Impact of Deficient Array Antenna Elements on Downlink Massive MIMO Performance in RIMP and Random-LOS Channels

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Abstract—The Over-the-Air (OTA) characterization of large antenna systems need to be time- and cost-efficient. Among others, two potential candidates are OTA measurement environments emulating the Rich Isotropic MultiPath (RIMP) and the Random Line-Of-Sight (Random-LOS) channels. In the present paper we study the capacity loss of downlink massive multi-user multiple-input multiple-output (Massive MIMO) narrowband systems in these channels due to deficient antenna elements. We evaluate the downlink ergodic sum rate for the Zero-Forcing (ZF) and the Matched-Filtering (MF) precoders in these channels. We consider a single-cell system comprising a base station with 50 equidistant isotropic source antenna elements placed 1/2 apart and 5 users each equipped with a single isotropic antenna. No coupling effects between the array elements have been assumed. In the present study we show that both the RIMP and the Random-LOS can be used to identify deficient performance of Massive MIMO arrays.

I. INTRODUCTION

5G wireless systems will deliver a plethora of quality-of-service enhancements, e.g., Gbps user throughput, sub-millisecond latency, as well as higher reliability and larger coverage and capacity. The Massive Multi-User Multiple-Input Multiple-Output (Massive MIMO) technology is being developed to provide for the above. In Massive MIMO, a base station (BS) is equipped with an array antenna with a very large number of antenna elements, and it serves many users in the same time-frequency resource. An advantage of Massive MIMO is that for so-called favorable propagation (FP) conditions, the channel vectors between the users and the BS are nearly pairwise orthogonal. Hence, the signal processing complexity can be considerably reduced since linear processing becomes nearly optimal [1], [2]. The BS can use linear receivers in the uplink and linear precoders in the downlink.

The Rich Isotropic Multipath (RIMP) propagation channel and the Random Line-Of-Sight (Random-LOS) propagation channel represent two limiting propagation channels. With the aim of providing time- and cost-efficient OTA characterization setup, the Kildal-conjecture for OTA device characterization was put forward in [3]: if a wireless device is proven to work well in RIMP and Random-LOS, it will work well in all real-life channels in a statistical sense. Similar ideas have been discussed later in the context of 5G and Massive MIMO systems in [4], [5], where it has been shown that both these channels provide FP conditions. Hence, it is sensible to anticipate that real propagation channels, which are likely to be in between the extremes, would also be favorable. Experimental validation of FP characteristics of Massive MIMO channels observed by real-life channel measurements have been reported, e.g., in [6].

To detect poor performance of large arrays is one of the key objectives of OTA characterization of 5G Massive MIMO. Therefore, here we present simulation results of the impact of deficient antennas in terms of the capacity loss of downlink Massive MIMO narrowband systems in the RIMP and the Random-LOS channels. More specifically, we assume that some of the antennas have a total embedded radiation efficiency that is less than 100%. The achievable ergodic sum rate capacity of a downlink Massive MIMO system is evaluated for the Zero-Forcing (ZF) and the Matched-Filtering (MF) linear precoders.

II. SYSTEM MODEL

Let’s consider the downlink of a single-cell narrowband Massive MIMO system. A BS equipped with \( M \) antenna elements with equal number of ports is serving \( K \) single-antenna users where \( M \geq K \). Let \( x \in \mathbb{C}^{M \times 1} \) denote the signal vector transmitted from the BS antenna array with normalized power \( E[\|x\|^2] = 1 \). The operation \( E[\|x\|^2] \) denotes computing the expected value of \( \|x\|^2 \), where \( \|x\| \) is the norm of the complex vector \( x \).

Then, the received signal vector \( y \in \mathbb{C}^{K \times 1} \) of the \( K \) users can be written as

\[
y = \sqrt{\text{SNR}_{\text{dill}}} H x + n, \tag{1}
\]

where \( \text{SNR}_{\text{dill}} \) is the average per-user signal-to-noise ratio (SNR); we drop the subscript in the following. \( n \in \mathbb{C}^{K \times 1} \) is the noise vector containing the Additive White Gaussian Noise (AWGN) components at each user, which we assume here to have unit variance. \( H \in \mathbb{C}^{K \times M} \) is the downlink Massive MIMO channel matrix satisfying the normalization \( E[\|H_{k,m}\|^2] = 1 \).

A. The Random-LOS channel

We assume that the receive antenna of the \( k \)th user and the \( m \)th transmit antenna element of the massive uniform linear array are in the far-field of each other. Their positions are given by \( r_k \) and \( r_m \), respectively. Thus, the coupling between


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the two antennas can be computed as
\[ H_{k,m} = \alpha_1 G_m(\hat{r}_{km}) \cdot G_k(-\hat{r}_{km}) \cdot e^{-j \frac{2\pi}{\lambda} |r_k - r_m|}, \quad (2) \]
where \( \lambda \) is the wavelength, \( G_m(\hat{r}_{km}) \) is the far-field function of the embedded array antenna element with index \( m \) when referred to the phase reference point \( r_m \) of that element and radiating in direction \( \hat{r}_{km} = (r_k - r_m) / |r_k - r_m| \). The far-field function of the \( k \)th user is given by \( G_k(-\hat{r}_{km}) \). The constant \( \alpha_1 \) is obtained from the channel matrix normalization given above. The randomness is assumed to stem from, first, the \( k \)th user’s position, \( r_k \), is random over the coverage area, and secondly, the far-field function of, e.g., the same \( k \)th user, \( G_k \), changes from position to position due to the user’s movement.

**B. The Rich Isotropic Multipath**

In addition to the assumption that the transmit and receive antennas are in each others far-fields, we assume that the scatterers and the antennas are in each others far-field too. The position of the \( i \)th scatterer associated with the \( k \)th user are denoted as \( r_{i,k} \). Thus, the coupling between the two antennas can be computed as
\[ H_{k,m} = \alpha_2 \sum_{i=1}^{N_w} G_m(\hat{r}_{i,k}) \cdot E_{i,k}(-\hat{r}_{i,k}), \quad (3) \]
where \( \lambda \) is the wavelength, \( G_m(\hat{r}_{i,k}) \) is the far-field function of the embedded array antenna element with index \( m \) when referred to the phase reference point \( r_m \) of that element and radiating in direction \( \hat{r}_{i,k} \). The far-field function of the scattered field associated with the \( k \)th user is given by the complex vector \( E_{i,k}(-\hat{r}_{i,k}) \), which has random amplitude, phase and polarization. The constant \( \alpha_2 \) is obtained from the channel matrix normalization given above.

**III. ERGODIC SUM RATE AND CAPACITY LOSS**

We assume perfect CSI (Channel State Information) at both the receiver and the transmitter. The signal transmitted from the \( M \) antennas, \( x \), is a linear combination of the symbols intended for the \( K \) users. The symbol intended for the \( k \)th user’s position, \( q_k \), such that \( E[|q_k|^2] = 1 \). Thus, the linearly precoded transmitted signal vector \( x \) can be represented as
\[ x = \sqrt{\beta} W q, \quad (4) \]
where \( q \in \mathbb{C}^{K \times 1} \), \( W \in \mathbb{C}^{M \times K} \) is the precoding matrix, and \( \beta = 1/E(\text{tr}(WW^H)) \) is a normalization constant chosen to satisfy the power constraint \( E[|x|^2] = 1 \). We further consider two linear precoding schemes: the Zero-Forcing (ZF) precoding and the Matched-Filtering (MF) precoding also known as Maximum Ratio Transmission (MRT). The precoding matrix is then given by
\[ W = \begin{cases} H^\dagger & \text{for MF}, \\ H(HH^\dagger)^{-1} & \text{for ZF}. \end{cases} \quad (5) \]

The signal to interference plus noise ratio of the \( k \)th user in the downlink, i.e., the \( k \)th-bitstream, can be computed as
\[ \text{SINR}_k = \frac{\beta \text{SNR} |H_k W_k|^2}{\beta \text{SNR} \sum_{k' \neq k} |H_k W_{k'}|^2 + 1}, \quad (6) \]
where \( H_k \) is the \( k \)th row of matrix \( H \) and \( W_k \) is the \( k \)th column of matrix \( W \).

The ergodic sum rate of the \( (M \times K) \) Massive MIMO system is
\[ \text{SR} = \sum_{k=1}^{K} E \{ \log_2(1 + \text{SINR}_k) \}, \quad (7) \]
where \( \text{SINR}_k \) is the SINR of the \( k \)th user computed above.

**A. Deficient antennas and capacity loss**

The deficient antennas are simulated assuming that a number \( N_{\text{ant, in loss}} \) of antenna elements have a total embedded radiation efficiency that is less than 100\%. The index \( l \) of the antenna elements in loss is chosen randomly, but to simplify this initial analysis we assume that each of the elements has the same performance loss factor \( P_\text{loss} \). Hence, the channel matrix entries corresponding to each the \( N_{\text{ant, in loss}} \) randomly chosen antenna elements are computed as following
\[ H_{k,l} = \frac{H_{k,l}}{\sqrt{P_\text{loss}}}, \quad (8) \]
Then using (8) we compute signal to interference plus noise ratio of the \( k \)th user when some of the BS array antenna elements are in loss, i.e., \( \text{SINR}_{k,\text{loss}}^k \). The capacity loss is then defined as following
\[ C_\text{loss} = 100 \left( 1 - \frac{\text{SR}_{\text{loss}}}{\text{SR}} \right), \quad (9) \]
where \( \text{SR}_{\text{loss}} \) and \( \text{SR} \) are the sum rate capacities computed with some of the antennas being deficient and 100\% efficient, respectively.

**IV. SIMULATION RESULTS**

In this section we specialize the above assumptions to specific values of parameters, statistical distributions and scenarios. We consider a single-cell narrowband Massive MIMO system operating at the \( f = 30 \) GHz frequency band. The uniform linear array at the BS has \( M = 50 \) vertically polarized isotropic source elements separated at the distance \( d = \lambda/2 = 0.005 \) m. Hence, the size of the array can be computed as \( L = M \lambda/2 = 0.25 \) m. In the Random-LOS case we assume a horizontal 120° 2D sector-cell. The \( K = 5 \) users are uniformly distributed over an area covering distances to the BS from 12.5 m to 300 m. Each user is assumed to be equipped with an isotropic antenna. The heights of the transmit and the receive antenna are chosen to be the same given that simulations are performed for users distributed over the horizontal plane only. In the RIMP case we assume that the amplitudes are all equal and the phases are uniformly distributed between 0 and \( 2\pi \). The directions-of-arrival, or rather, the directions-of-departure, of the waves are assumed to be isotropically
The sum rate capacity of Massive MIMO degrades as the ratio $M/K$ decreases. The largest losses are observed for the Random-LOS as compared to RIMP, i.e., deficient antennas have a major impact on system performance in Random-LOS channels. The smaller beamforming gain due to the smaller number of effectively contributing antenna elements results in lower per user SINR. A similar trend is observed when we compare MRF and ZF, since the former is more dependent on the actual SINR maximization as compared to ZF, which relies on interference mitigation. In the present study we show that both the RIMP and the Random-LOS can be used to identify deficient performance of Massive MIMO arrays for ideal linear precoders such as the ZF and the MRF precoders.

V. CONCLUSION

We have shown that the performance of ZF and MRF linear precoding is more sensitive to antenna element deficiency in terms of poor total embedded efficiency in the Random-LOS channel than in the RIMP channel. This has a great relevance to the design of Massive MIMO systems at 5G mmWave frequency bands. As the antenna element deficiency results in
both “effectively” smaller arrays and arrays with “effectively” less elements the performance of MRF is more sensitive to poor antenna efficiency than ZF. The MRF relies more on link gain maximization than ZF that relies on interference cancellation. Finally, we have shown that both the RIMP and the Random-LOS can be used to identify deficient performance of Massive MIMO arrays employing ZF and MRF precoding.

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