Conference on Mathematics of Wave Phenomena

February 14-18, 2022

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Germany

Book of Abstracts

Conference on Mathematics of Wave Phenomena 2022

held online at the Department of Mathematics, Karlsruhe Institute of Technology, Germany, February 14–18, 2022

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Wave phenomena analysis and numerics

Karlsruher Institut für Technologie

Preface

The mathematical modeling, simulation and analysis of wave phenomena entail a plethora of fascinating and challenging problems both in analysis and numerical mathematics. This statement remains as true today as four years ago, when we organized the first Conference on Wave Phenomena at KIT. In this time, the field has lost none of its appeal and richness of interesting problems that make it so attractive to researchers worldwide.

Our conference has grown significantly: again, eleven plenary lectures will be held, but the number of minisymposiae has grown from fourteen to twenty-two. More than 260 presentations are planned to take place, with subjects spanning the triangle between analysis of non-linear PDEs, numerical analysis and simulations of wave phenomena, and physical aspects. The summaries of all contributions can be found in this book, starting with the plenary lectures, followed by the minisymposia and, finally, the contributed talks. Topics include non-linear wave equations, waves in waveguides, water waves, waves in non-linear media, waves in metamaterials, direct and inverse scattering problems for acoustic, electro-magnetic or elastic waves, biomedical and seismic imaging, finite and boundary element methods, and both time-harmonic and transient wave propagation problems.

Unfortunately, the Corona pandemic with new waves and variants has frustrated our plans to hold the conference on the KIT premises in Germany. We are conscious of the concern many of our participants feel about exposing themselves to the risk of traveling and meeting with many other people. For others, quarantine regulations have made traveling abroad impossible to begin with. Thus, reluctantly, we have decided to hold the conference in an online format. In the heading of every abstract in this book, you will find a direct link to the corresponding virtual meeting room. In addition, there is the platform `wonder.me` for informal meetings and discussions (see the section at the end of this book). We hope very much that it will soon be possible to meet in person for such events.

The conference is organized at Karlsruhe Institute of Technology by members of the Collaborative Research Centre 1173 (CRC 1173) on Wave Phenomena. We gratefully acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) through CRC 1173.

Karlsruhe, January 2022

Tilo Arens
Willy Dörfler
Dirk Hundertmark
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Plenary Lectures

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Plenary Lecture **Monday, 9:15–10:15, Zoom Room 00, link tba**

Mortar coupling of FEM and BEM for the Helmholtz problem

Ilaria Perugia[1,*], **Christoph Erath**[2], **Lorenzo Mascotto**[3], **Jens M. Melenk**[4], **Alexander Rieder**[5]

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We present a mortar-type coupled finite element-boundary element discretization of the Helmholtz problem in three dimensions with variable coefficients, where the mortar variable is related to an impedance trace on an artificial boundary. In particular, we focus on discontinuous Galerkin discretizations in the interior domain [1]. Compared to the case of a conforming finite element discretization [2], here a modification in the discrete coupling condition is required in order to satisfy a discrete Gårding inequality. The resulting method presents a block structure with subblocks that are nonsingular, independently of the choice of the artificial boundary. This allows for the use of existing discretization techniques for these blocks. As a tool for the theoretical analysis, a discontinuous-to-continuous reconstruction operator on curvilinear meshes with optimal h- and p-stability and approximation properties is introduced, which may be of independent interest.

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Plenary Lecture	Monday, 10:15–11:15, Zoom Room 00, link tba

Stability of smooth travelling waves and instability of peaked travelling waves in the Camassa–Holm equations

Dmitry E. Pelinovsky[1,*]

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I will review recent results on stability of travelling solitary and periodic waves in the Camassa–Holm equation and its generalizations known as the b-family of the Camassa–Holm equations.

I will first show that perturbations to the peaked solitary and periodic waves grow in the $W^{1,\infty}$ norm and may blow up in a finite time even though they are bounded in the H^1 norm. This is related to the intristic linear and nonlinear instability of peaked travelling waves in the Camassa–Holm equation.

Spectral instability of solitary waves in the b-family is then proven from analysis of a linearized operator derived for perturbations in $H^1 \cap W^{1,\infty}$ and extended to weaker functions in L^2. The spectrum of this linearized operator covers a closed vertical strip of the complex plane for all $b \neq \frac{5}{2}$.

In the last part of my talk, I will show that the smooth solitary and periodic waves of the Camassa–Holm equation are stable with respect to smooth perturbations. The key to obtaining this result is to use an alternative Hamiltonian structure which is different from the standard Hamiltonian structure common to the Korteweg-de Vries equations. Moreover, the alternative Hamiltonian structure can be introduced for the b-family and can be used to generalize the stability result for the smooth solitary waves for all $b > 1$. In both cases, a precise stability condition is derived. The stability condition is generally verified only numerically in the open region where the smooth solitary and periodic waves exist.

The talk is based on the following series of papers.

References

[1] F. Natali and D.E. Pelinovsky, Instability of H^1-stable peakons in the Camassa-Holm equation, *Journal of Differential Equations*, **268** (2020), pp. 7342–7363.

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Plenary Lecture Tuesday, 9:00–10:00, Zoom Room 00, link tba

Challenges in the field of dispersive PDE

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The field of dispersive PDE has seen rapid advances in the last 20 - 30 years, and questions which seemed unassailable before then, such as classification of large solutions for certain types of geometric nonlinear wave equations, have been resolved. Given that many of the most far reaching results have a rather qualitative character, these advances have also raised a number of intriguing questions. Amongst these are the issue of obtaining more quantitative results, such as explicit a priori space time bounds for critical nonlinear wave equations, as well as the relation of smoothness of solutions to their dynamical behavior, and finally the issue of whether 'generic data' result in better behavior in some sense than 'pathological data'. This last point is also related to the technique of randomizing the data. The aim of this talk will be to provide an overview of some of these issues.

References

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ISBN : 978-3-03719-106-4
DOI : 10.4171/106,

Plenary Lecture Tuesday, 10:00–11:00, Zoom Room 00, link tba

Lorentzian Calderón problem under curvature bounds

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We introduce a method of solving inverse boundary value problems for wave equations on Lorentzian manifolds, and show that zeroth order coefficients can be recovered under certain curvature bounds. The set of Lorentzian metrics satisfying the curvature bounds has a non-empty interior in the sense of sufficiently smooth perturbations of the metric, and the interior contains the Minkowski metric. On the contrary, all previous results on this problem impose conditions on the metric that force it to be real analytic with respect to a suitably defined time variable. The analogous problem on Riemannian manifolds is called the Calderón problem, and in this case the known results require the metric to be independent of one of the variables. In particular, the Riemannian version of the problem is open near the Euclidean metric.

Our approach is based on a new unique continuation result in the exterior of the double null cone emanating from a point. The approach shares features with the classical Boundary Control method, and can be viewed as a generalization of this method to cases where no real analyticity is assumed.

Plenary Lecture **Wednesday, 9:00–10:00, Zoom Room 00, link tba**

Modelling sound-light interactions on the nanoscale

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The interaction between electromagnetic and elastodynamic waves has a long and distinguished history, dating at least from the work of Brillouin and Mandelstam in the early 20th century [1]. More recently physicists have begun to rediscover these interactions in the context of nanophotonics, in which material vibrations are trapped or guided within structures that possess features that are typically as large as the wavelength of light (and sound) in the material [2]. These interactions can lead to several interesting and unusual effects, including acoustic storage, by which information can be written holographically into long-lasting acoustic waves, and "slow-light", by which the speed of light is reduced to a fraction of its value in vacuum. However at these small scales the mathematics of the different types of waves, and of the forces that cause them to interact, can become complicated, and modelling of the interlinked PDEs is a difficult task [3]. This modelling challenge is greatly complicated by the presence of noise, which arises in a stochastic way from thermal phonons and which in general cannot be neglected. Here we discuss the journey towards a comprehensive and accurate mathematical description of light-sound interactions, and review recent progress in using these models for comparison with experiments.

References

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Plenary Lecture **Wednesday, 10:00–11:00, Zoom Room 00, link tba**

We need to talk about the impedance boundary condition

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The impedance boundary condition is the simplest (lowest order) absorbing boundary condition. Therefore, a ubiquitous model problem in the numerical analysis of wave problems is the Helmholtz equation posed in the exterior of an impenetrable obstacle, with the exterior domain truncated by an artificial boundary with an impedance boundary condition.

Over the last ∼15 years, there has been a large amount of research analysing numerical methods applied to this model problem, with the analyses valid in the high-frequency limit with the artificial boundary fixed. However, perhaps surprisingly, there have not been any rigorous frequency-explicit bounds proved on the error between the solution of this model problem and the true scattering solution, despite the decades-long interest in absorbing boundary conditions.

This talk presents sharp bounds on this error, showing how the error depends on the frequency, the shape of the artificial boundary, and (when the impedance condition is replaced by a more-general absorbing boundary condition) the order of the absorbing boundary condition.

Plenary Lecture **Thursday, 9:00–10:00, Zoom Room 00, link tba**

Directional H2-matrices for lossy Helmholtz problems

Steffen Börm[1], **María López-Fernández**[2,*], **Stefan Sauter**[3]

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The sparse approximation of high-frequency Helmholtz-type integral operators has many important physical applications such as problems in wave propagation and wave scattering. The discrete system matrices are huge and densely populated and hence their sparse approximation is of outstanding importance. We generalize the directional H2-matrix techniques from the "pure" Helmholtz operator, with imaginary frequency, to general complex frequencies with a positive real part. In this case, the fundamental solution decreases exponentially for large arguments. We develop a new admissible condition which contains the real part of the frequency in an explicit way and introduce the approximation of the integral kernel function on admissible blocks in terms of frequency-dependent directional expansion functions. We develop an error analysis which is explicit with respect to the expansion order and with respect to the real and the imaginary parts of the frequency. This allows us to choose the variable expansion order in a quasi-optimal way. The complexity analysis shows how higher values of the real part of the frequency reduce the complexity. In certain cases, it even turns out that the discrete matrix can be replaced by its near field part. Numerical experiments illustrate the sharpness of the derived estimates and the efficiency of our sparse approximation.

References

[1] S. Börm, M. López-Fernández and S. Sauter, Directional H2-matrices for lossy Helmholtz problems, *IMA Journal of Numerical Analysis* **41** (2021), pp. 2896–2935.

Plenary Lecture **Thursday, 14:00–15:00, Zoom Room 00, link tba**

On the nonlinear stability of shear flows and vortices

Alexandru Ionescu[1,*]

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In this lecture we will present some of the main result in our recent work (joint with Hao Jia) on the asymptotic stability of shear flows and vortices among solutions of the Euler equations in two dimensions.

More precisely, we will discuss a theorem on the nonlinear asymptotic stability of a large class of shear flows $(b(y), 0)$ in the finite channel $\mathbb{T} \times [0, 1]$, defined by strictly increasing Gevrey smooth functions b and a theorem on the nonlinear asymptotic stability of point vortex solutions of the Euler equation in $\mathbb{R}^2$.

Plenary Lecture **Friday, 9:00–10:00, Zoom Room 00, link tba**

Interplay between imaging algorithms and the analysis of interior transmission problems

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A large progress has been made in the last two decades on the analysis of so-called sampling methods (the linear sampling method, the factorization method, the generalized linear sampling method, etc...) which offered an original way to handle the imaging problem for inverse scattering problem without the need for any linearization assumption nor a forward solver. Beyond its practical aspect, this class of methods triggered interests for the theoretical analysis of an associated boundary value problem, the so-called interior transmission problem (ITP) and its associated (non linear and non self adjoint) spectral problem. The latter also revealed an interesting aspect of the duality between invisibility and resonance for penetrable media [1]. After a rapid survey of these elements, I would like to point out, through two examples, how new algorithms adapted to challenging configurations have been proposed exploiting this close relation with the ITP. The first example is the imaging of dense crack network. Although the problem seems to have nothing to do with the notion of transmission eigenvalues, the latter helped the design of an indicator function for the crack density, providing better results than classical approaches [2]. The second example is differential imaging. Monitoring a media through successive measurement campaigns offers the possibility to image newly born defects. The design of an indicator function capable of revealing these additional defects relies on the comparison between solutions to the interior transmission problem. Application of this procedure to cracks imaging in fracture elastic media has been applied in [3]. In my talk I will rather present a closely related problematic which is the imaging of defects in a periodic media, assuming that the periodic structure is not known a priori. For this case a single measurement campaign is needed. We show how to build an indicator function for the defect independently from the any prior reconstruction of the background. In an earlier work, we gave justification of this procedure assuming that the defect also have some (larger) periodicity scale [4]. We take the opportunity of this talk to present some recent elements of the analysis that helped removing this assumption.

References

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[4] F. Cakoni, H. Haddar, T.-P. Nguyen, New interior transmission problem applied to a single Floquet–Bloch mode imaging of local perturbations in periodic media, *Inverse Problems 35*, 1, 2019, p. 015009,

Plenary Lecture	Friday, 10:00–11:00, Zoom Room 00, link tba

Dynamical low-rank approximation for the treatment of kinetic Alfvén waves

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Many phenomena in both plasma physics and radiative transfer require a kinetic description. The corresponding model is an up to six-dimensional hyperbolic partial differential equations. The high-dimensional phase space, in particular, poses significant challenges for any numerical method. The associated unfavorable scaling (n^6) of the degrees of freedom with the number of grid points per direction (n) is often referred to as the curse of dimensionality. In addition, these kinetic equations have a number of specificities (in particular, small scale oscillations) that renders many of the well-known complexity reduction techniques (e.g. sparse grids) ineffective.

In this talk, we will employ a complexity reduction technique based on a low-rank approximation. This approach allows us to resolve small-scale oscillations while still drastically reducing the number of degrees of freedom required to obtain an accurate simulation. In order to effectively use these methods requires algorithmic improvements (e.g. to conserve mass and energy), efficient parallelization (to run dynamical low-rank schemes on supercomputers), and a better understanding of when these methods work well and how to choose the rank in practice. We will touch upon all of these aspects in this talk and provide some examples where dynamical low-rank algorithms can be used to solve interesting physics problems (such as kinetic Alfvén waves or plasma echos).

Plenary Lecture	Friday, 11:30–12:30, Zoom Room 00, link tba

The Benjamin–Ono equation with periodic boundary conditions

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This is a survey talk about recent results on the dynamics of the Benjamin–Ono equation on the torus, which are consequences of the construction of a nonlinear Fourier transform associated to the integrable structure. Among these results are sharp wellposedness of the initial value problem in Sobolev spaces, almost–periodicity of the trajectories, stability of multi–solitons, existence of non–smooth periodic solutions in the time variable, and high frequency approximation of solutions in connection with a gauge transformation introduced by T. Tao.

Minisymposia

List of Minisymposia

MS01 **Monday, 11:45–12:10, Zoom Room 01, link tba**

Computational and Experimental Challenges of 3D Ultrasound Tomography

Felix Lucka[1,*], **Mailyn Pérez-Liva**[2], **Bradley E. Treeby**[3], **Ben T. Cox**[3]

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New high-resolution, three-dimensional imaging techniques are being developed that probe the breast without delivering harmful radiation. In particular photo-acoustic tomography (PAT) and ultrasound tomography (UST) promise to give access to high-quality images of tissue parameters with important diagnostic value. However, the involved image reconstruction problems are very challenging from an experimental, mathematical and computational perspective. In this talk, we want to illustrate some of the UST image reconstruction challenges for the example of a prototype scanner for combined PAT and USCT that is currently tested in a clinical feasibility study. In particular, we will discuss Full Waveform Inversion (FWI) approaches for speed-of-sound imaging with UST. To obtain high-quality 3D images with 0.5mm resolution or less, PDEs modeling ultrasound propagation through domains with different material properties needs to be solved repeatedly and to a high accuracy. After reviewing our current results, we will discuss future developments that will broaden the range of clinical scenarios in which these techniques are viable.

MS01 **Monday, 12:15–12:40, Zoom Room 01, link tba**

Ray-based inversion accounting for scattering for reconstruction of the sound speed in weakly heterogeneous media

Ashkan Javaherian[1,*], **Ben Cox**[1]

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A novel approach for solving the inverse problem of the reconstruction of the sound speed of the interior of an object from the ultrasonic measurements made on its boundary is proposed [1]. First, a ray approximation to the heterogeneous Green's function which models the propagation of acoustic waves in the frequency domain is introduced and numerically compared to an open-source wave solver (k-Wave) [2] for a smooth refractive index distribution. The proposed ray-based model can account for refraction by using bent rays, geometric spreading through Green's law, and arbitrary absorption and dispersion, for example frequency power-law absorption described by Szabo's model. More importantly, we describe how the singly scattered waves can be incorporated into the image reconstruction of the sound speed via embedding our forward model in a ray-born inversion framework applied step-wise from low to high frequencies.

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References

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[2] http://www.k-wave.org/.

MS01 **Monday, 12:45–13:10, Zoom Room 01, link tba**

Breast imaging with 3D Ultrasound Tomography at KIT: system setup(s), challenges and potential future perspectives

Torsten Hopp[1,*], **Michael Zapf**[1], **Hartmut Gemmeke**[1], **Nicole V. Ruiter**[1]

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Ultrasound Tomography (USCT) is a promising new imaging modality for breast imaging. At Karlsruhe Institute of Technology (KIT) we are developing a full 3D Ultrasound Tomography (USCT) system, which surrounds the breast with approximately 2000 ultrasound transducers and thereby enables isotropic 3D imaging and very fast data acquisition. While being able to provide these unique features, setting up such system(s) and reconstructing high resolution 3D images poses a considerable challenge. In this talk we will address the solutions we developed in recent years in order to manufacture the world's first and updated second prototype for clinical trials with 3D USCT. This will cover on the one hand the system aspects such as mechanical challenges, transducer development and fast data acquisition and on the other hand the development and acceleration of image reconstruction algorithms for fast transmission and reflection imaging. On all aspects we will try to give an outlook on future perspectives in order to make 3D USCT a viable tool in clinical routine of breast cancer diagnosis and screening.

MS01 **Monday, 16:30–16:55, Zoom Room 01, link tba**

Ultrasound matrix imaging for aberration quantification and correction

William Lambert[1,*], **Mathias Fink**[2], **Alexandre Aubry**[2]

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Ultrasound imaging assumes that the medium under investigation is homogeneous, with a constant speed of sound. Although this assumption is required for real-time imaging, it may not be valid for some configurations where long-scale fluctuations of the speed of sound give rise to various distortions of the incident and back-scattered wave-fronts.

In this work, we apply a matrix approach of aberration quantification [1] and correction [2] to the *in-vivo* case of human calf imaging. By splitting the location of the transmit and receive focal spot, a focused reflection matrix $\mathbf{R_F}$ is built between virtual transducers at each depth. While the main diagonal of $\mathbf{R_F}$ yields the conventional ultrasound image, its off-diagonal elements measure the cross-talk between the virtual transducers. First, this information proves to be valuable for a quantitative assessment of the focusing quality. Spatial Fourier transforms of $\mathbf{R_F}$ are then successively performed along its rows and columns to analyze the distortion of the corresponding wave-fronts and retrieve the aberration phase laws both in transmit and receive, thereby providing a full-field image of the medium with diffraction-limited resolution.

References

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[2] W. Lambert, L. Cobus, T. Frappart, M. Fink, A. Aubry, *Proc. Natl. Acad. Sci.* 117 (26), 2020

MS01 **Monday, 17:30–17:55, Zoom Room 01, link tba**

The Story of Ultrasound Tomography: A Decades Long Journey from the Bench to the Clinic

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Ultrasound tomography (UST) is a new imaging modality that implements solutions of the acoustic wave equation to deliver 3D imaging of various tissue properties in support of breast cancer imaging. An overview of the technique is presented, along with a history of its development, including clinical and regulatory milestones spanning almost 20 years. A series of studies was used to test the efficacy of UST in a clinical setting, culminating in a recently completed, multi-center, national trial aimed at obtaining pre-market approval from the FDA for breast cancer screening. The results from these studies, relating to breast cancer screening, diagnostics, risk assessment and treatment monitoring are presented.

The challenges of translating UST from bench to bedside in a regulatory setting and the associated commercialization of the technology are described. Finally, future prospects for applications of UST and potential local collaborations are also discussed.

MS01 **Monday, 18:00–18:25, Zoom Room 01, link tba**

Parametrix for the inverse source problem of thermoacoustic tomography with reduced data

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We consider the inverse source problem of thermo- and photoacoustic tomography, with data registered on an open surface partially surrounding the source of acoustic waves. We present two efficient non-iterative methods for solving this problem.

The first approach [1] is valid if the speed of sound is constant. It is based on solving the exterior Dirichlet problem and performing the exterior Radon transform of the solution.

The second procedure [2] modifies the crudely approximate, time-reversed solution by two Hilbert transforms, one in time and one in a certain spatial variable. This techniques works for a smooth known speed of sound, subject to an additional geometric condition.

Both techniques produce microlocally accurate approximations to the sought initial condition. In certain geometries these methods can be implemented as fast algorithms.

References

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MS01 Monday, 17:00–17:25, Zoom Room 01, link tba

Ray tracing approximation to the acoustic propagation in weakly nonlinear regime with applications in HIFU

Matt G. Foster[1,*], **Ben T. Cox**[2], **Bradley E. Treeby**[2], **Marta M. Betcke**[1]

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High Intensity Focused Ultrasound (HIFU) is a therapy that uses ultrasound waves to non-invasively destroy malignant cells inside the human body. The technique works by sending a high-energy beam of ultrasound into the tissue using a focused transducer. Numerically modelling HIFU presents a problem due to nonlinear effects leading to the formation of harmonics of the source frequency. Each significant harmonic requires a finer grid to resolve, rapidly increasing computational complexity. We look to use the weakly non-linear ray theory framework[1] to reduce the nonlinear PDE in $\mathbb{R}^d$ to a set of one dimensional PDEs. We construct rays emanating from the transducer on which we calculate the phase of the waves via the Eikonal equation. In ray coordinates the amplitude can be found by solving the nonlinear transport equation along the ray[2]. This equation can be transformed into the Burger's equation which we then solve and transform back to obtain the amplitude along each ray.

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MS02 Monday, 11:45–12:10, Zoom Room 02, link tba

A Well-Conditioned Weak Coupling of Boundary Element and High-Order Finite Element Methods for Time-Harmonic Electromagnetic Scattering

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In this talk, we present an efficient weak coupling formulation between the boundary element method and the high-order finite element method for solving three-dimensional time-harmonic electromagnetic scattering problems. The approach is based on the use of a non-overlapping domain decomposition method involving quasi-optimal transmission operators. The associated transmission boundary conditions are constructed through a localization process based on complex rational Padé approximants of the nonlocal Magnetic-to-Electric operators [1]. The number of iterations required to solve this weak coupling is only slightly dependent on the frequency and the mesh refinement [2].

References

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MS02 Monday, 12:15–12:40, Zoom Room 02, link tba

FEM–BEM coupling for the Maxwell–LLG system via convolution quadrature

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We consider the Landau-Lifshitz-Gilbert-equation (LLG) on a bounded domain Ω coupled to the Maxwell equations on the whole 3D space.

In [2], we propose a numerical algorithm based on convolution quadrature and show a-priori error bounds in the situation of a sufficiently regular exact solution. To that end, we combine the convergence results for the LLG equation [1] via linearly implicit backward difference formulae and the leapfrog and convolution quadrature coupling for the Maxwell system [3]. The precise method of coupling allows us to reduce the full nonlinear system to a series of linear solves and still achieves the same convergence rates as in the uncoupled cases [1] and [3].

Numerical experiments via FEniCS and Bempp illustrate the theoretical results and demonstrate the applicability of the method.

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MS02 **Monday, 12:45–13:10, Zoom Room 02, link tba**

A new approach to Space-Time Boundary Integral Equations for the Wave Equation

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In this talk we present a new approach to formulate the boundary integral equations for the wave equation. This mathematical formulation allows us to prove that the associated boundary integral operators are continuous and satisfy inf-sup conditions in trace spaces of the same regularity, which are closely related to standard energy spaces with the expected regularity in space and time. This feature is crucial from a numerical perspective, as it provides the foundations to derive sharper error estimates and paves the way to devise efficient adaptive space-time boundary element methods.

MS02 **Monday, 15:00–15:25, Zoom Room 02, link tba**

Galerkin Boundary Element Methods for the Hodge-Laplacian

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We consider boundary value problems for the Euclidean Hodge-Laplacian in three dimensions, $\boldsymbol{\Delta}_{\mathrm{HL}} := \mathbf{curl}\,\mathbf{curl} - \mathbf{grad}\,\mathrm{div}$. Their weak formulations are posed in subspaces of $\boldsymbol{H}(\mathbf{curl}, \Omega) \cap \boldsymbol{H}(\mathrm{div}, \Omega)$, $\Omega \subset \mathbb{R}^3$ a bounded Lipschitz domain. Boundary value problems involving $\boldsymbol{\Delta}_{\mathrm{HL}}$ arise from Maxwell's equations in frequency domain when employing the Lorenz gauge.

Via a representation formula and Calderón identities we derive corresponding first-kind boundary integral equations set in traces spaces of $H^1(\Omega)$, $\boldsymbol{H}(\mathbf{curl}, \Omega)$, and $\boldsymbol{H}(\mathrm{div}, \Omega)$. They give rise to saddle-point variational formulations and feature kernels whose dimensions are linked to fundamental topological invariants of Ω.

Kernels of the same dimensions also arise for the linear systems generated by low-order conforming Galerkin boundary element (BE) discretization. On their complements, we can prove stability of the discretized problems, nevertheless. We prove that discretization does not affect the dimensions of the kernels and also illustrate this fact by numerical tests.

References

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MS02 **Monday, 15:30–15:55, Zoom Room 02, link tba**

Subdomain coupling by means of layer potentials in Optimised Schwarz Methods

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The Optimized Schwarz Method (OSM) is a well established domain decomposition (DDM) strategy for solving frequency domain wave propagation problems such as Helmholtz or Maxwell equations [1]. We will present a new variant of this method [2] where transmission conditions and the coupling between subdomains is based on layer potentials. This new approach, which can cope with the presence of so-called cross points, leads to strongly coercive formulations. We shall discuss in detail estimates on the coercivity constant, in particular with respect to frequency, and explain how various boundary conditions (Dirichlet, Neumann, Robin) can be taken into account at the external boundary of the computational domain.

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MS02 **Monday, 16:30–16:55, Zoom Room 02, link tba**

An OSRC Preconditioner for the EFIE

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The Electric Field Integral Equation (EFIE) is commonly used to solve high-frequency electromagnetic scattering problems. However, the EFIE being a First Kind Fredholm operator, needs a regulariser in order to use iterative solvers [1]. A regulariser alternative is the exact Magnetic-to-Electric (MtE) operator, which has the disadvantage of being as expensive as solving the EFIE. However, the authors in [2] have developed a local surface approximation of the MtE for time-harmonic Maxwell's equations that can be efficiently evaluated through the solution of sparse linear systems. In this research we demonstrate the preconditioning properties of the approximate MtE operator for the EFIE using a Bempp implementation and show a number of numerical comparisons against other preconditioning techniques like the Calderón Preconditioner.

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MS02 Monday, 17:00–17:25, Zoom Room 02, link tba

On the use of oblique quasi-Helmholtz projectors for regularizing the electric flux volume integral equations

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Volume integral equations are highly suited for analysing inhomogeneous and anisotropic objects, which are fundamental requirements for high fidelity bioelectromagnetic modeling. However, biomedical applications require the modeling of high-contrast human tissues at low frequencies, a scenario in which standard volume integral equations suffer from ill-conditioning and loss of accuracy. The preconditioned electric flux volume integral equation (D-VIE) presented in this talk is free from these numerical limitations. It is composed of volume quasi-Helmholtz projectors that enable correction of the mismatch between the solenoidal and non-solenoidal parts of the discretized D-VIE. A particular aspect of these new projectors, which differentiates them from standard quasi-Helmholtz strategies and makes them particularly suitable for inhomogeneous and anisotropic scenarios, is their obliqueness. In this talk we will introduce the new equation together with the theoretical framework justifying its performance. Numerical results in real case medical scenarios illustrating the impact of the new formulation for the medical practice are also included.

References

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MS03 Wednesday, 11:30–11:55, Zoom Room 01, link tba

SPDEs driven by non-gaussian fractional processes

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We study SPDEs in a large variety of most commonly used function Banach spaces, e.g. in Lebesgue spaces, Sobolev spaces, or, more generally, Besov and Triebel-Lizorkin spaces. The equations are driven by additive cylindrical non-gaussian processes including e.g. fractionally filtered generalized Hermite processes. Technically, we develop a stochastic integration and apply it to stochastic convolutions.

MS03 Wednesday, 12:00–12:25, Zoom Room 01, link tba

Nonlinear PDE models with stochastic fractional perturbation

Aurelien Deya[1]
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We consider the following quadratic SPDE model:

$$\mathcal{L}u = u^2 + \dot{B}, \quad t \in [0,T],\, x \in \mathbb{R}^d, \tag{1}$$

where $\dot{B}$ is a stochastic noise, and $\mathcal{L}$ can be either: (i) the heat operator $\mathcal{L}u = \partial_t u - \Delta u$; (ii) the wave operator $\mathcal{L}u = \partial_t^2 u - \Delta u$; (iii) the Schrödinger operator $\mathcal{L}u = \imath\partial_t u - \Delta u$.

The dynamics can thus be seen, on the one hand, as the most basic stochastic perturbation of the standard nonlinear PDE $\mathcal{L}u = u^2$, and on the other hand as the most elementary nonlinear extension of the standard SPDE $\mathcal{L}u = \dot{B}$.

Our objective is to study the influence of the roughness of $\dot{B}$ on the equation, and to this end, we rely on the great flexibility offered by the fractional noise model $\dot{B} := \frac{\partial^{d+1} B}{\partial t \partial x_1 \cdots \partial x_d}$, where B is a space-time fractional Brownian field of indexes $H_0, \ldots, H_d \in (0,1)^{d+1}$.

We are especially interested in "rough" situations where the equation can no longer be treated in a space of functions, but only in a space of general distributions.

References

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MS03 **Wednesday, 12:30–12:55, Zoom Room 01, link tba**

Subsonic limit for a stochastic Zakharov system

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The Zakharov system is an asymptotic model for the dynamics of the interaction between Langmuir waves, that describe rapid oscillations of the electron density, and ion sound waves, in a ionized plasma (like the ionosphere). The system couples a Schroedinger equation for the complex envelop of the electric field, and a wave equation for the deviation of the ion density from the equilibrium.

We will consider in this talk a Zakharov system where the ion density deviation is perturbed by a white noise in time correlated in space, and weak damping. Such a model is natural when taking into account external fluctuations in the ion density. It writes

$$\begin{cases} i\partial_t u = -\Delta u + nu \\ \frac{1}{c^2}\partial_t^2 n + \frac{\alpha}{c}\partial_t n = \Delta(n+|u|^2) + \phi\dot{W}, \end{cases}$$

where u represents the complex envelop of the electric field and n the deviation of the ion density, ϕ is a regularizing operator and $\dot{W}$ a space-time white noise. We restrict ourselves to space dimension one, and study the subsonic limit when the ion sound speed c goes to infinity. We prove the (strong) convergence to a multiplicative stochastic nonlinear Schroedinger equation. Note that in this limit the evolution of the energy becomes singular and the scaling is a diffusion-approximation regime, requiring the use a predictor-corrector method. Asymptotic preserving schemes are also discussed.

MS03 **Wednesday, 15:00–15:25, Zoom Room 01, link tba**

The Smoluchowski-Kramers approximation for SPDEs with variable damping

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We study the validity of a Smoluchowski-Kramers approximation for a class of wave equations in a bounded domain of $\mathbb{R}^n$ subject to a state-dependent damping and perturbed by a multiplicative noise. We prove that in the small mass limit the solution converges to the solution of a stochastic quasilinear parabolic equation where a noise-induced extra drift is created. Based on a joint work with Guangyu Xi.

MS03 Wednesday, 15:30–15:55, Zoom Room 01, link tba

Invariant measures for a stochastic nonlinear and damped 2D Schrödinger equation

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We consider a two-dimensional stochastic nonlinear defocusing Schrödinger equation with zero-order linear damping, where the stochastic forcing term is given by a combination of a linear multiplicative noise in Stratonovich form and a nonlinear noise in Itô form. We work at the same time on compact Riemannian manifolds without boundary and on compact smooth domains with either Dirichlet or Neumann boundary conditions. We construct a martingale solution using a modified Faedo-Galerkin's method, then by means of suitable Strichartz estimates we show the pathwise uniqueness of solutions. Finally, we prove the existence of invariant measures by means of a version of the Krylov-Bogoliubov method, which involves the weak topology, as proposed by Maslowski and Seidler. Some remarks on the uniqueness in a particular case are provided as well.
The talk is based on a joint work with B. Ferrario and Z. Brzeźniak.

MS03 Wednesday, 16:30–16:55, Zoom Room 01, link tba

2D Euler equations with transport noise: bounded and measure-valued vorticity

Zdzislaw Brzezniak[1], **Michele Coghi**[2], **Franco Flandoli**[3], **Mario Maurelli**[4,*]

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We consider the incompressible Euler equations in two dimensions perturbed by transport noise (on the vorticity). This choice of noise preserves the relation with the characteristics of the associated flow and hence it preserves the hyperbolic nature of the equation; it also preserves the enstrophy, as in the deterministic case. We show well-posedness among bounded vorticity solutions and existence among nonnegative H^{-1} vorticity solutions, thus generalizing to the stochastic case classical results by Yudovich and Delort. If time allows, we also show approximation of bounded solutions via a system of interacting vortices. Based on [1,2,3].

References

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MS03 **Wednesday, 17:00–17:25, Zoom Room 01, link tba**

Invariant Gibbs dynamics for the hyperbolic (wave) sine-Gordon equation

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In this talk, I will talk about the stochastic hyperbolic partial differential equation (SPDE); the sine-Gordon equation arises in the context of Yukawa gases. More precisely, we consider stochastic damped wave equation with sin nonlinearity on the two-dimensional torus. We construct the corresponding Gibbs measure and the global solution to this equation. Finally, we show the invariance of the Gibbs measure (the sine-Gordon measure) under the resulting dynamics by applying the Bourgain's invariant measure argument. This is a series of joint works with Tadahiro Oh (Edinburgh) Tristan Robert (Nancy), Philippe Sosoe (Cornell).

MS03 **Wednesday, 17:30–17:55, Zoom Room 01, link tba**

Stochastic geometric wave equation with rough data driven by a fractional Brownian sheet

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In this talk we will be concerned with the well-posedness theory for geometric wave equations, perturbed by a fractional Brownian sheet, on one dimensional Minkowski space $\mathbb{R}^{1+1}$ when the target manifold M is a compact Riemannian manifold and the initial data is rough in the sense that it has lower regularity than energy space i.e. $H^1 \times L^2$. Here, to achieve the existence and the uniqueness of a local solution we extend the theory of pathwise stochastic integrals in Besov spaces to two dimensional case. The global theory for this problem is the subject of ongoing work.

MS03 **Wednesday, 18:00–18:25, Zoom Room 01, link tba**

High-speed soliton ejection generated from the scattering of bright solitons by modulated reflectionless potential wells

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We investigate numerically and theoretically the conditions leading to soliton ejection stimulated through the scattering of bright solitons by modulated reflectionless potential wells. Such potential wells allow for the possibility of controlled ejection of solitons with significantly high speeds. At the outset, we describe the scattering setup and characterise the soliton ejection in terms of the different parameters of the system. Then, we formulate a theoretical model revealing the underlying physics of soliton ejection. The model is based on energy and norm exchange between the incident soliton and a stable trapped mode corresponding to an exact solution of the governing nonlinear Schrödinger equation. Remarkably, stationary solitons can lead to high-speed soliton ejection where part of the nonlinear interaction energy transforms to translational kinetic energy of the ejected soliton. Our investigation shows that soliton ejection always occurs whenever the incident soliton norm is greater than that of the trapped mode while their energy is almost the same. Once the incident soliton is trapped, the excess in norm turns to an ejected soliton in addition to a small amount of radiation that share translational kinetic energy. We found that higher ejection speeds are obtained with multi-node trapped modes that have larger binding energy. Simultaneous two-soliton ejection has been also induced by two solitons scattering with the potential from both of its sides. An ejection speed almost twice as that of single soliton ejection was obtained. Ejection outcome and ejection speed turn out to be sensitive to the relative phase between the two incoming solitons, which suggests a tool for soliton phase interferometry.

MS04 **Wednesday, 16:30–16:55, Zoom Room 06, link tba**

Exponential accuracy for the method of perfectly matched layers

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The method of perfectly matched layers (PML) is used to compute solutions to time harmonic wave scattering problems. This method can be seen as a numerical adaptation of the method of complex scaling in which the infinite exterior region is replaced by a Dirichlet condition on a finite region. In this talk, we recall the methods of complex scaling and PML and study the error produced by replacing the genuine scattering problem with the PML truncation. We show that this error decays exponentially as a function of the scaling angle, the scaling width, and the frequency.

MS04 **Wednesday, 17:00–17:25, Zoom Room 06, link tba**

Multitrace boundary integral formulations for elasticity transmission problems

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The accurate numerical modeling of highly oscillatory elastic phenomena is of great interest in medical or industrial applications. It is a timely research field with a large panel of applications, from seismology and non-destructive testing to medical ultrasound.

In this talk, we focus on the time-harmonic scattering of elastic waves by a bounded penetrable and composite object. We present multitrace boundary integral formulations for solving such transmission problems. Both theoretical and numerical aspects are addressed.

References

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MS04 **Wednesday, 17:30–17:55, Zoom Room 06, link tba**

Galerkin boundary element methods for high-frequency sound-hard scattering problems

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We propose two Galerkin boundary element methods for the solution of high-frequency sound-soft scattering problems in the exterior of smooth, compact, strictly convex obstacles [1]. As in their sound-soft counterparts [2,3], these methods demand an incremental increase of $\mathcal{O}(k^\epsilon)$, for any $\epsilon > 0$, in the number of degrees of freedom to ensure frequency independent precisions with increasing wavenumber k. Numerical results validate our theoretical findings. Moreover, from a theoretical point of view, for any fixed number of degrees of freedom the accuracy increases with increasing k provided sufficiently many terms in the asymptotic expansion are incorporated into the integral equation formulation.

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MS04 **Wednesday, 18:00–18:25, Zoom Room 06, link tba**

Initial value representation for solutions of systems of Schrödinger equations

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Initial value representations based on the use of gaussian wave packets have been introduced by theoretical chemists at the end of the 70s. They have only been studied by mathematicians in the beginning of the 2000s, for scalar equations, and both form theoretical and numerical points of view. We will describe here what is known for systems, with a special concern for those presenting eigenvalue crossings (also called conical intersections).

MS04 **Thursday, 10:30–10:55, Zoom Room 06, link tba**

Filament structure in random waves

Melissa Tacy[1,*]
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Numerical studies of random plane waves, functions

$$u = \sum_j c_j e^{\frac{i}{h}\langle x, \xi_j \rangle}$$

where the coefficients c_j are chosen "at random", have detected an apparent filament structure. The waves appear enhanced along straight lines. There has been significant difference of opinion as to whether this structure is indeed a failure to equidistribute, numerical artefact or an illusion created by the human desire to see patterns. In this talk I will present some recent results that go some way to answering the question. First we consider the behaviour of a random variable given by $F(x,\xi) = ||u||_{L^2(\gamma_{(x,\xi)})}$ where $\gamma_{(x,\xi)}$ is a unit ray from the point x in direction ξ. We will see that this random variable is uniformly equidistributed. That is, the probability that for any (x,ξ), $F(x,\xi)$ differs from its equidistributed value is small (in fact exponentially small). This result rules out a strong scarring of random waves. However, when we look at the full phase space picture and study a random variable $G(x,\xi) = ||P_{(x,\xi)}u||_{L^2}$ where $P_{(x,\xi)}$ is a semiclassical localiser at Planck scale around (x,ξ) we do see a failure to equidistribute. This suggests that the observed filament structure is a configuration space reflection of the phase space concentrations.

MS04 **Thursday, 11:00–11:25, Zoom Room 06, link tba**

Sharp bounds on Helmholtz impedance-to-impedance maps and application to overlapping domain decomposition

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This talk describes sharp bounds on certain impedance-to-impedance maps for the Helmholtz equation at high-frequency, with these bounds proved using semiclassical defect measures. Our main motivation for studying these maps is in the analysis of overlapping domain-decomposition methods for the high-frequency Helmholtz equation. The recent paper [1] studied a parallel overlapping Schwarz method for the Helmholtz equation with impedance boundary conditions on the subdomains (this method can be thought of as the overlapping analogue of the parallel non-overlapping method of Després). This paper related the error-propagation operator for this DD method to certain impedance-to-impedance maps, and thus formulated sufficient conditions for convergence of the DD method in terms of properties of these impedance-to-impedance maps.

References

[1] S. Gong, M.J. Gander, I.G. Graham, D. Lafontaine, E.A. Spence, Convergence of parallel overlapping domain decomposition methods for the Helmholtz equation, *Preprint*, `https://arxiv.org/abs/2106.05218`

MS04 Thursday, 11:30–11:55, Zoom Room 06, link tba

A wavelet approach to semiclassical Helmholtz equation

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Solving numerically the Helmholtz equation is a considerable challenge when the wave number k is large. On the one hand, the standard methods (like the Finite Element Method, or the Finite Difference Method) lead to a discrete problem with at least $O(k^d)$ degrees of freedom, which is often excessive. On the other hand, in the past decades, many *wave-based methods*, specially adapted to the Helmholtz equation, have been developed; those give discrete problems with a number of degrees of freedom greatly reduced, but described by matrices that are dense and ill-conditioned.

I will explain an alternative approach, relying on considerations from wavelet theory and semiclassical analysis. The discrete problem obtained from this approach has $o(k^d)$ degrees of freedom, but remains well-conditioned.

MS04 Thursday, 15:00–15:25, Zoom Room 06, link tba

Fully frequency-independent Hybrid Numerical Asymptotic Boundary Element Methods

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For certain geometries, the *Hybrid Numerical Asymptotic Boundary Element Method* (HNA BEM) can accurately represent solutions of high-frequency scattering problems in a frequency-independent number of DOFs.

In practice, HNA BEMs require the evaluation of highly-oscillatory (and sometimes singular) integrals, which can introduce unwelcome frequency-dependence to a numerical implementation. For the 2D screen problem, this was addressed in [1], via *Numerical Steepest Descent* (NSD). The idea is to deform the domain of integration into the complex plane, onto a contour where the integrand is non-oscillatory and exponentially decaying, hence better-suited to numerical quadrature. The resulting HNA BEM has frequency-independent cost.

In this talk I will introduce HNA BEM and NSD, with the aid of some numerical examples, initially focusing on the 2D screen problem. I will then consider scattering by a polygon and explain how branch cuts arise in the integrals, making it difficult to choose an optimal contour for NSD. Finally, I will propose some new ideas to address this difficulty.

References

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MS04 Thursday, 15:30–15:55, Zoom Room 06, link tba

On the Imaginary Part of Whispering Gallery Resonances of Transparent Obstacles : an Investigation using Quadruple Precision Computations

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Whispering Gallery resonances appear along the boundary of transparent obstacles subject to some convexity conditions mixing the curvature and the values of the optical index, possibly variable. Such resonances have the form $k = k_\mathrm{r} - ik_\mathrm{i}$ with k_r and k_i positive. When some longitudinal frequency m tends to ∞, k_i tends to 0 super-algebraically fast. By analogy with scattering by analytic potentials, one conjectures an exponential decay of the form $k_\mathrm{i} \simeq e^{-2S_0 m}$ where the positive number S_0 accounts for a tunneling effect [1].

We present computations in rotationally symmetric cases, reducing to various shapes of *discontinuous* potentials [2,3]. The exponential decay of k_i requires high precision calculations in order to resolve its asymptotic behavior, since k_r grows linearly with m. To this end, we use multiple extrapolations of finite difference computations in the Float128 format of `julia`.

References

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MS05 Tuesday, 11:30–11:55, Zoom Room 01, link tba

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Decompositions of high-frequency Helmholtz solutions via functional calculus, and application to the finite element method

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Over the last ten years, results from [1], [2], [3], and [4] decomposing high-frequency Helmholtz solutions into "low"- and "high"-frequency components have had a large impact in the numerical analysis of the Helmholtz equation. These results have been proved for the constant-coefficient Helmholtz equation in either the exterior of a Dirichlet obstacle or an interior domain with an impedance boundary condition.

In this talk, I will present our recent results [5, 6], obtaining analogous decompositions for scattering problems fitting into the very general black-box scattering framework of Sjöstrand-Zworski, thus covering Helmholtz problems with variable coefficients, impenetrable obstacles, and penetrable obstacles all at once. These results allow us to prove new frequency-explicit convergence results for finite-element methods applied to the Helmholtz equation.

References

[1] J. M. Melenk and S. Sauter. Convergence analysis for finite element discretizations of the Helmholtz equation with Dirichlet-to-Neumann boundary conditions. *Math. Comp*, 79(272):1871–1914, 2010.

MS05 Tuesday, 12:00–12:25, Zoom Room 01, link tba

A Hausdorff measure boundary element method for acoustic scattering by fractal screens

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We introduce and analyse a novel discretization method for acoustic scattering by fractal screens. In contrast to previous studies [1], in which a conventional boundary element method (BEM) was applied on a "pre-fractal" approximation of the fractal screen, here we work with BEM basis functions supported on the fractal screen itself, integrating with respect to Hausdorff measure rather than the conventional surface (Lebesgue) measure. Using function space results from [2,3] we prove convergence of our BEM, and obtain convergence rates under natural solution regularity assumptions. We also detail a strategy for the numerical evaluation of the required Hausdorff measure integrals, accompanied by a fully discrete convergence analysis.

References

[1] S. N. Chandler-Wilde, D. P. Hewett, A. Moiola, and J. Besson, Boundary element methods for acoustic scattering by fractal screens, *Numer. Math.*, 147 (2021), pp. 785–837.

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MS05 **Tuesday, 12:30–12:55, Zoom Room 01, link tba**

Acoustic scattering by fractal inhomogeneities: well-posedness and discretisation of volume integral equations

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We study time-harmonic acoustic scattering by an arbitrary compactly supported inhomogeneity in $\mathbb{R}^n$ $(n = 2, 3)$ by reformulating the inhomogeneous Helmholtz equation into the Lippmann-Schwinger equation (LSE). Unlike previous studies [1,2], we make no regularity assumptions on the inhomogeneity, permitting fractal inhomogeneities, and consider a general L^∞ refractive index. Well-posedness is derived in the standard way via Riesz-Fredholm theory with the associated integral operator of the LSE being shown to be compactly perturbed coercive in an appropriate function space setting. This then allows us to use the notion of Mosco convergence to investigate the convergence of discretisations on smoother "pre-fractals" to the solution of the continuous problem on the fractal, as in [3]. We also present numerical results illustrating our theory.

References

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MS05 **Tuesday, 15:00–15:25, Zoom Room 01, link tba**

How to solve numerically the time-harmonic Maxwell equations with one sign-changing coefficient?

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In electromagnetism, in the presence of a negative material surrounded by a classical material, the electric permittivity, and possibly the magnetic permeability, can exhibit a sign-change at the interface. In this setting, the study of electromagnetic phenomena is a challenging topic. We focus on the time-harmonic Maxwell equations in a bounded set of $\mathbb{R}^3$. The abstract framework to solve this model can be found in [2]. However, less is known on the numerical approximation of this model. We study the approximation of the electromagnetic fields by edge finite elements, for the model with one sign-changing coefficient. With the help of T-coercivity for scalar problems, we can prove convergence, and derive optimal a priori error estimates. Numerically, the T-coercivity theory relies on the use of T-conform meshes [3,1], so we study how the choice of the meshes impacts the convergence of the numerical method.

References

[1] A.-S. Bonnet-Ben Dhia, C. Carvalho and P. Ciarlet, Jr., Mesh requirements for the finite element approximation of problems with sign-changing coefficients, *Numer. Math.*, **138** (2018), pp. 801–838.

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MS05 Tuesday, 15:30–15:55, Zoom Room 01, link tba

Rational-based MOR methods for parametric-in-frequency Helmholtz problems with adapted snapshots

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This talk deals with the time-harmonic wave equation, where the parameter is the wavenumber that sweeps a given interval of interest. We are aim at (i) deriving an approximation to the Helmholtz frequency response map *for all the parameter values* ranging in the given interval of interest, and at (ii) identifying the possible *resonances* of the problem.

In principle, those issues could be addressed via the direct numerical evaluation of the frequency response map for the whole range of frequencies. However, this naive approach is prohibitive, because of its affordable computational cost.

This talk presents a Model order reduction (MOR) method that aims at alleviating the computational cost, by producing an approximation to the frequency response (the so-called *surrogate*) that presents the double advantage of being accurate and cheap to evaluate. We look for a *rational* surrogate: the roots of its denominator will be approximation to the resonances of the problem. The construction relies on the computation of a set of *snapshots* (offline information) by means of the adaptive finite element method.

References

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MS05 Tuesday, 16:30–16:55, Zoom Room 01, link tba

Multiscale FEM for light propagation through locally periodic complex photonic structures

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Interference of light in periodic photonic structures (crystals) represents a promising tool for achieving complete control of light propagation. However, this phenomenon is extremely complex and extensive computational modelling is required to understand the physics behind it. These computational models are notoriously difficult to solve, especially for realistically large, but finite, photonic structures, to the extent that they are untractable even on state-of-the-art supercomputers.

Here we propose a multiscale finite element method (MS-FEM) utilizing the local periodicity of the finite crystal to alleviate the computational complexity of the problem. The basic idea is to couple piecewise polynomials, which are defined on a macrogrid that does not resolve single unit cells, with Bloch modes that describe the propagation of light in an infinite crystal. As a result, our method becomes effectively size-robust. This opens new possibilities for research in photonic crystals, previously completely inaccessible.

MS05 Tuesday, 17:00–17:25, Zoom Room 01, link tba

MHM Method for Helmholtz Equation with a Quasi-Periodic Boundary Condition

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The multiscale hybrid-mixed (MHM) method is a two-level finite element method that arises from an hybridization procedure. Multiscale basis functions are locally computed via independent cell problems, and these basis functions are then used in a global problem on a coarse mesh. The MHM method has been adapted recently to solve the Helmholtz equation, and appears as very performant in the case of heterogeneous media using coarse meshes. In this work, we use the MHM method to solve Helmholtz equation with quasi-periodic boundary conditions, which represents a particular case of trapping geometries. We provide frequency-explicit error estimates, that show that the MHM method is more accurate and stable than the standard finite element method. In fact, we find that the lowest-order finite-element method is stable under the condition that $k^3H \leq C$, where k is the wavenumber and H the mesh size, while the MHM is stable under the condition that $k^2H \leq C$ with the same number of degrees of freedom.

In this talk, I will explain what are the key challenges associated with trapping geometries, and present the key arguments showing why the MHM method is more performant in this case. I will also present numerical experiments that highlights the main theoretical findings.

MS05 Tuesday, 17:30–17:55, Zoom Room 01, link tba

Multiscale Scattering in Nonlinear Kerr-Type Media

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Wave propagation in heterogeneous and nonlinear media has arisen growing interest in the last years since corresponding materials can lead to unusual and interesting effects and therefore come with a wide range of applications. An important example for such materials are Kerr-type media, where the intensity of a wave directly influences the refractive index. In the time-harmonic regime, this effect can be modeled with a nonlinear Helmholtz equation. If underlying material coefficients are highly oscillatory on a microscopic scale, the numerical approximation of corresponding solutions can be a delicate task.

We present a multiscale technique that allows one to deal with microscopic coefficients in a nonlinear Helmholtz equation without the need for global fine-scale computations, see also [1]. The method is based on an iterative and adaptive construction of appropriate multiscale spaces based on the multiscale method known as Localized Orthogonal Decomposition, which works under minimal structural assumptions.

References

[1] R. Maier and B. Verfürth, Multiscale scattering in nonlinear Kerr-type media, *ArXiv Preprint*, 2011.09168, 2020.

MS05 **Tuesday, 18:00–18:25, Zoom Room 01, link tba**

Stable Trefftz methods using evanescent plane waves

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Helmholtz solutions are known to be well approximated by a suitable superposition of plane waves $\mathbf{x} \mapsto e^{\imath \mathbf{k}\cdot\mathbf{x}}$, with $\mathbf{k} \in \mathbb{R}^2$, leading to successful Trefftz methods [2]. However, when too many plane waves are used, the computation of the approximation is known to be numerically instable. This is due to the presence of (exponentially) large coefficients in the expansion.

In this work, we show that any Helmholtz solution on a disk can be exactly represented by a continuous superposition of evanescent plane waves (with a wavenumber $\mathbf{k} \in \mathbb{C}^2$ hence exponentially decaying), generalizing the standard Herglotz representation. In addition, we prove the existence of a bicontinuous transform mapping the Helmholtz solution to its Herglotz density, therefore overcoming the instability observed with propagative plane waves. In view of practical implementations, we construct suitable finite sets of evanescent plane waves using some quasi-optimal sampling strategies in a parametric space. Provided one uses sufficient oversampling and regularization [1], the resulting discrete expansions are shown to be both controllably accurate and numerically stable, as supported by numerical evidence.

References

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MS06 **Thursday, 15:00–15:25, Zoom Room 07, link tba**

Photonic and Quantum Design Problems as QCQPs

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We show that a wide range of physical design problems can be transformed to quadratically constrained quadratic programs (QCQPs). This leads to a general framework for identifying bounds, or fundamental limits, to possible response. We show applications of this approach across nanophotonics – where we find bounds for perfect absorbers, analog optical computers, and broadband extinction – as well as to quantum optical control.

MS06 **Thursday, 15:30–15:55, Zoom Room 07, link tba**

Limits on electromagnetic scattering

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Much of the continuing appeal and challenge of electromagnetism stems from the same root: given some desired objective (enhancing radiation from a quantum emitter, the field intensity in a photovoltaic cell, the radiative cross section of an antenna) subject to some physical constraints (material compatibility, fabrication tolerances, or system size) there is currently no method for finding or assessing uniquely best wave solutions. While improvements in nanofabrication and computational methods have driven dramatic progress in expanding the range of achievable optical characteristics, they have also greatly increased design complexity [1]. These developments have led to heightened relevance for the study of fundamental limits on optical response. Here, we review recent progress in our understanding of these limits with special focus on an emerging theoretical framework that combines computational optimization with conservation laws to yield physical limits capturing all relevant wave effects [2]. Results pertaining to canonical electromagnetic problems such as thermal emission, scattering cross sections, Purcell enhancement, and power routing are presented.

References

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MS06 **Thursday, 16:30–16:55, Zoom Room 07, link tba**

The Figure of Merit Matters in Inverse Design

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In traditional engineering-design, carefully-tailored, (often) simplified, physics models, coupled with a few design parameters, are often employed to identify (locally optimal) solutions.

In topology optimization (TO) based inverse design, many restrictions from traditional design are lifted, as powerful numerical tools are used to solve the physics problem, coupled with the use of efficient gradient-based optimization algorithms to solve the design problem.

For wave-propagation problems we discuss that, even if it may seem to be the case, the use of inverse design is not mainly effort lifiting, but rather effort shifting. Obviously the effort is shifted from: constructing simplified design models and/or doing cumbersome trial and error, to: implementing inverse design tools. But, more fundamentally, the effort is shifted towards (re)thinking the measure of a successful design, i.e. carefully selecting the physics model and figures of merit. In order words, designing for what one truly wants to achieve, rather than a simplified surrogate measure, introduced under the constraints of traditional design.

As an example consider the observed improvement of $\approx 350\%$ when designing plasmonic nano-particles for enhanced Raman scattering, obtained by considering the problem as a two step process (excitation+emission) rather a single step process (only excitation/emission) [1].

References

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MS06 **Thursday, 17:00–17:25, Zoom Room 07, link tba**

Maximizing the electromagnetic chirality for metallic nanowires in the visible spectrum

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Any time-harmonic electromagnetic wave can be uniquely decomposed into left and right circularly polarized components. Electromagnetic chirality describes differences in the interaction of scattering objects with these two components. If the scattering behavior of an object with respect to incident waves of one helicity cannot be reproduced with incident fields of the opposite helicity, then the object is said to be electromagnetically chiral (em-chiral), otherwise it is called em-achiral. Em-chirality can be quantified by chirality measures that attain the value 0 for an em-achiral object and the value 1 for a maximally em-chiral object. Maximally em-chiral objects are invisible to fields of one helicity. Scattering objects with large chirality measures at visible frequencies would have interesting applications in the design of metamaterials.

In this talk, we investigate a shape optimization problem, where the goal is to construct thin metallic nanowires that exhibit large measures of em-chirality at a given frequency. We present a gradient based optimization method, which is based on an asymptotic perturbation formula for approximating scattered fields due to thin metallic scattering objects. We show numerical results obtained by this algorithm and discuss the optimized shapes.

This talk is based on joint work with Tilo Arens, Ivan Fernandez-Corbaton, Roland Griesmaier and Carsten Rockstuhl.

MS06 **Thursday, 17:30–17:55, Zoom Room 07, link tba**

Fast calculation of transmission along waveguides supporting propagation of higher order modes

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Simplified methods are favourable for fast modelling of large optical structures, for which a numerical solution to Maxwell's equations is no longer feasible. In this work we present a multi-mode approximation based on [1] to describe the light propagation in curved waveguides over length scales of hundreds of micrometers. This method intends to overcome limitations of the fundamental mode approximation [2] when the waveguides support multiple modes. In spatial regions with a sudden change in the radius of curvature, light is coupled with a notable amplitude to these higher-order modes, which affects the transmission of the fundamental mode. The work has practical applications for the fast design of photonic wire bonds.

References

[1] T. Paul, *Light propagation in optical metamaterials: a bloch modal approach*, Doctoral dissertation, 2012.

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MS06 **Thursday, 18:00–18:25, Zoom Room 07, link tba**

Acoustic Full-Waveform Inversion via Optimal Control

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Full-waveform inversion (FWI) is a recent technique in seismic tomography to reconstruct physical parameters sampled by waves. Compared with other methods relying only on partial waveform information such as travel times or phase velocities, FWI exploits the entire waveform content and iteratively minimizes the nonlinear misfit between synthetic and observed data. In this talk we discuss an optimal control method for acoustic FWI to reconstruct the speed wave parameter entering the PDE model in the coefficient of the second-order time derivative of the acoustic pressure. Based on our first-order and second-order analysis, the method seems to be promising as it could also treat non-smooth reconstruction with sharp interfaces as also confirmed by our numerical tests.

MS07 **Tuesday, 11:30–11:55, Zoom Room 02, link tba**

Optimizers in the Sobolev-curl inequality

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We study a Sobolev-type inequality involving the p-curl operator in $\mathbb{R}^3$. We show that there is a minimizer of the problem, which solves the p-curl-curl equation in the critical case. We present a concentration-compactness principle for the p-curl operator and investigate some symmetric properties of the equation. The problem is motivated by nonlinear Maxwell equations [1] as well as by three dimensional Dirac equations.

References

[1] J. Mederski, A. Szulkin, Sharp constant in the curl inequality and ground states for curl-curl problem with critical exponent, *Arch. Rational Mech. Anal.* **241** (2021), pp. 1815–1842.

[2] J. Mederski, Optimizers in Sobolev-curl inequalities, in preparation.

MS07 **Tuesday, 12:00–12:25, Zoom Room 02, link tba**

Breather Solutions for a Quasilinear $(1+1)$-dimensional Wave Equation

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We consider the (1+1)-dimensional quasilinear wave equation $g(x)w_{tt} - w_{xx} + h(x)(w_t^3)_t = 0$ on $\mathbb{R}\times\mathbb{R}$ which arises in the study of localized electromagnetic waves modeled by Kerr-nonlinear Maxwell equations. We are interested in time-periodic, spatially localized solutions. Here $g \in L^\infty(\mathbb{R})$ is even with $g \not\equiv 0$ and $h(x) = \gamma\,\delta_0(x)$ with $\gamma \in \mathbb{R}\backslash\{0\}$ and δ_0 the delta-distribution supported in 0. We assume that 0 lies in a spectral gap of the operators $L_k = -\frac{d^2}{dx^2} - k^2\omega^2 g$ on $L^2(\mathbb{R})$ for all $k \in 2\mathbb{Z}+1$ together with additional properties of the fundamental set of solutions of L_k. By expanding w into a Fourier series in time we transfer the problem of finding a suitably defined weak solution to finding a minimizer of a functional on a sequence space. The solutions that we have found are exponentially localized in space. Moreover, we show that they can be well approximated by truncating the Fourier series in time. The guiding examples, where all assumptions are fulfilled, are explicitely given step potentials and periodic step potentials g. In these examples we even find infinitely many distinct breathers.

References

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MS07 **Tuesday, 12:30–12:55, Zoom Room 02, link tba**

Exponential integrators for quasilinear hyperbolic evolution equations

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In this talk we propose two exponential integrators of first and second order applied to quasilinear hyperbolic evolution equations. We work in an analytical framework which is an extension of the classical Kato framework and covers quasilinear Maxwell's equations in full space and on a smooth domain as well as a class of quasilinear wave equations.

In contrast to earlier works, we do not assume regularity of the solution but only on the data. From this we deduce a well-posedness result upon which we base our error analysis.

References

[1] B. Dörich and M. Hochbruck. Exponential integrators for quasilinear wave-type equations. CRC 1173 Preprint 2021/12, Karlsruhe Institute of Technology, 2021. URL https://www.waves.kit.edu/downloads/CRC1173_Preprint_2021-12.pdf.

MS07 **Tuesday, 15:00–15:25, Zoom Room 02, link tba**

Rigorous Asymptotic Analysis for Quasilinear 2D Maxwell's Equations with Interface

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We consider Maxwell's Equations in 2D for two Kerr isotropic media with a planar interface. For space-dependent susceptibilities we formally construct an approximative solution with the method of amplitude equations where the envelope is given by a nonlinear Schrödinger equation. By extending an existing well-posedness result with the help of a bootstrapping argument we show the exact approximation properties on a large time-scale analytically.

The construction and numerical calculation of initial values close to the ansatz such that the displacement field is divergence free will also be discussed.

MS07 **Tuesday, 15:30–15:55, Zoom Room 02, link tba**

Determination of initial values with a divergence-free $\mathcal{D}$-field in Maxwell equations: a quasilinear transmission problem

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Maxwell equations in the free space with no charges require that initial data have a divergence free $\mathcal{D}$-field. If $\mathcal{D}$ nonlinearly depends on $\mathcal{E}$, as e.g. for Kerr materials, the equation $\nabla\cdot\mathcal{D}(\mathcal{E})$ has to be satisfied. In many applications, for instance in the modelling of wave-packets, the approximative asymptotic ansatz of the electric field $\mathcal{E}$ fulfils this condition only up to a small residual. The aim is hence to enforce it by means of a (small) correction of the ansatz.

We consider the problem in a rather general framework. We suppose that the dependence of $\mathcal{D}$ on $\mathcal{E}$ is polynomial, and hence the equation is quasilinear. We also allow for the medium to consist of two different materials, so that a transmission problem across an interface is produced. We show the existence of the correction term, together with a-priori estimates also for its derivatives. Finally we apply such results in the original wave-packet framework, to show that the correction term is indeed asymptotically smaller than the original ansatz.

MS07 **Tuesday, 16:30–16:55, Zoom Room 02, link tba**

Maxwell equations with rough coefficients

Robert Schippa[1,*], **Roland Schnaubelt**[1], **Rainer Mandel**[1]

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In this talk quasilinear Maxwell equations in two and three dimensions are considered. In two dimensions, we show Strichartz estimates for symmetric, positive-definite permittivity $\varepsilon \in C^s(\mathbb{R} \times \mathbb{R}^2; \mathbb{R}^{2\times 2})$ and permeability $\mu = 1$ with $0 < s \leq 2$ [1,2]. In three dimensions we have partial results under symmetry assumptions on ε.

The proof hinges on phase space analysis, making use of the FBI transform. The evolution is conjugated in phase space to half-wave equations, some of which are degenerate. We quantify the degenerate evolution with the charges.

Furthermore, we show that this diagonalization also gives L^p-L^q-resolvent estimates for time-harmonic Maxwell equations [3,4] with constant coefficients.

References

[1] R. Schippa and R. Schnaubelt, On quasilinear Maxwell equations in two dimensions, *arXiv e-prints* (05/2021), arXiv:2105.06146.

[2] R. Schippa, Well-posedness for Maxwell equations with Kerr nonlinearity in three dimensions via Strichartz estimates, *arXiv e-prints* (08/2021), arXiv:2108.07691.

[3] R. Schippa, Resolvent estimates for time-harmonic Maxwell's equations in the partially anisotropic case, *arXiv e-prints* (03/2021), arXiv:2103.16951.

[4] R. Mandel, R. Schippa, Time-harmonic solutions for Maxwell's equations in anisotropic media and Bochner-Riesz estimates with negative index for non-elliptic surfaces, *arXiv e-prints* (03/2021), arXiv:2103.17176.

MS07 **Tuesday, 17:00–17:25, Zoom Room 02, link tba**

Maxwell Obstacle Problems in Electromagnetic Shielding

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Electromagnetic (EM) shielding is a physical process of canceling or redirecting EM waves by means of obstacles made of conducting or magnetic materials. From the mathematical point of view, EM shielding falls into the class of obstacle problems: In the free region, the EM waves satisfy the Maxwell equations, whereas in the shielded area pointwise constraints are applied to the fields. This talk discusses recent well-posedness results on this research direction.

MS07 **Tuesday, 17:30–17:55, Zoom Room 02, link tba**

Quasilinear Maxwell Variational Inequality in Ferromagnetic Shielding: Optimal Control

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We aim to discuss the optimal control of a quasilinear variational inequality of the first kind in magnetostatics where first order differential constraints are imposed. While the existence of an optimal solution is achieved by well-known techniques, the derivation of necessary optimality conditions requires the introduction of a novel penalization and a careful analysis thereof. Here, as a result of the first order constraint, the main difficulty is to characterize the Lagrangian multiplier which is associated with the adjoint equation.
Specific monotone quasilinearities in the leading **curl curl**-term are used to model electromagnetic wave phenomena when ferromagnetic materials are in play. For the equation case, the corresponding analysis and control is well understood. Thus, the extension to a quasilinear obstacle problem is of interest to the physics of ferromagnetic shielding and hence deserves a detailed investigation.

MS07 **Tuesday, 18:00–18:25, Zoom Room 02, link tba**

Boundary Stabilization of Quasilinear Maxwell Equations

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We investigate an initial-boundary value problem for a quasilinear nonhomogeneous, anisotropic Maxwell system subject to an absorbing boundary condition of Silver & Müller type in a smooth, bounded, strictly star-shaped domain of $\mathbb{R}^3$. Imposing usual smallness assumptions in addition to standard regularity and compatibility conditions, a nonlinear stabilizability inequality is obtained by showing nonlinear dissipativity and observability-like estimates enhanced by an intricate regularity analysis. With the stabilizability inequality at hand, the classic nonlinear barrier method is employed to prove that small initial data admit unique classical solutions that exist globally and decay to zero at an exponential rate. Our approach is based on a recently established local well-posedness theory in a class of $\mathcal{H}^3$-valued functions.

References

[1] M. Pokojovy, R. Schnaubelt. Boundary stabilization of quasilinear Maxwell equations, *Journal of Differential Equations* **268**(2) (2020), pp. 784–812.

[2] R. Schnaubelt, M. Spitz, Local wellposedness of quasilinear Maxwell equations with absorbing boundary conditions, *Evolution Equations & Control Theory* **10**(1) (2021), pp. 155–198.

MS07 **Wednesday, 11:30–11:55, Zoom Room 02, link tba**

Interchanging space and time in nonlinear optics modeling and dispersion management models

Reika Fukuizumi[1], **Guido Schneider**[2,*]

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Interchanging the role of space and time is widely used in nonlinear optics for modeling the evolution of light pulses in glass fibers. A phenomenological model for the mathematical description of light pulses in glass fibers with a periodic structure in this set-up is the so called dispersion management equation. It is the purpose of this paper to answer the question whether the dispersion management equation or other modulation equations are more than phenomenological mod- els in this situation. Using Floquet theory we prove that in case of comparable wave lengths of the light and of the fiber periodicity the NLS equation and NLS like modulation equations with constant coefficients can be derived and justified through error estimates under the assumption that rather strong stability and non-resonance conditions hold. This is the first NLS approximation result documented for time-periodic dispersive systems. We explain that the failure of these conditions allows us to prove that these modulation equations in general make wrong predictions. The reasons for this failure and the behavior of the system for a fiber periodicity much larger than the wave length of light shows that interchanging the role of space and time for glass fibers with a periodic structure is at least a questionable modelling.

MS07 Wednesday, 12:00–12:25, Zoom Room 02, link tba

Stability and asymptotic properties of a linearized hydrodynamic medium model for dispersive media in nanophotonics

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In this talk, we will present a linearized hydrodynamical model describing the response of nanometric dispersive metallic materials illuminated by optical light waves, a situation occurring in nanoplasmonics. The model corresponds to the coupling between the Maxwell system and a PDE describing the evolution of the polarization current of the electrons in the metal. Fiirst by using semigroup theory its well posedness will be shown. Then using a frequency approach, polynomial (and optimal) stability results for different boundary conditions will be presented.

MS07 Wednesday, 12:30–12:55, Zoom Room 02, link tba

Analysis of Dispersive Electromagnetic Surface Waves in Weighted Spaces

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When modelling electromagnetic waves at interface geometries, like surface plasmon polaritons, in general one encounters retardation effects and nonlinearities. These are incorporated into Maxwell's equations, which we formulate as an evolutionary system in weighted function spaces.

In this framework we study decay properties of solutions in time and space. Exponential stability of small solutions may provide a novel tool for the approximation of nonlinear surface wave packets on long timescales.

MS08 Tuesday, 15:00–15:25, Zoom Room 08, link tba

Floating objects interacting with noninear dispersive waves

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The motion of the free surface of a fluid can be described either by the water waves equations or by simpler asymptotic models. In order to allow the presence of a partially immersed object, the approach proposed in [2] consists in decomposing the horizontal plane into two regions: the exterior region where the surface of the fluid is in contact with the air, and the interior region where it is in contact with the bottom of the object. In the exterior region, we have the standard equations, where the surface is free but the pressure is constrained (equal to the atmospheric pressure). In the interior region, this is the reverse: the pressure is free but the surface is constrained, which changes the structure of the equations. Finally, coupling conditions between both regions are needed. We show in [1] how to implement this program in the case where the waves are described by the nonlinear dispersive Boussinesq equations and for $d = 1$. This raises several issues, such as the analysis of initial boundary value problems for dispersive perturbations of hyperbolic systems, dispersive boundary layers, hidden regularity for the traces at the contact points, etc. We also show how this theoretical approach can be used to build an efficient numerical scheme for the description of wave-structure interactions.

References

[1] G. Beck, D. Lannes. Freely floating objects on a fluid governed by the Boussinesq equations, *Ann. IHP*, to appear.

[2] D. Lannes. On the dynamics of floating structures, *Annals of PDE* **3** (2017):11.

MS08 Tuesday, 15:30–15:55, Zoom Room 08, link tba

Numerical study of fractional Korteweg-de Vries and nonlinear Schrödinger equations

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We present a detailed numerical study of solutions to fractional Korteweg-de Vries and nonlinear Schrödinger equations. Solitary waves are constructed, and their stability is studied. Mechanisms for potential blow-up are discussed.

References

[1] C. Klein and J.-C. Saut, *A numerical approach to blow-up issues for dispersive perturbations of Burgers' equation*, Physica D: Nonlinear Phenomena (2015), 46-65, 10.1016/j.physd.2014.12.004

[2] C. Klein, C. Sparber and P. Markowich, *Numerical study of fractional Nonlinear Schrödinger equations*, Proc. R. Soc. A 470: (2014) DOI: 10.1098/rspa.2014.0364.

MS08 Tuesday, 16:30–16:55, Zoom Room 08, link tba

Stability of periodic traveling waves in the Camassa-Holm equation

Anna Geyer[1,*], **Renan Martins**[2], **Fábio Natali**[2], **Dmitry Pelinovsky**[3]

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I will present a novel method to study the spectral stability of periodic waves in the Camassa-Holm equation which is potentially applicable to other peakon-bearing equations. The key to obtaining this result is that the periodic waves can be characterized by an alternative Hamiltonian structure, different from the standard formulation common to the Korteweg–de Vries equation, which has the disadvantage that the period function is not monotone and the quadratic energy form may have two rather than one negative eigenvalues. By exploring the nonstandard formulation one can prove that the period function is monotone in this case, and that the quadratic energy form has only one simple negative eigenvalue. Finally, a precise condition for the spectral and orbital stability of the smooth periodic waves is deduced.

MS08 Tuesday, 17:00–17:25, Zoom Room 08, link tba

Transverse instability of solitary waves for fractional Kadomtsev-Petviashvili equation

Handan Borluk[1,*], **Gabriele Bruell**[2], **Dag Nilsson**[3]

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The fractional Kadomtsev-Petviashvili (fKP) equation is a two-dimensional extension of the fractional Korteweg–de Vries equation. For different values of the fractional order, the equation represents various special cases of Kadomtsev-Petviashvili (KP) equation such as the classical KP equation and the KP-version of Benjamin-Ono equation. Analogously to the classical KP equation the fKP equation comes in two versions: fKP-I and fKP-II. We show that the line solitary wave solutions of fKP-I equation are transversely linearly instable. We also perform numerical experiments to observe the stability dynamics of line solitary wave solutions for both fKP-I and fKP-II equations.

MS08 **Tuesday, 17:30–17:55, Zoom Room 08, link tba**

Symmetry of Periodic Traveling Waves for Nonlocal Dispersive Equations

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Of concern is the *a priori* symmetry of traveling wave solution of a general class of nonlocal dispersive equations

$$u_t + (u^2 + Lu)_x = 0,$$

where L is a Fourier multiplier operator with symbol m which can be homogeneous or inhomogeneous. We characterize a large class of symbols m guaranteeing that periodic traveling wave solutions are symmetric under a mild assumption on the wave profile. In contrast with the classically imposed setting in the water wave problem which assumes traveling waves to have a unique crest and trough per period or a monotone structure near troughs, we formulate a reflection criterion which does not presuppose a monotone structure on the wave profile. Thereby, the reflection criterion enables us to treat *a priori* solutions with multiple crests of different size per period. Moreover, our result applies not only to smooth traveling wave solutions, but also to those with singular crests around which some cancellation structure appear, including in particular waves with peaks or cusps. The proof relies on a so-called *touching lemma*, which is related to a strong maximum principle for elliptic operators, and a weak form of the celebrated *method of moving planes*.

MS08 **Tuesday, 18:00–18:25, Zoom Room 08, link tba**

A resonant Lyapunov Centre Theorem with an application to doubly periodic hydroelastic waves

Dag Nilsson[1,*], **Mark Groves**[1], **Rami Ahmad**[1]
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We give an abstract Lyapunov centre theorem for Hamiltonian systems with the spectrum of the linearization of the Hamiltonian vector field consisting of two geometrically double eigenvalues $\pm i\kappa_0$, and allowing for 0 to be an algebraically double eigenvalue or a point of the continuous spectrum. This theorem is then used to prove existence of doubly periodic hydroelastic waves in both finite and infinite depth.

MS08 **Wednesday, 11:30–11:55, Zoom Room 08, link tba**

Three-Dimensional Doubly Periodic Gravity-Capillary Waves with Small Vorticity

Douglas Svensson Seth[1], **Kristoffer Varholm**[1], **Erik Wahlén**[2,*]

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I will discuss an existence proof for small-amplitude three-dimensional gravity-capillary water waves with vorticity. The waves are symmetric and periodic in both horizontal coordinates. The proof is inspired by Lortz' construction of magnetohydrostatic equilibria in reflection symmetric toroidal domains, but the presence of the free surface introduces new challenges. In particular, the resulting free boundary problem is not elliptic and the involved mappings are only Fréchet differentiable with a loss of regularity. Nevertheless, a version of the Crandall-Rabinowitz local bifurcation method applies if one keeps track of the loss of regularity. The solutions bifurcate from uniform horizontal flows and therefore have small vorticity.

MS08 **Wednesday, 12:00–12:25, Zoom Room 08, link tba**

Bifurcation of solitary waves to Whitham-type equations

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Truong, Wahlén & Wheeler have recently verified Whitham's conjecture on the existence of a highest, cusped traveling-wave solution for the solitary-wave case [1]. We are therefore motivated to study the bifurcation of solitary waves for Whitham-type equations of the form $c\varphi - K * \varphi - |\varphi|^p = 0$, where $p = 2$ or $p \geq 3$. The limited smoothness of the nonlinear term introduces additional challenges. We show a local bifurcation result for all $p = 2$ or $p \geq 3$, and a global result for even-integer p. In the latter case, a highest traveling-wave solution is obtained as a limit along the main global bifurcation curve. This solution is positive, symmetric about its crest and has exponential decay. We provide a precise local Hölder regularity at the crest of a highest wave in the spirit of [2]. Finally, we show how our result extends to the bidirectional Whitham equation, and the capillary-gravity Whitham equation.

References

[1] T. Truong, E. Wahlén & M. H. Wheeler, Global bifurcation of solitary waves for the Whitham equation, *Mathematische Annalen*, in print.

[2] M. Ehrnström & E. Wahlén, On Whitham's conjecture of a highest cusped wave for a nonlocal dispersive equation, *Annales de l'Institut Henri Poincare, Analyse non lineaire* **36** (2019), pp. 1603–1637.

MS08 **Wednesday, 12:30–12:55, Zoom Room 08, link tba**

On the precise behaviour of extreme solutions to a family of nonlocal dispersive equations

Kristoffer Varholm[1,*], **Mats Ehrnstrom**[1], **Ola Maehlen**[1]
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We prove exact asymptotic behaviour at the origin for nontrivial solutions of a family of nonlocal equations. This family of equations includes those satisfied by the cusped highest steady waves for both the uni- and bidirectional Whitham equations. In particular, our results partially settle conjectures for such extreme waves posed in [1,2,3].

References

[1] M. Ehrnström, M. A. Johnson and K. M. Claassen, Existence of a highest wave in a fully dispersive two-way shallow water model, *Arch. Ration. Mech. Anal.* **231** (2019), no. 3, pp. 1635–1673.

[2] M. Ehrnström and E. Wahlén, On Whitham's conjecture of a highest cusped wave for a nonlocal dispersive equation, *Ann. Inst. H. Poincaré Anal. Non Linéaire.* **36** (2019), no. 6, pp. 1603–1637.

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MS08 **Wednesday, 15:00–15:25, Zoom Room 08, link tba**

Long-time asymptotics of non-Newtonian Taylor–Couette flows

Christina Lienstromberg[1,*], **Tania Pernas-Castaño**[1], **Juan J.L. Velázquez**[1]
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I will offer an insight into mathematical models describing the dynamic behaviour of non-Newtonian thin films. The resulting PDEs are in general nonlinear, degenerate, of fourth order, and with a possibly 'weak' dependence of the coefficients.

I will discuss recent results on such an evolution equation for the interface separating two viscous immiscible fluids, confined between two concentric cylinders rotating at a small relative velocity. In this so-called Taylor–Couette setting, two competing effects drive the dynamics of the interface – the surface tension and the shear stresses induced by the rotation of the cylinders. When the two effects are comparable, solutions behave, for large times, as in the Newtonian regime. For the regime in which surface tension effects dominate the stresses induced by the rotating cylinders, we prove local existence of positive weak solutions for both shear-thinning and shear-thickening fluids. In the case of a shear-thickening fluid, one observes that interfaces which are initially close to a circle converge to a circle in finite time.

References

[1] C. Lienstromberg, T. Pernas-Castaño & J.J.L. Velázquez, Analysis of a two-fluid Taylor-Couette flow with one non-Newtonian fluid, to appear in *Journal of Nonlinear Science*, 2022.

MS08 **Wednesday, 15:30–15:55, Zoom Room 08, link tba**

Conserved energies for the one dimensional Gross-Pitaevskii equation

Herbert Koch[1], **Xian Liao**[2,*]
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In this talk we will show the conservation of the energies at the H^s-regularity level by the one dimensional Gross-Pitaevskii flow, for all $s \geq 0$.

MS08 **Wednesday, 16:30–16:55, Zoom Room 08, link tba**

Large-amplitude solitary waves with constant vorticity

Susanna V. Haziot[1], **Miles H. Wheeler**[2,*]
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We consider the classical problem of steady waves propagating along the surface of an incompressible fluid with constant vorticity. Rather than using the elegant non-local formulation due to Constantin, Strauss, and Vărvărucă [1], we view the problem a local elliptic system for two scalar functions, one describing the conformal mapping of the fluid domain and another describing the motion inside the fluid. This local formulation turns out to have several advantages, which we use to prove a new result on the global bifurcation of solitary waves.

References

[1] Adrian Constantin, Walter Strauss, and Eugen Vărvărucă, Global bifurcation of steady gravity water waves with critical layers, *Acta Math.* **217** (2016), no. 2, pp. 195–262.

[2] Susanna V. Haziot, Miles H. Wheeler, Large-amplitude steady solitary water waves with constant vorticity, *arXiv preprint* arXiv:2110.04901 (2021).

MS08 **Wednesday, 17:00–17:25, Zoom Room 08, link tba**

Global well-posedness and scattering for the Dysthe equation in $L^2(\mathbb{R}^2)$

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The Dysthe equation is a higher order approximation of the water waves system in the modulation (Schrödinger) regime and in the infinite depth case. After reviewing the derivation of the Dysthe and related equations, we will focus on the initial-value problem. We prove a small data global well-posedness and scattering result in the critical space $L^2(\mathbb{R}^2)$. This result is sharp in view of the fact that the flow map cannot be C^3 continuous below $L^2(\mathbb{R}^2)$. Our analysis relies on linear and bilinear Strichartz estimates in the context of the Fourier restriction norm method. Moreover, since we are at a critical level, we need to work in the framework of the atomic space U_S^2 and its dual V_S^2 of square bounded variation functions. We also prove that the initial-value problem is locally well-posed in $H^s(\mathbb{R}^2)$, $s > 0$. Our results extend to the finite depth version of the Dysthe equation.

MS08 **Wednesday, 17:30–17:55, Zoom Room 08, link tba**

Modulational Dynamics of Lugiato-Lefever Periodic Waves

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We discuss the nonlinear modulational stability of spectrally stable periodic waves in the Lugiato-Lefever equation, a damped nonlinear Schrödinger equation with forcing that arises in nonlinear optics. So far, nonlinear stability of such solutions has only been established against co-periodic perturbations by exploiting the existence of a spectral gap. Here, we consider perturbations which are localized, i.e., integrable on the line. Such localized perturbations naturally yield the absence of a spectral gap, so we must rely on a substantially different method with origins in the stability analysis of periodic waves in reaction-diffusion systems. We obtain nonlinear stability of periodic steady waves against localized perturbations with precisely the same decay rates as predicted by the linear theory.

MS09 **Monday, 17:00–17:25, Zoom Room 07, link tba**

Scattering by finely layered obstacle: frequency-explicit stability and homogenization

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We consider the time-harmonic scattering of scalar waves by a penetrable obstacle D modeled by the Helmholtz equation in dimension $d = 2$ or 3, with frequency k. We assume that D satisfies a specific geometric condition (allowing convex obstacles as a particular case) and that the wavespeed in D is greater than the one in the surrounding (homogeneous) medium. The PDE coefficients describing the materials contained in D do not depend on the $\boldsymbol{x}_d$ variable, but they are allowed to strongly vary in every other direction with length scale ε. This situation models in particular obstacles made of finely layered or (if $d = 3$) finely fibred materials.

We establish a novel stability estimate for this Helmholtz problem, which controls the L^2 norm of the solution by the one of the right-hand side. Crucially, our bound is fully explicit in k, and uniform in ε. This estimate shows in particular that the norm of the solution grows at most linearly as the frequency is increased.

As an important application, we consider the case where the coefficients are periodic, with period ε. We employ our stability estimate to bound the difference between the solution u_ε and the solution u_0 to a homogenized problem where the (constant) coefficients in the obstacle have been suitably averaged. The resulting error estimate is fully explicit in terms of k and ε, which appears to be entirely novel.

MS09 **Monday, 17:30–17:55, Zoom Room 07, link tba**

Scattering from a thin coating of randomly distributed nanoparticles

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We study the time-harmonic scattering by a heterogeneous object covered with a thin layer of randomly distributed nanoparticles. The size of the particles, their distance between each other and the layer's thickness are all of the same order but small compared to the wavelength of the incident wave. Solving numerically Helmholtz's equation in this context is very costly. To circumvent this, we propose, via a multi-scale asymptotic expansion of the solution, an effective model where the layer of particles is replaced by an equivalent boundary condition. The coefficients that appear in this equivalent boundary condition depend on the solutions to corrector problems of Laplace type defined on unbounded random domains. Under the assumption that the particles are distributed given a stationary and mixing random point process [1], we prove that those problems admit a unique solution in the proper spaces with both homogeneous Dirichlet (for $d \geq 2$) and Neumann boundary conditions (for $d = 3$) on the inclusions. We then establish quantitative error estimates for the effective model and present numerical simulations that illustrate our theoretical results.

References

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MS09 **Tuesday, 11:30–11:55, Zoom Room 07, link tba**

Time-harmonic acoustic scattering from locally perturbed periodic curves

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For the Dirichlet rough-surface scattering problem in two dimensions, we prove that the Green's function defined with the angular spectrum representation radiation condition satisfies the Sommerfeld radiation condition over the half-plane. To the best of our knowledge, such an outgoing property has not been rigorously justified in the literature. We prove well-posedness for the time-harmonic acoustic scattering of plane waves from locally perturbed periodic surfaces. It will be shown that the scattered wave of an incoming plane wave is the sum of the scattered wave for the unperturbed periodic surface plus an additional scattered wave satisfying Sommerfeld's condition on the half-plane. Whereas the scattered wave for the unperturbed periodic surface has a farfield consisting of a finite number of propagating plane waves, the additional field contributes to the far field by a far-field pattern defined in the half-plane directions similarly to the pattern known for bounded obstacles.

MS09 **Tuesday, 12:00–12:25, Zoom Room 07, link tba**

PML and high-accuracy boundary integral equation solver for wave scattering by a locally defected periodic surface

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In this talk, we shall present a high-accuracy perfectly-matched-layer (PML) based boundary-integral-equation (BIE) solver for wave scattering in a half space of homogeneous medium bounded by a two-dimensional, perfectly conducting, and locally defected periodic surface. Along the vertical direction, we place a PML to truncate the unbounded domain onto a strip and prove that the PML solution converges linearly to the true solution in the physical subregion of the strip with the PML thickness. Laterally, we divide the unbounded strip into three regions: a region containing the defect and two semi-waveguide regions, separated by two vertical line segments. In both semi-waveguides, we prove the well-posedness of an associated scattering problem so as to well define a Neumann-to-Dirichlet (NtD) operator on the associated vertical segment. Either NtD operator is related to a Neumann-marching operator, governed by a nonlinear Riccati equation. The two Riccati equations are efficiently solved by a recursive doubling procedure and a high-accuracy PML-based BIE method so that the boundary value problem on the defected region can be solved efficiently and accurately. Numerical experiments demonstrate that the PML solution converges exponentially fast to the true solution in any compact subdomain of the strip.

MS09	Tuesday, 12:30–12:55, Zoom Room 07, link tba

Windowed Green function method for acoustic and electromagnetic wave scattering by periodic media

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This work introduces a novel boundary integral equation (BIE) method for the numerical solution of time-harmonic acoustic and electromagnetic scattering problems in the presence of periodic media in two and three dimensions. We consider both 1d- and 2d-periodic infinite arrays of bounded penetrable scatterers. Our approach is based on the windowed Green function method [1], and it relies on a direct BIE formulation using the free-space Green function instead of the quasi-periodic one. The resulting BIE system involves integrals over the (unbounded) unit cell's boundary. Such integrals are approximated via window integration that introduces errors that decay super-algebraically fast as the window size increases. The resulting second-kind BIE system is then discretized using a Nyström-based density interpolation method [2] that, away from Wood's anomalies, leads to well-conditioned linear systems.

References

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MS09	Tuesday, 15:00–15:25, Zoom Room 07, link tba

The Problem With the Limiting Absorption Principle for Periodic Waveguides

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We consider different types of limiting absorption principles (LAP) for the Helmholtz equation $\Delta u + k^2 nu = -f$ in $\mathbb{R}^2_+ = \{x \in \mathbb{R}^2 : x_2 > 0\}$ where the index of refraction $n \in L^\infty(\mathbb{R}^2_+)$ is periodic with respect to x_1 and equal to one for $x_2 > h_0$ for some $h_0 > 0$. Furthermore, Neumann's boundary condition $\partial u/\partial x_2 = 0$ is assumed for $x_2 = 0$. The classical LAP is to replace the wave number $k > 0$ by $k + i\varepsilon$ for $\varepsilon > 0$. Then a unique solution $u_\varepsilon \in H^1(\mathbb{R}^2_+)$ exists. The convergence as ε tends to zero results in a radiation condition which takes into account the proper treatment of guided modes. A second LAP can be considered where the refractive index n is replaced by $n + i\varepsilon q$ for some non-negative (and non-trivial) function $q \in L^\infty(\mathbb{R}^2_+)$ which is periodic with respect to x_1 and zero for $x_2 > h_0$. As a third LAP we consider the case where the Neumann boundary condition is replaced by $\partial u/\partial x_2 + i\varepsilon qu = 0$ for $x_2 = 0$ with some non-negative and periodic function $q \in L^\infty(\mathbb{R})$. We will treat all LAPs with the same abstract approach and show that they lead to different radiation conditions. The same problem occurs also for closed waveguides $W = \mathbb{R} \times (0, H)$ with some $H \geq h_0$ and where an additional boundary condition is assumed for $x_2 = H$.

MS09 **Tuesday, 15:30–15:55, Zoom Room 07, link tba**

Energy method approach to existence results for the Helmholtz equation in periodic wave-guides

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Abstract: We consider the Helmholtz equation $-\nabla \cdot (a\nabla u) - \omega^2 u = f$ in an unbounded wave-guide $\Omega := \mathbb{R} \times S \subset \mathbb{R}^d$, where $S \subset \mathbb{R}^{d-1}$ is bounded. We assume periodicity of the coefficient a outside a compact set and derive an existence result: For non-singular frequencies ω there exists a solution u to the Helmholtz equation with radiation conditions. Proofs of such results exist, they are based on analyticity arguments and operator theory. By contrast, our proof uses only energy methods. The flexibility of the new method allows also to study the case that two different media are used in the two unbounded directions.

References

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MS09 **Tuesday, 16:30–16:55, Zoom Room 07, link tba**

Guided waves in honeycomb periodic structures

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In this talk, we investigate the propagation of waves in a particular honeycomb structure made of thin tubes. Based on an asymptotic analysis, we prove that the dispersion surfaces associated with this system have conical points located at the vertices of the Brillouin zone: this is a well-known property for systems having hexagonal symmetry see (see for instance [1,2]). Then, introducing so call zig-zag perturbations in the structure generates guided waves propagating along the defect. More specifically, we show that the frequency of those guided modes may be independent of the quasi-periodicity parameter β, leading to almost flat dispersion curves. As in [3], those particular eigenvalues may be seen as invariant of dispersion curves appearing when then structure is dislocated. We present numerical results to illustrate our results.

References

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MS09 **Tuesday, 17:00–17:25, Zoom Room 07, link tba**

Asymptotic anatomy of the Berry phase for 2D periodic continua

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We deploy a recent asymptotic model for the interaction between nearby dispersion surfaces and respective eigenstates toward explicit evaluation of the Berry phase governed by the scalar wave equation in 2-dimensional (2D) periodic media. The model, featuring a pair of coupled Dirac equations, entails a 4-dimensional parametric space and endows the interacting Bloch eigenstates with an explicit gauge that caters for analytical integration in the wavenumber domain. Among the featured parameters, the one ($\mathfrak{s} \in [0, \frac{1}{2}]$) that synthesizes the phase information on the coupling term is shown to decide whether the Berry connection round the loop is singular ($\mathfrak{s} = 0$) or analytic ($\mathfrak{s} > 0$). The analysis demonstrates that the Berry phase for 2D lattices is π-quantal and topological when $\mathfrak{s} = 0$, equalling π (modulo 2π) when the contour encloses the apex of a Dirac cone and zero in all other situations (avoided crossings or line crossings). The analogous result is obtained, up to an $O(\mathfrak{s})$ residual, when $\mathfrak{s} \simeq 0$ and similarly for $\mathfrak{s} \simeq \frac{1}{2}$. In the interior of the $\mathfrak{s}$-domain, on the other hand, we find that the Berry phase either equals π or is not quantal. Beyond shedding light on the anatomy of the Berry phase for 2D periodic continua, the featured analysis carries a practical benefit for it permits single-wavenumber evaluation of the Berry phase.

MS10 **Monday, 11:45–12:10, Zoom Room 03, link tba**

Machine Learning Techniques for Optimization under Uncertainty

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Approaches to decision making and learning mainly rely on optimization techniques to achieve “best” values for parameters and decision variables. In most practical settings, however, the optimization takes place in the presence of uncertainty about model correctness, data relevance, and numerous other factors that influence the resulting solutions. For complex processes modeled by nonlinear ordinary and partial differential equations, the incorporation of these uncertainties typically results in high or even infinite dimensional problems in terms of the uncertain parameters as well as the optimization variables, which in many cases are not solvable with current state of the art methods. One promising potential remedy to this issue lies in the approximation of the forward problems using novel techniques arising in uncertainty quantification and machine learning.

We propose in this talk a general framework for machine learning based optimization under uncertainty and inverse problems. Our approach replaces the complex forward model by a surrogate, e.g. a neural network, which is learned simultaneously in a one-shot sense when estimating the unknown parameters from data or solving the optimal control problem. By establishing a link to the Bayesian approach, an algorithmic framework is developed which ensures the feasibility of the parameter estimate / control w.r. to the forward model.

MS10 **Monday, 12:15–12:40, Zoom Room 03, link tba**

Shape uncertainty quantification for the strongly damped wave equation

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The strongly damped wave equation is found in many applications for the modeling of viscoelastic materials. In this talk, we consider the effect of uncertainty in the shape of a scatterer on the solution to such equation. To handle the shape variations, we consider a mapping approach to a reference configuration with fixed shape. We show that, in the presence of the strong damping, the map from the high-dimensional parameter describing the shape uncertainties to the solution on the reference configuration is holomorphic. This allows to apply high order quadrature methods for the computation of moments of the solution, which are robust with respect to the dimension of the parameter characterizing the shape variations. The performance of one of these algorithms, namely dimension adaptive sparse grids, will be shown in numerical experiments.

MS10	Monday, 12:45–13:10, Zoom Room 03, link tba

On shape uncertainty in electromagnetic wave scattering

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We consider time-harmonic electromagnetic scattering problems on perfectly conducting scatterers with uncertain shape. Thus, the scattered field will also be uncertain. Based on the knowledge of the two-point correlation of the domain boundary variations around a reference domain we discuss several approaches to quantify the uncertainty in the scattered field, including a new perturbation analysis of the mean. Therefore, we compute the second shape derivative of the scattering problem for a single perturbation. Numerical experiments in three dimensions are presented.

References

[1] J. Dölz, A Higher Order Perturbation Approach for Electromagnetic Scattering Problems on Random Domains, *SIAM/ASA Journal on Uncertainty Quantification*, 8(2):748–774, 2020.

MS10	Monday, 15:00–15:25, Zoom Room 03, link tba

A Posteriori Error Control of Numerical Schemes for Random Conservation Laws

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We derive error estimates for numerical schemes approximating systems of hyperbolic conservation laws with random initial data and (possibly) including random flux functions. The discretization of these problems usually proceeds in two steps: Firstly, the stochastic space is discretized either by a spectral ansatz (stochastic Galerkin) or by collocation, leading to deterministic problems. Secondly, the deterministic problems are discretized in space and time using e.g. Runge-Kutta discontinuous Galerkin schemes.

Firstly, we present an error estimator for stochastic Galerkin schemes for random scalar hyperbolic conservation laws. One of its key features is that it admits an orthogonal splitting into estimators for the stochastic error and for the space-time discretization error, respectively.

Secondly, we discuss error estimates for stochastic collocation schemes. In this case the error estimator can, again, be split into different parts that correspond to different error sources but the splitting is no longer orthogonal.

References

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MS10 **Monday, 15:30–15:55, Zoom Room 03, link tba**

Adaptive Stratified Sampling for Non-smooth Problems

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Various sampling methods, including the multilevel Monte Carlo method, have been established as general-purpose procedures for efficiently quantifying uncertainties in computational models. The improved computational efficiency of these methods compared to vanilla Monte Carlo sampling is usually obtained by suitable variance reduction techniques. It is known, however, that these techniques may not provide performance gains when there is a non-smooth, in particular discontinuous, parameter dependence. Moreover, in many applications some key variance reduction ideas cannot be fully exploited, such as those of multilevel Monte Carlo when a hierarchy of computational models cannot be easily constructed (e.g., for transport problems in fractured porous media of relevance to carbon storage and wastewater injection). An alternative means to obtain variance reduction in these cases is offered by stratified sampling methods. In this talk, we will discuss various ideas on adaptive stratified sampling methods tailored to applications with a discontinuous parameter dependence. Examples exhibiting discontinuous solutions include hyperbolic PDEs under uncertainty, such as the Euler equations describing a high-speed flow, and the shallow water equations modeling dam breaks, flooding and atmospheric flow. For such discontinuous problems, the stochastic domain is adaptively stratified using local variance estimates, and the samples are sequentially allocated to the strata for asymptotically optimal variance reduction. The proposed methodology is demonstrated on discontinuous test cases from computational fluid mechanics and CO2 storage in subsurface reservoirs.

MS10 **Monday, 16:30–16:55, Zoom Room 03, link tba**

Prediction of Iced Airfoil Performance With Data-Driven Non-Intrusive Uncertainty Quantification

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Airfoil icing is a severe safety hazard in aviation and causes power losses on wind turbines. The shape of the ice is subject to large uncertainties, so uncertainty quantification (UQ) is needed for a reliable prediction of its effects. In this study, a set of experimentally measured wind tunnel ice shapes serves as input data. Using principal component analysis, a set of linearly uncorrelated geometric modes is constructed from the data, which serves as random input to the UQ simulation. For uncertainty propagation, polynomial chaos expansion (PCE), multi-level Monte Carlo (MLMC) and multi-fidelity Monte Carlo (MFMC) methods are employed and compared. Airfoil lift, drag and pressure distributions are considered as quantities of interest. Simulations are carried out with the in-house framework POUNCE (Propagation of Uncertainties). Its focus is on a high level of automation and practical efficiency considerations in a high performance computing environment. As a baseline model, large eddy simulations are carried out using the discontinuous Galerkin flow solver FLEXI. Due to the high number of samples, the simulation tool chain of the baseline model is also completely automatized, including a new structured surface mesh generator for highly irregular domain shapes. Results show that forces on the airfoil vary considerably due to the uncertain ice shape. All three methods prove to be suited to predict mean and standard deviation. The MFMC method performs best.

MS10 **Monday, 17:00–17:25, Zoom Room 03, link tba**

Robust and rank adaptive dynamical low-rank approximation for uncertainty quantification

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Problems from uncertainty quantification commonly exhibit a large phase space, leading to increased memory requriements and computational costs. One approach to cope with such high-dimensional phase spaces is dynamical low-rank approximation (DLRA) [1]. The method can be interpreted as a stochastic-Galerkin method which picks basis functions adaptively and thereby mitigates the curse of dimensionality. In this talk, we outline the DLRA method for uncertainty quantification and present computational results in various settings. Furthermore, we investigate stable and robust spatial discretizations of the DLRA evolution equations following [2]. Computations are enhanced by a novel rank adaptive integrator, which picks the rank adaptively in time [3].

References

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MS10 **Monday, 17:30–17:55, Zoom Room 03, link tba**

Quantifying Uncertainties in Electromagnetic Waves: Sparse Approximation and Multifidelity Analysis

Ulrich Römer[1,*], **Niklas Georg**[2], **Sebastian Schöps**[2], **Rolf Schuhmann**[3]
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Uncertainties arise frequently in the mathematical description of electromagnetic waves with Maxwell's equations. Dispersive material data and scattering surfaces for instance, exhibit variations which may result in large uncertainties in quantities of interest, such as scattering parameters. In this talk we report recent work on parametric surrogate modeling and uncertainty propagation for Maxwell's equations with random influences. We first address Maxwell's source problem and introduce a moderately high-dimensional case, where both material and geometry uncertainty is present. We construct sparse polynomial surrogate models and introduce adjoint error estimation and conformal mappings, which yield more accurate approximations. In the second part of the talk, we consider periodic structures, which are important for optical applications. When parameter uncertainties are independent for each individual cell, the parameter space is very high-dimensional. In this case, we present a method to decouple the uncertainty propagation problem, such that only a single surrogate model is required. With the local surrogate model, the individual cells are then coupled with the so-called scattering matrix approach and the bias is removed within a multifidelity Monte Carlo framework. Various numerical examples will be shown to illustrate the uncertainty propagation methods.

MS10 Monday, 18:00–18:25, Zoom Room 03, link tba

A Non-Intrusive Multilevel Uncertainty Quantification (UQ) Framework for Wave Equations with Random Input Data

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Multilevel Monte Carlo methods and Stochastic Collocation have been successfully applied to many problems by numerous authors. In this talk we will present a framework which can treat a wide range of PDEs with uncertain input data with various discretization combined with non-intrusive stochastic estimators. In particular, we want to discuss the combination of space-time discretizations and multilevel estimators for the acoustic wave equation with random coefficients and briefly want to introduce our setting for high performance computations.

References

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MS11 **Wednesday, 11:30–11:55, Zoom Room 03, link tba**

Localized ground states of the Gross-Pitaevskii eigenvalue problem under disorder

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In this talk, we consider Bose-Einstein condensates in disorder potentials, which are modeled by the Gross-Pitaevskii eigenvalue problem. Considering ultra-low temperatures close to 0K, such models allow to study certain quantum phenomena on a macroscopic observation scale. In the regime of weak particle interaction, we are able to quantify exponential localization of the ground state, depending on statistical parameters and the strength of the potential.

In the one-dimensional case, localization can be proven rigorously by an interpretation of the nonlinearity as a perturbation of the given disorder potential in combination with known results from the linear case. For higher dimensions, we formulate a conjecture when localization is to be expected. These findings will be supported by numerical experiments.

References

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MS11 **Wednesday, 12:00–12:25, Zoom Room 03, link tba**

Superconvergence of a multiscale method for ground state computations of Bose-Einstein condensates

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To compute ground states of Bose-Einstein condensates the stationary Gross-Pitaevskii equation (GPE) is often used. In this talk we revisit a two-level multiscale technique introduced in [1] for computing numerical approximations to the GPE. The method is based on localized orthogonal decomposition of the solution space, which exhibit high approximation properties and improves the convergence rates compared to classical finite element methods. This reduces the computational cost for computing the ground states significantly.

In the paper [1], high order convergence of the method was proven, but even higher orders were observed numerically. In this talk we show how to improve the analysis to achieve the observed rates, which are as high as $O(H^6)$ for the eigenvalues. We also show some numerical experiments for both smooth and discontinuous potentials.

References

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MS11 **Wednesday, 12:30–12:55, Zoom Room 03, link tba**

Highly oscillatory integrators for the Klein–Gordon equation at low regularity

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We propose a novel class of uniformly accurate integrators for the Klein–Gordon equation which capture classical c = 1 as well as highly-oscillatory non-relativistic regimes $c \gg 1$ and, at the same time, allow for low regularity approximations. In particular, the schemes converge with order τ and τ^2, respectively, under lower regularity assumptions than classical schemes, such as splitting or exponential integrator methods, require. The new schemes in addition preserve the nonlinear Schrödinger (NLS) limit on the discrete level. More precisely, we will design our schemes in such a way that in the limit $c \to \infty$ they converge to a recently introduced class of low regularity integrators for NLS.

MS11 **Wednesday, 15:00–15:25, Zoom Room 03, link tba**

A space–time quasi-Trefftz DG method for the wave equation with piecewise-smooth coefficients

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We propose a "*quasi-Trefftz*" method to approximate initial boundary value problems for the acoustic wave equation with piecewise-smooth wave speed. The key feature of the scheme is that all discrete trial and test functions are elementwise approximate solutions of the wave equation. The quasi-Trefftz scheme is framed in a space–time discontinuous Galerkin (DG) setting. We prove stability and high-order convergence, and we show that the number of DOFs needed to obtain a given accuracy is much smaller than for schemes based on classical polynomial spaces. The quasi-Trefftz basis functions are polynomials in the space–time variable and can be computed with a simple algorithm. The inspiration for this method comes from the generalised plane waves developed for time-harmonic problems with variable coefficients.

References

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MS11 **Wednesday, 15:30–15:55, Zoom Room 03, link tba**

A discontinuous Galerkin coupling for nonlinear elasto-acoustics

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In this talk, motivated by medical applications of high-intensity ultrasound, we will discuss a coupled elasto-acoustic problem with general acoustic nonlinearities of quadratic type as they arise, for example, in the Westervelt and Kuznetsov equations of nonlinear acoustics. In particular, we will focus on the convergence analysis of a finite element approximation to this coupled problem in a setting that involves different acoustic materials. Here a subdomain-based discontinuous Galerkin approach realizes the acoustic-acoustic coupling of different materials, whereas elasto-acoustic interface conditions are used for a mutual exchange of forces between the different models. The talk is based on [1].

References

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MS11 **Wednesday, 16:30–16:55, Zoom Room 03, link tba**

Homogenization and numerics for the Landau-Lifshitz equation with a highly oscillatory coefficient

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We consider a simplified version of the Landau-Lifshitz equation to study a composite ferromagnetic object that consists of magnetic materials with different interaction behavior. In the model this is represented by a rapidly varying material coefficient. Direct numerical simulation of the problem is expensive as the small scale must be resolved; however, the small scale cannot be ignored either, since it has a significant influence on the magnetization behavior on the coarse scale. We design an efficient numerical method for this case using the framework of heterogeneous multiscale methods (HMM). In this talk I will show some new homogenization results for the Landau-Lifshitz equation and use them to bound the upscaling error in the HMM method. I will also present numerical examples that confirm the error estimates.

MS11 **Wednesday, 17:00–17:25, Zoom Room 03, link tba**

A generalized finite element method for the strongly damped wave equation with rapidly varying data

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We propose a generalized finite element method for the strongly damped wave equation with highly varying coefficients. The proposed method is based on the localized orthogonal decomposition introduced in [1], and is designed to handle independent variations in both the damping and the wave propagation speed respectively. The method does so by automatically correcting for the damping in the transient phase and for the propagation speed in the steady state phase. Convergence of optimal order is proven in $L^2(H^1)$-norm, independent of the derivatives of the coefficients. We present numerical examples that confirm the theoretical findings

References

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MS11 **Wednesday, 17:30–17:55, Zoom Room 03, link tba**

Super-localization for Helmholtz problems

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In this talk, we study time-harmonic wave propagation in complex media modeled by the Helmholtz equation. Following [1], we introduce a novel super-localized orthogonal decomposition (S-LOD) that aims to find problem-adapted ansatz spaces with basis functions that decay super-exponentially. An abstract a priori error estimate shows that the new method is at least as good as the classical LOD for Helmholtz problems presented in [2]. Moreover, numerical experiments demonstrate the super-exponential decay of the localization errors and an optimal order of convergence.

References

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MS11 Thursday, 15:00–15:25, Zoom Room 03, link tba

Local absorbing boundary conditions for heterogeneous and convected time-harmonic acoustic problems

, Philippe Marchner[123,*]**Xavier Antoine**[1]**, Christophe Geuzaine**[2]**, Hadrien Bériot**[3]
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We address the problem of building local absorbing boundary conditions (ABCs) for heterogeneous and convected Helmholtz problems, based on the historical approach from Engquist and Majda [1]. ABCs are built as a local approximation of the Dirichlet-to-Neumann (DtN) operator, for which the total symbol is expressed as an asymptotic expansion in the microlocal sense. We use pseudodifferential calculus to compute the most significant symbols of the expansion and further propose well-suited local operators. We explain their practical implementation in an open source high-order finite element code. Simple numerical examples illustrate and assess the accuracy of the ABCs in media where heterogeneities and convective effects are important. We apply these conditions to non-overlapping domain decomposition, where the quality of the DtN approximation is crucial for the convergence of the method.

References

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MS11 Thursday, 15:30–15:55, Zoom Room 03, link tba

Learned infinite elements

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This talk deals with transparent boundary conditions for time-harmonic wave propagation in stratified media in the presence of strong reflections. The Sun represents an important example of such a medium. In order to develop transparent boundary conditions efficient and accurate approximations of the Dirichlet-to-Neumann (DtN) operator are required. While a plethora of suitable approximations for homogeneous media is available, the case of stratified media involving strong reflections seems to be less well understood.

In this talk we present learned infinite elements [1] as a possible approach to deal with such media utilizing the assumption of a separable geometry. In this case, the DtN map is separable, however, it remains a non-local operator with a dense matrix representation, which renders its direct use computationally inefficient. Therefore, we will approximate the DtN only indirectly by adding additional degrees of freedom to the linear system in such a way that the Schur complement w.r.t. the latter provides an optimal approximation of DtN and sparsity of the linear system is preserved. This optimality is ensured via the solution of a small minimization problem, which incorporates solutions of one-dimensional time-harmonic wave equations and allows for great flexibility w.r.t. properties of the medium.

References

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MS12 **Wednesday, 15:00–15:25, Zoom Room 05, link tba**

Some wave phenomena in coherent and time-modulated metasurfaces

S. A. Tretyakov[1,*], **M. S. Mirmoosa**[1], **X. Wang**[1], **F. S. Guesta**[1], **G. A. Ptitcyn**[1]

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In this talk we will review some of our recent results of studies of coherent and time-modulated metasurfaces.

Coherent metasurfaces are designed for operation under illumination by two or more coherent waves. These devices may offer better possibilities for control of metasurface response and show interesting wave effects. Many metasurfaces that act as engineered boundaries are formed by a patterned metal sheet (or an array of dielectric particles) over a reflector. We note that it is generally possible to create a corresponding coherent metasurface functioning under illumination of its both sides. Here, we will present the results of our investigations of coherent asymmetric absorbers, and discuss wave phenomena found in these devices both in free-space and waveguide environments.

In the second part, we will present an analytical derivation of the dispersion equation for surface waves travelling along reactive boundaries which are periodically modulated in time. Numerical results show that time modulation of boundaries leads to opening band gaps that can be controlled by engineering the modulation spectrum. Furthermore, we also point out an interesting effect of field amplification related to the existence of these band gaps for surface waves.

MS12 **Wednesday, 15:30–15:55, Zoom Room 05, link tba**

Multipolar Modeling of Spatially Dispersive Metasurfaces

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Currently, most approaches developed to model metasurfaces are based on dipolar descriptions of the metasurface scattering particles. In addition, these models usually also include a form of weak spatial dispersion to properly account for structural asymmetries of the scattering particles, which leads to chirality or asymmetric scattering. While these approaches have shown to be particularly effective at modeling metasurfaces illuminated within the paraxial approximation limit, their accuracy generally breaks down for illuminations with larger angles of propagation or for electrically large particles [1]. To overcome this limitation, we have derived zero-thickness sheet transition conditions, which include dipolar and quadrupolar responses, by using a distribution-based expansion of Maxwell equations. In order to further improve the efficacy of the model, we have also considered higher-order spatial dispersion related to the first-order spatial derivatives of the electric and magnetic fields. To complete the model, we have extended the Lorentz reciprocity and the Poynting theorems by deriving the associated relations of reciprocity and losslessness that apply to the considered higher-order spatially dispersive hypersusceptibility components. This work is expected to be essential for accurately modeling the angular scattering responses of metasurfaces and hence perform operations such as optical analog processing or spatial filtering.

References

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MS12 **Wednesday, 16:30–16:55, Zoom Room 05, link tba**

Axion bombs, Black holes and breaking global charge conservation.

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Charge conservation is a fundamental, and experimentally verified rule of electrodynamics. It turns out that only local charge conservation is needed and attested. Going from local charge conservation to global charge conservation requires either one of two assumptions, or both. Either the excitation fields D and H are well defined or spacetime has no holes. We can therefore break global charge conservation by both demanding spacetime has a hole and D and H are not defined. We can provide the spacetime hole by creating a black hole that evaporates. The latter is achieved by modifying Maxwell's equations to include an axion field. We have given an explicit solution to these modified Maxwell equations in a simplified holey spacetime namely Minkowski spacetime with a point removed. The solution can be given pictorially.

These results have appeared in Annalen der Physics [1] (where one of the pictures is on the cover). It also had an article in Netherland New Scientist [2].

References

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MS12 **Wednesday, 17:00–17:25, Zoom Room 05, link tba**

Multiple Scale Method Applied to Homogenization of Irrational Metamaterials: Asymptotics and numerics

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An asymptotic procedure based upon a two-scale approach is developed for homogenization of wave equations in quasiperiodic inhomogeneous media. Partial differential operators (gradient, divergence and curl) acting on periodic functions with m variables in a higher-dimensional space are projected onto operators acting on quasiperiodic functions with n variables in the physical space ($m > n$). We replace heterogeneous photonic quasicrystals (coined irrational metamaterials) by homogeneous media described by anisotropic permittivity and permeability, deduced from electrostatic annex problems on a periodic cell in higher dimensional space. Two-scale convergence can be applied to rigorously establish our asymptotic results [1,2].

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MS12 **Thursday, 10:30–10:55, Zoom Room 05, link tba**

Homogenization of domain perforations and sound absorption

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Abstract: We investigate limit equations for the Helmholtz equation in perforated domain, where the perforation is only included along an interface. The analysis starts with the lowest order approximation: We find that a Neumann sieve perforation is invisible to leading order. Non-trivial transmission conditions occur for the corrector. We derive these conditions with a direct method, which relies on L1-estimates and the study of limit measures. We generalize results of Delourme, Haddar, and Joly, and require only second order equations in the derivation. In a three scale geometry we derive a limit system that can explain sound absorption at perforated walls.

References

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MS12 **Thursday, 11:00–11:25, Zoom Room 05, link tba**

Homogenization in perforated domains revisited

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In this talk we revisit one of the classical problems in homogenization theory – homogenization of the Laplacian in a domain with a lot of tiny holes; for the Dirichlet Laplacian it is also known as *crushed ice problem*. We focus on the case when an extra potential arises in homogenized equation (the so-called *strange term*). First, we review the pioneer results obtained in this area from early 1960s to late 1980s. Then we discuss our recent contributions [1,2], where we improve those classical results deriving estimates on the rate of convergence in terms various operator norms. As a byproduct we establish estimates on the distance between the spectra of the associated operators.

References

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MS12 **Thursday, 11:30–11:55, Zoom Room 05, link tba**

Multiscale simulations for spatial metamaterials

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Electromagnetic metamaterials are receiving growing attention because of their unusual properties that may go beyond the known regime. Typically, such materials are characterized by fine spatial structures which imply rapidly varying coefficients in the PDE context.

The numerical simulation is very challenging, since standard discretization schemes require an enormous computational effort through the resolution of all fine structures. In this talk, we instead present numerical homogenization approaches that use either effective material parameters or problem-adapted so-called multiscale (ansatz) functions. We discuss the design of these schemes and present several numerical examples illustrating unusual effects in metamaterials. One particular focus lies on materials with a high physical constrast between the properties (e.g., permittivity) of the components.

The talk is based on joint works with Mario Ohlberger (WWU Münster) and Daniel Peterseim (University of Augsburg).

MS12 **Thursday, 15:00–15:25, Zoom Room 05, link tba**

Far-field Perfect Imaging with Time Modulated Gratings

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We study the capabilities of time-modulated diffraction gratings as imaging devices. It is shown that a time-dependent but transversally homogeneous slab can be used to make a perfect image of an object in the far-field, since all the evanescent modes couple to propagative time-diffracted orders. It is found that, if the image to be obtained is axially symmetric, it can be recovered by measuring the time-signal at a single point, without the need of performing a spatial scan, so that time gratings can act as well as single-pixel imaging devices. In the case of having an object without axial symmetry, the time-grating can be combined with a spatial grating, and then the full image can be recovered again with a measurement at a single point. We apply the theory of compressive sensing to optimize the recovery method and numerical examples are provided. We show therefore that time-modulated gratings can be used to perfectly recover the image of an object in the far field and after measuring at a single point in the space, being therefore a promising approach to superesoloution and ultra-fast imaging.

MS12 **Thursday, 15:30–15:55, Zoom Room 05, link tba**

Electromagnetic Waves in Periodic Heterostructures: Symmetry Breaking, "Bulk Transition" and Bloch Bands

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We address several topics related to the analysis and simulation of wave propagation and homogenization of periodic heterostructures.

First, we show that magnetoelectric coupling can reconcile the possible incompatible symmetries between a periodic sample and its homogenized version.

Secondly, we dispute the intuitive notion of material parameters converging to bulk characteristics as the number of layers in the sample passes a certain threshold. Rather, we demonstrate that, in a sense, one layer is already "bulk".

Finally, we present some advances in the efficient computation of photonic bands. These developments are based on novel computational versions of plane wave expansion on the one hand and adaptive finite element analysis on the other.

MS12 **Thursday, 16:30–16:55, Zoom Room 05, link tba**

Simulation of homogenized subwavelength metasurfaces

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We revisit the classical zero-thickness Generalized Sheet Transition Conditions (GSTCs) [2, 3] which are used to design metafilms able to control the flow of light in a desired way. We will present a modified formulation of these conditions in which the original metafilm is replaced by transition conditions that exclude the layer from the physical domain [1]. One advantage of our formulation is that it provide a well-posed set of equations, hence guaranteeing the stability of numerical schemes in the time domain. In the case of all-dielectric structures, the effective susceptibility tensors are derived thanks to an asymptotic analysis combined with homogenization technique and bounds for the susceptibilities entering the balance of energy are provided. Implementation and validation of our effective model will be presented using frequency and time dependent simulations in both two and three dimensions.

References

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MS12 **Thursday, 17:00–17:25, Zoom Room 05, link tba**

Wave propagation through a non-linear resonant metascreen

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Our study concerns the propagation of acoustic waves through a non-linear resonant metascreen made of a periodic arrangement of air bubbles in water. The nonlinear dynamics of the bubbles is obtained in the time domain using asymptotic analysis and homogenization technique. The resulting effective model is set in the water (the screen has disappeared) and it encapsulates the effect of the screen in a jump of the normal acoustic velocity. The jump is linked to the continuous version of the bubble radius which satisfies a non-linear equation of the Rayleigh–Plesset type. This allows us to highlight two important effects. Firstly, a bubble within the array has a much larger radiative damping than an isolated bubble. Secondly it perceives a pressure which differs from the acoustic pressure imposed by the source due to bubble–bubble interactions; it results in a term of mass correction deduced from the Green's function for a Laplace problem which accounts for the bubble arrangement.

References

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MS12 **Thursday, 17:30–17:55, Zoom Room 05, link tba**

Numerical simulation of variationnal formulation for resonant Maxwell's equations in cold magnetized plasma

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The modeling of electromagnetic waves in cold magnetized plasma leads to resonant equations in time-harmonic domain. In this model, because of the a priori continuity of the plasma density, the sign of some crucial coefficient of the resonant dielectric permittivity changes *continuously*, and it behaves as the signed distance to the interface. The singularity have already been characterized by limiting absorption principle for simplified cases[1,2] and a mixed variational formulation has been provided in previous works[3].

The next step consists in lowering some regularity assumption, designing new numerical methods and proving the limiting absorption principle used to define the singularity. Some numerical experiments will be provided to illustrate the problem. Then, extensions are studied.

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MS13 **Tuesday, 11:30–11:55, Zoom Room 03, link tba**

Monotonicity Principle in Tomography of Nonlinear Materials

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We treat an inverse electrical conductivity problem which deals with the reconstruction of nonlinear electrical conductivities, starting from boundary measurements in steady currents operations. In this framework, a key role is played by the Monotonicity Principle, which establishes a monotonic relation connecting the unknown material property to the (measured) Dirichlet-to-Neumann operator (DtN). Monotonicity Principle is at the foundation for a class of non-iterative and real-time imaging methods and algorithms.

Unfortunately, the Monotonicity Principle for the DtN operator holds only for the linear and p-Laplacian cases. In this contribution we prove that the Monotonicity Principle can be extended to nonlinear materials, by introducing a new operator, the Average DtN operator. The fundamental results for getting the Monotonicity Principle for the Average DtN operator are: (i) the Monotonicity of the Dirichlet Energy and (ii) the connection of the latter to boundary measurements corresponding to the Average DtN operator.

References

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MS13 **Tuesday, 12:00–12:25, Zoom Room 03, link tba**

The monotonicity method for the inverse crack scattering problem

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The monotonicity method for the inverse acoustic scattering problem is to understand the inclusion relation between an unknown object and artificial one by comparing the far field operator with the artificial operator. This talk introduces the development of this method to the inverse crack scattering problem. Our aim is to give the following two indicators: One is to determine whether an artificial small arc is contained in the unknown arc. The other one is whether an artificial large domain contains the unknown one. Finally, numerical examples are given. This talk is based on the paper [1].

References

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MS13 **Tuesday, 12:30–12:55, Zoom Room 03, link tba**

Monotonicity-Based Inversion Methods for Some Inverse Problems

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In this talk, we introduce the monotonicity-based inversion method for some inverse problems. The method can be applied to various mathemat- ical models: Linear, nonlinear and nonlocal equations.

MS13 **Tuesday, 15:00–15:25, Zoom Room 03, link tba**

Monotonicity method for extreme, singular and degenerate inclusions in electrical impedance tomography

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In electrical impedance tomography, the monotonicity method enables reconstructing both definite and indefinite inclusions under only mild assumptions on the conductivity perturbation and the background conductivity [1]. However, in its standard form the method requires the conductivity inside the inclusions to be bounded away from zero and infinity. This work generalizes the method for extreme inclusions that correspond to some parts of the domain being perfectly conducting or insulating [2]. The result is also extended to allow the conductivity perturbation to be the restriction of an A_2-Muckenhaupt weight in parts of the domain, thereby including singular and degenerate behavior in the governing elliptic equation [3].

References

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MS13 Tuesday, 15:30–15:55, Zoom Room 03, link tba

Reconstruction of piecewise constant layered conductivities

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In electrical impedance tomography, the outer shape of inclusions can be determined directly using the monotonicity method [2,3,4]; in many contexts this is sufficient. For layered inclusions, however, this will only find the outermost layer. In this talk, I will present a reconstruction method for piecewise constant layered conductivities from partial boundary measurements, based on a new uniqueness proof [1]. The method successively reconstructs one layer at a time, starting from the exterior and moving inwards. The shape of each layer is obtained through a modified version of the monotonicity method. The conductivity values in each connected component of a layer are found independently through basic one-dimensional optimization problems.

References

[1] H. Garde. Reconstruction of piecewise constant layered conductivities in electrical impedance tomography. *Comm. PDE*, 45(9):1118–1133, 2020.

[2] V. Candiani, J. Dardé, H. Garde, and N. Hyvönen. Monotonicity-based reconstruction of extreme inclusions in electrical impedance tomography. *SIAM J. Math. Anal.*, 52(6):6234–6259, 2020.

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MS13 Tuesday, 16:30–16:55, Zoom Room 03, link tba

Towards global convergence for inverse coefficient problems with finitely many measurements

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This talk presents new developments in monotonicity-based approaches that go beyond shape reconstruction. We will show that the fundamental monotonicity estimate is actually a convexity property. Combining this with localized potentials arguments allows us to explicitly estimate the number of measurements that is required to achieve a desired resolution in an inverse coefficient problem with finite data. We moreover obtain error estimates for noisy data, and overcome the problem of local minima by reformulating the inverse problem as a convex non-linear semidefinite optimization problem.

References

[1] B. Harrach, Uniqueness, stability and global convergence for a discrete inverse elliptic Robin transmission problem, *Numer. Math.* **147** (2021), pp. 29–70.

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MS13 **Tuesday, 17:00–17:25, Zoom Room 03, link tba**

Monotonicity in inverse scattering for Maxwell's equations

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We consider an inverse scattering problem for time-harmonic Maxwell's equations in unbounded free space. The goal is to recover the position and shape of compactly supported scatterers by means of far field observations of the scattered electromagnetic waves. The media are supposed to be penetrable, non-magnetic and non-absorbing but the electric permittivity may be inhomogeneous inside the scattering objects.

First, we establish monotonicity relations for the eigenvalues of the far field operator which maps plane wave incident fields to the far field patterns of the corresponding scattered waves. In addition, we discuss the existence of localized vector wave functions that have arbitrarily large energy in some prescribed region whereas at the same time having arbitrarily small energy in some other prescribed region. Combining the monotonicity principle and the localized vector wave functions leads to rigorous characterizations of the support of the scattering objects. Finally, we present shape reconstruction algorithms and give numerical examples to illustrate the reconstruction procedure.

This talk is based on a joint work with Roland Griesmaier[1].

MS13 **Tuesday, 17:30–17:55, Zoom Room 03, link tba**

Experimental Detection and Shape Reconstruction of Inclusions in Elastic Bodies via Monotonicity-Based Methods

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The main motivation of this problem is the non-destructive testing of elastic structures for material inclusions. We consider the inverse problem of recovering the shape of the inclusions from the Neumann-to-Dirichlet map. In doing so, we deal with the rigorously proven theory of the monotonicity methods developed for linear elasticity with the explicit application of the methods, i.e. the implementation and simulation of the reconstruction of inclusions in elastic bodies for both artificial and experimental data. More specifically, we give an insight into the monotonicity-based methods and take a look at a lab experiment. Finally, we present our reconstructions based on experimental data and compare them with the simulations obtained from artificial data, where we want to highlight that all inclusions can be detected from the noisy experimental data, thus, we obtain accurate results.

References

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MS13 **Tuesday, 18:00–18:25, Zoom Room 03, link tba**

Solving an inverse Robin transmission problem with a finite number of electrodes

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We introduce a method for solving the inverse Robin transmission problem where the corrosion function on the boundary of an interior object is to be determined from voltage current measurements on the boundary of an outer domain. We consider the shunt electrode model, where in contrast to the standard Neumann-Boundary condition the applied electrical current is only partially known. The aim of this talk is to determine the corrosion coefficient with finitely many measurements.

We will first study the well-posedness of the forward problem. Then we aim to derive a criterion that ensures uniqueness for the inverse problem and allows rewriting the problem as a convex non-linear semidefinite optimization problem. Our result extends the recent work of Harrach [1] to a realistic electrode model.

References

[1] B. Harrach, *Solving an inverse elliptic coefficient problem by convex non-linear semidefinite programming*, Optim. Lett, (2021)

MS14 **Monday, 11:45–12:10, Zoom Room 04, link tba**

An optimal control approach for the scalar transmission problem with sign-changing coefficients.

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We present a new numerical method to approximate the solution of the scalar transmission problem between a positive material Ω^+ and a negative one Ω^-. The method is based on the use of an optimal control reformulation of the problem that generalizes the one proposed in [3]. Compared to other available works [1, 2], our method is proved to be convergent without any particular assumptions neither on the considered sequence of meshs nor on the regularity of the solution. Our proof of convergence will be illustrated by some numerical experiments.

References

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MS14 **Monday, 12:15–12:40, Zoom Room 04, link tba**

On the approximation of dispersive electromagnetic eigenvalue problems in 2D

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We consider time-harmonic electromagnetic wave equations in composites of a dispersive material surrounded by a classical material. In certain frequency ranges this leads to sign-changing permittivity and/or permeability. Previously meshing rules were reported, which guarantee the convergence of finite element approximations to the related scalar source problems [1]. Here we generalize these results to the electromagnetic two dimensional vectorial equations and the related holomorphic eigenvalue problems [2]. We confirm our theoretical results with computational studies.

References

[1] A.S. Bonnet-BenDhia, C. Carvalho and P. Ciarlet, Mesh requirements for the finite element approximation of problems with sign-changing coefficients, *Numerische Mathematik* **138** (2018), pp. 801–838.

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MS14 **Monday, 12:45–13:10, Zoom Room 04, link tba**

Boundary element methods for plasmonic resonance problems

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We consider Galerkin boundary element methods for the approximation of plasmonic resonance problems. An analysis of the used boundary integral formulation and its numerical approximation is presented in the framework of eigenvalue problems for holomorphic Fredholm operator-valued functions. We employ recent abstract results [1] to show that the Galerkin approximation with Raviart-Thomas elements provides a so-called regular approximation of the underlying operators of the eigenvalue problems. This enables us to apply classical results of the numerical analysis of eigenvalue problems for holomorphic Fredholm operator-valued functions [2,3] which implies convergence of the approximations and quasi-optimal error estimates [4]. We present numerical results which confirm the theoretical findings.

References

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MS14 **Monday, 15:00–15:25, Zoom Room 04, link tba**

Finite element computation of optically resonant modes using contour integration techniques

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Recent research in nanophotonics shows an increasing interest in advanced numerical methods to find optically resonant modes. The latter are modeled by a nonlinear eigenvalue problem involving Maxwell's equations. We discretize Maxwell's equations with a finite element method (FEM), and we employ perfectly matched layers (PML) to account for the radiation condition at infinity [1]. We utilize two contour integration techniques in the complex plane to compute the discrete eigenvalues: the Beyn [2] and Feast [3] algorithms.

In this talk, we consider a series of numerical examples in physically relevant 2D nanophotonic systems including small dielectric and metallic particles. Our main contribution is to propose practical guidelines for selecting the FEM and PML parameters, the quadrature points in the complex contour, and choosing the Beyn or Feast algorithm.

References

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[2] W.-J. Beyn, An integral method for solving nonlinear eigenvalue problems, *Linear Algebra and its Applications* **436** (2012), pp. 3839–3863.

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MS14 Monday, 15:30–15:55, Zoom Room 04, link tba

Complex-scaling method for the complex plasmonic resonances of particles with corners

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This talk will present recent results on the existence and computation of complex plasmonic (CP) resonances $\varepsilon_n \in \mathbb{C}\backslash\mathbb{R}$ for planar particles whose boundaries are smooth except for a finite number of straight corners.

CP resonances are associated with strongly-oscillating fields that do not belong to H^1_{loc}, which prevents from directly using H^1-conforming 2D finite element (FE) approximations. However we show that CP resonances can be computed as eigenvalues of a *modified* plasmonic eigenvalue problem, obtained using a corner complex scaling, whose FE discretization yields a complex-symmetric linear generalized eigenvalue problem of the form $AU = \varepsilon BU$ [1]. Numerical results corroborate the study [2], which proved the existence of embedded plasmonic eigenvalues and discussed the construction of particles that exhibit CP resonances.

References

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MS14 Monday, 16:30–16:55, Zoom Room 04, link tba

Maxwell's equations with hypersingularities at a conical plasmonic tip

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In this talk, we are interested in the analysis of time-harmonic Maxwell's equations in presence of a conical tip of a material with negative dielectric constants. When these constants belong to some critical range, the electromagnetic field exhibits strongly oscillating singularities at the tip which have infinite energy. Consequently Maxwell's equations are not well-posed in the classical L^2 framework. The goal of the present work is to provide an appropriate functional setting for 3D Maxwell's equations when the dielectric permittivity (but not the magnetic permeability) takes critical values. Following what has been done for the 2D scalar case, the idea is to work in weighted Sobolev spaces, adding to the space the so-called outgoing propagating singularities. The analysis requires new results of scalar and vector potential representations of singular fields. The outgoing behaviour is selected via the limiting absorption principle.

References

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MS14 **Monday, 17:00–17:25, Zoom Room 04, link tba**

Electromagnetic scattering by slender bodies

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Nanoresonators showing a high-aspect ratio are ubiquitously used in nanophotonic systems. Owing to their geometry, their resonances properties are amenable to asymptotic analysis, which, in turn, furnishes closed form anaytical formulae for some non-trivial geometries and a convenient numerical scheme otherwise.

In this talk I will show how to use matched asymptotic expansions to obtain such reduced problems in three physical regimes of importance: nonlocal, quasi-static and full-wave.

References

[1] M. Ruiz and O. Schnitzer, Slender-body theory for plasmonic resonance, Proc. R. Soc. A.475:20190294

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MS14 **Monday, 17:30–17:55, Zoom Room 04, link tba**

The transmission eigenvalue problem for the Maxwell Equations

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Transmission eigenvalue problems appear in several mathematical areas, such as the inverse scattering theory and the analysis of the partial differential equations with time changing coefficients. Cakoni and Nguyen [1] recently proposed very general conditions on the coefficients of the Maxwell equations for which they established the discreteness of the set of transmission eigenvalues. In this talk, I discuss the completeness of the generalized eigenfunctions and an upper bound on the counting function under their conditions assuming additionally a weak smoothness condition on the coefficients. This is based on joint work with Hoai-Minh Nguyen [2].

References

[1] F.Cakoni and H-M. Nguyen, On the Discreteness of Transmission Eigenvalues for the Maxwell Equations, *SIAM J. Math. Anal.*, **53** (2021), no. 1, 888–913. MR 4209665.

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MS14 **Tuesday, 11:30–11:55, Zoom Room 04, link tba**

Electromagnetic energy decay rate in dispersive and dissipative media

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It is well-known that electromagnetic dispersive structures such as metamaterials can be modelled by generalized Drude-Lorentz models, see e.g. [1]. In this work, we are interested in dissipative Drude-Lorentz open structures and we wish to quantify the loss in such media in terms of the long time decay rate of the electromagnetic energy for the corresponding Cauchy problem. By using two different approaches; one based on (frequency dependent) Lyapounov estimates and the other on modal analysis, we show that this decay is polynomial in time. These results generalize a part the ones obtained for bounded media in [2] via a quite different method based on the notion of cumulated past history and semi-group theory. A great advantage of the approaches developed here is to be directly connected to the physics of the system via energy balances or modes behavior.

References

[1] M. Cassier, P. Joly and M. Kachanovska, Mathematical models for dispersive electromagnetic waves: an overview., *Computers & Mathematics with Applications* **74** 11 (2017), pp. 2792–2830.

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MS14 **Tuesday, 12:00–12:25, Zoom Room 04, link tba**

Time-domain analysis of perfectly matched layers for waveguides in dispersive media

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We consider the propagation of electromagnetic waves in dispersive waveguides. To be able to treat the unbounded domains numerically we apply so-called perfectly matched layers (PMLs). It is known (see [1]) that applying standard PMLs to such problems can lead to unstable solutions due to the possible occurrence of backward propagating waves. To obtain stable solutions we follow [1] and adapt the PMLs specifically to the material parameters in question.

Despite the fact that the use of PMLs is very popular only very few and recent results (e.g., [2] and [3]) on their convergence in time-domain (which requires considering non-constant damping functions) are available. Continuing the work done in [2] we consider Lorentz materials and study the stability of the modified PMLs, as well as their convergence with respect to the PML thickness and the choice of the damping function.

References

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MS14 **Tuesday, 12:30–12:55, Zoom Room 04, link tba**

Modal approximation for plasmonic resonators in the time domain

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We study the electromagnetic field scattered by a metallic nanoparticle with dispersive material parameters in a resonant regime. We consider the particle placed in a homogeneous medium in a low-frequency regime. We define modes for the non-Hermitian problem as perturbations of electro-static modes, and obtain a modal approximation of the scattered field in the frequency domain. The poles of the expansion correspond to the eigenvalues of a singular boundary integral operator and are shown to lie in a bounded region near the origin of the lower-half complex plane. Finally, we show that this modal representation gives a very good approximation of the field in the time domain. We present numerical simulations in two dimensions to corroborate our results.

References

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MS15 **Monday, 11:45–12:10, Zoom Room 05, link tba**

Slow absolute spectrum: creating eigenvalues in slow-fast nonlinear waves

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Motivated by pulse-replication phenomena observed in the FitzHugh–Nagumo and other reaction diffusion systems, we investigate traveling waves with slow-fast profiles. We identify and rigorously verify a general mechanism by which point eigenvalues of the PDE linearization about such waves accumulate as the slow scale parameter approaches zero. These limit sets are related to the so-called absolute spectrum of points on the slow manifold. Beyond previous results of [1], we also provide first insights into accumulation for non-real spectra.

References

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MS15 **Monday, 12:15–12:40, Zoom Room 05, link tba**

Nonlinear stability of fast invading fronts in a Ginzburg-Landau equation with an additional conservation law

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Consider a Ginzburg-Landau equation, which is coupled to an additional conservation law. This system appears generically as an amplitude equation in Turing pattern-forming systems admitting a conservation law structure such as the Bénard-Marangoni problem. In particular, the system exhibits invading fronts, which connect a stable invading state to an unstable ground state.

In this talk, I will present a nonlinear stability result for sufficiently fast invading fronts with respect to perturbations, which are exponentially localized ahead of the front. The stability proof is based on the use of exponential weights ahead of the front to stabilize the ground state. Furthermore, I will outline the novel challenges compared to previous results, namely the lack of a comparison principle and the fact that the invading state is only diffusively stable, i.e. perturbations of the invading state only decay polynomially in time.

References

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MS15 **Monday, 12:45–13:10, Zoom Room 05, link tba**

Sharp stability results for finite difference approximations of hyperbolic equations

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In this presentation, I will report on some recent works regarding the power boundedness of some specific operators which are typically obtained by approximating hyperbolic partial differential equations by means of finite difference schemes. In a first part, I will focus on the case of convolution operators in one space dimension and prove uniform generalized Gaussian bounds under some mild assumptions on the Fourier transform of the coefficients of the convolution operator extending earlier results by Thomé and Diaconis & Saloff-Coste. In a second part, I will consider the case of finite rank perturbations of Toeplitz operators that have simple eigenvalues on the unit circle. Our result gives a positive answer to a conjecture by Trefethen, Kreiss and Wu that only a weak form of the so-called Uniform Kreiss-Lopatinskii Condition is sufficient to imply power boundedness. In both cases, our techniques of proof rely on precise point-wise estimates of an associated Green's function. This is based on joint works with Jean-François Coulombel.

References

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MS15 **Monday, 15:00–15:25, Zoom Room 05, link tba**

Invasion into remnant instability: a case study of front dynamics

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We study the invasion of an unstable state by a propagating front in a peculiar but generic situation where the invasion process exhibits a remnant instability. Here, remnant instability refers to the fact that the spatially constant invaded state is linearly unstable in any exponentially weighted space in a frame moving with the linear invasion speed. Our main result is the nonlinear asymptotic stability of the selected invasion front for a prototypical model coupling spatio-temporal oscillations and monotone dynamics. We establish stability through a decomposition of the perturbation into two pieces: one that is bounded in the weighted space and a second that is unbounded in the weighted space but which converges uniformly to zero in the unweighted space at an exponential rate. Interestingly, long-time numerical simulations reveal an apparent instability in some cases. We exhibit how this instability is caused by round-off errors that introduce linear resonant coupling of otherwise non-resonant linear modes, and we determine the accelerated invasion speed.

MS15	Monday, 15:30–15:55, Zoom Room 05, link tba

Stability analysis of solutions to the b-family of Peakon Equations

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The Camassa-Holm equation with linear dispersion was originally derived as an asymptotic equation in shallow water wave theory. Among its many interesting mathematical properties, which include complete integrability, perhaps the most striking is the fact that in the case where linear dispersion is absent it admits weak multi-soliton solutions - 'peakons' - with a peaked shape corresponding to a discontinuous first derivative. There is a one-parameter family of generalized Camassa-Holm equations, most of which are not integrable, but which all admit peakon solutions.

In this talk, we establish information about the point spectrum of the peakon solutions and notably find that for suitably smooth perturbations there exists point spectrum in the right half plane rendering the peakons unstable for $b < 1$. The linearized operator is extended on the space of functions $L^2(\mathbb{R})$. For $b \neq \frac{5}{2}$, the spectrum of that extended linearized operator is proved to cover a closed vertical strip of the complex plane around the imaginary axis. For $b = \frac{5}{2}$, the strip shrinks to the imaginary axis, but an additional pair of real eigenvalues exists due to projections to the peakon and its spatial translation. We explore numerically these ideas in the realm of fixed-point iterations, spectral stability analysis and time-stepping of the model for the different parameter regimes. We also discuss recent results concerning the nonlinear stability of smooth solitary wave solutions that exist on a nonzero background. These results were obtained in collaboration with Efstathios G. Charalampidis, Panayotis G. Kevrekidis, Ross Parker, and Dmitry E. Pelinovsky.

MS15	Monday, 16:30–16:55, Zoom Room 05, link tba

Nonlinear stability and interactions of high-energy solitary waves in Fermi-Pasta-Ulam-Tsingou chains

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The dynamical stability of solitary lattice waves in non-integrable FPUT chains is a long standing open problem and has been solved only in the KdV limit, in which the waves propagate with near sonic speed, have large wave length, and carry low energy. There also interactions of such waves can be understood. In this talk I explain similar results in a complementary asymptotic regime of fast and strongly localized waves with high energy [1]. The spectrum of the linearized FPUT operator contains asymptotically no unstable eigenvalues except for the neutral ones that stem from the shift symmetry and the spatial discreteness. Then high-energy waves are linearly stable and nonlinear stability in some orbital is granted by the general, non-asymptotic part of works by Friesecke-Pego and Mizumachi. For the linear stability refined two-scale techniques relate the high-energy wave to a nonlinear asymptotic shape ODE and provide accurate approximation formulas. This yields the existence, local uniqueness, smooth parameter dependence, and exponential localization of fast lattice waves for potentials with algebraic singularity. Proper eigenfunctions can asymptotically be linked to unique normalized solutions of the linearized shape ODE, which disproves the existence of unstable eigenfunctions. First results on the interaction of such waves are sketched.

References

[1] M. Herrmann and K. Matthies. Stability of high-energy solitary waves in Fermi-Pasta-Ulam-Tsingou chains. *Trans. Amer. Math. Soc.*, 372(5):3425–3486, 2019.

MS15 Monday, 17:00–17:25, Zoom Room 05, link tba

On the stability of the periodic waves for the Benney system

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We analyze the Benney model for interaction of short and long waves in resonant water wave interactions. Our particular interest is in the periodic traveling waves, which we construct and study in detail. The main results are that, for all natural values of the parameters, the periodic dnoidal waves are spectrally stable with respect to perturbations of the same period. For another natural set of parameters, we construct the snoidal waves, which exhibit instabilities, in the same setup.

Our results are the first instability results in this context. On the other hand, the spectral stability established herein improves significantly upon the work [1], which established stability of the dnoidal waves, on a subset of parameter space, by relying on the Grillakis-Shatah theory. Our approach, which turns out to give definite answer for the entire domain of parameters, relies on the instability index theory. Interestingly, end even though the linearized operators are explicit, our spectral analysis requires subtle and detailed analysis of matrix Schrödinger operators in the periodic context, which support some interesting features.

References

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MS15 Monday, 17:30–17:55, Zoom Room 05, link tba

Uniform Nonlinear Subharmonic Dynamics of Lugiato-Lefever Periodic Waves

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We study the nonlinear dynamics of spectrally stable T-periodic stationary solutions of the Lugiato-Lefever equation (LLE), a damped nonlinear Schrödinger equation with forcing that arises in nonlinear optics. It is known that spectrally stable T-periodic solutions are nonlinearly stable to NT-periodic, i.e., subharmonic, perturbations for each $N \in \mathbb{N}$ with exponential decay rates of the form $e^{-\delta_N t}$. However, both the exponential rates of decay δ_N and the allowable size of initial perturbations tend to 0 as $N \to \infty$ so that this result is non-uniform in N and is, in fact, empty in the limit $N = \infty$. The primary goal of this talk is to build on recent methodologies developed by the authors in order to obtain a uniform (in N) stability result for subharmonic perturbations at the nonlinear level. The obtained uniform decay rates are shown to agree precisely with the polynomial decay rates of localized, i.e,. integrable on the real line, perturbations of such spectrally stable periodic solutions of LLE.

MS15 **Monday, 18:00–18:25, Zoom Room 05, link tba**

Universal dynamics of pulled fronts

Montie Avery[1,*], **Arnd Scheel**[2]
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We discuss universal features in the propagation of pulled fronts. It has long been known that solutions to the Fisher-KPP equation with sufficiently steep initial data converge to a critical front, up to a shift in the position which is logarithmic in time. A long history of experimental, numerical, and formal asymptotic work in the physics literature suggests this phenomenon is universal, holding for broad classes of equations and initial data. The difficulty in rigorously extending results beyond the Fisher-KPP equation is the lack of a comparison principle in most other interesting models. Here we present rigorous results establishing this universality, identifying conceptual assumptions which guarantee selection of a pulled front with a logarithmic delay from steep initial data. Our results apply to open classes of equations, including higher order equations without comparison principles.

MS15 **Tuesday, 11:30–11:55, Zoom Room 05, link tba**

An Evans function for 2D steady flows of the Euler equation on the torus

Robert Marangell[1,*], **Holger Dullin**[1]
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I will discuss an Evans function for the linearisation of the Euler equation for an ideal fluid on the torus about a shear flow. By separation of variables, the problem is reduced to the study of a complex Hill's equation. The construction of an Evans function relies on the Hill determinant formula, and allows for the complete characterisation of the point spectrum of the linearisation, as well as a count of the eigenvalues with non-zero real part.

MS15 **Tuesday, 12:00–12:25, Zoom Room 05, link tba**

Localized Structures in a Reaction-Diffusion System with Spatially Varying Coefficients

Martina Chirilus-Bruckner[1,*], **Robbin Bastiaansen**[2], **Arjen Doelman**[3]

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The study of localized structures (such as pulse or front solutions) for nonlinear partial differential equations often serves as a convenient starting point for unraveling the complete picture of the possible dynamics for such equations. We present the analysis of a new type of solution – namely, pulses and fronts with heterogeneous tails – for nonlinear reaction-diffusion system with spatially varying coefficients. Our construction is based on the theory of exponential dichotomies and is, hence, not limited to the choice of specific coefficients (e.g. periodic or localized ones). In the context of an extended Klausmaier model, we show existence and linear stability of such solutions and comment on generalizations of the presented results.

MS15 **Tuesday, 12:30–12:55, Zoom Room 05, link tba**

Stability of Decaying Cnoidal Waves in Nonlinear Schrödinger Equations with Linear Dissipation.

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We consider the cubic nonlinear schrodinger equation with a linear damping term on the one dimensional torus. It is well known that the Hamiltonian equation (i.e. without dissipation) admits a class of solitons, called cnoidal waves, which are anti symmetric in space with respect to the half-period. The linear damping forces any solution to decay to zero exponentially fast, hence the usual notion of orbital or asymptotic stability of solitons cannot be applied here. In my talk, I will show a stability result for suitably scaled cnoidal waves, where the scaling takes into account the dissipation effect. More precisely, by considering an initial datum which is a small anti-half-periodic perturbation of a cnoidal wave, we prove that the solution remains close to the family of cnoidal waves with decaying L^2-norm. The main mathematical difficulty to study our problem stems from the fact that the standard Lyapunov functional, constructed around the cnoidal wave, is not able to bound the H^1-norm of the perturbation uniformly in time. In order to provide a global control, we find a first order approximation of the soliton within the dissipative dynamics. The modified Lyapunov functional that controls the perturbation is then constructed around the approximated soliton.

MS16 Tuesday, 16:30–16:55, Zoom Room 09, link tba

Generalized Maslov indices for non-Hamiltonian systems

Graham Cox[1,*], **Thomas Baird**[1], **Paul Cornwell**[2], **Christopher Jones**[3], **Robert Marangell**[4]

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The Maslov index is a topological invariant naturally associated to a Hamiltonian dynamical system. This is a useful tool for linear stability analysis, provided the eigenvalue equation can be written in a Hamiltonian form. In this talk I will describe a recent generalization of the Maslov index to systems that lack this additional structure. This generalization allows one to study reaction-diffusion systems of activator-inhibitor type. As an application, I will show how this new index can be used to characterize the Turing instability.

MS16 Tuesday, 17:00–17:25, Zoom Room 09, link tba

Asymptotic perturbation theory for extensions of symmetric operators

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This talk concerns asymptotic perturbation theory for varying self-adjoint extensions of symmetric operators. First, we will discuss a symplectic version of the celebrated Krein formula for resolvent difference. Then we will switch to an asymptotic analysis of resolvent operators via first order expansion for the path of Lagrangian planes associated with perturbed operators. This asymptotic perturbation theory yields an Hadamard–Rellich-type variational formula for multiple eigenvalue curves bifurcating from an eigenvalue of the unperturbed operator. Applications will be given to quantum graphs, periodic Kronig–Penney model, elliptic second order partial differential operators with Robin boundary conditions, and heat equations with thermal conductivity. This talk is based on various joint projects with G. Berkolaiko (Texas A&M) and Y. Latushkin (Missouri/NYU).

MS16 **Tuesday, 17:30–17:55, Zoom Room 09, link tba**

Counting the Number of 'Unwanted' Eigenvalues. Challenges and Solutions

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In engineering design, negative eigenvalues of essentially positive operators can be a nuisance, e.g., due to resonance effects. There are various methods how to remove known eigenvalues by suitable small perturbations. Contrary to the procedures of linear algebra and to the treatment of linear systems of ordinary differential equations, in general there are no constructive methods of determining the eigenvalues of partial differential equations. Some time ago, the authors with collaborators (see also the work of Y. Latushkin and collaborators) have shown that the number of these "unwanted" eigenvalues can be identified with the Morse index and hence with the Maslov index of an associated curve of Fredholm Lagrangian subspaces in symplectic functional analysis, arising from a natural continuous variation of the original problem. That provided a constructive approach to determining the precise number of, e.g., all negative eigenvalues of a self-adjoint essentially positive operator with compact resolvent. These results, however, depend on technical preconditions regarding (A) the so-called weak inner Unique Continuation Property (wiUCP) and (B) the continuity of the Cauchy data spaces in all relevant Sobolev spaces. Precondition (A) is fulfilled for linear elliptic operators with constant coefficients, with analytric coefficients and for most geometrically defined operators like the Laplacian or operators of Dirac type. For elliptic operators with arbitrarily varying coefficients, the wiUCP is not valid in general. In this talk we shall report on recent joint work with other collaborators on the interconnection between these two technical requirements.

MS16 **Wednesday, 11:30–11:55, Zoom Room 09, link tba**

The Maslov index and the spectrum of the Nonlinear Schrödinger equation

Robert Marangell[1,*], **Mitch Curran**[1], **Graham Cox**[2], **Yuri Latushkin**[3]

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Standing waves in the 1+1 dimensional nonlinear Schrödinger (NLS) equation provide a nice, somewhat prototypical example to examine many features of the Maslov index that are often exploited in spectral calculations, but where monotonicity calculations do not hold. In this talk I will discuss the relationship between the crossing form used to compute the Maslov index and spectral curves that naturally arise in stability problems of NLS type (or really any 1+1 ODE on a compact domain) as well as how the Maslov index can serve as a link between various parameters.

MS16 Wednesday, 12:00–12:25, Zoom Room 09, link tba

Constrained variational problems and L-derivatives

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Given a constrained variational problem, a critical point and a subspace of variations it is possible to construct a Lagrangian plane in some symplectic space which contains information about the Hessian of the minimised functional. In particular, it is possible to control the growth of the Morse index after adding variations via an appropriate variant of the Maslov index. This technique is especially effective in the case when constraints are finite dimensional and its extension to the infinite dimensions raises many interesting questions and possible applications. This is a joint work with A. Agrachev [1].

References

[1] A. Agrachev, I. Beschastnyi. Jacobi fields in optimal control: one-dimensional variations. *J. Dyn. Control Syst.* **26** (2020), no. 4, 685–732.

MS16 Wednesday, 12:30–12:55, Zoom Room 09, link tba

Space versus energy oscillations of Prüfer phases for matrix Sturm-Liouville and Jacobi operators

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This talk considers Sturm oscillation theory for regular matrix Sturm-Liouville operators on finite intervals and for matrix Jacobi operators. The number of space oscillations of the eigenvalues of the matrix Prüfer phases at a given energy, defined by a suitable lift in the Jacobi case, is shown to be equal to the number of eigenvalues below that energy. This results from a positivity property of the Prüfer phases, namely they cannot cross -1 in the negative direction, and is also shown to be closely linked to the positivity of the matrix Prüfer phase in the energy variable. The theory is illustrated by numerical calculations for an explicit example.

References

[1] Hermann Schulz-Baldes and Liam Urban, Space versus energy oscillations of Prüfer phases for matrix Sturm-Liouville and Jacobi operators, *Electronic Journal of Differential Equations* **2020** (2020).

MS16 **Wednesday, 15:00–15:25, Zoom Room 09, link tba**

The Maslov index of solitary waves and symbolic dynamics

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Four-dimensionnal Hamiltoninan systems with a homoclinic orbit with decaying but oscillating tails features a symbolic dynamics [2], and this implies the existence of an infinity of other homoclinic orbits.

These orbits are of interest because some correspond to multi-pulse solitary wave solutions, for example to the Kawahara equation [1].

One can define a Maslov Index for these homoclinic orbits, and it can be related to spectral problems relevant to the stability of the corresponding solitary waves.

In this talk, we relate symbolic dynamics as described in [2] to the Maslov index of these orbits.

References

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[2] R.L. Devaney, Homoclinic orbits in Hamiltonian systems, *J. Differential Equations* **21** (1976), pp. 431–438.

[3] P.Howard, The Maslov Index and Spectral Counts for Linear Hamiltonian Systems on $\mathbb{R}$, *J. Dyn. Diff. Equat.* (2021).

MS16 **Wednesday, 15:30–15:55, Zoom Room 09, link tba**

Validated Spectral Stability via Conjugate Points and the Maslov Index

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Classical results from Sturm-Liouville theory state that the number of unstable eigenvalues of a scalar, second-order linear operator is equal to the number of associated conjugate points. Recent work has extended these results to a much more general setting, thus allowing for spectral stability of nonlinear waves in a variety of contexts to be determined by counting conjugate points. However, in practice, it is not yet clear whether it is easier to compute conjugate points than to just directly count unstable eigenvalues.

We address this issue by developing a framework for the computation of conjugate points using validated numerics. Moreover, we apply our method to a parameter-dependent system of bistable equations and show that there exist both stable and unstable standing fronts. This application can be seen as complimentary to the classical result via Sturm-Louiville theory that in scalar reaction-diffusion equations pulses are unstable whereas fronts are stable, and to the more recent result that symmetric pulses in reaction-diffusion systems with gradient nonlinearity are also necessarily unstable.

MS16 **Wednesday, 16:30–16:55, Zoom Room 09, link tba**

Fredholm determinants, Evans functions and Maslov indices for partial differential equations

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The Evans function is a well known tool for locating spectra of differential operators in one spatial dimension. In this paper we construct a multidimensional analogue, as the modified Fredholm determinant of a ratio of Dirichlet-to-Robin operators on the boundary. This gives a tool for studying the eigenvalue counting functions of second-order elliptic operators, which need not be self-adjoint. In the self-adjoint case we relate our construction to the Maslov index, another well known tool in the spectral theory of differential operators. This connection gives new insight into the Maslov index, allowing us to obtain crucial monotonicity results using methods of complex analysis.

MS16 **Wednesday, 17:00–17:25, Zoom Room 09, link tba**

The Equivariant Spectral Flow and Bifurcation of Periodic Solutions of Hamiltonian Systems

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In the first part of this talk, we review the construction of the spectral flow and its application to bifurcation theory. The second part of our talk introduces a generalisation of the spectral flow to operators that are equivariant under the action of compact Lie groups. Finally, we discuss an application to bifurcation of periodic solutions of autonomous Hamiltonian systems that are equivariant under a type of group action that was considered by Bartsch and Willem in 1994.

MS16 **Wednesday, 17:30–17:55, Zoom Room 09, link tba**

A generalized index theory for non-Hamiltonian systems

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The Maslov index and the spectral flow are topological invariants naturally associated to a Hamiltonian dynamical system. In this talk, we provide a generalization of these invariants for non-Hamiltonian system and we prove a new generalized spectral flow formula. As an application, we construct a generalized Morse index theory.

If time permits some applications to reaction-diffusion systems will be also discussed.

MS17 **Thursday, 10:30–10:55, Zoom Room 01, link tba**

Soliton resolution and asymptotic stability for the sine-Gordon equation

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In this paper, we study the long-time dynamics of the sine-Gordon equation

$$\partial_{tt} f - \partial_{xx} f + \sin f = 0,\ (x, t) \in \mathbb{R} \times \mathbb{R}^+.$$

Firstly, we use the nonlinear steepest descent for Riemann-Hilbert problems to compute the long-time asymptotics of the solutions to the sine-Gordon equation whose initial condition belongs to some *weighted Sobolev spaces*. Secondly, combining the long-time asymptotics with a refined approximation argument, we analyze the asymptotic stability for multi-soliton solutions to the sine-Gordon equation in weighted energy spaces. It is known that the obstruction to the asymptotic stability of kink solutions to the sine-Gordon equation in the energy space is the existence of small breathers which is also closely related to the emergence of wobbling kinks. Our stability analysis gives a criterion for the weight which is sharp up to the endpoint so that the asymptotic stability holds.

MS17 **Thursday, 11:00–11:25, Zoom Room 01, link tba**

Riemann problem for the Benjamin-Bona-Mahony equation

Thibault Congy[1,*], **Gennady A. El**[1], **Mark A. Hoefer**[2], **Michael Shearer**[3]

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The Benjamin-Bona-Mahony (BBM) equation $u_t + uu_x = u_{xxt}$ is a model for unidirectional, weakly nonlinear dispersive shallow water wave propagation, asymptotically equivalent to the Korteweg-de Vries (KdV) equation. The BBM dispersion relation is nonconvex, a property that gives rise to a number of intriguing features markedly different from those found in the KdV equation, providing the motivation for the study of the BBM equation as a distinct dispersive regularisation of the Hopf equation.

The dynamics of the step initial value problem or Riemann problem for BBM equation are studied using asymptotic methods and numerical simulations. I will present the emergent wave phenomena for this problem which can be split into two categories: classical (observed in convex KdV-type dispersive hydrodynamics) and nonclassical. Nonclassical features are due to nonconvex dispersion and include the generation of two-phase linear wavetrains, expansion shocks, solitary wave shedding, dispersive Lax shocks, DSW implosion and the generation of incoherent solitary wavetrains. This presentation is based on the work [1]

References

[1] T. Congy, G.A. El, M.A. Hoefer and M. Shearer, *Studies in Applied Mathematics* **147** (2021), pp. 1089–1145.

MS17 **Thursday, 11:30–11:55, Zoom Room 01, link tba**

Stability results of periodic multi-solitons for perturbed KdV equations

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I will present some recent results concerning the stability of periodic multi-solitons of the perturbed KdV equation. More precisely I will focus on the following two results: 1) The persistence of the majority of periodic multi-solitons under a sufficiently small perturbation. This is a KAM-type result. 2) I will show that for initial data sufficiently close to the periodic multi-soliton, the corresponding solution of the perturbed KdV equation remains close to the multi-soliton for long times. The proofs are based on several ingredients: 1) The construction of suitable Normal Form coordinates around the periodic multi-soliton, having an expansion in terms of Pseudo-differential operators. 2) KAM and normal form techniques based on Pseudo-differential and Para-differential operators.

This are joint works with Thomas Kappeler.

MS17 **Thursday, 15:00–15:25, Zoom Room 01, link tba**

Dirichlet-to-Neumann Map for Evolution PDEs on the Half-Line with Time-Periodic Boundary Conditions

Maria Christina van der Weele[1,*], Athanassios S. Fokas[1]
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For a well-posed boundary value problem, a certain number of boundary values must be prescribed as boundary conditions, while the rest of the boundary values are unknown. The task of determining the unknown boundary values in terms of the prescribed ones is called the computation of the "generalised Dirichlet-to-Neumann map". Here we elaborate on a new approach for finding the Dirichlet-to-Neumann map in the large time limit for evolution PDEs on the half-line, for the physically significant case of time-periodic boundary conditions [1].

The new method is illustrated both for linear PDEs (including the heat equation, the convection-diffusion equation and the linearised KdV equation) and for integrable nonlinear PDEs, in particular for the focusing NLS equation. It is shown that the time-dependent part of the Lax pair is instrumental in yielding, via an elegant algebraic calculation, the large t asymptotics of the periodic unknown boundary values in terms of the prescribed periodic boundary data. This method is based on earlier work by Lenells and Fokas [2], in which the NLS equation was treated via a more complicated approach.

References

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[2] J. Lenells and A. S. Fokas, The Nonlinear Schrödinger Equation with t-Periodic Data: II. Perturbative Results, *Proc. R. Soc. A* **471** (2015), 20140926.

MS17 **Thursday, 15:30–15:55, Zoom Room 01, link tba**

The defocusing NLS with step-like oscillatory initial data

Ronald Quirchmayr[1,*], **Samuel Fromm**[2], **Jonatan Lenells**[2]
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We present results concerning the Cauchy problem for the defocusing nonlinear Schrödinger (NLS) equation under the assumption that the solution vanishes as $x \to +\infty$ and approaches an oscillatory plane wave as $x \to -\infty$. This talk covers the corresponding direct and inverse scattering transform via a Riemann-Hilbert formalism, the existence of a global solution for the Cauchy problem and the derivation of its long-time behavior by means of a steepest descent analysis.

References

[1] S. Fromm, J. Lenells and R. Quirchmayr, The defocusing nonlinear Schrödinger equation with step-like oscillatory initial data, arXiv: 2104.03714.

MS17 **Thursday, 16:30–16:55, Zoom Room 01, link tba**

Multi-solitary waves of the Benjamin–Ono equation on the line

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Every multi-soliton manifold of the Benjamin–Ono equation on the line is invariant under the Benjamin–Ono flow. Its generalized action–angle coordinates allow to solve this equation by quadrature and we have the explicit expression of every multi-solitary wave.

If you have further questions, please do not hesitate to contact me at

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References

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MS17 **Thursday, 17:00–17:25, Zoom Room 01, link tba**

On universality for scalar nonlinear waves in 1+1 dimensions

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We present some results concerning the universality of the critical behaviour (in the sense of Dubrovin) for scalar nonlinear waves. More precisely, we consider the following model equation for 1-dimensional scalar unidirectional waves

$$u_t + a(u)u_x + N[u] = 0, \tag{1}$$

where $a(u)$ is a non constant function (in most relevant cases $a(u) = u$) and N is a nonlinear – and possibly nonlocal – differential operator, which admits a long-wave expansion, in such a way that in the long-wave regime (1) is a small perturbation of the Hopf equation.

We classify –by partially heuristic methods– the local behaviour of the shock-wave close to the gradient catastrophe of the Hopf equation (i.e. *the critical behaviour in the sense of Dubrovin*) and show that there exist exactly three integrable critical behaviours, each modelled explcitly in terms of string solutions of the Burgers, KdV or Bejamin-Ono equation.

References

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MS18 **Wednesday, 11:30–11:55, Zoom Room 04, link tba**

Global seismic tomography with millions of waveforms

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Since the advent of seismic tomography, the internal structure of Earth has been vastly sampled and imaged at a variety of scales. Recent advances in seismic data acquisition, methodology and computing power have drastically progressed tomographic models. Broad-band seismic waveforms can now be simulated up to the highest naturally occurring frequencies and consequently, measurement techniques can exploit seismic waves in their entire usable spectrum and in multiple frequencies.

Here, we present three new global tomography models of 3-D isotropic P-wave velocity in the earth's mantle. Multifrequency cross-correlation traveltimes are measured in passbands from 30 s dominant period to the highest frequencies that produce satisfactory fits ($\approx$3 s). Synthetics up to 1 s dominant period are computed by full wave propagation in a spherically symmetric earth using the spectral-element method AxiSEM. These models are generated using a linear inversion technique in which the traveltime observations are linearly linked to the discretized earth model (as parameterized by an adaptive tetrahedral grid with $\approx$400,000 vertices). In this talk, we discuss the impacts of three factors on the resulting models: (1) data selection; (2) (data-driven) parameterization; and (3) regularization parameters. We also show how the inclusion of a very large core-diffracted waves can result in tomographic resolution matching and exceeding that of global S-wave tomographies, which have long been the models of choice for interpreting lowermost mantle structure.

MS18 **Wednesday, 12:00–12:25, Zoom Room 04, link tba**

Local Multiscale Post-Processing of Seismic Data

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We present a multiscale post-processing method in exploration. Based on a physically relevant mollification technique involving the elasto-oscillatory Cauchy-Navier equation, we mathematically describe the extractable information within 3D migration models as is commonly used for geophysical exploration purposes. More explicitly, the developed multiscale approach extracts and visualizes geological features inherently available in signature bands of certain geological formations, such as aquifers, salt domes etc. by specifying suitable wavelet bands. We compare the presented approach with already existing methods such as Helmholtz multiscale decorrelation.

References

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MS18 **Wednesday, 12:30–12:55, Zoom Room 04, link tba**

A novel approach to travel time tomography for ray theory

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For linear(ized) inverse problems on the ball, there exist a variety of global as well as local basis systems, e.g. polynomial or finite element based ones. For a representation, we often decide for one type of trial function and, by this, accept their benefits and weaknesses. We developed alternative approximation methods for certain examples from the geosciences. In summary, they are called the Inverse Problem Matching Pursuits (IPMPs).
By construction, the IPMPs combine different basis systems. They build an approximation in a so-called best basis chosen iteratively from an intentionally overcomplete set of diverse trial functions, the dictionary. By reducing the Tikhonov functional, one dictionary element is chosen for the best basis in each iteration.
The methods proved to be working well, for instance, for the downward continuation of the gravitational potential as well as the MEG-/EEG-problem from medical imaging. These successes and the differences between existing models in travel time tomography inspired the application of the methods for seismological ray theory.
An earthquake produces body waves or rays. At arrival, seismometers at the surface measure the travel time which is influenced by the material the ray moved through. Thus, travel time tomography aims at understanding the interior of the Earth. Remodelling the IPMPs for travel time tomography includes: developing (dependent on the data) the operator, deciding for dictionary elements and applying the operator to them. Moreover, we have to establish termination criteria and the regularization. We compute the regularization terms analytically as far as possible. In this talk, we introduce the IPMPs and results from our remodelling.

MS18 **Wednesday, 15:00–15:25, Zoom Room 04, link tba**

Parsimonious Global Full-Waveform Inversion

Andreas Fichtner[1,*], **Solvi Thrastarson**[1], **Dirk-Philip van Herwaarden**[1], **Michael Afanasiev**[2], **Lion Krischer**[2], **Martin van Driel**[1], **Christian Boehm**[1]
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We present an ensemble of forward modelling and inversion strategies that can reduce the computational cost of global-scale full seismic waveform inversion by more than an order of magnitude. The two most important components are the following: (1) Wavefield-adapted spectral-element meshes, constructed in accord with prior knowledge on wavefield geometry in relatively smooth tomographic Earth models, which effectively turn the 3D forward modelling problem into a nearly-2D problem [1]. (2) Dynamic mini-batch optimisation that quasi-randomly selects data sub-volumes for each iteration, that are small but representative of the complete dataset [2]. In addition to the method itself, we present a real-data application that reveals details of 3D global Earth structure while requiring only a fraction of the computational resources required for comparable full-waveform models.

References

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MS18 **Wednesday, 15:30–15:55, Zoom Room 04, link tba**

Approximation Properties of the Double Fourier Sphere Method

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We investigate analytic properties of the double Fourier sphere (DFS) method, which transforms a function defined on the two-dimensional sphere to a function defined on the two-dimensional torus [2]. Then the resulting function can be written as a Fourier series yielding an approximation of the original function.

We show that the DFS method preserves smoothness [1]: it continuously maps spherical Hölder spaces into the respective spaces on the torus, but it does not preserve spherical Sobolev spaces in the same manner. Furthermore, we show sufficient conditions for the absolute convergence of the resulting series expansion on the sphere as well as results on the speed of convergence.

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MS18 **Wednesday, 16:30–16:55, Zoom Room 04, link tba**

Secondary Microseisms: The Generation of Love Waves and Direct Observations Using Mobile Hydrophone Arrays

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Secondary microseisms are ubiquitous ambient noise vibrations due to ocean activity. Their origin is a heterogeneous distribution of pressure fluctuations along the ocean surface. We present global-scale modelling of secondary microseisms using a spectral-element method, which accounts for surface sources, topography and bathymetry, and 3-D heterogeneity [1]. Seismic Love waves emerge ergodically once the system reaches steady state, due to lateral heterogeneities in Earth structure, which solves a 100-year-old conundrum in seismic wave phenomena. We also report on the first direct measurements of microseisms in the water column by autonomous profiling floats equipped with a hydrophone, MERMAID. In the period ranges between 1 and 10 s, the temporally resolved infrasonic ambient oceanic noise spectrum very closely matches predictions from an independently produced ocean-wave model, WAVEWATCH III, which is in line with the Longuet-Higgins–Hasselmann "frequency-doubling" mechanism.

References

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MS18 **Wednesday, 17:00–17:25, Zoom Room 04, link tba**

Realistic modeling of gravity strain due to prompt elasto-gravity signals (PEGS) – How simple can the Earth model be?

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As soon as the rupture of an earthquake starts, seismic waves begin to spread. These waves change the density distribution in the Earth, which results into a change in gravity. This change in gravity can be measured with gravimeters and seismometers even before the first seismic wave arrives.

Gravimeters and seismometers, however, not only measure the change in gravity mentioned, but also a ground acceleration that is caused by this change in gravity. The resulting signals partially cancel which makes the detection of so called prompt elasto-gravity signals (PEGS) a challenging task. Instruments that are sensitive to changes in gravity but not to ground acceleration would be advantageous.

Gravity strainmeters, that are currently under development would be such instruments. Studies have shown their promising capabilities to detect PEGS. These studies all modeled gravity strain due to PEGS in a homogeneous half space. The question of this presentation is, if this Earth model is too simple or if it can be even simpler to get realistic results, when modeling gravity strain of PEGS.

MS18 **Wednesday, 17:30–17:55, Zoom Room 04, link tba**

A tree of Indo-African mantle plumes imaged by regional seismic tomography

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Mantle plumes are commonly envisioned as thin, buoyant conduits rising vertically from the core mantle boundary (CMB) to the earth's surface, where they produce volcanic hot spots. Most hotspots are located in the sparsely instrumented oceans.

We present a 3-D P-wave tomography model, supplemented by a global data set of P-diffracted measurements, and by a selection of analyst-picked, teleseismic P arrival times. We achieve high image resolution beneath the Indian Ocean hemisphere, and especially beneath La Réunion. We observe the Large Low Velocity Province (LLVP) rising 800 km above the CMB, forming a cusp beneath South Africa. A low-velocity branch undulates obliquely from this cusp region towards the uppermost mantle beneath La Réunion. Hence La Réunion's connection to the lower mantle is more complex than previously envisioned.

Our results provide the first high-resolution image of a western Indian Ocean plume cluster, from the surface to the CMB. This represents a key advance for linking geophysical, geodynamic and geochemical observations.

MS19	Thursday, 10:30–10:55, Zoom Room 02, link tba

Asymptotics of Moore-Gibson-Thompson equations with memory

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The Moore-Gibson-Thompson (MGT) equation is a wave-type equation arising in acoustics when the paradox of the infinite speed is eliminated by considering the Maxwell–Cattaneo law instead of the Fourier one. In this talk, we consider the MGT equation with memory, a model which accounts for additional nonlocal effects due to molecular relaxation, and we investigate its asymptotic properties.

References

[1] F. Dell'Oro, I. Lasiecka and V. Pata, The Moore–Gibson–Thompson equation with memory in the critical case, *Journal of Differential Equation* **261** (2016), pp. 4188–4222.

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MS19	Thursday, 11:00–11:25, Zoom Room 02, link tba

On the Moore-Gibson-Thompson equation with memory with nonconvex kernels

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We consider the MGT equation with memory

$$\partial_{ttt}u + \alpha\partial_{tt}u - \beta\Delta\partial_t u - \gamma\Delta u + \int_0^t g(s)\Delta u(t-s)ds = 0.$$

We prove an existence and uniqueness result removing the convexity assumption on the convolution kernel g, usually adopted in the literature. In the subcritical case $\alpha\beta > \gamma$, we establish the exponential decay of the energy, without leaning on the classical differential inequality involving g and its derivative g', namely,

$$g' + \delta g \leq 0, \quad \delta > 0,$$

but only asking that g vanishes exponentially fast.

MS19 **Thursday, 11:30–11:55, Zoom Room 02, link tba**

On the Cauchy problem of the standard linear solid model with heat conduction: Fourier versus Cattaneo

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We consider the standard linear solid model in $\mathbb{R}^N$ coupled, first, with the Fourier law of heat conduction (see [1]) and, second, with the Cattaneo law (see [2]). We first give the appropriate functional setting to prove the well-posedness of both models in the critical and subcritical cases. Second, using the energy method in the Fourier space, we obtain the optimal decay rate of a norm related to the solution for each of the thermoviscoelastic models and, therefore, their asymptotic stability. The eigenvalues expansion method shows the optimality of the decay rate of the solution itself in some of the cases. We compare these results between them and with the Cauchy problem without heat conduction, both in the critical and subcritical cases. The regularity-loss phenomenon is discussed in all cases.

References

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MS19 **Thursday, 15:00–15:25, Zoom Room 02, link tba**

Shape sensitivity analysis for optimization problems in nonlinear acoustics

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In various biomedical applications, precise focusing of nonlinear ultrasonic waves is crucial for efficiency and safety. In this talk, we will discuss the shape sensitivity analysis for a class of optimization problems constrained by general quasi-linear acoustic wave equations that arise in high-intensity focused ultrasound (HIFU) applications. Within our theoretical framework, the Westervelt and Kuznetsov equations of nonlinear acoustics are obtained as particular cases. The quadratic gradient nonlinearity, specific to the Kuznetsov equation, requires special attention throughout. To discuss the existence of the Eulerian shape derivative within the variational framework of [1], we will review the local well-posedness and regularity of the forward problem under the hypothesis of small initial and boundary data. We will give an overview of the analysis of the corresponding adjoint problems for cost functionals of practical interest and conclude with the expression of a well-defined shape derivative. The talk is based on [2].

References

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MS19 **Thursday, 15:30–15:55, Zoom Room 02, link tba**

Time-fractional Moore–Gibson–Thompson equations

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It is well-known that using the classical Fourier temperature flux law in the derivation of second-order models of nonlinear acoustics may lead to the paradox of infinite speed of propagation. On the other hand, using the hyperbolic heat equation may violate the second law of thermodynamics. Fractional generalizations of the heat flux equation have emerged in the literature as a way of interpolating between the properties of these two laws; see, e.g., [1].

In this talk, we will discuss the derivation and analysis of several time-fractional generalizations of the Jordan–Moore–Gibson–Thompson (JMGT) equations in nonlinear acoustics. In particular, we will consider their local well-posedness and relate them to the classical third order in time JMGT equation via their respective limits as the fractional order tends to one. The talk is based on [2].

References

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MS19 **Thursday, 16:30–16:55, Zoom Room 02, link tba**

The Cauchy Problem for the JMGT Equation

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We consider the Cauchy problem of a third-order in time nonlinear equation known as the Jordan–Moore–Gibson–Thompson (JMGT) equation arising in acoustics as an alternative model to the well-known Kuznetsov equation. We show a local existence result in appropriate function spaces, and, using the energy method together with a bootstrap argument, we prove a global existence result for small data, without using the linear decay. Finally, polynomial decay rates in time for a norm related to the solution will be obtained.

MS19 **Thursday, 17:00–17:25, Zoom Room 02, link tba**

Global well-posedness for the solution of the Jordan–Moore–Gibson–Thompson equation with arbitrary large higher-order Sobolev norms.

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In this talk, we consider the nonlinear Jordan-Moore-Gibson-Thompson equation (JMGT) arising in acoustics. First, we prove that the solution exists globally in time provided that the lower order Sobolev norms of the initial data are considered to be small, while the higher-order norms can be arbitrarily large. This improves some recent result. Second, we prove a new decay estimate for the linearized model by removing the L^1-assumption on the initial data. The proof of this decay estimate is based on the high-frequency and low-frequency decomposition of the solution together with an interpolation inequality related to Sobolev spaces with negative order.

MS20 **Thursday, 15:00–15:25, Zoom Room 08, link tba**

Photoacoustic Reconstruction Using Sparsity in Curvelet Frame: Image versus Data Domain

Marta M. Betcke[1,*], **Bolin Pan**[1], **Simon R. Arridge**[1], **Felix Lucka**[2], **Ben T. Cox**[3], **Nam Huynh**[3], **Paul C. Beard**[3], **Edward Z. Zhang**[3]

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Curvelet frame is of special significance for photoacoustic tomography (PAT) due to its sparsifying and microlocalisation properties. In this talk we derive a one-to-one map between wavefront directions in image and data spaces in PAT which suggests near equivalence between the direct reconstruction of the initial pressure and two step approach with intermediate stage of PAT data recovery from compressed/subsampled measurements when assuming sparsity in Curvelet frame. As the latter approach is computationally more tractable, we investigate to which extent this equivalence holds. Effective representation of the photoacoustic data requires basis defined on the range of the photoacoustic forward operator. To this end we propose a novel *wedge-restriction* of Curvelet transform which enables us to construct such basis. Both recovery problems are formulated in a variational framework. As the Curvelet frame is heavily overdetermined, we use reweighted ℓ_1 norm penalties to enhance the sparsity of the solution. We compare and discuss the relative merits of the two approaches and illustrate them on 2D simulated and 3D real data in a fair and rigorous manner.

MS20 **Thursday, 15:30–15:55, Zoom Room 08, link tba**

Time-reversal for Full Field Photoacoustic Tomography with Variable Sound Speed

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Photoacoustic tomography (PAT) is a non-invasive imaging modality that requires recovering the initial data of the wave equation from certain measurements of the solution outside the object. In the standard PAT measurement setup, the used data consist of time-dependent signals measured on an observation surface. In contrast, the measured data from the recently invented full-field detection technique provide the solution of the wave equation on a full spatial domain at a single instant in time. While reconstruction using classical PAT data has been extensively studied, not much is known for the full-field PAT problem. In this paper, we settle mathematical foundations of the latter problem for variable sound speed and settle its uniqueness and stability. Moreover, we introduce an exact inversion method using time-reversal and study its convergence. Our results demonstrate the suitability of both the full-field approach and the proposed time-reversal technique for high resolution photoacoustic imaging.

MS20 **Thursday, 16:30–16:55, Zoom Room 08, link tba**

Electrical impedance tomography using virtual X-rays, deconvolution, and machine learning

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A fundamental connection between Electrical Impedance Tomography (EIT) and classical X-ray tomography was found in [1] by using microlocal analysis. There, it was shown that a one-dimensional Fourier transform applied to the spectral parameter of Complex Geometric Optics (CGO) solutions produces generalised projections, enabling a novel filtered back-projection type nonlinear reconstruction algorithm for EIT. This approach is called Virtual Hybrid Edge Detection (VHED).

One of the medically most promising applications of EIT is stroke imaging. The main difficulty in using EIT for head imaging is the resistive skull. Because of that, the relevant signal from the brain is weak and almost buried in noise. VHED offers a way to divide the information in EIT measurements into geometrically understood pieces. One could wish that those pieces are less sensitive to noise than a full reconstructed image of the conductivity. This presentation shows how machine learning can be used for classifying stroke based on VHED profiles. Examined are fully connected neural networks (FCNN), convolutional neural networks (CNN) and recurrent neural networks (RNN).

References

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MS20 **Thursday, 17:00–17:25, Zoom Room 08, link tba**

Propagation of singularities in subelliptic wave equations

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We consider the propagation of singularities in subelliptic wave equations. Firstly, revisiting ideas of R. Melrose, we show that singularities propagate only along null-bicharacteristics and abnormal extremal lifts of singular curves. This result makes a bridge with classical notions in sub-Riemannian geometry, since abnormal extremal lifts are known to be possible minimizers of the sub-Riemannian distance.

Secondly, we illustrate the previous general result in the particular Martinet case. We construct initial data whose singularities propagate along any Martinet singular curve at any speed between 0 and 1. This is in strong contrast with the propagation at speed 1 in Hörmander's theorem for PDEs with principal symbol of real principal type. This second part is a joint work with Yves Colin de Verdière.

References

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MS20 **Thursday, 17:30–17:55, Zoom Room 08, link tba**

Frame based regularizations in photoacoustic tomography

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In this talk we introduce the concept of frame based regularizaiton for inverse problems in Hilbert spaces as a generalization of classical filter methods that are based singular value decompositions. We will discuss that frames (as generalized singular system) allow for a better adaption to a given class of potential solutions. We also show that frame based filtered methods yield convergent regularization methods and discuss recent development of this relatively novel approach. In the context photoacoustic tomography, we will present an operator adapted frame decompositions for the PAT operator and present concrete regularization methods based on such frame decompositions.

MS20 **Thursday, 18:00–18:25, Zoom Room 08, link tba**

Microlocal analysis of Compton scattering imaging

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As revealed in [1], in which the author studied the spectral data in Compton scattering imaging (CSI) and its different scattering components in terms of smoothness properties, the analysis and the use of the first-order scattering is essential in order to achieve image reconstruction from CSI data.

A proper analysis shows that one viewpoint is not necessary in general to encode all the wavefront-sets of the sought-for object leading to limited data issues. We aim to design a proper configuration for a CSI scanner enabling a full reconstruction of the wavefront-sets and using multiple viewpoints. Such a configuration will then deliver data suited for reconstruction methods based on minimization problems [2] or even learning techniques.

References

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MS20 **Thursday, 18:30–18:55, Zoom Room 08, link tba**

Microlocal analysis of generalized Radon transforms from scattering tomography

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Here we present a novel microlocal analysis of generalized Radon transforms which describe the integrals of L^2 functions of compact support over surfaces of revolution of C^∞ curves q. We show that the Radon transforms are elliptic Fourier Integral Operators (FIO) and provide an analysis of the left projections Π_L. Our main theorem shows that Π_L satisfies the semi-global Bolker Assumption if and only if $g = q'/q$ is an immersion. An analysis of the visible singularities is presented, after which we derive novel Sobolev smoothness estimates for the generalized Radon FIO. Our theory has specific applications of interest in Emission Compton Scattering Tomography (ECST) and Bragg Scattering Tomography (BST). We show that the ECST and BST integration curves satisfy the Bolker Assumption and provide simulated reconstructions from ECST and BST data. Additionally we give example "sinusoidal" integration curves which do not satisfy Bolker and provide simulations of the image artifacts. The observed artifacts in reconstruction are shown to align exactly with our predictions.

References

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MS21 **Monday, 11:45–12:10, Zoom Room 06, link tba**

Multi-solitons for coupled Lowest Landau Level equations

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We consider a coupled system of nonlinear Lowest Landau Level equations. We first show the existence of multi-solitons with an exponentially localised error term in space, and then we prove a uniqueness result. We also show a long time stability result of the sum of traveling waves having all the same speed, under the condition that they are localised far away enough from each other. Finally, we observe that these multi-solitons provide examples of dynamics for the linear Schrödinger equation with harmonic potential perturbed by a time-dependent potential.

MS21 **Monday, 12:15–12:40, Zoom Room 06, link tba**

Uniform bound of solutions of the 2-D Benjamin-Ono equation

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We study the BO-ZK operator in the two-dimensional case. This operator contains both local and nonlocal terms and appears in a model describing the electromigration in thin nanoconductors on a dielectric substrate. It is also introduced in the half-wave-Schrödinger equation. We first prove the associated anisotropic Gagliardo-Nirenberg inequality and its best constant. Next, as an application, by considering the related evolution equations and their Cauchy problems, we show the uniform bound of solutions in the energy space.

References

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MS21	Monday, 12:45–13:10, Zoom Room 06, link tba

Well-posedness for the KdV hierarchy

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The Kortweg-de Vries equation (KdV) $u_t = -u_{xxx} + 6uu_x$ is part of an infinite family of nonlinear PDE, known as the KdV hierarchy, whose flows all commute with each other. For KdV itself recent work by Killip and Visan showed well-posedness in $H^{-1}(\mathbb{R})$ and $H^{-1}(\mathbb{T})$ [1]. Bringmann, Killip and Visan showed well-posedness for the fifth order KdV equation in $H^{-1}(\mathbb{R})$ and $L^2(\mathbb{T})$ [2]. Making use of a modified Miura transform and hence connecting KdV and Gardner hierarchies, we extend these results to the whole KdV hierarchy and show well-posedness for the Nth KdV (of order $2N+1$) in $H^{-1}(\mathbb{R})$ and $H^{N-2}(\mathbb{T})$.

References

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[2] B. Bringmann, R. Killip, and M. Visan, Global well-posedness for the fifth-order KdV equation in $H^{-1}(\mathbb{R})$, (2019), *arXiv:1912.01536 [math.AP]*

MS21	Monday, 15:00–15:25, Zoom Room 06, link tba

Solitons interaction for the damped non linear Klein-Gordon equation

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The damped Klein-Gordon equation is a model for dispersive and dissipative equations: it still admits stationary solutions, which are nonetheless unstable (and which we call soliton). We will present some recent works with Yvan Martel, Xu Yuan and Lifeng Zhao on the description of 2-solitons, and in one space dimension, on the resolution in solitons.

MS21 **Monday, 15:30–15:55, Zoom Room 06, link tba**

Soliton resolution for energy-critical equivariant wave maps

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We consider wave maps $\mathbb{R}^{1+2} \to \mathbb{S}^2$, under the assumption of equivariant symmetry. We prove that every solution of finite energy resolves, as time passes, into a superposition of harmonic maps (solitons) and radiation. It was proved in works of Côte [1], and Jia and Kenig [3], that such a decomposition holds along a sequence of times. We show that the resolution holds continuously in time via a "no-return lemma" based on the virial identity. The proof combines a modulation analysis of solutions near a multi-soliton configuration with the concentration-compactness method.

References

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[2] J. Jendrej and A. Lawrie, Soliton resolution for equivariant wave maps, *Preprint*, `https://arxiv.org/abs/2106.10738`.

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MS21 **Monday, 16:30–16:55, Zoom Room 06, link tba**

Large time well posedness for a Dirac–Klein-Gordon system

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In this talk, we discuss well posedness for a system coupling a nonlinear Dirac with a Klein-Gordon equation that represents a toy model for the Helium atom with relativistic corrections: the wave function of the electrons interacts with an electric field generated by a nucleus with a given charge density. One of the main ingredients we need is a new family of Strichartz estimates for time dependent perturbations of the Dirac equation. This is a joint work with F. Cacciafesta, Long Meng, Jérémy Sok (U. Padova).

MS21 **Monday, 17:00–17:25, Zoom Room 06, link tba**

Small data global results for a Cauchy problem related to biharmonic wave maps

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In this talk, we present the small data global wellposedness of bi-harmonic wave equations with quadratic nonlinearities of the form

$$\begin{aligned}\partial_t^2 u + \Delta^2 u = & \; Q_u(\partial_t u, \partial_t u) + 2Q_u(\nabla^2 u, \nabla^2 u) + Q_u(\Delta u, \Delta u) \\ & + \; 2Q_u(\nabla u, \nabla \Delta u) + 2Q_u(\nabla \Delta u, \nabla u)\end{aligned}$$

in dimension $n \geq 3$ in the scaling critical Besov space $\dot{B}^{\frac{n}{2}}_{2,1} \times \dot{B}^{\frac{n}{2}-2}_{2,1}$ with persistence of $\dot{H}^s \times \dot{H}^{s-2}$ regularity. This is achieved by a modification of Tataru's F, $\Box F$ spaces in the solution of the division problem for high dimensional wave maps [1], where the main obstruction in our work is to overcome the additional loss of regularity of the above fourth-order operator. To this end, we present a smoothing property, similarly used in the work on low regularity Schrödinger maps by Bejenaru, Ionescu-Kenig. In our case, we derive lateral Strichartz estimates by the calculations in [2], which then imply a gain of regularity in the bilinear estimates.

References

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[2] I. Bejenaru, Global results for Schrödinger maps in dimensions $n \geq 3$, *Comm. Partial Differential Equations*, **33.3** (2008), pp. 451-477.

MS21 **Monday, 17:30–17:55, Zoom Room 06, link tba**

On scattering for nonlinear dispersive equations with randomized data

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The local and global wellposedness theory of nonlinear dispersive equations with randomized data has attracted a lot of interest over the last years. In particular in the scaling-supercritical regime, where a deterministic wellposedness theory fails, randomization has become an important tool to study the generic behaviour of solutions. But also for questions beyond the reach of current methods, randomization has been used to advance the understanding of the problem.

In this talk, we present several randomization techniques from [1,2] and discuss the resulting (almost surely) improved space-time integrability of solutions of the linear Schrödinger and wave equation with randomized data. We then explain how these improved space-time estimates can be used to prove almost sure scattering for certain nonlinear equations.

References

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[2] M. Spitz, Randomized final-state problem for the Zakharov system in dimension three, *To appear in Comm. Partial Differential Equations.*

MS21 **Monday, 18:00–18:25, Zoom Room 06, link tba**

Stable blowup for the supercritical hyperbolic Yang–Mills equations

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We consider the Yang-Mills equations in $(1 + d)$-dimensional Minkowski spacetime. Under equivariance symmetry, this model reduces to a nonlinear radial wave equation, which admits in all supercritical dimensions, i.e., for $d \geq 5$, a closed form self-similar blowup solution, as discovered by Bizoń and Biernat. The significance of this discovery lies in the fact that this solution is conjectured to be the universal attractor for large equivariant data evolutions. In this talk we illustrate the first step in establishing this conjecture. Namely, we show that the blowup mechanism exhibited by the Bizoń-Biernat solution is stable.

MS21 **Tuesday, 15:00–15:25, Zoom Room 06, link tba**

Stability of self-similar blowup in supercritical wave equations

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I will review the state of the art in our understanding of the stability of blowup via self-similar solutions for supercritical wave equations. Furthermore, I will report on recent progress and new directions of research in this field. The talk is based on joint work with David Wallauch and Matthias Ostermann.

MS21 Tuesday, 15:30–15:55, Zoom Room 06, link tba

Collapsing-ring blowup solutions for the parabolic-elliptic Keller-Segel system

Charles Collot[1,*], **T.-E. Ghoul**[2], **N. Masmoudi**[2], **V. T. Nguyen**[2]
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The parabolic-elliptic Keller-Segel equation in three dimensions is a model for chemotaxis:

$$\text{(KS)} \qquad \begin{cases} \partial_t u = \nabla \cdot (\nabla u - u \nabla \Phi_u), \\ -\Delta \Phi_u = u, \end{cases} \qquad x \in \mathbb{R}^3.$$

Solutions concentrating their mass along an annulus that shrinks at a point in finite time were derived formally in [2] and observed numerically in [3]. The only known rigorous result for such dynamics concerns the mass supercritical nonlinear Schrödinger equation [1], where existence of such solutions (without stability study) was proved. We prove the existence and radial stability of such solutions for (KS), by stabilising a viscous Burgers-like traveling wave coming at the origin and understanding the role played by the inviscid region left behind.

References

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MS22 **Thursday, 10:30–10:55, Zoom Room 04, link tba**

A priori estimates on the rates of enhanced dissipation in chaotic passive scalar advection

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We are concerned with flow enhanced mixing of passive scalars in the presence of diffusion. Under the assumption that the velocity gradient is suitably integrable, we provide upper bounds on the exponential rates of enhanced dissipation. Recent constructions indicate the optimality of our results.

MS22 **Thursday, 11:00–11:25, Zoom Room 04, link tba**

Some mathematical aspects of the 2D Boussinesq equations

Roberta Bianchini[1,*], **Michele Coti Zelati**[2], **Michele Dolce**[3]

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This talk will concern the analysis of the stably stratified fluids system, where the velocity satisfies the incompressible Euler equations, coupled with a scalar term, called buoyancy. It is obtained by a linearization of the equations of incompressible non-homogeneous fluids around a background density profile $\rho(z)$ that increases with depth. Adding the Boussinesq approximation, according to which density variation is neglected except when it directly causes buoyancy forces, one obtains the Boussinesq system. In the first part, I will present some general properties of the 2D Boussinesq equations. The second part will be devoted to the presentation of the stability properties of the 2D Boussinesq equations linearized around the Couette flow $(y;0)$. The last part is based on joint works with Michele Coti Zelati and Michele Dolce (Imperial College, UK).

MS22 **Thursday, 11:30–11:55, Zoom Room 04, link tba**

An elementary proof of existence and uniqueness for the Euler flow in uniformly localized Yudovich spaces

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I will revisit Yudovich's well-posedness result for the 2-dimensional Euler equations. I will derive an explicit modulus of continuity for the velocity, depending on the growth in p of the (uniformly localized) L^p norms of the vorticity. If the growth is moderate at infinity, the modulus of continuity is Osgood and this allows to show uniqueness. I will also show how existence can be proved in (uniformly localized) L^p spaces for the vorticity. All the arguments are fully elementary, make no use of Sobolev spaces, Calderon-Zygmund theory, or Paley-Littlewood decompositions, and provide explicit expressions for all the objects involved. This is a joint work with Giorgio Stefani (Basel)

MS22 **Thursday, 15:00–15:25, Zoom Room 04, link tba**

Long-time behaviour in the 2D Euler-Boussinesq equations near a stably stratified Couette flow

Michele Dolce[1,*], Jacob Bedrossian[2], Roberta Bianchini[3], Michele Coti Zelati[1]
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Fluids in the ocean are often inhomogeneous, incompressible and, in relevant physical regimes, can be described by the 2D Euler-Boussinesq system. Equilibrium states are then commonly observed to be stably stratified, namely the density increases with depth. We are interested in considering the case when also a background shear flow is present. In the talk, I will describe quantitative results for small perturbations around a stably stratified Couette flow. The density variation and velocity undergo an $O(t^{-1/2})$ inviscid damping while the vorticity and density gradient grow as $O(t^{1/2})$. This is precisely quantified at the linear level [1]. For the nonlinear problem, the result holds [2] on the optimal time-scale on which a perturbative regime can be considered. Namely, given an initial perturbation of size $O(\epsilon)$, thanks to the linear instability, the perturbation become of size $O(1)$ on a time-scale of order $O(\epsilon^{-2})$.

References

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MS22 **Thursday, 15:30–15:55, Zoom Room 04, link tba**

Global axisymmetric Euler flows with rotation

Klaus Widmayer[1,*], **Yan Guo**[2], **Benoit Pausader**[3]

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We discuss the construction of a class of global, dynamical solutions to the 3d Euler equations near the stationary state given by uniform "rigid body" rotation. These solutions are axisymmetric, of Sobolev regularity and have non-vanishing swirl. At the heart of this result is the dispersive effect due to rotation, which is captured in our new "method of partial symmetries". This approach is adapted to maximally exploit the symmetries of this anisotropic problem, both for the linear and nonlinear analysis, and allows to globally propagate sharp decay estimates. This is joint work with Y. Guo and B. Pausader.

MS22 **Thursday, 16:30–16:55, Zoom Room 04, link tba**

Infinite energy solutions for dissipative active scalar equations

Niklas Knobel[1,*]

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We will consider the active scalar equation

$$(T)_\alpha : \begin{cases} \partial_t\theta + (u\cdot\nabla)\theta + \nu\Lambda^\alpha\theta = 0, & \forall x \in \mathbb{R}^n, t > 0 \\ u = \mathcal{T}[\theta], \\ \theta(0,x) = \theta_0(x), \end{cases}$$

with $\Lambda^s\theta = (-\Delta)^{\frac{s}{2}}\theta$ and each component of $\mathcal{T}$ is Calderon Zygmund operator with convolution kernel. The initial data θ_0 will not be necessary be in the L^2 space. Therefore, we consider the weighted fractional Sobolev spaces and show under which condition we obtain a solution in the critical ($\alpha = 1$) and subcritical ($\alpha > 1$) case.

References

[1] Omar Lazar and Pierre-Gilles Lemarie-Rieusset, Infinite energy solutions for a 1D transport equation with nonlocal velocity, *Dynamics of Partial Differential Equations* **13** (2016), pp. 107–131.

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MS22 **Thursday, 17:00–17:25, Zoom Room 04, link tba**

Global bifurcation of capillary-gravity water waves with overhanging profiles and arbitrary vorticity

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While the research on water waves modeled by Euler's equations has a long history, mainly in the last two decades traveling periodic rotational waves have been constructed with mathematical rigor by means of bifurcation theorems. In this talk, I will present a new reformulation of this traveling periodic water wave problem in two dimensions and in the presence of surface tension, gravity, and a flat bed. Using conformal mappings and a new Babenko-type reformulation of Bernoulli's equation, the problem is equivalently cast into the form "identity plus compact", which is amenable for Rabinowitz' global bifurcation theorem. The main advantages of this new reformulation are that no simplifying restrictions on the geometry of the surface profile and no simplifying assumptions on the vorticity distribution (and thus no assumptions regarding the absence of stagnation points) have to be made. Within the scope of this new formulation, local and global solution curves, bifurcating from laminar flows with a flat surface, are constructed. Moreover, I will further discuss the condition for local bifurcation and the possible alternatives for "endpoints" of the global curve.

MS22 **Thursday, 17:30–17:55, Zoom Room 04, link tba**

The asymptotic lake equations for an evanescent or emergent island

Lars Eric Hientysch[1,*], Christophe Lacave[2], Evelyne Miot[3]

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The lake equations arise as a geophysical two-dimensional model for the evolution of a fluid in a lake characterised by the geometry of its surface and its depth. Motivated by physical phenomena such as flooding, sedimentation and seismic activity, we investigate the stability of these equations under changes of both the geometry and the topography. More precisely, we first consider the singular limit for an evanescent island, namely an island shrinking to a point where the depth function vanishes. Second, we discuss the scenario of an emergent island. We obtain an asymptotic equation for both cases. In the former, a point vortex located at the point to which the island has collapsed is created. While the lake equations reduce to the two-dimensional incompressible Euler equations for a flat topography (constant depth), the lake equations are degenerate if the depth function vanishes at the boundary (beaches) or in the interior of the domain. We provide new uniform estimates in weighted spaces that enable us to prove the compactness result. This is joint work with Christophe Lacave and Evelyne Miot.

Contributed Talks

Contributed Talk **Monday, 11:45–12:10, Zoom Room 08, link tba**

High-frequency wave propagation in nonlinear media

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We consider semilinear hyperbolic systems

$$\partial_t \mathbf{u} + A(\partial)\mathbf{u} + \tfrac{1}{\varepsilon} E\mathbf{u} = \varepsilon T(\mathbf{u},\mathbf{u},\mathbf{u}), \qquad t \in [0, \tfrac{t_{\mathrm{end}}}{\varepsilon}],\ x \in \mathbb{R}^d, \tag{1}$$

with a small parameter $0 < \varepsilon \ll 1$ and highly oscillatory initial data

$$\mathbf{u}(0,x) = 2p(x)\cos\left(\tfrac{\kappa \cdot x}{\varepsilon}\right). \tag{2}$$

$T(\cdot,\cdot,\cdot)$ is a trilinear nonlinearity, $E \in \mathbb{R}^{n\times n}$ is skew-symmetric matrix, and $A(\partial)$ denotes the differential operator $A(\partial) = \sum_{\ell=1}^{d} A_\ell \partial_\ell$ with symmetric matrices $A_1, \ldots, A_d \in \mathbb{R}^{n\times n}$ $(d, n \in \mathbb{N})$. The initial data (2) involve a wave vector $\kappa \in \mathbb{R}^d \setminus \{0\}$ and a smooth envelope function $p : \mathbb{R}^d \to \mathbb{R}^n$.

Solving (1)-(2) numerically is a challenging task, because physically relevant solutions oscillate rapidly in time and space due to the small parameter ε which occurs both in the PDE (1) and in the initial data (2). Moreover, the problem is scaled in such a way that solutions have to be computed on a long time interval $[0, t_{\mathrm{end}}/\varepsilon]$ with some $t_{\mathrm{end}} > 0$.

The classical nonlinear Schrödinger approximation provides a possibility to approximate the solution $\mathbf{u}$ analytically up to an error of $\mathcal{O}(\varepsilon)$ without having to solve a highly oscillatory problem. We propose an alternative approach which reduces the error down to $\mathcal{O}(\varepsilon^2)$ and still avoids spatial oscillations completely. This makes the new strategy attractive for numerical computations.

Contributed Talk **Monday, 12:15–12:40, Zoom Room 08, link tba**

Multirate leapfrog-type methods for second-order semilinear ODEs

Constantin Carle[1,*], **Marlis Hochbruck**[1]

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In this talk we consider second-order semilinear differential equations, for which the stiffness is induced by only a few components of the linear part. For the leapfrog scheme such problems cause severe restrictions on the step size to ensure stability. To overcome this issue, we introduce a general class of multirate leapfrog-type methods [1]. Special attention is paid to explicit multirate methods, which are based on stabilized leapfrog-Chebyshev polynomials introduced in [2] and are related to the local time-stepping schemes investigated in [3]. For these multirate schemes we show that we can allow for step sizes which are independent of the stiff part of the equation. We further show that these schemes converge with order two in time, and confirm our theoretical results with some numerical examples.

References

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Contributed Talk **Monday, 12:45–13:10, Zoom Room 08, link tba**

Error analysis of St-LO

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In this talk, we introduce a new dynamical low-rank integrator for second-order matrix differential equations

$$A''(t) = F(A(t)), \qquad A(0) = A_0, \quad A'(0) = B_0, \quad t \in [0, T],$$

typically stemming from space discretizations of wave equations. The integrator is constructed by combining the projector-splitting integrator introduced in [1] with a Strang splitting ansatz. Further, we present a convergence result for the new integrator and discuss the main steps in the error analysis. Numerical experiments illustrate the performance of the new scheme.

References

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Contributed Talk **Monday, 11:45–12:10, Zoom Room 09, link tba**

hp-FEM for the Helmholtz equation in piecewise smooth media

Jens M. Melenk[1,*], **Maximilian Bernkopf**[2], **Théophile Chaumont-Frelet**[2]

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We consider the Helmholtz equation with piecewise analytic coefficients at large wavenumber k. In order to understand how k affects the convergence properties of discretizations of such problems, we develop a regularity theory for the Helmholtz equation that is explicit in k. At the heart of our analysis is the decomposition of solutions into two components: the first component is a piecewise analytic, but highly oscillatory function and the second one has finite regularity but features wavenumber-independent bounds. This decomposition generalizes earlier decompositions of [Melenk-Sauter 2011] which considered the Helmholtz equation with constant coefficients, to the case of piecewise analytic coefficients. This regularity theory for the Helmholtz equation allows for the analysis of high order Galerkin discretizations of the Helmholtz equation that are explicit in the wavenumber k. We show that quasi-optimality is guaranteed if (a) the approximation order p is selected as $p = O(\log k)$ and (b) the mesh size h is such that kh/p is sufficiently small.

References

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Contributed Talk **Monday, 12:15–12:40, Zoom Room 09, link tba**

Scattering by the local perturbation of an open periodic waveguide in the half plane

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We consider the scattering problem of the local perturbation of an open periodic waveguide in the half plane. Recently in [1], a new radiation condition was introduced in order to solve the unperturbed case of this problem. Under the same radiation condition with [1], we show the well-posedness of the perturbed scattering problem. This talk is based on the paper [2].

References

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Contributed Talk **Monday, 12:45–13:10, Zoom Room 09, link tba**

APPLICATION OF ADAPTED-BUBBLES TO THE HELMHOLTZ EQUATION WITH LARGE WAVE NUMBERS IN 2D

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An adapted bubble approach which is a modifiation of the residual-free bubbles (RFB) method, is proposed for the Helmholtz problem in 2D. A new two-level finite element method is introduced for the approximations of the bubble functions. Unlike the other equations such as the advection-diffusion equation, RFB method when applied to the Helmholtz equation, does not depend on another stabilized method to obtain approximations to the solutions of the sub-problems. Adapted bubbles (AB) are obtained by a simple modification of the sub-problems. This modification increases the accuracy of the numerical solution considerably. The AB method is able to solve the Helmholtz equation efficiently in 2D up to ch = 3.5 where c is the wave number and h is the mesh size. We provide analysis to show how the AB method mitigates the pollution error.

Contributed Talk **Monday, 16:30–16:55, Zoom Room 09, link tba**

Maximum norm error bounds for the full discretization of non-autonomous wave equations

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In the this talk, we discuss the full discretization of a non-autonomous wave equation

$$\partial_{tt}u(t,x) = \lambda(t,x)^{-1}\Delta u(t,x) + f(t,x), t \in [0,T],\ x \in \Omega, \tag{1}$$

by isoparametric finite elements in space and the implicit Euler method in time. Here, Ω is a convex bounded domain with a regular boundary. Building upon the work of Baker and Dougalis [1], we prove maximum norm estimates for the semi discretization in space and the full discretization. The key tool in the analysis is to trade in integrability, coming from the inverse of the discretized differential operator Δ_h, for time derivatives on the error in the $H^1 \times L^2$-norm. We show how the ideas of the semi discretization transfer to the fully discrete error analysis, yielding a good starting point for higher order in time and quasilinear problems.

References

[1] G. A. Baker and V. A. Dougalis, On the L^∞-convergence of Galerkin approximations for second-order hyperbolic equations, *Math. Comp.* **34** (1980), no. 150, pp. 401–424.

[2] B. Dörich, J. Leibold and B. Maier, Maximum norm error bounds for the full discretization of non-autonomous wave equations. CRC 1173 Preprint 2021/47, Karlsruhe Institute of Technology, 2021. URL `https://www.waves.kit.edu/downloads/CRC1173_Preprint_2021-47.pdf`.

Contributed Talk **Monday, 17:00–17:25, Zoom Room 09, link tba**

A unified error analysis for nonlinear wave-type equations with application to acoustic boundary conditions

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Acoustic boundary conditions are an effective model for a boundary that is subject to small oscillations in normal direction which are caused by a wave propagation in the interior of the domain. If the boundary is assumed to be porous, the coupling between the wave in the domain and the oscillations of the boundary becomes nonlinear. In this talk, we derive error bounds for a finite element discretization of wave equations with such nonlinear coupled acoustic boundary conditions.

Since acoustic boundary conditions include derivatives on the boundary, they are usually posed on domains with smooth boundary. Hence, the domain has to be approximated by the finite element method which renders the space discretization non-conforming and makes the error analysis much more involved.

To tackle this difficulty, we introduce a unified error analysis for non-conforming space discretizations of nonlinear wave equations. The unified error analysis is an abstract framework in which wave equations as well as their spatial discretizations are considered as evolution equations in Hilbert spaces. In this framework, the error analysis is performed which gives an abstract error bound in terms of approximation properties of the space discretization method.

Contributed Talk **Monday, 17:30–17:55, Zoom Room 09, link tba**

Time integration of Maxwell equations on heterogeneous media

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The Peaceman-Rachford alternating direction implicit (ADI) scheme is very attractive for the time integration of linear Maxwell equations on cuboids [2,3]. Indeed, it is unconditionally stable and has only linear complexity. During the error analysis, it however turns out that the error estimates depend heavily on the regularity of the solutions to the Maxwell equations.

We study a linear isotropic time-dependent Maxwell system on a cuboid, modeling a rectangular waveguide. The medium is assumed to consist of several smaller homogeneous subcuboids with different material properties. Here the solutions of the Maxwell equations can have lower regularity [1]. Based on regularity results for the Maxwell system, we present rigorous time-discrete error estimates for the ADI scheme. In particular, we make only assumptions on the material and the initial data, but none on the solution of the Maxwell system.

References

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[2] T. Namiki, 3-D ADI-FDTD method - unconditionally stable time-domain algorithm for solving full vector Maxwell's equations, *IEEE Trans. Microwave Theory Tech.* **48** (10) (2000), pp. 1743–1748.

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Contributed Talk **Tuesday, 11:30–11:55, Zoom Room 10, link tba**

The Complex-Scaled Half-Space Matching Method

Anne-Sophie Bonnet-Ben Dhia[1], **Simon N. Chandler-Wilde**[2], **Sonia Fliss**[1], **Christophe Hazard**[1], **Karl-Mikael Perfekt**[3], **Yohannes Tjandrawidjaja**[4]
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The Half-Space Matching (HSM) method is a method for the solution of 2D scattering problems with complex backgrounds, providing an alternative to Perfectly Matched Layers (PML) or other artificial boundary conditions. Based on half-plane representations for the solution, the scattering problem is rewritten as a system of integral equations in which the unknowns are restrictions of the solution to the boundaries of a finite number of overlapping half-planes contained in the domain: this integral equation system is coupled to a standard finite element discretisation localised around the scatterer. In the present talk (for detail see [1]), by combining the HSM framework with a complex-scaling technique in the spirit of PML, we provide a new formulation which is provably well-posed for real wavenumbers and has the attraction for computation that the solutions of the complex-scaled integral equation system decay exponentially at infinity. Effectiveness is illustrated by numerical results.

References

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Contributed Talk — Tuesday, 12:00–12:25, Zoom Room 10, link tba

High frequency scattering by multiple obstacles

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Standard Boundary Element Methods (BEM) for scattering problems ,with piecewise polynomial approximation spaces, have a computational cost that grows with frequency. Recent Hybrid Numerical Asymptotic (HNA) BEMs with enriched approximation spaces consisting of the products of piecewise polynomials with carefully chosen oscillatory functions have shown to be effective in overcoming this limitation for a range of problems, mostly (with some recent exceptions) focused on single convex scatterers or very specific non-convex or multiple scattering configurations. Here we present a novel HNABEM approach to the problem of 2D scattering by a pair of screens in an arbitrary configuration, which we anticipate may serve as a building block towards algorithms for general multiple scattering problems with computational cost independent of frequency.

Contributed Talk — Tuesday, 12:30–12:55, Zoom Room 10, link tba

CVEM-BEM coupling for exterior wave propagation problems

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We consider the wave equation defined on the exterior of a bounded 2D space domain, endowed with a Dirichlet condition on the boundary. We propose a numerical method that approximates the solution using computations only in an interior finite domain. This is obtained by introducing a curved smooth artificial boundary on which a non-reflecting boundary condition, defined by a boundary integral equation, is imposed. The approach we consider allows to solve the original problem by means of the coupling of an interior domain method with a boundary element one associated to the artificial boundary.

For the space discretization in the interior computational domain, we propose a Galerkin approach based on the Curvilinear Virtual Element Method (CVEM), and for the time discretization we use the classical Crank-Nicolson method. For the approximation of the non-reflecting condition on the artificial boundary, we apply a standard collocation Boundary Element Method (BEM) combined with a Lubich time convolution quadrature formula.

Some numerical results are presented to test the performance of the proposed approach and to highlight its effectiveness.

Contributed Talk **Tuesday, 11:30–11:55, Zoom Room 11, link tba**

The Cauchy problem and continuation of periodic solutions for a generalized Camassa-Holm equation

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In this work, we provide a global well-posedness result for space-periodic solutions of the Cauchy problem related to the three-parameter family of nonlinear equations with $p+1$-order nonlinearities. For certain choices of the parameters, this family of equations give rise to Camassa-Holm equation. Moreover, we give the conditions to prevent wave breaking in finite time. Last but not least, we prove unique continuation properties for some values of the parameters.

Contributed Talk **Tuesday, 12:00–12:25, Zoom Room 11, link tba**

Metastability in two-dimensional rectangular lattices

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In this talk I will review a recent result [1] about the dynamics of two-dimensional rectangular lattices with periodic boundary conditions. I will consider long-wavelength anisotropic initial data, and I will show that, in the continuous approximation, the resonant normal form of the system is given by integrable PDEs.

One can exploit the normal form in order to prove the existence of metastability phenomena for the lattices, in which the energy is shared among a packet of low-frequencies modes; such distribution of the energy spectrum remains unchanged up to the time-scale of validity of the continuous approximation.

This is the first analytical result about metastable phenomena in two-dimensional Hamiltonian lattices with periodic boundary conditions; in particular, this is the first rigorous result for two-dimensional lattices in which the dynamics of the lattice in a (suitable) two-dimensional regime is described by a system of two-dimensional integrable KP-II equations.

References

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Contributed Talk	Tuesday, 12:30–12:55, Zoom Room 11, link tba

Analysis of Coupled PDE Systems for Micro-Electro-Mechanical Systems

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In this talk, we study some mathematical models for a Micro-Electro-Mechanical System (MEMS) capacitor, consisting of a fixed plate and a flexible plate separated by a fluid. It investigates the wellposedness of solutions to the resulting quasilinear coupled systems, as well as the finite-time blow-up (quenching) of solutions. The models considered include a parabolic-dispersive system modelling the fluid flow under an elastic plate, a parabolic-hyperbolic system for a thin membrane, as well as an elliptic-dispersive system for quasistatic fluid flow under an elastic plate. Short-time existence, uniqueness and smoothness are obtained by combining wellposedness results for a single equation with an abstract semigroup approach for the system. Quenching is shown to occur if the solution ceases to exist after a finite time. We also present a study of self-similar quenching solutions and their stability for a simple hyperbolic membrane model for a MEMS capacitor.

Contributed Talk	Tuesday, 15:00–15:25, Zoom Room 10, link tba

Interfacial waves in a three-fluid system

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Dissipation of interfacial capillary-gravity waves in a three-fluid system is considered here. The system comprises air and water separated by an interfacial fluid layer of finite thickness. Waves in both the barotropic and baroclinic modes are considered. In recent studies [1], the interfacial fluid was assumed to be Newtonian, but here we generalize the theory to account for the complex rheology of the interfacial fluid. The composition of a typical interfacial layer on the ocean is investigated, and laboratory experiments are conducted to study its rheology. Using appropriate rheological models for the interfacial fluid, the dissipation rates of the barotropic and baroclinic waves are computed and compared with each other.

References

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Contributed Talk **Tuesday, 15:30–15:55, Zoom Room 10, link tba**

Reflection of plane waves in a memory-dependent nonlocal magneto-thermo-elasticity in an electrically conducting triclinic solid half-space

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The present research work is the analysis of the reflection problem in a magnetized electrically conducting thermo-triclinic solid half-space in a memory-dependent nonlocal magneto-thermo-elasticity. The governing equations of a triclinic magneto-thermoelastic medium in the context of a memory-dependent derivative nonlocal generalized thermo-elasticity are formulated and solved to obtain the velocity equation. Plane-wave solution indicates that three quasi plane waves namely quasi longitudinal displacement (qP) wave, quasi thermal (qT) wave and quasi shear vertical (qSV) wave propagate in the medium. The velocities of qP,qT and qSV waves are computed for the triclinic medium. The effect of nonlocal parameter, magnetic field parameter and frequency on the speed of qP,qT and qSV waves is investigated and shown graphically. A reflection problem at a thermally insulated /isothermal stress-free boundary is studied to obtain reflection coefficients. The expressions for the reflection coefficient and energy ratio are derived for the incidence of a coupled quasi plane wave. Reflection coefficients and energy ratios of various reflected qP,qT and qSV waves are computed numerically for a particular triclinic material and the effect of nonlocality, magnetic field, anisotropy, memory kernels and angle of incidence on the reflection coefficient and energy ratio is shown graphically.

Contributed Talk **Tuesday, 15:00–15:25, Zoom Room 11, link tba**

Exponential integrators for second-order in time partial differential equations

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Two types of second-oder in time partial differential equations, namely: semilinear damped wave equations with damping term, structural (visco-elastic) damping term and mass term; and the Euler–Bernoulli beam equations with Kelvin–Voigt damping, are considered. By spatial discretization, we get a matrix differential equation of the form

$$\dot{y}(t) = Ay(t) + F(y(t)), \qquad y(0) = y_0 = (p, q)'. \tag{1}$$

where $A = \begin{pmatrix} 0 & I \\ -\alpha S - \delta I & -\beta S - \gamma I \end{pmatrix}$ with $\alpha > 0$, $\beta, \gamma, \delta \geq 0$; and $S \in \mathbb{R}^{k\times k}$ is the discretized version of the operators $(-\Delta)$ or ∂^4_{xxxx}.

We present a new approach to compute the action of matrix exponential as well as matrix functions of A efficiently. With this at hand, the solution of linearized versions of (1) can be evaluated for arbitrary time $t > 0$ in a fast and reliable way. The solution of semilinear systems can be computed by employing exponential integrators (as in [1]).

References

[1] M. Hochbruck and A. Ostermann, Exponential Runge–Kutta methods for parabolic problems, *Applied Numerical Mathematics* **53** (2005), pp. 323–339.

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Contributed Talk **Tuesday, 15:30–15:55, Zoom Room 11, link tba**

An accurate and time-parallel rational exponential integrator for hyperbolic and oscillatory PDEs

Marco Caliari[1], **Lukas Einkemmer**[2], **Alexander Moriggl**[2,*], **Alexander Ostermann**[2]
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Rational exponential integrators (REXI) are a class of numerical methods that are well suited for the time integration of linear partial differential equations with imaginary eigenvalues. Since these methods can be parallelized in time (in addition to the spatial parallelization that is commonly performed) they are well suited to exploit modern high performance computing systems. In this paper, we propose a novel REXI scheme that drastically improves accuracy and efficiency. The chosen approach will also allow us to easily determine how many terms are required in the approximation in order to obtain accurate results. We provide comparative numerical simulations for a shallow water equation that highlight the efficiency of our approach and demonstrate that REXI schemes can be efficiently implemented on graphic processing units.

References

[1] M. Caliari, L. Einkemmer, A. Moriggl and A. Ostermann, An accurate and time-parallel rational exponential integrator for hyperbolic and oscillatory PDEs, in *Journal of Computational Physics, Volume 437, 2021´.*

Contributed Talk **Tuesday, 16:30–16:55, Zoom Room 10, link tba**

Validity of the Nonlinear Schrödinger Approximation for the Two-Dimensional Water Wave Problem With and Without Surface Tension

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We consider the two-dimensional water wave problem in an infinitely long canal of finite depth both with and without surface tension. In order to describe the evolution of the envelopes of small oscillating wave packet-like solutions to this problem the Nonlinear Schrödinger equation can be derived as a formal approximation equation. The rigorous justification of the Nonlinear Schrödinger approximation for the water wave problem was an open problem for a long time. In recent years, the validity of this approximation has been proven by several authors only for the case without surface tension.

In this talk, we present the first rigorous justification of the Nonlinear Schrödinger approximation for the two-dimensional water wave problem which is valid for the cases with and without surface tension by proving error estimates over a physically relevant timespan in the arc length formulation of the water wave problem. Our error estimates are uniform with respect to the strength of the surface tension, as the height of the wave packet and the surface tension go to zero.

References

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Contributed Talk **Tuesday, 17:00–17:25, Zoom Room 10, link tba**

The validity of the derivative NLS equation for extended systems with cubic nonlinearities.

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The generalized derivative Nonlinear Schrödinger (DNLS) equation can be derived as an amplitude equation via multiple scaling perturbation analysis from dispersive wave systems. It occurs when the cubic coefficient for the associated NLS equation vanishes for the spatial wave number of the underlying modulated wave packet. We prove that the DNLS equation makes correct predictions about a Klein-Gordon model with a cubic nonlinearity. The proof is based on energy estimates and suitably chosen normal form transformations.

Contributed Talk **Tuesday, 17:30–17:55, Zoom Room 10, link tba**

Validity of the KdV approximation for a Boussinesq equation posed on the periodic necklace graph

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We consider a Boussinesq equation posed on the infinite periodic necklace graph. In the long wave limit we derive a KdV equation and prove that it makes correct predictions about the dynamics of the Boussinesq equation. The proof is based on the construction of a suitable energy. The new difficulties come from the periodic structure and the non-smoothness of the solutions at the vertices of the graph.

References

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Contributed Talk **Tuesday, 18:00–18:25, Zoom Room 10, link tba**

On the Schrödinger approximation for the Helmholtz equation

Maximilian Klumpp[1,*], **Guido Schneider**[1]

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Time-harmonic electromagnetic waves in a medium are described by the Helmholtz equation $\Delta u + \omega^2 n u = 0$ for $(x, y, z) \in \mathbb{R}^3$, where $n = n(x, y, z)$ is the refractive index. A Schrödinger equation can be derived through a multiple scaling ansatz for the evolution of such waves along the z-axis in case of a z-independent refractive index. We proved the validity of this formal approximation in case that the refractive index is a step function $n = n(x)$. The proof uses mode filters and a spectral representation of the problem. The major difficulty comes from the fact that the Helmholtz equation is an ill-posed evolutionary system.

References

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Contributed Talk **Tuesday, 16:30–16:55, Zoom Room 11, link tba**

The Total Helicity of Electromagnetic Fields and Matter

Ivan Fernandez-Corbaton[1,*]

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The electromagnetic helicity of the free electromagnetic field and the static magnetic helicity are shown in [1] to be two different embodiments of the same physical quantity, the total helicity. The total helicity is the sum of two terms: The optical (electromagnetic) helicity, which measures the difference between the number of left-handed and right-handed photons of the free field, and another term that measures the screwiness of the static magnetization density in matter, proportional to the static magnetic helicity. Each term is the manifestation of the total helicity in different frequency regimes: $\omega > 0$ and $\omega = 0$, respectively. This new link between optics and magnetism establishes the theoretical basis for studying the conversion between the two embodiments of total helicity upon light-matter interaction. In my talk, I will explain this result in detail and focus on a particular area of potential application: Interaction of light with solid-state magnetic systems.

References

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Contributed Talk	Tuesday, 17:00–17:25, Zoom Room 11, link tba

Electromagnetic scattering from homogeneous, time-varying and dispersive spheres

A. G. Lamprianidis[1,*], G. Ptitcyn[2], T. Karamanos[1], V. S. Asadchy[3], R. Alaee[1], M. Müller[1], M. Albooyeh[4], M. S. Mirmoosa[2], S. Fan[3], S. A. Tretyakov[2], C. Rockstuhl[1]

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Metamaterials constitute a platform to shape electromagnetic waves at will, leading to a plethora of applications in photonics. Lately, a new degree of freedom in light-matter interactions is being explored: that of light interacting with time-varying, structured media [1].

Here, we study a canonical scattering problem, from which physical insights on light interaction with time-varying and structured media can be obtained. Specifically, we develop an analytical theory of light scattering by a sphere made from a time-varying and dispersive material. First, we discuss the properties of light inside time-varying and dispersive bulk media, and, then, we study the properties of the scattering system of a sphere made of such material. We highlight the fundamental property of such a system to transfer energy from the time-varying medium to the electromagnetic field. We verify our analytical method with full-wave numerical simulations.

References

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Contributed Talk	Tuesday, 17:30–17:55, Zoom Room 11, link tba

Group-theoretical formulation of electromangetic scattering and its application to time-dependent problems and conformal transformations

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The linear interaction of an object with dynamic electromagnetic fields can be completely described by its T-matrix (or equivalently the far-field operator) that connects the incident and scattered fields. The T-matrix method is usually applied for monochromatic scattering. We reformulate the method in the setting of group representation theory and generalize the description to the whole frequency domain. T-matrices are bijectively mapped to operators that act on the Hilbert space of irreducible representations of the Poincare group (group of isometries of the four-dimensional Minkowski space). This allows to consider time-dependent scattering and to conveniently describe Lorentz transformations. We use an existing connection between the massless representations of the Poincare group and representations of the conformal group to describe conformal transformations of the T-matrix.

Contributed Talk Wednesday, 11:30–11:55, Zoom Room 07, link tba

Wave propagation in buried waveguides - Part 1 : the forward problem

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Guided waves can be used to perform non destructive testing of partially buried elongated structures present in various industrial sectors such as civil engineering or oil and gas. A typical situation is the case of a cable which is partially embedded into another elastic medium. It may happen that some defects within the embedded part of the cable or at the interface between the cable and the surrounding medium have to be retrieved from measurements located on the only accessible part of the cable, that is its free part.
The talk addresses the forward problem while a second one tackles the inverse problem. We address the forward problem by using Perfectly Matched Layers in the transverse direction, which has the effect of closing the waveguide but of introducing a non-selfadjoint eigenvalue problem in the transverse direction. At the continuous level, we use the Kondratiev theory to establish well-posedness of the forward problem and to specify the asymptotic behaviour of solutions at infinity. In a view to compute a numerical solution, we also propose some transparent boundary conditions in the longitudinal direction of the waveguide, based on Dirichlet-to-Neumann operators. After proving that such artificial condition actually enables to approximate the exact solution, numerical experiments illustrate the quality of such approximation.

Contributed Talk Wednesday, 12:00–12:25, Zoom Room 07, link tba

Wave propagation in buried waveguides - Part 2 : imaging with a sampling method

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Guided waves can be used to perform non destructive testing of partially buried elongated structures present in various industrial sectors such as civil engineering or oil and gas. Several techniques exist to identify defects in closed waveguides; among them, the Linear Sampling Method (LSM) has given good results in many different configurations. However, there are few works on the modelling of the propagation of waves and on the identification of defects in buried waveguides. A first talk entitled "*Wave propagation in buried waveguides - Part 1 : the forward problem*" tackles theoretical and numerical aspects of the forward problem in such structures. In this second talk, we address the inverse problem which presents two main challenges: we can only access to one side of the zone to control, and the waves propagating in the structure will partially leak in the surrounding medium. A modal formulation of the LSM is obtained by decomposing the Green function of the buried waveguide on a basis of reference fields, which are the responses of the healthy structure to the incident modes. This technique enables an evaluation of the right-hand side of the so called "near field" equation of the LSM independent of the sampling point. First results of imaging are finally presented.

Contributed Talk **Wednesday, 12:30–12:55, Zoom Room 07, link tba**

Acoustic passive cloaking using thin resonant cavities

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In this work, we consider the propagation of acoustic waves in a 2D waveguide Ω which is unbounded in the (Ox) direction and which contains an obstacle. We fix the wavenumber $k \in (0, \pi)$ so that only the modes $w^{\pm}(x, y) = e^{\pm ikx}$ can propagate. We are interested in the diffraction of the incoming wave w^+ coming from $-\infty$, which generates a reflection and a transmission characterized by some scattering coefficients $R, T \in \mathbb{C}$.

The goal of this presentation is to do passive geometrical cloaking by working with thin ligaments of width $\varepsilon \ll 1$. More precisely, when adding ligaments, we create a new geometry Ω^ε and we denote by R^ε and T^ε the corresponding scattering coefficients. Our objective is to show that one can place the ligaments and tune their lengths to get, when ε tends to zero, $R^\varepsilon = o(1)$ and $T^\varepsilon = 1 + o(1)$, as if, approximately, there were no obstacle. This work is published in the article [1].

References

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Contributed Talk **Wednesday, 11:30–11:55, Zoom Room 10, link tba**

Nonlinear Helmholtz equations with sign-changing diffusion coefficient

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In this talk, we study nonlinear Helmholtz equations with sign-changing diffusion coefficients on bounded domains of the form $-\mathrm{div}(\sigma(x)\,\nabla u) - \lambda\, c(x)\, u = g(x, u)$. Using weak $\mathtt{T}$-coercivity theory, we can establish the existence of an orthonormal basis of eigenfunctions of the linear part $-c(x)^{-1}\,\mathrm{div}(\sigma(x)\,\nabla u)$. Then, all eigenvalues are proved to be bifurcation points and we investigate the bifurcating branches both theoretically and numerically. As a fundamental example, we look at some one-dimensional model, we obtain the existence of infinitely many bifurcating branches that are mutually disjoint, unbounded, and consist of solutions with a fixed nodal pattern.

References

[1] R. Mandel, Z. Moitier, and B. Verfürth, *Nonlinear Helmholtz equations with sign-changing diffusion coefficient*, arXiv:2107.14516 (2021).

Contributed Talk **Wednesday, 12:00–12:25, Zoom Room 10, link tba**

Time-periodic solutions of a semilinear wave equation

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We study the semilinear wave equation $V(x)u_{tt} - u_{xx} = f(x,u)$ on $(0,2l)\times\mathbb{R}$ with Dirichlet boundary conditions and look for time-periodic solutions by using variational methods. The main idea is to consider a Fourier expansion ansatz and to analyze the spectrum of the wave operator in dependence of V. With the help of variational methods one can find weak solutions as critical points of appropriate functionals.

Contributed Talk **Wednesday, 12:30–12:55, Zoom Room 10, link tba**

Asymptotic stability on a discrete necklace graph

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We are interested in nonlinear dispersive systems on continuous and discrete quantum graphs. Quantum graphs are used in nano-technological devices and photonic crystals. We consider a Klein-Gordon type equation on a continuous and discrete necklace graph. We investigate the existence of localized breather solutions and the nonexistence of localized solutions due to the coupling into the continuous dispersive modes.

Contributed Talk **Wednesday, 15:00–15:25, Zoom Room 07, link tba**

Low regularity error estimates for the time integration of the 2D NLS

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We study a filtered Lie splitting scheme for the cubic periodic nonlinear Schrödinger equation on the two-dimensional torus $\mathbb{T}^2$. This scheme overcomes the standard stability restriction in Sobolev spaces $H^s(\mathbb{T}^2)$ requiring $s > 1$. In particular, our analysis in Bourgain spaces now allows us to handle initial data in $H^s(\mathbb{T}^2)$ with $0 < s \leqslant 1$. Moreover, we establish low regularity error estimates in discrete (in time) Bourgain spaces, and prove convergence of order $\tau^{\frac{s}{2}}$ in $L^2(\mathbb{T}^2)$, where τ denotes the time step size of the scheme.

Contributed Talk **Wednesday, 15:30–15:55, Zoom Room 07, link tba**

A fully discrete low-regularity integrator for NLS

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In this talk, I will present a new fully discrete low-regularity integrator for the one dimensional cubic nonlinear Schrödinger equation on the torus. The method is convergent for NLS with nonsmooth initial data. For initial data in H^γ, $\frac{1}{2} < \gamma \leq 1$, the scheme provides an $\mathcal{O}(\tau^{\frac{3}{2}\gamma-\frac{1}{2}-\varepsilon} + N^{-\gamma})$ error bound in L^2. This scheme is explicit and can be computed efficiently by FFT. This is joint work with A. Ostermann.

References

[1] A. Ostermann, F. Yao, A fully discrete low-regularity integrator for the nonlinear Schrödinger equation, *arXiv:2108.04794*.

Contributed Talk **Wednesday, 15:00–15:25, Zoom Room 10, link tba**

Complex-scaled integral equation for water waves scattering

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We consider the two-dimensional linearized water waves scattering problem. The fluid domain, with a finite or an infinite depth, is supposed to be infinite in the horizontal direction. The scatterer may be some submerged or floating body, or any local variation of the depth profile. To solve this problem with a boundary integral method, a classical approach consists in computing a specific Green's function which incorporates the free-surface boundary condition, and then derive an integral equation reduced to the boundaries of the scatterers. In the spirit of [1] (see also [2]), we develop an alternative which avoids heavy calculations of Green's functions. It consists in applying a complex-scaling to the horizontal coordinate and then use the Green's function of the complex-scaled Laplace equation. This leads to an integral equation on both the boundaries of the scatterers and the infinite free surface. The integral on the free surface has to be truncated for numerical purposes. Numerical results show an exponential convergence with respect to the truncation distance.

References

[1] W. Lu, Y. Y. Lu, and J. Qian, Perfectly matched layer boundary integral equation method for wave scattering in a layered medium. *SIAM Journal on Applied Mathematics*, **78** (2018), pp. 246–265.

[2] O. P. Bruno, M. Lyon, C. Pérez-Arancibia and C. Turc, Windowed Green function method for layered-media scattering. *SIAM Journal on Applied Mathematics*, **76** (2016), pp. 1871–1898.

Contributed Talk **Wednesday, 15:30–15:55, Zoom Room 10, link tba**

Quadrature by Parity Asymptotic eXpansions (QPAX) for scattering by high aspect ratio particles

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The study of scattering by a high aspect ratio particle has important applications in nanophotonics problems, including sensing and plasmonic imaging. To illustrate the effect of parity and the need for adapted methods (in the context of boundary integral methods), we consider the scattering in two dimensions by a sound-hard, high aspect ratio ellipse. This fundamental problem highlights the main challenge and provides valuable insights to tackle plasmonic problems and general high aspect ratio particles. In this case we find that the boundary integral operator is nearly singular due to the collapsing geometry from an ellipse to a line segment. We show that this nearly singular behavior leads to qualitatively different asymptotic behaviors for solutions with different parities. Without explicitly taking this nearly singular behavior and this parity into account, computed solutions incur a large error. To address these challenges, we introduce a new method called Quadrature by Parity Asymptotic eXpansions (QPAX) that effectively and efficiently addresses these issues. We demonstrate the effectiveness of QPAX through several numerical examples, including the Dirichlet problem for Laplace's equation and scattering problems (sound-soft and sound-hard), and we discuss the application to plasmonic problems.

Contributed Talk **Wednesday, 16:30–16:55, Zoom Room 07, link tba**

Solving inverse scattering problems in attenuating media via multifrequency topological derivatives

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Most inverse algorithms for shape reconstruction using acoustic data assume that the attenuation coefficient is negligible. However, in some applications, attenuation effects cannot be discarded. For example, in medical studies of biological tissues, the attenuation coefficient provides important information for diagnosis. Similarly, sound attenuation in subbottom tomography for the seabed plays an important role.

Attenuation complicates the detection of buried scatterers to a great extent. Furthermore, in many applications, data can only be acquired in a limited part of the sounded region.

In this work, we develop a topological derivative-based multifrequency iterative algorithm to reconstruct objects buried in attenuating media from limited aperture data. We demonstrate the method on half-space configurations, where emitters and receives are only located on the accessible part of the sample. One-step implementations of the algorithm provide initial approximations, which are improved in a few iterations. We can locate object components of sizes smaller than the main components, or buried deeper inside. However, attenuation effects hinder object detection depending on the size and depth for fixed ranges of frequencies.

References

[1] A. Carpio and M.-L. Rapún, Multifrequency topological derivative approach to inverse scattering problems in attenuating media, *Symmetry* **13** (2021), art. 1702.

Contributed Talk **Wednesday, 17:00–17:25, Zoom Room 07, link tba**

Numerical solution of inverse generalized impedance boundary value problem for planar linear viscoelasticity

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We propose boundary integral equations methods for the direct and the inverse problem with impedance functions as unknowns [1]. These methods are based on representations of radiating solution to the Navier equation via a single layer-potential and Somigliana integral formula, correspondingly. The integral equations of the first kind with the Fredholm operator of index zero are solved numerically via the fully discrete spectral Galerkin method. Singularities appearing in integral operators are treated by a direct splitting off or via employing the tangential Gunter derivative and variational form [3]. The reconstructions of impedance functions are comparable to the acoustic case [2].

References

[1] O. Ivanyshyn Yaman and F. Le Louer, An inverse parameter problem with generalized impedance boundary condition for two-dimensional linear viscoelasticity, *SIAM Journal of Applied Mathematics* **81** (2021), pp. 1668–1690.

[2] R. Kress, Integral equation methods in inverse obstacle scattering with a generalized impedance boundary condition, *Contemporary Computational Mathematics - A Celebration of the 80th Birthday of Ian Sloan*, Springer, New York, 2018.

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Contributed Talk **Wednesday, 17:30–17:55, Zoom Room 07, link tba**

Modeling and analysis of an inverse boundary value problem in a two dimensional viscoelastic medium

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We consider the solution of a transmission problem at a thin layer interface of thickness $\varepsilon > 0$ in a two dimensional homogeneous viscoelastic medium. A multi-scale expansion for that solution as ε tends to 0 enables to replace the thin layer with a generalized impedance boundary condition [1]. This boundary condition involves a new second order surface symmetric operator with mixed regularity properties on tangential and normal components [3, Appendix]. Extending previous investigation for the Laplace equation case [2], the unique identification of the impedance parameters from measured data produced by the scattering of three independent incident plane waves is established.

References

[1] F. Caubet, D. Kateb and F. Le Louer, Shape sensitivity analysis for elastic structures with generalized impedance boundary conditions of the Wentzell type - Application to compliance minimization, *Journal of Elasticity* **136** (2019), pp. 17–53.

[2] F. Cakoni and R. Kress, Integral equation methods for the inverse obstacle problem with generalized impedance boundary condition, *Inverse problems*, **29** (2013), 015005.

[3] O. Ivanyshyn Yaman and F. Le Louer, An inverse parameter problem with generalized impedance boundary condition for two-dimensional linear viscoelasticity, *SIAM Journal of Applied Mathematics* **81** (2021), pp. 1668–1690.

Contributed Talk **Wednesday, 18:00–18:25, Zoom Room 07, link tba**

An accelerated level-set method for inverse scattering problems

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In this talk, we propose a rapid and robust iterative algorithm to solve inverse acoustic scattering problems formulated as a PDE constrained shape optimization problem. We use a level-set method to represent the obstacle geometry and propose a new scheme for updating the geometry based on an adaptation of the celebrated Nesterov accelerated gradient descent method. The resulting algorithm aims at reducing the number of iterations and improving the accuracy of reconstructions. To cope with regularization issues, we apply a moderate smoothing to the shape gradient using a single layer potential associated with ik where k is the wave number. Numerical experiments are given for several data types (full aperture, backscattering, phaseless, multiple frequencies) to show the effectiveness of our method comparatively to a non accelerated level set method.

Contributed Talk **Thursday, 10:30–10:55, Zoom Room 09, link tba**

On the well-posedness of the damped time-harmonic Galbrun equation and the equations of stellar oscillations

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We study the time-harmonic Galbrun equation describing the propagation of sound in the presence of a steady background flow. With additional rotational and gravitational terms these equations are also fundamental in helio- and asteroseismology as a model for stellar oscillations. For a simple damping model we prove well-posedness of these equations, i.e., uniqueness, existence, and stability of solutions under mild conditions on the parameters (essentially subsonic flows). The main tool of our analysis is a generalized Helmholtz decomposition.

Finally, we will briefly sketch potential analogous constructions at a discrete level which are currently studied in order to obtain stable and provably convergent numerical schemes for solution of these equations.

References

[1] M. Halla, T. Hohage, On the well-posedness of the damped time-harmonic Galbrun equation and the equations of stellar oscillations, *SIAM J. Math. Anal.* **53**(4), pp. 4068–4095 (2021)

Contributed Talk **Thursday, 11:00–11:25, Zoom Room 09, link tba**

Numerical treatment of the vectorial equations of solar oscillations

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The Galbrun's equation with additional rotational and gravitational terms model stellar oscillations. Recently, in [1] it was shown that the time-harmonic problem is well-posed, when incorporating a simple damping term. A suitable generalized Helmholtz decomposition plays a crucial role in the analysis. For the discretization, we aim to preserve a discrete version of the generalized Helmholtz decomposition, which is crucial for stability and helpful for the numerical analysis. We present an $H(\mathrm{div})$-conforming numerical method that respects the structural properties of the continuous problem and introduce the tools needed for the numerical analysis.

References

[1] M. Halla and T. Hohage, *On the well-posedness of the damped time-harmonic Galbrun equation and the equations of stellar oscillations,* SIAM J. Math. Anal. 53(4) (2021), pp. 4068–4095.

Contributed Talk **Thursday, 11:30–11:55, Zoom Room 09, link tba**

Quantitative passive imaging by iterative helioseismic holography

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We study the passive imaging inverse problem dealing with the estimation of interior solar parameters from measurements of correlations of line-of-sight fluctuations at the solar surface. The forward problem is defined in terms of a scalar time-harmonic wave equation. Whereas the noise level in correlation data is huge, very large amounts of data are available by high-resolution Doppler shift measurements of solar fluctuations, continuously over 25 years. Traditional approaches compute averages of correlation data in spatial in frequency directions (e.g., travel times) with acceptable signal-to-noise ratio in a preprocessing step to inversion. Holography, on the other hand, consists in first backpropagating data and then "correlating" in some appropriate way to image the interior quantity of interest, thus avoiding the computation of higher dimensional correlations. Although helioseismic holography has produced very promising results in terms of resolution, the reconstructions are not at all quantitative. In this talk we will interpret holographic backpropagation in terms of the adjoint of the Fréchet derivative of an appropriate forward operator mapping to covariance data. This naturally leads to iteration schemes which are convergent regularization methods and the first iteration steps of which correspond to traditional helioseismic holography. Our approach may also be interpreted as an implementation of nonlinear full waveform inversion. We will examplify our method by numerical results for the reconstruction of solar differential rotation.

Contributed Talk **Thursday, 10:30–10:55, Zoom Room 10, link tba**

Hardware acceleration for fast solution to shallow water system

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For the seismic event offshore Japan, it takes just nearly 20 minutes for tsunami wave to approach the nearest dry land. There are several tools to calculate the wave propagation over the real digital bathymetry. However, all of these tools require extended computational resources and, as a rule, valuable time. The authors have proposed and implemented FPGA (Field Programmable Gates Array) based hardware acceleration of numerical modelling of tsunami wave propagation using personal computer [1]. To enjoy advantages of the FPGA features, the stream processor architecture was proposed for the MacCormack algorithm implementation. The developed Calculator contains several processor elements connected into a chain and running as conveyer. Precision of the proposed solution scheme has been tested by a comparison of the computed solution with the known analytic solutions. FPGA based Calculator shows the same precision as the reference code and better tracing of the wave front. Numerical simulation in 3000×2000 nodes computational domain of 1 hour tsunami wave propagation, takes less than 1 min of processing time. So, the computation process takes 250–300 times less compared to sequential code execution at the same personal computer. This may be used to save time for decision making in case of a near field event.

References

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Contributed Talk **Thursday, 11:00–11:25, Zoom Room 10, link tba**

Analysis of a Monte-Carlo Nystrom method

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We present a Monte-Carlo Nystrom method for solving integral equations of the second kind of the form

$$z(y) + \int_\Omega k(y,y')z(y')\rho(y')\mathrm{d}y' = f(y), \qquad y \in \Omega, \tag{1}$$

whereby the values $(z(y_i))_{1\le i\le N}$ of the solution z at a set of N random and independent points $(y_i)_{1\le i\le N}$ are approximated by the solution $(z_{N,i})_{1\le i\le N}$ to the linear system

$$z_{N,i} + \frac{1}{N}\sum_{j\neq i} k(y_i,y_j)z_{N,j} = f(y_i), \qquad 1 \le i \le N. \tag{2}$$

Under the unique assumption that eq. (1) is well-posed, we prove the invertibility of eq. (2) for sufficiently large N and the convergence of the solution $(z_{N,i})_{1\le i\le N}$ towards the values $(z(y_i))_{1\le i\le N}$ at the rate $O(N^{-\frac{1}{2}})$. For particular choices of kernels, eq. (2) arises as the Foldy-Lax approximation for the scattered field generated by a system of N sources emitting waves at the points $(y_i)_{1\le i\le N}$. In this context, our result yields the well-posedness of the Foldy-Lax approximation and the convergence of the scattered field to the solution of the Lippmann-Schwinger equation eq. (1) characterizing the effective medium. The convergence of Monte-Carlo solutions at the rate $O(N^{-1/2})$ is numerically illustrated on 1D and 2D examples.

This presentation is an introduction to our submitted preprint

F. Feppon and H. Ammari, Analysis of a Monte-Carlo Nystrom method, *Submitted to Siam Journal of Numerical Analysis* (2021). HAL preprint hal-03281401.

Contributed Talk **Thursday, 11:30–11:55, Zoom Room 10, link tba**

Computational lower bounds of the Maxwell eigenvalues

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In this talk we propose a method to compute guaranteed lower bounds to the eigenvalues to the Maxwell system in two or three space dimensions. We apply the idea by Liu and Oishi [1] to numerically quantify the convergence of the Galerkin projection in the L^2 norm by solving an auxiliary eigenvalue problem based on the hypercircle principle. In order to be able to explicitly control the error arising from replacing the right-hand side in a source problem by some piecewise polynomial approximation, we employ the local bounded cochain projection introduced by Falk and Winther and compute upper bounds on its stability constant. This situation is different from the Laplace operator, where such a perturbation is easily controlled through local Poincaré inequalities. The practical viability of the approach is demonstrated in two-dimensional test cases.

References

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Contributed Talk **Thursday, 10:30–10:55, Zoom Room 11, link tba**

On the Periodic Wave Solutions of the Fractional Benjamin-Bona-Mahony Equation

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Currently, the studies on periodic traveling waves of the nonlinear dispersive equations are becoming very popular. In this study we investigate the spectral stability of the periodic waves for the fractional Benjamin-Bona-Mahony (fBBM) equation, numerically. For the numerical generation of periodic traveling wave solutions we use an iteration method which is based on a modification of Petviashvili's algorithm.

Contributed Talk **Thursday, 11:00–11:25, Zoom Room 11, link tba**

Stability of traveling oscillating fronts in complex Ginzburg Landau equations

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Traveling oscillating fronts (TOFs) are specific waves of the form $U_\star(x,t) = e^{-i\omega t}V_\star(x-ct)$ with a profile $V_\star$ which decays at $-\infty$ but approaches a nonzero limit at $+\infty$. TOFs usually appear in complex Ginzburg Landau equations of the type $U_t = \alpha U_{xx} + G(|U|^2)U$. In this talk we investigate the asymptotic stability of TOFs, where we allow the initial perturbation to be the sum of an exponentially localized part and a front-like part which approaches a small but nonzero limit at $+\infty$. The underlying assumptions guarantee that the operator, obtained from linearizing about the TOF in a co-moving and co-rotating frame, has essential spectrum touching the imaginary axis in a quadratic fashion and that further isolated eigenvalues are bounded away from the imaginary axis. The basic idea is to consider the problem in an extended phase space which couples the wave dynamics on the real line to the ODE dynamics at $+\infty$. The framework allows to derive nonlinear stability with respect to exponentially weighted norms with a decay rate weaker than the rate with which the wave approaches the limit at $+\infty$.

Contributed Talk Thursday, 11:30–11:55, Zoom Room 11, link tba

Surviving non-self-adjointness: a couple of tools

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Since around the turn of the millennium, there has been a flood of interest in the study of non-self-adjoint operators in Quantum Mechanics, not only for their physical interest, but also for the mathematical challenges faced in their study. The lack of tools like the variational methods and the Spectral Theorem, requires the implementation of other techniques, such as the method of the multipliers and the Birman-Schwinger principle, the latter being widely exploited in recent times in combination with resolvent estimates of independent interest. In this talk, we give a quick survey about the application of these methods to the eigenvalues confinement problem for many operators, with a special attention to the Dirac one, main topic of our Ph.D. dissertation.

Contributed Talk Thursday, 16:30–16:55, Zoom Room 09, link tba

Wave Propagation in Unbounded Quasiperiodic Media: a Numerical Approach

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Quasiperiodic media, defined as cuts of higher-dimensional periodic media, have drawn a lot of attention since the discovery of quasi-crystals. Partial differential equations with quasiperiodic coefficients have been extensively studied from a theoretical point of view, but aside from the homogenization setting, there has been much less work on their numerical resolution.

This talk is devoted to the numerical resolution of the time-harmonic scalar wave equation, with a nonreal complex frequency, in a one-dimensional unbounded quasiperiodic medium. Using the definition of quasiperiodicity, this problem can be lifted onto a higher dimensional non-elliptic problem with periodic coefficients. The periodicity of the new problem allows one to use adapted tools. However, the non-elliptic nature of the equation makes its mathematical and numerical analysis more delicate. Numerical results are provided to illustrate the method. Finally, the extension of the numerical method to the non-dissipative case will be discussed.

Contributed Talk	Thursday, 17:00–17:25, Zoom Room 09, link tba

Efficient numerical method for time domain electromagnetic wave propagation in thin co-axial cables.

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The numerical simulation in the time domain of electromagnetic wave propagation in thin coaxial cables represents a scientific challenge. Indeed, the cables have transverse dimensions that are relatively small compared to the longitudinal dimension, and, for realistic applications, it is necessary to be able to take into account complex situations, in particular the highly heterogeneous character of the internal structure of these cables.

We have to take into account that, in the considered applications, the wavelength λ is very large compared to δ (the rescaled diameter of the cable cross-section), and at the same time is very small compared to L (the size of the cable). This specificity has two impacts on the time discretization : first, an implicit scheme would be too costly given the size of the problem, second, an explicit scheme is to be avoided because the corresponding CFL condition would be too constraining.

In this work we will present an efficient hybrid numerical method for solving our problem. The idea is to use an anisotropic prismatic mesh, with a transverse step size h_T and a longitudinal step size h for the space discretization (with $h_T << h$), and a hybrid implicit-explicit in the longitudinal scheme for the time discretization. The main property of this method is that the stability condition only involves the longitudinal space-step h : $\Delta t \leq c\ h$.

Some numerical simulations will be presented.

Contributed Talk	Thursday, 17:30–17:55, Zoom Room 09, link tba

Adaptive Spectral Decomposition for Inverse Scattering Problems

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A nonlinear optimization method is proposed for inverse scattering problems, where the inverse medium problem is formulated as a PDE-constrained optimization problem and solved by a gradient based method. Instead of a grid-based discrete representation, the medium is projected to a finite-dimensional subspace, which is iteratively adapted during the optimization. Each subspace is spanned by the first few eigenfunctions of a linearized regularization penalty functional chosen a priori. The eigenfunctions are selected according to both their approximation properties and the cost functional's sensitivities. The (small and adaptively chosen) finite number of eigenfunctions effectively introduces regularization into the inversion and thus avoids the need for standard Tikhonov-type regularization and, in practice, appears robust to missing data or added noise.
Numerical results illustrate the accuracy and efficiency of the resulting adaptive spectral regularization for time-dependent inverse scattering problems.

References

[1] D. H. Baffet, Y. G. Gleichmann, and M. J. Grote, Error estimates for adaptive spectral decompositions, *arXiv preprint arXiv:2107.14513 (2021).*

[2] D. H. Baffet, M. J. Grote, and J. H. Tang, Adaptive spectral decompositions for inverse medium problems, *Inverse Problems 37.2 (2021): 025006.*

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