in the literature: input delays and communication delays. Input delays encapsulate the processing time to execute an input for each agent, whereas communication delays can be considered as the time it takes to transmit information from an agent to its destination. Many works have been focused on dealing with input delays, specifically with the objective of deriving an upper bound on the input delays such that agents can still achieve synchronization. See, for example [1, 15, 27, 23, 33]. Some research has been done for networks subject to communication delays. Fundamentally, there are two approaches in the literature for dealing with MAS subject to communication delays.

1) Standard state/output synchronization where the objective is to regulate the output to a constant trajectory.
2) Delayed state/output synchronization.

Both of these approaches preserve diffusiveness of the couplings (i.e. ensuring the invariance of the consensus manifold). Also, the notion of delayed output synchronization coincides with standard regulated output synchronization if the regulated output is required to be a constant trajectory. As such delayed synchronization can be viewed as the generalization of standard synchronization in the context of MAS subject to communication delay.

The majority of research on MAS subject to communication delay has been focused on achieving standard output synchronization by regulating the output to constant trajectory (see [3, 27, 32, 35] and references therein). It is worth noting that in all of the aforementioned papers, design of protocols requires at least some knowledge about the graph or the size of the network. We should also point out that [20, 21] give consensus conditions for networks with higher-order but require SISO dynamics. The paper [16] considers second-order dynamics, but the communication delays are assumed to be known. More recently, the notion of delayed synchronization was introduced in [7] for MAS with passive agents and strongly connected and balanced graphs where it

This work is supported by the Nature Science Foundation of Liaoning Province, PR China under Grant 2019-MS-116, the Fundamental Research Funds for the Central Universities of China under Grant N2004014 and the United States National Science Foundation under Grant 1635184.

Agents are said to be introspective when they have access to either exact or estimates of their states, otherwise they are called non-introspective [11].

is assumed that there exists a unique path between any two distinct nodes. Then, the authors extended their results in [5, 6] when they allowed multiple paths between two agents in strongly connected communication graphs. Although the synchronized trajectory in these papers is constant and standard definition of synchronization can be utilized, the authors motivation for utilizing delayed synchronization is exploring the possible existence of delayed-induced periodicity in synchronized trajectory of coupled systems. These solutions, provided they exist, can be valuable in several applications, as clarified in, for example, [24, 28]. It is worth to note that the protocol design in these papers do not need knowledge of the graph, since they are restricted to passive agents. An interesting line of research utilizing delayed synchronization formulation was introduced recently in [17, 18]. These papers considered a **dynamic** synchronized trajectory (i.e. non constant synchronized trajectory) and designed protocols to achieve regulated delayed state/output synchronization in the presence of communication delays under the condition that the communication graph was a spanning tree. However, the protocol design required knowledge of the graph and size of the network.

In this paper, we extend our earlier results of delayed synchronization by developing a **scale-free** framework utilizing localized information exchange for discrete-time heterogeneous MAS subject to unknown non-uniform and arbitrarily large communication delays to achieve delayed regulated synchronization when the synchronized trajectory is a *dynamic* signal generated by a so-called exosystem. The associated graphs to the communication networks are assumed to be a directed spanning tree (i.e., they have one root node and the other non-root nodes have in-degree one). We achieve scale-free delayed regulated output synchronization for heterogeneous discrete-time MAS with introspective agents. Our proposed design methodologies are scale-free, namely

- The design is independent of information about the communication network such as the spectrum of the associated Laplacian matrix or the size of the network.
- The collaborative protocols will work for any network with associated directed spanning tree, and can tolerate any unknown, non-uniform and arbitrarily large communication delays.

Notations and definitions

Given a matrix $A \in \mathbb{R}^{n\times m}$, A^{T} denotes its conjugate transpose and $\|A\|$ is the induced 2-norm. Let $\mathbf{j}$ indicate $\sqrt{-1}$. A square matrix A is said to be Schur stable if all its eigenvalues are in the open unit disc. It should be noted that the set of at most weakly unstable agents contains stable agents, neutrally stable agents as well as weakly unstable agents. We denote by $\mathrm{diag}\{A_1,\dots,A_N\}$, a block-diagonal matrix with $A_1,\dots,A_N$ as the diagonal elements. $A\otimes B$ depicts the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n\times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context.

To describe the information flow among the agents we associate a *weighted graph* $\mathcal{G}$ to the communication network. The weighted graph $\mathcal{G}$ is defined by a triple $(\mathcal{V},\mathcal{E},\mathcal{A})$ where $\mathcal{V}=\{1,\dots,N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix with non negative elements a_{ij}. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij}>0$ denotes an edge $(j,i)\in\mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij}=0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii}=0$. The *weighted in-degree* of a node i is given by $d_{in}(i)=\sum_{j=1}^N a_{ij}$. Similarly, the *weighted out-degree* of a node i, is given by $d_{out}(i)=\sum_{j=1}^N a_{ji}$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1,\dots,i_k\}$ such that $(i_j,i_{j+1})\in\mathcal{E}$ for $j=1,\dots,k-1$. A *directed tree* is a subgraph (subset of nodes and edges) in which every node has exactly one parent node except for one node, called the *root*, which has no parent node. The *root set* is the set of root nodes. A *directed spanning tree* is a subgraph which is a directed tree containing all the nodes of the original graph. If a directed spanning tree exists, the root has a directed path to every other node in the tree. For a weighted graph $\mathcal{G}$, the matrix $L=[\ell_{ij}]$ with

$$\ell_{ij}=\begin{cases}\sum_{k=1}^N a_{ik}, & i=j,\\ -a_{ij}, & i\neq j,\end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$ [8]. Moreover, if the graph contains a directed spanning tree, the Laplacian matrix L has a single eigenvalue at the origin and all other eigenvalues are located in the open right-half complex plane [26]. A matrix $D=\{d_{ij}\}_{N\times N}$ is called a row stochastic matrix if

1) $d_{ij}\geq 0$ for any i,j,
2) $\sum_{j=1}^N d_{ij}=1$ for $i=1,\dots,N$.

A row stochastic matrix D has at least one eigenvalue at 1 with right eigenvector $\mathbf{1}$. D can be associated with a graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$. The number of nodes N is the dimension of D and an edge $(j,i)\in\mathcal{E}$ if $d_{ij}>0$.

2 Heterogeneous MAS with introspective agents

Consider a heterogeneous MAS consisting of N non-identical linear agents:

$$\begin{aligned} x_i(k+1) &= A_ix_i(k)+B_iu_i(k),\\ y_i(k) &= C_ix_i(k), \end{aligned} \tag{1}$$

where $x_i\in\mathbb{R}^{n_i}$, $u_i\in\mathbb{R}^{m_i}$ and $y_i\in\mathbb{R}^p$ are the state, input, output of agent i for $i=1,\dots,N$. The agents are introspective, meaning that each agent has access to its own local information. Specifically, each agent has access to part of its state

$$z_i(k)=C_i^m x_i(k). \tag{2}$$

where $z_i\in\mathbb{R}^{q_i}$.

The network provides agent i with following information

$$\zeta_i(k)=\frac{1}{1+d_{in}(i)}\sum_{j=1}^N a_{ij}(y_i(k)-y_j(k-\kappa_{ij})), \tag{3}$$

where $\kappa_{ij}\in\mathbb{Z}_{\geqslant 0}$ represents an unknown communication delay from agent j to agent i where we assume that $\kappa_{ii}=0$. In the above $a_{ii}=0$ and $a_{ij}\geqslant 0$. This communication topology of the network, presented in (3), can be associated to a weighted graph $\mathcal{G}$ with each node indicating an agent in the network and the weight of an edge is given by the coefficient a_{ij}. The communication delay implies that it took κ_{ij} seconds for agent j to transfer its state information to agent i. Next, we rewrite ζ_i as

$$\zeta_i(k) = \sum_{j=1}^{N} d_{ij}(y_i(k) - y_j(k - \kappa_{ij})), \tag{4}$$

where $d_{ij} \geqslant 0$, and we choose $d_{ii} = 1 - \sum_{j=1, j\neq i}^{N} d_{ij}$ such that $\sum_{j=1}^{N} d_{ij} = 1$ with $i, j \in \{1, \ldots, N\}$. Note that d_{ii} satisfies $d_{ii} > 0$. The weight matrix $D = [d_{ij}]$ is then a so-called, row stochastic matrix. Let $D_{in} = \text{diag}\{d_{in}(i)\}$, for $i = 1, \ldots, N$, then the relationship between the row stochastic matrix D and the Laplacian matrix L is

$$(I + D_{in})^{-1} L = I - D. \tag{5}$$

We make the following definition.

Definition 1 *Let $\mathbb{G}^N$ denote the set of directed spanning tree graphs with N nodes for which the corresponding Laplacian matrix L is lower triangular. The corresponding Laplacian matrix L has the property that the entries of the first row are equal to zero and $\ell_{ii} > 0$ for $i = 2, \ldots, N$. We consider agent 1 as the root agent.*

Remark 1 *Note that any graph which is a directed spanning tree, has a possible lower triangular Laplacian matrix after reordering of the agents.*

For the graph defined by Definition 1, we have that row stochastic matrix D is lower triangular matrix with $d_{11} = 1$ and $d_{1j} = 0$ for $j = 2, \ldots, N$. Therefore, we have

$$D = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ d_{21} & d_{22} & 0 & \cdots & 0 \\ d_{31} & d_{32} & d_{33} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ d_{N1} & \cdots & d_{N,N-2} & d_{N,N-1} & d_{N,N} \end{pmatrix}.$$

Since the graph is equal to a directed spanning tree, in every row (except the first one) there are exactly two elements unequal to 0.

Our goal is to achieve delayed regulated output synchronization among all agents while the synchronized output dynamics of agents are equal to a time-shifted priori given trajectory generated by a so-called exosystem

$$\begin{aligned} x_r(k+1) &= A x_r(k), \quad x_r(0) = x_{r0} \\ y_r(k) &= C x_r(k) \end{aligned} \tag{6}$$

where $x_r \in \mathbb{R}^n$ and $y_r \in \mathbb{R}^p$. Clearly, we need some level of communication between the desired trajectory and the agents. According to the structure of communication network, we assume that only agent 1 has access to $y_r(k)$ with delay κ_{1r}. Since the graph is a spanning tree, there is a unique path between agent i and the exosystem which is connected to agent 1. We define κ_{ir} as the sum of delays from agent i to the exosystem through this path. Then, we can define:

$$\psi_i = \iota_i(y_i(k) - y_r(k - \kappa_{ir})), \quad \iota_i = \begin{cases} 1, & i = 1, \\ 0, & i = 2, \cdots, N. \end{cases} \tag{7}$$

Therefore, the information available for agent $i \in \{1, \ldots, N\}$ is given by:

$$\bar{\zeta}_i(k) = \frac{1}{2 + d_{in}(i)} \sum_{j=1}^{N} a_{ij}(y_i(k) - y_j(k - \kappa_{ij})) + \iota_i(y_i(k) - y_r(k - \kappa_{ir})). \tag{8}$$

For any graph $\mathbb{G}^N$, with the Laplacian matrix L, we define the expanded Laplacian matrix and stochastic matrix $\bar{D}$ as

$$\begin{aligned} \bar{L} &= L + diag\{\iota_i\} = [\bar{\ell}_{ij}]_{N\times N} \\ \bar{D} &= I - (2I + D_{in})^{-1} \bar{L} = [\bar{d}_{ij}]_{N\times N}. \end{aligned}$$

then, (8) can be rewritten as:

$$\bar{\zeta}_i(k) = y_i(k) - y_r(k - \kappa_{ir}) - \sum_{j=1}^{N} \bar{d}_{ij}(y_j(k - \kappa_{ij}) - y_r(k - \kappa_{ir})) \tag{9}$$

Each agent $i \in \{1, \ldots, N\}$ also has access to the localized information exchange among neighbors $\hat{\zeta}_i$, i.e.,

$$\hat{\zeta}_i(k) = \frac{1}{2 + d_{in}(i)} \sum_{j=1}^{N} \bar{\ell}_{ij} \xi_j(k - \hat{\kappa}_{ij}) = \xi_i(k) - \sum_{j=1}^{N} \bar{d}_{ij} \xi_j(k - \hat{\kappa}_{ij}) \tag{10}$$

where $\xi_j \in \mathbb{R}^n$ is a variable produced internally by agent j and to be defined in next sections while $\hat{\kappa}_{ij} \in \mathbb{Z}_{\geqslant 0}$ $(i \neq j)$ represents an unknown communication delay from agent j to agent i.

We introduce the following definitions.

Definition 2 *The agents of a heterogeneous MAS are said to achieve*

- *delayed output synchronization for all $i \in \{1, \ldots, N\}$ if*

$$\lim_{k\to\infty} \left[(y_i(k) - y_j(k - \kappa_{ij}) \right] = 0, \quad \textit{for all } j \textit{ such that } (j, i) \in \mathcal{E}, \tag{11}$$

- *and delayed regulated output synchronization if*

$$\lim_{k\to\infty} [(y_i(k) - y_r(k - \kappa_{ir})] = 0, \quad \textit{for all } i \in \{1, \ldots, N\}. \tag{12}$$

We formulate the scalable delayed regulated output synchronization problem for heterogeneous networks with unknown, nonuniform communication delays as follows.

Problem 1 *Consider a MAS* (1) *and* (9) *and the exosystem* (6)*. Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1. Then, the* ***scalable delayed regulated output synchronization problem based on localized information exchange utilizing collaborative protocols*** *for heterogeneous networks with unknown nonuniform and arbitrarily large communication delay is to find, if possible, a linear dynamic protocol for each agent $i \in \{1, \ldots, N\}$, using only knowledge of agent models, i.e. (C_i, A_i, B_i) of the form:*

$$\begin{cases} x_{i,c}(k+1) = A_{i,c} x_{i,c}(k) + B_{i,c} \bar{\zeta}_i(k) + C_{i,c} \hat{\zeta}_i(k) + D_{i,c} z_i(k) \\ u_i(k) = E_{i,c} x_{i,c}(k) + F_{i,c} \bar{\zeta}_i(k) + G_{i,c} \hat{\zeta}_i(k) + H_{i,c} z_i(k), \end{cases}$$

where $\hat{\zeta}_i$ is defined by (10) *with $\xi_i = H_c x_{i,c}$ and $x_{i,c} \in \mathbb{R}^{n_c}$ such that for any N, any graph $\mathscr{G} \in \mathbb{G}^N$, any communication delays $\kappa_{ij} \in \mathbb{Z}_{\geqslant 0}$ and $\hat{\kappa}_{ij} \in \mathbb{Z}_{\geqslant 0}$ we achieve delayed regulated output synchronization as stated by* (12) *in Definition 2.*

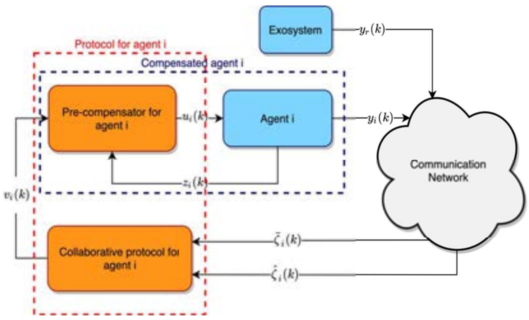

Figure 1: Architecture of scale-free protocols

We make the following assumptions.

Assumption 1 *For agents $i \in \{1, \ldots, N\}$,*

1) *(C_i, A_i, B_i) is stabilizable and detectable.*
2) *(C_i, A_i, B_i) is right-invertible.*
3) *(C_i^m, A_i) is detectable.*

Assumption 2 *For exosystem,*

1) *(C_r, A_r) is observable.*
2) *all the eigenvalues of A_r are in the closed unit disc.*

We design scale-free protocols to solve scalable delayed regulated output synchronization problem as stated in Problem 1. After introducing the architecture of our protocol, we design the protocols through four steps.

2.1 Architecture of the protocol

Our protocol has the structure shown below in Figure 1. As seen in the figure, our design methodology consists of two major modules.

1) The first module is remodeling the exosystem to obtain the target model by designing pre-compensators following our previous results in [29].
2) The second module is designing collaborate protocols for almost homogenized agents to achieve delayed regulated output synchronization.

2.2 Protocol design

To design our protocols, first we recall the following Lemma.

Lemma 1 ([29]) *There exists another exosystem given by:*

$$\begin{aligned} \check{x}_r(k+1) &= \check{A}_r \check{x}_r(k), \quad \check{x}_r(0) = \check{x}_{r0} \\ y_r(k) &= \check{C}_r \check{x}_r(k), \end{aligned} \tag{13}$$

such that for all $x_{r0} \in \mathbb{R}^r$, there exists $\check{x}_{r0} \in \mathbb{R}^{\check{r}}$ for which (13) *generate exactly the same output y_r as the original exosystem* (6). *Furthermore, we can find a matrix $\check{B}_r$ such that the triple $(\check{C}_r, \check{A}_r, \check{B}_r)$ is invertible, of uniform rank n_q, and has no invariant zero, where n_q is an integer greater than or equal to maximal order of infinite zeros of $(C_i, A_i, B_i), i \in \{1, \ldots, N\}$ and all the observability indices of (C_r, A_r). Note that the eigenvalues of $\check{A}_r$ consists of all eigenvalues of A_r and additional zero eigenvalues.*

Protocol 1: Heterogeneous MAS

Step 1: remodeling the exosystem First, we remodel the exosystem to arrive at suitable choice for the target model $(\check{C}_r, \check{A}_r, \check{B}_r)$ following the design procedure in [29] summarized in Lemma 1.

Step 2: designing pre-compensators In this step, given the target model $(\check{C}_r, \check{A}_r, \check{B}_r)$, by utilizing the design methodology from [29, Appendix A], we design a pre-compensators for each agent $i \in \{1, \ldots, N\}$ of the form

$$\begin{cases} \xi_i(k+1) = A_{i,h}\xi_i(k) + B_{i,h}z_i(k) + E_{i,h}v_i(k), \\ u_i(k) \quad = C_{i,h}\xi_i(k) + D_{i,h}v_i(k), \end{cases} \tag{14}$$

which yields the compensated agents as

$$\begin{aligned} x_i^h(k+1) &= \check{A}_r x_i^h(k) + \check{B}_r(v_i(k) + \rho_i(k)), \\ y_i(k) &= \check{C}_r x_i^h(k), \end{aligned} \tag{15}$$

where $\rho_i(k)$ is given by

$$\begin{aligned} \omega_i(k+1) &= A_{i,s}\omega_i(k), \\ \rho_i(k) &= C_{i,s}\omega_i(k), \end{aligned} \tag{16}$$

and $A_{i,s}$ is Schur stable. Note that the compensated agents are homogenized and have the target model $(\check{C}_r, \check{A}_r, \check{B}_r)$.

Step 3: designing collaborative protocols for the compensated agents Collaborative protocols based on localized information exchanges are designed for the compensated agents $i = 1, \ldots, N$ as

$$\begin{cases} \hat{x}_i(k+1) = \check{A}_r \hat{x}_i(k) - \check{B}_r K \hat{\zeta}_i(k) + H(\bar{\zeta}_i(k) - \check{C}_r \hat{x}_i(k)), \\ \chi_i(k+1) = \check{A}_r \chi_i(k) + \check{B}_r v_i(k) + \check{A}_r \hat{x}_i(k) - \check{A}_r \hat{\zeta}_i(k), \\ v_i(k) \quad = -K\chi_i(k), \end{cases} \tag{17}$$

where H and K are matrices such that $\check{A}_r - H\check{C}_r$ and $\check{A}_r - \check{B}_r K$ are Schur stable. The exchanging information $\hat{\zeta}_i$ is defined as (10) and $\bar{\zeta}_i$ is defined as (9).

Step 4: obtaining the protocols The final protocol which is the combination of module 1 and 2 is

$$\begin{cases} \xi_i(k+1) = A_{i,h}\xi_i(k) + B_{i,h}z_i(k) - E_{i,h}K\chi_i(k), \\ \hat{x}_i(k+1) = \check{A}_r \hat{x}_i(k) - \check{B}_r K \hat{\zeta}_i(k) + H(\bar{\zeta}_i - \check{C}_r \hat{x}_i(k)), \\ \chi_i(k+1) = (\check{A}_r - \check{B}_r K)\chi_i(k) + \check{A}_r \hat{x}_i(k) - \check{A}_r \hat{\zeta}_i(k), \\ u_i(k) \quad = C_{i,h}\xi_i(k) - D_{i,h}K\chi_i(k), \end{cases} \tag{18}$$

We design our protocols through the four steps as following.

Then, we have the following theorem for scalable regulated output synchronization of heterogeneous MAS.

Theorem 1 *Consider a heterogeneous network of N agents* (1) *satisfying Assumption 1 with local information* (2) *and the associated exosystem* (6) *satisfying Assumption 2. Then, the scalable delayed regulated output synchronization problem as defined in Problem 1 is solvable. In particular, the dynamic protocol* (18) *solves the scalable delayed regulated output synchronization problem based on localized information exchange for any N, any graph $\mathcal{G} \in \mathbb{G}^N$, any communication delays $\kappa_{ij} \in \mathbb{Z}_{\geqslant 0}$ and $\hat{\kappa}_{ij} \in \mathbb{Z}_{\geqslant 0}$.*

Proof: Due to space limitation the proof is omitted and provided in [22]. ■

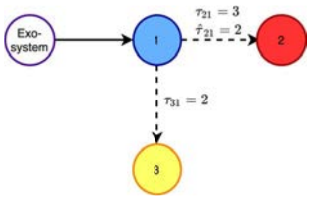

Figure 2: Communication graph of the network with 3 nodes

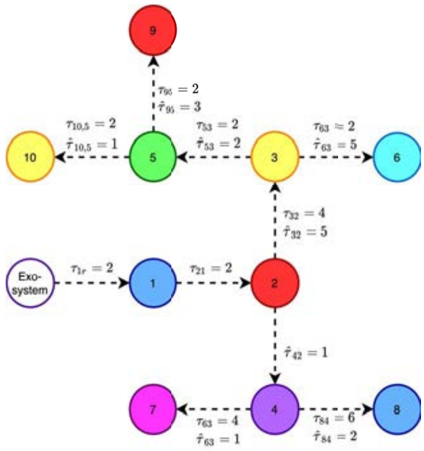

Figure 3: Communication graph of the network with 10 nodes

3 Numerical Example

We show that our protocol design is scale-free and it works for any graph $\mathcal{G} \in \mathbb{G}^N$ with any number of agents. Consider the agents models (1) with

$$A_i = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, C_i = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}, C_i^m = I$$

for $i = 1, 6$, and

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, C_i^m = I,$$

for $i = 2, 7$, and

$$A_i = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}, B_i = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, C_i = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix},$$
$$C_i^m = I,$$

for $i = 3, 4, 8, 9$, and

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C_i = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, C_i^m = I,$$

for $i = 5, 10$. Note that $\bar{n}_d = 3$, which is the degree of infinite zeros of (C_2, A_2, B_2). In this example, our goal is delayed output regulation to a non-constant signal generated by

$$\dot{x}_r = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix} x_r, \quad y_r = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x_r.$$

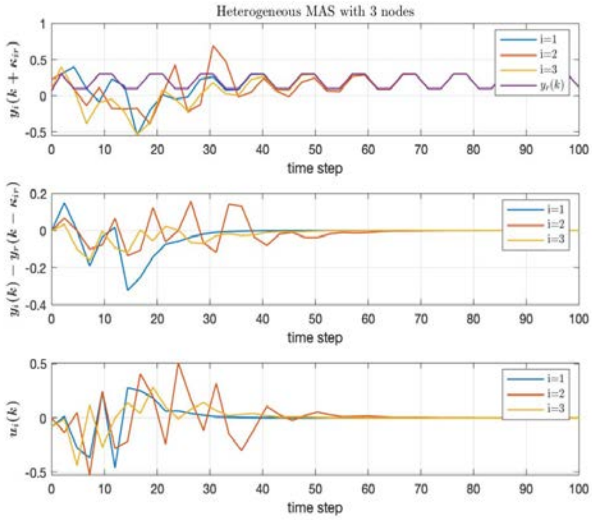

Figure 4: Scale-free delayed regulated output synchronization for heterogeneous MAS with 3 nodes

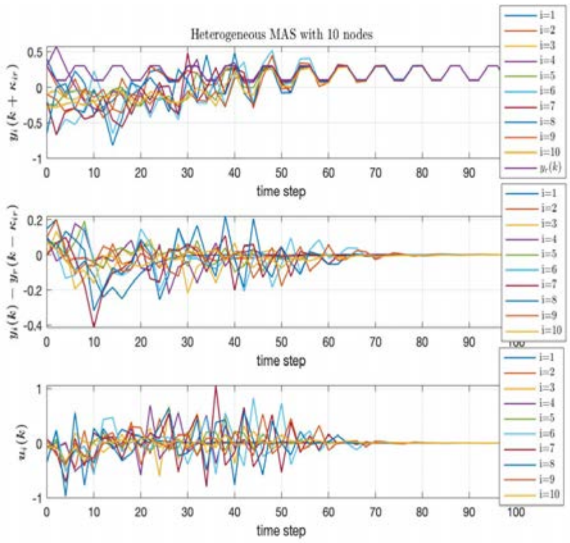

Figure 5: Scale-free delayed regulated output synchronization for heterogeneous MAS with 10 nodes

Utilizing Lemma 1, we choose $(\check{C}_r, \check{A}_r, \check{B}_r)$ as

$$\check{A}_r = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}, \quad \check{B}_r = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \check{C}_r = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

and $K = \begin{pmatrix} 1.006 & -0.99 & 0.6 \end{pmatrix}$ and $H = \begin{pmatrix} 0.9 & -0.35 & -0.225 \end{pmatrix}^{\mathrm{T}}$.

To show the scalability of our protocol, we consider two heterogeneous MAS with different number of agents and different communication topologies (more examples are provided in [22]).

Case I: Consider a MAS with 3 agents with agent models (C_i, A_i, B_i) for $i \in \{1, \ldots, 3\}$, and directed communication topology shown in Figure 2 where the values of communication delays are shown in the figure.

Case II: Next, we consider a MAS with 10 agents (C_i, A_i, B_i) for $i \in \{1, \ldots, 10\}$ and directed communication topology, shown in Figure 3 where the values of communication delays are shown in the figure.

The simulation results are illustrated in Figure 4–5. The simulation results show that our one-shot-designed protocol works for any MAS with any size of the network and any communication delays.

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