Robust Energy Management for a Microgrid

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2022 IEEE 7th International Energy Conference (ENERGYCON) | 978-1-6654-7982-0/22/$31.00 ©2022 IEEE | DOI: 10.1109/ENERGYCON53164.2022.9830301

Abstract—**Due to the increasing penetration of photovoltaic (PV) systems, electric vehicles (EV) and other smart devices on a household level, the role of consumers changes from pure consumption to production and storage of electricity. These prosumers will also directly participate in future electricity markets. To compensate for the small scale and the fluctuations in their demand and production, one promising approach for prosumers is to form small energy communities or microgrids, and participate in the electricity markets as one entity. A challenge for these microgrids is to find an optimal energy management strategy, mainly due to the uncertainty in electricity prices, in PV generation as well as in the prosumer loads. To integrate this uncertainty into the planning, an adaptive robust optimization approach using linear decision rules is proposed in this paper. The linear decision rules allow for a delayed determination of some of the decisions and can therefore adapt to realizations of the uncertainty. Three different uncertainty scenarios are used to evaluate and compare the proposed approach in a case study and to get more structural insights into the efficiency of the approach.**

Index Terms—**adaptive robust optimization, microgrid, energy management, uncertainty, energy transition**

I. Introduction

In recent years, the ongoing energy transition has led to a drastic increase of renewable energy sources and new smart devices, especially in the distribution grid. This in principle positive development has also a downside, since in particular the increase of PV systems as well as the growing popularity of EVs and other devices such as e.g. heat pumps has led to an increased burden on the low-voltage (LV) grid due to larger peaks in demand and supply. Furthermore, the intermittent nature of renewable energy sources poses new problems to the operation of LV grids.

A promising approach to this problem is to use robust planning and optimization techniques to ensure feasibility of operation regardless of future realizations of the uncertainty in demand and supply. This may allow to integrate more PV systems and EVs without the need to upgrade current LV grids.

In recent research, robust optimization has already been successfully applied to various settings, from the classical unit commitment problem to trading between microgrids. Due to its focus on feasibility, robust optimization fits well with the risk-averse rules of the current energy system. For example, static robust optimization has been used to deal with different sources of uncertainty in the energy system, from *i)* price uncertainty [1] over *ii)* uncertainty within household or EV load to *iii)* uncertainty in renewable energy generation, such as e.g. PV and wind generation [2], [3]. Adaptive robust optimization on the other hand offers an appealing way to include the time aspect of decision making into the optimization process. Instead of directly making all the decisions, the concrete decision for some of the variables may only need to be determined in a later stage. These situations often can be modeled as two-stage robust optimization problems and one way to solve these problems is to split them up into an inner and an outer subproblem. In [4], this approach, combined with Benders' decomposition, is applied to a security-constraint unit commitment problem. In the last few years, this approach has become quite popular in energy management systems and the algorithm has been modified to also include other types of uncertainty, such as e.g. uncertain EV arrivals and departures [5] or generator failures [6], new types of constraints, such as e.g. active and reactive power [7] or energy transactions between microgrids [8]. In addition, decentralized versions of the algorithm have been proposed to better comply with the microgrid structure as well as with privacy aspects of the prosumers [9].

In this work, we propose and analyze a specific adaptive robust energy management problem which occurs on a microgrid level. Based on results in [10], the calculation of an optimal solution to such a problem can be computationally expensive. To overcome this issue, we propose to use affine linear decision rules (LDRs) at the cost of approximating optimality. To first get structural insights into the suggested approach, we only use base models for the different devices, while details can be integrated in a later stage.

The paper is structured as follows. In Section II, we introduce the considered model of a microgrid. We focus hereby on devices, which offer most flexibility to the systems. Such devices must likely form the core components of any future energy management system. In addition, we assume that the microgrid has a connection with limited capacity, which allows the microgrid to participate in different electricity markets. This connection is considered as a bottleneck of many current and nearby future LV grids. In Section III, we shortly introduce the main concepts of (adaptive) robust optimization and then deal with the uncertainty considered in this paper. In Section IV we verify and analyze our proposed algorithms in a small case study and compare them with a static robust optimization

This research is supported by the Netherlands Organization for Scientific Research (NWO) grant 645.002.001.

approach. Section V concludes the work and gives an outlook on future research directions.

II. Setting and Model

In this section, we first introduce the setting of the considered energy management problem with its players and devices and then present a deterministic mathematical formulation for the operation of the microgrid. This forms the basis for the adaptive robust approach considered in the following sections.

A. Devices and Entities

Throughout this paper, we consider a time horizon which is split up into a set of time slots $\mathcal{T}$ with $T = |\mathcal{T}|$. For any two time slots $t_1, t_2 \in \mathcal{T}$, $t_1 < t_2$ denotes that time slot t_1 ends before time slot t_2 starts. The considered microgrid consists of a microgrid operator (MGO), and a set $\mathcal{N}_P$ of prosumers. In addition, the microgrid is connected to two different electricity markets, where the MGO can buy and sell electricity:

- In the *day-ahead* market, the MGO can buy and sell electricity the day before the actual realization. The price for buying and selling electricity at time slot t from the day-ahead market are assumed to be given and denoted by $\pi_t^{DA,b}$, and $\pi_t^{DA,s}$.
- In the *intraday* market, the MGO can buy and sell electricity directly within a time slot. The prices for buying and selling electricity at the intraday market at time slot t are denoted by $\pi_t^{ID,b}$ and $\pi_t^{ID,s}$.

The microgrid is connected to the markets via a line with a limited capacity C^{grid}, representing e.g. the capacity constraint of the considered LV grid.

Each prosumer has given a fixed load and is possibly equipped with different devices, such as a PV system, a battery or an EV. The MGO can be equipped with a (communal) battery, a PV system and with distributed generators (DG). Let $\mathcal{N}_{PV}$, $\mathcal{N}_B$, $\mathcal{N}_{EV}$ and $\mathcal{N}_{DG}$ denote the overall sets of PV systems, batteries, EVs and distributed generators respectively and let $f : \mathcal{N}_{PV} \cup \mathcal{N}_B \cup \mathcal{N}_{EV} \to \mathcal{N}_P$ be a function mapping each device to its corresponding prosumer. In the following, we shortly describe the modeling of the different device types and the flexibility, these offer to the system:

- Each prosumer $i \in \mathcal{N}_P$ has a *fixed load* $p^{L,i} \in \mathbb{R}^T$, with $p_t^{L,i}$ being the load during time slot t. This load cannot be curtailed or shifted.
- A PV system $j \in \mathcal{N}_{PV}$ has an output $p^{PV,j} \in \mathbb{R}^T$, with $p_t^{PV,j}$ being the output during time slot t. The PV generation can only be controlled by curtailment.
- A battery $k \in \mathcal{N}_B$ has four different parameters, the capacity $C^{B,k} \in \mathbb{R}_+$, a discharging limit $DL^{B,k} \in \mathbb{R}_+$, a charging limit $CL^{B,k} \in \mathbb{R}_+$ and an initial state of charge $SOC^{B,k} \in \mathbb{R}_+$ at the beginning of the time horizon.
- An EV $h \in \mathcal{N}_{EV}$ is modeled as a battery with some additional parameters. $C^{EV,h}$, $DL^{EV,h}$, $CD^{EV,h}$ and $SOC^{EV,h}$ again denote the capacity, the discharging and charging limit and the initial state of charge of EV h. Furthermore $at_v^{EV,h} \in \mathcal{T}$ and $dt_w^{EV,h} \in \mathcal{T}$ denote the v-th arrival and w-th departure time of EV h during the time horizon and $p_t^{EV,h}$ the electricity usage during time slot t. Let $\mathcal{I}^h$ denote the set of time slots in which EV h is not connected to the microgrid.
- A generator $g \in \mathcal{N}_{DG}$ is specified by a capacity $C^{DG,g}$, a ramp-up limit $RU^{DG,g}$ and a ramp-down limit $RD^{DG,g}$. In addition, there are variable generation costs $c^{DG,g} \in \mathbb{R}^T$ per unit of produced electricity.

B. Model

In this section we introduce the energy management problem within the microgrid by specifying its decision variables, the objective and the constraints.

1) Variables:

- $y_t^{DA,buy}$ and $y_t^{DA,sell}$ denote the amount of energy the MGO buys or sells at the day-ahead market during time slot t.
- $x_t^{ID,buy}$ and $x_t^{ID,sell}$ denote the interactions of the MGO with the intraday market.
- $x_t^{PV,j}$ denotes the amount of electricity from PV system j during time slot t which is not curtailed.
- $x_t^{B,k}$ denotes the amount of charged electricity of battery k during time slot t. If $x_t^{B,k} > 0$, the battery is charged, while for $x_t^{B,k} < 0$, the battery is discharged.
- $x_t^{EV,h}$ represents the charging and discharging of EV h and has the same assumptions as the battery.
- $x_t^{DG,g}$ denotes the amount of produced electricity by generator g.

2) Objective Function: The objective function (1) represents the cost associated with the microgrid over the complete time horizon.

$$\min \sum_{t\in\mathcal{T}} \Big(\pi_t^{DA,b} y_t^{DA,b} - \pi_t^{DA,s} y_t^{DA,s} + \pi_t^{ID,b} x_t^{ID,b} - \pi_t^{ID,s} x_t^{ID,s} + \sum_{g\in\mathcal{N}_{DG}} c^{DG,g} x_t^{DG,g} \Big) \quad (1)$$

3) Demand-Supply-Balancing Constraint: The demand-supply-balancing constraint (2) ensures the balance between demand and supply within the microgrid for every time slot t.

$$\sum_{j\in\mathcal{N}_{PV}} x_t^{PV,j} + \sum_{g\in\mathcal{N}_{DG}} x_t^{DG,g} - \sum_{k\in\mathcal{N}_B} x_t^{B,k} - \sum_{h\in\mathcal{N}_{EV}} x_t^{EV,h} + y_t^{DA,b} + x_t^{ID,b} - y_t^{DA,s} - x_t^{ID,s} = \sum_{i\in\mathcal{N}_P} p_t^{L,i}. \quad (2)$$

4) PV Constraint: The following constraint restricts the influence on the PV production to curtailment.

$$0 \le x_t^{PV,j} \le p_t^{PV,j} \quad \forall t \in \mathcal{T}, j \in \mathcal{N}_{PV} \quad (3)$$

5) Battery Constraints: Constraint (4) ensures that the state of charge of a battery after time slot t, is between 0 and the total capacity of the battery, while constraint (5) ensures the limitations on charging and discharging.

$$0 \le SOC^{B,k} + \sum_{s=1}^{t} x_s^{B,k} \le C^{B,k} \quad \forall t \in \mathcal{T}, k \in \mathcal{N}_B \quad (4)$$

$$-DL^{B,k} \le x_t^{B,k} \le CL^{B,k} \quad \forall t \in \mathcal{T}, k \in \mathcal{N}_B \quad (5)$$

6) EV Constraints: As an EV h can be modeled as a battery, constraint (6) is similar to constraint (4). The only difference in the state of charge is the incorporation of the used energy $p_s^{EV,h}$ of EV h, and constraint (8), which ensures that EV h can only be charged or discharged when connected to the grid. Note that constraints (6) - (8) have to be introduced for each EV $h \in \mathcal{N}_{EV}$.

$$0 \leq SOC^{EV,h} + \sum_{s=1}^{t} x_s^{EV,h} - p_s^{EV,h} \leq C^{EV,h} \quad \forall t \in \mathcal{T} \quad (6)$$

$$-DL^{EV,h} \leq x_t^{EV,h} \leq CL^{EV,h} \forall t \in \mathcal{T} \quad (7)$$

$$x_t^{EV,h} = 0 \qquad \forall t \in \mathcal{I}^h \quad (8)$$

7) Generator Constraints: Constraint (9) ensures that the capacity of each generator $g \in \mathcal{N}_{DG}$ is not exceeded, while constraint (10) enforces the ramping-up and ramping-down limits for each generator $g \in \mathcal{N}_{DG}$.

$$0 \leq x_t^{DG,g} \leq C^{DG,g} \qquad \forall t \in \mathcal{T} \qquad (9)$$

$$-RD^{DG,g} \leq x_t^{DG,g} - x_{t-1}^{DG,g} \leq RU^{DG,g} \quad \forall t \in \mathcal{T} \quad (10)$$

8) Market and Grid Constraints: Constraints (11) and (12) ensure that the line capacity between the markets and the microgrid is not violated.

$$0 \leq y_t^{DA,buy} + x_t^{ID,buy} \leq C^{grid} \quad \forall t \in \mathcal{T} \quad (11)$$

$$0 \leq y_t^{DA,sell} + x_t^{ID,sell} \leq C^{grid} \quad \forall t \in \mathcal{T} \quad (12)$$

$$y_t^{DA,sell}, y_t^{DA,buy}, x_t^{ID,sell}, x_t^{ID,buy} \geq 0 \qquad \forall t \in \mathcal{T} \quad (13)$$

Objective function (1), together with constraints (2) – (13) form the deterministic model for the energy management of the considered microgrid.

III. Adaptive Robust Optimization

Robust optimization offers a practical way to include uncertainty of the parameters into the model. Within the scope of the above microgrid model, the uncertainty in the parameters stems mainly from prediction errors, either due to human behavior (e.g. the load) or due to the intermittent nature of renewable energy sources (e.g. PV generation). In robust optimization, for each uncertain parameter, an uncertainty set is defined as the set of possible true realizations of this parameter. Robust optimization now tries to find the best solution, which is feasible for every possible realization of the uncertain parameters. To do so, it reformulates constraints containing uncertain parameters into a computationally tractable form. In this way it directly includes the uncertainty into the model, see e.g. [11] for more details.

Disadvantages of this (static) robust optimization approach are its often quite conservative solution, as well as the fact that all decisions are made upfront, i.e. already for the whole time period. However, in settings with multiple time slots often some of the decisions in later time slots do not have to be made at the beginning of the planning horizon, but can be delayed until earlier uncertainty has been realized. If this additional information can be used, the resulting solution will likely perform better compared to the static case. Adaptive robust optimization (ARO) is a methodology for such multi-stage robust problems, in which not all decisions have to be made directly, see [12] for an overview of different techniques and examples.

ARO divides the variables of a multi-stage robust optimization problem into *here-and-now* and *wait-and-see* variables. The values of the here-and-now variables are to be determined directly, while the decisions of the wait-and-see variables can be postponed until the start of their corresponding time slot. Although even simple multi-stage RO problems are often NP-hard [10], there are techniques using Bender's decomposition or column and constraint cuts to speed up the computation. Nevertheless, for larger instances, even these advanced techniques may struggle to find the optimal solution within a reasonable time. One way how to overcome this issue is to drop the optimality requirement and to only approximate an optimal solution. This e.g. can be done by replacing the wait-and-see variables by a (affine linear) function depending on the uncertain parameters. In the remainder, we refer to these affine linear functions as linear decision rules (LDRs). It has been shown that this type of approximation performs quite well in many different settings, see e.g. [13] for further applications and examples.

A. Uncertainty Sets

The key components of any form of robust optimization are the uncertainty sets, which define the uncertainty in the parameter. Within the scope of this microgrid, there are a various sources of uncertainty:

- **Load:** The demand of the prosumers is highly influenced by the behavior of the prosumers.
- **PV:** The output of a PV system strongly depends on the weather. Perfect predictions even over a short or mid-term time horizon with small time slots are nearly impossible to achieve.
- **EV:** The electricity demand of an EV depends on a number of external factors, such as temperature, distance, speed and acceleration as well as the vehicle heating or cooling. In addition, arrival and departure times are often uncertain as well, in particular in setting with short time slot lengths, e.g. as 15 minutes.
- **Market:** The electricity prices of both, day-ahead and intraday market are subject to constant fluctuations.

In the following, we use the PV generation as an example to introduce the uncertainty sets in more detail. The uncertainty sets for the remaining parameters can be dealt with in a similar way.

The set of all PV systems is modeled as one single PV system in order to take into account the correlation between the output of neighboring PV systems. Instead of directly defining the uncertainty set with possible PV output values, we modify the uncertain PV output into a certain (known) part and an uncertain part,

$$p_t^{PV} = \hat{p}_t^{PV}(1 + \alpha_t^{PV} u_t^{PV}),$$

where $\hat{p}_t^{PV}$ is the deterministic expected or nominal value of the PV output. The expression $\alpha_t^{PV} u_t^{PV}$ describes the fraction by which the PV production $\hat{p}_t^{PV}$ might deviate, i.e. increase or decrease. This part consists of a given (known) fraction $\alpha_t^{PV} \in [0, 1]$ of the PV production, which may depend on the weather forecast or other external factors and a random variable u_t^{PV} specifying how much of this fraction is really added or subtracted. The latter variable represents the uncertain part of the parameter and hence, the uncertainty of the PV output can be specified by the uncertainty set

$$U^{PV} = \left\{ u^{PV} \in \mathbb{R}^{|\mathcal{T}|} | \|u^{PV}\|_\infty \leq 1, \|u^{PV}\|_1 \leq \Gamma^{PV} \right\}.$$

This uncertainty set is often referred to as a *budget* uncertainty set. It restricts the uncertainty for each time slot between -1 and 1, and in case $\Gamma^{PV} < |\mathcal{T}|$, the $L1$-norm constraint does not allow all uncertainty to be at extreme points (here -1 or 1). The budget uncertainty set therefore represents scenarios in which not all possible uncertainty per time interval can be realized, but at most a certain amount Γ^{PV}. This type of uncertainty set is a frequently used set for uncertainty and there exist robust optimization techniques to derive computationally tractable robust counterparts for such sets, see [14] for a detailed overview.

Due to the standardized uncertain part u, most of the uncertainty sets are very similar to each other and only have an alternative budget parameter Γ, depending on the corresponding parameters and setting. Note, that the uncertainty parameter α allows for a quick and easy adaptation of the uncertainty within the model.

The only uncertain parameters within our microgrid model which cannot be modeled by means of a budget uncertainty set are the arrival and departure times of the EVs. Here, a simple uncertainty interval is an appropriate choice

$$U^{EV,A,h} = \left\{ at^{EV,h} \in \mathcal{T} | \underline{at}^{EV,h} \leq at^{EV,h} \leq \overline{at}^{EV,h} \right\},$$

whereby values $\underline{at}^{EV,h}$ and $\overline{at}^{EV,h}$ are given lower and upper bounds on the considered times.

B. Linear Decision Rules

In the following, we highlight how the uncertainty of the parameters can be included into the linear decision rules to design an adaptive robust model. Similar to the uncertainty sets, we use the PV system to present the linear decision rule in a more detailed way; applying them to the other variables can be done in a similar way. Given a PV system j and time slot t, the variable $x_t^{PV,j}$ describes the amount of electricity which is used, i.e. not curtailed. Note, that the decision how much to curtail can be made directly before the time slot. Thus, the corresponding LDR for $x_t^{PV,j}$ is designed as follows

$$x_t^{PV,j}(u) = \beta_t^{PV,j} + \beta_{t,t}^{PV,j,PV} u_t^{PV} + \beta_{t,t-1}^{PV,j,L,F(j)} u_{t-1}^{L,F(j)},$$

where $\beta_t^{PV,j}, \beta_{t,t}^{PV,j,PV}$ and $\beta_{t,t-1}^{PV,j,L,F(j)}$ serve as the parameters of the LDR. Within this LDR, $\beta_t^{PV,j}$ represents the part of the decision, which is independent of any future realizations, while $\beta_{t,t}^{PV,j,PV}$ and $\beta_{t,t-1}^{PV,j,L,F(j)}$ represent the influence of the realization of the uncertain PV and load parameter. Note, that $\beta_{t,t-1}^{PV,j,L,F(j)}$ depends on the realization of the load uncertainty of time slot $t-1$, while for $\beta_{t,t}^{PV,j,PV}$ we assume that it is possible to perfectly predict the PV production of the next time slot t. When replacing the variable $x_t^{PV,j}$ by the LDR $x_t^{PV,j}(u)$, the parameters of the LDR become here-and-now variables in the robust model. Note that replacing every wait-and-see variable by a LDR results in a final model with only here-and-now variables, and hence well-known standard techniques from static robust optimization can be applied.

TABLE I
DEVICE PARAMETER

Type	C	CB/RU	DB/RD	SOC
EV	54	1.9	1.9	0
Battery	10	5	5	0
DG	1	5	5	-

IV. Computational Study

The goal of this computational study is to demonstrate the practical feasibility of the proposed approach by means of a small case study. Hereby, we focus on two main questions. The first question is how much better the LDR approach performs compared to the static robust model. In a second step, we focus more on the effect of the different components of LDRs on the objective value. Unless explicitly mentioned, we apply the linear decision rules as described in Section III to all decisions of the deterministic model in Section II apart from the day-ahead market variables. That is, we replace $x_t^{ID,buy}$, $x_t^{ID,sell}$, $x_t^{PV,j}$, $x_t^{G,g}$, $x_t^{B,b}$ and $x_t^{EV,h}$ each with their respective LDR.

A. Data

Throughout this computational study, we consider a microgrid over a time horizon of 2 days, split up into 192 time slots, each of 15 minutes length. In the base scenario, we consider a microgrid of 10 prosumers, one distributed generation unit, one communal battery and 5 EVs (see Table I for the parameter settings of the EVs, the battery and the DG). Furthermore, the day-ahead and intraday market prices are taken from the EPEX market from the 8th and 9th of September 2021 (see Figure 1). The prosumer load profiles are modeled to have an average electricity consumption of 8 to 10 kWh per day. The aggregated load profile as well as the PV production over the complete time horizon are specified as depicted in Figure 2.

Regarding the uncertainty sets, we introduce three scenarios, A, B and C, each representing a different level of uncertainty. The corresponding uncertainties are given in Table II.

TABLE II
UNCERTAINTY SCENARIOS

Scenario	α^L	α^{PV}	α^{EV}	α^{DA}	α^{ID}
A	0.1	0.15	0.05	0.05	0.1
B	0.2	0.25	0.1	0.1	0.2
C	0.35	0.4	0.2	0.2	0.3

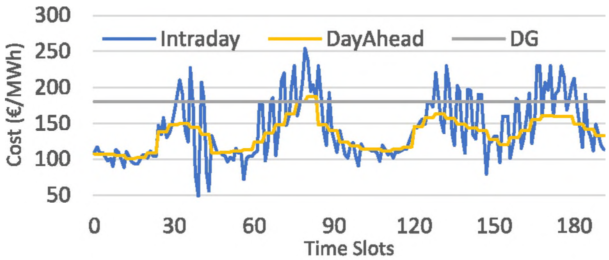

Fig. 1. Market Prices and DG cost

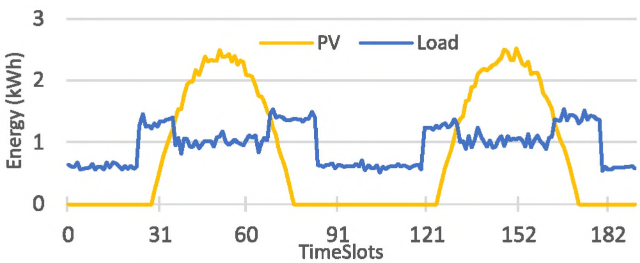

Fig. 2. Aggregated PV and Load profiles

B. LDRs vs. Static Robust Model

To get some insight in the working of the proposed approach, we compare the solution by the adaptive robust LDR approach with the solution of the static approach for the three different scenarios. As the LDR approach heavily depends on the true realizations of the uncertainty, directly comparing the objective values of both models with each other does not lead to a fair comparison due to the adaptive behavior of our proposed approach. Hence, we will randomly draw 100 realizations of the uncertainty and use the LDRs as well as the solution of the static robust model to compute the actual costs of the approaches.

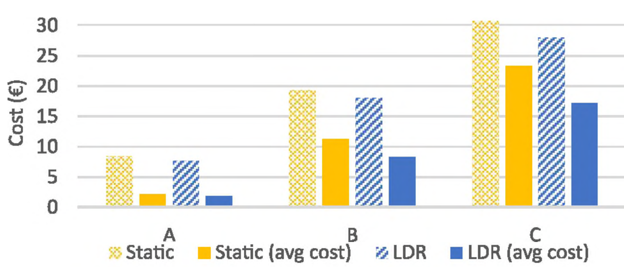

Fig. 3. Static vs. Adaptive Approaches

The results are displayed in Figure 3, where the three groups represent the three different scenarios A, B and C. Furthermore, the columns *Static* and *LDR* represent the objective value of the static robust, respectively the LDR model, while the addition of *(avg cost)* represents the average actual cost of the 100 random realizations. Note, that due to the uncertainty in the market prices, the average actual costs of both approaches are better than the objective value of the respective models, as these assume the worst case realizations of the prices. Focusing on the objective values of the models first, we note that the additional flexibility gained by replacing the wait-and-see variables by LDRs results in small improvements of 5.4% to 6.9%. Comparing the average actual costs on the other hand, we note that for scenario A, the LDRs are 10.70% better than the static solution, for scenario B, this increases to 24.41% and in scenario C, the LDRs are better by 24.65%.

As expected, the real advantage of the LDR approach over the static robust model only reveals itself when considering the actual costs instead of the objective value of the models, as the LDR can only then display their adaptive behavior. In addition, the LDR approach is particularly useful for scenarios with larger uncertainty sets, although even in cases of only minor uncertainty it already outperforms the static robust optimization model. This improvement for larger uncertainty sets can be attributed to the, on average, larger differences between the worst case realizations and the actual realizations.

C. Structure of LDRs

As the adaptive robust approach performs (significantly) better than the static robust model, in this subsection, we analyze the LDRs in more detail. One disadvantage of the proposed approach is the size of the resulting LP formulation, which is several (hundred) times that of the static robust model. As a larger LP formulation often implies a longer running time of the solver, reducing the number of variables and constraints without changing the set of feasible solutions can be a first step to shorten the running time. By identifying which LDRs have a larger impact on the objective value, we are able to derive more compact models which still perform better than the static robust model. This subsection consists of two parts. In the first, we derive and test various versions of LDR models and in the second, we analyze the differences between the two best performing LDR models.

To identify which LDRs have the largest impact on the objective value, we modify the full LDR model to only include LDRs for one variable type. The resulting five models (PV, EV, Battery, intraday market, DG) are tested using base scenario B. In addition to the single LDR models, we also test a model in which only the EV, the PV and the intraday market variables are replaced by LDRs. This choice stems from the optimal solutions of the full LDR model, in which only these three LDRs had non-zero parameters corresponding to the uncertainties. Hence, the full LDR model and the EV+PV+ID model should perform equally well. Once again, the objective value of the model and the average actual cost are used to compare the models with each other, see Table III.

Focusing on the objective values first, we notice that both, the EV model and the EV+PV+ID model, achieve the same objective value as the full LDR model. Regarding the EV+PV+ID model, this is no surprise, as the optimal solution of the full LDR model is also feasible for the more restricted model. Regarding the EV model, the optimal solution of the full LDR model is not feasible, but nevertheless the objective value is the same. This implies that replacing only the EV variables

TABLE III
COMPARISON OF SINGLE LDR MODELS

Model	obj. value	avg actual cost
Static Robust model	19.05	11.01
full LDR model	18.02	8.32
PV	19.05	11.04
EV	18.02	10.09
Battery	19.05	11,04
intraday market	19.05	11.04
DG	19.05	11.04
EV+PV+ID	18.02	9.73

by LDRs can already achieve the same effect as replacing all wait-and-see variables and hence offers a promising direction into decreasing the size of the model.

Shifting the focus from the 'worst-case' oriented objective value to the actual cost, we see a difference between the previously equally good models. Both, the EV model and the EV+PV+ID model perform worse compared to the full LDR model. The reason for this is that the EV model only has a single LDR, and therefore, it can adapt less of its decisions to the actual realizations of the uncertainty. Regarding the EV+PV+ID model, this does not hold true, as also the full LDR model only makes use of the LDRs of the EV, the PV and the intraday market. Hence, it might be expected that both models should perform equally well w.r.t. the actual cost. However the full model performs better by about 16.88%.

This difference can be explained by the structure of the underlying set of feasible solutions. Due to the additional variables introduced via the LDRs, there are often multiple optimal solutions. Even though the optimal solution of the full LDR model is also feasible (and optimal) for the EV+PV+ID model, the solver produced another optimal solution. Hence, although both solutions have the same objective value w.r.t. the objective function, evaluating them w.r.t. the random realizations leads to different actual costs. Using the optimal solution of the EV model, which is feasible and also optimal for the full LDR model, the difference in actual cost between two optimal solutions can be as much as 21.24%.

One possibility to get a 'good' optimal solution regarding the actual realizations is the following approach, which is based on solving two similar models.

1) Run the original model giving the optimal objective value of the problem.
2) Modify the model by adding a constraint, considering only solutions having this objective value into the model. Now replace the objective function by a function which uses e.g. average or expected prices.

Introducing the objective function as a constraint into the model restricts the set of feasible solutions to the set of optimal solutions. Solving the second model then only considers optimal solutions w.r.t. the original objective function, while also minimizing the average of the expected cost. Furthermore, note that this approach does not guarantee a better solution, as there may always be realizations in which the original optimal solution performs better. Note, that all LDR models within this computational study are using the afore mentioned two stage approach. For the full LDR model and scenario B this leads to an improvement of 16.18% between the average actual cost of the two stage approach (8.32) and the average actual cost of the solution of the first model run (9.93).

V. CONCLUSION

In this paper, a new adaptive robust optimization approach was introduced for a microgrid energy management problem. The proposed approach uses affine linear functions to replace variables and thereby enables an adaptive behavior in the model. The numerical results show that the proposed adaptive approach performs significantly better than the static robust model, while still ensuring feasibility.

The proposed approach still leaves further open research directions. When introducing the two stage approach, we briefly mentioned an option on how to integrate further knowledge, in particular stochastic information, such as e.g. distribution functions of the prices. This is one way how to combine robust optimization with aspects of stochastic programming, such as optimizing over expected values. Another way to develop the approach even further is by combining the model with a Rolling Horizon technique to reflect on the time-dependency of the uncertainty.

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