Nonlinear Dynamical Control Systems

Henk Nijmeijer · Arjan van der Schaft

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Henk Nijmeijer
Eindhoven University of Technology
Eindhoven
The Netherlands

Arjan van der Schaft
University of Groningen
Groningen
The Netherlands

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Preface

This textbook on the differential geometric approach to nonlinear control grew out of a set of lecture notes, which were prepared for a course on nonlinear system theory, given by us for the first time during the fall semester of 1988. The audience consisted mostly of graduate students, taking part in the Dutch national Graduate Program on Systems and Control. The aim of this course is to give a general introduction to modern nonlinear control theory (with an emphasis on the differential geometric approach), as well as to provide students specializing in nonlinear control theory with a firm starting point for doing research in this area.

One of our primary objectives was to give a self-contained treatment of all the topics to be included. Since the literature on nonlinear geometric control theory is rapidly expanding this forced us to limit ourselves in the choice of topics. The task of selecting topics was further aggravated by the continual shift in emphasis in the nonlinear control literature over the last years. Therefore, we decided to concentrate on some rather solid and clear-cut achievements of modern nonlinear control, which can be expected to be of remaining interest in the near future. Needless to say, there is also a personal bias in the topics we have finally selected. Furthermore, it was impossible not to be influenced by the trendsetting book "Nonlinear Control Systems: an Introduction", written by A. Isidori in 1985 (Lecture Notes in Control and Information Sciences, 72, Springer).

A second main goal was to illustrate the theory presented with examples stemming from various fields of application. As a result, Chapter 1 starts with a discussion of some characteristic examples of nonlinear control systems, which will serve as illustration throughout the subsequent chapters, besides several other examples.

Thirdly, we decided to include a rather extensive and self-contained treatment of the necessary mathematical background on differential geometry. Especially the required theory on Lie brackets, (co-)distributions and Frobenius' Theorem is covered in detail. However, some rudimentary knowledge about the fundamentals of differential geometry (manifolds, tangent space, vectorfields) will greatly facilitate the reading of the book. Furthermore, the reader is supposed to be familiar with the basic concepts of linear system theory; especially some acquaintance with linear geometric control theory will be very helpful.

Modern nonlinear control theory, in particular the differential geometric approach, has emerged during the seventies in a rather successful attempt to deal with basic questions in the state space formulation of nonlinear control systems, including the problems of controllability and observability, and (minimal) realization theory. It was also motivated by optimal control theory, in particular the Maximum Principle and its relation with controllability issues. The theory gained strong impetus at the end of the seventies and beginning of the eighties by the introduction of several new concepts, most of them having as their crucial part nonlinear *feedback*. Let us illustrate this with two papers, which can be seen as benchmarks in this development. First, there is the paper by Brockett on "Feedback invariants for nonlinear systems" (Proc. VIIth IFAC World Congress, Helsinki, pp. 1115-1120, 1978), which deals with the basic question to what extent the structure of a nonlinear control system can be changed by (static state) feedback. A direct outgrowth of this paper has been the theory on feedback linearization of nonlinear control systems. Secondly, in the paper "Nonlinear decoupling via feedback: a differential geometric approach" by Isidori, Krener, Gori-Giorgi & Monaco (IEEE Trans. Automat. Control, AC-26, pp. 341-345, 1981) the concept of a controlled invariant distribution is used for various sorts of decoupling problems (independently, a similar approach was taken by Hirschorn ("(A,B)-invariant distributions and disturbance decoupling of nonlinear systems", SIAM J. Contr. Optimiz. 19, pp. 1-19, 1981)). It is worth mentioning that the concept of a controlled invariant distribution is a nonlinear generalization of the concept of a controlled invariant subspace, which is the cornerstone in what is usually called linear geometric control theory (see the trendsetting book of Wonham, "Linear Multivariable Control", Springer, first edition 1974, third edition 1985). In fact, a substantial part of the research on nonlinear control theory in the eighties has been involved with the "translation" to the nonlinear domain of solutions of various feedback synthesis problems obtained in linear geometric control theory. Connected with the concept of (controlled) invariant distributions, the above mentioned IEEE paper also stressed the usefulness of special choices of state space coordinates, in which the system structure becomes more transparent. The search for various kinds of nonlinear normal forms, usually connected to some algorithm such as the nonlinear D^*-algorithm, the Hirschorn algorithm or the dynamic extension algorithm, has been another major trend in the eighties.

At this moment it is difficult to say what will be the prevailing trends in nonlinear control theory in the near future. Without doubt the feedback stabilization problem, which has recently obtained a strong renewed interest, will be a fruitful area. Also adaptive control of nonlinear systems, or, more modestly, the search for adaptive versions of current nonlinear control schemes is likely going to be very important, as well as digital implementation (discretization) of (continuous-time based) control strategies. Moreover, it seems that nonlinear control theory is at a point in its development where more attention should be paid to the special (physical) structure of some classes of nonlinear control systems, notably in connection with classical notions of passivity, stability and symmetry, and notions stemming from bifurcation theory and dynamical systems.

The contents of the book are organized as follows:
Chapter 1 starts with an exposition of four examples of nonlinear control systems, which will be used as illustration for the theory through the rest of the book. A few generalities concerning the definition of nonlinear control systems in state space form are briefly discussed, and some typical phenomena occurring in nonlinear differential (or difference) equations are touched upon, in order to put the study of nonlinear control systems also into the perspective of nonlinear dynamics. **Chapter 2** provides the necessary differential geometric background for the rest of the book. Section 2.1 deals with some fundamentals of differential geometry, while in Section 2.2 vectorfields, Lie brackets, (co-)distributions and Frobenius' Theorem are treated in some detail. For the reader's convenience we have included a quick survey of Section 2.1, as well as a short summary of Section 2.2 containing a list of useful properties and identities. In **Chapter 3** some aspects of controllability and observability are treated with an emphasis on nonlinear rank conditions that generalize the well-known Kalman rank conditions for controllability and observability of linear systems, and on the role of invariant distributions in obtaining local decompositions similar to the linear Kalman decompositions. **Chapter 4** is concerned with various input-output representations of nonlinear control systems, and thus provides a link with a more input-output oriented approach to nonlinear control systems, without actually going into this. Conditions for invariance of an output under a particular input, which will be crucial for the theory of decoupling in later chapters, are derived in the analytic as well as in the smooth case. In **Chapter 5** we discuss some problems concerning the transformation of nonlinear systems into simpler forms, using state-space and feedback transformations, while **Chapter 6** contains the full solution of the local feedback linearization problem (using static state feedback). In **Chapter 7** the fundamental notion of a controlled invariant distribution is introduced, and applied to the local disturbance decoupling problem. **Chapters 8 and 9** are concerned with the input-output decoupling problem; using an analytic, respectively a geometric approach. In **Chapter 10** some aspects of the local feedback stabilization problem are treated. **Chapter 11** deals with the notion of a controlled invariant submanifold and its applications to stabilization, interconnected systems and inverse systems. In **Chapter 12** a specific class of nonlinear control systems, roughly speaking mechanical control systems, is treated in some detail. Finally, in **Chapters 13** and **14** a part of the theory developed in the preceding chapters is generalized to general continuous-time systems $\dot{x} = f(x,u)$, $y = h(x,u)$, respectively to discrete-time systems.

At the end of every chapter we have added bibliographical notes about the main sources we have used, as well as some (very partial) historical information. Furthermore we have occasionally added some references to related work and further developments. We like to stress that the references are by no means meant to be complete, or are even carefully selected, and we sincerely apologize to those authors whose important contributions were inadvertently not included in the references.

As already mentioned before, many topics of interest could not be included in the present book. Notable omissions are in particular realization theory, conditions for local controllability, observer design, left- and right-invertibility, global issues in decoupling and linearization by feedback, global stabilization, singular perturbation methods and high-gain feedback, sliding mode techniques, differential algebraic methods, and, last but not least, nonlinear optimal control theory. (We also like to refer to the very recent second edition of Isidori's "Nonlinear Control Systems" (Springer, 1989) for a coverage of some additional topics.)

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Henk Nijmeijer
Arjan van der Schaft

In the second printing of the book a number of minor corrections have been included.

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Arjan van der Schaft

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Arjan van der Schaft

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Eindhoven, November 2015 *Henk Nijmeijer*
Groningen, November 2015 *Arjan van der Schaft*

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