Energy-Aware Control of Euler–Bernoulli Beams by Means of an Axial Load

Federico Califano, Associate Member, IEEE, Alexander Dijkshoorn, Student Member, IEEE, Sander Roodink, Stefano Stramigioli, Fellow, IEEE, and Gijs Krijnen, Senior Member, IEEE

Abstract—In this article, we present a novel energy-based control architecture on Euler–Bernoulli beams equipped with a variable stiffness mechanism. To prove the methodological validity of the approach, two control laws are developed using the power balance of the system, explicitly encoded in its infinite-dimensional port-Hamiltonian formulation. The laws are designed to stabilize the beam and to induce limit cycles on it, respectively, increasing damping by removing energy from the system and counteracting damping by injecting energy into the system. The variable stiffness mechanism is realized through a distributed axial load, applied by means of a wire on a winch, and is able to achieve effective stiffness variation due to softening. An experimental setup is designed to validate the theory. 3-D-printed, embedded, piezoresistive strain gauges are used as sensing units for closed-loop control. We show how the developed approach conveniently deals with such sensors, overcoming potential problems arising from their nonideal response. Experimental results show the validity and the robustness of the proposed control laws. High speed videos are used to validate the measurements.

Index Terms—3-D-printed sensors, energy-based control, limit cycles, port-Hamiltonian (pH) systems, stiffness modulation.

I. INTRODUCTION

In the last years, evidences that controlled mechanical systems would benefit from the possibility of actively changing their stiffness properties have been accumulated in the literature. These benefits include energy-efficiency, prevention of safety hazards, and more generally the possibility of providing new functionalities to the controlled system [1]–[3]. For an extensive discussion regarding this theme see [4], where the authors indicated their vision on distributed variable stiffness actuation combined with embedded sensing, which is embodied in this work using as plant the continuum mechanics of an Euler–Bernoulli beam. The distributed control input associated with the variable stiffness mechanism that we propose and realize in this study, consists of an adjustable axial loading of the beam, generated by a dc-motor-controlled tension of a wire attached longitudinally along the beam. The contribution of this work is twofold.

Methodological Contribution: We analyze the advantages of designing control strategies using energy-based-control, in the context of port-Hamiltonian (pH) theory [5]. This framework grew out of the interest to extend the applicability of traditional Hamiltonian theory and has proven to be very effective for the description of distributed-parameter systems [6]. The main aspects of this methodology which are relevant in this work are: 1) the pH formulation of a physical system directly embeds information on the total energy of the system, i.e., the Hamiltonian, in its dynamic model and 2) the variation of the Hamiltonian in time, that is the power flow toward the system, can be easily calculated using the pH model, and its expression contains terms that physically represent the various mechanisms able to exchange energy with the system. This procedure induces a family of control methods [7], sometimes called energy-aware [8], in which the objective is to steer and control physical power flows, rather than regulate and track signals. The main advantages are: 1) the methodology is equally valid for linear and nonlinear systems in any spatial dimension, allowing for an extension to structures undergoing large deformations and nonlinear elasticity, e.g., soft robots; 2) the pH modeling allows for a modular extension of the control strategy when one or more components are added to the system, i.e., if the same system undergoes a new additional input (e.g., interaction with an environment, change in boundary conditions, etc.) the control objective can be shaped by observing the contribution of the new input at a power balance level, without the need of starting a new design; and 3) once the methodology is cast into an optimization framework, it possesses a clear biomimetic perspective since it provides a natural way to optimize for metrics based on physical energy, like safety and energy efficiency.

We apply this methodology, as a proof of concept, in two cases, oriented at, respectively, stabilizing and inducing limit cycles on the beam. We experimentally validate the resulting control laws, designed on the basis of the nontrivial effect that the variable stiffness mechanism has on the power flow of the closed-loop system.

Manuscript received 30 March 2022; revised 16 May 2022 and 29 June 2022; accepted 16 July 2022. Recommended by Technical Editor Kunpeng Zhu and Senior Editor W. J. Chris Zhang. This work was supported by the PortWings project funded by the European Research Council under Grant 787675. (Federico Califano, Alexander Dijkshoorn, and Sander Roodink contributed equally to this work.) (Corresponding author: Federico Califano.) The authors are with the Robotics and Mechatronics Department, University of Twente, 7522 NB Enschede, The Netherlands (e-mail: f.califano@utwente.nl).

This article has supplementary material provided by the authors and color versions of one or more figures available at https://doi.org/10.1109/TMECH.2022.3192324. Digital Object Identifier 10.1109/TMECH.2022.3192324

1083-4435 © 2022 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See https://www.ieee.org/publications/rights/index.html for more information.
Technological Contribution: We combine different, already existing, technical components into one controlled system. We realize a practical implementation of an infinite-dimensional, energy-based control law through the use of active stiffness variation with cable actuators and the use of 3-D-printed, embedded, piezoresistive strain sensors for measuring the curvature. The energy-based control law suggests a sensing strategy that is conveniently realized by 3-D-printing the beam together with its sensing unit through state-of-the-art 3-D-printing technology. In fact 3-D-printing can be used for embedded sensing, which is an upcoming field of research [9]–[11]. The capability of 3-D-printed embedded sensors to perform dynamic strain measurements was proven up to 800 Hz in [12], showing negligible piezoresistive nonlinearities for the 3-D-printed sensors in the linear-response region of a beam under vibrations. In [13], vibration durability self-awareness with similar sensors and processes for reliable fatigue estimation was demonstrated. Furthermore, the sensing unit offers advantages in terms of geometric freedom for the sensor design and the possibility of collocation. These works pave the way for new applications of 3-D-printed piezoresistive embedded sensors in which dynamic measurements are used for control. Despite the steps made in this field, the use of 3-D-printed sensors for control still presents limitations [14], which we aim at tackling in this work presenting a full closed-loop design using this technology.

Related work: We recognize related works using energetic arguments to design vibration suppressing controllers for beams. In [15], a bang–bang boundary feedback controller is proposed to actively suppress vibrations on a slender beam with clamped ends using a time dependent force at one boundary, which is free to move in horizontal direction. In [16], a control strategy is proposed for a cantilever slender beam using a finite-dimensional model approximation. In [17], vibration suppression of frame structures (and in [18] for cantilever beams) has been investigated using cable-supplied forces. In [19], an actuation mechanism based on an axially loaded cantilever beam by means of intermittently attached tendons, is proposed and numerically analyzed, but no control is addressed. In this work, we do not use any finite-dimensional approximation of the beam, but stay at a partial differential equation (PDE) level and exploit the pH formulation for designing the controller and the 3-D printed sensing unit. It is worth mentioning that, contrarily to the cited references, we account for natural dissipation of the beam at a modeling stage, which provides analytic conditions for formation of limit cycles.

The rest of this article is organized as follows. In Section II, the pH model for a Euler–Bernoulli beam is derived and the energy-aware control laws are proposed. In Section III, the experimental setup and methodology are presented. The results are presented and discussed, respectively, in Sections IV and V. Finally, Section VI concludes this article.

II. PORT-HAMILTONIAN MODEL AND ENERGY-AWARE CONTROL

In this section, we review the pH formulation of the Euler–Bernoulli beam and then design the proposed energy-aware control laws. We remark that we do not go into functional analytic treatment of PDEs, that is, we do not introduce specific functional spaces for the variables, nor address the problem of well posedness, i.e., existence and smoothness of solutions. We just assume that all variables belong to a sufficiently smooth functional class such that the formally presented operations can be performed and the described integrals are well defined. Readers interested in the functional analytic framework, in which the pH formulation of Euler–Bernoulli beam is presented, see [20]–[22].

A. Modeling

Consider the beam shown in Fig. 1(a). The vertical displacement $\omega(z,t)$ of the beam is a function of both time ($t > 0$) and space ($z \in [0,L], L > 0$). We denote $\partial_i$ the partial derivative operator where $i \in \{z,t\}$ denotes the space or time variable. Multiple indices indicate higher order derivatives with respect to the indicated variables. The Euler–Bernoulli model for a beam is given by the following PDE:

$$\mu(z)\partial_t \omega(z,t) = -\partial_{zz}(EI(z)\partial_{zz} \omega(z,t)) - b\partial_z \omega(z,t) + Q(z,t)$$  \hspace{1cm} (1)

where $\mu(z)$ is the beam mass per unit length, $b$ is the positive viscous damping coefficient, $E$ is the Young modulus and $I(z)$ the inertia of the section at $z$, and $Q$ is an external, vertical, distributed load on the beam. The proposed model-based control technique makes use of a different representation of the Euler–Bernoulli beam, i.e., its pH model. This formulation is based on the total mechanical energy of the beam, expressed as the sum of kinetic and potential energy

$$E_{\text{mech}} = \frac{1}{2} \int_0^L [\mu(z)(\partial_z \omega(z,t))^2 + EI(z)(\partial_{zz} \omega(z,t))^2]dz.$$  \hspace{1cm} (2)
The state variables in the pH formulation, the so-called energy variables, are defined: $c(z,t) = \partial_z\omega(z,t)$, representing the curvature; and $p(z,t) = \mu(z)\partial_t\omega(z,t)$, representing the linear transversal momentum. It is now possible to define the Hamiltonian functional, representing the map from the space of energy variables to the total energy of the system

$$H(c,p) = \frac{1}{2} \int_0^L [p^2/\mu(z) + EI(z)c^2]dz. \quad (3)$$

Now the so-called coenergy variables are defined as the variational (or functional) derivatives of the Hamiltonian (indicated with the symbol $\delta_H$) with respect to the energy variables. In particular we have $\delta_c H = EI(z)c$, representing the angular moment, and $\delta_p H = p/\mu(z) = \partial_t\omega(z,t)$, representing the vertical velocity of a beam element. The following (first order in time) equations yield the pH model for the Euler–Bernoulli beam:

$$\left(\begin{array}{c}
\partial_c c \\
\partial_p p
\end{array}\right) = \left(\begin{array}{c}
0 \\
-\partial_z z - b
\end{array}\right) \left(\begin{array}{c}
\delta_c H \\
\delta_p H
\end{array}\right) + \left(\begin{array}{c}
0 \\
Q
\end{array}\right). \quad (4)$$

Notice that the first equation represents just a kinematic identity while the second equation corresponds to $(1)$. We refer to [22] for a thorough description of the functional analytic and geometric properties of (4) and highlight here the practical control theoretic perspective that this model offers by computing the time derivative of the Hamiltonian, i.e., the mechanical power flowing in/out the system. In particular, applying derivation by parts under the integral sign, one obtains that power can be decomposed in the sum of three terms

$$\partial_t H = \int_0^L [\delta_c H \cdot \partial_z c + \delta_p H \cdot \partial_z p]dz$$

$$= \int_0^L \frac{\delta_c H \cdot \partial_z (\delta_p H) - \delta_p H \cdot \partial_z (\delta_c H)}{\mu(z)}dz \bigg|_0^L$$

$$\text{Power Injection at the Boundary: } := \partial \Gamma$$

$$+ \int_0^L b(\delta_p H)^2dz + \int_0^L [\delta_p H \cdot Q]dz. \quad (5)$$

The first term characterizes power flow at the boundary of the spatial domain $\partial \Gamma$, and is the only term in case no external load ($Q = 0$) and no internal friction ($b = 0$) are present. To physically characterize this expression we observe that $\partial_z (\delta_p H) = \partial_z\omega(z,t)$ represents the angular velocity of a beam element (conjugated to the angular moment), while $\partial_z (\delta_c H) = \partial_z (EI\partial_z\omega(z,t))$ represents the shear force (conjugated to the vertical velocity). Specific boundary conditions on the beam are needed to complete the model, and they impose constraints on the terms in $\partial \Gamma$. For example in case of a clamped-free beam we have $\delta_p H(0) = \delta_z(H(H)(0) = 0$ (the velocities at the clamped end are zero) and $\delta_p H(L) = \delta_z\omega(H)(L) = 0$ (force and moment at the free end are zero), producing globally $\partial \Gamma = 0$, and

1) Improving Stabilization: In this example, we want to stabilize the beam by means of control, thus improving the convergence to the zero state of the beam. Inspired by the idea of damping injection, developed in passivity based control and in the pH framework [7], stability is achieved by implementing a power flow such that the mechanical energy of the system is always nonincreasing, i.e., we impose $\partial_t H \leq 0$ along the solutions of the system. The implementation of the pulling axial force produces in general a dependency on $P$ of the boundary terms in $\partial \Gamma$, the control objective turns into determining an expression for $P$ such that $\partial \Gamma + \mathcal{C} \leq 0$ holds true. In the rest of the work, we consider the clamped left end on the beam and the axial actuation on the right. For this implementation, we obtain $\partial \Gamma = 0$, but the procedure can be generalized to more complex actuation inputs which are not perfectly axial and as a confirming energy conservation $\partial_t H = 0$ in case of an unloaded and frictionless beam. The second term, denoted by $\mathcal{C} \geq 0$, corresponds to the power dissipated by the damping forces in the beam. The third term contributing to the power flow in the beam is the distributed term $\mathcal{C} = \int_0^L \delta_p H \cdot Q$ and if we interpret $Q$ as the control input, it corresponds to the power injected in the system by means of the controller. In the next section we show how we take advantage of the computed power balance to design the proposed feedback law.

B. Control

We focus on an application in which the control input is characterized by a pulling axial force $P$ on the beam as represented in Fig. 1(b). A simple force balance analysis [23] shows that the vertical load $Q$ induced by the effect of such a pulling axial force is given by

$$Q(z,t) = P(t)\partial_z\omega(z,t) = P(t)c(z,t). \quad (6)$$

A constant application of such a force ($\partial_t P = 0$) can be regarded as a variable stiffness effect since the softening/hardening effect of the pulling force effectively shifts the vibrational eigenfrequencies of the beam with respect to the unloaded case. As a matter of fact substituting (6) in (1) produces another PDE that can be characterized (in case of spatially constant parameters) in the frequency domain using, e.g., separation of variables methods [23]. In this sense, we refer to the control method as variable stiffness control, and an empirical characterization of the effective stiffness variation for constant pulling forces $P$ is shown in Section IV-B. Nevertheless, we stress that the proposed control design is methodologically unrelated to frequency analysis, and will be carried out using energetic considerations, exploiting the pH model and the associated power balance (5). In particular, specializing the latter expression to the case of a pulling axial force (6), the power balance specializes in an instance of (5) with

$$\mathcal{C} = P \int_0^L [\partial_t\omega \cdot \partial_\omega\omega]dz \quad (7)$$

where the axial force $P$ has been pulled out of the integral since it does not depend on the spatial variable $z$. Starting from this power balance we propose and validate two different controllers.

1) Improving Stabilization: In this example, we want to stabilize the beam by means of control, thus improving the convergence to the zero state of the beam. Inspired by the idea of damping injection, developed in passivity based control and in the pH framework [7], stability is achieved by implementing a power flow such that the mechanical energy of the system is always nonincreasing, i.e., we impose $\partial_t H \leq 0$ along the solutions of the system. The implementation of the pulling axial force produces in general a dependency on $P$ of the boundary terms in $\partial \Gamma$, the control objective turns into determining an expression for $P$ such that $\partial \Gamma + \mathcal{C} \leq 0$ holds true. In the rest of the work, we consider the clamped left end on the beam and the axial actuation on the right. For this implementation, we obtain $\partial \Gamma = 0$, but the procedure can be generalized to more complex actuation inputs which are not perfectly axial and as a
consequence induce nonzero terms in ∂T. With this assumption, a possible expression for the pulling axial force can be simply given as the bang–bang control law

\[ P(\partial_t \omega, \partial_z \omega) = -k \cdot \text{sgn} \left( \int_0^L [\partial_t \omega \cdot \partial_z \omega] dz \right) \]  

(8)

where \( k \) is some positive constant determining the damping injection rate and \( \text{sgn}(\cdot) \) is the function producing the sign of its argument. With this design, the only relevant information that needs to be observed along the control is the sign of the integral in (8), and not its value. Notice that a substitution of (8) back in (1) through (6) would produce a nonlinear PDE, practically impossible to analyze with classical methods, while looking at the energy balance stemming from the pH model it is clear that this feedback law extracts energy from the system, and hence improves its stabilization, i.e.,

\[ \partial_t H = - \mathcal{G} + k \cdot \text{abs} \left( \int_0^L [\partial_t \omega \cdot \partial_z \omega] dz \right) \leq 0 \]  

(9)

where \( \text{abs}(\cdot) \) returns the absolute value of its argument. The thus defined control law produces an additional control induced damping term \( \mathcal{G}_c \geq 0 \), which improves the stabilization of the beam to its zero state. A formal stability analysis would require functional analytic tools, which are outside the scope of this article, but a sketch of the proof can be given as follows. Assuming existence and precompactness of trajectories of the closed-loop system, asymptotic stability of the zero state would follow as a simple corollary of LaSalles invariance principle for infinite-dimensional systems [24]. In fact, using the Hamiltonian \( H \) as Lyapunov candidate functional, the greatest invariant set in the phase space such that \( \partial_t H = 0 \) and compatible with the boundary conditions is constituted by the zero state \((c, p) = (0, 0)\), which would then qualify as an asymptotically stable equilibrium.

2) Inducing Limit Cycles: Inspired by the proposed energetic approach, we propose an alternative control objective, namely, the generation of stable limit cycles on the closed loop system by means of power flow control. The intuition is that, in order to stabilize the system on a periodic motion, at every period the controller must inject in the system the same amount of energy which is naturally damped by the system. In this case, the control induced power \( \mathcal{G}_c \) must act as positive power source for the system, and not as dissipation, i.e., \( \mathcal{G}_c \leq 0 \) must hold. We propose the same control law as in the stabilization case (8), but now with a negative control gain \( k < 0 \). Necessary condition for the formation of a limit cycle is that the dynamic evolution of the system obeys to the constraint

\[ \int_\tau \partial_t H dt = \int_\tau (-\mathcal{G}_c - \mathcal{G}) dt = 0 \]  

(10)

where the period \( \tau \) of the cycle is parametrized by time. In words, the work per cycle done by the closed loop system must be zero. Notice that this condition does not imply \( -\mathcal{G}_c = \mathcal{G} \), which would represent a pointwise (in time) cancelation of the natural dissipation by the controlled power flow, but the weaker condition \( \int_\tau \mathcal{G}_c dt = \int_\tau \mathcal{G}_c dt = \int_\tau \mathcal{G}_c dt \), which would allow a possible nonzero instantaneous variation of the energy functional, which is nevertheless zero over the entire cycle [25]. For the proposed control law the condition translates into

\[ b \int_\tau \int_0^L [(\partial_t \omega)^2] dz dt = k \int_\tau \text{abs} \left( \int_0^L [\partial_t \omega \cdot \partial_z \omega] dz \right) dt. \]  

(11)

In this work, we do not address the problem of designing the stable limit cycle, for which condition (11) is only a necessary condition, but we are interested in understanding if the proposed control law is qualitatively able to stabilize the closed loop system on these periodic orbits. We obtained a positive answer experimentally.

III. SETUP DESCRIPTION AND METHODOLOGY

In this section, we present the system that has been constructed to validate experimentally the proposed control strategy. We will describe it in a top–down perspective, in which a detailed explanation of the various subcomponents will follow a general overview. A comparison with the state-of-the-art manufacturing approaches is carried through along the description.

A. Setup

Fig. 2 shows a picture of the setup and its related schematic overview. A 3-D printed beam with two pairs of embedded strain gauges is placed on top of an electromechanical shaker (MB Dynamics PM-50 A). The base of the beam is clamped in a holder connected to a dc-motor (Maxon 142733 DC-motor), which can apply an axial load to the tip of the beam through
a wire on a winch. The control law and data processing are performed with 20-Sim, a multiphysics numerical simulation package from Controllab [26]. An in-house developed hardware board with field-programmable gate array (Altera Cyclone III EP3C40Q240C8N) and input/output functionality called the RaM-stix, which is compatible with 20-Sim, is used to connect all components. A motor controller (Maxon ads50/5 4-q-dc) is used to control the current which is sent to the dc-motor, and voltage dividers with amplifiers are used to measure the strain gauge signals, where the whole system is sampled at 5000 Hz.

The following subsections give a detailed description of the 3-D-printed strain gauges, the variable stiffness mechanism, and the implementation of the control strategy.

1) 3-D-Printed Beam With Strain Gauges: The beam is 3-D-printed using fused deposition modeling (FDM) because of the geometric freedom, as well as affordable, easy-accessible and multimaterial capabilities [11]. The multimaterial capabilities allow for embedding of piezoresistive strain sensors, which can be used for measuring the curvature of the beam.

The piezoresistive strain sensors are composed of a conductive polymer composite, namely, polylactic acid (PLA) filled with carbon black nanoparticles of the brand Proto-pasta [27]. PLA is chosen for both the conductive and nonconductive material for the relatively high Young’s modulus of 3.5 GPa and low damping in comparison to, e.g., thermoplastic polyurethane. The similar mechanical properties for the conductive and nonconductive material enables uniform mechanical properties along the beam and reduces stress concentrations at the locations where the sensors are embedded [28], minimally influencing the beam dynamics. A differential sensing layout, with strain sensors on both sides of the beam, is used to improve the linearity of the polymer sensors by ideally compensating for the odd orders of nonlinearity [29]. In this way, differential sensing helps compensating for disturbances like changes in temperature and axial compression. Similar to [12] and [30], meandering strain gauges are used, increasing the total length of the gauge channel.

The strain gauges are designed with a constant width over the entire path. The resistance of 3-D-prints is anisotropic due to the infill raster, where the resistance perpendicular to tracks is higher than parallel to tracks [31], [32]. Therefore, the meandering channels are connected by curved bridges, to achieve continuous 3-D-printed tracks from contact pad to contact pad. A close-up of a sensor can be seen in Fig. 3 on the top left. Within the beam, two meandering strain gauge pairs are embedded at the points of the highest curvature for a clamped-free beam in first and second resonance mode, respectively, at the base of the beam and halfway the beam, with strain gauges on the top and bottom side of the beam.

The printing is performed with a Diabase H-series multimaterial 3-D-printer with PLA as nonconductive material and conductive PLA from Proto-Pasta as conductor. Computer-aided design (CAD) is performed in Solidworks software and the printing commands are generated with slicer software from Simplify3D [33]. The beam dimensions are 300 mm (plus an additional part for clamping) by 15 by 5 mm. After printing the beam is placed in an oven at 150 °C for 10 min to decrease the residual stress and to remove any significant offset in curvature. Channels, through which the electrical wires are routed, are included in the printed beam, as shown in Fig. 3, top right. The electrical wires are molten into the sensor connection pads at the end of the strain gauges by means of locally heating the wire with a soldering iron, following the methodology in [34].

A Wheatstone half-bridge configuration is used for the differential measurement per sensor pair. The beam with sensors is then placed in a frame together with the variable stiffness mechanism.

2) Variable Stiffness Mechanism: To implement the axial load, a wire is attached to the tip of the beam to apply a pulling force. Such a mechanism can be lightweight along the beam length, minimally influencing the beam dynamics. To actively control the axial load, the wire tension is made adjustable by means of a winch, driven by a dc-motor, as seen in Fig. 3 at the bottom left. In order to implement the proposed control law, it must be possible to both decrease and increase the axial load with respect to a reference stress condition. This is achieved by pretensioning the wire to a fixed value \( P \), and implementing the control law (8) with gain \( k \) around this pretensioned value. Additionally, the pretension circumvents slack in the wire that can cause delays [18]. The pretension and gain \( k \) are normalized to achieve a maximum load below the critical buckling load. We will comment on the effects of the pretension implementation in the discussion section. The winch diameter is determined by taking into account the buckling limit of the beam and the maximum torque of the motor, yielding an arm length of 10 mm. The brackets and frame are designed to have their first eigenfrequency far above the second eigenfrequency of the beam. As pulling wire, a braided Dyneema fishing line (Spiderwire) is used for its low elasticity and low weight.
3) **Controller Implementation**: In order to apply the proposed control the curvature and velocity of the beam need to be determined. During the experimental validation, the beam is excited with frequencies exciting its first eigenmode. An analytic model of the first eigenmode curvature of a Euler–Bernoulli beam \([35]\) is fitted to the data of both sensor pairs (filtered with a low-pass Butterworth filter with cut off at 16 Hz) and used to determine the curvature function. This yields a fit for the curvature function \(\partial_2 \omega(z,t)\) and hence for the beam displacement \(\omega(z,t)\) within the controller. A state-variable filter is subsequently used to determine the velocity function \(\partial_t \omega(z,t)\) of the beam. From the determined curvature and the velocity function the integral of the proposed control laws can be calculated, and the required sign in the control law can be determined.

**B. Methodology**

We now list the steps that have been executed to characterize the setup and perform/discuss the experiments. These will be analyzed in detail in the next section. We recall that even if two sensor pairs are embedded in the beam to determine the curvature and the velocity, in the following discussion of the experiments we show plots only from the sensor pair located at the base of the beam, since the signals from the second sensor pair follow a qualitatively similar trend.

1) **Sensor Characterization**: To characterize the sensor response of a piece of beam with a single pair of sensors, the tip of the beam is displaced with a sinusoidal trajectory, similarly to \([36]\). A linear actuator (SMAC LCA25-050-15F) is used to displace the tip of the beam, while the differential voltage of the sensors is measured. This method can be used to characterize the sensor response and to analyze the possible presence of viscoelastic effects in conductive polymer composite-based sensors, such as hysteresis and creep \([11]\). A flexible connection between the linear actuator and the beam is used to relieve all directions except for bending.

2) **Dynamic Behavior**: To characterize the dynamics of the beam for various amounts of pretension, the electromechanical shaker (MB Dynamics PM-50 A) is used to excite the beam with a frequency sweep from 0 to 120 Hz, while measuring the sensor signals.

3) **Experiments**: In every experiment, the beam is excited for 0.5 s with the shaker after which the controller is activated. Both positive and negative control gains are tested, respectively, for stabilizing the zero position and for inducing limit cycles. The axial load sweeps around a pretension of \(P = 0.5\). The actual control gain \(k\) in \((8)\) ranges from \(-0.45\) to 0.45. With this choice we exploit a full range between an almost untensioned wire \((P = 0.05)\) and a tension close to the buckling limit \((P = 0.95)\). We remind that the nondimensional units are due to normalization with respect to the buckling load.

4) **High-Speed Video Validation**: To validate the sensor measurements and discuss the experiments, high-speed videos are taken for control gains \(k \in \{-0.3, 0, 0.3\}\), filming at 300 fps with a camera (Casio Exilim Pro EX-F1). A compilation of these videos can be found in the supplementary information. The videos are analyzed with open-source video analysis software, Kinovea \([37]\), The tip of the beam can be tracked over time and can be compared to the sensor data.

**IV. Experimental Results**

**A. Sensor Characterization**

Fig. 4 shows the differential voltages from the strain gauges at the base of the beam under sinusoidal tip deflection (top graph). The differential voltage is also presented in Fig. 4 as a function of the tip deflection (bottom graph, low-pass filtered with a cutoff frequency of 16 Hz), showing a slight nonlinear relation between sensor deformation and sensor output. Since the sensor characterization does not show any significant drift or hysteresis, the sensors are suited for determining the sign of the curvature in the controllers.

**B. Dynamic Behavior**

Fig. 5 shows the dynamic response of the sensor pair at the base of the beam for various pretension values. The insets show a close-up around the first resonance peak, where clearly a shift in frequency can be recognized for different pretension values, as described in literature \([19]\). Notice that also the second resonance peak is clearly visible in all the measurements, and that a third resonance peak can be recognized around 43 Hz. This coincides with the first eigenmode of the planar oscillation of the beam, which might be present due to imperfect excitation and/or beam imperfections.

In the following experiments a pretension characterized by \(P = 0.5\) is used. This corresponds to the yellow curve in Fig. 5, where the first resonance frequency can be found around 14.2 Hz.
C. Experiments

At the beginning of each experiment, the beam is excited by means of the electromechanical shaker for 0.5 s, after which the proposed control laws are applied. Figs. 6 and 7 represent, respectively, the set of experiments corresponding to stabilization of the beam and to limit cycle induction. The shaker acts between 1.5 and 2 s, and the time evolution of the vibrating beam for different control gains, as measured with the sensor pair, is then shown. The top graph of Fig. 6 (resp. Fig. 7) demonstrates the stabilization of the beam (resp. the induction of limit cycles) with different positive (resp. negative) control gains. As predicted by the theory, an increase in damping is observed when increasing the positive control gains, and limit cycles are induced for some negative control gains, where a lowest negative gain yields a larger sustained oscillation amplitude. The central plots in Figs. 6 and 7 show the rate of change of voltage versus the voltage over time, measured by the sensor pair for $k = 0$ (uncontrolled beam), $k = 0.3$ and $k = -0.3$. This is not a proper phase space plot, but it gives a good indication of the evolution of the states of the beam since the voltage is proportional to the curvature, whereas its time derivative is proportional to the velocity. For the cases with control, the effect of the axial load can be recognized every half period as a jump. It is clearly visible that the jump induced by the discontinuous control law acts in the moment the beam exhibits its maximum kinetic energy (corresponding to $V = 0$, i.e., minimum curvature) in case the goal is to extract energy ($k = 0.3$); and conversely in the moment in which the potential energy exhibits its maximum ($\frac{\partial}{\partial t} V = 0$, i.e., maximum curvature) in case the goal is to inject energy ($k = -0.3$) in the system to produce a limit cycle.

Fig. 8 shows the peaks (the periodic maximum values) of the sensor signal in Figs. 6 and 7 as a function of time. The peaks are plotted on a logarithmic scale, to show lines in the case of exponential decay (constant damping). The approximately straight lines in the logarithmic plot until 3 s indeed indicate a constant magnitude of damping with different control gains, as also found in the literature for different axial loads [16], [38]. After 3 s, the experiments with lowest negative gains, show the peaks converging to constant oscillation amplitudes, indicating the presence of limit cycles. However, not for all negative control gains this can be observed. Smaller control gains only show a reduction in damping, instead of sustained oscillations. This outcome will be discussed in the discussion section.
The presence of the pretension effect does not cause any problems for the effective control law $L = b - \eta$ makes the effective control law $ISCUSSION = 0 \cdot \omega$ rather than $\omega$.

Fig. 8. Positive peaks (periodic maximum values) of the signals in Figs. 6 and 7 displayed as a function of time. The maximum amount of damping is achieved for the largest positive gain and limit cycles with a sustained oscillation are induced for the lowest negative gains. The thick black line indicates the measurement without control.

Fig. 9. High speed video validation measurements. (a) Merged high-speed camera footage of beam in two different deflection states; the green tip is tracked for validation. (b) Comparison of the sensor measurement with the tip analysis from the high-speed video.

D. High-Speed Video Validation

High-speed videos are taken for the validation of the sensor measurements for $k \in \{-0.3, 0, 0.3\}$ at 300 Hz. Fig. 9(a) shows two merged frames of the beam deformation in the high-speed video. In the analysis, the green piece of tape on the tip of the beam is tracked as marker for the validation. Fig. 9(b) shows the data from the sensor pair compared to the analyzed tip deflection from the high-speed video data for $k = 0.3$. It is important to note that these data are not collocated, since the plot compares the signals from the sensor pair at the base of the beam to the deflection of the tip. The analysis of the tip deflection is done in Kinovea [37], where the deflection is given in the number of pixels. From the white reference stick it was determined that one pixel corresponds to roughly 0.625 mm. So the maximum deflection is approximately 34.4 mm, or 11.4% of the total beam length.

The last plots in Figs. 6 and 7 display the data from the video in a phase space perspective. The sensor and video data display qualitatively similar results, showing the same peaks and trends over time and confirming the validity of the proposed control strategies.

V. DISCUSSION

The results shown in the previous sections validate the proposed energy-aware methodology for both proposed control objectives. Fig. 8 shows that the energy damped by the controller increases for increasing gains as the theory predicts. From the video validation in Fig. 6, which provides an accurate trajectory on the phase space of the tip of the beam, it can be appreciated how the controller stabilizes the beam in fewer cycles than those required in the uncontrolled case. The controlled system is inherently robust to parametric disturbances, since the net effect of the control is to extract energy from the system as long as inequality (9) remains valid. Concerning the second set of experiments, we interestingly observe that for some negative gains $k$ in Fig. 8, the controller is able to stabilize the system on a stable limit cycle, experimentally confirming the theoretical claim. It is worth noticing that this means that at steady state a constraint like (11) is satisfied, which could be used in principle to identify physical parameters of the beam like the damping coefficient $b$. This can be clearly appreciated from the plot in Fig. 7 reconstructed through video validation, where we see the system approaching a limit cycle. The development of the latter within a finite range of the control parameter is an empirical evidence of some degree of robustness of the proposed control law. We stress that being able to induce, identify, and optimize limit cycles over controlled mechanical systems can give great insight on energy efficient forms of periodic locomotion, providing a biomimetic perspective to the design. It is also shown that the sensor response in Fig. 4 does not cause any problems for the control, showing the convenience in using the 3-D-printed, embedded sensors. The implementation of the control law only requires measuring the sign of the curvature locally, putting less strict requirements on the sensor performance compared to a more classical control approach that requires precise, quantitative measurements. This demonstrates the potential of these 3-D-printed, embedded sensors in a control perspective in an unprecedented way.

We conclude the discussion of the experiments explaining two practical deviations from the theoretical description that are crucial to properly comment the results and address future research directions.

1) Pretensioning Effect: The presence of the pretensioning force $P$ makes the effective control law $P = P - k \cdot \text{sgn}\left(\int_0^L [\partial_t \omega \cdot \partial_{zz} \omega] dz\right)$ rather than (8). It is easy to verify that, at a power balance level, the effect of $P$ produces an additional term, undefined in sign, which makes the control power flow

$$G_c = \text{abs}\left(\int_0^L [\partial_t \omega \cdot \partial_{zz} \omega] dz\right)$$
\[
\times \left( k - P \cdot \operatorname{sgn} \left( \int_{0}^{L} [\partial_{t} \omega \cdot \partial_{z} \omega] \, dz \right) \right). \quad (12)
\]

What follows is that a true damping injection (resp. positive power injection) strategy, i.e., one that produces a positive (resp. negative) \( G \), at any time instant, would require \( k \geq P \) (resp. \( k \leq -P \)). This requirement is not realizable in the presented setup, since the implementation of the control mechanism can only pull and not push on the wire. The reason why the experiments show a successful validation of the theory lies in the quasi periodic behavior of the beam, that makes the net power flow over a period effectively damp (resp. increase) the overall energy in the system also with the chosen positive (resp. negative) gains \( k \).

In other terms, even if \( \text{abs}(k) < P \) the term \( k - P \) in (12) is indefinite in sign, it results that over a period \( \tau \), the term \( \int F \cdot \dot{G} \) remains perfectly valid. This explanation is furthermore consistent with the "weak" damping effect we observe for small positive gains \( k \), as shown in Fig. 6, and with the fact that the effect of the pretension becomes less predictable when second-order mode excitation is present in the beam.

2) Nonzero Boundary Terms: The wire is solely fixed at the tip and guided through the base of the beam. Hence, upon an axial load and bending of the beam, the wire keeps straight like the string of a bow. This yields an angle between the axial load and the tip of the beam, which deviates from energy-based design of the control law in the fact that some nonzero boundary terms in \( \partial T \) might be induced at the free end of the beam. In this work, we neglected this effect since the beam underwent small angles.

Future implementations of the variable stiffness mechanism, able to overcome the disadvantages induced by pretensioning effects and to extract and inject energy in a faster way, are currently under study and are necessary when moving to more complicated nonlinear beam models in which large deformations are allowed.

VI. CONCLUSION

In this article, we have presented a novel energy-aware methodology to control Euler–Bernoulli beams. The pH formulation of the system encodes its power balance, used to design control laws by extracting or injecting energy in the system. The technological realization of the setup comprises novel 3-D printing technology for the sensors embedded in the plant, conveniently used in relation to the proposed approach, validated experimentally with the help of high-speed camera recordings.

Ongoing research focuses on casting this energy-aware approach into an optimization framework, aiming at finding control laws that induce a desired behavior in the controlled system. In this respect we are investigating the use of neural networks to learn maps implementing state feedback laws which are much more general than (8), while keeping the same energetic interpretation. Furthermore, we are working toward extending the approach to general flexible 3-D systems, in order to design control laws achieving optimal periodic behaviors in term of energy-efficiency, a metric which naturally fits into the used methodology.

ACKNOWLEDGMENTS

The authors would like to thank Ing. Marcel Schwitz and Ing. Remco Sanders for their invaluable technical support for the experiments.

REFERENCES

Sander Roodink received the master’s degree in systems and control from the University of Twente, Enschede, The Netherlands, in 2020. He is currently working with Safan Darley, Loc hern, Netherlands, improving their automated bending solutions project. His research interests include finding a way to implement mechatronic ideas into the real world.

Stefano Stramigioli (Fellow, IEEE) received the M.Sc. degree with Hons. (cum laude) in electrical engineering from the University of Bologna, Bologna, Italy, in 1992, and the Ph.D. degree with Hons. (cum laude) in robotics from the Delft University of Technology, Delft, The Netherlands, in 1998.

He is currently a Full Professor of Advanced Robotics. He currently serves as the Vice President for Research of eRobotics, the private part of the PPP cooperation with the European Commission known as SPARC, the biggest robotic civil program worldwide. He has authored or coauthored more than 300 publications including four books.

Dr. Stramigioli is an ERC Advance Grant Laureate and a Member of the Royal Holland Society of Science and Humanities. He has been the 2009 recipient of the IEEE-RAS distinguish service award. He has been teaching Modeling, Control and Robotics to under and post-graduates. He has been Editor in Chief for the IEEE Robotics and Automation Magazine, which he brought from the seventh to the first place in the ranking of the Impact Factor among all journals on Robotics. He has furthermore been Editor in Chief for the IEEE IEEE Intelligent Transportation Systems Conference Newsletter and Guest Editor for others. He is a member of the Editorial Board of the Springer Journal of Intelligent Service Robotics. He has been an IEEE Robotics and Automation Society officer for many years. He has been an AdCom member of the IEEE Robotics and Automation Society, and has been the founder and chair of the Electronic Products and Services of the IEEE Robotics and Automation Society and has been serving as Vice President for Membership of the same society for two consecutive terms.

Sander Roodink

Stefano Stramigioli

Federico Calilano (Associate Member, IEEE) was born in 1991. He received the master's degree (cum laude) in automation engineering and the Ph.D. degree in automatic control and operational research from the University of Bologna, Bologna, Italy, in 2015 and 2019, respectively. He is currently an Assistant Professor with theFaculty of Electrical Engineering, Mathematics and Computer Science, Robotics and Mechatronics, University of Twente, Enschede, The Netherlands. His research interests include port-Hamiltonian modeling and control, infinite-dimensional system theory, geometric and nonlinear control, and energy-aware robotics.

Alexander Dijkstra (Student Member, IEEE) received the M.Sc. degree in electrical engineering, in 2019, with a focus on characterising anisotropic electrical properties of conductive 3-D-prints from the University of Twente, Enschede, The Netherlands, where he is currently working toward the Ph.D. degree in 3-D-printing embedded sensing and actuation for aerial robotics as part of the PortWings project with the Robotics and Mechatronics Group.

His research interests include embedded sensing, 3-D-printing of sensors and electronics, smart materials, and variable stiffness structures.

Federico Calilano

Alexander Dijkstra

Gijs Krijnen (Senior Member, IEEE) received the Ph.D. degree (cum laude) in nonlinear integrated optics devices from the University of Twente, Enschede, the Netherlands. He is currently with the Robotics and Mechatronics group, University of Twente. He has coauthored over 115 refereed journal papers, 11 book chapters, and 250 conference contributions on a variety of subjects including nonlinear integrated optics, micro-mechanical sensors and actuators, biomimetic flow and inertial sensors and parametric and nonlinear transduction. His interests include bio-inspired transducers, parametric sensing schemes, and additive manufacturing (embedded sensing).

Dr. Krijnen was a Fellow of the Royal Netherlands Academy of Arts and Sciences. He was the recipient of the VICI grant by the Netherlands Organisation for Scientific Research, in 2005, for research on bio-inspired flow-sensors (BioEARS).

Gijs Krijnen

Authorized licensed use limited to: UNIVERSITY OF TWENTE. Downloaded on August 02,2022 at 06:40:46 UTC from IEEE Xplore. Restrictions apply.