Design tools for circular overbraiding of complex mandrels

DESIGN TOOLS FOR CIRCULAR OVERBRAIDING OF COMPLEX MANDRELS

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This research project was financially supported by the Netherlands Enterprise Agency and in kind by Eurocarbon BV.

Design tools for circular overbraiding of complex mandrels,
Van Ravenhorst, Johan Hendrik,
PhD Thesis, University of Twente, Enschede, The Netherlands,
September 2018

ISBN 978-90-365-4586-0

DOI 10.3990/1.9789036545860

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Printed by Ipskamp Drukkers BV, Enschede, The Netherlands

The front cover shows a snapshot of the circular overbraiding process, generated by Braidsim. The realized product is shown on page 258.

DESIGN TOOLS FOR CIRCULAR OVERBRAIDING OF COMPLEX MANDRELS

DISSERTATION

to obtain
the degree of doctor at the University of Twente,
on the authority of the rector magnificus,
Prof. dr. T.T.M. Palstra,
on account of the decision of the graduation committee,
to be publicly defended
on Thursday, 13 September 2018 at 14:45.

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Summary

Circular braiding is increasingly used to manufacture free-form tubular composites that replace metal primary structural components. Recent examples are car frame parts like a-pillars and rockers. A high repeatability and the simultaneous deposition of hundreds of yarns makes braiding suited for automated series production of composite preforms. The inhomogeneous and anisotropic material properties of composites add to the design degrees of freedom, allowing tailoring for specific requirements. This also makes composites more challenging for designers, especially when taking into account the manufacturing constraints of the braiding process.

Various braiding process models are used to address these challenges. Where analytical models lack accuracy and finite element models lack a short calculation time, kinematic models can offer a compromise, describing the braiding process by the three-dimensional geometry, yet without material deformations and interactions. This can be especially useful in a conceptual or preliminary design phase when a high level of detail is not yet needed.

The objective of this work is to develop braiding design charts for simple braided component designs and to develop fast kinematic braiding process simulation software for more complex designs. In this regard, the following three problems are addressed in an increasing level of detail:

First, circular overbraiding on complex mandrels currently lacks automatic generation of machine control data that provides the braid angle distribution matching the braid's required structural properties. To solve this limitation, an inverse kinematics-based procedure was designed and implemented for circular braiding machines with optional guide rings. This procedure generates the braiding machine control data for a given braid angle distribution on mandrels with complex 3D shapes including

non-axisymmetric, optionally eccentric cross-sections that can vary in shape and size along an optionally curved mandrel centerline. This procedure results in a computation time of seconds, fit for Computer Aided Engineering process chain integration. Experimental validation showed a deviation from the model that was mainly attributed to the neglect of yarn interaction and guide ring contact friction in the model, leading to an incorrectly modeled convergence zone length. To assess the systematic error caused by the neglect of yarn interaction, a novel axisymmetric biaxial braiding process model was proposed that includes the yarn interaction with friction. Both this model and experimental validation with carbon yarns and a 144 carrier machine confirm a convergence zone length decrease of 25% with respect to a model without yarn interaction for the case analyzed.

Secondly, removing the correct spools from a machine for a desired braid pattern to braid, for example, a small product on a large machine, can be error-prone. Novel procedures were presented for converting braid patterns to and from spool patterns to avoid trial-and error for finding the correct spool pattern for a given required braid pattern.

Thirdly, an incorrect braid thickness can lead to problems when inserting the braided preform in the mold for downstream resin injection, and it can affect the structural properties and the weight. For this purpose, the braid meso-geometry geometry was parametrized, enabling a braid characterization for symmetric and non-symmetric, bi- and triaxial, open- and closed-mesh braids with mono-material and hybrid bias yarn groups and arbitrary distributions of the braid angle and the yarn count on arbitrary continuous or discrete mandrel surfaces. This approach can be readily integrated in analytical, kinematic, or finite element models.

To avoid simulation for simple designs, novel design charts were proposed for biaxial braids, showing, amongst others, the feasible design space, bounded by jamming on the one hand and insufficient coverage on the other hand. The charts enable a quick feasibility assessment prior to or instead of braiding process modeling or physical experimentation. For more complex designs, the kinematic models proposed in this work have been implemented in the braiding process simulation software 'Braidsim', taking the braiding machine control data as input and generating the composite layup or vice versa. In both cases, Braidsim can generate animations of the braiding process and a full 3D composite layup for downstream structural analysis.

Samenvatting

Vlechten wordt in toenemende mate gebruikt voor het produceren van buisvormige composieten ter vervanging van metalen structurele componenten. Recente voorbeelden zijn onderdelen van autoframes zoals A-zuilen en kokerbalken. Een goede reproduceerbaarheid en het gelijktijdig deponeren van honderden draden maakt vlechten geschikt voor geautomatiseerde serieproductie van omvlochten kernen. De inhomogene en anisotrope materiaaleigenschappen van composieten voegen vrijheidsgraden aan het ontwerp toe, wat het mogelijk maakt om aan specifieke eisen te voldoen. Dit maakt het gebruik van composieten echter ook moeilijker voor ontwerpers, voornamelijk wanneer ook rekening wordt gehouden met de productierandvoorwaarden van het vlechtproces.

Verschillende vlechtprocesmodellen worden gebruikt om deze uitdaging aan te gaan. Waar analytische modellen een lage nauwkeurigheid hebben en eindige elementenmodellen een hoge rekentijd, kunnen kinematische modellen een compromis bieden door het vlechtproces te beschrijven met driedimensionale geometrie, maar zonder materiaal deformaties en -interacties. Dit kan met name nuttig zijn in de conceptuele of voorlopige ontwerpfase waarin een hoog detailniveau nog niet relevant is.

De doelstelling van dit werk is de ontwikkeling van ontwerpdiagrammen voor het ontwerpen van eenvoudige gevlochten componenten en de ontwikkeling van snelle kinematische vlechtprocessimulatiesoftware voor complexere ontwerpen. In dit verband worden de volgende drie vraagstukken behandeld, geordend op een toenemend detailniveau:

Ten eerste ontbreekt het bij vlechten over complexe kernen aan het geautomatiseerd genereren van machinebesturing die resulteert in de vlechthoekverdeling overeenkomstig de vereiste structurele eigenschappen van de vlecht. Om deze beperking weg te nemen, is een procedure

gebaseerd op inverse kinematica ontwikkeld en geïmplementeerd voor vlechtmachines met optionele vlechtringen. Deze procedure genereert de machinebesturing voor een gegeven vlechthoekverdeling op kernen met complexe driedimensionale vormen inclusief niet-axisymmetrische, optioneel excentrische doorsnedes langs een optioneel gekromde hartlijn. Deze procedure heeft een rekentijd van enkele seconden, zodat het geschikt is voor integratie in de procesketen van computer-ondersteund ontwerpen. Experimentele validatie liet een afwijking van het model zien die hoofdzakelijk kan worden toegeschreven aan de verwaarlozing van draadinteractie en wrijving met de vlechtringen in het model, leidend tot een incorrect gemodelleerde lengte van de convergentiezone. Om de systematische fout door de verwaarlozing van het gebrek aan draadinteractie te analyseren, is een nieuw biaxiaal vlechtprocesmodel inclusief draadinteractie met wrijving beschreven. Zowel dit model als een experimentele validatie met koolstof draden en een 144-spoels machine bevestigden een 25% afname van de convergentiezonelengte voor het onderzochte geval in vergelijking met het model dat draadinteractie verwaarloost.

Ten tweede is het verwijderen van de juiste spoelen uit de machine voor het verkrijgen van het juiste vlechtpatroon, bijvoorbeeld wanneer een klein product op een grote machine wordt gevlochten, foutgevoelig. Nieuwe procedures voor het converteren van vlechtpatronen van en naar spoelpatronen zijn beschreven om langdurig zoeken naar het juiste spoelpatroon voor een gegeven vlechtpatroon te voorkomen.

Ten derde kan een afwijkende vlechtdikte leiden tot problemen bij het inleggen van de omvlochten kern in de mal voor harsinjectie. Ook kan een dikte afwijking invloed hebben op de structurele eigenschappen en het gewicht. Om deze reden is de vlecht mesogeometrie geparameteriseerd voor de karakterisatie van symmetrische en asymmetrische, bi- en tri-axiale, open- en gesloten-maas vlechten met mono-materiaal en hybride bias draadgroepen en willekeurige verdelingen van de vlechthoek en de maaswijdte op willekeurige continue of gediscrètiseerde oppervlakken. Deze aanpak is te integreren in zowel analytische, kinematische als eindige-elementen modellen.

Ter voorkoming van het uitvoeren van simulaties voor eenvoudige ontwerpen zijn nieuwe ontwerpdiagrammen voor biaxiale vlechten voorgesteld. Deze tonen onder andere de realiseerbare ontwerpruimte, begrensd door een geblokkeerde vlecht aan de ene kant en onvoldoende

bedekking van de kern aan de andere kant. De diagrammen maken een snelle haalbaarheidsanalyse mogelijk, voorafgaand aan of in plaats van modellering van het vlechtproces of fysieke experimenten. Voor complexere ontwerpen zijn de kinematische modellen uit dit werk geïmplementeerd in de vlechtprocessimulatiesoftware 'Braidsim', waarbij de vlechtmachinebesturingsdata als input dient en de composietstructuur als output of vice versa. In beide gevallen kan Braidsim zowel vlechtprocesanimaties als driedimensionale vlechtcomposietstructuren voor structurele analyse gegenereert.

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Chapter 1

Introduction

1.1 Braiding

Braiding is well known as a hair styling technique that can be traced back to ancient civilizations. It is also known as an old technique for rope making. Braiding was mechanized by the textile industry centuries ago to make ropes, shoelaces and complex fabrics like lace. Common examples of braids are shown in Figure 1.1. Since the middle of the twentieth century, braiding has been used to manufacture fiber-reinforced plastic (FRP) composite products. A composite is here defined as a material consisting of multiple materials that are combined to outperform its constituents. The fibers function as the main load-bearing components, embedded in a matrix material that holds them in place. An example is shown in Figure 1.2. In various industries, FRP composites replace metal components to reduce weight.

Figure 1.1 Common braid examples: A shoestring and the outer conductor of a coaxial cable.

Figure 1.2 Schematic representation of a braided composite material. The matrix consists of a plastic material, a cured epoxy for example.

Braiding machines come in various shapes and sizes. Highly customized braided structures can be generated on a range of machines like flat, lace, rotary, circular and 3D-braiding machines. This work is dedicated to ‘circular’ braiding machines that produce tubular braids. This machine type is also referred to as a maypole-, tubular-, or a 2D-braiding machine. For more information about other braiding machines, refer to e.g. [1–4].

This chapter first describes the circular braiding process using the most important braiding terminology. Subsequently, the integration of braiding in a typical composite component production chain is outlined. Next, recent applications are shown, identifying current industry trends. Based on these trends, the motivation of this work is presented. Finally, the remainder of this thesis is outlined.

1.2 Circular overbraiding

A typical circular braiding process setup used for the production of braided ‘preforms’ is shown in Figure 1.3. The process is shown step-by-step in Figure 1.4. The circular braiding process can be used for ‘overbraiding’ when a core or ‘mandrel’ is used to define the inner surface shape of the preform. Without a mandrel, the machine produces a rope or a ‘braided sleeve’.

Usually, either the machine [5] or the mandrel moves while the other remains fixed. In the most common setup, the machine is fixed and a traverse moves the mandrel over a single axis through the machine

Figure 1.3 Circular braiding machine, moving a mandrel with a 'traverse'.
Photo courtesy Eurocarbon BV.

like in Figure 1.3. Alternatively, one or two articulated arm robots can move the mandrel and/or the machine using multiple axes [6]. For some specialized circular braiding machines, the 'carrier track ring', shown in Figure 1.3, can be opened and closed [7], making it possible to load and unload mandrels with a closed curve as a centerline like a traditional diamond bicycle frame.

The mandrel moves through the machine with an axial 'take-up speed' v as shown in step 4 of Figure 1.4. The term 'take-up' originates from the vertically oriented axis of braiding machines used for rope making. Meanwhile, two 'groups' of spools, also known as 'bobbins', move around the mandrel on 'carriers' shown in Figure 1.5. The term 'group' applies to either yarns, spools or carriers. The warp and weft groups are also referred to as 'bias' groups. The 'warp' (X) spools move clockwise while 'weft' (O) spools move counter-clockwise. Both groups have the same average speed ω around the process axis. The carriers are passed between the slots of 'horn gears' and follow a serpentine path through the carrier track ring.

Figure 1.4 Typical circular overbraiding process steps. An anchor indicates a fixed position within a process step.

The moving mandrel pulls the warp and weft yarns from the moving bias spools. To compensate for yarn length fluctuations due to the change in radial spool position, each carrier has a tensioning mechanism shown in Figure 1.6. Pulley 2 moves back and forth to provide a yarn length buffer to reduce yarn slack. When enough yarn is drawn out, pulley 2 levers the pawl to unblock the ratchet, releasing some yarn length until spring 1 is short enough to let spring 2 re-block the ratchet. For more machine construction details and variants, refer to e.g. [8–10]. The two bias yarn groups interlock, forming a biaxial fabric on the mandrel. Braiding in this manner can be regarded as tubular weaving because some common patterns can be manufactured by both. In contrast to weaving, however, the warp and weft yarns are deposited simultaneously.

Optionally, as illustrated in step 1 of Figure 1.4, a third group of ‘stem’ yarns can be inserted through the horn gear shafts, shown in more detail in Figure 1.6, to form a triaxial braid. Both braid types are shown in Figure 1.7, where the stem yarns run through ‘tunnels’ created by the

Figure 1.5 Serpentine carrier movement through carrier tracks, guided by the sliders shown in Figure 1.6. In step 2, horn gears are hidden to show the tracks. In step 3, the tracks are hidden too to show the horn gear cogwheels. In step 4, a cogwheel is hidden to show the motor.

Figure 1.6 Schematic construction of a typical carrier loaded with a spool. The 'slider' is guided by a serpentine track behind the horn gears, shown in Figure 1.5.

bias yarns as shown before in Figure 1.2. The stem yarns are deposited in the mandrel length direction, providing improved structural properties for axial loading and bending.

Define the 'braid angle' α as the angle, measured on the mandrel surface, that a deposited warp or weft yarn makes with the mandrel centerline projection as shown in Figure 1.7. A single braid layer is deposited in an 'overbraiding run' or simply 'run'. For thick-walled components, the mandrel can be repeatedly overbraided using multiple runs in forward and reverse direction along the full component length or only locally.

The optional use of guide rings has multiple purposes. Firstly, it reduces

Figure 1.7 A bi- and triaxial braid, showing braid angle α . In the triaxial braid, the stem yarns run through the tunnels created by the bias yarns.

Figure 1.8 Reverse braiding on an axial (left) and a radial (right) braiding machine.

the distance from the spools to the ‘fell front’, i.e. the location where the yarns come in contact with the mandrel, shown in step 4 of Figure 1.4. Secondly, it increases the response to changes in take-up speed. Lastly, it enables ‘reverse braiding’, i.e. braiding with a negative take-up speed as shown in step 7 of Figure 1.4. In Figure 1.3, a single guide ring is used for forward braiding only. In Figure 1.4, two guide rings are used: the outer one for forward braiding and the inner one to enable reverse braiding. Reverse braiding without a guide ring would lead to interference of yarns and carriers, possibly damaging the machine. The guide ring does not need to be circular. Different shapes can be used to influence the fiber distribution [11, 12]. A guide ring vibration unit can reduce the effects of friction on the yarn deposition and distribution on the mandrel.

Classical braiding machines as described above are categorized as ‘axial’ braiding machines because the spool axes are parallel to the process axis. For ‘radial’ braiding machines, the spool axes are perpendicular to the process axis as shown in the right half of Figure 1.8. Radial braiding machines were recently developed to reduce process-induced carbon yarn damage [3, 4]. For radial braiding machines, the guide rings are usually located at both sides of the bias spools. The next section describes how the resulting braided preform can be processed further.

Figure 1.9 A typical process chain for an overbraided composite component, viewed along the mandrel axial direction.

1.3 Braided component production process chain

The overbraiding process is typically part of a composite production process chain as shown in Figure 1.9. After overbraiding, the preform is usually impregnated and cured with a thermoset resin using resin transfer molding (RTM) with processing times in the order of several minutes with some resin systems. Optionally, the mandrel is removed afterwards. It is also possible to remove the mandrel directly after braiding, followed by a forming process of the braid prior to resin injection [6].

As an alternative to thermoset resins, the use of commingled thermoplastic fibrous yarns has been explored for improved recyclability and faster consolidation, optionally integrated in the braiding machine using infrared heaters [8].

A ‘virtual’ equivalent of the physical production process chain, generally represented by computational simulation models, can be used when designing the component and its manufacturing process. These models can include the simulation of the braiding process and downstream manufacturing processes like forming or resin injection. The output of these models is typically used in a structural analysis to assess the performance of the cured overbraided component. Initially, the resin injection simulation can be skipped for simplicity. Instead, the braid properties generated by the braiding process simulation can be combined with the given structural properties of the used fiber and the cured resin to estimate the resulting properties and to directly define the cured composite layup on a full component scale.

The current work is dedicated to the overbraiding process and its potential for integration in the virtual composite component production

process chain. For further information about braiding process integration, refer to e.g. [13, 14].

1.4 Advantages of braiding

The braiding process offers the following advantages. The interlaced fiber structure reduces the tendency of the yarns to slip off the mandrel after deposition during production. It also allows the yarn path to have a higher curvature on the mandrel surfaces without occurrence of slip compared to non-interlaced yarn deposition like filament winding [15], allowing more complex mandrel shapes. Interlacement can provide a higher damage tolerance [16] with a more controlled crack propagation or a more benign failure behavior compared to composites without interlacement [17]. Braids can also be used to increase the specific energy absorption, i.e. increasing the amount of energy absorbed per unit mass of component material [18–21].

The braiding process can offer a high repeatability compared to more traditional preform production like hand-layup, and is suited for (semi-) automated series production. When higher production rates are required at increasing series sizes, circular braiding can be considered as an alternative to filament winding because hundreds of yarns are deposited simultaneously in up to three directions, leading to a high material deposition rate. Braiding can also provide the integration of product features such as holes and flanges without fiber cutting.

1.5 Applications of braiding

Overbraiding is mainly applied to manufacture tubular components that optionally vary in cross-section along their length and consist of one or multiple applied braid layers, possibly alternated with filament-wound layers or non-crimp fabric patches. Commonly used materials are fibrous yarns, optionally combining distinct yarn materials as a ‘hybrid’ braid, for example to improve the toughness as compared to using carbon fiber yarns only [16, 22, 23]. Braiding is increasingly used for primary structural components in many industries as is shown by the following examples: The automotive industry braids drive shafts, rockers, A-pillars as shown in Figure 1.10, window frames [24], and crumple zone

Figure 1.10 Braided A-pillar, indicated by the arrow, in a series-produced 2016 BMW 7 Series at the booth of SGL Automotive Carbon Fibers at the 2016 JEC fair in Paris.

parts for maximum specific energy absorption. Recently, the feasibility of braiding pressure vessels for natural gas and hydrogen storage has been researched [25, 26], traditionally the field of filament winding. The aerospace industry braids engine parts [19], fuselage frame parts, helicopter blades, landing gear [27, 28] and propellers. The sports industry braids high-end baseball bats, bike frame parts, hockey sticks and skis. For more examples, refer to e.g. [3, 4, 23, 29, 30].

1.6 Motivation

As shown in the previous section, braiding can be considered as a candidate for production of one or more components in a larger product like a car or airplane. Such products are designed using an engineering design process of which many variations exist. A simplified example is shown in Figure 1.11 [31]. In reality, this process can be far more

Figure 1.11 *Engineering design process design phases (simplified).*

complex with more feed-back loops. At a higher resolution, a similar process flow applies at least to some degree to the design and fabrication of each component of the product, for example when comparing metal forming, filament wound composite and braided composite variants as the component manufacturing options. Each of such manufacturing techniques usually involves a large amount of specialized knowledge. An obstacle for braiding in the composites industry was [5] and is the lack of a 'knowledge-base' for manufacturing process, structure and performance parameters. Although there exists no universally accepted definition of a knowledge base, it can be interpreted as the combination of a database and the software to access it, for example to search the structural braid properties for a given combination of mandrel, machine and material. It can be feasible to create such a knowledge-base for frequently used and well-researched combinations of parameters, but it is cost-prohibitive to evaluate all cases due to the vast number of combinations of mandrel shapes, machines, materials, braid patterns, etc. Moreover, although specialized organizations may possess limited knowledge bases, there seem to be none that are public available, except tables containing stiffness data for standardized braided sleeves as provided by some braid manufacturers. Instead, the performance of a braided component can be calculated for specific combinations of

mandrel, machine and material using computational models for braiding process simulation that are currently under development [32–35].

The use of braiding simulation and optional downstream structural analysis aims to reduce lead-time and cost related to engineering, physical testing, over-dimensioning, waste of materials and energy, labour due to spool replacement and re-spooling [21], and commissioning inappropriate capital-intensive equipment. For example, a design can cause unexpected yarn slip after deposition at rapid mandrel cross-sectional changes, requiring a mandrel redesign and re-manufacture. Or worse, the preform thickness exceeds the resin injection mold cavity size, requiring a braid thickness reduction or expensive and time-consuming mold adjustments. Even worse, the finished prototype component does not satisfy the structural requirements, possibly requiring a redesign of all manufacturing process steps.

Braiding process models can be based on analytical expressions, finite elements, or kinematics. Analytical models usually consist of a few formulas on a spread-sheet to describe the braiding process, providing the shortest calculation times, yet possibly providing too coarse results for typical free-form mandrel components used in practice. Instead, finite element (FE) models can offer a more thorough simulation [7, 36–39]. FE allows modeling of yarn tension and bending, yarn interaction including friction, guide ring contact friction, gravity, slip over the mandrel after deposition and discontinuous yarn deposition that accounts for yarn ‘bridging’ over concave (hollow) mandrel regions, i.e. a local lack of contact between the braid and the mandrel. FE approaches still involve several modeling simplifications. The yarn cross-section is often a circular approximation. Friction-dominated transverse yarn deformation is usually neglected, as well as orientation- and pressure-dependency of the friction as described in [40]. Also the effects of broken and entangled fibers is not taken into account. Hence, FE results can still contain significant systematic errors. The added modeling functionality of FE over analytical solutions is at the cost of calculation time. FE typically requires calculation times in the order of days per overbraiding run [38]. Finally, access and experience with respect to the specific FE software may not be readily available.

As a compromise between the speed of analytical models and the accuracy of FE models, braiding can also be simulated with ‘kinematic’ models [5, 41, 42]. The main difference between kinematic and FE

Figure 1.12 Braiding process simulation time using different modeling approaches for a single braid layer.

models is that kinematic models neglect yarn-to-yarn interaction, yarn slip after deposition, and transverse yarn deformation, so kinematic models represent each yarn by a straight line from the spool or guide ring contact point to the mandrel. Kinematic models typically require seconds of calculation time per run as illustrated in Figure 1.12. This offers advantages in earlier design phases of the manufacturing process when the high level of detail from FE is not yet needed.

Braiding process models typically take the machine control parameters as input and generate the braided structure as output. From the perspective of the designer and braider, usually the opposite is desired: the designer creates a braided component design, and machine control data must somehow be generated to actually braid it. Performing multiple trial-and error FE simulations to achieve the desired control data would be impractical due to the long calculation time while still offering a limited accuracy. The speed of kinematic models is a key property used to address this issue.

The designer can require a specific braid pattern, but it should not be his primary concern whether, and if so, how, such a pattern can be realized by the distribution of spools over the carriers. It can also be important to know whether a small product can be braided on a large machine in case no small machine is available. This may be feasible by leaving the correct carriers empty while still achieving the desired braid pattern. Therefore it seems useful if the relation between braid patterns and spool patterns is integrated in a braiding design tool.

The kinematic braiding process is of limited practical use to a designer without information about the resulting braid material properties that are typically needed in downstream structural analysis. Examples

of important material properties are the interlacement pattern, fiber direction and thickness. These properties, the mandrel design, and the braiding process are highly interdependent as illustrated in Figure 1.13. In addition, braiding manufacturing constraints should be taken into account.

This work addresses these challenges to lower the threshold for adopting circular braiding production technology by composite component designers in particular and mechanical engineers in general.

1.7 Objective

The principal objective of this work is to develop two practical design tools to assist in the design of overbraided components with complex-shaped mandrels. The first tool is a set of design charts for cases where braiding simulation is not desired. The second tool is fast kinematic braiding process simulation software. For this purpose, the following three goals are identified to address easily noticeable problems in braiding composite components. Meeting these goals constitutes the core of this work.

The first problem is to produce the appropriate braid angle distribution that is required for the structural properties of the braided component. The corresponding goal is to develop a procedure to calculate the braiding machine control data for a given braided component design with a required braid angle distribution, and to experimentally assess the accuracy of the braid angle distribution that is produced on a braiding machine controlled with this data.

The second problem is mainly related to producing the correct braid pattern and producing small products on a given large machine. The corresponding goal is to find the relation between the spool pattern on the braiding machine and the corresponding produced braid pattern.

The third problem mainly involves calculating the thickness of the braided component that translates to the mold dimensions of the downstream resin injection. The corresponding goal is to calculate the braid thickness for bi- and triaxial braids, and to experimentally assess the accuracy of the thickness predicted by the model.

Figure 1.13 Design-process-material triangle showing the content of each chapter and showing the current chapter underlined.

1.8 Outline

The emphasis of each chapter is shown in Figure 1.13, although strictly speaking, each chapter contains at least some information about all areas of design, process and material. The current chapter provided a general description of the braiding process as used in the remainder of this work. The main focus of the following chapters is as follows:

In Chapter 2, a novel kinematic ‘inverse solution’ is proposed, taking a prescribed braid angle distribution as input to calculate the machine control data. As a building block of the inverse solution, a kinematic ‘forward solution’ is presented as well, taking the machine control data as input to calculate the braid angle and fiber distribution. The inverse solution is experimentally validated with a complex mandrel.

In Chapter 3, the inverse solution is applied to various mandrel shapes to learn what take-up speed profiles can be expected as the output and how these compare to analytical solutions.

In Chapter 4, a biaxial yarn interaction model is proposed and experi-

mentally validated to assess the systematic error due to the neglect of yarn interaction in the kinematic model from Chapter 2.

In Chapter 5, the relation between spool patterns and braid patterns is analyzed, focusing at the relation between the construction of circular braiding machines and removal of spools on the one side and the braid pattern on the other.

In Chapter 6, the meso-geometry of mono-material and hybrid bi- and triaxial braids is analyzed to calculate the braided preform thickness. This is needed to design a resin injection mold that fits the preform shown in Figure 1.9 and for structural modeling of final impregnated and cured component. Only geometrical parameters are used to reduce calculation times. This approach can be at the cost of a more generic and possibly more concise FE formulation, but this is more appropriate for earlier manufacturing process design phases where a global design feasibility assessment is preferred over a high level of detail.

In Chapter 7, a design chart is proposed with the main manufacturing constraints for biaxial braiding. For more complex designs, the kinematic braiding process simulation software 'Braidsim' is introduced, implementing the models from the preceding chapters.

In Chapter 8, the results of the preceding chapters are discussed.

In Chapter 9, the main findings of this work are summarized and recommendations for further research are given.

Chapter 2

Circular braiding take-up speed generation using inverse kinematics*

2.1 Introduction

This chapter focuses on the braiding process and the overbraided component design as schematically shown on the path in Figure 2.1. Simulation of the circular braiding process is possible using analytical, kinematic and finite element models. Analytical models generally output the braid angle as a function of a constant carrier and take-up speed and a constant mandrel cross-section, supporting only basic mandrel cross-sectional shapes. In contrast, kinematic and FE approaches can calculate braid angle distributions based on variable speeds and more complex mandrel shapes. Most models require the machine control parameters as input and generate the braid structure as output. This is a ‘forward solution’. The reverse order with the fiber distribution on an arbitrary mandrel as input and the machine control data as output is called the ‘inverse solution’ [5, 41].

*Based on [43].

Figure 2.1 Design-process-material triangle showing the current chapter underlined.

Problem and objective

An automated method that calculates the inverse solution for complex mandrels is not publicly available to the authors' knowledge. An acceptable inverse solution can sometimes be obtained using rules of thumb, spreadsheets and machine operator experience. In some cases, robotized braiding machines facilitate in generating an inverse solution using so-called teach-in functionality that allows a machine operator to guide the mandrel relative to the machine while recording its trajectory to a program that is used for later control in production. Trial-and-error using multiple teach-in attempts may be required to obtain a satisfactory inverse solution. Mandrels with increasingly complex shapes, which are common practice, can make it more difficult to obtain acceptable process settings.

The objective in this chapter is to automatically generate control data for circular braiding machines with optional guide rings, resulting in an optimum deposition of warp and weft yarns on mandrels with complex

3D shapes. In this work, a ‘complex’ mandrel shape is defined as a mandrel that can include non-axisymmetric, possibly eccentric cross-sections that can vary in shape and size along an optionally curved mandrel centerline, possibly requiring a curved machine movement relative to the mandrel. The method should be fast and fit for Computer Aided Engineering (CAE) process chain integration. The resulting output can be used in process simulation like resin injection, structural analysis, aerodynamics and crash performance.

Outline

First, kinematic approaches from earlier work are discussed. Next, a generalized kinematic forward solution is described. In turn, this forward solution is used as a component in a novel inverse solution to generate the take-up speed profile. Finally, the inverse solution is experimentally validated using a complex mandrel by comparing the distribution of the modeled and the experimental braid angle.

2.2 Earlier work[†]

This section discusses kinematic models from earlier work, followed by a summary of what sets the novel inverse kinematic model apart from these earlier models.

Kinematic models describing the overbraiding process made their appearance in the 1990s and are treated in chronological order. These models neglect the serpentine spool movement, yarn slip over the mandrel surface after deposition and branching mandrels. The first two models provided an inverse solution:

Recognizing the need for representing more complex mandrel shapes than only cylinders, Michaeli, Rosenbaum, and Jehrke [42] modeled the axisymmetric braiding process without yarn interaction. A procedure was presented for direct control of the machine, taking the braid angle distribution and the carrier speed as input and generating the take-up speed profile as output. It was assumed that the angular speed of the fell point (the point where the yarns come into contact with the mandrel)

[†]Based on [23].

equals that of the carrier, although generally, this is incorrect as shown in Appendix A.

Similar to the previous approach, Du and Popper [5] modeled the braiding process while approximating the mandrel by a series of centered conical segments. Given the braid angle distribution, either the carrier speed or the take-up speed was chosen to be generated, while keeping the other speed constant. Again, it was assumed that the angular speed of the fell point equals that of the carrier around the process axis. At least partly due to this assumption, the generated machine speed profiles needed manual adjustments to obtain a braid angle distribution closer to that required.

Other models provided a forward solution, using the speeds as input and the braid angle as output:

In an attempt to improve the accuracy of braiding process modeling, Zhang, Beale, and Broughton [21, 44] included yarn interaction with friction in their model for the axisymmetric circular braiding process in the steady state with a cylindrical mandrel. The convergence zone is approximated with a developable surface, mapped to a plane, resulting in a 2D problem. They stated that the fell point angular speed is smaller than the angular carrier speed when inter-yarn friction is significant. Due to the steady state of the process under consideration, both speeds are actually equal as shown in Appendix A. They also stated that the braid angle decreases with increasing friction. This can only hold in a transient state, for example at the braiding start, and depends on the initial convergence zone length. In the steady state, the friction has no effect on the braid angle because the braid angle is only a function of the mandrel radius and the ratio of carrier and take-up speed.

In an attempt to generalize braiding process modeling beyond axisymmetry excluding yarn interaction, Kessels and Akkerman [41] proposed a closed-form analytical model to express the braid angle distribution around the mandrel as a function of the amount of eccentricity, i.e. the distance between the center of the machine and the mandrel. This model implicitly assumes that the axial speed of the fell point equals the take-up speed, which is incorrect, because the process is not in a steady state. Instead, the fell point speed varies periodically with each spool revolution. They also proposed a 3D model that neglects yarn interaction, but allows non-circular mandrel cross-sections and sharp mandrel curvatures, assuming that the mandrel cross-sectional change

in axial direction is only gradual. Experimental validation suggested that this model performed better for the experiments described in their work compared to the analytical solutions regarding the effects of mandrel shape variations along the mandrel length.

In contrast to the previous work, the following authors analyzed the simplest possible mandrel shape for cases without a steady state. Nishimoto et al. [45] analyzed the transient effects without yarn interaction on a centered cylindrical mandrel that can occur in the start-up phase of the braiding process or after stepwise take-up speed. They show that the deposition speed in axial direction temporarily differs from the take-up speed and that the braid angle temporarily deviates from the target value, depending on the initial convergence zone length. In this case, the angular yarn deposition speed equals the carrier speed because yarn interaction is neglected.

Continuing on complex mandrels, Guyader, Gabor, and Hamelin [46] recognize the importance of distinguishing between the deposition speed and carrier speed in both angular and axial direction. A 3D kinematic model is proposed without yarn interaction, implicitly assuming that the minimum and maximum mandrel curvature correspond to the mandrel length and hoop direction, respectively, which is not necessarily the case in general. They also state that the angular speed of the fell point changes with increasing friction, in turn decreasing the braid angle, which does not hold for the steady state and which depends on the initial conditions of the transient state.

As a complement to the aforementioned kinematic models, and tightly related to machine control, Lian et al. [47] proposed a machine vision system for inspection of the fiber directions and mentioned that it can be used for on-line feedback control.

Apart from this braiding-specific literature, machine control data can also be generated with the help of general-purpose computational optimization techniques, for example minimizing the braid angle error, given a prescribed braid angle distribution. Generic literature exists on using multiple simulations to achieve (near) optimal results as studied in recent years for metal forming processes [48]. Examples, ranging from brute-force to more subtle approaches, are branch-and-bound optimization, genetic programming and gradient-based methods [49], usually requiring large calculation times depending on the simulation complexity. Hence, it is beneficial if such an approach is not needed.

Figure 2.2 Kinematic process model of the circular overbraiding process.

The novel inverse solution presented in this chapter differs from the earlier solutions in two major ways: Firstly, it generalizes the problem as described earlier [5, 42] from scalar to vector form to generate a take-up speed profile assuming a constant given carrier speed. This extends the analysis from straight to curved mandrel centerlines, from linear to curved machine paths, from centered to eccentric mandrel cross-sections, and from circular to non-circular mandrel cross-sections for multiple fibers simultaneously, including the effect of optional guide rings. Secondly, the assumption that the yarn deposition speeds equal those of the spools is not used. Instead, they are a function of the combination of the mandrel shape as well as the trajectory and momentary speeds of the machine relative to the mandrel using a contact model and inverse kinematics.

2.3 Kinematic braiding process model

The kinematic braiding process model is schematically shown in Figure 2.2. All bias (warp and weft) yarns are deposited on the mandrel outer surface in the first overbraiding run. An approximation of the braid outer surface from the previous run is used for subsequent runs. The surface of deposition is called the 'substrate surface' or simply 'substrate'.

Figure 2.3 Kinematic process model for a single yarn.

For the kinematic braiding process model, it is assumed that

- trajectories of deposited yarns are continuous,
- trajectories of deposited yarns are coincident with the substrate,
- yarns are fixed after the first contact with the substrate,
- yarns do not interact, and
- friction between yarns and guide rings is negligible.

Based on these assumptions, each yarn is modeled independently as shown in Figure 2.3, making it similar to simultaneous filament winding [50] of multiple yarns.

Throughout this work, schematic close-ups of the substrate can show a continuous surface for explanation purposes. However, in the kinematic model, The substrate is approximated by a surface mesh S of triangular facets. The input mandrel mesh can be provided using, for example, the STL file format. This is a simple and widely used format that can be generated by most CAE software. S must be a closed shell without holes (as in a torus) and must be non-branching, but small protrusions perpendicular to the mandrel centerline are allowed. S is defined in

the global Cartesian mandrel coordinate system with unit axes $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ shown in Figure 2.3. The surface region to be overbraided is bounded by a start and end contour that do not necessarily coincide with the mandrel end contours.

The machine parametrization is described next and illustrated in Figure 2.3. Both radial and axial braiding machines shown in Figure 1.8 are represented by the kinematic model in Figure 2.2. The number of yarns per yarn group is given by n_y and equals the number of spools per yarn group. The spools are modeled to move over a ‘spool plane circle’ with radius r_{sp} , neglecting the serpentine component of their motion.

The inner and outer guide ring are optional. Guide rings are modeled as circular, although sometimes different shapes are used [11, 12]. The inner and outer guide ring have a radius $r_{gr,in}$ and $r_{gr,out}$, with $r_{gr,in} < r_{sp}$ and $r_{gr,out} < r_{sp}$. The guide rings have ‘heights’ $h_{gr,in}$ and $h_{gr,out}$ shown in Figure 2.3, where $h_{gr,in} > 0$ and $h_{gr,out} > 0$ for axial braiding machines, while $h_{gr,in} < 0$ and $h_{gr,out} > 0$ for radial braiding machines. The counter-intuitive term ‘height’ matches the term take-‘up’ because historically, the process axis used to be vertical.

The machine origin $\mathbf{m}$, given its initial position $\mathbf{m}_0$, coincides with the arbitrary mandrel centerline curve L in this implementation. The machine orientation is represented using a local Cartesian machine coordinate system with unit axes $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. In this implementation, the machine axis is required to coincide with one of the global mandrel axes or with the local mandrel centerline direction, i.e. $\mathbf{w}(t) \in \{\pm\mathbf{x}, \pm\mathbf{y}, \pm\mathbf{z}, \pm\mathbf{t}(t)\}$, where $\mathbf{t}$ is the unit centerline tangent. The machine rotational degree of freedom around its axis is determined using $\mathbf{u} \cdot \mathbf{n}_g = 0 \mid \mathbf{n}_g \in \{\pm\mathbf{x}, \pm\mathbf{y}, \pm\mathbf{z}\}$ where $\mathbf{n}_g$ can be regarded as the ‘zenith’, pointing opposite to the gravitational acceleration direction, although gravity itself is not modeled. In a future extension to the model, the machine position and orientation can be made variable as well.

The carrier rotation speed ω around the process axis is assumed constant and given. The take-up speed v , as a function of time t , is either given as input in the forward solution, or it is calculated as an output in the inverse solution as explained later.

The smallest circle that is in contact with a yarn is defined as the ‘creating circle’ of that yarn. This is either the spool plane circle or one of the optional guide rings. In the forward overbraiding run in Figure 1.4, the

outer guide ring circle is the creating circle of all yarns. For the reverse run, the inner guide ring is the creating circle.

Define the ‘supply point’ of a yarn as the point on the yarn in contact with the creating circle. Depending on the mandrel shape and its motion through the machine, the set of all supply points can be simultaneously distributed over both guide rings and the spool plane circle, resulting in multiple creating circles. For specific cases or for explanation purposes, multiple creating circles are sometimes represented by a single one.

Define the ‘fell point’ as the contact point, located between the substrate and a yarn, that is closest to the spool as measured along the yarn. The set of fell points creates the ‘fell front’ as explained in Section 1.2.

The yarns move from the spools to the substrate, passing through the funnel-shaped ‘convergence zone’ of length H , defined as the region between the creating circle and the fell front, shown in Figures 2.2 and 2.3. For complex mandrels and variable speeds, H varies. The convergence zone does not include the region between the optional guide ring and the spool plane circle in case of guide ring contact. For reverse braiding as shown in Figure 1.4, this definition applies to the region between the inner guide ring and the fell front.

Finally, define the ‘free segment’ as the length of a yarn between its supply point and fell point.

2.4 Forward solution

This section describes the forward solution with the take-up speed profile as input and the braid angle distribution as an output. A time-stepping method with a constant user-defined time step size Δt is used. Define $\mathbf{p}$ as the fell point, $\mathbf{q}$ as the spool position and $\mathbf{r}$ as the point of possible guide ring contact as shown in Figure 2.3.

Assuming a constant carrier speed ω , the serpentine spool trajectory for the i th spool of a bias yarn group is approximated with a helix-like curve Q_i , described as a function of time in a formal way by

$$\mathbf{q}_{X,i}(t) = \mathbf{m}_0 + r_{\text{sp}} \begin{bmatrix} \cos(\varphi_{0,i}) \\ \sin(\varphi_{0,i}) \\ 0 \end{bmatrix}^T \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{v}(t) \\ \mathbf{w}(t) \end{bmatrix} + \int_{t=t_0}^{t=t_e} \begin{bmatrix} -\omega r_{\text{sp}} \sin(\varphi_{0,i} + \omega r_{\text{sp}} t) \\ \pm \omega r_{\text{sp}} \cos(\varphi_{0,i} + \omega r_{\text{sp}} t) \\ v(t) \end{bmatrix}^T \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{v}(t) \\ \mathbf{w}(t) \end{bmatrix} dt \quad (2.1)$$

where X and O denote warp and weft, respectively, corresponding to the + and – sign in the second row, r_{sp} is the spool plane radius as shown in Figure 2.3, and φ is the spool plane angle. Irrespective of carriers that are optionally left empty or removed, the spools are distributed evenly over the spool plane circle with

$$\varphi_{0,i} = \frac{2\pi i}{n_y}. \quad (2.2)$$

For the optional stem yarns, Equations (2.1) and (2.2) apply with ω set to zero.

2.4.1 Yarn deposition model

The yarn deposition model is illustrated in Figure 2.4. For each time step, given an existing fell point $\mathbf{p}_j$ on yarn centerline Y, the next fell point $\mathbf{p}_{j+1}$ is calculated as follows: First, the spool position is updated to $\mathbf{q}_i$ according to Equation (2.1). Next, contact between the yarn and the optional guide rings is checked geometrically. In case of contact, assuming no friction between guide rings yarns, the contact point $\mathbf{r}_i$ is obtained by minimization of the yarn length between fell point $\mathbf{p}_j$ and spool position $\mathbf{q}_i$ using Halley-iteration. The supply point $\mathbf{s}_i$ for this time step is obtained as

$$\mathbf{s}_i = \begin{cases} \mathbf{q}_i & \text{if no guide ring contact,} \\ \mathbf{r}_i & \text{if guide ring contact.} \end{cases} \quad (2.3)$$

The sequence of $\mathbf{s}(t)$ results in the ‘supply point trajectory’ T. Supply point $\mathbf{s}$ can alternate between the spool plane circle and guide ring circles. When $\mathbf{s}_i$ has first passed the plane of the appropriate adjacent facet F of $\mathbf{p}_j$, then $\mathbf{s}_i$ is projected on F along the substrate surface normal $\mathbf{n}_s$ to yield $\mathbf{s}'_i$. The intersection of the line segment from $\mathbf{p}_j$ to $\mathbf{s}'_i$ and the appropriate

Figure 2.4 *Yarn deposition model.*

edge E is the next fell point p_{j+1} .

If, instead, p_{j+1} is taken as the point corresponding to the shortest distance between E and the ‘free segment’ shown in Figure 2.4, then this would result in small artificial in-plane yarn curvatures on planar mandrel regions. Note that instead using the shortest distance between s_i and E is generally inaccurate.

This deposition model is a simplified variation to deposition using the local ‘geodesic path’ as described in [51] with a geodesic path as the shortest path between two points on a surface, and yields the same solution if the time step size approaches zero.

A limitation of this deposition algorithm is that it only searches for contact at the edges of the facets that are connected to the fell point edge, so it performs a yarn-substrate contact search only in the neighborhood of the fell point instead of a global search over the full substrate. As a consequence, bridging is not taken into account. For example, if the mandrel has sharp grooves in the circumferential direction, then the deposition model allows free yarn segments to temporarily, artificially, pierce the mandrel. This effect is reduced for larger braid angles.

This deposition procedure can continue until the mandrel end is reached. As a result, this forward solution generates the yarn centerline trajectory Y on the substrate, and, implicitly, the convergence zone length H as a function of the fell point position or spool trajectory Q.

The deposition model is extended to include axial yarns as follows. First, the curves of the bias yarn centerlines on the substrate are evaluated using the previously described deposition model. Next, the intersections of the bias yarns are calculated. These intersections are used in combination with information of the triaxial braid pattern to find the topological and

geometrical bounds of each stem yarn. Series of these bounds form ‘tunnels’ through the biaxial braid for stem yarns as shown in Figure 1.7. Hence, it is assumed that the position of the axial yarns is defined as a function of the bias yarn intersections and that the axial yarns do not influence the position of the bias yarns.

Once the virtual braiding has finished, the following process-related output parameters can be evaluated.

2.4.2 Braid angle

The ‘braid angle’ α is defined as the angle between a bias yarn and the mandrel centerline projection on the substrate. Analytical closed form solutions exist for a few simple process configurations. For a centered cylindrical mandrel in a rotation symmetrical process in a ‘steady state’ as defined in Appendix A, the braid angle is expressed using the ‘classical solution’ [8, 29, 30],

$$\alpha = \arctan \left(\frac{\omega r_m}{v} \right) \quad (2.4)$$

with ω as the average carrier speed around the mandrel center, r_m as the (equivalent) mandrel radius, and v as the take-up speed. As soon as a mandrel varies in cross-section along its length, analytical solutions can be very difficult or impossible to derive, even for relatively simple geometries as shown by [52–54]. For this purpose, a more generalized approach is needed like the kinematic model proposed here.

As a consequence, for complex mandrels, the braid angle is more conveniently and generically expressed in vectorized form. This is shown in Figure 2.5 for point $\mathbf{c}$ on a bias yarn centerline Y , in turn on the substrate surface S , and a curved mandrel centerline L .

Let $\mathbf{n}_s$ be the unit normal of S at $\mathbf{c}$. Let $\mathbf{f}$ be the local unit tangent of Y at $\mathbf{c}$. Let $\mathbf{t}$ be the unit tangent of L at point $\mathbf{l}$ on L in the run direction, and let $\mathbf{l}$ be the closest point to $\mathbf{c}$. Let $\mathbf{i}$ be the unit projection of $\mathbf{t}$ on the tangent plane of S at $\mathbf{c}$,

$$\mathbf{i} = \frac{\mathbf{t} - (\mathbf{t} \cdot \mathbf{n}_s) \mathbf{n}_s}{\|\mathbf{t} - (\mathbf{t} \cdot \mathbf{n}_s) \mathbf{n}_s\|}. \quad (2.5)$$

Let $\mathbf{j}$ be the local unit tangential (circumferential) direction on S at $\mathbf{c}$. For

Figure 2.5 Definition of the braid angle. *L* represents the mandrel centerline, *S* the substrate, and *Y* a yarn centerline. This example shows a weft (O) yarn in forward braiding.

forward braiding,

$$\mathbf{j}_X^O = \pm \frac{\mathbf{n}_s \times \mathbf{i}}{\|\mathbf{n}_s \times \mathbf{i}\|}. \quad (2.6)$$

Remember that the weft (O) yarns always rotate counter-clockwise around *L* and the warp (X) clockwise when viewed as in Figure 2.2, both at forward and reverse braiding. For reverse braiding, the sign of *t* swaps and in Equation (2.6), $\pm$ is replaced by $\mp$. Finally,

$$\alpha = \arctan \left| \frac{\mathbf{f} \cdot \mathbf{j}}{\mathbf{f} \cdot \mathbf{i}} \right|. \quad (2.7)$$

The braid angle definition as used in this work does not include local variations related to yarn interlacement and the corresponding superimposed yarn undulation in mandrel radial direction as described in e.g. [55].

Figure 2.6 Yarn pitch p , schematically showing only a single yarn group, neglecting interlacement and undulation, viewed perpendicularly to the substrate with perimeter P_m .

2.4.3 Yarn pitch

A special type of braid defined here is a ‘symmetric braid’, not necessarily using a mandrel, defined as a braid where the bias (warp and weft) yarn groups contain the same

- material(s),
- braid angle, and
- yarn-to-yarn spacing,

along the braid length and circumference. An ‘axisymmetric braid’ is defined as a symmetric braid with a circular cross-section.

Define the yarn centerline pitch or simply ‘yarn pitch’ p as the local spacing between the projection of adjacent yarn centerlines within the same yarn group on the substrate shown in Figure 2.6. For a symmetric biaxial braid, p is expressed in closed-form for each bias yarn group using the geometry from Figure 2.6 as

$$p = \frac{P_m}{n_y} \cos \alpha. \quad (2.8)$$

with α as the braid angle, P_m as the (mandrel) substrate perimeter, and n_y as number of yarns per group, hence $p > 0$. For the optional stem yarns in a symmetric triaxial braid, Equation (2.8) is used with $\alpha = 0$.

The yarn pitch is generally obtained as an output of braiding process simulations. However, it can also be used as an input, in turn resulting in an approximation of the corresponding required braid angle distribution using Equation (2.8) after rearrangement, given the mandrel perimeter

distribution P_m and the number of yarns per group n_y . The accuracy of this approach decreases with an increasing mandrel complexity.

2.4.4 Slip tendency

Undesired yarn slip over the substrate after deposition can cause undesired fiber distributions including for example a locally low degree of surface coverage. A 'slip tendency' c_{slip} and a slip direction $\mathbf{d}_{\text{slip}}$ are used to assess the yarn placement stability [56]. Geometrically, c_{slip} resembles a fictitious slope φ of a ramp supporting an object that can slide off. The steeper the slope, the larger the slip tendency. When assuming Coulomb friction, the slope has a critical value φ_{cr} where the object starts slipping. Just before slipping, force equilibrium shows that

$$\mu = \frac{W}{N} = \tan \varphi_{\text{cr}} \quad (2.9)$$

with W and N as the friction and normal force, and μ as the static coefficient of friction between the object and the slope. Expressing the slip tendency as [57]

$$c_{\text{slip}} = \tan \varphi, \quad (2.10)$$

a criterion for the yarn placement stability can be defined as

$$c_{\text{slip}} \leq \mu. \quad (2.11)$$

where μ can be determined experimentally, for example a value of 0.2 for inter-yarn contact of perpendicular carbon yarns [38, 58]. For a point on the deposited and tensioned yarn centerline curve, in either continuous or discrete form, on a smooth enough convex (curved outwards) substrate surface shown in Figure 2.7, φ can be obtained geometrically with

$$\cos \varphi = -\mathbf{n}_c \cdot \mathbf{n}_s \quad (2.12)$$

with $\mathbf{n}_c$ as the principle normal of the curve [59] and $\mathbf{n}_s$ the substrate surface normal at that point. The unit slip direction on the substrate is

$$\mathbf{d}_{\text{slip}} = \frac{\mathbf{n}_c - \mathbf{n}_c \cdot \mathbf{n}_s}{\|\mathbf{n}_c - \mathbf{n}_c \cdot \mathbf{n}_s\|}. \quad (2.13)$$

2

2

2

2

2

Figure 2.8 A simplified schematic representation of the actual and modeled yarn payout s_y , and the fictitiously rewound yarn length s_{rew} as a function of time t .

Generally, the yarn buffer does not prevent slack for phenomena such as:

- take-up speed reversals corresponding to sudden braid angle changes,
- take-up speed reversal between overbraiding runs,
- eccentric mandrels, and
- mandrel movements in the radial machine direction.

The onset of yarn slack is a process boundary. For this purpose, a criterion is used to indicate if yarn slack occurs. The modeled unwound yarn length or 'yarn payout' s_y can be evaluated as a function of time as shown in Figure 2.8. Express the yarn payout ds_y during time dt , as

$$\frac{ds_y}{dt} = \frac{ds_{dep}}{dt} + \frac{ds_{free}}{dt} \quad (2.14)$$

where s_{dep} is the deposited yarn length so $\frac{ds_{dep}}{dt}$ is the speed of the fell point over the substrate and s_{free} the free yarn length between the fell point and the spool. The onset of a negative yarn payout corresponds with

$$\frac{ds_y}{dt} = 0 \wedge \frac{d^2s_y}{dt^2} < 0. \quad (2.15)$$

Neglecting yarn movement after the first contact with the substrate, parameter s_{dep} increases monotonically as long as the corresponding yarn is being deposited on the substrate,

$$\frac{ds_{dep}}{dt} \geq 0. \quad (2.16)$$

A temporarily reversed take-up speed sign can temporarily reduce s_{free} , so $\frac{ds_{\text{free}}}{dt} < 0$ in Equation (2.14). If the reversal is smooth enough, then $\frac{ds_{\text{dep}}}{dt} > -\frac{ds_{\text{free}}}{dt}$, so the left-hand side of Equation (2.14) is positive, preventing yarn slack. Hence, a temporary take-up speed sign reversal does not necessarily lead to yarn slack.

For the kinematic models treated in this work, the geometry of the slack yarns between the spools and the fell front is not modeled and the carrier movement is assumed circular without the serpentine oscillation. The modeled yarn segment length between the fell point and the spool, optionally contacting a guide ring, is always minimized as explained in Section 2.4.1, so carriers fictitiously ‘rewind’ any slack yarn length as schematically shown in Figure 2.8. Still, a statement can be made about the risk of slack. Let the current yarn payout maximum $s_{y,\text{max},\text{cur}}$ occur at t_0 and let t_1 , $t_1 > t_0$ be the first time where $s_y(t) = s_{y,\text{max},\text{cur}}$. For the maximum amount of fictitiously rewound yarn length $s_{\text{rew}}(t)$ that could actually cause slack in reality,

$$s_{\text{rew}}(t) = s_{y,\text{max},\text{cur}} - s_y(t), \quad t_0 < t < t_1. \quad (2.17)$$

If the fell point remains unchanged while the slack length changes, then the maximum rewound length $s_{\text{rew},\text{max}}$ can be found for that point with

$$s_{\text{rew},\text{max}}(t) = \max(s_{\text{rew}}(t)). \quad (2.18)$$

The value of $s_{\text{rew},\text{max}}$ can be evaluated for all fell points through time and visualized as a scalar field on the substrate for each yarn group as a indicator for slack.

2.5 Inverse solution

The goal of the inverse solution is to generate the take-up speed profile $v(t)$ for a given required braid angle distribution α_{req} at a constant carrier speed ω .

First, the inverse solution is introduced for a single yarn. Subsequently, it is extended to multiple yarns. Finally, the overbraiding run initialization is described as well as the finalization at the run end. The resulting approach can be applied to the full process, often consisting of multiple overbraiding runs. The proposed procedure is independent of optional

Figure 2.9 Fiber Direction Field and a Fiber Stream Line.

axial yarns. Geometrically, it is the inverse of the previously described forward solution. Now the spool trajectory Q is calculated as a function of yarn trajectory Y , closely resembling ‘inverse kinematics’ as used for robotic motion planning.

For the inverse solution, it is additionally assumed that

- a constant non-zero carrier speed ω is provided as input,
- the machine position and orientation relative to the mandrel is continuous and differentiable,
- the instantaneous geometry of the supply point trajectory is a helix on a cylindrical surface with the radius of the creating circle and the axis coincident with the instantaneous machine $\mathbf{w}$ -axis.

The last assumption implies that spool trajectory Q is also a helix that is defined analogously to the supply point trajectory.

The following definitions are needed to proceed. First, define a ‘fiber direction vector field’ (FDF) as a vector field on the substrate surface S that contains the required warp or weft fiber direction vectors $\mathbf{f}_X$ or $\mathbf{f}_O$, respectively, shown in Figure 2.9. An FDF can be non-conservative as shown in Figure 2.10. For complex mandrels with an arbitrary surface mesh, such cases are common and should be taken into account by a robust implementation.

Given α_{req} , the direction of braiding, L , S and the appropriate bias yarn group, $\mathbf{f}$ is solved from Equations (2.5) to (2.7) for each facet on the substrate S . Define a ‘fiber streamline’ (FSL) as a curve that is instantaneously tangent to the fiber directions in the FDF as illustrated in Figure 2.9. Given an initial fell point position of a bias yarn in the FDF as a boundary condition, the corresponding FSL represents the required trajectory of the yarn’s centerline Y . Define the spool cylinder C_{sp} as the extrusion of the spool plane circle along the instantaneous machine axis

Figure 2.10 Example of a locally non-conservative FDE, caused by the combination of a required braid angle that varies along the mandrel length and the orientation of an edge and that of the connected facets. As a result, in this case the edge is a 'sink' because the FDE of both connected facets have a component toward the edge.

w , shown in Figure 2.11. Hence, Q lies on the surface of C_{sp} . Analogously define the guide ring cylinder C_{gr} using the guide ring circle for each guide ring.

2.5.1 Algorithm

First, the inverse solution is described for a single yarn at an arbitrary point in time. Next, the contributions of the individual yarns are used to get an averaged take-up speed.

Single yarns

The inverse problem is reduced to finding the geometrical parameters defining the instantaneous helix curves describing spool trajectory Q of each individual spool. These curves are constructed incrementally and yield the take-up speed $v(t)$ as described next. This is illustrated in Figure 2.11 using the simple case of a centered cylindrical mandrel for ease of explanation, although applicable to complex mandrel geometries with curved centerlines. The algorithm is represented by a flow chart in Figure 2.12.

Figure 2.11 Geometry for the inverse kinematics.

Take the local machine coordinate system as the frame of reference. Following a small amount of deposition with the forward solution described in Section 2.4, the actual fell point $\mathbf{p}_j$ and its corresponding FSL tangent $\mathbf{f}_j(\mathbf{p}_j)$ are known, representing the local *desired* yarn direction. However, for an arbitrary mandrel geometry and multiple yarns, $\mathbf{f}_j(\mathbf{p}_j)$ is usually close to, but not necessarily coincident with the tangent of the *actual* yarn trajectory Y at $\mathbf{p}_j$. Hence, the *desired* spool position and guide ring contact position of the yarn under consideration generally differ from the current state as well. Next, contact with optional guide rings is detected. If the yarn is in contact with a guide ring, then the corresponding guide ring contact position $\mathbf{r}_j$ is calculated in closed form as the intersection of the ray from $\mathbf{p}_j$ in direction $\mathbf{f}_j$ with cylinder C_{gr} . Subsequently, the corresponding spool position $\mathbf{q}_j$ is obtained iteratively assuming frictionless force equilibrium at $\mathbf{r}_j$.

This procedure is repeated for a second point $\mathbf{p}_{j+1}$ on the same FSL at a small distance Δs_j downstream from $\mathbf{p}_j$, yielding $\mathbf{q}_{j+1}$. Together, $\mathbf{q}_j$ and $\mathbf{q}_{j+1}$ result in the instantaneous increments of the spool plane angle $\Delta\varphi_j$ and axial displacement Δz_j shown in Figure 2.11. Given the constant carrier speed ω , the new instantaneous take-up speed for the i th bias yarn

Figure 2.12 Flowcharts for the inverse solution in simplified form.

at $\mathbf{p}_j$ is calculated as

$$v_i = \omega \frac{\Delta z_j}{\Delta \phi_j}. \quad (2.19)$$

If no guide ring contact occurs, then the only cylinder being intersected is C_{sp} directly yielding $\mathbf{q}$. For arbitrary mandrel geometries, the angular increment on the guide ring and spool plane circle are generally not equal, except for special cases like a centered cylindrical mandrel in a steady state as defined in Appendix A.

Alternatively, instead of taking the FSL tangent for $\mathbf{f}_j(\mathbf{p}_j)$, the tangent of the last actual deposited yarn segment on Y can be taken. In that case $\mathbf{r}_j$ and $\mathbf{q}_j$ follow directly from the current deposition state. A larger difference between the actual and desired fiber direction results in a larger take-up speed correction. This approach is here defined to include 'feedback', resulting in a faster response at the cost of stability.

The value of Δs is a trade-off between a value small enough for accurate fiber deposition and large enough to damp noise effects from surface mesh irregularities that can prevent a smooth movement of the machine. In other words, Δs is used to discriminate between mesh noise and a design feature. For a planar mandrel surface region with straight FSLs, two points on the same FSL in that region yield $\mathbf{q}_j = \mathbf{q}_{j+1}$, leaving v undefined. Therefore Δs is made adaptive as a function of the mandrel geometry for a valid solution.

If a sharp edge in circumferential direction or a braid angle change larger than the 'smoothness threshold' $\Delta \alpha_{\max}$ is crossed in the FSL increment Δs , then large machine accelerations relative to the mandrel can be the result. For the moment, these crossings are discarded as invalid and replaced by the first encountered valid Δs downstream of that discontinuity on the same FSL. If the resulting sign of $\Delta \phi$ in Equation (2.19) opposes that of the yarn group rotation direction, then the take-up speed is discarded and recalculated downstream as well.

Figure 2.13 Example cross-sectional view of a rectangular mandrel where $\mathbf{y}$ is chosen as the master normal $\mathbf{n}_m$ to indicate the master side. The master side surface region is bounded by $\gamma_{\max}$

Multiple yarns

The actual braid angle generally varies over the mandrel circumference and per yarn group, for example if the mandrel has:

- a rectangular cross-sections,
- a position eccentric in the machine, shown in [41], or
- a curved centerline.

Hence, generally each combination of a mandrel side, a yarn group and its required braid angle requires a separate take-up speed. However, only a single instantaneous take-up speed can be generated for all bias yarns due to the machine construction as shown in Chapter 1. Therefore a single combination of a mandrel side and an optional yarn group can be specified as ‘master’ to drive the take-up speed calculation. The designation of a master side is implemented by selection of a ‘master normal’,

$$\mathbf{n}_m \in \{\pm \mathbf{x}, \pm \mathbf{y}, \pm \mathbf{z}\}. \quad (2.20)$$

Using the mandrel CS as reference, a facet is a part of the master side if

$$\angle(\mathbf{n}_{s\perp}, \mathbf{n}_m) < \gamma_{\max} \quad (2.21)$$

with $\mathbf{n}_{s\perp}$ as the substrate surface normal component perpendicular to the mandrel centerline tangent $\mathbf{t}$,

$$\mathbf{n}_{s\perp} = \frac{\mathbf{n}_s - \mathbf{n}_s \cdot \mathbf{t}}{\|\mathbf{n}_s - \mathbf{n}_s \cdot \mathbf{t}\|}, \quad (2.22)$$

with $\gamma_{\max}$ as the maximum angle between the two vectors, chosen as 45° , illustrated in Figure 2.13. A weight factor $w(f, g)$ is assigned to each combination of mandrel facet f and yarn group g . Without master

designations, all weight factors equal 1, so the yarns from all groups at all sides participate. If a master side is selected, then only the weight factors of facets on the master side equal 1, the rest equals 0. If a master side *and* a master yarn group are designated, then only the weight factors of the facets on the master side corresponding to the master yarn group equal 1, the rest equals 0.

The instantaneous optimum process take-up speed $\bar{v}$ is the weighted average of the individual take-up speeds v of each yarn i in each bias yarn group g ,

$$\bar{v} = \frac{\sum_{g=1}^{n_{ag}} \sum_{i=1}^{n_{y,g}} w(f(i), g) v_{g,i}}{\sum_{g=1}^{n_{ag}} \sum_{i=1}^{n_{y,g}} w(f(i), g)} \quad (2.23)$$

with n_{ag} as the number of active bias yarn groups (usually two), $n_{y,g}$ as the number of yarns in the g th group, and $w(f(i), g)$ denoting the weight factor corresponding to the combination of the first facet f that is located downstream of the fell point of yarn i and yarn group g . As a result, the instantaneous optimum take-up speed $\bar{v}$ is now known.

Next, the machine is moved relative to the mandrel using the calculated value of $\bar{v}$ and the constant value of ω , depositing a small thresholded amount of yarn with the forward solution. After that, $\bar{v}$ is recalculated using the updated fell front and machine position. This alternation is repeated until the mandrel end is reached, finishing the inverse solution for the current overbraiding run. Hence, the forward solution is a building block for the inverse solution.

The same procedure applies to reverse braiding. This inverse solution can be interpreted as a vectorized form of the two dimensional axisymmetric solution as proposed in [5]. This novel approach expands the possibilities to process complex mandrel geometries from industrial practice, allowing it to integrate well with existing CAE tools in the process chain for design and manufacture. The computational time complexity is in the same order of the forward solution algorithm in Section 2.4.

Start of a run

In practice, the mandrel and braid usually have an excess length to attach the yarns to avoid effects of sharp mandrel end edges and to allow the braid to converge from an arbitrary initial braid angle to the correct braid

angle through a transient regime [45]. In order to minimize waste from the excess length, attention is paid to the start of the process.

In the model, at the start of the first overbraiding run, the machine origin is arbitrarily assumed to coincide with a plane fit through the run start contour. For each yarn i in group g , the initial fell point positions $\mathbf{p}_{i,0}$ are evenly spaced on the run start section contour and are positioned such that the sum of their extracted lengths is minimized,

$$\min \sum_{g=1}^{g=n_{ag}} \sum_{i=1}^{i=n_{y,g}} \|\mathbf{q}_{g,i,0} - \mathbf{s}_{g,i,0}\| + \|\mathbf{s}_{g,i,0} - \mathbf{p}_{g,i,0}\| \quad (2.24)$$

with n_{ag} as the number of active yarn groups including optional stem yarns and $n_{y,g}$ as the number of yarns of the g th group. For the initial spool positions $\mathbf{q}_{i,0}$, the initial distribution is given by Equation (2.1). In case of guide ring contact, according to Equation (2.3), $\mathbf{s}_{i,0}$ equals $\mathbf{r}_{i,0}$, calculated using frictionless force equilibrium.

For each overbraiding run, the machine starts moving as shown in Figure 2.14 with $\omega = 0$ along curve I_i to meet the instantaneous optimum helix trajectory that is obtained as follows: The inverse solution is applied with the FSL through each $\mathbf{p}_{i,0}$, resulting in $\mathbf{q}_{i,2}$, $Q_{i,0}$, and, collectively, $\bar{v}_0$. Let $\mathbf{q}_{i,1}$ be the intersection between $Q_{i,0}$ and I_i closest to $\mathbf{q}_{i,2}$. This results in the i th machine moving distance, collectively resulting in a weighted average. After moving this distance with idle carriers, the carriers start rotating with ω and the machine's take-up speed is set to $\bar{v}_0$, resulting in a braided interlacement with a braid angle close to the required one right at the start of the braid.

End of a run

The overbraiding run is finished as soon as the last bias fell point passes the mandrel end section contour. At that instant, both speeds are set to zero. The corresponding spool positions are taken as the initial positions for the next overbraiding run. The intersections of the deposited yarns with the run end section contour are, after being offset with the run thickness, taken as the initial fell points for the next run and optional guide ring contact is taken into account. Surface S equals the mandrel outer surface in the first run only. After each run, S is offset by the run braid thickness distribution before the next run starts.

Figure 2.14 Overbraiding run start for the inverse solution, showing the carrier trajectory for a single yarn. The helix is stretched in axial direction for visualization purposes.

2.6 Experiment

The mandrel shown in Figure 2.15 was used to validate the inverse kinematic model. In some respects, this mandrel, as used previously in [41], is a more difficult shape compared to many practical cases where the cross-sectional size is kept as constant as possible. However, it contains many features that result in relevant results as shown later.

Let the mandrel skeleton curve or ‘skeleton’ be defined as the union of centroids from the planar cross-sectional contours perpendicular to the local mandrel length direction. For typical process states, the fell front is a closed contour around the skeleton. The centroid of that contour generally roughly coincides with the skeleton. A practical rule of thumb states that the fell front and its centroid should stay concentric with the machine for an appropriate fiber distribution. In this experiment, this rule is not followed because the origin and axis of the machine are constrained to coincide with the mandrel z -axis that is chosen as the centerline for simplicity.

The overbraiding experiment was performed at Eurocarbon on the braiding machine described in Table 2.1. The given braid angle applies to all sides simultaneously, roughly corresponding to a quasi-isotropic lay-up for a triaxial braid. The process take-up speed was generated while all mandrel sides and bias yarns equally contributed to the take-up speed calculation. A single forward braiding run was performed in the positive mandrel z -direction.

Figure 2.15 Mandrel with dimensions in millimeter. The four sides are numbered for reference.

2.6.1 Take-up speed profile generation

The inverse solution was implemented as a single-threaded application that required a few minutes of computation time on a 1.4 GHz single core PC for the current mandrel consisting of approximately 50000 facets. The threshold for the detection of sharp edges in circumferential direction was set at a dihedral angle of 45 degrees, which is large enough to identify all edges in this mandrel as smooth. The generated take-up speed profile is shown in Figure 2.16. The generated take-up speed profile had to be reduced in size because of limitations in the control system of the particular braiding machine. Discrete velocity steps of 7 mm/s were used in the machine code for the experiment as shown in the coarsened profile in Figure 2.16. This resulted in a stepped approximation with regions having too low values. Figure 2.17 shows the predicted fiber distribution corresponding to the original take-up speed profile.

The modeled slip tendency is shown in Figure 2.18, suggesting slip at the tapered mandrel regions toward the smaller circumference. The modeled yarn slack showed to be negligible for the current process settings.

Table 2.1 Relevant braiding parameters of the experiment.

parameter	value
spool plane radius r_{sp}	840 mm
inner guide ring radius $r_{gr,in}$	200 mm
inner guide ring height $h_{gr,in}$	350 mm
outer guide ring radius $r_{gr,out}$	200 mm
outer guide ring height $h_{gr,out}$	385 mm
number of yarns per group n_y	48
carrier speed ω	$36^\circ/\text{s}$
required braid angle α_{req}	60°
master side	none
master yarn group	none
bias primary yarn material	800 tex Toray T700 carbon
bias secondary yarn material	300 tex PPG 1383 glass
stem yarn material	2 x 800 tex Toray T700 carbon

Figure 2.16 Take-up speed profile as output and coarsened for machine input.

2.6.2 Virtual overbraiding

The simulated braid angles on sides 1 to 4, as indicated in Figure 2.15, are plotted in Figure 2.19, showing deviations of up to approximately 7° when compared against the required braid angle of 60° . In the interval from 650 to 1100 mm, both too high and too low values of α occur, showing the compromise as a result of the averaging over the four sides. In order to assess the effect of the coarsening, the generated speed profile is fed back as input for a process simulation using the forward solution. The simulation output for the coarsened take-up speed profile is illustrated in Figure 2.20 for the warp yarn group on side 2. This shows

Figure 2.17 An impression of the fiber distribution on sides 1 (top) and 2 (bottom), visualized in Gmsh [60].

Figure 2.18 The modeled slip direction for the weft (O) yarns with the vector length as the slip tendency c_{slip} . The yarns are not shown for clarity.

that the coarsening with lower values for the take-up speed around 600 and 1000 mm results in an increase of α up to approximately 5° , as to be expected from the classical solution (2.4). This example emphasizes the importance of a coarsening step that is sufficiently small to achieve accurate results.

2.6.3 Physical overbraiding

The physical overbraiding resulted in the preform shown in Figure 2.21. The experimentally obtained braid angle was measured at the same locations used at the virtual braid with close-range multi-image photogrammetry [61]. This allows an inspection of the 3D orientation of fiber segments on surfaces with arbitrary orientation relative to any camera image plane as illustrated in Figure 2.22. The measurement procedure was performed in a semi-automated manner as follows. First, the relative camera orientations are obtained automatically from photogrammetry software Photomodeler [62] using coded targets. Next, in each photo,

Figure 2.19 The modeled braid angle α for all yarn groups at all sides for the original take-up speed profile.

Figure 2.20 The modeled braid angle α for the warp yarn group at side 2 for the original and coarsened take-up speed profile, respectively.

the same yarn segment is identified by manually selecting its start and end point in the 2D image plane using a mouse. Finally, the intersection of the corresponding planes through each 2D segment and their camera perspective center yields the yarn segment in 3D. The estimated accuracy is in the order of $\pm 2^\circ$ and is mainly caused by the manual selection.

2.7 Results and discussion

2.7.1 Experimental results

In general, no significant slip was observed between the yarns and the mandrel during experiments, in contrast to Figure 2.18. This was confirmed after examination of the video footage taken of the

Figure 2.21 The overbraided mandrel.

Figure 2.22 Schematic overview of inspection using photogrammetry.

overbraiding runs. In previous experiments [41] with the same mandrel but without stem yarns, significant slip was observed. Apparently, the structure of a triaxial braid significantly reduces slip on the mandrel. Hence, in the case of a triaxial braid, the assumption of no slip after deposition holds for this complex mandrel. The only observed slip occurred at the sharp mandrel edge corners of the mandrel start as shown in Figure 2.23.

In Figure 2.24, the the coarsened model braid angle is compared to the experimental braid angle α_{exp} . The deviation of α_{exp} in the z-interval from 0 to 50 mm is mainly attributed to the yarn slip over the mandrel edges. Rotational misalignment of the braid funnel around the mandrel axis can be excluded as the cause because this would result in the braid angle to

Figure 2.23 Close-up of the braid start. Yarn slip over the sharp mandrel corners result in the small uncovered regions at the corners.

deviate with an equal sign at each side per yarn group, which is not the case. The rest of this interval shows acceptable values.

Upbraiding, i.e. braiding from small to large mandrel radius, in the first tapered interval from 200 to 350 mm caused α_{exp} to rapidly increase and exceed the modeled value for both warp and weft yarn groups on all sides. The classical solution in Equation (2.4) suggests that the corresponding value for v was too small, given α too large and ω constant. In other words, the generated ramp-up of v arrived too late. Hence, the experimental fell front crossed the 200 mm transition much earlier than anticipated by the model. The main reason is presumed to be a much shorter actual convergence zone compared to the model, which is likely caused by neglect of yarn interaction including friction and guide ring contact friction, as described in e.g. [30, pg. 4-5] and as schematically shown in Figure 2.25. On the other hand, the convergence zone length increases with increasing stem yarn tension, possibly partly canceling out this effect. The delay is not caused by the coarsening because it did not result in a shift in time of $v(t)$ on a global scale.

Braiding along the largest cross-section interval from 350 to 650 mm yielded an acceptable braid angle, except for the end of this interval where a rapid decrease occurs.

Downbraiding, i.e. braiding from large to small mandrel radius, in the first half of the second tapered interval from 650 to 850 mm yields an α_{exp} below the target value for 1X (denoting the side 1 X-yarns), 1O, 2O and 4X. Analogous to the first tapered interval it could be stated, although

Figure 2.24 The braid angle for the coarsened model and experiment.

not as strongly, that the ramp-down of v , again, arrived too late for these yarns. An explanation for the lagging effect being smaller is the larger mandrel radius which reduces the yarn density and interaction. This results in a smaller decrease of the convergence zone length. For 2O and 4X, a second cause of the lower α_{exp} is attributed to the skeleton not being parallel to the centerline in this interval as shown in Figure 2.15.

Figure 2.25 Convergence zone length in the model and experiment, here shown schematically and simplified for a single yarn on a centered cylindrical mandrel without guide rings. Only the upper half of the braiding process is shown.

Figure 2.26 Side 2 at the interval from 650 to 850 mm with fiber directions $\mathbf{f}_X$, $\mathbf{f}_O$ and $\mathbf{f}_S$ for warp, weft and stem yarns respectively, and the resulting asymmetric result in α .

The average stem yarn orientation on sides 2 and 4 tends to follow the skeleton curve as shown in Figure 2.26, thereby constraining the bias yarns, resulting in the tendency of the bias yarns to orient such that the sum of their unit direction, $\mathbf{f}_X + \mathbf{f}_O$, is parallel to the stem yarn direction $\mathbf{f}_S$. For 2O and 4X, this results in a smaller braid angle. For 2X and 4O, the braid angle increases, possibly partially canceling out the effect of the delayed ramp-down arrival, yielding α_{exp} closer to the target value.

Around 850 mm, a peak occurs for 4X. Examination of video footage showed that the machine operator manually pushed the braid toward the mandrel to avoid bridging, influencing the results. This was done single-handed, possibly exerting an asymmetric pressure on the braid.

In the eccentric interval from 850 to 1000 mm, the braid angle is affected by eccentricity, a phenomenon that was described earlier in [41].

In the interval from 1000 to 1100 mm, stem yarns impose the same constraint as in the downbraided interval, again resulting in a relatively large deviation in braid angle on sides 2 and 4. The maximum deviation

Figure 2.27 Result overview, showing, up to 650 mm, the experimental scatter of the experimental braid angle for both yarn groups at all sides.

of approximately 10 degrees occurs here for 4O.

In the last interval from 1100 to 1300 mm, the braid angle is acceptable but also deviates at the end due to edge effects similar to the start of the first interval.

Based on symmetry arguments, theoretically all braid angles should be equal from 0 to 650 mm. This is not the case, as shown in Figure 2.27. Also the following braid angle pairs should be equal along the full mandrel length: {1X, 1O} and {3X, 3O} as can be compared in Figure 2.24 and {2X, 4O} and {2O, 4X} as shown in Figure 2.28. The large deviation in the pair {2O, 4X} around 850 mm suggests to confirm an asymmetric effect of the machine operator interference. Apart from that outlier, a large experimental scatter of around 7 degrees is observed while for the non-tapered mandrel regions, the braid angle error between the coarsened model and the experiment is around 3 degrees.

2.7.2 Validity of assumptions

The assumption of continuous yarn trajectories holds. The assumption that the yarn trajectories coincide with the mandrel surface holds for most of the mandrel surface except at the mandrel interior edges in circumferential direction where a small amount of yarn bridging was observed in the experiment. It should be noted that the manual machine operator interference reduced bridging near 850 mm, possibly concealing a local higher level of bridging. Near the concave edges, the deposition model allows the free yarn segment to temporarily pierce the mandrel.

Figure 2.28 Symmetry check for experimental results on sides 2 and 4, showing overlap in the pairs {2X, 4O} and {2O, 4X}.

Locally, this can artificially influence the fiber distribution.

The assumption of the yarns remaining fixed after contact with the mandrel was observed to be valid for this case using a triaxial braid, but is not always valid, especially for a biaxial braid.

The assumption that the yarns do not interact, combined with neglect of yarn thickness, is not valid and leads to an over-estimation of the convergence zone, yielding systematic errors in the generated control data, in turn contributing to the error in the braid angle. Physical experiments or more complex braiding process models could provide correction factors to improve the accuracy.

The assumption of no friction between yarns and guide rings is difficult to discuss because this phenomenon has not been investigated independently from other phenomena. It can be stated, however, that the use of a vibration unit attached to the guide ring, although not used here, may lower the friction from static to kinematic, thus approaching the assumption. However, reverse braiding can lead to a larger contact angle on the guide ring, significantly increasing the effect of friction.

2.8 Conclusions

A novel kinematic model is proposed and implemented to generate the take-up speed profile for circular braiding machines with optional guide rings, given a braid angle distribution on mandrels with complex 3D shapes. These shapes include non-axisymmetric, optionally eccentric

cross-sections that can vary in shape and size along an optionally curved mandrel centerline, allowing a curved machine movement.

The fell point velocity components in axial and circumferential direction are not assumed to equal those of the spool. This offers an essential generalization that distinguishes the proposed model from earlier publications. The yarn trajectories of optional stem yarns are generated using the topology of the bias yarns. The computational time complexity is in the order of seconds per overbraiding run, making it suitable for use in earlier design phases and for CAE process chain integration. The procedure accommodates rapid changes in braid angle or mandrel geometry, possibly resulting in local take-up speed reversals. Finally, the model includes the optional assignment of a master side and a master yarn group because sometimes the braid angle cannot be satisfied on all mandrel sides simultaneously.

Experimental validation with the complex shaped 'exhaust' mandrel in Section 2.6 results in an experimental scatter of 7 degrees and an error around 3 degrees in the centered, non-tapered mandrel regions and a maximum error around 10 degrees in tapered regions. The neglect of yarn interaction in the convergence zone is among the largest causes of systematic errors in the proposed kinematic model. As a consequence, the convergence zone length is over-estimated. Other causes of errors are the neglect of guide ring contact friction, the neglect of the influence of stem yarns in regions where the mandrel skeleton curve direction deviates from the global mandrel centerline, and the fact that the required braid angle cannot be satisfied at all sides simultaneously. Hence, in general, experiments remain necessary to fine-tune the generated take-up speed for an accurate enough braid-angle distribution.

Chapter 3

The effect of the mandrel shape on the take-up speed profile

3.1 Introduction

This chapter focuses on the braiding process with increasingly complex mandrel shapes as shown on the path in Figure 3.1. The previous chapter discussed how a take-up speed profile can be generated for complex mandrel shapes using inverse kinematics, given the mandrel, the machine and the braid angle distribution. However, that chapter was mainly focused on the experimental validation of the presented kinematic model in terms of the braid angle error. Less attention was paid to braiding process properties, like the shape of the take-up speed profile, in relation to various practical mandrel shapes like mandrels with a large variation of the cross-sectional shape and size along the length and mandrels with curved centerlines.

Problem and objective

The ‘inverse solution’ presented in the previous chapter has not yet been applied to a variety of mandrel shapes. Hence, the behavior of this proposed approach is largely unknown. This makes it difficult to know what results should be expected for practical mandrel designs.

Figure 3.1 Design-process-material triangle showing the current chapter underlined.

The objective is to improve the understanding of the capabilities of the inverse solution by applying it to case studies of complex mandrel shapes and to subsequently visually illustrate the results. More specifically, and as the main focus, the generated take-up speed profile is plotted as a function of the mandrel geometry and the required braid angle distribution. Analytical solutions are used for comparison purposes.

Outline

First, analytical expressions of the braid angle are presented as a function of the machine speeds and a mandrel cross-section that is either circular, rectangular, square, or a regular polygon. Next, the analytical model for the rectangular cross-section is compared qualitatively to the kinematic model from the previous chapter and to experiments. Subsequently, the inverse solution is applied to case studies using an axisymmetric braiding process and various mandrel shapes, roughly in the order of increasing complexity. Next, a case study using a mandrel with a curved centerline

is presented. Finally, the phenomena encountered in these case studies are summarized to provide new insights in braiding arbitrary geometries.

3.2 Analytical solutions

3.2.1 Cylindrical mandrels

For a cylindrical mandrel in an axisymmetric process in steady state as defined in Appendix A, the braid angle and the take-up speed are expressed with the ‘classical solution’ [8, 29, 30] and the ‘inverse classical solution’, respectively,

$$\alpha = \arctan \left(\frac{\omega r_m}{v} \right) \quad (3.1) \quad \Leftrightarrow v = \frac{\omega r_m}{\tan \alpha} \quad (3.2)$$

with α as braid angle in radians, ω as the average carrier rotation speed around the machine axis in radians per second, r_m as the mandrel radius, optionally increased by half the braid thickness, and v as take-up speed.

The mandrel radius in Equations (3.1) and (3.2) is sometimes [41, 42, 63] approximated for mandrels with arbitrary cross-sections using

$$r_{m,eq} = \frac{P_m}{2\pi} \quad (3.3)$$

with $r_{m,eq}$ as the equivalent mandrel radius and P_m as the mandrel cross-sectional perimeter.

For special cases of transient states using a cylindrical mandrel in an axisymmetric braiding process, analytical solutions for the braid angle as a function of time may exist, like for the case of a stepwise braid angle change [45].

3.2.2 Rectangular mandrels

The analytical steady-state braid angles can be determined for a centered rectangular cross-sectioned mandrel with unequal sides as shown in the Figures 3.2 and 3.3, when neglecting yarn interaction, yarn slip after deposition, and guide rings.

Figure 3.2 A centered rectangular mandrel.

If, in contrast, frictionless slip over the mandrel edges is assumed while still neglecting yarn interaction, then all yarns follow geodesic (shortest) paths, leading to a uniform braid angle around the circumference.

The input parameters are the carrier rotation speed ω , the take-up speed v , spool plane radius r , rectangle width w and height h , all of which are assumed constant. The following equations are derived from Figures 3.2 and 3.3:

$$\beta_1 = \arcsin\left(\frac{w}{2r}\right) \quad (3.4) \quad \beta_2 = \arccos\left(\frac{h}{2r}\right) \quad (3.5)$$

$$\gamma_w = \beta_1 + \beta_2 \quad (3.6) \quad \gamma_h = \pi - \gamma_w \quad (3.7)$$

$$m = \frac{\gamma_h v}{\omega} \quad (3.8) \quad n = \frac{\gamma_w v}{\omega} \quad (3.9)$$

$$p = \sqrt{r^2 - \left(\frac{w}{2}\right)^2} - \frac{h}{2} \quad (3.10) \quad q = \sqrt{r^2 - \left(\frac{h}{2}\right)^2} - \frac{w}{2} \quad (3.11)$$

Combining the geometrical parameters above and using the geometry

Figure 3.3 The rectangular mandrel from Figure 3.2 showing the deposited yarn geometry.

from Figure 3.3,

$$\frac{p+h}{\tan(\alpha_h)} = m + \frac{q}{\tan(\alpha_w)}, \quad (3.12) \quad \frac{q+w}{\tan(\alpha_w)} = n + \frac{p}{\tan(\alpha_h)}. \quad (3.13)$$

After substituting $(\tan \alpha_h)^{-1} = a_h$ and $(\tan \alpha_w)^{-1} = a_w$ and solving for a_h and a_w ,

$$\tan(\alpha_h) = \frac{wp + hq + hw}{mq + wm + nq}, \quad (3.14) \quad \tan(\alpha_w) = \frac{wp + hq + hw}{mp + np + hn}. \quad (3.15)$$

As a final result,

$$\alpha_h = \arctan \left(\frac{\omega}{v} \cdot \frac{w\sqrt{r^2 - \left(\frac{w}{2}\right)^2} + h\sqrt{r^2 - \left(\frac{h}{2}\right)^2}}{\pi \left(\sqrt{r^2 - \left(\frac{h}{2}\right)^2} - \frac{w}{2} \right) + w\gamma_h} \right) \quad (3.16)$$

and

$$\alpha_w = \arctan \left(\frac{\omega}{v} \cdot \frac{w\sqrt{r^2 - \left(\frac{w}{2}\right)^2} + h\sqrt{r^2 - \left(\frac{h}{2}\right)^2}}{\pi \left(\sqrt{r^2 - \left(\frac{w}{2}\right)^2} - \frac{h}{2} \right) + h\gamma_w} \right). \quad (3.17)$$

Equations (3.16) and (3.17) show that the narrow and wide rectangle sides generally have different braid angles and the largest braid angle is on the narrow side as shown in Figure 3.4. The difference between the two braid angles increases with an increasing mandrel size relative to the value of the spool plane radius r . This could occur if r is used to represent the radius of the guide ring instead of the assumed spool plane circle. However, the angular speed of a supply point along the guide ring does not equal that of the spool and is not constant. Instead, it shows a periodical variation along the guide ring circumference when neglecting all friction. Hence, if r is taken as the guide ring radius, then an additional systematic error is introduced.

The approximation using the classical solution (3.1) with the equivalent radius from Equation (3.3) does not equal this exact solution.

If $h \ll w$, then only Equation (3.17) approaches the classical solution (3.1) in conjunction with Equation (3.3) where $P_m \approx 2w$. For practical cases with a width-to-height ratio of 10 and a spool plane radius that is 10% larger than the mandrel circumscribing circle radius, α_h is roughly 5° larger than α_w .

It is sometimes assumed that the axial advancement of a spool equals that of the corresponding fell point, so $\Delta z_h = m$ and $\Delta z_w = n$ [63, 64]. However, using the braid angles obtained from Equations (3.16) and (3.17), it follows that in general $\Delta z_h \neq m$ and $\Delta z_w \neq n$. Both are equal only for special cases, for example for a centered regular polygon or if the spool plane radius r equals the circumscribing circle of the mandrel cross-section.

Finally, although this analytical model may seem theoretical, it can be useful in practical cases that match the model assumptions like filament winding of high-tack yarns.

Comparison to experiments

Experiments on rectangular cross-sectioned mandrels have been documented in scientific literature. Nishimoto et al. [64] find the largest braid angle on the wide side, which opposes the analytical solution in Section 3.2.2. A similar result is mentioned by Ebel and Hans [12], where a typical composite braiding process setup with a guide ring is used, resulting in a full mandrel surface coverage. Taking the guide ring for the radius in the analytical solution would cause a systematic error as

Figure 3.4 Exaggerated schematic braid angle for a centered rectangular mandrel as in Figure 3.3. The mandrel faces have been flattened to a common plane. The experimental trends were obtained from [12, 64].

explained in Section 3.2.2. When performing a kinematic simulation as described in Chapter 2, including guide ring contact, instead of using the analytical model, then the side with the largest braid angle matches the physical experiment as shown in Figure 3.4. Additionally, the yarn follows an S-curve on each side due to yarn interaction and slip over the mandrel, which is not included in the analytical model nor in the kinematic model.

3.2.3 Square and regular polygon mandrels

When using a square mandrel cross-section, then it follows for the parameters in Section 3.2.2 that $w = h$, so $m = n$, $p = q$, $\gamma_h = \gamma_w = 90^\circ$ and $\alpha_h = \alpha_w$. Substitution in Equation (3.16) and canceling out terms yields

$$\alpha = \arctan \left(\frac{4\omega h}{2\pi v} \right), \quad (3.18)$$

which equals the classical solution (3.1) when using an equivalent mandrel radius $r_{m,eq}$ according to Equation (3.3) to represent r_m ,

$$r_{m,eq} = \frac{4h}{2\pi}, \quad (3.19)$$

with the square perimeter $P_m = 4h$. Hence, the classical solution also holds for a centered square cross-sectioned mandrel when using the equivalent radius $r_{m,eq}$ from Equation (3.3), despite the fact that formally, this configuration is never in a steady state as defined in Appendix A.

Analogous to the square mandrel cross-section, the classical solution

Table 3.1 Default braiding parameters as used for all case studies, unless overridden for the case study at hand. The given required bias braid angle can be used to obtain a ‘quasi-isotropic’ layup.

parameter	value
guide rings	none
max. machine take-up speed $v_{\max}$	40 mm/s
mandrel position in the machine	centered
braid type	regular
stem yarns included	yes
required braid angle α_{req}	60°
master side	none
master yarn group	none
smoothness threshold $\Delta\alpha_{\max}$	5°
time step size	0.1s
feed-back	yes

Equation (3.1) is exact for any centered regular polygon mandrel cross-section [45] when using the equivalent radius from Equation (3.3).

3.3 Braiding take-up speed generation case studies*

In this section, the ‘inverse solution’ from the previous chapter is applied to generate the take-up speed profile for various mandrel shapes and a given required braid angle. The take-up speed is here expressed as a function of the position of the machine origin on the mandrel centerline. Alternatively, the take-up speed can be expressed as a function of the fell front position. However, for complex mandrel shapes, the fell front is generally an arbitrary curve on the mandrel surface around the centerline, hence not corresponding to a unique centerline position. The default input parameters are given in Table 3.1 and apply to all case studies unless specified otherwise. For each case, the machine is first moved from the indicated mandrel start section to the optimum braiding start location with the carriers idle to minimize transient braid angle errors due to start-up effects as described in Section 2.5.1.

*Based on [65].

Figure 3.5 Axisymmetric ‘cylinder’. Only half of the axisymmetric geometry is shown. Dimensions are in mm.

Table 3.2 Cylinder braiding parameters in addition to those from Table 3.1.

parameter	value
required braid angle α_{req}	$45^\circ; 60^\circ$
spool plane radius r_{sp}	840 mm
inner guide ring radius $r_{\text{gr,in}}$	100 mm
inner guide ring height $h_{\text{gr,in}}$	300 mm
outer guide ring radius $r_{\text{gr,out}}$	100 mm
outer guide ring height $h_{\text{gr,out}}$	400 mm
number of yarns per group n_y	48
carrier speed ω	$36^\circ/\text{s}$
feed-back	no and yes

3.3.1 Cylinder

In this case study, a stepwise braid angle change is required on a centered cylindrical mandrel shown in Figure 3.5. The mandrel is overbraided using the parameters in Tables 3.1 and 3.2. The braid angle smoothness threshold $\Delta\alpha_{\text{max}}$ in Table 3.1 asserts that the take-up speed is recalculated downstream of the stepwise braid angle transition as described in Section 2.5.1 to reduce the chance of large accelerations of the machine. Two take-up speed profiles are generated, one with and one without ‘feed-back’ as defined in Section 2.5.1. Remember that *with* feed-back, the *actual* deposited fiber directions are used to calculate the take-up speed instead of the required fiber direction field (FDF). In contrast, *without* feed-back, only the *required* fiber directions are used. Both take-up speed profiles are shown in Figures 3.6 and 3.7. The steady-state take-up speed values match those of the inverse classical solution (3.2).

The inverse solution without feed-back leads to a delayed response of the braid angle, shown in Figure 3.6. This response has an exponential trend that is close to the analytical solution mentioned in [45], using $r_{\text{gr,out}}$ as the creating circle radius. Mesh refinement and time step reduction let

Figure 3.6 Left: Cylinder take-up speed profile without feed-back. Right: The corresponding analytical solution and the kinematic model braid angle for both yarn groups.

Figure 3.7 Left: Cylinder take-up speed profile with feed-back. Right: The corresponding kinematic model braid angle for both yarn groups. The curves overlap.

the model approach the analytical solution. In contrast, when using feed-back, a short period of 'backward' braiding is generated, leading to the much faster response shown in Figure 3.7.

In Figure 3.8, the slip tendency as defined in Section 2.4.4 is shown. It suggests that in the case with feed-back, the yarns have a more localized yet larger maximum tendency to slip after deposition. If the tendency to slip exceeds the coefficient of friction μ , slip can occur. When assuming $\mu \approx 0.2$ for carbon yarns, the slip indicator predicts more slip for the case with feed-back. However, the occurrence and the extent is also influenced by the braid structure. The presence of stem yarns, for example, can prevent most slip.

Finally, note that the feasibility of this solution is not shown with respect

Figure 3.8 Close-up of the distribution and the slip tendency of the warp yarns for the take-up speed profile without (left) and with (right) feed-back. The slip tendency and direction is indicated by a vector field (note the different scales).

Figure 3.9 Axisymmetric ‘cone’. Only half of the axisymmetric geometry is shown. Dimensions are in mm. Dimensions between parentheses are driven.

to yarn slack. The risk of slack is expected to be higher for the case including feed-back due to the temporary backward movement of the machine.

3.3.2 Cone

The ‘cone’ in Figure 3.9 is a mandrel with non-smooth transitions in the mandrel length direction. It is designed to be overbraided using the parameters in Tables 3.1 and 3.3.

For this and all following cases, the take-up speed profile is generated with ‘feed-back’ for a rapid response to mandrel geometry changes.

Table 3.3 Nozzle and cone braiding parameters in addition to those from Table 3.1.

parameter	value
spool plane radius r_{sp}	1382 mm
number of yarns per group n_y	72
carrier speed ω	8.33 deg/s (100 ‘picks / min.’)

Figure 3.10 Left: Cone take-up speed profile. Right: The corresponding modeled braid angle for both yarn groups. The measurement path is the curve from start to end section, obtained as the intersection of the mandrel surface and the yz -plane at the positive y -side.

Figure 3.11 Cone fiber distribution.

The generated profile shown in Figure 3.10 shows a counter-intuitive spike at the start and end of the conical segment. The steady-state values corresponding to the cylindrical regions match those of the inverse classical solution (3.2).

The resulting braid angle is axisymmetric as shown in Figure 3.11. The braid angle deviates no more than one degree from the required braid angle as shown in Figure 3.10. The tendency to slip was approximately 0.25 on the tapered surface, so slip could be an issue, at least in case of a biaxial braid.

Figure 3.12 Axisymmetric 'nozzle'. Only half of the axisymmetric geometry is shown. Dimensions are in mm.

Figure 3.13 Left: Nozzle take-up speed profile. Right: The modeled braid angle for both yarn groups for the generated take-up speed profile. The measurement path is the curve from start to end section, obtained as the intersection of the mandrel surface and the yz -plane at the positive y -side.

3.3.3 Nozzle

The 'nozzle' in Figure 3.12 is a mandrel with a smooth surface. It is designed to be overbraided using the parameters in Tables 3.1 and 3.3.

The generated take-up speed shown in Figure 3.13 shows that take-up speed regression (reverse braiding) occurs when the fell point crosses the 'downbraiding' region in the z -interval from approximately 300 to 400 mm. This can lead to slack yarns, which is not taken into account. The steady-state values corresponding to the cylindrical regions match those of the inverse classical solution (3.2).

The resulting braid angle distribution is axisymmetric as shown in Figure 3.14. The resulting braid angle deviates no more than one degree from the required braid angle as shown in Figure 3.13.

Yarn slip is predicted to occur at least for biaxial braids as shown in Figure 3.15.

Figure 3.14 Nozzle fiber distribution.

Figure 3.15 The slip tendency and direction for the warp (X) yarn group.

3.3.4 Torus

The quarter ‘torus’ in Figure 3.16, a mandrel with a curved centerline, is designed to be overbraided with the parameters in Tables 3.1 and 3.4. The take-up speed profile is obtained using four configurations with different constraints:

1. not specifying a master side or a master yarn group,
2. specifying a master side,
3. specifying both a master side and a master yarn group, and
4. only specifying an offset centerline.

First, the results without master designations are discussed. The generated take-up speed profile as shown in Figure 3.17 has an average value

Figure 3.16 Quarter 'torus'. Positions along the centerline are measured with arc length s . Dimensions are in mm.

Table 3.4 Torus braiding parameters in addition to those from Table 3.1.

parameter	value
spool plane radius r_{sp}	1000 mm
inner guide ring radius $r_{gr,in}$	100 mm
inner guide ring height $h_{gr,in}$	-100 mm
outer guide ring radius $r_{gr,out}$	100 mm
outer guide ring height $h_{gr,out}$	100 mm
number of yarns per group n_y	48
carrier speed ω	$45^\circ/s$
master side	none and top

that matches that of the inverse classical solution (3.2). The corresponding fiber and braid angle distributions are shown in Figures 3.18 and 3.19. The braid angle significantly varies over the mandrel circumference for both yarn groups because the modeled braid angle cannot be satisfied at all sides simultaneously in this configuration. In reality, this effect may be different due to the neglect of yarn interaction in the model. The modeled braid angle is closest to the target at the mandrel inside and outside for both bias yarn groups because these sides remain the least influenced by the machine coordinate system rotation relative to the mandrel. Focusing at the mandrel top side, both yarn groups have a significant braid angle error.

Suppose, for demonstration purposes in configuration 2, that the structural performance of the mandrel top side is critical. This calls for specifying this mandrel side as the master side. However, after doing

Figure 3.17 Torus take-up speed as a function of the position of the machine origin, expressed as the arc length s on the mandrel centerline.

Figure 3.18 Torus fiber distribution, showing a small amount of stem yarn waviness.

so, all results remain the same as the case without master designations. Hence, even at a single mandrel side, the modeled braid angle cannot be satisfied by both bias yarn groups simultaneously.

When additionally assigning the warp yarn group as the master yarn group in the third configuration, then the take-up speed profile changes as shown in Figure 3.17 and the top side warp braid angle improves significantly as shown in Figure 3.19. The bottom weft braid angle improves as well due to symmetry, but the rest still deviates significantly.

As shown in Table 3.4, feed-back is always enabled for this mandrel geometry, so the *actual* deposited fiber directions are used to calculate the take-up speed. If, instead, feed-back is *not* used, so only the *required* fiber directions are used, then the master designations have no effect. This is explained by the observation that the feed-back results in a correction

Figure 3.19 The modeled braid angle for the bias yarn groups for the generated take-up speed profile. The dashed lines correspond to the case without master mandrel side and without master yarn group, the solid lines to the case with the warp (X) yarns at the top specified as 'master'. For this case, $\alpha_{\text{req}} = 60^\circ$.

for the effects of the machine coordinate system rotation relative to the mandrel: the process geometry used to generate the take-up speed assumes that the carriers travel over an instantaneous cylindrical surface around the mandrel as shown in Figure 2.11. That assumption is only an approximation for this case due to the rotation. Hence, for cases with a machine coordinate system that rotates relative to the mandrel, the use of feed-back is advised.

The take-up speed profiles in Figure 3.17 show a significant amount of noise. The profile with the master designations shows the most noise because the master designations reduce the number of yarns that contribute to the calculation of the average take-up speed, i.e. only the warp yarns that instantaneously have their fell point at the top side, so the effects of mandrel mesh irregularities are not averaged out as much.

The previous three configurations showed the purpose of master designations. Instead of using master designations, a nearly globally constant modeled braid angle at the prescribed value can be achieved for all

Figure 3.20 The modeled braid angle for both bias yarn groups on the four mandrel sides without master designation with 15mm centerline offset and $\alpha_{\text{req}} = 60^\circ$.

sides and yarn groups simultaneously by offsetting the centerline an appropriate amount and in the appropriate direction. This is explained as follows. Without the offset, the braid angle variation around the mandrel is periodic as a function of the machine rotation only. Adding the offset results in eccentricity, in turn providing a second periodic variation [41]. The offset can be tuned so the second variation is in counter-phase with the first, in effect reducing or canceled out the braid angle variation. As a consequence, the optimum centerline for a constant braid angle may not coincide with the mandrel skeleton curve as defined in Section 2.6.

For this case study, in the fourth configuration, a nearly constant braid angle is achieved with an offset of approximately 15mm in the zx -plane shown in Figure 3.16 toward the outside of the mandrel, resulting in a larger centerline radius. The resulting take-up speed roughly equals that without master designation from configuration 1 in Figure 3.17, and, again, matches the inverse classical solution (3.2). The resulting braid angle distributions are shown in Figure 3.20. However, the question remains whether this theoretical result reproduces in practice.

3.4 Conclusions

The generation of take-up speed profiles using the inverse kinematics model from Chapter 2 has been applied in case studies involving various mandrel shapes, including mandrels with curved centerlines and step-wise changes in either the braid angle or the mandrel slope in length direction. The modeled results are summarized in Table 3.5. This table partly shows to what degree the analytical (inverse) classical solution applies and how the braid angle deviates from the analytically predicted value when the assumptions of the analytical model are not satisfied.

Table 3.5 *Summary of the observed phenomena in the profiles of the generated take-up speed and the corresponding braid angle.*

case	Differences from the (inverse) classical solution for take-up speed (v) and braid angle (α) profiles
centered cylinder with stepwise change in α	v : regression if feed-back is enabled; α : shorter response dist. if feed-back is enabled
mandrel with rectang. cross-section	α : kinematic model matches experiments qualitatively better than analytical model
variable radius (cone)	v : includes counter-intuitive spikes at mandrel slope transitions
variable radius (nozzle)	v : includes regression at rapid mandrel radius change
curved centerline (torus)	v, α : can depend on master designations and centerline offset

Although these case studies were not experimentally validated, these trends can be useful for qualitative comparisons and phenomenological analysis.

The take-up speed profiles generated throughout the case studies show local deviations from what might be expected and what would be obtained when using only analytical solutions like the inverse classical solution (3.2). These profiles result in modeled braid angle distributions that are close to the required ones. In turn, these profiles, based on a model without yarn interaction and slip over the mandrel after deposition, can serve as an initial guess to guide the machine operator to find the best take-up speed profile on a physical machine.

The following counter-intuitive practical phenomena can be abstracted for the case studies, yet this listing is by no means exhaustive.

Although the fell point position increases monotonously along the mandrel centerline, the position of the machine may temporarily reverse with respect to the mandrel. Hence, fell point monotonicity does not warrant machine monotonicity. Also, the fell point speed generally does not equal the take-up speed, and a negative take-up speed does not necessarily correspond to a negative fell point speed. Analytical solutions do not take these phenomena into account, but the inverse kinematics approach does.

The generated take-up speed profile contains noise due to the mandrel surface mesh discretization error. This noise increases with a decreasing number of participating yarns. The more yarns participate, the more the noise is suppressed by the averaging effect over all yarns. For typical composite braiding machines, the number of carriers is usually large enough to result in a sufficiently high signal-to-noise ratio.

A take-up speed regression can locally increase the risk of slack yarns and slip after deposition. This underlines the utility of an indicator for yarn slack and yarn slip.

The resulting take-up speed profile and the resulting braid angle distribution can depend on whether feed-back is enabled in inverse kinematics. It is shown that use of feed-back can speed up the response of the braid angle. The use of feed-back is preferred to account for the effects of possible braiding machine coordinate system rotations relative to the mandrel.

Finally, the effect of assigning a master mandrel side and a master yarn group was demonstrated. The modeled braid angle can vary significantly around the mandrel circumference, despite keeping the fell front centered in the machine. The modeled braid angle error can be reduced on a master side for a master yarn group if specified. However, comparison against experiments is required to validate this braid angle variation. Besides, a centerline offset may provide an alternative solution, but this theoretical prediction requires validation as well.

Chapter 4

A yarn interaction model for circular braiding*

4.1 Introduction

This chapter focuses on the biaxial braiding process, but in more detail than the previous chapters, taking into account the used yarn material as shown on the path in Figure 4.1.

Problem and objective

In Section 2.6, a braid was manufactured on a machine controlled with instructions that were generated using a model that neglects yarn interaction. This resulted in deviations up to 10 degrees from the required braid angle. The main cause was assumed to be an incorrect convergence zone length, caused by the neglect of yarn interaction in the convergence zone. Hence, to reduce the error, yarn interaction must be taken into account. Apart from a convergence zone length change, the yarn interaction also increases yarn damage by shearing broken fibers off the yarns, in turn influencing the yarn interaction behavior [67] and the component quality. In order to reduce the number of manufacturing iterations to achieve a satisfactory braid angle, a more accurate value of the convergence zone length is required. The objective of this chapter is to model the yarn interaction for the biaxial axisymmetric braiding process

*Based on [66].

Figure 4.1 Design-process-material triangle showing the current chapter underlined.

including friction, resulting in the coordinates of each interlacement point in the convergence zone and the convergence zone length.

Outline

First, yarn interaction models from earlier work are discussed. Next, the yarn interaction including friction is modeled for the special case of an axisymmetric biaxial braid in a steady state with a single yarn material. The model calculates the non-linear yarn geometry in the convergence zone and the convergence zone length as a function of a prescribed braid angle on a mandrel with a given radius. Finally, the model is experimentally validated with a larger 144 carrier machine, a common size for vehicle structural components, increasing the effects of yarn interaction.

4.2 Earlier work

This section discusses yarn interaction analyses from earlier work, followed by a summary of what sets the novel model apart from these earlier models.

In their early contribution to the field on braiding analysis in the early 1990s, Du and Popper [5] reported that braiding over conic mandrel segments requires manual adjustment of the take-up speed profile that was generated using kinematics that neglects yarn interaction and transient effects.

In the last two decades, various models capable to represent yarn-to-yarn interaction were developed:

Zhang, Beale, and Broughton [21, 44] modeled the axisymmetric braiding process with a cylindrical mandrel and a 64-carrier machine. The spool tension was taken as input, as well as parameters describing Howell friction [68]. The yarns were modeled by their centerlines as 2D curves on a plane approximating the flattened version of the generally non-developable convergence zone surface. The yarn cross-section was modeled as an ellipse, but transverse yarn deformation was not taken into account. In general, these assumptions do not hold due to the significant yarn curvature in the convergence zone and the optionally decreasing yarn width caused by lateral compression by adjacent yarns. The serpentine spool path, tension fluctuation and yarn mass, were, among others, ignored. The interaction model resulted in a braid angle that is approximately 10° smaller than that of their kinematic model. However, in a steady state, the braid angles from both models should be the same. Hence, the reported difference should be attributed to other causes like transient braiding start-up effects instead of to the new braiding model features.

As mentioned in Section 1.6, the braiding process can also be modeled using a finite element approach, enabling the modeling of features that are ignored in kinematic approaches like yarn-to yarn friction, yarn deformations, slip after deposition and gravity at the cost of computation time.

Providing a more generic model than for braiding alone, Kanno and Ohsaki [69] propose a mathematical framework based on second-order cone programming for the analysis of 3D cable networks, including

a global optimum search, non-smooth large deformations, inter-cable contact, rigid body contact and cable slack while taking into account the cable network topology. Although this method may be able to model the braiding process, it lacks friction, a non-zero cable thickness and transverse cable deformations. Hence, it is expected that the resulting systematic errors call for extensions of the method to include these lacking features.

Taking a computationally simpler approach than the previous ones, Mazzawi [70] introduced an ‘interlacing parameter’, based on physical steady-state experiments, to account for the resulting convergence zone length change using complex mandrels.

The main novelty in this chapter consists of modeling the double curvature of the convergence zone surface with a short computation time compared to FE instead of approximating it by a developable surface as in [21, 44]. Additionally, the optional change of the yarn cross-sectional shape is taken into account.

4.3 Model

After providing the main modeling assumptions, a theoretical braid with only a single interlacement point is analyzed to explain the effect of friction on the convergence zone length. Next, the analysis is extended to an axisymmetric biaxial braiding process with an arbitrary number of interlacement points. Finally, two numerical implementations of that axisymmetric model are proposed.

4.3.1 Assumptions

Assuming a negligible yarn weight compared to the yarn tensions, yarn mass is neglected and the process is modeled as quasi-static, entailing the neglect of gravity and inertia effects. The inter-yarn friction dependency on pressure, relative speed and the relative fiber orientation [40] is neglected. For the latter, if two contacting and untwisted yarns are moved increasingly parallel to each other at a very small angle, then the coefficient of friction rapidly increases. In general, such small angles do not occur in the braiding process due to the interlacement, partly justifying this assumption. Coulomb friction is used to model friction

at interlacement points, neglecting stick-slip and viscous-like friction. Howell friction is not used due to the lack of the corresponding material characterization data. For Coulomb friction, an 'average apparent dynamic inter-yarn coefficient of friction' μ_{ap} is used.

The yarns are modeled with a rectangular cross-section for explanation purposes and a constant cross-sectional area. The yarn trajectory in the convergence zone is represented by a polyline, created by its interlacement points, starting at the fell point.

In this chapter, only the region between the fell front and the creating circle is taken into account.

4.3.2 Single interlacement point

This section describes a fictitious braid with only a single interlacement point to explain the qualitative effect of friction on the convergence zone length. Under the assumptions of Section 4.3.1, a single interlacement point **a** for one warp and one weft yarn is analyzed. Denote the machine coordinate system with origin **m** and axes $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

The cases with a zero, intermediate (finite and positive non-zero) and infinite value of μ_{ap} are compared in Figure 4.2, resulting in the interlacement points $\mathbf{a}_0$, **a** and $\mathbf{a}_\infty$, respectively, and convergence zone lengths H_0 , H and H_∞ . When yarn interaction is neglected, the convergence zone length follows from geometry as

$$H_0 = \frac{\sqrt{r_{sp}^2 - r_m^2}}{\tan \alpha} \quad (4.1)$$

with r_{sp} as the spool plane circle radius, r_m as the mandrel radius and α as the braid angle.

In Figure 4.3, the instantaneous kinematics are shown, including a 'kink angle' β and the interlacement point **a**. The relative velocity $\mathbf{v}_{rel,X}$ of the warp (X) yarn is equal to the difference of $\mathbf{v}_X$ and $\mathbf{v}_O$, corresponding to the instantaneous velocities of fiber material particles at **a** on the X and O yarn, respectively, rotating around their instantaneous fell points. As indicated earlier [21], $\mathbf{v}_{rel,X}$ is directed tangentially around the process axis and the friction acts in its opposite direction.

Figure 4.2 Interaction between two yarns without guide rings. Points in parentheses are visually obstructed by other points.

Figure 4.3 Instantaneous kinematics in the plane parallel to the machine $\mathbf{v}$ -axis, and through the coplanar points $\mathbf{a}$, $\mathbf{p}_O$, $\mathbf{p}_X$, $\mathbf{s}_O$ and $\mathbf{s}_X$.

For the extreme cases, $\beta = 0$ for $\mu_{ap} = 0$ and $\beta = \frac{\pi}{2} - \alpha$ for $\mu_{ap} = \infty$ at the instant of the interlacement point touching the mandrel. This shows that the fell point $\mathbf{p}$ shifts toward the spool plane with an increasing μ_{ap} , thereby reducing the convergence zone length, matching earlier statements [30, 43, 70, 71]. Hence, the convergence zone length is bounded from below at infinite friction and from above at zero friction.

A more generic and detailed biaxial yarn interaction model is proposed to provide a theoretical framework in Appendix B. The theoretical case discussed in the next section is a special case of this generalized model.

4.3.3 Multiple interlacement points for axisymmetry

In this section, the analysis of the biaxial braiding process is extended to an axisymmetric biaxial braiding process with an arbitrary number of interlacement points.

Assuming that the braiding process is axisymmetric as shown in Figure 2.2, the spool movement is circular and one modeled yarn represents all. The process is assumed to be in a steady state as defined in Appendix A.

Define the ‘fell point coordinate system’ as the global Cartesian coordinate system with the axes $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ and its origin on the process axis and

Figure 4.4 Schematic detail geometry of a yarns in the convergence zone.

closest to the fell point as shown in Figure 4.4. Unless specified otherwise, the local machine coordinate system and all coordinates are expressed in the fell point coordinate system. Let the machine axis $\mathbf{w}$ be collinear with $\mathbf{z}$.

Define n_{ipt} as the number of interlacement points per yarn, including the fell point and the supply point (replacement) as described later. Force equilibrium is applied to derive the positions of the successive interlacement points on a yarn in the convergence zone, from the fell point with the prescribed fiber orientation up to the supply point. Locally, at each interlacement point, the forces are approximated as coplanar, neglecting the normal components. There are three quasi-static force equilibrium equations for each ‘interior’ interlacement point, i.e. excluding the fell and supply point, in the Cartesian three-dimensional space,

$$\Sigma \mathbf{F}_i \equiv F_i \mathbf{f}_i - G_i \cdot \mathbf{f}_{i-1} - W_i \cdot \mathbf{t}_i = 0 \text{ for } i \in \{1, \dots, n_{ipt} - 2\} \quad (4.2)$$

with $\mathbf{f}$ and $\mathbf{t}$ in bold as the unit direction vectors, F and G as the yarn tension magnitudes, and W as the friction force magnitude. This results in $3(n_{\text{ipt}} - 2)$ equations. Neglecting bending, the struts are two-force members subjected to a tensile force that increases from the fell point to the spool, so

$$F_i = G_{i+1}. \quad (4.3)$$

Next, the unknowns are identified. The number of interlacement points n_{ipt} is unknown, just as the interior interlacement point positions and the supply point position. An initial guess of n_{ipt} can be readily obtained using the process geometry without yarn interaction. Each interlacement point, including a known fell point, lies on a ‘constraint plane’ through the process axis $\mathbf{z}$ shown in Figure 4.5, spaced at an angular interval

$$\delta = \frac{\pi}{n_y} \quad (4.4)$$

with n_y the number of yarns per yarn group [71]. Following any yarn from its fell point into the convergence zone, the i th interlacement point lies on the same ‘interlacement circle’, centered around the process axis. Due to the axisymmetry, the interlacement points can be conveniently expressed in cylindrical coordinates (r, φ, z) shown in Figures 4.4 and 4.5. Given a known fell point $\mathbf{a}_0$ of a yarn, previously denoted by $\mathbf{p}$ in Figure 4.2, the i th interlacement point on that yarn has the angle

$$\varphi_i = \varphi_0 + \text{sign}(\omega \mathbf{w} \cdot \mathbf{z}) i \cdot \delta \quad (4.5)$$

where φ_0 corresponds to the fell point. The sign function is used to indicate that the direction of rotation around the process axis while traversing along the yarn from fell point to supply point depends on the combination of the signed yarn group dependent carrier rotation speed ω and the machine orientation in the fell point coordinate system, indicated by the dot product $\mathbf{w} \cdot \mathbf{z}$. As a consequence, given the fell point, the only interlacement point degrees of freedom $\mathbf{x}$ left are in the r and z directions:

$$\mathbf{x} = (r_1, z_1, \dots, r_{n_{\text{ipt}}-1}, z_{n_{\text{ipt}}-1}). \quad (4.6)$$

Figure 4.5 Front view showing interlacement circles and interlacement point constraint planes at intervals δ . In this case, the number of interlacement points per yarn n_{ipt} is 16, including the fell point and the virtual interlacement point.

The boundary conditions to be prescribed are the fell point $\mathbf{a}_0$ and its yarn direction $\mathbf{f}_0$, corresponding to the prescribed braid angle. The strut between $\mathbf{a}_0$ and $\mathbf{a}_1$ is constrained to be in direction $\mathbf{f}_0$ using the corresponding direction cosine,

$$\Delta_{\cos} \equiv \frac{\mathbf{a}_1 - \mathbf{a}_0}{\|\mathbf{a}_1 - \mathbf{a}_0\|} \cdot \mathbf{f}_0 - 1 = 0. \quad (4.7)$$

Generally, the supply point is not an interlacement point and does not lie on a constraint plane. For a valid solution, one ‘virtual’ interlacement point (at the supply point end) must lie outside the creating circle radius r_{cc} as shown in Figure 4.5. The supply point is the intersection between the creating circle extrusion and the yarn polyline end segment (strut) connected to the virtual point. The virtual point is included in n_{ipt} , replacing the actual supply point. The yarn tension magnitude must be prescribed as a boundary condition at the virtual point.

The number of variables in r and z to be solved is $2(n_{ipt} - 1)$, excluding the known fell point and including the virtual interlacement point. This number is exceeded by the $3(n_{ipt} - 2)$ force equilibrium equations (4.2),

Figure 4.6 Simplified yarn interaction. Generally, $\eta \neq 90^\circ$.

making the system of equations overdetermined. However, approximate solutions can be obtained using non-linear optimization techniques as shown in the next section. In the remainder of this section, the constituents of Equation (4.2) are derived, mainly with the help of Figure 4.4.

Coulomb friction is used to obtain the friction force magnitude,

$$W_i = \mu_{ap} N_i \quad (4.8)$$

with μ_{ap} as the average apparent dynamic inter-yarn coefficient of friction and N_i as the local normal force magnitude, approximated as shown in Figure 4.6 by

$$N_i = (F_i + G_i) \sin(\theta_i), \quad (4.9)$$

assuming only a small rate of change of F , G and ‘crimp angle’ θ along the yarn. As shown in Figure 4.7, θ_i is approximated by

$$\theta_i = \frac{\arctan\left(\frac{t_{y,i}}{l_i}\right)}{n_{float}} \quad (4.10)$$

with

$$l_i = \|\mathbf{a}_{i+1} - \mathbf{a}_i\| \quad (4.11)$$

as the ‘strut length’, n_{float} as the constant ‘float length’, and $t_{y,i}$ as the local yarn thickness,

$$t_{y,i} = \frac{A_y}{w_{y,i}} \quad (4.12)$$

with $w_{y,i}$ as the local yarn width and A_y as the constant rectangular yarn

Figure 4.7 Cross-section of the convergence zone, viewed perpendicularly to both the local yarn centerline direction and the local yarn thickness direction. From top to bottom: A ‘diamond braid’ having $n_{\text{float}} = 1$, and three ‘regular braids’ having $n_{\text{float}} = 2$ with a decreasing yarn-to-yarn spacing, resulting in a yarn thickness increase that can occur when traveling to the fell point.

cross-sectional area,

$$A_y = \frac{\rho_{l,y}}{v_{f,y}\rho_f} \quad (4.13)$$

with $\rho_{l,y}$ as the yarn linear density, $v_{f,y}$ the yarn fiber volume fraction and ρ_f the fiber density. Neglecting yarn spreading and allowing the yarn to reduce in width without deformation resistance, the local yarn width $w_{y,i}$ in Equation (4.12) is modeled using

$$w_{y,i} = \min(w_{y,\text{ini}}, p_i) \quad (4.14)$$

with $w_{y,\text{ini}}$ as the initial yarn width on the spool and p_i as the local yarn pitch, assuming only a small rate of change for p_i . Between the i th and $(i+1)$ th interlacement point,

$$p_i = 2d_i \sin \psi_i, \quad (4.15)$$

using the geometrical symbols shown in Figure 4.4. For d_i and ψ_i , the following parameters are required. The interlacement circle tangent $\mathbf{t}$ points in the yarn rotation direction when traveled from fell to spool,

corresponding to the relative yarn sliding direction as described in Section 4.3.2,

$$\mathbf{t}_i = \text{sign}(\omega) \frac{\mathbf{w} \times \mathbf{a}_i}{\|\mathbf{w} \times \mathbf{a}_i\|}. \quad (4.16)$$

Define $\mathbf{d}_i$ as the local unit direction vector from $\mathbf{a}_i$ to the adjacent interlacement point on the same interlacement circle, having the same sense as $\mathbf{t}_i$. Express $\mathbf{R}$ as the constant rotation matrix that rotates $\mathbf{t}$ around $\mathbf{w}$ by an angle δ to yield $\mathbf{d}$,

$$\mathbf{R} = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.17)$$

with the coefficients $c = \cos[\text{sign}(\omega \mathbf{w} \cdot \mathbf{z})\delta]$ and $s = \sin[\text{sign}(\omega \mathbf{w} \cdot \mathbf{z})\delta]$. Now

$$\mathbf{d}_i = \mathbf{R} \mathbf{t}_i. \quad (4.18)$$

In Equation (4.15), the angle ψ_i is the ‘pseudo-braid angle complement’,

$$\psi_i = \arccos(\mathbf{f}_i \cdot \mathbf{d}_i), \quad (4.19)$$

and d_i is half the local Euclidean distance between two adjacent interlacement points on the same interlacement circle,

$$d_i = r_i \sin \delta \quad (4.20)$$

with δ from Equation (4.4) and radius r_i as the distance between $\mathbf{a}_i$ and the process axis,

$$r_i = \|\mathbf{a}_i - (\mathbf{a}_i \cdot \mathbf{w})\mathbf{w}\|. \quad (4.21)$$

4.3.4 Implementation

An implementation involving non-linear optimization techniques is presented, here referred to as ‘NLO’, followed by a novel ‘frontal approach’. No generic concise analytical expression of the convergence zone length as a function of the input parameters was found. Instead, only case-specific numerical simulations can be performed using the proposed approaches.

Non-linear optimization approach

The problem in Section 4.3.3 can be classified as a non-linear optimization problem. The Levenberg-Marquardt (LM) algorithm is a widely used approach to solve such problems. This algorithm is here used to iteratively solve the objective function

$$\min(\|\Sigma \mathbf{F}_1\|^2 + \cdots + \|\Sigma \mathbf{F}_{n_{\text{ipt}}-2}\|^2 + \Delta_{\cos}^2), \quad (4.22)$$

where all except the last term are the resultant force residuals from Equation (4.2) in Cartesian coordinates and the last term is the residual direction cosine from Equation (4.7). The unknowns are the cylindrical interlacement point coordinates in $\mathbf{x}$ from Equation (4.6). In other words, the NLO calculates the convergence zone geometry corresponding to force equilibrium for the prescribed braid angle. LM varies the values of $\mathbf{x}$ until it has found a minimum where each residual term in Equation (4.22) should vanish.

In this chapter, ALGLIB's [72] implementation of LM is used. This implementation offers a scheme that requires, as the main input for the problem at hand, a function that evaluates the terms in Equation (4.22) as a function of both $\mathbf{x}$ and the known constant process parameters listed in Table 4.1. The derivatives of the terms in Equation (4.22) are not needed as input because these are numerically approximated by this scheme.

In essence, the function that evaluates the terms in Equation (4.22) calculates the following for each iteration's $\mathbf{x}$: First, all interlacement point coordinates are transformed from cylindrical to Cartesian coordinates. Next, the convergence zone geometry is evaluated using Equations (4.10) to (4.21). Finally, the terms of Equation (4.22) are readily evaluated with Equation (4.2) and Equation (4.7).

An NLO generally has $\mathcal{O}(n^3)$ time complexity, where n is the number of unknowns. It can have zero, one or multiple solutions, depending on the applied friction model and the boundary condition magnitudes. The emerging solution depends on the initial guess of the interlacement point positions.

Frontal approach

An approximate solution can also be obtained using a computationally faster 'frontal sweep' for a single interlacement point at a time, starting

Table 4.1 Constant input parameters of the problem, matching the experimental values.

parameter	value
yarn description	Teijin Toho Tenax IMS65 E23 24k carbon
yarn fiber density ρ_f	1780 kg/m ³
yarn linear density $\rho_{l,y}$	830 · 10 ⁻⁶ kg/m
yarn fiber volume fraction $v_{f,y}$	0.7
yarn initial width $w_{y,ini}$	4 · 10 ⁻³ m
spool plane radius r_{sp}	1.382 m
number of yarns per group n_y	72
number of floats n_{float}	2
spool tension F_{sp}	4.7 N
apparent avg. dyn. coeff. of friction μ_{ap}	0.2
machine axis w	(0, 0, -1) m
mandrel radius r_m	75 · 10 ⁻³ m
fell point coordinates a ₀	(r_m , 0, 0) m
braid angle α	60°
fell point tangent f ₀	(0, sin α , cos α) m
yarn group (X or O)	X

at the fell point and progressing through the convergence zone until a virtual interlacement point is found outside of the creating circle radius r_{cc} . The supply point is obtained as the intersection between the last strut and the surface of the creating circle extrusion. The same boundary conditions from the NLO approach apply. As an initial guess, the tension magnitude $F_0 = F_{sp}$ is used at the known fell point **a**₀ with the known fiber direction **f**₀. For the approximation of the normal force N , only the tension at the fell point side is used, and the crimp angle θ of the previous point is used, assuming only a gradual change, replacing Equation (4.9) with

$$N_{i+1} = 2F_i \sin(\theta_i). \quad (4.23)$$

Similarly, instead of calculating the interlacement pitch l_i using two interlacement points in Equation (4.11), using Figure 4.4,

$$l_i = \frac{d_i}{\cos \psi_i}. \quad (4.24)$$

From the fell point to the spool, the next interlacement point is obtained by

$$\mathbf{a}_{i+1} = \mathbf{a}_i + l_i \mathbf{f}_i, \quad (4.25)$$

eliminating Equation (4.7). The evaluation order of the equations at each interlacement point is shown in Figure 4.8 and is traceable using Figure 4.4. The implementation of this evaluation sequence has an $\mathcal{O}(n_{\text{ipt}})$ time complexity, and therefore offers a large computation time reduction compared to the NLO, although obviously slower than the $\mathcal{O}(1)$ time complexity without yarn interaction. There is no need for an initial guess of the interlacement point coordinates and they inherently coincide with the constraint planes. If Coulomb friction is used, then the friction force W scales with the same factor as the tension force F . As a consequence, at each interlacement point the yarn kink angle is independent of the tension magnitude, and, in turn, the yarn geometry and convergence zone length are independent of the spool tension. Given the solution geometry, the corresponding tension distribution can be evaluated as a post-processing step.

Figure 4.8 Simplified flow charts summarizing the function evaluations for each interlacement point in the frontal approach.

4.4 Numerical case study

Both approaches are compared to assess whether the faster frontal approximation yields a solution that is close enough to the more rigorous NLO approach.

A centered cylindrical mandrel is overbraided without the use of guide

Figure 4.9 Bias yarn centerlines in the convergence zone as calculated by the frontal approach. All interlacement points are shown for both yarn groups. Each i th internal interlacement point on a yarn coincides with the i -th interlacement point on a yarn in the opposite group.

rings. The used parameter values are listed in Table 4.1 and correspond as good as practically possible to the physically equivalent experiment described in the next section. The expected coefficient of friction of approximately 0.2 is based on perpendicular tow-to-tow friction measurements from [38, 40] and is primarily used for the numerical comparison. The result is a non-jamming braid and a full mandrel surface coverage, which is usually desired. The resulting yarn centerlines in the convergence zone are shown in Figure 4.9, clearly showing that the yarn curvatures increase toward the fell front. Figures 4.10 and 4.11 represent the positions of the successive interlacement points, showing that for this case the systematic error of the frontal approach is negligible compared to that of the NLO approach. A parametric study resulted in a substantial convergence zone length decrease of 50 mm per 0.1 difference in μ_{ap} , shown in Figure 4.12. Variation of the other parameters resulted in relatively small changes.

Figure 4.10 Interlacement point r -parameter values at their constraint plane angles. The angular increment of φ between two interlacement points is 2.5° .

Figure 4.11 Interlacement point z -parameter values at their constraint plane angles. The angular increment of φ between two interlacement points is 2.5° .

4.5 Experiment

The experimental setup consists of a hot-wire cut styrofoam cylindrical mandrel with its axis coinciding with the braiding machine axis. The mandrel is clamped on a aluminum tube with a bending stiffness high enough to limit the gravity-induced deflection below a millimeter, asserting the axisymmetry assumption. A Eurocarbon 144 carrier machine without guide rings as shown in Figure 4.13 was used with Teijin Toho Tenax IMS65 E23 24k carbon yarn material for both yarn groups. The corresponding values are listed in Table 4.1.

It was made sure that the process was in steady state to avoid non-prescribed transient braid angles [45]. For this purpose, a distance of 500 mm was overbraided and the braid angle was asserted to be

Figure 4.12 Yarn centerline shapes for a variable μ_{ap} . Length units: mm.

60 degrees using a goniometer. It is noted that immediately after stopping the machine, the yarns briefly continue to show a viscous-like yarn motion which is not modeled. Fiber breakage and entanglements occurred at the scale ranging from single fibers to yarns, sometimes leading to situations as shown in Figure 4.14, significantly influencing the yarn geometry in the convergence zone. Besides the entanglement that can be visualized in photographs, the entire convergence zone is permanently covered with a very fine web of detached fibers.

Close-range photogrammetry was used to trace the 3D trajectory of yarns. During this measurement, the yarns were not touched. For this purpose, a tubular frame was built and put around the mandrel to hold coded targets as reference points for the measurement. Using Photomodeler [62], a generic photogrammetry software, ten warp yarn curves were extracted as piecewise linear curves with a negligible measurement error. Close to the fell points, individual yarns segments could not be properly distinguished and were not processed further.

Figure 4.13 *Experimental setup at Eurocarbon. Top right: Fell front with pairwise yarn clustering. The region bounded by the dashed box is magnified in Figure 4.14.*

4.6 Results and discussion

The yarns from Figure 4.12 were transformed to emerge from a single spool as shown in Figure 4.15. Analogously, the experimentally obtained yarn polylines were added to the same view for comparison. In the model and experiment, the yarn curves in the convergence zone are not planar. The yarn curvature is relatively large near the fell front and rapidly decreases towards the spools. The experimental yarns are closest to the modeled yarn with $\mu_{ap} = 0.3$. This value is higher than the value of 0.2 reported in literature and used in the numerical comparison, possibly caused by fiber entanglement. As shown in Figure 4.16, the corresponding average convergence zone decrease, as

Figure 4.14 A magnification of the region indicated in Figure 4.13, showing a cluster of broken fibers, emphasized by the dashed line, and the effect after entanglement with yarns, causing the yarns to kink as indicated by the white arrows. The yarn moving direction is indicated by the black arrows.

compared to $\mu_{ap} = 0$, is around 200 mm, 25% of the convergence length for frictionless conditions. The maximum average difference between this modeled yarn and the experiment, measured in the machine $\mathbf{v}$ -direction, was about 30 mm. Hence, for this case study and experiment, the proposed model is limited to represent the actual braiding process with an accuracy of this order. At the region of the maximum difference, the experimental yarns clearly show a larger curvature than the model in the spool plane projection. This is visualized in more detail in Figures 4.16 and 4.17. No significantly better match was found by a drastic change of model parameter values, including the yarn linear density $\rho_{l,y}$, yarn fiber volume fraction $v_{f,y}$, initial yarn width $w_{y,ini}$, and spool tension F_{sp} . Also a generalization to other yarn cross-sectional geometries and Howell friction [68] using various values did not improve the match.

Detached fibers, accumulated and entangled with broken fibers of yarns of the other yarn group, are not taken into account by the model. It is not clear whether these phenomena always increase friction. Perhaps a decrease occurs as well in certain regimes, e.g. due to caterpillar track-like rolling of fibers. Detailed modeling of these phenomena requires

Figure 4.15 Model yarns (grey) and experiment yarns (black) in the machine coordinate system after transformation to the same spool position. Length units: mm.

Figure 4.16 Machine w -coordinate of the yarns in the model and experiment as a function of the yarn arc length from the spool.

micro-scale fiber interaction, which is computational cost prohibitive in the context of design optimization.

When increasing the radius of an optional guide ring located at a fixed distance from the spool plane and a given mandrel size, the convergence zone length increases as shown by Equation (4.1). This increases the yarn length of the interlacement point slip, in turn increasing the accumulated fiber damage. Simultaneously, the contact angle with the guide ring decreases, decreasing the local capstan friction, in turn locally decreasing fiber damage. Hence, the effect of the creating circle size on μ_{ap} depends on the contributions of both effects.

The possibility of the thicknesses exceeding the width is clearly a limitation of this model. The model does not include resistance of the

Figure 4.17 Machine v -coordinate (not to be confused with the take-up speed v) of the yarns in the model and experiment as a function of the yarn arc length from the spool.

yarns against width reduction, so it is assumed that its error increases for (nearly) jammed braids.

The circular spool movement assumption ignores the actual serpentine spool movement as shown in Figure 4.18 and the effects of the spool carrier ratchet and pawl mechanism described in Section 1.2, with the following consequences: A modeled yarn interlacement point is created as soon as two spools of opposite moving groups pass each other. An actual interlacement point is created later due to the difference in spool radial position. The modeled distance between yarns of the same group in tangential direction is constant. The actual distance, however, alternates due to pairwise yarn clustering as shown in Figure 4.13, caused by violating axisymmetry. This leads to the two distinct groups (each containing warp and weft yarns) of experimental data points in Figure 4.16 and Figure 4.17. All modeled yarns are subjected to the same constant tension. The actual tension fluctuates, optionally influenced by empty carriers. For the fictitious case of a machine radius that is much larger than the mandrel radius and the neglect of ratcheting, the tension waveform would be similar to the spool amplitude waveform. For radial braiding machines, this geometrical fluctuation is almost eliminated. Consequently, the yarn interlacement geometry shown in Figure 4.7 is a fairly coarse approximation.

For centered mandrels, the axisymmetry assumption is violated if the effect of gravity results in a non-axisymmetric yarn displacement distribution. When fictitiously neglecting yarn interaction, the instantaneously

Table 4.2 Example input parameters.

parameter	value
yarn description	Toray T700SC-24K
yarn fiber density ρ_f	1800 kg/m ³
yarn linear density $\rho_{l,y}$	1650 · 10 ⁻⁶ kg/m
yarn fiber volume fraction $v_{f,y}$	0.7
yarn initial width $w_{y,ini}$	5 · 10 ⁻³ m
spool plane radius r_{sp}	1.382 m
number of yarns per group n_y	72
number of floats n_{float}	2
spool tension F_{sp}	3 N
apparent avg. dyn. coeff. of friction μ_{ap}	0.2

The resulting braid angle distribution corresponds to a known fiber direction field (FDF) on the mandrel surface. Next, the yarn shape, convergence zone length, and the yarn payout, i.e. the unwound yarn length, are calculated at a series of points along a fiber streamline (FSL) in the FDF. This calculation is performed without yarn interaction, resulting in straight yarns in the convergence zone as in the previous chapters, and with yarn interaction using the frontal approach from Section 4.3.4. In this example, the calculation without yarn interaction is conveniently carried out using the frontal method as well, setting the friction coefficient μ_{ap} to zero, resulting in straight yarns. The braiding parameters are shown in Table 4.2 and no guide rings are used.

The resulting of the convergence zone length H is shown in Figure 4.19. The values at the start and end sections match the analytical solution from Figure 4.2 for $\mu_{ap} = 0$. The convergence zone length for the reverse braiding direction, starting at section 2, is roughly the same. The predicted convergence zone length is significantly smaller for the model including yarn interaction.

The resulting yarn payout is shown in Figure 4.20 for the scenarios where braiding is started at either section 1 or 2. The shown maximum amounts of rewind yarn length $s_{rew,max}$ represent the predicted amount of yarn slack as explained in Section 2.4.5. The amount of slack in the model including yarn interaction is significantly lower compared to the straight yarn model, so the latter may be too conservative for this case. Although yarn slack seems unavoidable, the amount of slack is minimal when starting at mandrel section 1.

Figure 4.19 The convergence zone length H versus the fell front z -coordinate in the mandrel CS. The braiding process starts at mandrel section 1.

This example shows that the yarn interaction model can help to choose the appropriate braiding direction. However, further experimental validation is desirable, because the yarn interaction model neglects many physical phenomena like stick-slip, yarn bending, and a transverse yarn deformation. Although a series of yarn shape force-equilibrium states can be calculated along an FSL of any axisymmetric mandrel, the model does not predict whether the *non*-steady-state transition (see Appendix A) between any one state to the next, in this case at the varying mandrel radius, is physically feasible.

4.8 Conclusions

To assess the influence of yarn interaction, a yarn interaction model was developed for the axisymmetric biaxial braiding process, implemented with non-linear optimization techniques and a computationally faster frontal approach. Comparison of the two approaches shows no significant difference in the resulting yarn geometry. A parametric study using the frontal approach shows that the result is mainly influenced by the coefficient of friction for the case under consideration.

The main limitations of this yarn interaction model and many other braiding simulation models, including finite element models, are that they do not capture the effect of broken, detached and entangled fibers

Figure 4.20 The yarn payout s_y versus the fell front z -coordinate in the mandrel CS. Braiding is started at either mandrel section 1 or 2, modeling the process either without ($\mu_{app} = 0.0$) or with ($\mu_{app} = 0.2$) yarn interaction.

with a significant effect on the inter-yarn forces. In addition, the yarn interaction model neglects non-axisymmetric features, especially tension fluctuation and pairwise yarn clustering.

Experimental validation of the yarn interaction model using carbon yarns shows that a modeled coefficient of friction value around 0.3 provides the closest match between model and experiment. For this value, both the model and the experiment show a significant convergence zone decrease around 25% with respect to the frictionless model for the studied case. This confirms that yarn interaction significantly influences the convergence zone length. Hence, when generating machine control data for accurate results, neglect of this change in convergence zone length can cause significant braid angle errors.

Chapter 5

Braid patterns*

5.1 Introduction

This chapter focuses on the spool arrangement on the braiding machine and the corresponding braid pattern or ‘topology’ as shown on the path in Figure 5.1. The arrangement of spools over a machine precedes the actual braiding as described in the previous two chapters and involves the combination of the component design and the braiding process. One of the first considerations for a braided composite product is an appropriate fiber distribution, usually of sufficiently high fiber content. A specific fiber structure may be required, based on various product requirements like thermo-mechanical properties, ability to drape, aesthetics, failure mechanisms and braiding process limitations. Like weaving, the interlacing structure of the yarns can be described by a braid pattern. The most common biaxial braid patterns are 2/2 twill, plain and, to a lesser extent [10], 3/3 twill, also known as ‘regular’, ‘diamond’ and ‘Hercules’ braids, respectively, shown in Figure 5.2. The braid pattern is needed to properly set up the braiding machine for production. The regular and diamond braid can be produced on a ‘conventional braiding machine’ [74], having four horns per gear as shown in Figures 1.3 and 5.3. This is the de-facto standard type of circular braiding machine in the composites industry. Similar to weaving and in contrast to filament winding, braiding prevents yarns of the same yarn group, defined in Section 1.2, to overlap each other due to the interlaced structure as shown in Figure 5.2.

*Based on [73].

Figure 5.1 *Design-process-material triangle showing the current chapter underlined.*

For general series-production of braids, all carriers are equipped with a spool. Empty carriers are usually removed to reduce the electrical power required to drive the carriers and to reduce carrier wear. For small series and prototypes of braided preforms, the available machines can have an inappropriate size while the purchase or development of a new machine can be cost-prohibitive. If an under-sized machine is available, this problem is sometimes solved using larger or more strands per spool. For over-sized machines, it is sometimes possible to use it in ‘partial occupation’, either by removing carriers or leaving some carriers empty. The choice depends on what is most economical or practical for the case at hand. Hence, the number of carriers can be less than the maximum possible number of carriers, and the number of spools can be less than the number of installed carriers.

For each yarn group, the arrangement of spools over the carriers is defined as the ‘spool pattern’ for that group. The spool pattern determines the braid pattern, and, combined with the number of carriers on the machine, influences the amount of deposited material on the

Figure 5.2 Common braid patterns represented using the 'canvas method'. Yarns of the same group cannot overlap each other.

mandrel. The machine construction puts a limit on the number of available braid patterns and generally only allows a single braid pattern per braid layer as defined in Section 1.2.

Problem and objective

From the braid manufacturer's point of view, the main problem is to find the correct spool pattern for a product with a required biaxial braid pattern design. An a-priori relation between the spool pattern and the resulting braid pattern can prevent a time consuming trial-and-error of finding the correct spool pattern. During the preparation of a braiding machine for the production of a prescribed braid pattern, an incorrect removal of spools from carriers or the incorrect placement of carriers in the machine can lead to problems like an incorrect braid pattern, unexpected areas with a low degree of coverage, or the absence of interlacement altogether, potentially leading to fabric disintegration upon handling [3, 36, 75]. The objective of this chapter is to address a procedure that evaluates whether a given biaxial braid pattern is feasible on a given braiding machine and to convert bi- and triaxial braid patterns to and from spool patterns.

Figure 5.3 Circular braiding machine at Eurocarbon (left). Initial carrier positions and two example carrier paths (right). Stem yarns emerge from the horn gear centers. Photo courtesy Eurocarbon BV.

Outline

First, earlier work on braid patterns is reviewed. Next, the conversion from a given machine construction including the carrier placement to the resulting biaxial braid pattern is explained, assuming that each carrier is assigned a spool. Furthermore, the influence of spool removal on the resulting biaxial braid pattern is addressed. Subsequently, the reverse procedure is addressed: a spool pattern is searched for a given biaxial braid pattern design. Finally, triaxial braids are analyzed. After the analysis, common braiding machine setups are analyzed using the proposed procedures.

5.2 Earlier work

Only a few works on relating braid patterns to spool patterns have been found.

Kyosev [10] presents a procedure to derive the braid pattern from a

flat or circular braiding machine with a given construction and carrier arrangement and introduces a notation system for braid patterns.

In [3], Kyosev mentions two sources [76, 77], written in German, that treat partial machine occupation, where the former source was written to educate textile technicians. Kyosev treats partial occupation of biaxial braiding machines, based on these German sources. He also observes that a trial-and-error approach, creating various spool patterns to find the resulting braid pattern with software [35], is not effective due to the very large number of possible combinations. From the braider's standpoint, this identifies the need for an automated procedure to transform a braid to a spool pattern.

Nawaz [74] formalized the description of spool removal using a matrix multiplication notation and uses Matlab to show the resulting topology in a user-defined color pattern. The generated patterns topologically match physical experiments. The inverse, generating a spool pattern for a given braid pattern, was not addressed.

5.3 Analysis

A few terms and definitions are needed for the analysis. Like weaving, braiding produces periodic fabrics. The smallest repetitive element of a periodic fabric is defined as a 'repeating unit cell' or simply 'repeat'. The repeat size is defined by the number of warp and weft yarns it contains. Using the 'canvas method', one or multiple weave repeats are visualized as a matrix as illustrated in Figure 5.2. A canvas matrix element represents the intersection between a warp and a weft yarn. In general, a given spool pattern results in an interlaced braid, so such an intersection corresponds to an 'interlacement point'. However, spool patterns exist that result in a complete loss of interlacement, in which case an intersection corresponds to a crossing instead of an interlacement point. A drawn yarn segment, possibly spanning multiple matrix elements, is called a 'top float'. The bands running in the braid axial direction, shown in Figure 5.2, are called 'wales' [6] or 'ridges' [10]. Although the yarn groups are drawn perpendicularly, the angle can vary from near-zero up to almost 180°. However, the used angle is irrelevant for describing the braid topology.

Assume, for axial and radial circular braiding machines as described in Section 1.2, including but not limited to conventional braiding machines as defined in Section 5.1, that

- the horn gears are assembled in a single closed loop,
- the number of horn gears is even, and
- the carriers do not collide when switching horn gear.

For arbitrary circular braiding machines, the collision assumption can be checked using specialized software for arbitrary braiding machine designs [35]. The number of warp and weft group carriers is $n_{c,X}$ and $n_{c,O}$, respectively. Generally, $n_{c,O} = n_{c,X}$ as in Figure 5.3, and this assumption is used for explanation purposes, dropping the subscripts X and O, and denoting the number of carriers of each group by n_c . However, all procedures are also applicable if $n_{c,O} \neq n_{c,X}$. Usually n_c increases with the machine dimensions, but machines designed to carry hundreds of carriers can also be equipped with just a few. Hence, n_c is also used to denote the ‘machine size’ and vice versa.

Define a braiding machine as ‘full’ if it contains the maximum possible number of carriers and all carriers are ‘full’, meaning that a spool has been assigned to each.

In this work, the distribution of the carriers over the slots is assumed to be given. This assumption simplifies the more generic problem of searching an appropriate distribution of the carriers over the slots that enables a desired braid pattern without carrier collisions. For further simplification, the removal of a spool from a carrier and the removal of a carrier from the braiding machine are both represented by the removal of a spool from a carrier.

For each yarn group, the carriers are numbered in the order of passing horn gear 1 as shown in Figure 5.3, irrespective of whether it has a spool. The numbering is independent of the amount of empty slots between two successive carriers of the same yarn group.

Define a ‘machine repeating unit cell’ (machine RUC) as the smallest repetitive set of horn gears including the maximum number of carriers in the horn gear slots as shown in Figure 5.4. The following parameters are specified for a machine RUC: the number of horn gears n_g , the number of slots per horn gear n_{sl} , the slots where the carriers are initially located, and the maximum number of carriers for each yarn group. The number of spools depends on the number of empty carriers. By convention, in

Figure 5.4 Examples of machine RUCs: a) Conventional braiding machine with carriers in 'regular formation'. b) Idem in 'tandem formation'. c) Six-slot horn gear machine with carriers in 'Hercules formation'. d) Idem in 'tandem formation'.

each machine RUC drawing, the warp ('X') carriers move globally to the right and the weft ('O') carriers to the left. The machine RUC shown in Figure 5.4a corresponds to a conventional braiding machine. Enough slots must be left empty to allow simultaneous inter-horn gear transfer of the carriers without collisions. However, this is not necessarily every other slot. Machine RUCs have their carriers in a unique arrangement called a 'formation' as shown in Figure 5.4.

A machine RUC is mapped to the corresponding braid pattern using a procedure called 'carrier tracking'. This procedure generalizes an existing approach from [10], and assumes a full braiding machine, so each carrier is equipped with a spool. The concept of carrier tracking is explained using the machine RUC of Figure 5.4a and with Figure 5.5. For this purpose, define n_{rev} as the number of horn gear revolutions required to move a carrier through the period of one machine RUC,

$$n_{\text{rev}} = \frac{n_g}{2} \quad (5.1)$$

with n_g as the number of horn gears in the machine RUC. Carriers of opposite groups can pass each other in the machine circumferential direction. For example, in step 3 of Figure 5.5, each carrier is passing a carrier of the opposite group. Note that in general, depending on the carrier arrangement, only a subset of all carriers may pass each other at any time. All possible states where carriers pass each other must be taken into account for any number of slots per horn gear n_{sl} to provide a generic approach. When assuming n_{sl} as even for explanation purposes, then a horn gear rotation increment $\Delta\varphi$ of *half* the central angle between two adjacent slots is required to account for all pass instances as shown in Figure 5.5. In that case, the resulting number of increments per horn

Figure 5.5 Example of converting the machine RUC from Figure 5.4a with $n_g = 2$ and $n_{sl} = 4$ to a scattered canvas matrix. This matrix is readily transformed to the braid canvas matrix in Figure 5.4a by a counter-clockwise rotation of 135° and the subsequent removal of empty rows and columns.

gear revolution $n_{incr/rev}$ follows from

$$n_{incr/rev} = 2n_{sl}. \quad (5.2)$$

with n_{sl} as the number of slots per horn gear. The number of increments n_{incr} for a period of one machine RUC is

$$n_{incr} = n_{rev} \cdot n_{incr/rev}. \quad (5.3)$$

Substituting the right-hand side of Equation (5.3) with Equations (5.1)

and (5.2), for n_{sl} as even,

$$n_{incr} = n_g \cdot n_{sl}. \quad (5.4)$$

A square ‘scattered canvas matrix’ shown in Figure 5.5 with n_{incr} rows and columns is needed to represent all possible relative circumferential carrier positions at all possible pass instances that occur while traversing one machine RUC.

After each incremental rotation $\Delta\varphi$, the machine RUC state is evaluated. At each pass instance, the radially outward yarns are added to the corresponding row as top floats using their relative position. These top floats appear at the outside of both the machine and the produced braid, assuming forward braiding for the latter.

Note that the scattered canvas matrix can contain multiple braid repeats. Although carrier tracking results in a relatively large amount of empty scattered canvas matrix elements (hence its name), which can be redundant for simple cases, it is generically applicable and accommodates horn gears other than four slots per horn gear just as well.

After completing a full period of the machine RUC, the scattered canvas matrix is transformed from the braid coordinate system to the carrier ID coordinate system, both shown in Figure 5.5. Visually, this amounts to a counter-clockwise rotation of 135° of the canvas. The result, after removal of empty rows and columns, is shown in the carrier ID coordinate system in Figure 5.6a.

Define a ‘machine braid repeat’ as the repeat of a braid produced on a full circular braiding machine. All canvases shown in Figure 5.6 are machine braid repeats because they are the result of the carrier tracking procedure, in turn using a full machine as the key assumption.

The topology of a machine braid repeat is identical to the corresponding scattered canvas matrix, but generally with a different braid repeat shape and size as shown when comparing Figure 5.6a with Figure 5.5.

The machine braid repeats from Figure 5.6 show that common names like ‘regular’, ‘plain’ and ‘Hercules’ are not sufficient to accurately describe the full range of possible braid patterns. This also holds for weaving-derived names like a ‘plain weave’, which could non-uniquely map to both Figure 5.6a and d. A more generic system of notation for braid patterns is proposed by Kyosev [10]. This notation is structured as X:X-Y, to be read as ‘over X groups : under X groups - as groups of Y yarns’.

Figure 5.6 Examples of machine braid repeats that are represented using the canvas method, corresponding to the machine RUCs from Figure 5.4, and obtained using the carrier tracking procedure. The shown pattern names follow the conventions of Kyosev [10].

Here, a ‘group’ is shorthand for the ‘float group size’, indicating the amount of yarns that follow the same topological path. This should not be confused with ‘yarn groups’ defined in Section 1.2. Examples of this naming convention are shown in Figure 5.6.

5.3.1 Biaxial spool pattern to braid pattern

The removal of a spool from a full machine corresponds to removing its yarn, in turn corresponding to its row or column, from the corresponding canvas. This can result in a different braid pattern with a different repeat size. An example of this ‘spool removal procedure’ is given in Figure 5.7. The canvas representing the full machine can contain an arbitrary repetition of machine braid repeats. For simple and periodic spool removal as in Figure 5.7, a single machine braid repeat can be sufficient to globally represent spool removal. For more complex removals, a larger canvas is required. This procedure can be readily executed on a general purpose spreadsheet without specialized software.

Define a ‘spool pattern’ to specify the spools to be removed. For the warp and weft groups, a spool pattern is represented by a ‘bit sequence’ (b_X) and (b_O), respectively, of length $n_{\text{bit},O}$ and $n_{\text{bit},X}$, indicating empty carriers with ‘0’ and full carriers with ‘1’, starting at carrier ID 1. The values of a sequence instance are enclosed in square brackets and are not separated by commas, for example [10] for a full carrier followed by an empty one. The distinction from literature references can be made using the context.

Figure 5.7 Example spool removal from a machine and its canvas. Lower left: Four machine braid repeats redundantly represent the full machine for explanation purposes. Middle: Spools are removed from carriers. This is represented by removal of their corresponding rows and columns. Right: A plain weave or ‘diamond braid’ after matrix compaction.

The term ‘spool pattern’ is also used as shorthand for the set of spool patterns for all yarn groups. To simplify notation, the yarn group subscript (X or O) is dropped if both yarn groups have the same spool pattern: $(b_O) = (b_X) = (b)$ and $n_{bit,O} = n_{bit,X} = n_{bit}$. A spool pattern can be ‘normalized’, describing it by the smallest repetitive bit subsequence. In the example of Figure 5.7, after spool removal, the spool pattern for both yarn groups is $[10101010]$ and can be normalized to $[10]$ because the latter is the smallest repetitive bit subsequence of the former. For all machine braid repeats, by definition using full machines, each carrier contains a spool. Hence, for the canvas matrices shown in Figure 5.6, all spool patterns are $[1]$.

For the number of spools n_{sp} in a group,

$$n_{\text{sp}} = \frac{n_{\text{c}}}{n_{\text{bit}}} \sum_{i=1}^{n_{\text{bit}}} b_i \mid n_{\text{c}} = k \cdot n_{\text{bit}}, k \in \{1, 2, 3, \dots\} \quad (5.5)$$

with b_i as an element in (b) . For an example including the most important symbols used so far, refer to Figure 5.7. Equation (5.5) requires that n_{bit} must be a divisor of n_{c} to be ‘compatible’ with that machine size. Consequently, $n_{\text{bit}} \leq n_{\text{c}}$. The bias yarn groups usually have the same number of spools, so $n_{\text{sp},\text{O}} = n_{\text{sp},\text{X}}$, to avoid braid ‘twisting effects’ as mentioned in [3], leading to an oval shape of the braid cross-section that rotates around the braid axis along the braid length as observed when braiding without a mandrel. So given a spool pattern as input, the bias braid pattern is obtained using the spool removal procedure for each empty spool in each yarn group.

In the example of Figure 5.8, the spool removal results in the loss of all interlacement, theoretically resulting in a ‘non-crimp fabric’ (NCF), where ‘crimp’ refers to the out-of-plane yarn waviness related to interlacement. Hence, all points in the shown canvas matrix are yarn crossings instead of interlacement points. This example shows that the canvas method can be used to provide the resulting number of layers, despite the loss of interlacement.

In practice, the machine operator can perform the spool removal procedure for one bias yarn group first. For the other bias yarn group, the spool removal must be started at the correct carrier relative to the starting point of the removal of the first group, otherwise an incorrect braid pattern may result. Generally, the reason is that the braid pattern is not invariant to spool bit pattern ‘rotation’. This can be understood by examining the examples in Figure 5.9. The NCF at the top right of this figure is identified as such analogously to the example in Figure 5.8. At the bottom left, a ‘basket weave’-like structure exists before rotation and another structure emerges after rotation with a constant and an alternating number of yarns per float, respectively.

5.3.2 Biaxial braid pattern to spool pattern

For design purposes, usually a specific braid pattern is required, represented by **B**. Assume that the yarn material is also prescribed, as well

Figure 5.8 Example spool removals applied to a single machine braid repeat of Figure 5.6a, resulting in a four-layered NCF.

as the required number of yarns n_y in each of the warp and weft yarn groups, where n_y must be large enough to fully cover the mandrel surface for a high fiber content. Assume, for explanation purposes, that n_y is the same for both yarn groups.

Let a braiding machine be available, characterized in full occupation by a machine braid repeat denoted by $\mathbf{M}$. Again, assume that both yarn groups have the same number of carriers, so $n_{c,O} = n_{c,X} = n_c$. If $\mathbf{B}$ equals $\mathbf{M}$, then the required braid pattern can be manufactured on the machine. However, if $n_y < n_c$, so the mandrel is relatively small to be produced on the full machine, then there is a chance that the machine can still be used to produce the desired pattern in partial occupation, realized by the spool removal procedure from Section 5.3.1.

The search for the optimum spool pattern must take the following given constraints into account:

- the required braid repeat $\mathbf{B}$,
- the available full machine braid repeat $\mathbf{M}$,
- the available machine size characterized by n_c , and
- the required number of yarns n_y per yarn group.

Figure 5.9 Top: Example of the effect of bit pattern rotation on braid repeats, applied to the machine braid repeat of Figure 5.6a. The left side shows a plain weave repeat. Rotation of (b_O) while keeping (b_X) constant yields a three-layered NCF repeat shown at the right. Bottom: Using Figure 5.6d and the same spool pattern for both groups.

A theoretical brute-force search is described next. Let the full machine canvas matrix $\mathbf{M}^*$ be sized $n_c \times n_c$ with $\mathbf{M}$ as its repeating unit to represent all carriers in the machine. This size is for explanation purposes only: for most analyses and implementations, a single machine braid repeat suffices.

First, a search in $\mathbf{M}^*$ selects all candidate spool patterns that result in the braid topology of $\mathbf{B}$. For all possible spool patterns, $n_{\text{bit}} = n_c$, so it is specified whether a spool is present on each carrier on the machine. The number of possible spool patterns per yarn group to be evaluated in this search grows exponentially with the machine size: 2^{n_c} . Typical composite braiding machines have n_c in the order of 72, so a brute-force implementation is computationally prohibitive, hence the ‘theoretical’ nature of this approach.

Next, only those spool patterns are selected from the candidates whose number of spools n_{sp} best matches the required number of yarns per group n_y . This eliminates spool patterns that result in the deposition of too little or too much material around the mandrel circumference. Application of this criterion may still result in multiple candidate

Table 5.1 Examples of unspecified input criteria, indicated by '?', resulting in multiple spool pattern solutions, using the canvas from Figure 5.6a for $\mathbf{M}$. *Denoted in normalized form for brevity.

n_c	n_y	braid pattern	distribution	output spool patterns* (b)
72	24	2/2 twill	?	[100]; [110000]; ...
72	36	?	?	[10] (plain); [1100] (NCF); ...
72	?	2/2 twill	?	[1] ($n_{sp} = 72$); [100] ($n_{sp} = 24$); ...

Figure 5.10 Two valid spool pattern solutions (of many) for the first example in Table 5.1. Left: Homogeneous distribution, or (b) = [100]. Right: Clustered distribution, (b) = [110000].

solutions as shown in the first example from Table 5.1, also shown in Figure 5.10.

Subsequently, a criterion is introduced to select the spool patterns with the most homogeneous distribution of spools over the carriers. A non-homogeneous distribution of the spools over the carriers can result in a 'clustered spool pattern' as shown in the right spool pattern of Figure 5.10, possibly leading to an inhomogeneous yarn distribution and uncovered mandrel regions. Still, multiple solutions may exist, although less likely.

Finally, an arbitrary solution can be chosen from the remaining set. Missing selection criteria can result in multiple spool pattern solutions as shown in Table 5.1.

Acceptable spool pattern calculation times can be achieved by modification of the theoretical procedure above. For this purpose, the search in $\mathbf{M}^*$ is reduced by evaluating only a small subset of all possible spool patterns. The disadvantage of this 'practical' approach is the potential discard of more optimal or even all spool pattern solutions, depending on the number of evaluated spool patterns. To alleviate this disadvantage, the algorithm in the next section determines whether $\mathbf{B}$ is feasible on $\mathbf{M}^*$.

Figure 5.11 A canvas matrix, represented at the right with the value 0 for a warp (X) and 1 for a weft (O) yarn as top float.

Braid feasibility algorithm

In this section, a novel ‘braid feasibility algorithm’ is proposed to determine the feasibility of a braid pattern and, when feasible, to return a solution spool pattern for a given machine braid repeat without taking into account the machine size n_c and the braided component. This can also be used as a computationally cheap pre-processing step to detect whether a solution exists at all, optionally followed by a more extensive search for candidate spool pattern solutions as described previously.

For numerical implementation, let canvas matrices **B** and **M** contain the value 0 for a warp and 1 for a weft yarn as top float, as illustrated in Figure 5.11. For simplicity, assume that

- each repeat contains at least one 0 and one 1,
- each repeat is square, and
- the diagonal of **B** corresponds to the component axial direction.

For non-square repeats, a square matrix can always be obtained by concatenation (stacking) of repeats in both dimensions. If **B** contains wales that are intended to run in the component axial direction, then these wales should run along the matrix diagonal, i.e. from the lower left to the upper right, to conform the machine braid repeats like that of Figure 5.6a.

Denote the sizes of **B** and **M** with $n_B \times n_B$ and $n_M \times n_M$, respectively. The algorithm for checking the feasibility of **B** on a machine represented by **M** is outlined in Algorithm 1 and illustrated with an example in Figure 5.12. The algorithm takes **B** and **M** as input and outputs feasibility and, when feasible, one arbitrary spool pattern solution. In Figure 5.12, the required braid **B** is a diamond braid and the machine braid repeat **M** is that of a conventional braiding machine with the carriers in the regular formation.

Algorithm 1 Braid feasibility algorithm. Convert a braid pattern $\mathbf{B}$ to carrier ID sequences (s_O) and (s_X) , given $\mathbf{M}$. The symbol ' $\leftarrow$ ' denotes a value assignment.

```

1: function CONVERT( $\mathbf{B}, \mathbf{M}$ )
2:    $(s_{O,1}, s_{X,1}) \leftarrow (0, 0)$  ▷ 1st diag. el. of  $\mathbf{B}$ 
3:    $\forall p, q \in \{1, \dots, n_M\}$ 
4:     if  $\mathbf{M}_{pq} = \mathbf{B}_{11}$  then
5:        $(s_{O,1}, s_{X,1}) \leftarrow (p, q)$ 
6:       goto line 7
7:   if  $(s_{O,1}, s_{X,1}) == (0, 0)$  then return no solution
8:    $\forall d \in \{2 \dots n_B\}$  ▷ 2nd, 3rd, ... diag. els. of  $\mathbf{B}$ 
9:      $C \leftarrow \emptyset$  ▷ reset
10:     $\forall \Delta p, \Delta q \in \{1, \dots, n_M\}$ 
11:       $p \leftarrow s_{O,d-1} + \Delta p$ 
12:       $q \leftarrow s_{X,d-1} + \Delta q$ 
13:      if  $\mathbf{M}_{pq} == \mathbf{B}_{dd} \wedge$ 
14:         $\forall r \in \{1, \dots, d-1\}, \mathbf{M}_{s_{O,r} q} == \mathbf{B}_{rd} \wedge$ 
15:         $\forall c \in \{1, \dots, d-1\}, \mathbf{M}_{p s_{X,c}} == \mathbf{B}_{dc}$  then
16:         $C \leftarrow C \cup \{(p, q)\}$ 
17:      if  $C == \emptyset$  then return no solution
18:       $(s_{O,d}, s_{X,d}) \leftarrow f(C)$  ▷ see Equation (5.6)
19:  return  $(s_O)$  and  $(s_X)$ 

```

Matrix index notation is used to refer to the canvas matrix elements. Subscripts r and c index $\mathbf{B}$ and subscripts p and q index $\mathbf{M}$. Subscript d iterates over the diagonal elements in $\mathbf{B}$. Define (s_O) as the sequence of full weft carrier IDs with length n_B . Each sequence element $s_{O,i}$ maps the yarn at the i -th row in $\mathbf{B}$ to a row representing a carrier in $\mathbf{M}$. Analogously, (s_X) maps the columns. For each diagonal element of $\mathbf{B}$, valid candidate elements in $\mathbf{M}$ are identified in a region limited to the size of $\mathbf{M}$. A larger search space is redundant due to the repetition of the machine braid repeat. Let p and q be shorthand for $p \bmod n_M$ and $q \bmod n_M$, respectively, to keep them within the bounds of $\mathbf{M}$.

The candidate elements are temporarily stored in the initially empty set C using 'ordered pairs' containing the element indices (p, q) . Function f selects the 'best' element from C . The best element corresponds to the

Figure 5.12 Example of Algorithm 1 to convert a diamond braid with $n_B = 2$ to spool ID sequences on a conventional braiding machine with the carriers in the regular formation and $n_M = 4$. Four repeats of **M** are shown for explanation. This gives $(s_O) = (1, 3)$ and $(s_X) = (2, 4)$. Next, Equation (5.7) gives $(b_{O,n}) = [1010]$ and $(b_{X,n}) = [0101]$.

most homogeneous distribution of spools over the carriers, obtained by

$$f(C) = \operatorname{argmin}_{(p,q) \in C} |(p - s_{O,1}) - (q - s_{X,1})|. \quad (5.6)$$

This selects the candidate element with the closest distance to the diagonal through the first element $(s_{O,1}, s_{X,1})$ in **M**. If there are still multiple solutions, then the first encountered element with a minimum distance to the origin of **M** is arbitrarily selected. If Algorithm 1 has a solution, then the required braid is feasible. If not, then there exists no spool pattern on the given machine that results in the required braid.

Define $\operatorname{Im}((s))$ as the image of the carrier ID sequence (s) . If a solution is found, then each carrier ID sequence (s) is converted to a spool pattern bit sequence (b) , padded with trailing zeros, using

$$b_i : 1 \leq i \leq n_{\text{bit}} = \begin{cases} 1, & \text{if } i \in \operatorname{Im}((s)) \\ 0, & \text{otherwise} \end{cases} \quad (5.7)$$

with n_{bit} as a multiple of n_M ,

$$n_{\text{bit}} = \left\lceil \frac{\max(\text{Im}((s)))}{n_M} \right\rceil n_M \quad (5.8)$$

with $\lceil \dots \rceil$ as the ‘ceiling’ function, i.e. the smallest integer greater than or equal to the argument. The padding asserts the correct braid pattern after concatenating multiple instances of the solution spool pattern. The generated spool pattern is only compatible with machines sizes that satisfy the constraint used in Equation (5.5): $n_c = k \cdot n_{\text{bit}}, k \in \{1, 2, 3, \dots\}$. When the generated spool pattern is applied to a machine of a given size n_c that satisfies this constraint, then the resulting number of spools n_{sp} may mismatch the required number of yarns n_y from the component design because the search takes n_c nor n_y into account.

5.3.3 Triaxial spool pattern to braid pattern

For triaxial braids, the stem yarn group is added to the bias yarn groups, increasing the pattern complexity. The canvas representation can be used again due to its simplicity for visualization and implementation. This is achieved by turning the biaxial canvas pattern by 45 degrees and adding the stem yarn intersections, resulting in the ‘triaxial braid topology matrix’ $\mathbf{T}$ as illustrated by the ‘regular triaxial braid’ shown in the example of Figure 5.13. The horizontal and vertical axes correspond to the mandrel axis and mandrel circumference, respectively. Each matrix element contains zero or one intra-yarn-group intersection.

By convention, $\mathbf{T}$ represents the full machine, so it contains all yarns from the maximum number of spools and it is sized $4n_c \times 4n_c$. As a consequence, $\mathbf{T}$ has the following properties: it contains all intersections for a single braid period around the mandrel circumference, it can be traversed as a toroidal graph in both directions as shown in Figure 5.13, and for each combination of a warp and a weft yarn, it has one intersection in both the left and the right half.

By convention, the braid is created from left to right. The initial position of the fell front as defined in Section 1.2 is represented by any curve through $\mathbf{T}$ in circumferential direction. For numerical implementation purposes, this curve is most conveniently taken to be between the last

Figure 5.13 Left: A triaxial braid from a conventional braiding machine with the carriers in the regular formation with yarn centerline intersection points. Right: A triaxial braid topology matrix T . The curved arrows illustrate the concept of 'toroidal traversal', provided that T represents the full machine.

Figure 5.14 The two types of yarn interaction at bias intersections in Figure 5.13

and first column, i.e. not coinciding, but just before the first series of intersections. However, the positions of the stem yarn intersections relative to others in T do not necessarily correspond to their physical equivalents. Stem yarns may be 'free' to move in transverse direction 'through' bias yarn intersections as shown in the left case of Figure 5.14, changing the positions of the stem yarn intersections relative to those of the bias yarns. Other bias yarn intersections can bound the position of a stem yarn, shown in the right case of Figure 5.14. As a consequence, the repeat shown in Figure 5.13 is not unique to describe the triaxial topology. Two alternative representations, obtained after moving the stem yarns relative to the bias yarns, are shown in Figure 5.15. The actual position of the stem yarns relative to the bias yarns depends on the complex interaction between all yarn groups, in turn depending, among others, on the yarn structural and frictional properties, the applied spool tension per yarn group, the local mandrel geometry where the yarns are deposited,

Figure 5.15 Topologically equivalent canvas matrix representations of the repeat shown in Figure 5.13. The dotted line at the start of the arrows shows the stem yarn position before shifting.

and the path that the mandrel follows through the braiding machine. Nonetheless, all bounding bias intersections are given for each stem yarn by **T**. For the bias braid patterns like the regular or the Hercules patterns, these bounds in turn define the tunnels through which the stem yarns run. This enables the calculation of the stem yarn centerlines over the mandrel as a function of the deposited bias yarns.

The spools with the stem yarns are mounted on creels as shown in Figure 1.4 that can be used in partial occupation, just like the bias yarn groups. Removal of spools again corresponds to removing the corresponding series of intersections, either running diagonally for bias yarns or horizontal for stem yarns. Subsequently, the matrix can be compacted by the removal of the appropriate rows and columns, similar to the matrix compaction in Section 5.3.1. As for biaxial braids, the resulting pattern can be sensitive to bit pattern rotation, and certain combinations of bias and stem spool patterns result in the total loss of bias bounds for stem yarns, theoretically resulting in a triaxial NCF.

5.3.4 Triaxial braid pattern to spool pattern

Triaxial braids usually correspond to the structure shown in the left half of Figure 5.13 or a similar structure based on the Hercules bias braid pattern. Other triaxial braid patterns are not used in industrial practice to the best knowledge of the author. A possible reason for this is that other bias braid patterns like the 1 : 1 do not provide the tunnels to embed the stem yarns like the 2 : 2 and the 3 : 3 do, resulting in a lower degree of surface coverage which is usually undesired. Hence, the conversion from

other triaxial braid pattern to spool pattern is outside the scope of this work.

For mandrels that are relatively small for a given machine, it is often desirable that each stem yarn still runs through its own tunnel such that the structure like that in the left half of Figure 5.14 is retained. In practice, this means that the same fraction of spools is to be removed for all three groups. However, an arbitrary removal of stem spools may not result in this desired pattern. Hence, a search for an appropriate spool pattern is necessary using the spool removal procedure. However, there is a key difference with respect to the biaxial search: for each candidate triaxial spool pattern, it should be checked if the corresponding *candidate* triaxial braid pattern matches *any* topological equivalent of the *desired* triaxial pattern repeat to avoid discarding valid solutions. For example, each equivalent in Figure 5.15 should be checked instead of only that of Figure 5.13.

5.4 Results and discussion

5.4.1 Spool pattern to braid pattern

Given the procedures for translating between biaxial spool patterns and braid patterns, it is now possible to assess how flexible common braiding machines are to produce a specific braid pattern in either a full or partial occupation for large or small mandrels, respectively. For this purpose, the effect of spool removal for a range of spool patterns is tabulated. Although these tables are not meant to be exhaustive, braiding software can use similar yet larger tables to find the best machine setup. Alternatively, a machine operator can consult these tables directly to find the correct spool pattern for a given overbraided component design.

The spool removal procedure is applied for various spool patterns on a conventional braiding machine having four slots per horn gear and the less common braiding machine with six horns per gear, assuming the same spool pattern for both yarn groups. This results in the braid patterns shown in Tables 5.2 and 5.3. It is possible that the tabulated braid pattern has a variable float group size as defined in Section 5.3 and as is the case in the bottom right of Figure 5.9 and in Figure C.2. In these cases, the pattern is too complex for the used system of notation and is named ‘other’. Entries within parentheses do not correspond directly to

Table 5.2 *Biaxial spool pattern examples on a conventional braiding machine with the carriers in the ‘regular’ and ‘tandem’ formation, using the machine braid repeats from Figure 5.6a and -b, respectively. Subscripts refer to figure numbers. NCFs are suffixed by the number of layers.*

spool pattern	regular formation braid pattern	tandem formation braid pattern	material fraction
[1]	2:2-1 _{5.6a}	1:1-2 _{5.6b}	100%
[10]	1:1-1 _{5.7}	1:1-1	50%
[100]	2:2-1	1:1-2	33%
[1000]	NCF-2 _{C.1a}	NCF-2	25%
[110]	other	other	67%
[1100]	NCF-4 _{5.8}	NCF-2 (NCF-3; 1:1-1)	50%
[11000]	2:2-2 _{C.3a}	1:1-4	40%
[1110]	other	other	75%
[11100]	other	other	60%
[111000]	other _{C.1b}	1:1-3	50%
[11110]	other	other	80%
[111100]	other	1:1-4 (other)	67%
[1111000]	other	other	57%

the given spool pattern, but apply after a spool pattern bit rotation for one or both spool groups. The tables also show the material fraction left after spool removal. This can be used to evaluate if smaller parts can be produced on larger machines.

5.4.2 Spool pattern search examples

From Tables 5.2 and 5.3, it appears that both regular and Hercules braids are not feasible on machines in tandem formation. Algorithm 1 shows that this is indeed impossible. Analogously, it can be shown that a Hercules braid is not feasible on a conventional braiding machine, i.e. a machine with four slots per gear, either with the carriers in the regular or the tandem formation.

An interlacement structure used often in composites engineering for their drapability is ‘satin’ as shown in Figure 5.16. For all machine braid repeats in this chapter, including those with the carriers in tandem formation, Algorithm 1 shows that a 4-harness satin is not feasible. However, this pattern is feasible on a lace braiding machine. Such

Table 5.3 *Biaxial spool pattern examples on a braiding machine with six slots per horn gear and carriers in ‘Hercules’ and ‘tandem’ formation, using the machine braid repeats from Figure 5.6c and -d, respectively. Subscripts refer to figure numbers. NCFs are suffixed by the number of layers.*

spool pattern	Hercules formation braid pattern	tandem formation braid pattern	material fraction
[1]	3:3-1 _{5.6c}	1:1-3 _{5.6d}	100%
[10]	other	other	50%
[100]	1:1-1	1:1-1	33%
[1000]	other	other	25%
[10000]	3:3-1	1:1-3	20%
[100000]	NCF-2	NCF-2	17%
[110]	2:2-1 (1:1-2)	1:1-2	67%
[1100]	other	other	50%
[11000]	other	other	40%
[110000]	NCF-4 (NCF-2)	NCF-2 (NCF-3; 1:1-1)	33%
[1100000]	3:3-2 _{C.3b}	1:1-6	29%
[1110]	other	other _{C.2}	75%
[11100]	other	other	60%
[111000]	NCF-6 (other)	NCF-2 (NCF-3; other)	50%
[1110000]	other	other	43%
[11100000]	other	other	38%
[11110]	other	other	80%
[111100]	other	other _{5.9} (1:1-2)	67%
[1111000]	other	other	57%
[11110000]	other	other	50%

machines can produce continuously variable patterns in both the braid axial and circumferential direction as mentioned in [2], but this is outside the scope of this work.

Suppose that a triaxial braid as shown in the left of Figure 5.13 is desired for a component that is relatively small for a given braiding machine. Assume, for explanation purposes, that [100] is used as the spool pattern for both bias yarn groups as listed in Table 5.2, resulting in a regular braid pattern. Next, stem spool removal shows that two solutions exist for the same fraction of stem yarns: [100] and [001], corresponding to the middle and the left canvas in Figure 5.15, respectively. Hence, the flexibility of using a machine in partial occupation for biaxial braids can also apply to triaxial braids.

Figure 5.16 A 4-harness satin.

5.5 Conclusions

In practice, a machine must sometimes be used in partial occupation because a small product is braided on a relatively large machine, or because the full machine does not result in the desired braid pattern. For this purpose, spools or carriers need to be removed from a braiding machine. The arbitrary removal of spools can result in the incorrect braid pattern and possibly the total loss of interlacement. An incorrect removal includes the incorrect carrier at which the removal is started, although the sequence of empty and full carriers is correct for each individual yarn group. Given a range of available braiding machines, the desired braid pattern may not even be feasible at all. However, it can be very difficult to judge this based on a visual inspection of a machine. The physical and virtual trial-and-error required to find an answer for the feasibility may be time- or cost-prohibitive.

To minimize the waste of time, material, and frustration that result from trial and error, the relation between braid patterns and spool patterns has been analyzed. First, a generalized 'carrier tracking' approach is proposed to convert the construction of an arbitrary circular braiding machine to a biaxial braid pattern. Next, a 'spool removal procedure' is presented to convert a bi- or triaxial spool pattern to a braid pattern. Subsequently, this procedure is used in both a theoretical and a practical search to find the bias spool pattern that results in the desired biaxial braid pattern. This includes a novel braid feasibility algorithm to check the feasibility of a biaxial braid pattern for a given circular braiding machine, generating a corresponding spool pattern if feasible. Finally, the principles for a biaxial braid are applied to triaxial braids as well.

As a result, the translation between biaxial spool and braid patterns is tabulated for a range of spool patterns on commonly used circular braiding machines with four and six slots per gear. These tables can be used directly in practice, and similar but larger tables can be used in

braiding simulation software. The braid feasibility algorithm was applied to confirm that certain combinations of machines and braid patterns are not feasible, preventing the need for further trial-and-error.

Chapter 6

Meso geometry of biaxial and triaxial braids

6.1 Introduction

This chapter focuses on the braided material meso geometry as shown on the path in Figure 6.1. The main input braid parameters that locally fully define the braid geometry on the meso-scale are shown in Figure 6.2. The yarn pitch and direction are an output of *any* braiding process model, including analytic, kinematic, and finite element analyses. In this work, they are obtained as an output from the kinematic braiding process model from Chapter 2, using the area of a facet introduced in Section 2.3 to define the ‘local’ scale.

A braid can contain either a single or multiple yarn materials per yarn group such as the combination of carbon and glass as shown in Figure 6.3, for example for increased toughness and tailored thermal properties as used in the automotive [6, 16, 78] and aerospace industry [22]. Such braids containing two different materials are called ‘hybrid braids’ and make an analysis of the resulting braid more complex compared to ‘mono-material’ braids containing only a single yarn material per yarn group.

For both theoretical and practical downstream process steps, the component thickness distribution is key. The thickness is needed for sizing the resin injection mold shown schematically in the process chain in Figure 1.9. An undersized cavity can require a large force to insert the

Figure 6.1 Design-process-material triangle showing the current chapter underlined.

preform, possibly resulting in braid distortions due to the shear between the preform and the mold, or even damage to the preform or the mold. On the other hand, an oversized cavity can result in a resin-rich outer surface of the component, in turn increasing weight, partly defying the usual purpose of replacing metals by composites. The latter problem can sometimes be partly overcome using an inflatable elastomer bladder as inner mold [30] that pushes the preform against the outer mold. However, for biaxial braids, a braid diameter increase can only be accomplished with a simultaneous reduction in component length. For triaxial braids, a diameter increase is difficult to achieve because the stem yarns counteract the lengthwise change [79].

The thickness is also key for building a model for downstream structural analysis [80]. A thickness under or over-estimation can result in structural performance under or over-estimations, respectively. Usually a uniform or only gradually changing thickness is desired at a full mandrel surface coverage to minimize stress concentrations, realized by a ‘closed-mesh’ braid shown in Figure 6.4. If too little material is deposited on the

Figure 6.2 Inputs and outputs of parameters that allow a braid to be described on different scales.

Figure 6.3 A hybrid triaxial braid with alternating carbon and glass fiber yarns in the bias yarn groups.

mandrel, then uncovered regions remain, resulting in an ‘open-mesh’ braid [16, 81], also shown in Figure 6.4. On the other hand, too much material can lead to ‘jamming’. Jamming occurs when too much material is forced to be deposited onto a given mandrel circumference, resulting in a small yarn width and a large yarn thickness, and a braid that does not fit tightly around the mandrel. Figure 6.4 shows that a braid angle increase for a fixed mandrel circumference reduces the yarn pitch and width. A maximum feasible braid angle exists that corresponds to the onset of jamming, here considered to be a process boundary.

Hence, for most downstream analyses, the braid thickness is a key parameter that is highly dependent of the braiding process settings. In order to calculate the thickness, many other meso-scale parameters like the yarn width and yarn thickness need to be evaluated as well. Therefore the meso geometry must be analyzed in a more general sense, conveniently offering a context to define relevant braiding-specific parameters like jamming and the degree of coverage.

Figure 6.4 A biaxial braid on a mandrel with a fixed circumference and a fixed number of yarns, and a braid angle ranging from a small value, resulting in open-mesh braids (leftmost two cases), to a large value, resulting in closed-mesh braids (rightmost two cases) and a ‘jammed’ braid (rightmost case).

Problem and objective

For cylindrical braids produced in an axisymmetric braiding process, the full braid geometry can be readily represented by a single repeating unit cell (RUC) on the meso-scale, representing the periodicity of the braid, assuming no variation in the braid properties. In contrast, complex shaped mandrels usually have braid properties that vary from point to point on the mandrel surface and possibly through the thickness in case of multiple braid layers.

For this reason, the braid on the component scale must be described using parameter distributions that vary in value over the substrate instead of using a single RUC to represent all. Above each mandrel facet described in Section 2.3 and shown in Figure 6.4, and for each braid layer deposited above that facet, the braid properties can be different. Hence, the resolution of the distribution is determined by the facet size.

The objective is to present a computationally fast thickness distribution calculation for symmetric and non-symmetric, bi- and triaxial, open- and closed-mesh braids with mono-material and hybrid bias yarn groups and arbitrary variable braid angles and yarn pitches on complex mandrels for any location on the overbraided component. This approach takes into account the mutual interaction between the yarns of all yarn groups without using calculation time-intensive FE. The method should be fast and fit for CAE process chain integration, just like the model in Chapter 2. To the best knowledge of the author, this is a novel approach.

Outline

First, earlier work dedicated to braids on the meso-scale is reviewed. Next, the yarn material used in the braid meso model is described along with material specific assumptions. Additionally, the modeling assumptions are proposed. Subsequently, the yarn width of all yarns is calculated, making use of the yarn material description and the other inputs listed in Figure 6.2. This width calculation is the key component of the hybrid triaxial braid meso model. Next, the yarn thickness and aspect ratio, the layer and component thickness, and the fiber volume fraction readily follow. For completeness, expressions for the areal density and cover factor are also presented. A parametric overview of the dependencies between the discussed meso-geometry input and output parameters is given at the end of the modeling sections. The model is used to calculate the simultaneous degrees of surface coverage for all yarn groups. The model is validated using overbraided mandrels using a single as well as multiple braid layers.

6.2 Earlier work

The meso geometry of braids is often analyzed to generate models for structural analysis. In general, the architecture of braids is considered to be too complex to be described analytically [82], so numerical methods are often used instead. For a historical overview of braid meso models, refer to [16] and [82]. The following works are limited to those that are generic enough to include triaxial braids instead of only biaxial braids. All mentioned models allow arbitrary braid angles, but assume a regular braid for the bias yarns. All works were developed to generate a RUC without the purpose of representing variations in yarn pitch and direction over complex mandrels on the full component scale.

Naik [82] provides a model for axisymmetric bi- and triaxial braids, so the braid angle and yarn pitch are constrained to be the same for both bias yarn groups. This model supports only a single yarn material per yarn group.

Robitaille et al. [83] proposes a formal procedure for describing arbitrary interlacing patterns, including bi- and triaxial braids, with the possibility to account for arbitrary preform manufacturing process settings.

However, this model contains no logic regarding yarn deformation and interaction.

Gao, Ko, and Hu [32] describe triaxial braids with the same limitations as Naik [82] and present an expression for the available space for the stem yarns as a function of the braid angle, bias yarn width and pitch. Their approach allows braids to be analyzed for a series of different mandrel perimeters and is made available with a graphical user interface.

Pickett, Sirtautas, and Erber [36] generate all yarn pitch and direction distributions with a FE braiding process analysis. These parameters are subsequently used to model the RUC of a triaxial braid, again using FE but now combined with Smooth Particle Hydrodynamics, resulting in a model with a high level of detail, including transverse yarn deformation, at the cost of calculation time.

Birkefeld et al. [80] use the WiseTex and TexComp software [84–86] to model the RUC of an axisymmetric triaxial braid. This software also enables the generation of detailed models of bi- and triaxial braids with different yarn pitches for the warp and the weft yarn groups, yet its main purpose is to generate highly detailed RUCs. For complex interlacements like triaxial braids, this software still requires manual editing, for example to avoid volumetric intersections of yarns and to adjust yarn cross-sectional shapes.

In this work, bi- and triaxial braids are described with variable parameters across the surface of one or multiple braid layers on complex mandrel shapes, instead of representing the braid by a single unit cell. Also included is the possibility to model two different yarn materials per bias yarn group and to model both open- and closed-mesh braids. The model takes into account a variable yarn cross-sectional width-to-thickness ratio for a rectangular, elliptical or lenticular yarn cross-sectional shape and minimizes volumetric yarn intersections, taking into account the interaction between all yarn groups at a low computational cost.

6.3 Yarn material input parameters

The material used for braiding preforms must be provided as an input. Yarns usually consist of one or more strands, supplied on large spools that generally do not fit on the braiding machine carriers. The strand

Figure 6.5 Simplified yarn geometry using different model cross-sections with the same width w_y and area A_y and different thicknesses t_y . The actual strand cross-sectional shape, twist, and their relative position can vary.

material is generally re-spooled onto smaller bobbins. In the remainder of this work, ‘bobbins’ and ‘spools’ are used interchangeably to denote bobbins. During re-spooling, multiple strands can be combined into a single yarn in a multitude of combinations. Hence, some restrictions are proposed for the scope of this work. First, it is assumed that a bobbin contains a single yarn material. Also assume that a yarn consist of a single ‘strand’ material and that each strand consists of a single filament material, here regarded as the smallest constituent, shown in Figure 6.5. Hence, material mixing within bobbins, yarns and strands is not modeled.

A yarn group consisting of a single yarn material is a ‘mono-material’ yarn group. A ‘hybrid’ yarn group contains two materials: a ‘primary’ and a ‘secondary’ yarn material, denoted by the numbers 1 and 2, respectively. For a hybrid yarn group, it is assumed that the two yarn materials alternate between the bobbins within that group as is the case in Figure 6.3. A braid containing only mono-material yarn groups is a mono-material braid. A braid with at least one hybrid yarn group is a hybrid braid. In this work, both bias yarn groups can be mono-material or hybrid, or one can be mono-material and the other hybrid, but the stem yarn group is always assumed to be mono-material.

The yarns are assumed to be or to have

- made of fibers,
- a constant cross-sectional area A_y ,
- a cross-sectional shape that is rectangular, elliptical or lenticular,
- free of broken and detachment fibers caused by braiding, and
- unaffected by entanglement with broken filaments of other yarns in the braiding process as discussed in Chapter 4.

The yarn cross-sectional shape depends on the case at hand and is used to provide a simple relation between width reduction and simultaneous thickness increase at a constant area A_y ,

$$A_y = f_{\text{shape}} w_y t_y \quad (6.1) \quad \Leftrightarrow t_y = \frac{A_y}{f_{\text{shape}} w_y} \quad (6.2)$$

with f_{shape} as the measured (or estimated) input yarn shape factor, w_y as the yarn width and t_y as the yarn thickness. For a rectangular, elliptical and lenticular cross-section, where the latter is modeled as two parabolas, the value of f_{shape} is readily known or evaluated as 1, $\frac{\pi}{4}$ and $\frac{2}{3}$, respectively. Hence, given a fixed yarn width, for the modeled yarn thickness, $t_{y,\text{rec}} : t_{y,\text{ell}} : t_{y,\text{len}} \approx 1.0 : 1.3 : 1.5$ as shown in Figure 6.5. In this work, a yarn cross-section is sometimes drawn as a rectangle for explanation purposes only although other shapes may be a more accurate representation.

Define the yarn fiber cross-sectional area $A_{f,y}$ as the area that only takes into account filament material without air, matrix or voids,

$$A_{f,y} = \frac{n_s \rho_{l,s}}{\rho_f} \quad (6.3)$$

with n_s as the given number of strands per yarn, $\rho_{l,s}$ as the strand linear density, e.g. in kg/m, and ρ_f as the strand volumetric fiber density, e.g. in kg/m³. Both are usually provided by the strand supplier. For the yarn cross-sectional area *including* voids or matrix material,

$$A_y = \frac{A_{f,y}}{v_{f,y}} \quad (6.4)$$

with $v_{f,y}$ as the input yarn fiber volume fraction, also known as the ‘packing density’, ‘packing factor’ or ‘packing fraction’, which should be obtained by measurement if possible, see also Appendix F.1. The

yarn fiber volume fraction $v_{f,y}$ does not need to be equal to that of its strands. The value of $v_{f,y}$ depends on parameters like the strand cross-sectional geometry before re-spooling it into a yarn, the nesting of the strands within the yarn, and the amount of twist. Finally, the initial yarn width $w_{y,ini}$ must be known. The densities are the only physical material input parameters. All others are either of geometrical nature or process-related.

Rosenbaum [8] introduces the ‘critical reduced yarn width’, a fraction of the initial yarn width corresponding to the onset of jamming described in Section 6.1. However, this value is not always known in earlier design phases. In addition, define the dimensionless yarn width to thickness aspect ratio or ‘yarn aspect ratio’ $\mathcal{R}_y$ [32] with

$$\mathcal{R}_y \equiv \frac{w_y}{t_y}. \quad (6.5)$$

Combining Equations (6.2) and (6.5),

$$\mathcal{R}_y = \frac{f_{shape} w_y^2}{A_y} \quad (6.6) \quad \Leftrightarrow w_y = \sqrt{\frac{A_y \mathcal{R}_y}{f_{shape}}} \quad (6.7)$$

The initial yarn aspect ratio $\mathcal{R}_{y,ini}$ follows from Equation (6.6) using $w_y = w_{y,ini}$. Define the ‘critical yarn aspect ratio’ $\mathcal{R}_{y,cr}$, as the smallest possible yarn aspect ratio at the onset of ‘jamming’ of the braid. If $\mathcal{R}_y < \mathcal{R}_{y,cr}$, then the yarns cannot be transversely compressed any further, indicating possible jamming. Because $\mathcal{R}_y$ is directly derived from the local braid meso geometry, this jamming indicator is independent of the used braiding process simulation approach.

A practical maximum value for the aspect ratio value is roughly 1000, based on a yarn width of 10 mm and a yarn thickness equal to one filament diameter of 10 μm . $\mathcal{R}_{y,cr}$ depends on many parameters and can be determined experimentally or through FEA [36], provided that the yarns are modeled with ample filaments. For $\mathcal{R}_y < 1$, the yarn thickness exceeds the width. This is usually undesirable and physically improbable, so the value of 1 is used as a lower bound. In practice, roughly $1 < \mathcal{R}_{y,cr} < 10$ for dry untwisted fibrous yarns. If no value is available for $\mathcal{R}_{y,cr}$, then a lower bound of 1 can be used as a theoretical rule of thumb.

Define the ‘critical yarn width’ $w_{y,cr}$ as the corresponding smallest

Table 6.1 List of material parameters and the typical source.

Parameter	Symbol	Units (SI)	Source
strand linear density	$\rho_{l,s}$	kg/m	supplier
strand vol. fiber density	ρ_f	kg/m ³	supplier
yarn number of strands	n_s	-	design decision
yarn shape factor	f_{shape}	-	measurement
yarn initial width	$w_{y,\text{ini}}$	m	measurement
yarn ini. dry fiber vol. frac.	$v_{f,y}$	-	measurement
yarn critical aspect ratio	$\mathcal{R}_{y,\text{cr}}$	-	estimation

possible yarn width at the onset of jamming. Using $\mathcal{R}_{y,\text{cr}}$, Equation (6.7) gives $w_{y,\text{cr}}$. Alternatively, the value of $w_{y,\text{cr}}$ can be measured in an intentionally jammed braid. The critical yarn aspect ratio $\mathcal{R}_{y,\text{cr}}$ follows from Equation (6.6) using $w_y = w_{y,\text{cr}}$. Jamming should not be confused with ‘locking’ [87] in ready-made woven fabrics because the latter does not have inter-yarn slip like the braiding process has. Jamming can be avoided by decreasing either the number of spools per group, the yarn size, and the braid angle, or by increasing the mandrel perimeter.

An overview of the yarn material input parameters is listed in Table 6.1.

6.4 Modeling assumptions

In addition to the assumptions from the previous section, assume that

- the substrate is meshed with small enough facets.

The facets are small enough if they allow an accurate representation of changes of the braid parameters over the substrate, especially in regions with large curvatures of the mandrel surface or the yarns.

Furthermore, it must be possible to locally evaluate the braid properties on any point of a complex mandrel substrate. The complexity of these evaluations is simplified by *locally*, that is, within the area of a facet, approximating

- the substrate surface as developable,
- the yarns as parallel in each yarn group, and
- the run thickness as much smaller than the local substrate radius.

The developability assumption means that locally, the surface can be fictitiously flattened with zero in-plane strain, and asserts that angles and

distances measured on the substrate equal those after flattening.

A run thickness that is much smaller than the local substrate radius allows the yarn pitch to be measured on the substrate surface instead of on the plane that runs midway through the run thickness.

As a first approximation for fibrous yarns, neglect

- yarn spreading,
- intra-group overlap of adjacent yarns, and
- influence of the bias braid pattern on the yarn width.

The neglect of yarn spreading results in $R_y \leq R_{y,ini}$.

In contrast to e.g. filament winding, overlap of yarns within the same yarn group in a braid layer is largely absent due to their interlacement. Instead, yarns within the same yarn group can be in lateral contact. As a consequence, yarns can deform by a decrease of width w_y and a simultaneous increase of thickness t_y , provided that the yarn allows such deformation, which is generally the case for fibrous yarns. In the case of rigid yarns such as preconsolidated tapes, this is generally not possible, unless a thermoplastic matrix is used with in-line heating to soften or melt it while braiding.

For triaxial braids, assume that:

- the bias braid pattern is a regular braid, and
- the stem yarn group is mono-material.

The braid pattern input depends on the used spool pattern as discussed in Chapter 5. In this chapter, a regular braid is assumed for the bias yarns because this pattern is used the most in both research and commercial production.

6.5 Output parameters

6.5.1 Yarn width

In this chapter, the yarn width calculation is key in the analysis of the braid thickness and most other braid properties. First, the bias yarn width(s) are calculated without taking into account the optional stem yarns. Next, the stem yarn width is calculated as a function of the parameters of the bias yarns groups, temporarily assuming rigid bias yarns. Both of these yarn width calculations use closed-form expressions

Figure 6.6 A triaxial braid with hybrid warp and weft yarn groups. p_x is the average yarn pitch. The sections show: An open-mesh, as in the photo (top); Dropping the yarn group subscript: the transition from the initial yarn widths to reduced widths (middle); The onset of jamming (bottom).

only, mostly based on geometrical observations. Subsequently, these calculations are used as building blocks in a generalized iterative analysis that accounts for the mutual influence between bias yarns and stem yarns. The resulting model is called the ‘interaction model’.

Bias yarn width

In each bias yarn group of hybrid braids, the secondary yarn is not necessarily positioned midway between the larger primary yarns as shown in Figure 6.6. As a result, the the yarn pitch p alternates. This alternation is observed in both bi- and triaxial hybrid braids based on a regular braid for the bias yarns, either symmetric or non-symmetric. In triaxial braids, the alternation is influenced by the interaction with stem yarns. This phenomenon is not taken into account in the kinematic model from Chapter 2 because all yarns are modeled by their centerlines, resulting in a distribution of the average yarn pitch p .

For hybrid braids, the initial yarn aspect ratios can vary between the yarn materials. The different yarn materials need to be parametrized individually because they vary in cross-sectional size by a factor 10 or more as shown in industrial projects executed by Eurocarbon BV [24]. One of the initial ratios can be small, such as material 2 in Figure 6.6,

providing a relatively large resistance to width reduction compared to material 1 with the larger aspect ratio. Hence, for each yarn material, the yarn width can reduce at a different rate with a decreasing yarn pitch.

Dropping the yarn group subscript, define the ‘available width’ w_{av} as the average space that is locally available per yarn in the yarn width direction for a single yarn group. For biaxial braids, w_{av} equals the local yarn pitch p of that group as defined in Section 2.4.3. For triaxial braids, w_{av} may be smaller than p due to the presence of the stem yarns.

The yarn widths are, for the moment, calculated independently for each bias yarn group. At the onset of yarn width reduction, shown in the center cross-section of Figure 6.6,

$$2w_{av,tr} = w_{y,ini,1} + w_{y,ini,2}, \quad (6.8)$$

with the subscript ‘tr’ stands for ‘transition’, $w_{y,ini}$ as the initial yarn width, and subscripts ‘1’ and ‘2’ denoting the primary and secondary material, respectively. At the onset of jamming shown in the bottom cross-section of Figure 6.6,

$$2w_{av,cr} = w_{y,cr,1} + w_{y,cr,2}, \quad (6.9)$$

with $w_{y,cr}$ from Equation (6.7).

If $w_{av} < w_{av,cr}$, then the braid is jammed as defined in Section 6.1 and not feasible. The ‘design space’ for design purposes is $w_{avcr} \leq w_{av} \leq w_{av,tr}$ to achieve a full surface coverage without jamming. In this range, a ‘compatibility constraint’ applies,

$$2w_{av} = w_{y,1} + w_{y,2}. \quad (6.10)$$

The yarn width is linearized for $0 \leq 2w_{av} \leq 2w_{av,tr}$ and yarn spreading is neglected, resulting in

$$w_{y,i} = \min(a_i + b_i \cdot 2w_{av}, w_{y,ini,i}). \quad (6.11)$$

with

$$b_i = \frac{w_{y,ini,i} - w_{y,cr,i}}{2w_{av,tr} - 2w_{av,cr}} \quad (6.12)$$

Figure 6.7 Yarn width w_y approximation as a function of the available width w_{av} . The yarns show different width reduction rates. The ‘actual width’ (dashed curve) is a schematic example that depends on the yarn material and the process conditions.

and

$$a_i = w_{y,ini,i} - b_i \cdot 2w_{av,cr}. \quad (6.13)$$

Equation (6.11) is plotted in Figure 6.7. Note that the curves of $w_{y,i}$ do not necessarily cross the origin. The yarn width for hybrid biaxial braids is expressed in closed form in Equation (6.11) as a function of w_{av} , $w_{y,ini,1}$, $w_{y,ini,2}$, $w_{y,cr,1}$, and $w_{y,cr,2}$. This model allows the yarn width to decrease to near-zero, although in reality, it is likely limited by $w_y \geq w_{y,cr}$. The model also neglects yarn spreading, although actually, yarns may spread out, resulting in $w_y > w_{y,ini}$. Both model limitations are schematically shown in Figure 6.7.

For biaxial braids, for each bias yarn group, $w_{av} = p$. In addition, for symmetric mono-material biaxial braids, $w_{y,ini,1} = w_{y,ini,2}$, $w_{y,cr,1} = w_{y,cr,2}$, reducing Equation (6.11) to

$$w_y = \min(p, w_{y,ini}), \quad (6.14)$$

evaluating to p and $w_{y,ini}$ for closed- and open-mesh braids, respectively.

Stem yarn width

In Chapter 5, a ‘repeat(ing unit cell)’ (RUC) describes the periodicity of braids with an emphasis on the pattern and not the geometry. For the geometrical analysis of triaxial braids, a different unit cell is needed for a more tractable geometry. For this purpose, define a ‘geometrical

Figure 6.8 A photo of a hybrid triaxial braid, oriented such that one of the bias yarn groups is aligned horizontally. The indicated RUC has the same topology as the repeat indicated in Figure 5.13. Two adjacent GUCs, bounded by the dash-dotted lines, are used for geometrical analysis.

unit cell' (GUC) as the smallest repetitive element of the *projection* of a periodic fabric on the substrate. The GUC does not necessarily fully define the topological periodicity like a RUC does as shown in Figures 6.8 and 6.9. To describe braids that are geometrically non-periodic across the component as is usually the case for complex mandrels, the periodicity in the GUC definition applies only locally, that is, on the area defined by a facet of the substrate.

The primary bias yarns are used as a fixed reference frame that applies to each GUC as shown in Figure 6.9. This fixes points D, E, F and G.

Figure 6.9 Two GUCs, bounded by the shown centerlines, of a non-symmetric triaxial braid, based on the braid shown on Figure 6.8. For explanation purposes, the secondary yarns are arbitrarily positioned midway between the primary yarns. Underlining denotes input as shown in Figure 6.2.

Figure 6.10 Symmetric triaxial braids, based on a regular mono-material biaxial braid pattern. Left: A small bias yarn width-to-pitch ratio enables stem-to-stem yarn contact when neglecting the bias yarn thickness, and $w_{av,S} = p_S$. Right: A large ratio prevents this contact, and $w_{av,S} = w_{tun}$.

Due to the GUC periodicity and assuming a single material for the stem yarn group, the parameters governing the properties of a single stem yarn can be used to represent all stem yarns within the facet bounds. Expressions for the geometric parameters describing a GUC shown in Figure 6.9 are given in Appendix E. Designating the yarn with the maximum width as either the primary or secondary yarn in either one or both bias groups does not affect the outcome of this analysis. The procedure presented here can also be applied to a mono-material bias yarn group using $w_{y,1} = w_{y,2}$.

The stem yarn width $w_{y,S}$ is obtained, neglecting yarn spreading, as

$$w_{y,S} = \min(w_{y,S,ini}, w_{av,S}). \quad (6.15)$$

with $w_{y,S,ini}$ as the given initial stem yarn width and $w_{av,S}$ as the 'available width' for the stem yarns. An expression for $w_{av,S}$ is determined next.

The available stem yarn width $w_{av,S}$ can be limited by the stem yarn pitch p_S , assuming that overlap between adjacent stem yarns is not possible due to the interlacement of the triaxial structure shown in Figure 6.10. Measured perpendicularly to f_S ,

$$p_S = q_X \sin \zeta. \quad (6.16)$$

with q_X and ζ derived in Appendix E. p_S is fully defined by the primary bias yarns and does not necessarily correspond to the mandrel tangential

direction, so p_S does not necessarily equal the yarn pitch of symmetric braids as expressed by Equation (2.8).

The available stem yarn width $w_{av,S}$ can also be limited by the ‘tunnel width’ w_{tun} , measured perpendicularly to f_S , and defined as the width of the tunnel through which each stem yarn runs as shown in Section 5.3.3. It is assumed that all three yarns touch at a common point in the right case in Figure 5.14 and that the secondary yarns remain locally parallel to the primary yarns. As shown in Figure 6.9, each tunnel is bounded by points B and/or G at one side and by points A and/or C at the other side. Each of the four combinations of these point pairs have their own spacing in the stem yarn width direction, and the tunnel width w_{tun} equals the minimum of these spacings,

$$w_{tun} = \min(w_{tun,GA}, w_{tun,GC}, w_{tun,BA}, w_{tun,BC}). \quad (6.17)$$

The four spacings in Equation (6.17) are fully defined by the coordinates (b_O, b_X) of point B in the given reference frame. As a consequence, the position of the secondary yarns relative to the primary yarns fully defines all parameters in Equation (6.17). Expressions for these parameters are derived in Appendix E.2 and their derivatives are listed in Appendix E.3. As a result, Equation (6.17) is plotted in Figure 6.11. The tunnel widths can overlap each other as shown in Figures 6.9 and 6.11. A key observation is that a position for B exist where

$$w_{tun} = w_{tun,GA} = w_{tun,GC} = w_{tun,BA} = w_{tun,BC} \quad (6.18)$$

as shown in the examples in Figures 6.11 and 6.12, corresponding to the constraints

$$AC \parallel DE \wedge B' : DE \quad (6.19)$$

with $\parallel$ and $:$ as a parallel and collinear constraint, respectively. The position of B' obeying Equation (6.19) is derived in Appendix E.4.

Figure 6.10 shows that it is possible that $w_{tun} > p_S$, so for the available width for a stem yarn,

$$w_{av,S} = \min(p_S, w_{tun}). \quad (6.20)$$

A key assumption is that the braiding process follows the path of least resistance while (for the moment) assuming bias yarns with a fixed cross-

Figure 6.11 The two GUCs from Figure 6.9 with the secondary yarns positioned such that Equation (6.18) applies. Equation (6.17) has the shape of a skewed pyramid, shown by iso-curves. Its side projections indicate which of the bounding point pairs, i.e. GA, GC, BA and BC, leads to the minimum tunnel width when B would be located in that area.

section. For this purpose, the GUC geometry is solved for the state with the least amount of transverse stem yarn deformation. In turn, this minimum deformation is approximated to correspond to a global maximum of the available width for the stem yarns.

A key decision is to let the initial guess of point B correspond to the global maximum tunnel width so that Equation (6.18) applies. As shown in the example of Figure 6.12, the initial guess can result in physically impossible overlap between adjacent secondary and primary yarns of the same bias yarn group. To prohibit this overlap, the solution space for a valid position of B is bounded as shown in Figures 6.11 and 6.12. Dropping the yarn group subscripts, for each bias yarn group, the non-

Figure 6.12 Two GUCs with large secondary yarns positioned for a maximum tunnel width.

6

overlap constraint is equivalent to

$$0 \leq b \leq g - a. \quad (6.21)$$

The initial guess of B corresponds to a solution with ‘unbounded values’, denoted using stars: b_O^* and b_X^* , in turn corresponding to the unbounded tunnel width w_{tun}^* as shown in Figure 6.12. As the second and last iteration, the bounded solution (b_O, b_X) , corresponding to the bounded global maximum tunnel width w_{tun} , is obtained by clamping the unbounded value b^* according to Equation (6.21),

$$b = \max(0, \min(b^*, g - a)). \quad (6.22)$$

The clamping operation can reduce the tunnel width and Equation (6.18) may no longer hold. However, it guarantees a global maximum for the tunnel width, possibly part of a plateau, coinciding with the bound of the (b_O^*, b_X^*) solution space. An example is shown in Figure 6.13 after applying Equation (6.22) to the case in Figure 6.12.

Figure 6.13 Braid configuration after clamping the unbounded state in Figure 6.12 with Equation (6.22), removing overlap.

Coupled yarn width

In the previous sections, the stem yarn width is expressed as a function of, among others, bias yarns with a fixed cross-section. In reality, the bias yarns can deform too. This is especially the case in braids where the initial stem yarn width exceeds its pitch and tunnel width. To account for this mutual influence, the previously described bi- and triaxial yarn width models are combined in an enclosing optimization to simultaneously deform all yarns. Ideally, this involves an approach that accounts for complex transverse yarn deformation mechanisms like FEA on the filament scale. However, for design purposes on the full component scale, this is computationally prohibitive. Instead, a coarse yet computationally cheap approach is proposed.

Define a single ‘yarn width compression factor’ c in the domain $0 \leq c \leq 1$, simultaneously applied to the local yarn segments of all yarn groups.

Given the local yarn pitches p_O , p_X and p_S , the value $c = 0$ corresponds with the fictitious state where the bias yarn widths are evaluated as if the

stem yarns are left out, while the stem yarn widths are evaluated as if the bias yarns are left out. The subscript 'max' is used for the yarn widths in this state.

The value $c = 1$ corresponds with the fictitious state where all yarn widths correspond to their given critical yarn aspect ratio $R_{y,cr}$. The subscript 'cr' is used for the yarn widths in this state.

A value of c is searched where all yarns are simultaneously and sufficiently compressed such that the stem yarn tunnel width equals the stem yarn width, so $w_{av,S}(c) = w_{y,S}(c)$. This corresponds to minimizing a residue,

$$\operatorname{argmin}_{0 \leq c \leq 1} |w_{av,S}(c) - w_{y,S}(c)|. \quad (6.23)$$

The two states at $c = 0$ and $c = 1$ are interpolated with a linear function that simultaneously varies the available space for each yarn group by varying $2w_{av,O}(c)$, $2w_{av,X}(c)$ and $w_{y,S}(c)$. Hence, it is assumed that all yarns are subject to the same yarn width compression factor value of c . In reality, each combination of yarn material and yarn group can have a different value, depending on their relative resistance to transverse deformation in the local GUC.

For each bias yarns group, dropping the subscripts, the interpolation function is

$$2w_{av}(c) = w_{y,1,max} + w_{y,2,max} - c [(w_{y,1,max} + w_{y,2,max}) - (w_{y,1,cr} + w_{y,2,cr})] \quad (6.24)$$

where $w_{y,i,max}$ is provided as the left-hand side of Equation (6.11) while on its right-hand side, w_{av} is substituted by the known yarn pitch p . In Equation (6.24), $w_{y,i,cr}$ is provided by Equation (6.7), given $R_y = R_{y,cr}$. In turn, the output $2w_{av}(c)$ from Equation (6.24) is used to calculate the primary and secondary bias yarns widths, again using Equation (6.11). Next, these bias yarn widths are used to calculate the tunnel width for Equation (6.20) to obtain $w_{av,S}$.

Similarly, the interpolation function for the stem yarns is

$$w_{y,S}(c) = w_{y,S,max} - c(w_{y,S,max} - w_{y,S,cr}) \quad (6.25)$$

with $w_{y,S,max}$ from Equation (6.15), given $w_{av,S,max} = p_S$ and $w_{y,S,cr}$ again

Figure 6.14 Cross-section of a triaxial braid with and without nesting, showing the ‘peaks’ with the local thickness maximum.

from Equation (6.7). If the residue does not vanish at $c = 1$, then a critically large bias yarn compression does not result in sufficient space for the stem yarns, so the braid is likely jammed and not feasible.

6.5.2 Yarn thickness

Once the yarn width w_y has been evaluated for each yarn group using the interaction model from the preceding section, the yarn thickness t_y is readily evaluated for each combination of yarn group and yarn material after combining Equations (6.1), (6.3) and (6.4),

$$t_y = \frac{n_s \rho_{l,s}}{f_{\text{shape}} w_y v_{f,y} \rho_f}. \quad (6.26)$$

6.5.3 Yarn aspect ratio

Given the yarn width from Section 6.5.1 and the yarn thickness from the preceding section, also the yarn aspect ratio $\mathcal{R}_y$ of the yarns in the braid is readily evaluated as an output parameter for each yarn material in each yarn group using Equation (6.5). This can be used to indicate the possibility of jamming as discussed in Section 6.3.

6.5.4 Run thickness

The run thickness distribution t_r is here represented by the local maximum braid layer thickness, despite thickness variations and the possible gaps between the yarns. Such a maximum occurs at a ‘peak’, a location where yarns of all groups overlap each other as shown in Figure 6.14. It is assumed that each instance of a GUC in the braid layer contains a peak. This implies a globally homogeneous and dense enough distribution of

these peaks. The run thickness t_r is approximated by summation of the maximum yarn thickness among the primary and optional secondary yarn over all yarn groups,

$$t_r = \sum_{i=1}^{n_{ag}} \max(t_{y,i,1}, t_{y,i,2}) \quad (6.27)$$

with $t_{y,i,j}$ obtained as described in Section 6.5.2, i as iterator over the yarn groups, and n_{ag} as the number of active yarn groups, which is either 2 or 3 for for a bi- or triaxial braid, respectively.

Note that the non-rectangular yarn cross-sectional models from Section 6.3 use t_y as the maximum thickness at the center of a yarn, so for these shapes, Equation (6.27) is only correct when the centerlines of yarns from all groups cross at the same point, which is generally not the case as shown in Figure 6.10. Hence, Equation (6.27) is expected to be an over-estimation and upper bound for these yarn cross-sections. Open-mesh braids can defeat the overlap assumption too, reducing the accuracy of Equation (6.27).

6.5.5 Component thickness

An optimum nesting of subsequent braid layers is advantageous for a reduced component thickness and a high fiber volume fraction. Sub-optimal nesting can be partially compensated by e.g. autoclave precompaction cycles, applying a cyclical overpressure on the bagged preform prior to resin injection. For low-cost series-production, it is beneficial if such operations can be avoided, and precompaction is neglected here. For an optimum nesting of a biaxial braid, all interlacement points of a braid layer coincide with the inter-yarn gaps of the preceding run. However, to avoid that interlacement points are stacked instead of nested, a deposition tolerance smaller than the yarn width is required, which is generally difficult to achieve for the full component, in contrast to e.g. filament winding. Hence, nesting between yarns is neglected.

For regular triaxial braids, [80] shows that the mandrel can be rotated by $\frac{360}{2n_y}$ degrees after each overbraiding run to improve nesting of the wales defined in Section 5.3 and shown in the left case of Figure 6.14, where a whale is defined as the bulge along each stem yarn. However, if the stem yarns are not kept parallel to those on the previous braid layer, then

Figure 6.15 Left: Resin Transfer Molding (RTM). Right: Vacuum-Assisted Resin Transfer Molding (VARTM). The braid is cut open and draped for explanation purposes only. Similar component thickness changes can occur for cylindrical mold systems.

optimum nesting is again difficult to achieve over the full component. Hence, whale nesting is neglected too. Besides the meso-scale geometry, nesting may also depend on the amount of yarn compaction in the braid [80].

Lastly, bridging is neglected, i.e. braids not touching a locally concave substrate, possibly resulting in a larger thickness including a resin-rich pocket.

The component thickness distribution t_c is approximated after all overbraiding runs have been finished by the summation over all run thicknesses from Equation (6.27),

$$t_c = \sum_{i=1}^{n_r} t_r \quad (6.28)$$

where n_r is the number of overbraiding runs, which can vary over the mandrel.

Hence, from a resin injection perspective, it is implicitly assumed that the outer mold or vacuum bag is only in contact with the peaks of the previous braid layer shown in Figure 6.14, in contrast to alternative cases shown in Figure 6.15 where the actual component thickness $t_{c,act}$ deviates from t_c .

6.5.6 Fiber volume fraction

Define the ‘homogenized fiber thickness’ distribution $t_{f,hom}$ as the thickness of a fictitious layer of pure fiber material ($v_f = 100\%$) This is only a temporary parameter as input for the calculation of, among others, the fiber volume fraction. The volume of this layer on a finite substrate area dA_s is $dA_s \cdot t_{f,hom}$ and equals the volume of the fibers of the braid

Figure 6.16 The modeled fiber volume fractions, showing $v_{f,c} < v_{f,r} < v_{f,y} < v_{f,s}$

layer deposited on dA_s . For the combination of a yarn group and a yarn material,

$$t_{f, \text{hom}} = \frac{A_{f,y}}{n_{\text{mat}} p} \quad (6.29)$$

with $A_{f,y}$ from Equation (6.3), n_{mat} the given number of materials per yarn group, and p as the given yarn pitch.

Voids on the strand and yarn scale as shown in Figure 6.16 are taken into account using the yarn fiber volume fraction $v_{f,y}$ as proposed in Section 6.3. For closed-mesh braids, voids on the run and component scale are neglected. For open-mesh braids, the gaps between the yarns are significant and are taken into account: Given the run thickness t_r and the homogeneous fiber thickness distribution $t_{f, \text{hom}}$, and using the assumption from Section 6.4 that the run thickness t_r is much smaller than the local substrate radius, the fiber volume fraction distribution of an overbraiding run $v_{f,r}$ is approximated by

$$v_{f,r} = \frac{\sum_{i=1}^{n_{\text{ag}}} \sum_{m=1}^{n_{\text{mat}}} t_{f, \text{hom}, i, m}}{t_r} \quad (6.30)$$

with n_{ag} as the number of active yarn groups, n_{mat} the number of materials per yarn group, and $t_{f, \text{hom}, i}$ from Equation (6.29).

6.5.7 Homogenized thickness

For downstream structural analysis like an elastic analysis for stiffness-critical applications, a braiding run can be approximated by a stack of non-crimp (zero waviness) fabric ‘plies’ with undulation-degraded engineering constants, where each ply represents one material for one yarn group. The ‘homogenized thickness’ t_{hom} is defined as the thickness of such a ply,

$$t_{\text{hom}} = \frac{t_{f,\text{hom}}}{v_{f,r}} \quad (6.31)$$

with $t_{f,\text{hom}}$ from Equation (6.29) and $v_{f,r}$ from Equation (6.30).

6.5.8 Areal density

The areal fiber density distribution $\rho_{A,r}$ is calculated for each combination of one yarn group and one yarn material,

$$\rho_A = \rho_f \cdot t_{f,\text{hom}} \quad (6.32)$$

with ρ_f as the given material fiber density from Section 6.3 and $t_{f,\text{hom}}$ from Equation (6.29).

6.5.9 Cover factor

The ‘cover factor’ CF is defined in literature as the fraction of the substrate area covered by the *bias* yarns. A cover factor for triaxial braids involves the fictitious absence of the stem yarns. A value of $CF = 1$ is usually desired for structural reasons as mentioned in Section 6.1.

The calculation of the degree of coverage for a triaxial braid would be more complex to express analytically, because it must take into account the stem yarn positions relative to the bias yarns. When the stem yarns have a width smaller than the tunnel width w_{tun} as shown in Figure 6.8, then the stem yarns can laterally slide through the biaxial braid, directly influencing the degree of coverage. Instead, for triaxial braids, low values of the fiber volume fraction as defined by Equation (6.30) can be used as an aid to identify regions of a low surface coverage.

Figure 6.17 Braid parameter dependencies. Underlined and boxed symbols represent global input and output parameters, respectively.

A generic analytical expression for the cover factor, applicable to any mandrel shape, prevents the need for analytical solutions that only apply to specific mandrel shapes like cones [70]. In Appendix D, such an expression is derived for non-symmetric hybrid biaxial braids.

In addition to the mentioned definition of the cover factor, 'the stem yarn cover factor' CF_S is defined as the fraction of the substrate area covered by the stem yarns while fictitiously leaving out the bias yarns.

6.5.10 Braid parameter dependencies

Figure 6.17 shows an overview of the braided component parameter dependencies from the filament to the component scale with the symbols used throughout this chapter.

Figure 6.18 The bias cover factor (CF) and the maximum possible stem yarn cover factor (CF_s) as a function of the dimensionless primary and secondary yarn widths, assuming an axisymmetric braid with hybrid bias yarn groups.

6.6 Numerical results - axisymmetric triaxial braids

For the special case of an axisymmetric triaxial braid with *hybrid* bias yarn groups, both the bias and stem yarn cover factor as defined in Section 6.5.9 are plotted in Figure 6.18. Both are conveniently independent of the braid angle if they are expressed as a function of dimensionless bias yarns widths. The graph shows that an increasing bias cover factor roughly corresponds with a decrease of the stem yarn cover factor. A bias cover factor of 1 can be combined with a stem yarn cover factor of 0.5 at the most. On the other hand, a stem yarn cover factor of 1 can be combined with a bias cover factor of 0.75 at the most. Hence, a simultaneous full coverage with both bias and stem yarns is impossible. If one of the dimensionless yarn widths is zero, then the braid is a diamond braid (having a plain weave pattern) for the bias yarns. The graph is symmetric across the diagonal, corresponding to the invariance of assigning the maximum yarn width to either the primary or secondary yarn as mentioned earlier on page 145.

When assuming only mono-material yarn groups besides the axisymmetry, then the input parameters are reduced to

- $\alpha_O = \alpha_X = \alpha$,
- $p_O = p_X = p_{OX}$, and
- $w_{y,O,1} = w_{y,X,1} = w_{y,O,2} = w_{y,X,2} = w_{y,OX}$.

For the intermediate geometrical GUC parameters in Equation (E.3) to Equation (E.14), $\zeta = \delta = \alpha$. The parameters q , i , a , g , and b are the same for both bias yarn groups and $i = a$. In turn, Equations (E.11) to (E.14) all reduce to the same expression,

$$w_{\text{tun}} = \frac{\frac{3}{2}p_{OX} - w_{y,OX}}{\cos \alpha}. \quad (6.33)$$

Substitution of Equation (2.8) in Equation (6.33), application of Equation (6.15), and finally dividing by the stem yarn pitch p_S cancels out the braid angle and results in

$$CF_S = \min \left(1, \frac{3}{2} - \frac{w_{y,OX}}{p_{OX}}, w_{y,S,\text{ini}} \right), 0 \leq w_{y,OX} \leq p_{OX}, \quad (6.34)$$

indicated with the dashed diagonal in Figure 6.18, assuming $w_{y,S,\text{ini}} > p_S$.

In the two examples of hybrid triaxial braids shown in Figure 6.10, the left example corresponds to a point on the diagonal with $0 < \frac{w_{y,OX}}{p_{OX}} < 0.5$ and $\frac{w_{\text{tun}}}{p_S} > 1$, so the stem yarns laterally contact each other along most of their length while roughly covering the full substrate. The right example also corresponds to a point on the diagonal with $0.5 < \frac{w_{y,OX}}{p_{OX}} < 1$ and $\frac{w_{\text{tun}}}{p_S} < 1$, so the stem yarns cannot contact each other.

6.7 Experiment

The interaction model presented in this chapter is experimentally validated using an axisymmetric triaxial braid with hybrid bias yarn groups. First, a single-layer braid is validated. Next, a four-layer braid is used to additionally validate the assumption that nesting can be neglected. This braid has the same braid pattern, materials, and number of yarns per group, but a different mandrel diameter and different braid angles, resulting in different yarn pitches. The input material values are listed in Table 6.2.

Table 6.2 *Material input values of the experimental thickness validation.*

parameter	value
Primary (1) bias yarn material	
strand supplier and name	Toray carbon fiber T700
strand fiber density $\rho_{l,s}$	1800 kg/m ³
strand linear density ρ_f	$800 \cdot 10^{-6}$ kg/m
number of strands per yarn n_s	1
yarn fiber volume fraction $v_{f,y}$	0.61
yarn initial width $w_{y,ini}$	2.5 mm
yarn cross-sectional shape	lenticular
Secondary (2) bias yarn material	
strand supplier and name	PPG E-Glass fiber; Sizing type 1383
strand fiber density $\rho_{l,s}$	2540 kg/m ³
strand linear density ρ_f	$300 \cdot 10^{-6}$ kg/m
number of strands per yarn n_s	1
yarn fiber volume fraction $v_{f,y}$	0.63
yarn initial width $w_{y,ini}$	1 mm
yarn cross-sectional shape	lenticular
Stem yarn material	
strand supplier and name	Toray carbon fiber T700
strand fiber density $\rho_{l,s}$	1800 kg/m ³
strand linear density ρ_f	$800 \cdot 10^{-6}$ kg/m
number of strands per yarn n_s	2
yarn fiber volume fraction $v_{f,y}$	0.75
yarn initial width $w_{y,ini}$	4 mm
yarn cross-sectional shape	lenticular
number of yarns per group n_y	48

The yarn fiber volume fraction and the yarn cross-sectional shape were obtained from the four-layer braid containing these yarns as described in Appendix F. The fiber volume fraction varies between the bias and stem yarns and, to a lesser degree, between the primary and secondary bias yarns. The higher value of the stem yarns can be explained by the compression by the bias yarn groups. The yarn cross-sectional shape appeared to be predominantly lenticular for all yarns.

The actual run thickness $t_{r,exp}$ of the first run is measured with a fiber-reinforced measurement tape. The tape thickness t_{tape} is compensated

for, so

$$t_{r,\text{exp}} = \frac{P_{\text{out}}}{2\pi} - t_{\text{tape}} - r_{\text{m}} \quad (6.35)$$

with P_{out} as the measured braid outer perimeter and r_{m} as the mandrel radius. All thicknesses and fiber volume fractions on the component scale are here based on the assumption that the component outer surface after impregnation and cure equals the inner surface of the measurement tape.

Simplified model

Define the ‘simplified model’, for comparison with the interaction model presented in this chapter, as the braid geometry model that neglects both the interaction between the yarn groups and the difference in deformation behavior between yarn materials within a group. The same equations are used, except that for each combination of yarn group and yarn material, the yarn width w_y from Section 6.5.1 is replaced by

$$w_y = \min(p, w_{y,\text{ini}}) \quad (6.36)$$

with p as the yarn pitch and $w_{y,\text{ini}}$ as the yarn initial yarn width.

6.8 Results and discussion

The single-layer braid is shown in Figure 6.19. The measured perimeter was uniform over the mandrel length, leading to a uniform run thickness. It follows that $CF < 1$ and $w_{\text{tun}} > w_{y,\text{S,ini}}$, so $w_y = w_{y,\text{ini}}$ for all yarns. Hence, the results of the interaction model equal those of the simplified model. Using

- $t_{y,X,1,\text{ini}} = t_{y,O,1,\text{ini}}$,
- $t_{y,X,2,\text{ini}} = t_{y,O,2,\text{ini}}$, and
- $t_{y,X,1,\text{ini}} > t_{y,X,2,\text{ini}}$,

the modeled run thickness $t_{r,\text{mod}}$ from Equation (6.27) reduces to

$$t_{r,\text{mod}} = 2t_{y,X,1,\text{ini}} + t_{y,S,\text{ini}}. \quad (6.37)$$

Table 6.3 *Single layer braid input and output. Both models result in the same output.*

parameter	value
mandrel radius r_m	49 mm
measurement tape thickness t_{tape}	0.5 mm
braid angle α	60°
braid outer perimeter P	320 mm
measured run thickness $t_{r,\text{exp}}$	1.4 mm
modeled run thickness $t_{r,\text{mod}}$	1.3 mm
error, $\frac{t_{r,\text{mod}} - t_{r,\text{exp}}}{t_{r,\text{exp}}}$	-8%

Figure 6.19 *Perimeter measurement for thickness validation.*

The numerical results for the single-layer braid are shown in Table 6.3. The run thickness model in Section 6.5.4 is expected to be an over-estimation. The component thickness model from Section 6.5.5, neglecting nesting, is expected to be an over-estimation. In the case of the single-layer braid, the modeled thickness is an under-estimation, which was unexpected. This can be caused by, among others,

- measurement errors,
- a too large input yarn fiber volume fraction,
- a too large input yarn initial width,
- an incorrect yarn cross-sectional shape, and
- the neglect of yarn bending stiffness.

If further experiments suggest that this underestimation is systematic, then the modeled run thickness can be adjusted to match the actual thickness using an empirical correction factor, resulting in a lower run

Table 6.4 Four-layer measurement values. *The mandrel diameter is 45mm and the braid is jammed, resulting in a braid inner diameter increase of 2mm, in turn increasing the 'virtual' mandrel radius.

parameter	value
measured mandrel radius r_m	$\frac{45+2}{2}$ mm*
measurement tape thickness t_{tape}	0.5 mm
measured braid angle α	$\frac{97^\circ}{2}$

Figure 6.20 Mandrel with four hybrid triaxial braid layers.

fiber volume fraction. However, such a factor may only apply to a small range of process, material and design parameter values.

The four-layer braid is shown in Figure 6.20. The used materials and measurement procedure equal those from the single run experiment. The measurement results are tabulated in Tables 6.4 and 6.5 and plotted in Figure 6.21. For the calculation of the modeled thickness of each individual run $t_{r,\text{mod}}$, the *measured* outer perimeter of the preceding run is used instead of the modeled equivalent. For the modeled cumulative braid thickness $t_{\text{cum},\text{mod}}$, the modeled outer perimeter of the preceding run is used.

The thickness results from the interaction model is discussed first. The data are tabulated in Table 6.6 and plotted in Figure 6.21. Figure 6.22 plots the calculated braid geometry over the actual geometry, showing a qualitative match between both. Unlike the single-layer experiment,

Table 6.5 Measured values for the braid shown in Figure 6.20. Used symbols: P_{out} : measured braid outer perimeter; t (hickness); r (un); exp (erimental); cum (ulative). Units: millimeter.

layer	P_{out}	$t_{r,exp}$	$t_{cum,exp}$
1	160	1.5	1.5
2	168	1.3	2.7
3	176	1.3	4.0
4	182	1.0	5.0

Table 6.6 Values of the 'interaction model' for the braid shown in Figure 6.20. Used symbols: t (hickness); r (un); exp (erimental); mod (el); e (rror); cum (ulative). Units: millimeter.

layer	$t_{r,mod}$	e: $\frac{t_{r,mod}-t_{r,exp}}{t_{r,exp}}$	$t_{cum,mod}$	e: $\frac{t_{cum,mod}-t_{cum,exp}}{t_{cum,exp}}$
1	1.8	23%	1.8	23%
2	1.7	35%	3.5	28%
3	1.6	29%	5.1	27%
4	1.6	66%	6.6	34%

the thickness of individual runs are now over-estimated by the model. The error of the modeled cumulative braid thickness shows an increasing trend as the number of braid layers increases. This can be explained by the thickness over-estimation related to the neglected 'gap' between runs as shown in Figure 6.14 due to the assumed absence of nesting. As a result, the 'nesting factor' [80], defined as the ratio between the component thickness without (fictitiously) and with nesting, is 1.3, suggesting that in this case roughly a quarter of the modeled volume is actually used for nesting.

The results using the simplified model of Equation (6.36) are shown in Table 6.7 and plotted in Figure 6.21. This model neglects nesting, resulting in an artificial thickness increase, and it neglects yarn interaction, resulting in an artificial thickness decrease. These two errors partially cancel each other out. As a result, the simplified model more closely matches this specific experiment than the interaction model.

The fiber volume fractions for both the experiment and both models are shown in Table 6.8. The modeled values are significantly lower than the experimental values, again most probably caused by the neglect of nesting. The large difference between the high fiber volume fraction in

Figure 6.21 Graphical representation of the data in Tables 6.5 to 6.7.

Figure 6.22 The 3rd layer on the mandrel in Figure 6.20, showing the braid calculated using the interaction model. Units: millimeter and degree.

Table 6.7 Values of the ‘simplified model’ for the braid shown in Figure 6.20. Used symbols: See Table 6.6.

layer	$t_{r,mod}$	e: $\frac{t_{r,mod}-t_{r,exp}}{t_{r,exp}}$	$t_{cum,mod}$	e: $\frac{t_{cum,mod}-t_{cum,exp}}{t_{cum,exp}}$
1	1.7	13%	1.7	13%
2	1.6	22%	3.2	17%
3	1.5	16%	4.6	16%
4	1.4	48%	6.0	21%

Table 6.8 Experimental and modeled fiber volume fraction, as percentage, for each run, and the average value for the component. The experimental average component fiber volume fraction $v_{f,c}$ is calculated using the measured thickness.

run	experiment	interaction model	simplified model
1	36%	31%	34%
2	40%	31%	34%
3	38%	31%	34%
4	49%	30%	34%
component avg.	40%	31%	34%

the yarns (~70%) and the low value in the runs (~40%) suggests that there may be ample room for nesting, backing the statement that nesting is significant. Although compaction on previously deposited layers can play a role as well, these data do not provide direct information regarding the extent.

The cover factors are shown in Table 6.9. According to Section 6.6, the simplified model has an unfeasible combination of cover factors because a 100% stem yarn cover factor can only occur in combination with a maximum bias yarn cover factor of 75%.

6.9 Conclusions

A novel meso-scale model is proposed to describe symmetric and non-symmetric, bi- and triaxial, open- and closed-mesh braids with mono-material and hybrid bias yarn groups, assuming a regular braid. Apart from the used yarn materials, the only required inputs for this model are the yarn pitch and the fiber direction for each yarn group. These data can be generated by any braiding model, including analytical, kinematic and finite element models. Hence, the model is generically applicable as an

Table 6.9 *Modeled bias and stem yarn cover factors as percentages.*

run	interaction model		simplified model	
	CF	CF _S	CF	CF _S
1	91%	81%	94%	100%
2	89%	83%	93%	100%
3	88%	85%	92%	100%
4	86%	87%	91%	100%

intermediate step between braiding process simulation and subsequent simulations like those for resin injection in a mold or for structural performance. The yarn fiber volume fraction can be chosen based on experiments, just like the yarn cross-sectional shape which is free to be chosen as long as a relationship between the width and thickness is specified.

As a modeling result, the feasible range of the degree of surface coverage for all yarn groups is visualized for axisymmetric triaxial braids with hybrid bias yarn groups, showing that a full bias yarn coverage cannot be combined with a full stem yarn coverage.

Experimental validation shows that the modeled thickness is underestimated by 8% for a symmetric triaxial open-mesh hybrid braid. The model over-estimates the braid thickness by 34% for a symmetric triaxial open-mesh hybrid braid with four overbraiding runs, likely caused by the neglect of nesting. Hence, for further analysis, nesting must be added to the model for more accurate results. Without a nesting model, the interaction model does not necessarily add value over simpler models.

Chapter 7

Design and demonstration

7.1 Introduction

This chapter focuses on the design of overbraided components as shown on the path in Figure 7.1. An overbraided component design involves many factors to account for. A full surface coverage is often important for a high fiber volume fraction and for avoiding uncovered mandrel regions that can result in resin-rich areas, in turn leading to stress concentrations that can locally weaken the structure. At the same time, an important braiding manufacturing constraint is that the braid does not jam to the extent that the braid does not fit tightly around the mandrel. Given the mandrel geometry, the machine size, and the yarn properties, these constraints and requirements can usually be translated to a range of feasible braid angle distributions, in turn used for generating the take-up speed profile for the braiding machine as described in Chapter 2.

Problem and objective

Simultaneously taking into account multiple design parameters as mentioned above is generally difficult, especially for complex mandrels with a large variation of the mandrel circumference along its length. The effort of using models to assist in this task, either kinematic or finite elements, is sometimes deemed too high [13]. For this reason, resorting to simpler approaches like a design chart may be desirable in at least the earliest design phases.

Figure 7.1 Design-process-material triangle showing the current chapter underlined.

The braiding process provides some flexibility to accommodate the mentioned constraints when using fibrous yarns, because these yarns can be compressed in width below their initial width, enabling a range of braid angles instead of only one, given the mandrel perimeter and the number of yarns. However, the reduction in yarn width usually results in a simultaneous increase in yarn thickness as explained in Section 6.3, in turn increasing the out-of-plane yarn waviness or ‘crimp’, possibly degrading structural properties like strength. To the best knowledge of the author, currently, all overbraiding design charts with the braid radius as a variable assume a fixed yarn width.

The objective of this chapter is to propose a biaxial braiding design chart for mandrels with a variable cross-section, indicating the degree of surface coverage, jamming, the effect of a yarn width reduction, and yarn crimp.

Due to the limited use of such a chart for more complex design cases, software including the numerical implementation of a braiding process model based on all preceding chapters is proposed as well.

Outline

First, earlier work on design for braiding is reviewed. Next, a new design chart is proposed for mono-material biaxial braids, including the effect of the optional yarn width reduction. Furthermore, the design chart is used for an overbraided design case study. Subsequently, the software Braidsim is introduced, implementing most of the models from the earlier chapters. Next, Braidsim is applied to a more complex example design. Finally, an overall discussion is presented.

7.2 Earlier work

The following authors have published braiding design charts in chronological order. All charts with a variable braid radius assume a fixed yarn width.

Ko [29] proposed several design charts, yet only for a single fixed-width 12K carbon yarn.

Rosenbaum [8] proposed a design chart for a single braid diameter, showing the minimum and maximum possible braid angle as a function of a full surface coverage and jamming, respectively, where jamming is represented by a maximum yarn width reduction.

Lee [88] proposed a design chart with the maximum braid angle as a function of the dimensionless braid diameter for different cover factors and the corresponding fiber volume fractions.

Carey [23] proposed a design chart with the maximum braid angle as a function of the dimensionless braid diameter for different cover factors, similar to [88].

The following authors have published braiding simulation software.

Gao, Ko, and Hu [32] described software for modeling axisymmetric braids using a series of approximating 2D segments with a constant radius. Inputs are the braid type (either bi- or triaxial), the number of carriers as the machine size, the yarn and matrix material, and the radius, length and braid angle for each mandrel segment. For the yarn properties, the engineering constants and densities are input, as well as the minimum and maximum yarn width-to-thickness aspect ratio to evaluate jamming and the fiber volume fraction. The output for each mandrel segment is the thickness, fiber volume fraction, cover factor,

jamming, the bias yarn length and the elastic properties of the braid. The latter approximates the yarn groups as unidirectional lamina. The segment-wise machine speeds are output using the classical solution from Equation (2.4).

Y. Kyosev [89] developed ‘TexMind Braider Standard’ and ‘TexMind Braiding Configurator’. A kinematic model is used to simulate the braiding process for circular and flat braids with a constant cross-section on the component scale. It provides a horn gear and spool configurator for any braiding machine, so it is not limited to circular braiding machines. Machines are either predefined from a database or user-defined. The software can be used, for example, to detect possible spool collisions and it enables the 3D meso-scale visualization of the resulting braid pattern. The meso-scale yarn centerlines are generated by tracing the spool path through the horn gears while the virtual machine is moved through space. This is primarily for visualization purposes, yarn interaction is not included. The yarn surfaces can be exported as triangulations in various formats for further downstream processing.

Dassault Systèmes [34] is developing the software ‘Composites Braiding Designer’, a kinematic braiding process software capable of processing complex shaped mandrels using a graphical user interface, embedded in CATIA V5-6 and 3DEXPERIENCE. Outputs are, among others, the fiber paths, the braid angle error, thickness, degree of coverage, and concavity. It features the optimization of the take-up speed profile as well as the path of the mandrel through the machine for more accurate braid angles and it integrates with downstream structural analysis tools.

7.3 Biaxial braiding design chart

In order to simultaneously see the effect of parameters like the machine size, yarn width, mandrel perimeters and braid angles, a dimensionless design chart is proposed for the relatively simple set of mono-material biaxial braids, extending the chart proposed by Lee [88]. Using dimensionless parameters only, this graph can help to select, among others, a yarn size that satisfies the manufacturing constraints.

Before presenting the design chart, define the dimensionless ‘yarn width

reduction factor' $f_{w,y}$, similar to [8],

$$f_{w,y} \equiv \frac{w_y}{w_{y,\text{ini}}} = \sqrt{\frac{\mathcal{R}_y}{\mathcal{R}_{y,\text{ini}}}}, \quad (7.1)$$

with w_y as the yarn width, $w_{y,\text{ini}}$ as the initial yarn width, $\mathcal{R}_y$ as the yarn width-to-thickness aspect ratio and $\mathcal{R}_{y,\text{ini}}$ as the initial yarn width-to-thickness aspect ratio. The equality in Equation (7.1) is inferred from Equation (6.7), assuming a constant yarn cross-sectional area A_y . Analogously to $\mathcal{R}_{y,\text{cr}}$ defined in Section 6.3, $f_{w,y,\text{cr}}$ denotes the critical (minimum) possible yarn width reduction factor.

For a symmetric biaxial closed-mesh mono-material braid, Equation (6.14) reduces to $w_y = p$. Combining this with Equations (2.8) and (7.1) and solving for α ,

$$\alpha = \arccos \left[f_{w,y} \cdot \left(\frac{P_m}{n_y w_{y,\text{ini}}} \right)^{-1} \right], \quad (7.2)$$

with the fraction as the 'dimensionless mandrel perimeter' with P_m as the mandrel perimeter, n_y as the number of yarns per yarn group, and $w_{y,\text{ini}}$ as the initial yarn width. These parameters are usually fixed for the full duration of an overbraiding run. Equation (7.2) is plotted in Figure 7.2.

Assuming a theoretical minimum of $\mathcal{R}_y = \mathcal{R}_{y,\text{cr}} = 1$ when jamming and a theoretical maximum of $\mathcal{R}_{y,\text{ini}} = 1000$ due to yarn manufacturing constraints as motivated in Section 6.3, the area with $f_{w,y} < \sqrt{1/1000}$ is excluded. To prevent open-mesh braids as motivated in Section 6.1, and neglecting yarn spreading, the area with $f_{w,y} > \sqrt{1/1}$ is excluded too. Hence, in general, the chart area corresponding to $\sqrt{1/1000} \leq f_{w,y} \leq \sqrt{1/1}$ is the maximum possible *theoretical* design space without jamming and with a full surface coverage. The benefit of the theoretical nature is that no physical experiments are required. The practical design space is more restricted as determined next.

For a design problem at hand, the curve of the theoretical critical yarn width reduction factor $f_{w,y,\text{cr}}$ provides the maximum theoretically possible braid angle to avoid jamming conform Section 6.1, further bounding the design space. This curve is obtained using Equation (7.1) with $\mathcal{R}_{y,\text{ini}}$ for the chosen yarn and $\mathcal{R}_y = \mathcal{R}_{y,\text{cr}} = 1$ for fibrous yarns as the theoretical onset of jamming. Hence, an increase of $\mathcal{R}_{y,\text{ini}}$ increases

Figure 7.2 Biaxial braiding design chart with the braid angle as a function of the dimensionless mandrel perimeter using Equation (7.1) with different yarn width reduction factors $f_{w,y}$ at various values of $R_{y,ini}$ and using Equation (7.4) with different crimp factors f_{crimp} , showing the crimp in both open- and closed-mesh braids.

the design space, suggesting a yarn that is initially as flat and wide as possible. However, the actual initial yarn width is limited by yarn manufacturing constraints as well as the size of the carrier eyelets that guide the yarns through the tensioning mechanism shown in Figure 1.6. Commercial off-the-shelf yarns as used in experiments throughout this work generally allow a value of $f_{w,y,cr}$ up to roughly 3.

A decreased yarn width, for example caused by an increasing braid angle shown earlier in Figure 6.4, usually corresponds to an increased yarn thickness, in turn increasing the amplitude of the yarn out-of-plane waviness. This waviness or ‘crimp’ can be an important design parameter due to its influence on the structural properties of the braid. The crimp model used here is a sine function as shown in Figure 7.3. In reality, the yarns follows a more complex path, so the model is a coarse approximation. The yarn crimp is quantified by the tangent of the maximum crimp angle as shown in Figure 7.3. The governing equations are derived in Appendix G. As a result, the braid angle can be expressed

Figure 7.3 Yarn crimp in a symmetric biaxial open-mesh mono-material regular braid. The crimp is coarsely approximated by a sine function.

as a function of the yarn crimp,

$$\alpha = \begin{cases} \arctan \left(f_{\text{crimp}} \cdot \left(\frac{P_m}{n_y w_{y,\text{ini}}} \right)^2 \right) & \text{if closed-mesh, } w_y = p \Leftrightarrow f_{w,y} < 1 \\ \arcsin \left(f_{\text{crimp}} \cdot \frac{P_m}{n_y w_{y,\text{ini}}} \right) & \text{otherwise, i.e. } w_y = w_{y,\text{ini}} \Leftrightarrow f_{w,y} = 1, \end{cases} \quad (7.3)$$

with f_{crimp} defined as the ‘crimp factor’,

$$f_{\text{crimp}} \equiv \frac{n_{\text{float}} \mathcal{R}_{y,\text{ini}}}{\pi} \tan \theta, \quad (7.4)$$

containing only constants in the fraction. Equation (7.3) is also plotted in Figure 7.2. These curves show the crimp gradient, providing a direction for minimizing crimp if required for structural reasons. Around $\alpha = 45^\circ$ and 90° , the gradient of $\alpha(f_{\text{crimp}})$ approximates that of $\alpha(f_{w,y})$.

Given a mandrel geometry, the number of yarns per group and the yarn material, the chart can be used to check whether each combination of a local (dimensionless) mandrel perimeter and the corresponding local required braid angle results in a data point that lies within the bounds of the design space.

In general, a small crimp angle is desired to improve structural properties, favoring small f_{crimp} values. For a simultaneous full surface coverage without a specified braid angle, the design space reduces to the curve of $f_{w,y} = f_{w,y,\text{cr}} = 1$, corresponding to yarns without width reduction, in effect uniquely mapping the local dimensionless perimeter to the braid angle. For rigid yarns, $\mathcal{R}_y = \mathcal{R}_{y,\text{ini}}$, also reducing the design space area

to this curve.

Define the ‘mandrel perimeter ratio’ $R_{p,m}$, as the ratio of the largest to the smallest mandrel perimeter,

$$R_{p,m} \equiv \frac{P_{m,max}}{P_{m,min}}. \quad (7.5)$$

If a constant braid angle is required, then a mandrel perimeter that is as constant as possible generally makes it easier to obtain a fully covering and non-jamming braid along the mandrel length. This corresponds to keeping $R_{p,m}$ as close to 1 as possible. Depending on the location of optional overbraiding reversals, $R_{p,m}$ generally has a maximum at the first overbraiding run and decreases over subsequent runs, although this decrease may be insignificant.

Generalized use of the design chart in Figure 7.2 for non-symmetric braids, i.e. braids with distinct properties for the bias yarn groups on mandrels with arbitrary cross-sections changing along the length may offer an acceptable approximation in earlier design phases. This can be at the cost of accuracy, but this is diminished by the practical guideline to move the mandrel through the machine while the fell front remains centered in the machine for a more homogeneous yarn distribution. Also the risk of yarn slack and slip after deposition from Section 2.4.4 is not taken into account. Slip depends on the chosen path between two chronological data points in Figure 7.2. Sudden braid angle transitions can cause large in-plane geodesic curvatures, in turn causing slip. This can sometimes be reduced by a more gradual braid angle transition.

7.3.1 Error in braid angle and thickness

Significant deviations of the braid angle from the nominal value are generally inevitable for complex braiding setups in practice as shown in Chapter 2. As a consequence, the thickness usually has an error too. The goal is to graph the thickness error as a function of the expected braid angle error and vice versa as a quick practical reference. The thickness error can be used to see if it falls within the required tolerance bounds related to e.g. the mold cavity design or the assembly of the braided component into the embedding structure.

For mono-material biaxial braids, a coarse approximation of the sensitivity of the yarn pitch p for a braid angle error $\Delta\alpha$ is presented next. Define

the ‘relative yarn pitch error’ e_p as

$$e_p \equiv \frac{\Delta p}{p_{\text{nom}}} \quad (7.6)$$

with p_{nom} as the nominal yarn pitch as shown in Figure 2.6 and Δp as the yarn pitch error, i.e. the difference between the nominal and actual yarn pitch. After substituting p_{nom} with Equation (2.8) and Δp with $\frac{\partial p}{\partial \alpha} \Delta \alpha$ where $\Delta \alpha$ is the braid angle error, in turn substituting $\frac{\partial p}{\partial \alpha}$ with the derivative of Equation (2.8),

$$e_p = -\tan(\alpha) \Delta \alpha. \quad (7.7)$$

Hence, the relative yarn pitch error e_p can be conveniently approximated using braid angle data only. Similarly, define the ‘relative thickness error’ e_t as

$$e_t \equiv \frac{\Delta t}{t_{\text{nom}}} \quad (7.8)$$

with t_{nom} as a nominal thickness and Δt as the thickness error, i.e. the difference between the nominal and actual thickness, where ‘thickness’ refers to either the yarn, run, or component thickness. Assuming a closed-mesh braid, combining Equations (6.2), (6.14) and (7.7), and rearranging,

$$e_t = \frac{\tan(\alpha) \Delta \alpha}{1 - \tan(\alpha) \Delta \alpha}. \quad (7.9)$$

Equation (7.9) is plotted in Figure 7.4, and, after solving for $\Delta \alpha$, in Figure 7.5. These relative errors are independent of the mandrel perimeter, the machine size, and the yarn material, shape and size. For actual braiding configurations, optionally non-axisymmetric, this analysis is a simplification that neglects, among others, nesting and braid angle changes near mandrel edges parallel to the mandrel centerline as shown earlier in Section 3.2.

An example of the application of the error analysis involves the exhaust overbraiding experiment in Section 2.6. If this experiment would have had a mono-material braid with a full surface coverage and $\alpha_{\text{req}} = 60^\circ$, then Figure 7.4 shows that the measured braid angle error of $\Delta \alpha_{\text{max}} \approx 10^\circ$ would correspond to a relative thickness error $e_{t,\text{max}} \approx 45\%$.

Figure 7.4 The relative thickness error e_t as a function of the braid angle α at various signed values of the braid angle error $\Delta\alpha$, assuming an axisymmetric biaxial closed-mesh mono-material braid for both the nominal and actual braid.

7.3.2 Design example*

Pressure vessel manufacturing is currently dominated by the filament winding process [15, 91]. To avoid slip over the mandrel in filament winding, the yarn trajectory must be close enough to the geodesic path on the mandrel, i.e. corresponding to the shortest path between two points on the surface. At the vessel poles, the structural optimum fiber angle is 90° , while at the cylindrical part, the optimum angle is around 54.6° as a result from netting analysis [92], here taken slightly lower for a maximum burst pressure [93]. For long vessels made with by conventional filament winding, it is generally not possible to satisfy both angles without slip. However, the interlacement of a braid allows a larger deviation from the geodesic path, better enabling the transition between the two angles with a reduced chance of yarn slip.

*Based on [25, 90].

Figure 7.5 The braid angle error $\Delta\alpha$ as a function of the braid angle α at various values of the signed relative thickness error e_t , assuming an axisymmetric biaxial closed-mesh mono-material braid for both the nominal and actual braid.

The braiding design chart is used for the case of the pressure vessel. The vessel is a biaxially overbraided cylinder with spherical end caps as shown in Figure 7.6 and parametrized with the values in Table 7.1. Each yarn group is mono-material. The design chart with the corresponding design space is shown in Figure 7.7. The complexity for analyzing the following design configurations is relatively low because the choice of the machine and the braiding materials is not yet explicitly needed.

Design configuration 1 starts with satisfying the requirements at the cylindrical segment as shown in Figure 7.7. The right point, corresponding to this segment, is fully defined by $\alpha = 50^\circ$ and $f_{w,y} = 1$. Next, shift a factor $R_{p,m} = 4$ to the left and intersect with the curve

Table 7.1 Pressure vessel design parameters.

parameter	value
mandrel perimeter ratio $R_{p,m}$ from Equation (7.5)	4
braid angle α at pole ($r_{m,min}$)	90° [92]
braid angle α at cylinder ($r_{m,max}$)	50° [93]
critical yarn width reduction factor $f_{w,y,cr}$ from Equation (7.1)	$\sqrt{1/10}$

Figure 7.6 Schematic pressure vessel liner (mandrel) geometry. Only one half of the axisymmetric vessel geometry is shown.

$f_{w,y} = f_{w,y,cr} = \sqrt{1/10}$ to obtain the point corresponding to the pole. As a result, the maximum possible pole braid angle without jamming is roughly 36° for this configuration, which is far off the optimum 90° . An error analysis using Figure 7.4 shows that a hypothetical braid angle error of $\Delta\alpha_{max} = 10^\circ$ at the cylinder and pole corresponds to the relative thickness errors of roughly 15% and 25%, respectively.

Alternatively, the number of yarns per group n_y and the initial yarn width $w_{y,ini}$ can be reduced to enable a larger pole braid angle. Design configuration 2 makes use of this approach, this time starting at the pole, requiring that $\alpha = 80^\circ$ at $f_{w,y} = f_{w,y,cr} = \sqrt{1/10}$. After shifting a factor $R_{p,m} = 4$ to the right, the dimensionless mandrel perimeter corresponding to the cylindrical segment is obtained. For this segment, the requirement of $\alpha = 50^\circ$ is retained for structural reasons at the cost of a full coverage. In this case, a full surface coverage on the cylindrical segment can only be obtained by relying on nesting of subsequent braid layers. An error analysis shows that the thickness at the pole is very sensitive to braid angle errors, resulting in a thickness error of 100% when assuming $\Delta\alpha_{max} = 5^\circ$. For an acceptable thickness error of 10%, Figure 7.5 shows that the braid angle error can be at most roughly 1° , which is difficult to achieve as shown by the experimental results in Chapter 2. For the cylinder, the presented error analysis cannot be used because the assumption of a full surface coverage is not satisfied.

All possible configurations for a given value of mandrel perimeter ratio $R_{p,m}$ have the same horizontal shift between the minimum and maximum

Figure 7.7 Design chart for the first overbraiding run of the pressure vessel. The pole and cylindrical region are indicated for two design configurations, given a fixed mandrel geometry.

dimensionless mandrel perimeters due to the log scale of the graph. This makes it easy to visually evaluate any possible configuration without the need for equations.

In any configuration of the given example, the crimp angle at the pole is relatively large compared to other locations in the design space as shown by the f_{crimp} -curves. Hence, this analysis shows that it is impossible to achieve an optimum composite layup for a pressure vessel using circular overbraiding.

This example shows that the design chart can be used to obtain a quick assessment of the potential of braiding for different braiding design configurations without the need of the machine size or the yarn width while requiring the mandrel perimeter ratio $R_{p,m}$ as the only parameter to describe the mandrel.

Table 7.2 Simplified Braidsim input parameter list. Legend: α : braid angle; ω : carrier rotation speed; yarn: see Table 6.1; E: Young's modulus; G: shear modulus; ν : Poisson ratio; L: longitudinal; T: transverse; ρ : matrix mass density; r_{sp} : spool plane radius; n_c : number of carriers per group; h_{gr} : guide ring height; r_{gr} : guide ring radius

input	parameter(s)
mandrel	*.stl; *.inp (Abaqus) file format
for each group	spool pattern; material pattern (either mono-material or hybrid)
centerline	x/y/z-axis or a curved polyline
for each run	start and end plane; α ; optionally a constant ω
for each yarn	ρ_f ; $\rho_{l,s}$; n_s ; $v_{f,y}$; $w_{y,ini}$; $R_{y,cr}$; E_L ; E_T ; G_{LT} ; G_{TT} ; ν_{LT}
matrix	ρ ; E; G; ν
machine	r_{sp} ; n_c ; ω ; $h_{gr,in}$; $h_{gr,out}$; $r_{gr,in}$; $r_{gr,out}$
spool motion	serpentine / circular
mode	conversion / simulation / optimization
time step size	typically 0.01 to 0.1s
output	element type: shell / continuum shell / continuum
ply stacking	top ply (X/O) or symmetrical layup

7.4 Braidsim[†]

Design charts such as proposed in Section 7.3 are difficult to create for more complex braids with hybrid yarn groups and stem yarns on complex mandrels. Instead, the braiding process can be simulated for individual design configurations. For this purpose, the models presented in this work have been implemented in the software 'Braidsim' [33]. Braidsim takes the data listed in Table 7.2 as input, and can output

- the machine numerical control,
- the full component finite element layup,
- braiding process animations, and
- 3D plots of the distributions of the parameters in Table 7.3.

The animations and 3D plots are output in the format of the free and open source pre- and post-processing software 'Gmsh' [60].

The mandrel must be provided as a surface mesh of triangular or quadrilateral facets as shown in Figure 7.8. Quadrilateral elements are split into a pair of triangles before processing, and for each output

[†]Based on [23].

Table 7.3 Simplified list of visual Braidsim output per combination of a substrate surface facet and a: (M)aterial within a yarn group; yarn (G)roup; (R)un; (C)omponent. The list is in alphabetic order. Equation numbers are given where possible.

output	M	G	R	C
areal density ρ_A	6.32			
braid angle α		2.7		
centerline arc length s			•	
cover factor CF			D.5	
fiber unit direction $\mathbf{f}$		•		
fiber volume fraction v_f			6.30	
mass of fiber, matrix, total			•	•
optim. weight factor f_{wt}			•	
perimeter length P			•	
segment ID			•	
slip direction $\mathbf{d}_{slip}$		2.13		
slip tendency c_{slip}		2.11		
thickness	6.26		6.27	6.28
yarn aspect ratio $\mathcal{R}_y$	6.5			
yarn pitch p		•		
yarn slack length $s_{rew,max}$		2.18		
yarn width w_y of bias, stem	6.11,6.15			

parameter they are assigned the average value of their triangle pair. The mandrel centerline can be optionally curved and is followed by the braiding machine center.

Section planes can be used to define the start and end of overbraiding runs or for defining segments with different braid properties. The section planes can be used to split intersecting facets, using the generated intersection edges for the purpose of run or segment boundaries. If the input mesh is not allowed to be modified due to requirements in downstream processing, then the facets can be left unsplit, in which case the facet edges closest to the section plane are used instead.

The machine can be an axial or radial conventional circular braiding machine with four horns per gear with a default or a custom spool pattern.

Braidsim can run in three modes as shown in Table 7.4: conversion, simulation, and optimization. In conversion mode, the braiding process is

Figure 7.8 *Triangular mandrel surface mesh and section planes containing contours for bounding overbraiding runs.*

approximated as axisymmetric and analytical solutions are used instead of kinematics, so the process is not modeled using time stepping. This mode is typically used to generate a composite layup for a downstream structural analysis, resulting in a smooth braid and preventing the artificial braid property fluctuations that are inherent to the other modes. The required braid angle distribution is provided as an input, either explicitly or implicitly by requiring a full mandrel coverage, in turn defining the braid angle according to the curve of $f_{w,y} = 1$ in Figure 7.2. Given the mandrel perimeter P_m as a function of the mandrel centerline position, Equation (2.8) is used to calculate the yarn pitch distribution that is assumed uniform in the mandrel circumferential direction, which is only a coarse approximation for non-axisymmetric configurations. Consequently, all resulting output braid properties are circumferentially uniform too. The output of the conversion model is therefore generally highly idealized. If structural analysis shows that such a braided design does not pass the structural requirements, then a redesign of the braided component can be prioritized over performing more detailed kinematic simulations, although exceptions to this rule of thumb may exist. This

Table 7.4 *Available Braidsim modes with simplified inputs and outputs. Here α is the braid angle distribution as a function of the mandrel centerline position s . Instead of $\alpha(s)$, a cover factor $CF = 1$ can be specified. The carrier speed $\omega(t)$ is assumed constant and given.*

mode	input	output
conversion	$\alpha(s)$	-
simulation	$v(t)$	$\alpha(s)$
optimization	$\alpha(s)$	$v(t)$

Figure 7.9 Braidsim process animation snapshot with a triaxial braid. The carriers follow a serpentine path for visualization purposes only.

mode is especially useful for cases with dozens of overbraiding runs where any other simulation approach is unfeasible.

The ‘simulation’ and ‘optimization’ modes use the kinematic models called the ‘forward’ and ‘inverse’ solution, respectively, from Chapter 2. A kinematic process animation snapshot is shown in Figure 7.9. Figure 7.10 schematically shows the process steps for both kinematic modes. An overview of the main practical purposes of the different modes is shown in Table 7.5.

Table 7.5 The main purposes for the conv(ersion), sim(ulation), and opt(imization) modes in Braidsim.

Goal↓ Mode→	conv	sim	opt
generate input for structural analysis	•	•	•
generate <i>idealized</i> input for structural analysis	•		
evaluate the effect of different take-up speed profiles		•	
generate braiding machine take-up speed profiles			•
generate braiding process animations		•	•

Figure 7.10 Simplified schematic overview of the kinematic process steps in Braidsim with an example mandrel and braid angle distribution. Only the upper half of an axisymmetric process is shown.

7.4.1 Example

A single biaxial mono-material overbraiding run is processed in Braidsim with the exhaust mandrel and the machine as used in Chapter 2 and the primary bias yarn material from Table 6.2 as the only material.

The objective is a full mandrel coverage with a run thickness that is as uniform as possible across the component with a minimum yarn crimp, in turn corresponding to a minimum reduction in the yarn width. For an axisymmetric mandrel, this would correspond to the curve of $f_{w,y} = 1$ in Figure 7.2.

For comparison purposes, Braidsim is run in all three modes. In ‘conversion mode’, the process is modeled as axisymmetric, and the braid angle is varied to achieve the uniform component thickness at a full surface coverage.

In ‘simulation mode’, the ‘forward solution’ from Chapter 2 is applied, requiring the machine take-up speed profile as input, given the known carrier speed from Table 6.2. Although trial and error can be used to generate this profile, the take-up speed was input as constant for simplicity. The take-up speed is calculated as follows. The average of the minimum and maximum mandrel perimeter is used to represent the approximate overall mandrel perimeter P_m . Next, Equation (2.8) is solved for the braid angle α , requiring that the yarn pitch equals the given initial yarn width from Table 6.2 to obtain a full coverage, so $p = w_{y,ini}$. Finally, given this braid angle and the carrier speed from Table 6.2, the take-up speed follows from the classical solution in Equation (2.4). The start section of the overbraiding run is the leftmost section shown in Figure 7.8. The initial machine position relative to the mandrel start section as shown in Figure 7.10 (step 3) is obtained using trial and error. The time step size Δt is initially chosen as shown in Table 7.6. The resulting simulation mode parameters are shown in Table 7.6.

Finally, in ‘optimization mode’, the ‘inverse solution’ from Chapter 2 is

Table 7.6 *Simulation mode input parameters.*

parameter	value
start offset	500mm
take-up speed v	12.4mm/s
time step size Δt	0.01s

Figure 7.11 The component thickness in both kinematic modes. In conversion mode, the thickness is uniformly 0.58 mm.

applied. This mode generates the initial machine offset from the run's start section as well as the take-up speed profile. The same time step size is used.

The results of the three modes are compared and discussed next.

7.5 Results and discussion

The resulting component thickness is shown in Figure 7.11. For both kinematic modes, this figure shows diagonal bands of variable thickness. In the conversion mode, the thickness is uniform without fluctuations and equals twice the initial yarn thickness to account for both bias yarn groups.

Similar bands appear for the cover factor in Figure 7.12. In conversion mode, the surface is fully covered without fluctuations. The bands again show for the fiber volume fraction in Figure 7.13. In conversion mode, the fiber volume fraction equals that of the yarn, again without fluctuations. Note that the thickness and fiber volume fraction shown in these figures do not linearly map to each other: For example, an area of uniform thickness may contain a variable fiber volume fraction due to a variable cover factor. The mentioned fluctuations may look like the result of yarn slip, but this is not the case because yarn slip is not modeled. The fluctuations are discussed next in greater detail because they can have a significant impact in downstream processes in the virtual design chain like structural analysis.

Figure 7.12 The cover factor in both kinematic modes. In conversion mode, the value is uniformly 1.

Figure 7.13 The component fiber volume fraction in both kinematic modes. In conversion mode, the value is uniformly 0.61.

Changing the time step size Δt while keeping all other parameters the same can have a large effect of the results such as the component thickness. This effect is numerically illustrated in Figure 7.14. This graph plots the maximum thickness of the braid on the mandrel. The minimum component thickness is less interesting because it is fixed at the value around 0.58 mm due to the way the thickness is calculated as explained in Chapter 6, including the neglect of yarn spreading. Above $\Delta t = 0.1s$, Figure 7.14 shows that the maximum thickness and its fluctuation increase significantly.

Similarly, the effect of the time step size on the fiber volume fraction is shown in Figure 7.15. This graph shows that the minimum fiber volume fraction decreases significantly with an increasing time step size.

The fluctuation of the thickness and the fiber volume fraction are

Figure 7.14 The maximum component thickness as a function of the chosen time step size in Braidsim.

Figure 7.15 The minimum fiber volume fraction as a function of the chosen time step size in Braidsim.

examples of fluctuations that are caused by the corresponding fluctuation of the yarn pitch shown in Figure 7.16, reducing the smoothness of the yarn pitch distribution across a large part of the mandrel surface. This yarn pitch fluctuation cannot be explained by the braid angle error from Equation (7.7) because the yarn pitch errors are an order of magnitude larger than those predicted by this equation for the case at hand. To examine this phenomenon further, a snapshot in time of the process state corresponding to the generation of the pair of low and high pitch bands in Figure 7.16 is shown in Figure 7.17. This illustrates that the pitch fluctuations are caused by the applied deposition model from Chapter 2: The deposition model only detects yarn-to-mandrel contact in the direct

Figure 7.16 Close-up showing the yarn centerlines and the thickness fluctuation corresponding to the shown yarn pitch fluctuation using a small and a very large time step size.

neighborhood of the fell point, neglecting possible contact away from the fell point and neglecting the corresponding physically impossible intersections shown in Figure 7.17. In this case, the locally concave (hollow) shape of the mandrel triggers these intersections.

A yarn deposition model that checks for contact globally instead of only locally could, in some cases, result in a more realistic fiber distribution. In reality, a concave mandrel region can also be subject to bridging as introduced in Section 1.6, depending on, among others, the extent of yarn slip. A global contact search may also provide an indicator of bridging. However, in general, yarn slip over the substrate should be taken into account as well because the first globally detected contact may only be the initial position of a yarn that may subsequently slip to a different position on the mandrel. Modeling such complex behavior requires time-consuming computational effort that defeats the purpose of a fast braiding process model. Hence, the artificial yarn pitch fluctuation is the price paid for a fast braiding process model.

In this example, the carrier speed is $36^\circ/\text{s}$ around the mandrel. Choosing a relatively large time step size of e.g. 1s results in a relatively large

Figure 7.17 Weft yarns temporarily intersect the mandrel surface at the indicated locations, resulting in the fluctuation indicated in Figure 7.16 for $\Delta t = 0.01s$. The warp yarns are left out for clarity.

carrier movement increment. In turn, this increases the yarn length that artificially intersects the mandrel. This phenomenon at very large time steps does not restrict itself to complex mandrel shapes, but also manifests itself for centered axisymmetric mandrels. For the current example, the result is a relatively large systematic error of the output parameters above $\Delta t = 0.1mm$ shown in Figures 7.14 and 7.15. For typical braiding machine carrier speeds, a time step size of between 0.01 and 0.1s is usually small enough to significantly reduce such systematic errors, although this choice can also be influenced by the mandrel geometry and the mesh quality of the mandrel surface. Convergence studies for the cases in this work show that a time step size of 0.01s is an acceptable compromise between accuracy and the calculation time, in all cases at most a few seconds per overbraiding run.

In turn, the fluctuation of the yarn pitch can also affect the distribution of the optional stem yarns. The stem yarn centerline positions are calculated as a function of the bias yarns, so bias yarn irregularities can result in stem yarn irregularities, predominantly resulting in artificial in-plane stem yarn waviness, in turn artificially reducing subsequently predicted structural properties in the mandrel length direction like stiffness and perhaps buckling strength.

The problem of yarn pitch fluctuations shown in this example can be

avoided by using analytical solutions instead of a yarn-by-yarn deposition, providing the primary motivation for proposing the 'conversion mode' as described in Section 7.4. Alternatively, if the yarn pitch fluctuation only consists of very localized outliers, then this fluctuation can be numerically 'smoothed' by Braidsim. If the chosen number of smoothing passes is very high, then the smoothing algorithm can result in inaccurate and nearly globally uniform values of the yarn pitch and all other braid properties. Hence, the chosen amount of smoothing is a trade-off between smoothness and accuracy. An acceptable number of smoothing passes depends on various factors such as the mandrel geometry and the mesh quality.

In reality, local yarn slip over the substrate, for example near mandrel corners, can lead to yarn pitch fluctuations too, in turn resulting in locally low degrees of coverage. Hence, the modeled pitch fluctuations are caused by numerical side-effects while the actual fluctuations are caused by physical slip. A clear understanding of the modeling approach is necessary to distinguish between these two phenomena. It can be tempting to use the numerical pitch fluctuations as a representation of the actual ones. However, although the fluctuation magnitude can be in the same order for both types, their location on the mandrel surface generally does not match. It depends on the specific case at hand to what extent the modeled braid can provide an accurate structural representation of the actual braid.

7.6 Conclusions

A novel dimensionless design chart is proposed for biaxial mono-material braids, showing the feasible design space, bounded by the generally undesirable phenomena of jamming and insufficient surface coverage, for a given mandrel size, machine size, material, braid angle and crimp. The extent to which a yarn can be reduced in width is a key parameter in this chart, showing the possible flexibility of the braiding process. The chart enables a quick feasibility assessment prior to any effort of modeling or experimentation.

Charts were proposed that map the braid angle error to the thickness error and vice versa as a function of the nominal braid angle for biaxial mono-material braids. All charts can be used for any braid, yet at

a decreasing accuracy for braids that increasingly deviate from the assumption of axisymmetry.

The kinematic braiding process model without yarn interaction as proposed in this work was implemented in the software 'Braidsim'. Braidsim can generate both the forward and inverse solution, so the braiding process can be simulated with the machine speeds as input and the braid angle distribution as output and vice versa. The output of the kinematic models shows artificial fluctuations of the output parameters over the mandrel surface. These fluctuations are mainly caused by yarn pitch fluctuation, in turn caused by the yarn deposition model. The influence of the time step size was assessed, suggesting that a sufficiently small time step can significantly reduce these fluctuations of the braid properties. The fluctuations can be further reduced by smoothing or even avoided using analytical instead of kinematic models, approximating the process configuration as axisymmetric. A 'conversion mode' was added to Braidsim to implement this analytical approach.

Chapter 8

Discussion

8.1 Overview

The objective of this research as defined in Section 1.7 was the development of tools for the design of overbraided components with complex shaped mandrels.

For this purpose, the circular overbraiding process was modeled in an increasing level of detail throughout this work, resulting in braiding process design charts and a fast kinematic braiding process model. This model enables the simulation of braiding over complex mandrel shapes with symmetric and non-symmetric, bi- and triaxial, open- and closed-mesh braids with mono-material and hybrid bias yarn groups and arbitrary variable braid angles and yarn pitches, and evaluating braid property distributions like the thickness and the fiber volume fraction of these braids. Supported complex mandrel shapes include non-axisymmetric, optionally eccentric cross-sections that can vary in shape and size along an optionally curved mandrel centerline.

The individual constituents of the model are discussed next, following the spiraling path in Figure 8.1.

Figure 8.1 *Design-process-material triangle showing the content of each chapter.*

8.2 Circular braiding take-up speed generation using inverse kinematics

In Chapter 2, a novel ‘inverse kinematic’ model was proposed to generate the take-up speed profile for circular braiding machines with optional guide rings, given a braid angle distribution on mandrels with complex 3D shapes. This model is independent of the chosen yarn materials and the chosen braid pattern. This chapter mainly focused on biaxial braids and neglects yarn interaction and slip after deposition. As a consequence, the computational time complexity can be in the order of seconds per overbraiding run, making it fit for CAE process chain integration and for use in earlier design phases.

Experimental validation with a complex shaped mandrel shows an experimental scatter of 7 degrees and an error up to 10 degrees, mainly caused by the neglect of yarn interaction in the model, in turn artificially increasing the convergence zone length. For conceptual component designs, such an accuracy is likely sufficient. For a detailed design, this is less likely the case. In either case, this depends on the design problem at hand.

The spool movement was modeled as circular. The superposition of the serpentine spool motion was found to have negligible effect on the modeled results for the cases studied in this work.

The machine origin is constrained to coincide with one of the global mandrel coordinate system axes or the mandrel centerline in the current implementation. The machine z-axis is constrained to remain either tangent to the mandrel centerline or one of these axes. In other words, the path and orientation of the machine coordinate system are inputs. This can result in an eccentric position of the fell front for complex mandrels, which is usually undesirable. A more centered fell front can be obtained by removing the mentioned constraints and generating the path of the machine origin and perhaps also the machine orientation as an output as a function of time. A recent effort of such an approach is demonstrated in [94].

8.3 The effect of the mandrel shape on the take-up speed profile

In Chapter 3, the generation of take-up speed profiles using the inverse kinematic model from Chapter 2 was applied to case studies to illustrate and understand its capabilities. The case studies involved various mandrel shapes, including mandrels with large cross-sectional size changes, curved centerlines and step-wise changes in either the braid angle or the mandrel slope in length direction. The model accommodates these features and includes the optional assignment of a master side and a master yarn group because sometimes the braid angle cannot be satisfied on all mandrel sides simultaneously, at least not according to the model. The generated take-up speed profiles showed local deviations from what might be expected and what would be obtained when using only analytical solutions like the classical solution. For example, the take-up speed profile can contain local reversals of the take-up speed direction. These profiles resulted in modeled braid angle distributions that are close to the required ones. In turn, these profiles can serve as an initial guess to guide the machine operator to find the best take-up speed profile on a physical machine. The difference between the predicted take-up speed profile and the best profile that is found by further experimental refinement is expected to increase with the yarn density and the mandrel

shape complexity due to the neglect of yarn interaction and slip after deposition in the model.

The carrier speed was always assumed to be constant while braiding throughout this work. However, if there are step-wise changes within the required braid angle distribution or in the mandrel slope in the length direction of the mandrel, then it may be more practical that the carrier speed is temporarily set to zero while continuing with a non-zero take-up speed in order to achieve acceptable accelerations.

Small irregularities in the mandrel surface mesh propagate to the fiber streamlines on that surface. The generated take-up speed profile contains noise which is a consequence of a high sensitivity to these irregularities. Mesh refinement can partly solve this problem. To further reduce this noise, the fiber streamlines can be smoothed by e.g. spline-fitting.

8.4 A yarn interaction model for circular braiding

In Chapter 4, a yarn interaction model was developed for the axisymmetric biaxial braiding process to calculate the effect of yarn interaction as compared to the kinematic model without yarn interaction. The model calculates the yarn shape in the convergence zone and the convergence zone length. A parametric study using the frontal approach suggests that the difference is mainly influenced by the coefficient of friction. Experimental validation of the yarn interaction model shows that an apparent coefficient of friction value around 0.3 provides the closest match between model and experiment. This resulted in a convergence zone decrease around 25% with respect to the frictionless model for the studied case. This confirms that yarn interaction significantly influences the convergence zone length. Hence, when generating machine control data for accurate results, neglect of this change in convergence zone length can cause significant braid angle errors.

This yarn interaction model only describes axisymmetric biaxial braiding. Generalization to more complex braiding configurations requires algorithms that are probably too slow for design purposes and too specialized to replace existing general-purpose FE approaches with similar calculation times. Hence, more braiding process-specific development in this direction probably adds little value for designing braided components.

Instead of developing more complex models, the forward and inverse solution as well as the meso-scale models proposed in this work require further experimental validation, possibly providing empirical case-specific correction factors to adjust the current modeling results. These correction factors can account for bi- or triaxial yarn and guide ring interaction at complex mandrel shapes at acceptable calculation times, including optional stem yarn tension effects. This may reduce the required number of manual adjustments of the generated take-up speed profile, in turn reducing physical production trials.

The main limitations of this yarn interaction model and many other braiding simulation models, including FE models, are that they do not capture the effect of broken, detached and entangled fibers from other yarns. It is unlikely that these phenomena will soon be simulated with short calculation times on the full braiding process scale, but empirical correction factors can be used to perhaps improve models.

For larger braiding machines, gravity effects can become more pronounced, leading to a varying braid angle around the mandrel. A finite element approach is preferred to capture this phenomenon if such a level of detail is required.

Although a series of force-equilibrium-based yarn shape states can be calculated along an FSL on an axisymmetric mandrel with a variable radius, the model does not predict whether the *non*-steady-state transition between any one state to the next is physically feasible.

8.5 Braid patterns

In Chapter 5, procedures were proposed to enable a theoretical and a practical search to find the spool pattern that results in the desired bi- or triaxial braid pattern. This includes a novel algorithm to check the feasibility of a biaxial braid pattern for a given circular braiding machine, generating a corresponding spool pattern if feasible. As a result, the translation between biaxial spool patterns and braid patterns is tabulated for a range of spool patterns on commonly used circular braiding machines. Such tables can be used in practice or in braiding simulation software. Another benefit of the pattern analysis is that the yarn trajectories of optional stem yarns can be readily generated as a post-processing step using the topology of the bias yarns combined with the

bias yarn centerline paths as generated using kinematic braiding process simulation.

The approach for converting a required biaxial braid to a spool pattern, given the machine and component size, requires further numerical implementation. Generalization is possible by generating the machine size as an output for a given mandrel size instead of an input, or by generating the number of slots per horn gear, the initial carrier positioning over the slots, etc, based on the required properties of the braided component. The ultimate generalization would be the ‘synthesis’ of a machine construction that produces a given required braid pattern where, optionally, each gear has a different number of slots, or the proof that such a solution does not exist for a given braid pattern. For example, a circular braiding machine capable of producing a 4-harness satin has not been identified in this work, nor has it been shown that such machine construction does not exist. A reduction of computation time in FE braiding process models may be feasible by using the provided braid topology to narrow down the search space for inter-yarn contact analysis.

8.6 Meso geometry of bi- and triaxial braids

In Chapter 6, a novel meso-scale model was proposed to describe symmetric and non-symmetric, bi- and triaxial, open- and closed-mesh braids with mono-material and hybrid bias yarn groups and arbitrary variable braid angles and yarn pitches on complex mandrels, assuming a regular braid. The used yarn materials, the yarn pitch and the fiber direction of each yarn group are the input as generated by any braiding model. Hence, the model is generically applicable as an intermediate step between braiding process simulation and subsequent simulations. Modeling results show that a full bias yarn coverage cannot be combined with a full stem yarn coverage.

Experimental validation showed that the modeled thickness was underestimated by 8% for a symmetric triaxial open-mesh hybrid single-layer braid. The model over-estimated the braid thickness by 34% for a symmetric triaxial open-mesh hybrid braid with four overbraiding runs. The main cause of the error is deemed to be the neglect of nesting in the model.

The braid meso model proposed in this work assumes a mono-material

stem yarn group. For further generalization, the model can be extended to include a hybrid stem yarn group. The yarn width calculation as currently used for mutual yarn interaction between and within yarn groups only roughly asserts that yarns of different groups do not artificially intersect each other. In addition, the model can account for a more physically correct distribution of the transverse yarn deformation energy by taking into account properties like the local ratio of yarn segment lengths within the unit cell. Another improvement involves the representation of nesting of contacting braid layers, resulting at least in part in a lower predicted run thickness in general. The degree of nesting can possibly be approximated with the aid of the evaluated braid meso geometry, for example increasing the modeled degree of nesting with a decreasing cover factor.

8.7 Design and demonstration

In Chapter 7, the end of the spiraling path Figure 8.1 is met. This chapter aimed at the integration of the preceding chapters. The possibly high threshold for use of any computational braiding process model was recognized. As a partial substitute, a novel dimensionless design chart was proposed for biaxial mono-material braids. The chart visually shows the feasible design space, bounded by the generally undesirable phenomena of jamming and insufficient surface coverage. The chart enables a quick feasibility assessment prior to the effort of any modeling or experimentation. Other charts were proposed that map the braid angle error to the thickness error and vice versa as a function of the nominal braid angle for biaxial mono-material braids. All charts can be used for any braid, yet at a decreasing accuracy for braids that increasingly deviate from the assumptions of axisymmetry and a single yarn material per group.

The kinematic braiding process model and the braid thickness model proposed in this work were implemented in the software 'Braidsim'. Braidsim can generate both the forward and the inverse solution, so the braiding process can be simulated with the machine speeds as input and the braid angle distribution as output and vice versa. Currently, Braidsim is able to represent arbitrary spool patterns on a conventional braiding machine with four horns per gear and the carriers in the 'regular formation'. Further generalization involves different carrier formations

and a different number of horns per gear. However, for practical usage, Braidsim must then also be extended to model the corresponding braid patterns and meso-geometries.

Compared to existing modeling approaches, Braidsim offers significant improvements. That does not alter the fact that Braidsim's assessment of manufacturing feasibility is coarse. The resulting composite layup for downstream structural analysis is only an approximation, resulting in structural properties that possibly only indicate the order of magnitude. Nonetheless, the results can be used for comparative studies.

The mandrel surface usually represents the inner surface of the braided component and is used as the starting point for braiding process modeling. However, sometimes the outer component surface is given instead for a given installation space. For this purpose, the model can be generalized to accept the outer surface as input instead of an output, or to accept both surfaces as an input to generate e.g. the corresponding number of braid layers that fit between these two surfaces as an output.

Currently, the free and open-source Gmsh [60] is used as the graphical 3D graphical user-interface for pre- and post-processing of Braidsim. Acceptance from industry can be increased by instead or additionally integrating the kinematic model into existing desktop or cloud-based CAE platforms that already have an integrated or browser-based 3D GUI. However, internally, Braidsim heavily depends on customized code libraries for e.g. topology, geometry, signal processing, statistics, mathematical solvers, and interfacing with external programs. For a more 'manageable' development, these libraries can be replaced by more mature and recognized third-party alternatives that are likely already integrated, at least partly, in the existing platforms.

Braidsim currently interfaces with Abaqus for downstream structural analysis. This can be augmented by interfacing with downstream resin injection simulation software, using the composite fiber layup including the fiber orientation, width, and thickness distribution as input. This should be feasible because similar software already exists for layups generated with composite forming process simulation software, refer to e.g. [95]. Other examples involve interfacing with machines from various manufacturers and interfacing with meso-scale structural analysis tools for a local structural analysis of critical regions in more detail like structural failure analysis. Finally, the expressions for common braiding constraints like a full surface coverage or avoidance of jamming can be

solution type → mandrel shape ↓	forward: $\alpha = f(v)$				inverse: $v = f(\alpha)$			
	excl. yarn interac.		incl. yarn interac.		excl. yarn interac.		incl. yarn interac.	
	biax	triax	biax	triax	biax	triax	biax	triax
centered cylindrical*	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE
centered rectangle*	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE
axisymmetric	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE
complex [†]	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE
branching	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE	ana. kin. FE

*steady state [†]variable cross-section and/or curved centerline

Figure 8.2 Simplified overview of the available modeling approaches by mandrel shape. Model abbreviations: ‘ana.’: analytical model; ‘kin.’: kinematic model; FE: Finite Element model. The shaded boxes in the columns excluding and including yarn interaction correspond to the model of Chapters 2 and 4, respectively.

used to reduce the solution space of component weight-minimization in structural optimization software.

The contribution of the newly developed kinematic model with respect to analytical and finite element models is shown in Figure 8.2. The empty boxes in this table show that in general, even the set of all modeling approaches has limitations with respect to what designers may require. Although not every possible braiding process configuration can be processed by the proposed kinematic model, the feasible range of configurations includes many mandrel shapes used in industrial practice. For cases involving branching mandrels, a coarse conceptual kinematic simulation can be performed by temporarily removing branches from the virtual mandrel and/or to simulate each mandrel branch separately.

Finally, benchmarking of the proposed kinematic models against finite element models can be performed to make more accurate quantitative comparisons in terms of accuracy and speed.

Chapter 9

Conclusions and recommendations

9.1 Summary of the objectives

The principal objective of this work as defined in Section 1.7 is to develop overbraiding design tools in the form of design charts and kinematic braiding process simulation software. To achieve this, three goals were defined:

1. Develop a procedure to calculate the braiding machine control data for a given braided component design with a required braid angle distribution, and experimentally assess its accuracy.
2. Find the relation between the spool pattern on the braiding machine and the corresponding produced braid pattern.
3. Calculate the resulting braid thickness for bi- and triaxial braids, and experimentally assess its accuracy.

9.2 Conclusions

The following conclusions are drawn from this work regarding the principal objectives:

Novel dimensionless design charts were developed for biaxial mono-material braids. These charts can be used for any braid, yet at

a decreasing accuracy for braids that increasingly deviate from the assumption of axisymmetry and a single yarn material per group.

A fast kinematic braiding process model was implemented in the novel software 'Braidsim'. Braidsim simulates the braiding process on circular braiding machines with or without guide rings for complex shaped mandrels, accepting the braid angle distribution as input and providing the machine speeds as output or vice versa. This model supports braids that are either symmetric or non-symmetric, bi- or triaxial, open- or closed-mesh, and mono-material or hybrid. Braidsim requires a few seconds of computation time per overbraiding run and is fit for CAE process chain integration.

The following conclusions are drawn from this work regarding the stated goals:

1. A novel 'inverse kinematic' model was developed to calculate the take-up speed profile for circular braiding machines, given a braid angle distribution on mandrels with complex 3D shapes. Yarn interaction and yarn slip after deposition are neglected in the model. The calculation time is in the order of seconds per overbraiding run, making it suitable for fit for CAE process chain integration and use in earlier design phases at an accuracy that is likely acceptable. The generated take-up speed profiles show significant local deviations from what might be expected by intuition and what would be obtained when using only analytical solutions.

A novel yarn interaction model was developed, showing that the actual convergence zone length is significantly smaller than predicted by the inverse kinematic model. This seems primarily due to the neglect to yarn interaction, in turn mainly governed by the apparent coefficient of friction. The main limitation of this yarn interaction model and many other braiding simulation models, including finite element models, is that they do not capture the effect of broken, detached and entangled fibers.

2. A novel procedure was proposed to search for a spool pattern that results in the desired bi- or triaxial braid pattern. As a result, the translation between biaxial spool patterns and braid patterns is tabulated for a range of spool patterns on commonly used circular braiding machines. The yarn trajectories of stem yarns on the mandrel can be generated using the topology of the bias yarns

combined with the bias yarn centerline paths as generated using kinematic braiding process simulation.

3. A novel meso-scale model was proposed to describe symmetric and non-symmetric, bi- and triaxial, open- and closed-mesh braids with mono-material and hybrid bias yarn groups, assuming a regular braid. The used yarn materials, the yarn pitch and the fiber direction of each yarn group are the input as generated by any braiding model. The model uses a yarn cross-sectional geometry that is either rectangular, elliptical or lenticular. The model outputs, among others, the thickness and fiber volume fraction of individual braid layers as well as the braided component.

9.3 Recommendations

The recommendations for further research and overbraiding process model development are the following:

1. Extend the kinematic braiding process model with a variable carrier speed in addition to a variable take-up speed.
2. Take the inertia of the mandrel and/or machine into account to bound the generated accelerations, given the available electrical power.
3. Smooth the fiber streamlines on the discretised mandrel surface to try to reduce take-up speed profile noise.
4. Optimize the path and the orientation of the mandrel relative to the braiding machine in tandem with the machine speeds, resulting in a fell front that remains as centered in the machine as possible.
5. Further experimentally validate the forward and inverse solution as well as the meso-scale models from this work to provide empirical correction factors, in turn possibly reducing the number of production trials.
6. Benchmark the proposed kinematic models against finite element models to make more accurate quantitative comparisons in terms of accuracy and speed.

7. Improve the braid meso model with a more physically correct distribution of the transverse yarn deformation.
8. Improve the braid meso model regarding nesting of contacting braid layers.
9. Improve easy access to the developed kinematic model with a graphical user interface.

Appendix A

Steady state

This section gives special attention to the definition of the ‘steady state’ because it is often implicitly or explicitly used in braiding process models and their underlying assumptions, affecting the systematic errors.

Strictly speaking, the serpentine spool motion causes the braiding process never to reach a steady state. However, when neglecting the serpentine component of the motion, a theoretical ‘steady state’ can be defined for modeling purposes. Neglect guide rings for the ease of explanation. The principles explained here also apply after the addition of guide rings.

For an axisymmetric circular braiding process including yarn interaction, a ‘steady state’ is defined as a state where the carrier rotation speed ω , take-up speed v , free segment length l as shown in Figure A.1, and yarn shape do not change with time t . Therefore

$$\frac{\partial \omega}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial l}{\partial t} = \frac{\partial \mathbf{f}(s)}{\partial t} = 0, \quad (\text{A.1})$$

where $\mathbf{f}(s)$ is the yarn unit tangent at arc length s measured from the spool as shown in Figure A.1, expressed in the spool coordinate system that rotates with that spool at the angular speed ω around the process axis.

Define the ‘fell point’ as the contact point between the substrate and a yarn that is closest to the spool as measured along the yarn. In the spool

Figure A.1 Schematic overbraiding geometry for steady state with a fictitiously fixed mandrel, showing one yarn to represent all. The yarn is curved due to interaction with yarns of the opposite group

coordinate system, express the position $\mathbf{p}$ of a fell point as

$$\mathbf{p} = \int_{s=0}^{s=l} \mathbf{f}(s) ds. \quad (\text{A.2})$$

In a steady state, (A.1) shows that this integral is constant through time, so the fell point $\mathbf{p}$ follows the spool $\mathbf{q}$ at a constant offset in all cylindrical directions. As a consequence, in a steady state, the fell point axial speed $v_{\text{fell,a}}$ must equal the take-up speed v and the fell point angular speed ω_{fell} must equal carrier rotation speed ω .

If axisymmetric braiding is performed on a mandrel segment with a variable radius, then the process is not in a steady state at that segment, because Equation (A.1) is not satisfied with respect to at least the free segment length l .

As an example, let the local mandrel radius profile equal that of the convergence zone surface as shown in Figure A.2. The convergence zone surface then ‘collides’ with the mandrel surface at a time t_c , leading to a sudden decrease of the free segment length in the convergence zone. In the infinitesimally small time interval $[t_c - \frac{1}{2} dt, t_c + \frac{1}{2} dt]$, the fell

Figure A.2 A collision of the convergence zone with the mandrel.

point axial speed $v_{\text{fell},a} = \lim_{dt \downarrow 0} \frac{\Delta z}{dt} = \infty$ and the fell point angular speed $\omega_{\text{fell}} = \lim_{dt \downarrow 0} \frac{\Delta \phi}{dt} = \infty$ while ω and v have finite values. Although this is an extreme case, it shows that for an axisymmetric process with a varying mandrel radius, $\omega_{\text{fell}} \neq \omega$ and $v_{\text{fell},a} \neq v$. Hence, if it is assumed that $\omega_{\text{fell}} = \omega$ or $v_{\text{fell},a} = v$ when generating machine control data for centered axisymmetric mandrels, then this can result in a decreasing braid angle accuracy for increasing mandrel tapers. Also, as a consequence of Equation (A.1), a non-axisymmetric circular braiding process cannot be in a steady state.

For the special case of an axisymmetric process just after an overbraiding run start or after a stepwise take-up speed change as shown in Figure A.3, $\omega_{\text{fell}} = \omega$ holds, but only when neglecting yarn interaction. However, the take-up speed and the convergence zone length H change, corresponding to a change in the free segment length l . Hence, the braiding process cannot be in a steady state during these events.

Figure A.3 Response for a stepwise increase of the take-up speed v at time t_0 , viewed from the spool coordinate system at the spool.

Appendix B

Generic biaxial braiding model

A detailed non-axisymmetric biaxial yarn interaction model is proposed to provide a theoretical framework for reference. The convergence zone geometry of the biaxial braiding process is described under the assumptions that

- spools move in a circle around the mandrel,
- an interlacement point is created only when two spools cross,
- an interlacement point is removed only when it gets into contact with the mandrel surface,
- friction at interlacements from relative yarn rotation is negligible, and
- yarn slack is negligible.

This implies that yarns never lose contact once it is formed, thereby avoiding the need for contact detection. However, this makes this model highly ‘synthetic’. A single interlacement point from the convergence zone shown in Figure 2.2 is analyzed using Figure B.1. Define a ‘strut’ as a yarn segment between two adjacent interlacement points on the same yarn. The superscript index denotes points in time. For variables that apply to an interlacement point, the first and second subscript denote the corresponding weft (O) and warp (X) yarn, respectively. For variables that apply to a strut or a point in a strut’s interior, the first and second subscript denote the yarn of interest and the interlacing yarn at its fell point side, respectively. Note that the weft yarn indexing

Figure B.1 Interlacement point displacement for an arbitrary biaxial braid.

direction differs from the braid and spool pattern analysis in Chapter 5 for implementation purposes.

During time increment Δt between time n and $n + 1$, each interlacement point moves to a new position $\mathbf{a}$,

$$\mathbf{a}_{ij}^{n+1} = \mathbf{a}_{ij}^n + \mathbf{u}_{ij}^{n+1} \quad (\text{B.1})$$

with $\mathbf{u}_{ij}^{n+1}$ as a small interlacement point displacement. A material particle in yarn i , located at $\mathbf{a}_{ij}^n$ at time n , moves to $\mathbf{p}_{ij}^{n+1}$ at time $n + 1$. Simultaneously, a particle in yarn j moves from $\mathbf{a}_{ij}^n$ to $\mathbf{p}_{ji}^{n+1}$.

Dropping the superscript, the strut length is

$$l_{ij} = \|\mathbf{a}_{ij+1} - \mathbf{a}_{ij}\|, \quad (\text{B.2})$$

where i and j can be swapped for the opposing yarn group strut lengths, here omitted for brevity. The strut length change is

$$\Delta l_{ij}^{n+1} = l_{ij}^{n+1} - l_{ij}^n, \quad (\text{B.3})$$

which is usually negative because the interlacement points move closer together while traveling towards the fell front. The signed arc length

distance s of $\mathbf{p}_{ij}^{n+1}$ relative to the closest interlacement point at the fell point side is

$$s_{ij}^{n+1} = -(\Delta l_{ij-1 \bmod n_y}^{n+1} + \Delta l_{ij-2 \bmod n_y}^{n+1} + \dots), \quad (\text{B.4})$$

with n_y the number of yarns per group and with the summation ending with the last strut fixed to the fell point. For $s_{ij}^{n+1} < l_{ij}^{n+1}$ to hold, Δt must be chosen small enough. The value of s can be negative, but this case is neglected for explanation purposes.

For the relative material particle displacement $\mathbf{d}$,

$$\mathbf{d}_{ij}^{n+1} = s_{ij}^{n+1} \mathbf{f}_{ij}^{n+1} - s_{ji}^{n+1} \mathbf{f}_{ji}^{n+1} \quad (\text{B.5})$$

with $\mathbf{f}$ as the unit strut direction. The friction force W_{ij} at $\mathbf{a}_{ij}$ acting from yarn j on yarn i works in the opposite direction of the relative velocity, i.e. opposite to $\mathbf{d}_{ij}$.

For each time step, the unknown displacements $\mathbf{u}$ can be evaluated in an optimization that minimizes frictional energy dissipation while satisfying force equilibrium at each interlacement point, including the yarn tension magnitudes F and G as well as the friction W opposite to $\mathbf{d}$ and the normal force, approximated using meso-geometry including the local yarn thickness. This requires boundary conditions including given initial fell point positions $\mathbf{p}$ and the yarn tension at the spools. For the initial guess, $\|\mathbf{u}\| > 0$ to avoid singularities that leave the friction direction undefined with $\mathbf{d}_{ij} = 0$. The resulting dissipative problem is non-conservative and non-linear. Numerical implementation attempts showed significant numerical instabilities. This can be explained, although only partly, by the observation that the system has unconstrained degrees of freedom, best described as a wave motion in the braid axial direction when braiding without a mandrel. This motion involves ‘scissoring’ of yarns, i.e. $l_{ij}^{n+1} \approx l_{ij}^n$, so $\mathbf{d}_{ij} \approx 0$, in turn resulting in a negligible work of friction. This motion can be constrained by introducing yarn bending stiffness and possibly mass.

The tractability of this problem requires implementations with long calculation times, defeating the purpose of developing a computationally fast braiding design tool and is therefore beyond the scope of this work. For special cases, a fast implementation is possible as presented in Chapter 4.

Appendix C

Example braid patterns

This appendix shows example results of the spool removal procedure.

Figure C.1 Example spool removals applied to a single machine braid repeat of Figure 5.6a. For case a), this results in a two-layered NCF, showing a layer of weft (O) yarns on top that crosses and covers a layer of warp (X) yarns. For case b), a canvas sized 12×12 ($3 \times 3 = 9$ machine braid repeats) was used, resulting in a pattern that should not be confused with the Hercules braid shown in Figure 5.6c. See also Table 5.3.

Figure C.2 Spool removal using the shown pattern on a braiding machine with six slots per horn gear and carriers in the 'tandem' formation results in a variable number of yarns per float (two and three). See also Table 5.3.

Figure C.3 a) Spool removal result using the shown spool pattern on a conventional braiding machine with carriers in the 'regular' formation. See also Table 5.2. b) Result on a machine with six slots per horn gear and carriers in the 'Hercules' formation. See also Table 5.3.

Appendix D

Cover factor

The ‘cover factor’ CF is defined as the fraction of the substrate area covered by the *bias* yarns. In this section, an analytical expression for the cover factor is derived. It is applicable to non-symmetric biaxial braids with optionally different values of the braid angle α and yarn pitch p for each yarn group. Such a braid geometry can emerge on, for example, complex mandrels. The cover factor can be evaluated at each point on the substrate under the assumptions of Section 6.4. Following the cover factor definition,

$$CF = \frac{A_{\text{cov}}}{A_{\text{tot}}} \quad (\text{D.1})$$

with A_{cov} and A_{tot} as the covered and the total substrate area, respectively. Using the empty area A_{em} and $A_{\text{cov}} = A_{\text{tot}} - A_{\text{em}}$, Equation (D.1) becomes

$$CF = 1 - \frac{A_{\text{em}}}{A_{\text{tot}}}. \quad (\text{D.2})$$

Using the geometrical unit cell as introduced in Section 6.5.1 and as shown in Figure D.1,

$$A_{\text{tot}} = \frac{4p_X p_O}{\sin(\alpha_X + \alpha_O)} \quad (\text{D.3})$$

Figure D.1 The geometrical unit cell, bounded by the shown centerlines, for the calculation of the cover factor of a non-symmetrical hybrid biaxial braid. The yarn pitches p_X and p_O are measured perpendicularly to the yarn.

and

$$A_{em} = \frac{(2p_X - w_{y,X,1} - w_{y,X,2})(2p_O - w_{y,O,1} - w_{y,O,2})}{\sin(\alpha_X + \alpha_O)} \quad (D.4)$$

with w_y as the yarn width for the primary (1) and optionally secondary (2) yarns. Assume no overlap between yarns of the same group, so $w_{y,1} + w_{y,2} \leq 2p$ for each yarn group. Substitution of Equations (D.3) and (D.4) in Equation (D.2) yields

$$CF = 1 - \frac{(2p_X - w_{y,X,1} - w_{y,X,2})(2p_O - w_{y,O,1} - w_{y,O,2})}{4p_X p_O}. \quad (D.5)$$

The yarn widths and yarn pitches are the only required input parameters. In general, $p_X \neq p_O$ and $\alpha_X \neq \alpha_O$, enabling a cover factor evaluation for non-symmetric braids, with an arbitrary position of the secondary yarns relative to the primary ones.

Note that a value of $CF = 1$ does not guarantee that *both* bias yarn groups fully cover the mandrel. For example, one yarn group can consist of wide carbon yarns while the other consists of small ‘stitch yarns’ as shown in [6].

For mono-material braids, $w_{y,1} = w_{y,2}$ for each yarn group. For

axisymmetric mono-material biaxial braids,

- $p_X = p_O = p$,
- $w_{y,X,1} = w_{y,O,1} = w_{y,X,2} = w_{y,O,2} = w_y$, and
- $\alpha_X = \alpha_O = \alpha$.

Using Equation (2.8), Equation (D.5) reduces to

$$CF = 1 - \left(1 - \frac{w_y n_c}{2\pi r_m \cos \alpha}\right)^2 \quad (D.6)$$

with w_y the yarn width, n_c the number of full carriers per yarn group, r_m the mandrel radius and α the braid angle. This result is well-known [8, 30, 52, 53, 96]. Hence, Equation (D.5) offers a generalization and can be used for complex mandrels instead of Equation (D.6).

Appendix E

Meso geometry

E.1 GUC geometry

The geometry of a hybrid triaxial braid from Figure 6.9 is derived in this section. The stem yarn fiber direction follows from geometrical continuity of the GUC,

$$\mathbf{f}_s = \frac{q_X \mathbf{f}_O + q_O \mathbf{f}_X}{\|q_X \mathbf{f}_O + q_O \mathbf{f}_X\|} \quad (\text{E.1})$$

with q_X and q_O as derived below and $\mathbf{f}_X$ and $\mathbf{f}_O$ as the given warp and weft unit fiber direction vectors, respectively, shown in Figures 6.8 and 6.9. The orientation $\mathbf{f}_s$ of the stem yarns is not necessarily parallel to the mandrel centerline.

The following parameters only depend on the given braid angle α , the average yarn pitch p , and yarn widths for the bias yarn groups:

$$\beta = \pi - (\alpha_X + \alpha_O), \quad (\text{E.2})$$

$$q_O = \frac{2p_O}{\sin \beta}, \quad (\text{E.3}) \quad i_O = \frac{w_{y,O,1}}{\sin \beta}, \quad (\text{E.4})$$

$$a_O = \frac{w_{y,O,2}}{\sin \beta}, \text{ and} \quad (\text{E.5}) \quad g_O = q_O - i_O. \quad (\text{E.6})$$

Equations (E.3) to (E.6) apply analogously to the X yarn group. Next, using the cosine rule,

$$d = \sqrt{q_O^2 + q_X^2 - 2q_O q_X \cos \beta}. \quad (\text{E.7})$$

Using the sine rule,

$$\frac{q_X}{\sin \delta} = \frac{d}{\sin \beta} \quad (\text{E.8}) \quad \Leftrightarrow \delta = \arcsin \left(\frac{q_X \sin \beta}{d} \right). \quad (\text{E.9})$$

Finally,

$$\zeta = \alpha_X + \alpha_O - \delta. \quad (\text{E.10})$$

E.2 Tunnel width

The parameters in Equation (6.18) are derived using the geometry from Figure 6.9. As a graphical aid, all dimensions within the parentheses are ordered and geometrically adjacent, starting at the first point in the subscript of w_{tun} , and going downward along in the O-direction in Figure 6.9.

$$w_{\text{tun,GA}} = \left(g_X + \frac{q_X}{q_O} (g_O - b_O - a_O) \right) \sin \zeta, \quad (\text{E.11})$$

$$w_{\text{tun,GC}} = \left(g_X - b_X - a_X + \frac{q_X}{q_O} g_O \right) \sin \zeta, \quad (\text{E.12})$$

$$w_{\text{tun,BA}} = \left(q_X + b_X - \frac{q_X}{q_O} a_O \right) \sin \zeta, \quad (\text{E.13})$$

and

$$w_{\text{tun,BC}} = \left(q_X - a_X + \frac{q_X}{q_O} b_O \right) \sin \zeta. \quad (\text{E.14})$$

Given that for each bias yarn, $w_y > 0$ and that for each bias yarn group, $w_{y,1} + w_{y,2} \leq 2p$, it follows that Equations (E.11) to (E.14) are always non-zero, positive, and vary linearly with either b_O or b_X .

E.3 Tunnel width derivatives

All non-zero partial derivatives with respect to b_O and b_X are constants that only depend on the input braid angles and bias yarn pitches,

$$-\frac{\partial w_{\text{tun,GA}}}{\partial b_O} = \frac{\partial w_{\text{tun,BC}}}{\partial b_O} = \frac{q_X}{q_O} \sin \zeta \quad (\text{E.15})$$

and

$$-\frac{\partial w_{\text{tun,GC}}}{\partial b_X} = \frac{\partial w_{\text{tun,BA}}}{\partial b_X} = \sin \zeta. \quad (\text{E.16})$$

Given the opposite signs in the non-zero derivatives in Equations (E.15) and (E.16), it follows that the linear Equations (E.11) to (E.14) intersect at a single point, showing that this intersection point is unique and not an element in a plateau. Given the minimizer in Equation (6.17), it follows that this point corresponds to the maximum tunnel width.

E.4 Constraints

Using AC||DE from Equation (6.19), an expression for b_O^* is obtained,

$$\frac{b_O^* + a_O}{b_X^* + a_X} = \frac{q_O}{q_X} \Leftrightarrow \quad (\text{E.17})$$

$$b_O^* = \frac{q_O}{q_X} (b_X^* + a_X) - a_O. \quad (\text{E.18})$$

Using B : DE in Equation (6.19) an expression for b_X^* is obtained,

$$\frac{g_O - b_O^*}{b_X^* + i_X} = \frac{q_O}{q_X} \Leftrightarrow \quad (\text{E.19})$$

$$b_X^* = \frac{q_X}{q_O} (g_O - b_O^*) - i_X. \quad (\text{E.20})$$

Substitution of b_X^* in Equation (E.18) by Equation (E.20) and rearranging yields

$$b_O^* = \frac{1}{2} \left(g_O + \frac{q_O}{q_X} (a_X - i_X) - a_O \right). \quad (\text{E.21})$$

Appendix F

Yarn cross-sectional shape

F.1 Yarn fiber volume fraction

The yarn fiber volume fraction $v_{f,y}$ can be determined theoretically using various strand packing models as shown in Figure F.1 [97]. Cox and Flanagan [98] suggest $0.70 \leq v_{f,y} \leq 0.80\%$ as a realistic range for commercially available materials. However, it is suggested that the actual value for use in composite models should always be measured [98]. For example, Birkefeld et al. [80] report measured values of yarns embedded in a braid as $58\% \leq v_{f,y} \leq 72\%$.

F.2 Measurements

In order to obtain an estimate of the yarn cross-sectional shape, size and fiber volume fraction, a sample piece of braid was cut out of the braid

Figure F.1 *Fiber packing models.*

Figure F.2 *Keyence VHX 5000 series digital microscope used to measure the braid surface.*

covering the mandrel and put under the confocal microscope shown in Figure F.2. This microscope and its software generate a 3D surface model of the braid top surface and subsequently make 2D cross-sections of this surface as shown in Figures F.3 to F.8. The bottom surface of the yarns cannot be visualized, yet the full yarn cross-sectional shape is approximated as described next.

The bias yarns shown in Figures F.3 to F.5 are measured as follows. First, a 2D profile of the top surface of a bias yarn is measured along a stem yarn. Next, a spline is manually fit through that profile. This fit is performed manually because it quickly allows to exclude ‘artificial roughness’ noise caused by broken filaments that lie far above the yarns. The discrimination between the actual yarn surface and broken filaments is performed visually using the 3D view shown in the top left of the measurement figures. Next, the spline is mirrored by a centerline, defined by a start and end point that are again obtained manually using the 2D profile and the 3D view. Finally, the mirrored bottom spline is clipped by the linear interpolation of the stem yarn below the profiled bias yarns,

assuming that the stem yarn crimp is negligible. Finally, the resulting cross-sectional area is projected along the bias yarn centerline to obtain the area perpendicular to the yarn, given the measured braid angle.

The stem yarns shown in Figures F.6 to F.8 are measured as follows. First, the top profile of a stem yarn is measured parallel to a bias yarn group. Next, a spline is fit and mirrored, and the area is projected analogously to the bias yarns.

The location of the 2D sections is chosen such that it contains a minimum amount of noise due to the broken filaments, so their location is not necessarily centered between yarns or coincident with a yarn centerline.

Using the measured cross-sectional area in combination with the yarn material data from Table 6.2, the yarn fiber volume fraction is obtained after combining Equations (6.3) and (6.4) and after projection,

$$v_{f,y} = \frac{n_s \rho_{l,s}}{A_{y,m} \rho_{f,s}} \sin \alpha, \quad (\text{F.1})$$

with n_s as the number of strands per yarn, $\rho_{l,s}$ as the linear density, $A_{y,m}$ as the yarn cross-sectional area in the measurement plane and $\rho_{f,s}$ as the volumetric strand fiber density as defined in Section 6.3. The sine term projects the measured area to the area normal to the yarn centerline as shown in Figure F.3.

The results are summarized in Table F.1. In this case, the cross-sectional shape of the yarns seems predominantly lenticular and the stem yarns seem to have a higher fiber volume fraction than the bias yarns, perhaps because the stem yarns are compressed more between the two bias yarn groups.

Table F.1 Measured yarn cross-sectional areas, the braid angle and the average fiber volume fraction per yarn material.

Material	$A_{y,m}$ (mm ²)
bias carbon 1 of 3	1.04
bias carbon 2 of 3	0.90
bias carbon 3 of 3	0.98
bias glass 1 of 3	0.23
bias glass 2 of 3	0.24
bias glass 3 of 3	0.28
stem 1 of 3	1.75
stem 2 of 3	1.58
stem 3 of 3	1.44
braid angle α	$\frac{97^\circ}{2}$
$v_{f,y}$ bias carbon (avg)	61%
$v_{f,y}$ bias glass (avg)	63%
$v_{f,y}$ stem (avg)	75%

Figure F.3 Bias yarn cross-sectional shape fit sample 1 of 3.

Figure F.4 Bias yarn cross-sectional shape fit sample 2 of 3.

Figure F.5 Bias yarn cross-sectional shape fit sample 3 of 3.

Figure F.6 Stem yarn cross-sectional shape fit sample 1 of 3.

Figure F.7 Stem yarn cross-sectional shape fit sample 2 of 3.

Figure F.8 Stem yarn cross-sectional shape fit sample 3 of 3.

Appendix G

Yarn crimp

An increasing braid angle generally results in a decreased yarn width as shown in Figure 6.4, in turn increasing the yarn thickness, in turn increasing the amplitude of the yarn out-of-plane waviness or yarn ‘crimp’. The amount of crimp is an important parameter for the structural performance. For example, an increasing crimp reduces the braid’s stiffness in the yarn length direction. For stiffness-critical applications, the stiffness reduction caused by the yarn undulation can be calculated using several elastic meso-scale models, taking the braid parameters as input [16, 23]. This reduction of a material property is sometimes also called a ‘knock-down’ or ‘translation’. This section will derive the relationship between the braid angle and the amount of crimp.

Define the crimp angle θ the maximum angle between a yarn and the substrate surface [22, 98]. For a regular mono-material symmetric biaxial braid, coarsely approximate the yarn centerline by a sine function as shown in Figure G.1,

$$z = \frac{t_y}{2} \sin\left(\frac{\pi x}{n_{\text{float}} l}\right) \quad (\text{G.1})$$

with t_y as the yarn thickness, n_{float} as the ‘float length’, i.e. the number of ends, e.g. $n_{\text{float}} = 1$ for a plain weave and 2 for a 2/2-twill, and l as the ‘strut length’, the distance between two adjacent interlacement points on the same yarn,

$$l = \frac{p}{\sin(2\alpha)}. \quad (\text{G.2})$$

Figure G.1 Yarn crimp in a symmetric biaxial open-mesh mono-material regular braid. The crimp is coarsely approximated by a sine function.

The amount of crimp is characterized by the maximum of the crimp angle tangent, in turn equal to the maximum of the derivative of Equation (G.1),

$$\tan \theta = \max \left(\frac{dz}{dx} \right) = \frac{\pi t_y}{2n_{\text{float}} l}. \quad (\text{G.3})$$

For a closed-mesh braid, t_y and l in Equation (G.3) are substituted with

$$t_y \stackrel{6.1}{=} \frac{A_y}{f_s w_y} \stackrel{6.14}{=} \frac{A_y}{f_s p} \stackrel{2.8}{=} \frac{A_y}{f_s \frac{P_m}{n_y} \cos \alpha} = \frac{n_y A_y}{f_s P_m \cos \alpha} \quad (\text{G.4})$$

and

$$l \stackrel{G.2}{=} \frac{p}{\sin(2\alpha)} \stackrel{2.8}{=} \frac{\frac{P_m}{n_y} \cos \alpha}{2 \sin \alpha \cos \alpha} = \frac{P_m}{2n_y \sin \alpha}, \quad (\text{G.5})$$

yielding

$$\tan(\theta) = \frac{\pi \frac{n_y A_y}{f_s P_m \cos \alpha}}{2n_{\text{float}} \frac{P_m}{2n_y \sin \alpha}} \Leftrightarrow \tan(\theta) = \frac{\pi n_y^2 A_y}{f_s n_{\text{float}} P_m^2} \tan \alpha \stackrel{6.14}{\Leftrightarrow}$$

$$\tan(\theta) = \frac{\pi n_y^2 t_{y,\text{ini}} w_{y,\text{ini}}}{n_{\text{float}} P_m^2} \tan \alpha \Leftrightarrow \tan(\theta) = \frac{\pi n_y^2 t_{y,\text{ini}} w_{y,\text{ini}}^2}{n_{\text{float}} P_m^2 w_{y,\text{ini}}} \tan \alpha \stackrel{6.5}{\Leftrightarrow}$$

$$\tan(\theta) = \frac{\pi}{n_{\text{float}} R_{y,\text{ini}}} \left(\frac{n_y w_{y,\text{ini}}}{P_m} \right)^2 \tan \alpha \Leftrightarrow$$

$$\alpha = \arctan \left[\frac{n_{\text{float}} R_{y,\text{ini}}}{\pi} \tan \theta \cdot \left(\frac{P_m}{n_y w_{y,\text{ini}}} \right)^2 \right]. \quad (\text{G.6})$$

For an open-mesh braid, substituting t_y and l in Equation (G.3) by $t_{y,\text{ini}}$ and Equation (G.2), respectively, yields

$$\tan \theta = \frac{\pi t_{y,\text{ini}}}{2 n_{\text{float}} \frac{P_m}{2 n_y \sin \alpha}} \Leftrightarrow \quad \tan \theta = \frac{\pi t_{y,\text{ini}} n_y}{n_{\text{float}} P_m} \sin \alpha \Leftrightarrow$$

$$\tan \theta = \frac{\pi t_{y,\text{ini}} n_y w_{y,\text{ini}}}{n_{\text{float}} P_m w_{y,\text{ini}}} \sin \alpha \stackrel{6.5}{\Leftrightarrow} \quad \tan \theta = \frac{\pi}{n_{\text{float}} R_{y,\text{ini}}} \frac{n_y w_{y,\text{ini}}}{P_m} \sin \alpha \Leftrightarrow$$

$$\alpha = \arcsin \left(\frac{n_{\text{float}} R_{y,\text{ini}}}{\pi} \tan \theta \cdot \frac{P_m}{n_y w_{y,\text{ini}}} \right). \quad (\text{G.7})$$

In Equations (G.6) and (G.7), the amount of crimp is expressed as a function of the mandrel perimeter, the machine size and the yarn material.

The crimp analysis can be expanded to include the effect of stem yarns. Crimp is neglected in the stem yarns, assuming that each stem yarn is uniformly sandwiched between both bias yarn groups as shown in Figures 1.7 and 6.14. This is a coarse approximation, because stem yarn crimp can be significant in the final composite product as discussed in [99]. To take the presence of stem yarns into account for the calculation of the bias yarn crimp, a simple approximation is to increase the crimp amplitude by half the stem yarn thickness: In Equation (G.1), replace t_y by $t_{y,\text{OX}} + t_{y,\text{S}}$ with $t_{y,\text{OX}}$ as the appropriate bias yarn thickness and $t_{y,\text{S}}$ as the stem yarn thickness.

The equations presented in this section can be readily used to map an amount of crimp to a braid angle and vice versa. This relationship is used in the design chart in Chapter 7.

Nomenclature

This nomenclature lists the most important abbreviations, entity names and symbols. If multiple purposes apply to a symbol, then these are separated by a semicolon, and the appropriate purpose is determined by the context of the main text. For brevity, not all used combinations of symbols and their optional sub- and superscript symbol(s) are listed here. Nonetheless, some combinations are included because their purpose may not be easily inferred from their constituents only. All units are SI by default unless overridden in the main text. Dummy indexes and symbols that locally represent temporary variables for derivations are not listed here and can override the nomenclature purpose.

Abbreviations

CAE	Computer Aided Engineering
CF	Cover Factor
CS	Coordinate System
FDF	Fiber Direction vector Field
FE	Finite Element
FEA	Finite Element Analysis
FRP	Fiber-Reinforced Plastic(s)
FSL	Fiber StreamLine
GUC	Geometrical Unit Cell
ID	IDentifier
NCF	Non-Crimp Fabric
NLO	Non-Linear Optimization
RTM	Resin Transfer Molding
RUC	Repeating Unit Cell
STL	STereoLithograpy (file format)
VARTM	Vacuum-Assisted Resin Transfer Molding

Entity names

C	cylinder; set of candidate solutions
cc	creating circle
cz	convergence zone
gr	guide ring
ipt	interlacement point
E	facet edge
F	facet
L	mandrel centerline
O	weft, rotating counter-clockwise
OX	O and/or X
Q	spool trajectory
S	stem; mandrel surface
sp	spool plane
T	supply point trajectory
X	warp, rotating clockwise
Y	yarn centerline

Scalars

A	cross-sectional area	[m ²]
$\mathcal{R}_y$	yarn width-to-thickness aspect ratio	[-]
c_{slip}	slip tendency	[-]
d	interlacement point half-distance on an interlacement circle	[m]
$f_{w,y}$	yarn width reduction factor	[-]
F	tensile force magnitude	[N]
G	tensile force magnitude	[N]
H	converge zone length	[m]
h	height measured along z	[m]
l	strut length	[m]
N	normal force magnitude	[N]
n_{ag}	number active yarn groups	[-]
n_{bit}	number of bits in a spool pattern bit sequence	[-]
n_{c}	number of carriers per group	[-]
n_{float}	float length	[-]
n_{g}	number of horn gears in a machine RUC	[-]
n_{incr}	number of increments	[-]

$n_{\text{incr/rev}}$	number of increments per revolution	[-]
n_{ipt}	number of yarn interlacement points	[-]
n_{rev}	number of horn gear revolutions	[-]
n_s	number of strands per yarn	[-]
n_{sl}	number of slots per horn gear	[-]
n_{sp}	number of spools per group	[-]
n_y	number of yarns per group	[-]
P	perimeter	[m]
p	yarn pitch	[m]
r	radius; 1 st coordinate in a cylindrical CS	[m]; [m]
s	arc length	[m]
t	thickness; time	[m]; [s]
v	take-up speed	[m/s]
v_f	fiber volume fraction	[-]
W	friction force magnitude	[N]
w	width; weight factor	[m]; [-]
z	3 rd coordinate in a CS	[m]
α	braid angle	[rad]
β	yarn kink angle	[rad]
γ	maximum angle between substrate surface normal and master normal projection along the centerline	[rad]
Δ_{cos}	direction cosine difference	[rad]
Δ	increment	[-]
δ	angle around $\mathbf{w}$ between two adjacent interlacement points	[rad]
η	angle between two yarns at an interlacement point	[rad]
θ	crimp angle	[rad]
μ	coefficient of friction	[-]
μ_{ap}	apparent average dynamic coefficient of friction	[-]
ρ_A	areal fiber density	[kg/m ²]
ρ_l	linear density	[kg/m]
ρ_f	fiber density	[kg/m ³]
φ	2 nd coordinate in a cylindrical CS	[rad]
ψ	pseudo-braid angle complement	[rad]
ω	carrier rotation speed	[rad/s]

Vectors

a	interlacement point	[m]
c	yarn segment centroid	[m]
d	unit slip direction; unit direction to next interlacement point on the same interlacement circle	[m]; [m]
F	force	[N]
f	unit yarn centerline tangent	[m]
i	facet unit mandrel longitudinal direction	[m]
j	facet unit mandrel transverse direction	[m]
l	closest point to c on L	[m]
m	machine CS origin	[m]
n_c	curve principal normal	[m]
n_s	unit surface normal	[m]
n_g	unit zenith direction	[m]
n_m	unit master side direction	[m]
p	fell point	[m]
q	spool point	[m]
r	guide ring contact point	[m]
s	supply point	[m]; [-]
t	unit mandrel centerline tangent; unit interlacement circle tangent at an interlacement point	[m]
u	1 st machine CS axis; displacement	[m]; [m]
v	2 nd machine CS axis	[m]
v_{rel}	relative material particle velocity	[m/s]
v_X	X material particle velocity	[m/s]
v_O	O material particle velocity	[m/s]
w	3 rd machine CS axis	[m]
x	1 st mandrel CS axis; 1 st fell point CS axis	[m]; [m]
y	2 nd mandrel CS axis; 2 nd fell point CS axis	[m]; [m]
z	3 rd mandrel CS axis; 3 rd fell point CS axis	[m]; [m]

Sequences

(<i>b</i>)	spool pattern bit sequence	[-]
(<i>s</i>)	spool pattern carrier ID sequence	[-]

Matrices

B	braid repeat	[-]
M	full machine braid repeat	[-]
R	rotation matrix	[-]
T	triaxial braid topology matrix	[-]

Subscripts

Entity names may also be used as subscript.

0	initial state
1	primary yarn material
2	secondary yarn material
ag	active yarn groups
ap	apparent
av	available
bit	bit
c	component; carrier
cr	critical
cos	cosine
cov	covered
dep	deposited
e	end state
em	empty
exp	experimental
f	fiber; facet
float	float
g	gear; yarn group
hom	homogenized
in	inner
incr	increment
ini	initial
ipt	interlacement point
m	mandrel; material
mod	model
oc	transition between open- and closed-mesh braid
out	outer

r	run
rel	relative
req	required
rev	revolution
s	strand
sl	slot
sp	spool
tot	total
trans	transition
tun	tunnel
y	yarn

Superscripts

n	time step number
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Acknowledgments

I want to acknowledge the intellectual inheritance upon which I draw. As Sir Isaac Newton wrote in 1676: 'If I have seen further, it is by standing on the shoulders of giants', referring to what John of Salisbury wrote during 1159 in *The Metalogicon*: 'Bernard of Chartres used to compare us to dwarfs perched on the shoulders of giants. He pointed out that we see more and farther than our predecessors, not because we have keener vision or greater height, but because we are lifted up and borne aloft on their gigantic stature.' For me, the skin of those shoulders is the work, time, and dedication of the students and researchers before me at the [University of Twente](#) (UT), especially Blasimir Villa Rodriguez.

After several years of silence regarding braiding research at the UT, braiding research was revived near the end of 2010 with the 'OBODAS' project. This project was financially supported by the [Netherlands Enterprise Agency](#) (Rijksdienst voor Ondernemend Nederland, at the time called 'Agentschap NL'). The project was supported in-kind by Eurocarbon BV and coordinated by the [National Aerospace Laboratory](#) NLR. This project finished after roughly a year and the braiding research was continued at the UT, partly with the help of commercial project partners from the automotive industry. The input and feedback of these partners were highly appreciated because it enabled a reprioritization of the lengthy list of functionality requests. After 2014, the work continued primarily through self-employment, using input from scientific work and from conversations with industrial project partners. Throughout all phases, improvements related to the braiding research were gratefully achieved.

I would like to thank my promotor, partners from industry, affiliated volunteers, colleagues at UT, and family and friends:

Foremost, my promotor, Prof. Remko Akkerman from the UT, for his

enthusiasm, time, input, feedback, and support.

Ton Bor and Wouter Grouve from the UT, for participating in the reading committee of this work.

Stephan Voskamp from [Eurocarbon](#), for his help, materials, and time in order to make it possible to carry out the braiding experiments.

Sybren Jansma and Ivo Vrooijink for assisting me as paranymphs during the defense.

Martin Sprenkeler from the UT, for his help in preparing experiments and the showcase shown below that has been decorating the Horst building at the UT.

Tyrone Klaver from the UT, for his help with the design of the showcase.

The employees of the model- and mechanical engineering workshop from the UT for their help in conducting experiments.

Cock and Annelies van Driel and Eelke Vrielink from [3EL-Company](#), for their help in the manufacturing of the hot-wire cut styrofoam mandrels.

Debbie Vrieze-Zimmerman van Woesik, Belinda Bruinink and Martina Tjapkes from the UT for, among others, helping me get through UT procedures and making travel arrangements.

Prof. Christophe Geuzaine and his colleagues from the University of Liège and co-developers, for the software [Gmsh](#), which enabled a professional and easy-to-use graphical user interface for pre- and post-processing of the braiding simulations. Also, a special thank you for their ability and willingness to quickly fix bugs and apply improvements on request.

Sergey Anatolyevich Bochkhanov from the [ALGLIB](#) Project, for providing the ALGLIB library, including easy access to non-linear solvers used to model yarn interaction.

Paul Bourke from [PaulBourke.net](#), for supplying many algorithms for a.o. computational geometry.

The volunteers from [StackExchange.com](#), for patiently answering my questions about math and $\text{\LaTeX}$.

My colleagues from the UT, Andriejus, Arnoud, Axel, the Berts, Bo, Christiaan, Devi, Diane, Emiel, Gert-Jan, Iqbal, Ismet, Ivo, Jos, Joost, Jurnan, Kirill, Laura, Laurent, Lily, Luca, Marieke, Muhammad, René, Roy, Sebastiaan, Sybren, Ted, Ton, Uli, Vaidotas, Vitaly, and the Wouters

for their time, conversations, and feedback, both personally and professionally; I enjoyed our conversations and soccer games.

And last, but not least, my wife, family, and friends. My wife, Koreen, for her constant joy and love, delightful friendship, intellectual camaraderie, physical rivalry, and support. My relationship with her would not have existed without the ASME conference visit related to this PhD; and yet, in spite of the time and brain drain associated with the PhD, the relationship with my wife bloomed amidst it. My family, and especially my mom, for allowing me to live in her 'basement' (actually, the attic) and sit in front of a computer screen all day while she provided wonderful meals, company, conversation, and support. And my family, friends and roommates who took time out of their busy schedules to enjoy an energizing exercise, meal, or conversation. They all provided their ongoing support, encouragement, and a much-needed listening ear after a week of intense software development.

Overbraided nozzle.

Publications

Journal articles

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Awards

JEC Europe 2014 innovation award, category software, JEC Europe, Paris, Mar. 2014

Software development

Braidsim, see www.braidsim.com

SharpCAE (mathematical libraries used by Braidsim)