SIZE-GRADATION EFFECT ON SAND TRANSPORT RATES UNDER OSCILLATORY SHEET-FLOWS

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Abstract: New experiments with graded sand was conducted in the large oscillating tunnel of Delft Hydraulics. The measured transport rates show that size gradation of sand particles has a minor effect on the net transport rates. On the other hand the transport rates of the fine fraction is reduced due to the presence of coarse particles. Coarse particles are more transported with different velocities resulting in a coarser composition for the transported sand compare to the original sand bed. Comparing the new results with three different transport models show that using the hiding/exposure correction factor of Day with the model of Ribberink leads to a better predictions for the composition of the transported sand. Using the model of Dibajnia & Watanabe overestimates the transport rates of the two fractions under investigation.

INTRODUCTION

Field observations show that sediment particles are hardly ever uniform, but consist of a mixture of different grain sizes. For example, the Dutch coast consists mainly of sand with median grain-size in the range of 125 – 600 µm. Moreover, observations show sorting of sediment grains along the cross-shore profile, i.e. coarse material is generally present near the shore-line and fine material is present in deep water. Until now little attention was paid to investigate the effect of sediment gradation on the transport process. Most of the available models assume the uniformity of sand mixture, which means that sand is characterised by a single sand size and that the size distribution is fully neglected. This assumption is acceptable in case of relatively

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uniform sand. However, when the sand has a wide range of sizes it is generally not realistic to simulate the sand with one size only. Therefore, present study is trying to:

gain more insight into the selective transport processes under sheet-flow conditions;

study the effect of graded sediment on the net total transport rate and the net transport rate per fraction and verify three existing transport models for graded sediment transport with use of the newly obtained data.

LABORATORY EXPERIMENTS

New experiments were conducted in the Large Oscillating Water Tunnel of Delft Hydraulics. The LOWT is a large facility that allows full-scale simulation of the near bed processes. The tunnel has U-shaped tube with a long horizontal test section and two vertical risers. One of them is open to the air and the other riser contains a piston to generate oscillating water motion in the test section. The range of oscillatory velocities is 0.2–1.8 m/s and the range of periods is 4-12 s. The test section is 14 m long; 0.3 m wide and 1.1 m high. A 0.3 m thick sand bed can be brought into the test section, leaving 0.8 m for water. Underneath both risers a sand tarp is constructed.

Three different wave conditions were studied, within the sheet-flow regime, each 2\textsuperscript{nd} order Stokes waves with different velocities (U_{\text{rms}} = 0.6, 0.7 and 0.9 m/s) and the same wave period (T = 6.5 s). The sand that was used consisted of a bimodal mixture of 70% uniform fine sand (D_{50}=0.21 mm) and 30% uniform coarse sand (D_{50}=0.97 mm). The wave conditions and sand characteristics were chosen in correspondence with previous wave tunnel experiments with uniform fine sand, with the same D_{50} = 0.21 mm, in order to compare the results and to study the effects of sand gradation.

The overall measuring program consisted of time-averaged measurements (net transport rates, sediment concentrations and sediment composition) and time-dependent measurements (flow velocities and concentrations inside the sheet-flow layer). Hassan et al. (1999) reported all the data. The present study focuses only on the measured total transport rates and size composition of the transported sand.

A Pump Bed Load Trap sampler (PBLT), developed by Van Rijn (1997) was used to measure the time-averaged bed load concentrations during onshore and offshore phase of the wave motion (inside the sheet-flow layer). Figure 1 shows the outline of the PBLT. Two nozzles (8 mm internal diameter)
are connected to two plastic hoses for pumping water and sediment. The nozzles are
opened and closed alternatively by valves through the action of oscillatory flow drag
on a metal pivoting plate connected by thin steel rods to the valves.

The total transport rates are measured with a mass balance technique, using the
sand collected in the two sand traps and the measured bed level changes before and
after each test. A bed level sounding system was used to measure the bed level
changes. Finally, flow velocities outside the boundary layer were measured using
Laser Doppler Anemometer (LDA).

EXPERIMENTAL RESULTS
Total transport rates and per size fraction
Experimental results show that net transport rates are all positive in the direction of
the wave propagation (onshore direction) and net transport rates are increasing for
increasing flow velocities. Figure 2 shows the measured net transport rates and the
transport rate of each size fraction (measured by the PBLT) versus $U_{\text{rms}}$.

![Graph showing net transport rates and per size fraction versus $U_{\text{rms}}$.]

The net transport rate of the coarse fraction appears to be almost equal to the net
transport rate of the fine fraction. This is remarkable, as the proportion of both
fractions in the sand bed is 70% fine fraction and 30% coarse fraction. At higher
velocities the transport rate of the coarse fractions seems to increase more than that of
the fine fraction. Selective transport is occurring. It seems that the coarse fraction is
more exposed to the flow than the fine fraction. This could be an indication for
unsteady effects of the fine fraction.

Onshore and offshore bed load composition
Using the PBLT gives the possibility to determine the composition of the sand
transported in the sheet-flow layer (lower 8 mm), averaged of the half wave-cycle
(onshore and offshore direction). Figure 3 shows the percentage of the coarse fraction
in both the onshore and offshore motion. This figure includes the results of individual
tests and the average values of each flow condition. Results show that the bed load
transported during the onshore phase of the wave motion is coarser than the bed load
transported during the offshore phase of the wave motion, as an effect of the asymmetric wave motion. During the onshore motion the percentage of coarse fraction is about 45-50% and during the offshore motion about 35-40%. During both wave cycles the percentage of the transported coarse material is larger than in the sand bed. Apparently, selective transport processes are occurring very close to the bed.

Fig. 3. Onshore and offshore bed load composition versus $U_{rms}$, measured by PBLT

COMPARISON WITH UNIFORM SAND

Figure 4 shows a comparison between the measured net transport rates with graded sand and those of previous wave tunnel experiments with 100% uniform fine sand ($d_{50}=0.21$ mm) and comparable hydraulic conditions. For the chosen gradation the net transport rates are systematically somewhat lower than the uniform sand cases. Especially at low velocities the difference is relatively large (about 20%). At higher flow velocities the difference decreases.

Fig. 4. Comparison between measured net transport rates of experiments with uniform sand and graded sand with almost the same $D_{50}$
The transport rates of the fine fraction at the present experiments have been compared with the transport rates of uniform fine sand at previous experiments (i.e. experiments with comparable hydraulic conditions). The transport rates of the present experiments have been corrected for the different ratio of occurrence of the fine sand in the sand bed (present experiments: 70% fine sand, previous experiments: 100% fine sand). It was concluded that, the fine fraction is hindered by the coarse fraction. The fine fraction in the mixture (present experiments) is transported more difficult than uniform fine sand (previous experiments). This is called hiding and resulted in a much smaller transport rate of the fine sand than expected (based on the different bed composition between the present and previous experiments).

Whether the coarse fraction is influenced by the fine fraction is not known, since there are no transport data available for uniform coarse sand. Although tests were performed with uniform coarse sand \((d_0=0.97 \text{ mm})\), no flat bed situation developed during these experiments (but large dunes) and negative transport rates were measured.

**Sand transport modelling**

**Models used**

Model of Bailard (1981)

The quasi-steady model of Bailard (1981) is a total load model based on Bagnold’s energetic approach (1963). This implies that the available fluid power for sediment transport is equal to the local rate of energy dissipation. Bailard assumed that the time-dependent bed shear stress to be an instantaneous function of the near bed velocity. The total transport load for a horizontal bed (as in the oscillating water tunnels) is given by:

\[
q_b(t) = \frac{\lambda f_w e_b}{\Delta g \tan \phi} u^3(t) + q_s(t) = \frac{\lambda f_w e_s}{\Delta g W_s} |u(t)|^3 u(t)
\]  

(1)

Where \(q_b(t)\) is a time-dependent bed load transport; \(q_s(t)\) is a time-dependent suspended load transport; \(f_w\) is wave friction factor; \(\Delta\) is the relative density = \((\rho_s - \rho_m)/\rho_m\)
\( \rho_w/\rho_w \); \( g \) is the gravity acceleration; \( \phi \) is an angle of internal friction of sediment; \( u(t) \) is the flow velocity and \( W_s \) is settling velocity of sediment particles.

Bailard used two efficiency factors \( \varepsilon_b \) and \( \varepsilon_s \), for bed load and suspended load transport, respectively. These factors account for the fraction of the energy spent to the sediment transport process. Both are obtained by calibration of the net transport rates against field data. Bailard (1982) found that \( \varepsilon_b = 0.1 \) and \( \varepsilon_s = 0.02 \). A general wave friction factor \( f_w \) of Jonsson (1966) has been used in the model.

**Model of Ribberink (1998)**

Ribberink (1998) assumed that sediment transport is a function of the effective shear stress \( \theta' \), the difference between the actual time-dependent bed shear stress and the critical bed shear stress \( \theta_{cr} \). The quasi-steady model of Ribberink reads:

\[
q_b(t) = m \sqrt{gAD_{50}^3} \left( \| \theta'(t) \| - \theta_{cr} \right)^n \frac{\theta'(t)}{\theta(t)}
\]

Where \( q_b(t) \) is a dimensionless instantaneous sediment transport rate; \( m \) and \( n \) are empirical coefficients; \( \theta' \) is effective Shields parameter and \( \theta_{cr} \) is the critical Shields parameter. The critical value of the Shields parameter depends on the non-dimensional grain-size as described by Van Rijn (1993). Under waves the sediment transport direction is determined by \( \theta'/|\theta| \). The coefficients \( m \) and \( n \) in equation (2) were determined by curve fitting of many experimental data with steady and oscillatory flows and different sand sizes. Ribberink (1998) found for a large data set that \( m = 11 \) and \( n = 1.65 \).

**Dibajnia & Watanabe (1992)**

This model is a 'semi-unsteady' model able to take into account unsteady effects, like time lag between velocity and concentration. The model considers the part of sand entrained during a positive half wave cycle may remain in suspension and be carried by the velocity of the successive negative half cycle into the negative direction and vice versa. For example the negative velocity could has two groups of sand to carry; one is the sand which is entrained by the negative velocity itself; and the other one is the sand remaining is suspension from the previous positive half cycle. For the exact formulations, the reader is referred to the original paper.

**Modelling of graded sand**

**Size fraction approach**

In the models described before the sand is characterised by the mean diameter only \( (D_{50}) \). This is satisfying in the case of uniform sediment, but variations in grain-size in the bed material can not be taken into account. In order to take into account the variation of grain-sizes in the bed material a simple size fraction method can be used. The size-fraction method divide the sand bed into a number of size-fractions, each characterised by a certain diameter \( D_i \) and a volume percentage of occurrences in the bed material \( p_i \). The transport rates of each fraction \( q_i \) can be calculated by:

\[
q_i = p_i \cdot q_{ui}
\]

Where \( q_{ui} \) is transport rate of fraction \( i \) as if the sand bed was composed of sand with a uniform distribution with \( D_{50} = D_i \).

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The total net transport rate in the case of \( N \) size-fractions, \( q_N \), is equal to the sum of the transport rates of all fractions. It is clear that using the size-fraction approach does not include any interaction between the different sediment fractions.

**Hiding/exposure correction factors**

In sand mixture different grain-sizes can influence each other and change its transportation mechanisms. Smaller grains can be sheltered or hidden by larger grains, while larger grains can be more exposed to the flow than in case of only coarse grains. Therefore, a hiding/exposure correction is needed for the transport formulas. In literature, hiding/exposure correction can be applied in two different ways: a) Correction of the effective Shields parameter, \( \theta_{\text{eff}} \). This implies reducing its value for the finer fractions and increasing it for the coarser fractions; b) Correction of the critical Shields parameter, \( \theta_{cr} \). This implies increasing its value for the finer fractions and reducing its value for the coarser fractions. In most available formulations hiding/exposure corrections river flow was considered rather than oscillating flow. The following two correction factors have been used with model of Ribberink in the present study.

**Egiazaroff (1965)**

Egiazaroff derived an expression for correcting the critical bed shear stress \( \theta_{cr} \) of each size fraction using a balance of forces on individual grains in a mixture on a flat bed with uniform flow. This expression reads:

\[
\xi_{cr,j} = \frac{\theta_{cr,\text{corrected}}}{\theta_{cr,\text{Shields}}} = \left[ \frac{\log 19}{\log \left( 19 \frac{D_i}{D_m} \right)} \right]^{2}
\]

Where \( \xi_{cr} \) is correction factor for critical shear stress; \( D_m \) mean diameter of the bed material and \( D_i \) mean diameter of fraction \( i \). Egiazaroff verified his formula indirectly using experimental transport data from other researchers and found good results.

**Day (1980)**

A correction factor for only the effective Shields parameter \( \xi_{\text{eff}} \) is given by Day. Day corrected the mobility number, which is a kind of dimensionless bed-shear stress, using a large number of experimental data. This factor can be transformed into a correction factor for the effective Shields parameter:

\[
\xi_{\text{eff}} = \left[ \frac{0.4}{\left( \frac{D_i}{D_A} \right)^{0.3}} + 0.6 \right] \quad \text{and} \quad \frac{D_A}{D_{50}} = 1.6 \left( \frac{D_{84}}{D_{16}} \right)^{-0.28}
\]

Where \( D_A \) is a grain-size in the mixture that needs no correction; \( D_A \) does not have to be equal to \( D_{50} \) or \( D_m \), but depends on the gradation of the mixture.
Between the transport formulas used in this study only the model of Ribberink uses these two parameters ($\theta_c$ and $\theta_{ef}$). Thus, only for this model hiding/exposure corrections can be applied.

**Verification of transport models**

Three different models have been verified in the present study, using the new tunnel data with graded sand. The results show that using the size fraction approach does not improve the predicted transport rates and it is not dependent on the number of size fractions.

The use of size fraction approach leads to minor effects on the predicted transport rates. This effect was found to be with a maximum of 5%.

Figure 4 shows the comparison between the measured and predicted total transport rates and the transport of the two size fractions.

The model of Bailard strongly underestimated the transport of the coarse fraction while considerably overestimated the transport of the fine fraction.

The model of Ribberink gives better agreements than the model of Bailard. The model underestimates the transport rate of the coarse fraction (about a factor 2) while the transport rates overpredicted with almost a factor 1.5.

The transport rates of both fine and coarse fractions are overestimated with about a factor two by the model of Dibajnia & Watanabe.
The correction factor of Egiazaroff has no influence on the results of the model of Ribberink. The transport rate of the fine fraction remains overestimated and the transport of the coarse fraction still strongly underestimated. In sheet-flow conditions the critical shear stress is very small compared to the effective shear stress (for the present experiments the difference is about a factor 10). Therefore, correcting only the critical Shields parameter has a limited impact on the predicted transport rates.

Applying the correction factor of Day to both fractions gives the best prediction for the transport rates of the two fractions. However, the total transport rate is overestimated a bit (about 5%).

![Graph showing results of the model of Ribberink with different correction factors](image)

Fig. 7. Results of the model of Ribberink with different hiding/exposure correction factors

Figure@@ shows the contribution of the fine fraction ($P_{\text{fine}}$) to the total transport rate, as predicted by each of the three models, is compared with the measured contribution of the fine fraction to the total transport rate.

![Graph showing measured and predicted $P_{\text{fine}}$ versus $u_{\text{rms}}$](image)

Fig. 8. Measured and predicted $P_{\text{fine}}$ versus $u_{\text{rms}}$
It is clear that model of Bailard largely overestimates the contribution of the fine fraction to the total transport rate. The model of Ribberink, with the correction factor of Day applied to all fractions, seems to give a good representation of the contribution of the fine fraction to the total transport rate. But when higher flow velocities appear the prediction is less accurate.

The model of Dibajnia & Watanabe overestimates the contribution of the fine fraction to the total transport rate a little. But on the other hand, it is the only model that gives a good qualitative prediction of the change of composition of the transported material, as a function of the flow velocity. For the present wave tunnel experiments it was found that the transported material became coarser when the flow velocities increased.

CONCLUSION
Laboratory experiments in the LOWT of Delft Hydraulics shows that size gradation of sand has small effects on the total transport rates (about 5-20% reduction). Fine particles in a sand mixture are less transported than in a uniform case (30-40%). Total transport rates are predicted rather well by Model of Ribberink. Predictions of the model of Dibajnia & Watanabe can be improved with different calibration factor. Composition of the transported sand can be predicted rather well by the model of Ribberink (1998) using hiding/exposure correction factor of Day Model of Dibajnia & Watanabe with different calibration factor

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