A Stochastic Model for Inter-terminal Container Transportation*

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October 16, 2013

Abstract

We propose a novel stochastic model for the inter-terminal transportation problem where multiple container terminals use a common fleet of vehicles (ALV, AGV, multi-trailers and barges) to transport containers between terminals. We model the problem using a semi-open queueing network, where the vehicles have multiple container carrying capacities. Our solution approach is decomposition-based where we solve the overall queueing network as a closed and an open queueing network, with bulk service capacities. To the best of our knowledge, this is the first work that considers bulk service in semi-open queueing networks. We numerically validate our model using simulated data where we find that our model results in errors of less than 5% for vehicle utilization. We then apply our model to real-world data from the Port of Rotterdam and show that our model can be used to analyze the throughput time trade-offs with alternate dwell point policies.

Keywords: *port and container terminal operations, semi-open queueing network, multi-class and bulk service queues*

1 Introduction

The growth in containerized shipping around the world has had a direct impact on the size of ports, with ports becoming ever larger in order to handle the increasing container

*The authors would like to thank the Port of Rotterdam Authority for sharing data on *Maasvlakte 1* and *Maasvlakte 2*, that was used in the numerical experiments. The authors would also like to thank René de Koster for his feedback and comments, which significantly improved the quality of this paper.

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flows. This has resulted in ports having multiple terminals that serve container ships, railways, barges and other forms of hinterland transportation. Examples of such ports include the Port of Rotterdam and the Port of Hamburg that have multiple terminals for container ships, along with rail and barge terminals for hinterland transportation. There is a significant amount of movement of containers within the port as containers are often transferred between terminals when they are transshipped between different modes of transportation. For instance, upon the completion of *Maasvlakte 2* at the Port of Rotterdam, the volume of this internal transport is estimated to be between 2 and 5 million TEU (Twenty-foot Equivalent Unit) per year. This internal movement of containers within a port is referred to as the *inter-terminal transportation* (ITT) problem. The ITT problem is depicted in Figure 1.

Figure 1: Illustration of the ITT problem.

ITT presents several challenges for the port authorities and the container terminal operators. In their paper, Tierney et. al. (Tierney, Voss & Stahlbock 2013) discuss some of the different aspects of the ITT problem, such as type of vehicle planning, expected congestion in the system given an infrastructure and vehicle configuration, and the dispatching and routing of vehicles for the transfer of containers. Typically, various types of vehicles such as automated guided vehicles (AGV), automated lift vehicles (ALV), multi-trailer system (MTS) or barges can be used for ITT, with each vehicle type having some pros and cons with respect to container carrying capacity, travel speed and container lifting ability (Tierney, Voss & Stahlbock 2013). The overall objective of an ITT system

is to minimize the delay in transferring containers between terminals.

In this work, we focus on the vehicle planning and system congestion aspects of the problem, and study how the number and type of vehicles, and related vehicle dwell point strategies influence the amount of time that containers spend in the ITT system. Specifically, we propose a stochastic model for analyzing the container throughput time, vehicle utilization, and other performance measures for an ITT system. We believe this work is the first attempt to develop a stochastic model for the ITT problem. We model the problem using a semi-open queueing network, with containers at the different terminals modeled as ‘customers’ in the queue, and vehicles as ‘servers’. In order to keep the model general with regards to the different types of vehicle, we model the servers with bulk service capacity. We then solve the overall queueing network by decomposing it into a closed and an open queueing network.

A significant number of studies have looked at the movement of containers within ports and container terminals [see (Steenken, Voss & Stahlbock 2004) and (Stahlbock & Voss 2008) for a review of this literature]. However, these have primarily been in the context of *intra-terminal* transport, where vehicles travel inside a container terminal short distances on designated routes without external traffic influence [see (Angeloudis & Bell 2011) for an overview; other references include (Briskorn & Hartmann 2006) and (Nguyen & Kim 2009)]. As pointed out in (Tierney, Voss & Stahlbock 2013), intra-terminal movement of containers is significantly different from ITT where vehicles need to travel longer distances in order to deliver containers and interact with outside traffic. Thus, the models developed for intra-terminal transportation are in general, not applicable to ITT.

There have been some simulation approaches to ITT [(Duinkerken, Dekker, Kurstjens, Ottjes & Dellaert 2006); (Ottjes, Veeke, Duinkerken, Rijsenbrij & Lodewijks 2006)] that have looked at the ITT problem at the Port of Rotterdam. These studies focussed on the number of containers arriving late at their destination terminal (i.e., past their due time). While a simulation approach is able to capture many details of the ITT operations, and can be used for the purpose of fine-tuning parameter values, it is unable to evaluate trade-offs or to optimize across policies. In contrast, an analytical model can help quickly evaluate, for instance, various vehicle and dwell point policies that could, if necessary, be further explored using simulation.

Recently, (Tierney, Voss & Stahlbock 2013) have proposed a deterministic optimization model for ITT. Their approach is to model ITT using network flows and they

formulate an integer programming model to minimize container delivery delay. They optimize infrastructure and vehicle configuration based on average travel times on arcs in the network. In contrast, the model in this paper is stochastic where we explicitly model the arrival of containers into the ITT system and estimate the total time that they spend in the system.

Another line of research that is relevant for this work is the literature on semi-open queueing networks (SOQN) [see (Jia & Heragu 2009)]. SOQN have seen applications in several manufacturing and logistical settings [see, for instance, (Heragu & Srinivasan 2011) and (Roy, Pazour & de Koster 2013)], although, they have not been applied to port related operations. In addition to being a novel application of SOQN, this paper provides the solution for multi-class bulk service capacities in semi-open queueing networks.

Our contribution is three-fold. First, we propose a novel stochastic model for analyzing an ITT system that explicitly captures the uncertainties in the arrival times of containers, the handling times of containers at the different terminals and in the travel times of vehicles between terminals. The model also captures the interaction between containers and vehicles in the system. Thus, our model is richer and more realistic than earlier models for the ITT problem, and should result in better modeling of congestion in the system and correspondingly, should result in better estimates for the expected delay in container transfers and other measures of system performance.

Our second contribution is to the literature on semi-open queueing networks. In this paper, we develop a solution approach for semi-open queueing networks with multi-capacity resources (multi-class, bulk-service semi-open queues). To the best of our knowledge, this is the first paper that considers bulk-service in the context of semi-open queueing networks. Thus, although our queueing model is stylized for the ITT problem, our solution framework for semi-open queues with bulk service could be applied to several other applications involving the analysis of vehicle-passenger transport.

Our third contribution is that we perform extensive numerical experiments to validate our model and provide analysis of the ITT problem at the Port of Rotterdam in The Netherlands. Based on simulation experiments using the AutoMod[®] software, we show that our model results in reasonable errors, and that the errors do not increase significantly as the model size increases. Thus, we numerically show that our model is scalable. We then use our model to calculate expected throughput times and vehicle utilization based on estimated container flow data at the Port of Rotterdam, and show how the model can be used to analyze throughput time trade-offs with different vehicle

dwelling point policies.

The remainder of the paper is organized as follows. In Section 2 we present our stochastic model for the ITT problem. Here, we first describe the basic model with two terminals and unidirectional container flows in Section 2.2, before extending the analysis to the more general multi-terminal case in Section 2.3. We next validate our model using simulation experiments in Section 3. In Section 4, we perform numerical experiments with data from the Port of Rotterdam. Finally, we conclude in Section 5 and discuss directions for future work.

2 Model Description

We model each container arrival stream as customers to the terminal for inter-terminal transport. Vehicles are modeled as fixed resources that circulate in the network of terminal handling stations and the vehicle dwelling point, in order to transfer the containers from their respective origin to destination terminals. If a vehicle is available for internal transport, then the vehicle travels from its dwelling point to reach the origin station, picks up containers (upto its capacity) at the handling stations, and travels to the destination terminal for unloading. After unloading the containers, the vehicle returns to the central dwelling point. The synchronization of vehicles and containers at a terminal is precisely modeled using a synchronization station. The vehicle travel times between any two terminals (or between a terminal and the central dwelling point) is modeled using infinite server (IS) stations with exponential travel times. The handling stations are modeled using single server exponential queues.

2.1 System description and modeling assumptions

We consider a container terminal system with the transport between terminals carried out by vehicles which can transport several containers simultaneously. We use queueing theory to obtain the throughput time of a container request, that is, the time it takes after a request is made, until the container is delivered to its destination. We assume that vehicles after delivering a container to the destination terminal return to a central dwelling point (CDP), that vehicles are never idle when there is a request waiting, and that the bulk-capacity is only used simultaneously for containers which have the same origin–destination pair (ODP). Our goal is to obtain the container waiting times at the

synchronization station, to find the expected throughput time of a request and to estimate the utilization of the vehicles.

To start the modeling, we assume that the terminals are numbered $1, \dots, N$, and there are K vehicles, all with capacity c . By assumption, the handling times at the terminals do not depend on the number of handled containers and are exponentially distributed with rate μ_j . At a terminal, only one vehicle can be handled at a time. For instance, the container might be handled with a reach stacker equipment. The transport times over routes (or lanes) are deterministic, and the number of vehicles in a route at a time instant can vary between 0 and K . The routes that are used to transport a container from terminal i to terminal j are fixed. The requests for a ODP (i, j) arrive according to a Poisson process with rate λ_{ij} .

In the following analysis, we first start by assuming that there is only one ODP and we refer to it as the single stream case. We then extend the analysis to the more general case with multiple terminals and multiple streams (several different ODP's).

2.2 Single stream case

In this section, we analyze the two terminal single stream case as depicted in Figure 2. Note that we only consider unidirectional container flows between terminals A and B. From a queueing perspective, this case can be modeled as illustrated in Figure 3. The routes on which the vehicles travel are assumed to be infinite servers, whereas the handling process at the terminals are modeled as single server queues with FCFS scheduling. As mentioned earlier, the handling times are assumed not to depend on the number of containers carried by the vehicle. The requests arrive at the CDP with rate λ .

Figure 2: The single stream case

Once the vehicle is paired with the container request at terminal i , the vehicle travels from the CDP to the handling station at terminal i . This travel time is modeled using an infinite server station (IS_1). Then the container is loaded on the vehicle at the single

server handling station, denoted as T_i . The vehicle then proceeds to the destination terminal; this inter-terminal travel time is also modeled using an infinite server station IS_2 . The container is unloaded at the handling station of the destination terminal, T_j . Once the vehicle is free it proceeds to the CDP, and this travel time is modeled using an infinite server station denoted by IS_3 .

The performance measures that we study are the utilization of the vehicles and the expected throughput time of a request, or, by Little's Law, equivalently, the expected number of requests in the system, inclusive the external queue..

Figure 3: The queueing model for the single stream case

The analysis focuses on the external queue for the requests and the number of vehicles in use. By this analysis, we find an approximation of the expected number of requests in the entire system which will eventually lead to the expected throughput time of a container. First we distinguish two separate periods in the process: in one period all vehicles are busy, the so called B-period; in the other period, denoted by F-period, at least one vehicle is free. The B- and F-periods are depicted in Figure 4. Note that during an F-period at least one vehicle is at the CDP, and correspondingly, at most $K - 1$ vehicles are on route. In the following analysis, we want to exclude the influence of the B-period over the F-period and vice-versa. To do this, we define an F-system as the system in which at least one vehicle is free and a B-system as the system in which all vehicles are busy. Essentially, the F-system is a series of F-periods and the B-system a series of B-periods.

The analysis of the F-system (see Figure 5) boils down to applying Marginal Distribution Analysis [MDA, see (Buzacott & Shanthikumar 1993)] to the entire system with

Figure 4: Illustration of the B- and F-periods. Here L represents the number of containers in the queue and N_C represents the number of vehicles at the central dwell point.

$K-1$ vehicles, where the CDP is modeled as an exponential server with rate λ . In an F-period, the external queue is empty. This analysis of the closed queueing network gives us the marginal distribution of $N_{V_i}^F$, the number of vehicles at each node i (that is the CDP, a terminal handling station or an inter-terminal travel path). From this marginal distributions, we find the expectations

$$EN_{V_i}^F = \sum_{k=1}^M kP(N_{V_i}^F = k), \quad i = 0, \dots, 5. \quad (1)$$

Note that the number of unused vehicles during an F-period $N_C = 1 + N_{V_0}^F$.

Analyzing the process of the B-system, is more elaborate since now we have to include the external queue. First we focus on the vehicles. Since they are all busy, no vehicle is at the CDP. We can analyze this internal process of the vehicles by MDA but now the number of busy vehicles is K and the CDP is modeled as an infinite rate server because a vehicle that visits the CDP immediately departs again. In this case, we denote the number of vehicles at node i by $N_{V_i}^B$. Next consider requests waiting in the external queue. Once in a while, an arbitrary vehicle returns to the CDP to pick up the waiting

Figure 5: Depiction of the F-period.

requests but at most c . The time between the arrivals of vehicles can be seen as service times of a single server which can handle batches of size c . In the following analysis we assume that these service times are independent. For now, we also take the service time distribution as known. At the end of the section, we come back to the distribution of the inter arrival times of the vehicles.

Consider a $M/G^Y/1$ system with bulk service where the server can serve upto c customers at a time. When the server ends a service and less than c customers are waiting, then the server starts a service with these customers (even if there are no customers at all). Let $\hat{S}(s)$ be the Laplace Stieltjes transform of the service times. Then the z-transform of the number of customers that arrive during the n^{th} service time (denoted by A_n) satisfies

$$\hat{A}(z) = E(z^{A_n}) = \hat{S}(\lambda(1 - z)) \quad (2)$$

where, λ is the arrival rate. We assume that $\lambda ES < c$ where $ES = -\hat{S}'(0)$ is the expected service time. Let L_n denote the queue length after the n^{th} service has started then $L_{n+1} = \max(L_n + A_n - c, 0)$. For the z-transform of the stationary distribution of L we find (3) using existing results [see (Chaudhry & Templeton 1983) or (van Ommeren 1991)]

$$\hat{L}(z) = \left(\frac{c - \lambda ES}{\prod_{j=1}^{c-1} (1 - z_j)} \right) \left(\frac{(1 - z) \prod_{j=1}^{c-1} (z - z_j)}{\hat{A}(z) - z^c} \right). \quad (3)$$

where, the z_i , $i = 1, \dots, c - 1$ are the complex roots of the denominator within the complex unit circle ($|z| < 1$). Note that $P(L = 0) = \hat{L}(0)$ and that the expected queue length just after the departure of a vehicle $EL = \hat{L}'(1)$. In the sequel, we need to know the expectation of L_T , that is the expected queue length at an arbitrary point in time. By conditioning on the service time, we find that

$$EL_T = EL + \frac{1}{2} \lambda (1 + c_S^2) ES, \quad (4)$$

where $c_S^2 = \hat{S}''(0)/\hat{S}'(0)^2 - 1$.

At this point, we have analyzed the F-system and the B-system during the F-period and the B-period, respectively (see Figure 6). To complete the analysis, we need to know the lengths of these periods and the fraction of the capacity of the vehicles that is used. Here we assume that the consecutive B-periods and F-periods are independent and that they behave as in the corresponding systems.

The length of the F-period can be found in the following way: consider the closed

system with the CDP a single server with rate λ , but now with K vehicles and analyze this system by MDA. By this method, we find the probability that all vehicles are busy, that is $P(N_{V0}^K = 0)$ where N_{V0}^K is the number of vehicles at the CDP. On the other hand, by renewal theory, we can express this probability also as the fraction of time that all vehicles are busy and the time between two epochs at which all vehicles become busy. Note that the expected time that all vehicles are busy is just ES , the expected service time during the B-period and that the expected length of an F-period is just the time between the moments that the first vehicle becomes idle until all vehicles are busy again. Together this gives that the expectation of T_F , the length of an F-period, is given by

$$ET_F = \frac{P(N_{V0}^K > 0)}{P(N_{V0}^K = 0)} ES. \quad (5)$$

The expected length of the B-period is found by first looking at the busy period of the server defined as the time elapsed between two consecutive moments at which the server leaves an empty queue, that is $L = 0$. During this period the expected number of *service starts* equals $1/P(L = 0)$, so the expected length of this busy period is $ET_{BP} = ES/P(L = 0)$. A B-period ends when the server leaves and no one enters during the following service time. The probability that no requests enter during a service time is given by $\hat{A}(0) = \hat{S}(\lambda)$. The number of busy periods during B-period is geometrically distributed with success probability $\hat{S}(\lambda)$ so the expectation of T_B , the length of a B-period, is given by

$$ET_B = \frac{ET_{BP}}{\hat{S}(\lambda)} = \frac{ES}{\hat{S}(\lambda)P(L = 0)}. \quad (6)$$

The probability P_F that the system is in an F-period at an arbitrary time point is now given by

$$P_F = \frac{ET_F}{ET_F + ET_B}. \quad (7)$$

The MDA algorithms provide us with the expected number of vehicles at every node (cf. eq. (1)), both during an F-period and a B-period. To apply Little's Law however, we do need to know the expected number of requests in the system. Since a vehicle in

Figure 6: Illustration of the system cycle.

the network can carry more containers, we compute the expected capacity used by the vehicles (which is at most c). We first find the number of vehicles that leave during an F-period and the following B-period. By assumption, every arrival during the F-period is served immediately, so the expected number of vehicles that depart during this period, equals the expected number of arrivals during this period which is λET_F . During the B-period, the number of departing vehicles is $\left(\hat{S}(\lambda)P(L=0)\right)^{-1}$, including the first vehicle, which has already been counted in the F-period. Correspondingly, on average, the capacity used by a vehicle equals

$$EC_V = \frac{\lambda(ET_F + ET_B)}{\lambda ET_F + \left(\hat{S}(\lambda)P(L=0)\right)^{-1} - 1}. \quad (8)$$

Now we can approximate the expectation of L_P , the number of requests being processed, by multiplying the expected number of vehicles processing a request by EC_V or

$$EL_P = \sum_{i=1}^{M-1} (P_F EN_{V_i}^F + (1 - P_F) EN_{V_i}^B) EC_V.$$

The expectations $EN_{V_i}^F$ and $EN_{V_i}^B$ follow from the MDA. By applying Little's Law, we find

$$EW = \frac{EL_P + EL_T(1 - P_F)}{\lambda},$$

where EL_T , the expected number of requests in the external queue during a B-period, is given in eq. (4)

Finally, we look at the other performance measure, namely the vehicle utilization which is the expected fraction of vehicles that is not idling at the CDP. During a B-period all K vehicles are busy, whereas in the F-period $K - EN_C$ are busy. So the utilization U is

$$U = 1 - \frac{EN_C}{K} \cdot P_F. \quad (9)$$

We finish this section by some remarks on the service time distribution during the B-period. The expectation is readily found by the MDA. The second moment, however, is already much more difficult to find, let alone the whole distribution. It is possible by doing a detailed analysis of the underlying product form network, but in this case of two terminals and one stream, the second moment is already hard to find. In the sequel, some choices of distributions are given, based on the first two moments. For now, assume that we know ES and $c_S^2 = \text{var}(S)/(ES)^2 \leq 1$. To approximate $\hat{S}(s)$, we have to make

some assumption on the service time distribution. We next present several options for modeling the service time during the B-period.

- The service time is the sum of a constant and an exponential random variable. It is easily found that

$$\hat{S}(s) = \frac{\mu}{\mu + s} e^{-sD}, \quad (10)$$

with $D = ES(1 - c_S)$ and $\mu^{-1} = c_S ES$.

- The service time is either constant D (with probability p) or an exponential random variable with expectation $1/\mu$. It is easily found that we may take $D = 1/\mu = ES$ and $p = (1 - c_S^2)$. This gives

$$\hat{S}(s) = p e^{-sD} + (1 - p) \frac{\mu}{\mu + s}. \quad (11)$$

- The service time is the sum of an Erlangian(k, ν) random variable and an exponential random variable with expectation $1/\mu$. It is easily found that

$$\hat{S}(s) = \frac{\mu}{\mu + s} \left(\frac{\nu}{\nu + s} \right)^k \quad (12)$$

with $k \geq \frac{1-c_S^2}{c_S^2}$, $\mu = (\alpha ES)^{-1}$ and $\nu = k((1-\alpha)ES)^{-1}$, where $\alpha = \frac{\sqrt{c_S^2 k^2 - (1-c_S^2)k+1}}{k+1}$.

- The service time is either an Erlangian(k, ν) random variable with probability p or an exponential random variable both with expectation $k/\nu = 1/\mu = ES$. It is easily found that we may take

$$\hat{S}(s) = \frac{\mu}{\mu + s} \left(\frac{\nu}{\nu + s} \right)^k \quad (13)$$

with $k \geq c_S^{-2}$ and $p = \frac{1-c_S^2}{1-1/k}$.

Our numerical experiments indicate that the results are not very sensitive to the choice of the distribution in use. Furthermore, we observed that using the mix of a deterministic and exponential distribution, see eq. (11) with $c_S^2 = (1 - K^{-1/2})^2$, gives a good approximation.

2.3 Multi-stream case

In this section we consider the case with multiple terminals and multiple streams. As an example, see Figure 7. We assume that the vehicles are not idle when a request is

Figure 7: The multi-stream case

waiting. If all vehicles are busy, incoming requests are queued. When a vehicle returns to the CDP, it will pick up the first request in the queue and all requests with the same ODP up to its capacity c . Requests with ODP i arrive according to a Poisson process with rate λ_ℓ , $\ell = 1, \dots, M$, where M is the number of different ODP's. The total arrival rate $\lambda = \sum_{\ell=1}^M \lambda_\ell$. All ODP requests of type ℓ have a fixed route through the network (so the vehicles servicing a type ℓ request to), reflected in the routing matrix R_ℓ . After a vehicle has delivered the containers at the destination terminal, the type of the vehicle changes to '0', not servicing a request. Note that by this definition, the arrival rate of type 0 vehicles is $\lambda_0 = \lambda$. The visit ratios of a vehicle of type ℓ to a node i is $V_{\ell i}$.

For the analysis, we again focus on the F-periods and the B-periods. During an F-period, every incoming requests is processed immediately. The analysis of this period is again performed using the MDA approach (mentioned in Section 2.2) but now with different types of container requests, so with an aggregated routing matrix which reflects the different types; more explicitly, we have that the entries of R , the routing matrix for an arbitrary vehicle, are given by

$$R_{ij} = \sum_{\ell=0}^M p_{\ell i} R_{\ell, ij}, \quad (14)$$

where $p_{\ell i}$, the probability that a vehicle at station i is of type ℓ is given by

$$p_{\ell i} = \frac{\lambda_{\ell} V_{\ell i}}{\sum_{\ell=0}^M \lambda_{\ell} V_{\ell i}}. \quad (15)$$

The MDA gives us the marginal distribution of N_{Vi}^F , the number of vehicles at node i , from which the EN_{Vi}^F follows.

Only during the B-period, we need to model the external queue. We model the system with $M = K(K - 1)$ different ODP's as M queues, one for every stream. For instance, with respect to Figure 7, we consider $M = 6$ queues, each corresponding to a stream. Once a vehicle returns to the CDP, it goes to the queue that has the earliest arrived customer (or correspondingly, the longest waiting customer) and takes at most c requests from this queue. We assume that the probability that of all nonempty queues, requests are taken from the queue by this returning vehicle, is proportional to the arrival rate. More formally, suppose that when the server returns to the CDP, a number of queues are not empty. Let M_{NE} be the set of nonempty queues upon arrival of a vehicle, so $M_{\text{NE}} \subset \mathcal{M} = \{1, \dots, K\}$. A queue $m \in M_{\text{NE}}$ is served with probability $p_m = \lambda_m / \Lambda$ where $\Lambda = \sum_{m \in M_{\text{NE}}} \lambda_m$. With these rules, let q_m denote the probability that queue m is empty after a service and assume that this queue is empty independent of the state of the other queues. Then

$$P(M_{\text{NE}}) = \prod_{m \in M_{\text{NE}}} (1 - q_m) \prod_{m \in M_{\text{NE}}^C} q_m. \quad (16)$$

Together, this gives the following conditional probability that queue ℓ , given that this queue is non empty, is served

$$\tilde{p}_{\ell} = \lambda_{\ell} \sum_{M_{\text{NE}} \in \mathcal{M}_{\ell}} \frac{P(M_{\text{NE}})/(1 - q_{\ell})}{\sum_{m \in M_{\text{NE}}} \lambda_m}. \quad (17)$$

In eq. (17), $\mathcal{M}_{\ell}$ denotes set of configurations with queue ℓ non empty.

To analyze the queues, we assume that a non empty queue i is always served when the server gets free, but that the number of served requests is zero with probability $1 - \tilde{p}_i$. In other words, the z-transform of the number of requests served from queue i is

$1 - \tilde{p}_i(1 - z^c)$. The z-transform of the queue length at queue ℓ is given by (cf. eq. (3))

$$\hat{L}_\ell(z) = \left(\frac{c\tilde{p}_\ell - \lambda_\ell ES}{\prod_{j=1}^{c-1}(1 - z_j)} \right) \left(\frac{(1 - z) \prod_{j=1}^{c-1}(z - z_j)}{\hat{A}_\ell(z)((1 - \tilde{p}_\ell)z^c - \tilde{p}_\ell) - z^c} \right). \quad (18)$$

where the z_j , $j = 1, \dots, c - 1$ are the complex roots of the denominator within the complex unit circle and $\hat{A}_\ell(z) = \hat{S}(\lambda_\ell(1 - z))$. The service times are found by the MDA (as mentioned in Section 2.2) where we assume that during a B-period a departing vehicle is processing a type i request with probability λ_i/λ . By this assumption, the routing matrix for an arbitrary departing vehicle is R , defined in eq. (15). The MDA also provides us the marginal distribution of N_{Vi}^B , the number of vehicles at node i in the B-system, from which the EN_{Vi}^B follows.

Note that $\hat{L}_\ell(z)$ depends on $\tilde{p}_\ell$ and $\tilde{p}_\ell$ depends on $q_k = \hat{L}_k(0)\hat{A}_k(0)$, $k = 1, \dots, M$. We can find a combination of $\tilde{p}_\ell$ and q_ℓ that satisfy these dependency relations, using an iterative approach. This is formally stated as a lemma below.

Lemma 1. *The following scheme converges:*

1. Set $n = 1$ and $\tilde{p}_\ell^{(1)} = 1$, $\ell = 1, \dots, M$; this assigns all capacity to all queues.
2. Compute $\hat{L}_\ell^{(n)}(z)$ with $\tilde{p}_\ell^{(n)}$, $\ell = 1, \dots, M$;
3. Set $n = n + 1$ and $q_\ell^{(n)} = \hat{L}_\ell^{(n-1)}(0)\hat{A}_\ell(0)$, $\ell = 1, \dots, M$;
4. Use $q_\ell^{(n)}$ to compute $\tilde{p}_\ell^{(n+1)}$ by eq. (17);
5. Increase n and iterate step 2, 3 and 4 until convergence.

Proof. To prove that the above scheme converges, we first note that if $\tilde{p}_\ell^{(n-1)} \geq \tilde{p}_\ell^{(n)}$ then $q_\ell^{(n)} \geq q_\ell^{(n+1)}$. Thus, if we show that $q_\ell^{(n-1)} \geq q_\ell^{(n)}$ implies that $\tilde{p}_\ell^{(n)} \geq \tilde{p}_\ell^{(n+1)}$, we can see that we would then get decreasing sequences $\tilde{p}_\ell^{(n)}$ and $q_\ell^{(n+1)}$, for $n = 1, 2, \dots$ of positive numbers that should converge. As can be seen from equation (17), $\tilde{p}_\ell$ is a function of $q_1, \dots, q_M$; and in order to make this explicit, we denote it as $\tilde{p}_\ell(q_1, \dots, q_M)$. Further, let q_ℓ^* be a probability less than q_ℓ . We want to show that $\tilde{p}_\ell(q_1, \dots, q_M) \geq \tilde{p}_\ell(q_1^*, \dots, q_M^*)$. Without loss of generality, we will show this for $\ell = 1$.

Now first assume that only $q_M \geq q_M^*$. Let M_Z denote any subset of $\{2, \dots, M - 1\}$

and $P(M_Z) = \prod_{m \in M_Z} (1 - q_m) \prod_{m \in M_Z^C} q_m$ where $M_Z^C \subset \{2, \dots, M-1\}$. Then

$$\tilde{p}_1(q_1, \dots, q_M) \quad (19)$$

$$= \lambda_1 \sum_{M_Z} P(M_Z) \left(\frac{q_M}{\sum_{m \in \{1\} \cup M_Z} \lambda_m} + \frac{1 - q_M}{\lambda_M + \sum_{m \in \{1\} \cup M_Z} \lambda_m} \right) \quad (20)$$

$$\geq \lambda_1 \sum_{M_Z} P(M_Z) \left(\frac{q_M^*}{\sum_{m \in \{1\} \cup M_Z} \lambda_m} + \frac{1 - q_M^*}{\lambda_M + \sum_{m \in \{1\} \cup M_Z} \lambda_m} \right) \quad (21)$$

$$= \tilde{p}_i(q_1, \dots, q_{M-1}, q_M^*). \quad (22)$$

We can repeat this argument to show that

$$\tilde{p}_1(q_1, \dots, q_M) \geq \tilde{p}_1(q_1, \dots, q_{M-1}^*, q_M) \geq \tilde{p}_1(q_1, \dots, q_{M-1}^*, q_M^*). \quad (23)$$

By repeating the above argumentation we get

$$\tilde{p}_1(q_1, \dots, q_M) \geq \tilde{p}_1(q_1, q_2^* \dots, q_M^*) = \tilde{p}_1(q_1^*, \dots, q_M^*). \quad (24)$$

This proves the lemma. ■

Now that we can analyse the queues for the different ODP's, we can give expressions for the performance measure for the multi-stream case. Most of the expressions are equal to those in case of a single stream, but for completeness, we also give them here. Again we first give the length of the F-period and B-period, then we derive the utilization of the vehicles and the used capacity per trip of a vehicle. Finally we give the expected throughput time of a request. The length of the F-period is given by (see eq. (5)),

$$ET_F = \frac{P(N_C^* > 0)}{P(N_C^* = 0)} ES. \quad (25)$$

The length of the B-period is found by first looking at the busy period of the server defined as the time elapsed between two consecutive moments at which the server leaves all queues empty, that is $L_m = 0$, $m = 1, \dots, M$. During this period the expected number of *service starts* equals $1 / \prod_{m=1}^M P(L_m = 0)$, so the expected length of this busy period is $ET_{BP} = ES / \prod_{m=1}^M P(L_m = 0)$. A B-period ends when the server leaves an empty system and no one enters during the following service time. The probability that no requests enter during a service time is given by $\hat{A}(0) = \hat{S}(\lambda)$. Thus the number of busy periods during a B-period is geometrically distributed with success probability $\hat{S}(\lambda)$

so the expectation of T_B , the length of a B-period, is given by

$$ET_B = \frac{ET_{BP}}{\hat{S}(\lambda)} = \frac{ES}{\hat{S}(\lambda) \prod_{m=1}^M P(L_m = 0)}. \quad (26)$$

The probability P_F that the system is in an F-period at an arbitrary time point is now given by $P_F = \frac{ET_F}{ET_F + ET_B}$.

During a B-period all K vehicles are busy, whereas in the F-period, on average, $K - EN_C$ are busy. Therefore, the vehicle utilization U is

$$U = 1 - \frac{EN_C}{K} P_F. \quad (27)$$

Next, we consider the fraction of capacity used by a vehicle for stream m , $m = 1, \dots, M$. By assumption, every arrival during the F-period is served immediately, so the expected number of vehicles that depart during this period carrying stream m containers, equals the expected number of arrivals of type m during this period which is $\lambda_m ET_F$. During the B-period, the number of departing vehicles is $\left(\prod_{m=1}^M P(L_m = 0)\right)^{-1}$, including the first vehicle, which has already been counted in the F-period. Each such vehicle is with probability λ_m/λ carrying stream m containers. Correspondingly, on average, the capacity used by a vehicle of type m equals

$$EC_V = \frac{\lambda_m(ET_F + ET_B)}{\lambda_m ET_F + (\lambda_m/\lambda) \left(\left(\prod_{m=1}^M P(L_m = 0)\right)^{-1} - 1\right)}. \quad (28)$$

Note that the used capacity does not depend on the type. Now we can approximate the expected number of type i requests being processed, by multiplying the expected number of vehicles processing a request of type m by EC_V , where the processing time of a request is the time from the moment that a vehicle leaves the CDP until the delivery of the container(s). From the MDA of the F-period and the B-period we see that EN_{Vi} defined as the expected number of vehicles at a station (that is a terminal or a lane) in the network, is given by

$$EN_{Vi} = EN_{Vi}^F P_F + EN_{Vi}^B (1 - P_F). \quad (29)$$

To find the expected number of type i vehicles at this station we have to multiply EN_{Vi} by p_{im} , the probability that a vehicle at this station is of type m . This probability is

given by

$$p_{im} = \frac{\lambda_m V_{mi}}{\sum_{j=0}^M \lambda_j V_{ji}}, \quad (30)$$

where V_{ji} denotes the visit ratio of a type j vehicle to station i , where we assume that a vehicle without a request (that is traveling back to the CDP), is of type 0. The expected throughput time of a type m request is given by

$$\frac{EL_{Tm}(1 - P_F) + \sum_{i=1}^K p_{im} EN_{Vi}}{\lambda_m}, \quad (31)$$

where EL_{Tm} , the number of type m requests waiting in the external queue, is given by

$$EL_{Ti} = EL_i + \frac{1}{2} \lambda_i (1 + c_S^2) ES. \quad (32)$$

3 Numerical Validation

We developed a detailed simulation of the ITT system using AutoMod[®] software. We are able to realistically model the vehicle travel paths and the container handling stations using a path mover system. The vehicles are modeled as a multi-trailer system, where each trailer denoted 1 TEU capacity. The originating and the destination terminal locations, and the vehicle dwell point location are denoted using control points. The handling station is modeled as a control point with finite capacity (only one vehicle can be handled at a time). The handling times are i.i.d random variables with exponential distribution. Depending on the direction of the flow, we use a uni-directional or a bi-directional travel path.

If a container doesn't find a vehicle on arrival, it waits at an orderlist queue to be assigned to a vehicle. Once a vehicle is available, the vehicle is assigned to the container, and then the vehicle travels from its CDP (control point) to the pick-up (originating) terminal and retrieves the container. If the vehicle has multi-capacity then it picks up all containers (upto its capacity) and travels to the destination terminal. In the case of a multi-terminal with multi-stream flows, the vehicle identifies the container index that is waiting for the longest time at a terminal for transport operations. The vehicle then travels to the terminal to retrieve the waiting container (along with other containers queued at the terminal upto vehicle's capacity) and transports them to the destination terminal.

In Figure 8, we illustrate the three-terminal layout that we adopt for numerical validation of the proposed analytical models for the ITT problem. The direction of vehicle travel connecting two terminals depends on the number of distinct container flows in the path. For instance, if containers need to be transported from Terminal A to Terminal B as well as from Terminal B to Terminal A, then the travel path is bi-directional.

Figure 8: Topology of the inter-terminal transport area considered for numerical experiments

We validate our analytical model with four experiment types. Here we define a container flow between any two pair of stations as balanced if the flow rates are equal.

1. Single stream of uni-directional container flow
2. Multiple streams of uni-directional container flow (balanced)
3. Multiple streams of uni-directional container flow (unbalanced)
4. Multiple streams of bi-directional container flow (unbalanced)

In this set of experiments with a single stream of container flow, we only consider one stream of container flow that originates from terminal A and travels to terminal B. We run the simulation experiments for a 5 day simulation period with a warm-up period of 5 hrs. The number of replications for each experiment is 15. The 95% confidence intervals is about 2% from its mean. The percentage errors ($(\frac{A-S}{S}) \times 100\%$, where A and S are the performance measure estimates obtained from the analytical and the simulation model

respectively) in vehicle utilization and the expected container throughput times are low for different values of vehicle capacity ($c=2,4,10$), see Tables 1-3. However, the errors in expected queue lengths at the external buffer (expected number of containers waiting for vehicle availability) are somewhat larger (upto 36%). The high figures are a result of the open queuing approximations in the solution approach. Further, any deviation from the low values of the expected queue lengths also result in larger percentage errors. Note that exact analysis of the queue length at the external queue is complex and existing methods for a single capacity semi-open queuing networks also report a high percentage errors [upto 48%, see (Jia & Heragu 2009)].

Table 1: Numerical results for a single container flow from Terminal A to Terminal B

Here $\lambda_{AB}=18$ containers per hour, $c=2$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer (expected number of containers waiting for vehicle availability) and $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[T_{AB}]$ (A) (sec)		
	A	S	% E	A	S	% E	A	S	% E
2	94.5%	94.7%	0.2%	2.1	2.1	-2%	910.7	901.4	-1%
3	82.3%	82.2%	-0.1%	0.57	0.63	10%	607.1	615.3	1%
4	71.2%	70.9%	-0.4%	0.24	0.29	18%	550.3	556.3	1%
5	61.6%	61.3%	-0.4%	0.11	0.14	24%	530.9	533.5	0%
6	53.5%	53.4%	-0.3%	0.05	0.06	26%	524.3	525.6	0%
7	46.9%	46.8%	-0.1%	0.02	0.03	36%	522.5	523.1	0%
8	41.4%	41.4%	-0.2%	0.01	0.01	34%	522.3	521.2	0%
9	37.0%	37.0%	0.0%	0.00	0.00	-6%	522.5	521.8	0%
10	33.4%	33.5%	0.2%	0.00	0.00	NA	522.7	523.1	0%

In the set of experiments with multiple streams of uni-directional container flow (balanced), we consider two balanced streams (Terminal A to Terminal B, and Terminal C to Terminal B) with $\lambda_{AB}=10.8$ containers per hour, $\lambda_{BA}=10.8$ containers per hour. We consider the container handling times at the terminals as 180 seconds. Similar to the uni-directional stream experiment, we observe that the percentage errors in expected container throughput times and vehicle utilizations are much smaller compared to the percentage errors in the expected queue lengths. We also repeat this set of experiments with an unbalanced container flow between the two streams of container flow ($\lambda_{AB}=5.4$ containers per hour, $\lambda_{BA}=16.2$ containers per hour), see Tables 4 and 5.

Finally, we test the model fidelity by increasing the complexity of interactions among the terminals. Since we have three terminals, we consider all the six possible bidirectional

Table 2: Numerical results for a single container flow from Terminal A to Terminal B

Here $\lambda_{AB}=24$ containers per hour, $c=4$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer (expected number of containers waiting for vehicle availability) and $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[T_{AB}]$ (A) (sec)		
	A	S	% E	A	S	% E	A	S	% E
2	93.8%	95.3%	1.6%	1.54	1.34	-15%	713.8	678.0	-5%
3	87.3%	88.5%	1.3%	0.68	0.76	10%	596.6	600.2	1%
4	80.6%	81.1%	0.6%	0.37	0.46	20%	561.2	566.9	1%
5	73.9%	74.0%	0.1%	0.21	0.28	26%	547.6	552.0	1%
6	67.3%	67.2%	-0.2%	0.12	0.17	30%	543.3	546.1	1%
7	61.1%	61.0%	-0.2%	0.06	0.09	33%	543.2	545.4	0%
8	55.4%	55.3%	-0.2%	0.03	0.05	33%	544.8	546.3	0%
9	50.3%	50.3%	0.1%	0.02	0.03	37%	546.8	548.2	0%
10	45.8%	45.8%	-0.1%	0.01	0.01	28%	548.4	548.4	0%

Table 3: Numerical results for a single container flow from Terminal A to Terminal B

Here $\lambda=36$ containers per hour, $c=10$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer (expected number of containers waiting for vehicle availability) and $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[T_{AB}]$ (sec)		
	A	S	% E	A	S	% E	A	S	% E
2	97.1%	97.8%	0.7%	1.75	1.91	8%	657.8	665.8	1%
3	94.4%	94.8%	0.5%	1.06	1.31	19%	601.1	612.8	2%
4	91.3%	91.4%	0.1%	0.71	0.93	24%	580.3	587.7	1%
5	87.9%	87.8%	-0.1%	0.50	0.69	28%	573.2	578.6	1%
6	84.3%	84.0%	-0.3%	0.35	0.50	29%	573.6	576.2	0%
7	80.5%	80.1%	-0.6%	0.25	0.36	31%	578.3	580.4	0%
8	76.6%	76.2%	-0.5%	0.17	0.26	33%	585.7	588.1	0%
9	72.7%	72.4%	-0.4%	0.12	0.18	33%	594.5	597.2	0%
10	68.8%	68.6%	-0.3%	0.08	0.13	35%	603.6	608.2	1%

Table 4: Numerical results for two balanced streams of uni-directional container flow

Here we have uni-directional container flow between Terminal A to Terminal B, and Terminal C to Terminal B. $\lambda_{AB}=10.8$ containers per hour, $\lambda_{BA}=10.8$ containers per hour, $c=2$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer (expected number of containers waiting for vehicle availability) and $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[L_{BA}]$			$E[T_{AB}]$			$E[T_{BA}]$		
	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E
15	79%	78%	2%	0.25	0.36	-30%	0.25	0.36	-30%	2331.6	2277.9	2%	2295.0	2168.9	6%
16	79%	78%	1%	0.24	0.33	-28%	0.24	0.34	-28%	2456.7	2391.2	3%	2420.2	2282.0	6%
17	78%	76%	2%	0.23	0.31	-25%	0.23	0.30	-22%	2583.9	2479.7	4%	2547.5	2375.3	7%
18	78%	76%	2%	0.23	0.31	-28%	0.23	0.31	-28%	2713.2	2624.9	3%	2676.8	2508.3	7%
19	78%	76%	3%	0.22	0.30	-26%	0.22	0.30	-27%	2844.4	2706.9	5%	2808.0	2623.3	7%
20	77%	75%	3%	0.21	0.31	-30%	0.21	0.30	-30%	2977.5	2866.7	4%	2941.1	2750.8	7%
21	77%	76%	2%	0.21	0.28	-26%	0.21	0.28	-26%	3112.4	3005.1	4%	3076.1	2884.9	7%
22	77%	76%	1%	0.20	0.29	-29%	0.20	0.29	-30%	3249.2	3174.7	2%	3212.9	3067.5	5%
23	77%	76%	1%	0.20	0.30	-34%	0.20	0.30	-32%	3387.7	3326.3	2%	3351.4	3200.0	5%
24	77%	76%	2%	0.20	0.28	-30%	0.20	0.29	-31%	3528.0	3415.5	3%	3491.7	3310.4	5%

Table 5: Numerical results for two balanced streams of uni-directional container flow

Here we have uni-directional container flow between Terminal A to Terminal B, and Terminal C to Terminal B. $\lambda_{AB}=5.4$ containers per hour, $\lambda_{BA}=16.2$ containers per hour, $c=2$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer (expected number of containers waiting for vehicle availability) and $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[L_{BA}]$			$E[T_{AB}]$			$E[T_{BA}]$		
	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E
15	80%	79%	2%	0.18	0.19	-6%	0.33	0.54	-40%	2147.7	2114.9	2%	2467.2	2327.5	6%
16	80%	78%	2%	0.17	0.18	-3%	0.31	0.53	-41%	2261.0	2215.1	2%	2594.9	2459.6	6%
17	79%	78%	2%	0.16	0.17	-2%	0.30	0.50	-41%	2377.6	2344.0	1%	2724.0	2572.6	6%
18	79%	78%	2%	0.16	0.17	-7%	0.29	0.51	-44%	2497.1	2468.2	1%	2854.6	2728.2	5%
19	79%	77%	2%	0.15	0.17	-10%	0.28	0.49	-43%	2619.5	2588.1	1%	2986.7	2838.4	5%
20	79%	77%	2%	0.15	0.15	-4%	0.27	0.45	-41%	2744.5	2690.0	2%	3120.3	2963.2	5%
21	78%	77%	1%	0.14	0.15	-3%	0.26	0.44	-40%	2872.0	2806.6	2%	3255.5	3088.8	5%
22	78%	78%	1%	0.14	0.15	-8%	0.25	0.47	-46%	3001.9	2981.4	1%	3392.1	3274.5	4%
23	78%	78%	0%	0.14	0.15	-8%	0.25	0.46	-47%	3134.1	3133.3	0%	3530.4	3445.0	2%
24	78%	76%	2%	0.13	0.14	-2%	0.24	0.39	-37%	3268.5	3135.8	4%	3670.1	3426.5	7%

container flows between any two adjacent terminals (see Tables 6-9). We see that the increase in network interactions do not increase the percentage errors in the performance measures by a large amount. We also observe that by increasing the vehicle capacity (from $c = 2$ to $c = 5$), the expected container throughput times decreases by about 1% (at low vehicle utilizations) to about 6% (at high vehicle utilizations). Further, we also see that the vehicle utilization does not change much with varying carrying capacities.

Table 6: Numerical results (utilization and expected queue lengths) for six streams of bidirectional container flow

Here $\lambda_{AB}=5.4$ containers per hour, $\lambda_{BA}=1.8$ containers per hour, $\lambda_{AC}=3.6$ containers per hour, $\lambda_{CA}=3.6$ containers per hour, $\lambda_{BC}=3.6$ containers per hour, $\lambda_{CB}=3.6$ containers per hour, $c=2$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer, columns A and S are the performance measure estimates from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[L_{BA}]$			$E[L_{AC}]$			$E[L_{CA}]$			$E[L_{BC}]$			$E[L_{CB}]$		
	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E
6	93%	93%	0%	0.62	0.57	9%	0.19	0.20	-4%	0.39	0.38	3%	0.39	0.38	2%	0.39	0.38	2%	0.39	0.38	2%
7	89%	89%	-1%	0.41	0.40	2%	0.13	0.14	-8%	0.27	0.28	-4%	0.27	0.27	-3%	0.27	0.27	-3%	0.27	0.28	-5%
8	85%	85%	-1%	0.29	0.30	-5%	0.10	0.11	-10%	0.19	0.20	-7%	0.19	0.20	-5%	0.19	0.21	-10%	0.19	0.21	-7%
9	81%	82%	-1%	0.21	0.23	-8%	0.07	0.08	-16%	0.14	0.16	-13%	0.14	0.16	-11%	0.14	0.16	-11%	0.14	0.16	-12%
10	78%	78%	0%	0.16	0.18	-9%	0.05	0.07	-17%	0.11	0.12	-10%	0.11	0.12	-13%	0.11	0.12	-11%	0.11	0.12	-13%
11	75%	75%	-1%	0.13	0.14	-13%	0.04	0.05	-14%	0.08	0.10	-15%	0.08	0.10	-16%	0.08	0.10	-16%	0.08	0.10	-17%
12	72%	72%	-1%	0.10	0.11	-14%	0.03	0.04	-23%	0.06	0.08	-15%	0.06	0.08	-18%	0.06	0.08	-16%	0.06	0.08	-16%
13	69%	69%	-1%	0.08	0.09	-19%	0.03	0.03	-25%	0.05	0.06	-14%	0.05	0.06	-16%	0.05	0.06	-18%	0.05	0.06	-16%
14	66%	67%	-1%	0.06	0.07	-17%	0.02	0.02	-18%	0.04	0.05	-18%	0.04	0.05	-21%	0.04	0.05	-16%	0.04	0.05	-17%
15	63%	64%	-1%	0.05	0.06	-18%	0.02	0.02	-25%	0.03	0.04	-20%	0.03	0.04	-22%	0.03	0.04	-19%	0.03	0.04	-23%

Table 7: Numerical results (expected throughput times) for six streams of bidirectional container flow (contd.)

Here $\lambda_{AB}=5.4$ containers per hour, $\lambda_{BA}=1.8$ containers per hour, $\lambda_{AC}=3.6$ containers per hour, $\lambda_{CA}=3.6$ containers per hour, $\lambda_{BC}=3.6$ containers per hour, $\lambda_{CB}=3.6$ containers per hour, $c=2$, $h_v=30$ km/hr. $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	$E[T_{AB}]$			$E[T_{BA}]$			$E[T_{AC}]$			$E[T_{CA}]$			$E[T_{BC}]$			$E[T_{CB}]$		
	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E
6	1445.3	1393.1	4%	1416.8	1402.7	1%	1516.3	1494.5	1%	1482.8	1468	1%	1403.3	1379.4	2%	1393.9	1368.3	2%
7	1365.2	1338.7	2%	1362.1	1347.7	1%	1452.2	1459.9	-1%	1418.9	1417.8	0%	1339	1344.9	0%	1329.8	1328.7	0%
8	1343.7	1342.8	0%	1349.6	1341.6	1%	1437	1436.6	0%	1404	1400	0%	1323.8	1341.4	-1%	1314.9	1331.1	-1%
9	1348.7	1361.1	-1%	1357.6	1359.9	0%	1445	1465.2	-1%	1412.2	1426.7	-1%	1331.7	1349.1	-1%	1323	1345.9	-2%
10	1366.4	1380.1	-1%	1375.5	1376.6	0%	1464.1	1465.2	0%	1431.4	1438.8	-1%	1350.7	1356.6	0%	1342.2	1366.9	-2%
11	1390.7	1406.6	-1%	1398.6	1377.4	2%	1488.8	1508.5	-1%	1456.3	1482.6	-2%	1375.4	1401.3	-2%	1367	1379.8	-1%
12	1417.7	1448.5	-2%	1423.5	1434.7	-1%	1515.7	1546.8	-2%	1483.3	1522.2	-3%	1402.3	1418.2	-1%	1394	1412.5	-1%
13	1445.2	1464.2	-1%	1448.6	1454.7	0%	1542.8	1560.3	-1%	1510.6	1530.3	-1%	1429.4	1456.4	-2%	1421.2	1454.8	-2%
14	1471.8	1503.3	-2%	1472.6	1474.7	0%	1569	1605	-2%	1536.8	1585.2	-3%	1455.5	1460.2	0%	1447.4	1463.6	-1%
15	1497.2	1514.9	-1%	1495.4	1495.8	0%	1593.7	1615.8	-1%	1561.6	1581.5	-1%	1480.2	1519	-3%	1472.1	1507.9	-2%

Table 8: Numerical results (utilization and expected queue lengths) for six streams of bidirectional container flow

Here $\lambda_{AB}=5.4$ containers per hour, $\lambda_{BA}=1.8$ containers per hour, $\lambda_{AC}=3.6$ containers per hour, $\lambda_{CA}=3.6$ containers per hour, $\lambda_{BC}=3.6$ containers per hour, $\lambda_{CB}=3.6$ containers per hour, $c=5$, $h_v=30$ km/hr. U_V is the vehicle utilization, $E[L_{AB}]$ is the expected queue lengths at the external buffer, columns A and S are the performance measure estimates from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	U_V			$E[L_{AB}]$			$E[L_{BA}]$			$E[L_{AC}]$			$E[L_{CA}]$			$E[L_{BC}]$			$E[L_{CB}]$		
	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E
6	94%	91%	3%	0.33	0.39	-16%	0.31	0.27	15%	0.15	0.15	-1%	0.29	0.28	7%	0.29	0.28	7%	0.29	0.27	7%
7	91%	88%	3%	0.24	0.30	-21%	0.23	0.22	3%	0.11	0.11	-8%	0.21	0.21	0%	0.21	0.21	-2%	0.21	0.22	-3%
8	87%	84%	4%	0.18	0.23	-22%	0.17	0.16	5%	0.08	0.09	-11%	0.16	0.16	-5%	0.16	0.16	-3%	0.16	0.16	-3%
9	84%	81%	4%	0.14	0.19	-25%	0.13	0.13	-1%	0.06	0.07	-16%	0.12	0.13	-8%	0.12	0.13	-8%	0.12	0.13	-10%
10	80%	78%	3%	0.11	0.15	-26%	0.10	0.10	-4%	0.05	0.05	-14%	0.09	0.10	-12%	0.09	0.10	-12%	0.09	0.10	-9%
11	77%	74%	3%	0.08	0.12	-27%	0.08	0.08	-9%	0.04	0.04	-19%	0.07	0.08	-13%	0.07	0.08	-15%	0.07	0.08	-13%
12	74%	72%	2%	0.07	0.10	-30%	0.06	0.06	-7%	0.03	0.04	-23%	0.06	0.07	-18%	0.06	0.07	-15%	0.06	0.07	-15%
13	70%	69%	2%	0.05	0.08	-30%	0.05	0.05	-13%	0.02	0.03	-22%	0.04	0.06	-20%	0.04	0.06	-21%	0.04	0.05	-20%
14	67%	66%	2%	0.04	0.06	-31%	0.04	0.04	-15%	0.02	0.02	-26%	0.03	0.04	-16%	0.03	0.04	-16%	0.03	0.04	-17%
15	65%	64%	2%	0.03	0.05	-34%	0.03	0.04	-20%	0.01	0.02	-26%	0.03	0.04	-23%	0.03	0.03	-21%	0.03	0.04	-24%

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Table 9: Numerical results (expected throughput times) for six streams of bidirectional container flow (contd.)

Here $\lambda_{AB}=5.4$ containers per hour, $\lambda_{BA}=1.8$ containers per hour, $\lambda_{AC}=3.6$ containers per hour, $\lambda_{CA}=3.6$ containers per hour, $\lambda_{BC}=3.6$ containers per hour, $\lambda_{CB}=3.6$ containers per hour, $c=5$, $h_v=30$ km/hr. $E[T_{AB}]$ is the expected container throughput times in seconds. Columns A and S are the performance measure estimates obtained from the analytical and the simulation models respectively, and E is the error of the analytical model with respect to the simulation model.

V	$E[T_{AB}]$			$E[T_{BA}]$			$E[T_{AC}]$			$E[T_{CA}]$			$E[T_{BC}]$			$E[T_{CB}]$		
	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E	A	S	% E
6	1253.5	1258.1	0%	1340.9	1289.3	4%	1425.3	1361.7	5%	1394.2	1336.9	4%	1311.6	1257.1	4%	1304.5	1252.1	4%
7	1261.9	1282.0	-2%	1323.2	1300.4	2%	1408.7	1382.1	2%	1377.2	1358.7	1%	1295.1	1275.5	2%	1287.6	1275.5	1%
8	1286.0	1291.4	0%	1331.0	1300.0	2%	1417.7	1387.8	2%	1386.0	1361.0	2%	1304.2	1287.6	1%	1296.5	1272.9	2%
9	1317.8	1327.6	-1%	1351.7	1330.3	2%	1439.8	1417.6	2%	1408.0	1393.2	1%	1326.3	1308.7	1%	1318.5	1312.2	0%
10	1352.7	1359.9	-1%	1378.5	1347.9	2%	1468.1	1448.0	1%	1436.3	1423.5	1%	1354.6	1340.9	1%	1346.8	1335.1	1%
11	1388.2	1378.8	1%	1407.4	1380.4	2%	1498.8	1482.4	1%	1466.9	1451.4	1%	1385.3	1389.9	0%	1377.4	1379.6	0%
12	1422.6	1425.9	0%	1436.2	1424.5	1%	1529.4	1508.2	1%	1497.5	1488.2	1%	1415.9	1440.7	-2%	1408.0	1422.1	-1%
13	1454.8	1449.9	0%	1463.6	1436.7	2%	1558.7	1553.3	0%	1526.8	1524.4	0%	1445.2	1457.4	-1%	1437.3	1447.2	-1%
14	1484.4	1469.5	1%	1489.0	1472.2	1%	1586.0	1597.9	-1%	1554.1	1578.0	-2%	1472.5	1476.4	0%	1464.6	1479.7	-1%
15	1511.2	1523.4	-1%	1512.0	1500.0	1%	1610.8	1596.5	1%	1578.9	1573.8	0%	1497.3	1509.5	-1%	1489.4	1505.0	-1%

4 Experiments with Data from the Port of Rotterdam (PoR)

The port of Rotterdam functions as a hub for the Baltic area and the UK. The *Maasvlakte 1* (MV1) houses some of the biggest terminals such as APMT, ECT, and Euromax. With the recent construction of the *Maasvlakte 2* (MV2), several new terminals will be set up to support the estimated growth of 75% in the number of container flows through the PoR. Significant growth in the percent container flows will demand greater efficiencies and throughput time performance in the inter-terminal transport.

The terminals have specific functions in the distribution process of containers to the Hinterland. These functions are deep sea terminal, common rail terminal, common barge terminal, empty depot and customs facilities. The deep sea terminal handles the ocean going vessels and has own facilities for truck, rail and barge for large consolidated cargo flows to the Hinterland. Smaller deep sea terminals focus more on short sea cargo and do not handle the big ocean going vessels. The common rail terminal and common barge terminal bundle small cargo flows for Hinterland destinations. The bundling of cargo flows increases the frequency of the connections to the Hinterland. A high frequent connection with Hinterland connection is likely to have higher utilization of trains and barges.

Through ITT, containers will be transported between the deep sea terminal, the common rail terminal, and common barge terminal. MV1 facilitates one Common Rail Terminal (Rail Terminal West) and one Common Barge Terminal (Barge Service Center Hartelhaven). MV2 will have a Common Rail Terminal and a Common Barge Terminal. The decision to handle a cargo flow at the deep sea terminal or at the common barge/rail terminal is based on the call size. Support activities are needed for a well-functioning logistic chain of containers. These activities are cleaning and pre-voyage check of the containers, storage of empty containers, repair of damaged containers and inspection of the container loads by customs. The support activities generate container transport executed by ITT. In Figure 9, we show the geographical position of the existing MVQ terminals and the proposed MV2 terminals. We also highlight the terminals that we consider for the ITT analysis.

The MV1 and MV2 area of PoR houses 9 deep sea terminals (ECT Delta, EuroMax, APM MV1, Rotterdam World Gateway, APM MV2 Terminal, T3, T4, ECT Delta barge Feeder terminal, Delta Container Services), two rail terminals (Common Rail Terminal and Rail Terminal West), two barge terminals (Rotterdam Container Terminal Hartel-

Figure 9: Topology of the inter-terminal transport area from the Port of Rotterdam

haven and Common Barge Service Center), and four empty container depots (Kramer Delta depot, Van Doorn Container depot, Empty depot MV1, Empty depot MV2) and one customs depot. To reduce the modeling complexity, we aggregate terminals that are in proximity and accessible by road/sea.

We adopt the following approach to obtain the candidate terminal list for our anal-

ysis:

1. We aggregate multiple terminals adjacent to each other, and replace the individual terminals with a proxy terminal whose location is in-between the two original terminals.
2. Consider aggregated streams of inter terminal container flow only when the flow rate between the two terminals exceeds 10000 TEU per year.
3. Cluster terminals into zones and analyze each zone separately. We also assume that cross docks at the interface of two zones buffer containers that require inter zone movement. So a container that needs to travel from a terminal in Zone 1 to a terminal in Zone 3 may have to travel via Zone 2. We denote the cross-dock areas between Zone 1 and Zone 2 as CD1, and between Zone 2 and Zone 3 as CD2.

We combine the two deep sea terminals, Euromax and T4 to an aggregated terminal, N1. Likewise, we combine T3 and Rotterdam World Gateway to N2; combine RTW and BSC Harterhaven to N3, combine ECT Delta and APM MV1 to N4. In addition to the four aggregated terminals, N1, N2, N3, and N4, we also include the four terminals, Common Barge Service Center (CBS), ED MV2, APMV2, and Common Rail (CR). Finally, we allocate the eight terminals into three zones. Zone 1 includes N1, CBS, and N2 with a combined inflow of 521207 TEU and an outflow of 553604 TEU. Likewise, Zone 2 includes ED, APM MV2, and CR with a combined inflow of 510930 TEU and an outflow of 444120 TEU. Zone 3 includes N4 and N3 with an inflow of 675952 TEU and an outflow of 710365 TEU. For each zone, we also identify a central dwelling point where the vehicles dwell after completing each inter-terminal container transport. Table 10 shows the different terminals (including newly formed aggregated terminals) and the annual inter-terminal container flow (in TEU) between any origin-destination pair. The vehicle central dwelling point for the Zones 1, 2, and 3 lie between terminal CBS and N2, at APM MV2, and between N3 and N4 respectively.

We now discuss the analytical findings for the zone-wise throughput time analysis and alternate dwell point choice for the ITT problem in the PoR (see Tables 11-12). Note that though we present the results for a three-zone case, with appropriate network simplification and flow aggregation, two zones or four zones can be formed. Because of the choice of the dwell point in Zone 1, we see that the expected throughput time between the aggregated terminal N1 and the central dwelling point, CD is highest (about

Table 10: Annual projected container flows between a pair of origin and destination terminals. The origin terminals are in the rows while the destination terminals are the columns.

	N1	CBS	N2	ED	APM MV2	CR	N4	N3
N1	0	23276	4410	18437	2544	50385	6751	85635
CBS	19117	0	11977	22925	13820	0	50481	0
N2	4536	14639	0	23102	3280	63132	8700	94060
ED	27655	15284	34652	0	0	0	57033	32018
APM MV2	2676	16891	3354	0	0	0	5133	54266
CR	66714	0	77162	0	0	0	118092	0
N4	7737	44809	9262	35355	5592	96620	0	89188
N3	68949	0	100504	50945	57983	0	109008	0

15 mins). Some of the lowest throughput times are in the range of 7 minutes. Further, the expected throughput time for the container flows in Zone 2 is as high as 20 mins for the incoming streams from Zone 3. Likewise, the maximum expected throughput times in Zone 3 is about 20 mins. Our models can be used to size the vehicles and their capacity for ITT in MV1 and MV2. In Table 14, we analyze the effect of an alternate dwell point policy on Zone 1 throughput time performance. We see that if the dwell point is changed from midpoint of CBS and N2 to the CD1 (cross-dock area of Zone 1), the vehicle utilization decreases marginally. The expected throughput times for some streams increases significantly (for instance, $E[T_{N1,CBS}]$ increases by 35%-40%) whereas the expected throughput times for other streams decreases significantly (for instance, $E[T_{CD,N2}]$ decreases by about 50%). This suggests that our model can be used to analyze the throughput time tradeoffs with alternate dwell point policies.

Table 11: Expected throughput times for the Port of Rotterdam inter-terminal container flows in Zone 1

This table shows the expected utilization U_V of vehicles and the expected throughput times for containers $E[T_{N1,CBS}]$ in Zone 1 (corresponding to a given origin-destination pair). The central dwelling point is assumed to be the midpoint between terminals CBS and N2. The values in this table are based on the flow data in Table 10 for ITT at the Port of Rotterdam, and estimated using the semi-open queueing model proposed in Section 2.

V	U_V	$E[T_{N1,CBS}]$	$E[T_{N1,CD}]$	$E[T_{CBS,N1}]$	$E[T_{CBS,N2}]$	$E[T_{CBS,CD}]$	$E[T_{N2,CBS}]$	$E[T_{N2,CD}]$	$E[T_{CD,N1}]$	$E[T_{CD,CBS}]$	$E[T_{CD,N2}]$
15	97.0%	701.9	950.8	517.4	533.8	587.6	537.3	431.4	956.9	781.9	606.6
16	96.6%	699.4	948.3	514.9	532.3	582.8	535.5	429.2	953.6	776.1	604.4
17	96.0%	697.8	946.6	513.3	531.8	578.6	534.7	428.0	951.3	771.0	603.2
18	95.5%	696.9	945.7	512.4	532.2	575.0	534.8	427.7	949.7	766.5	603.0
19	94.8%	696.5	945.4	512.1	533.5	571.9	535.8	428.4	949.0	762.6	603.8
20	94.2%	696.7	945.7	512.5	535.5	569.3	537.6	429.8	948.8	759.1	605.4
21	93.5%	697.4	946.5	513.4	538.2	567.1	540.0	431.9	949.2	756.0	607.7
22	92.7%	698.5	947.8	514.7	541.6	565.1	543.2	434.7	950.1	753.3	610.8
23	91.9%	700.0	949.4	516.5	545.5	563.5	546.9	438.2	951.4	750.9	614.4
24	91.0%	701.8	951.4	518.6	550.0	562.2	551.2	442.2	953.2	748.8	618.7

Table 12: Expected throughput times for the Port of Rotterdam inter-terminal container flows in Zone 2

This table shows the expected utilization (U_V) of vehicles and the expected throughput times for containers $E[T_{CD1,ED}]$ in Zone 2 (corresponding to a given origin-destination pair). The central dwelling point is assumed to be at terminal APM MV2. The values in this table are based on the flow data in Table 10 for ITT at the Port of Rotterdam, and estimated using the semi-open queueing model proposed in Section 2.

V	U_V	$E[T_{CD1,ED}]$	$E[T_{CD1,APMV2}]$	$E[T_{CD1,CR}]$	$E[T_{CD1,CD2}]$	$E[T_{ED,CD1}]$	$E[T_{ED,CD2}]$	$E[T_{APMV2,CD1}]$	$E[T_{APMV2,CD2}]$	$E[T_{CR,CD1}]$	$E[T_{CR,CD2}]$	$E[T_{CD2,ED}]$	$E[T_{CD2,APMV2}]$	$E[T_{CD2,CR}]$	$E[T_{CD2,CD1}]$
33	97.9%	562.9	613.2	1005.3	975.0	374.8	725.7	361.4	585.4	1104.9	636.0	1141.9	1065.7	761.9	1202.7
34	97.8%	561.6	610.4	1006.6	972.7	373.8	724.7	358.9	583.3	1106.4	637.7	1140.6	1063.1	763.4	1200.4
35	97.6%	560.5	607.7	1008.3	970.6	373.0	723.9	356.6	581.3	1108.3	639.8	1139.5	1060.7	765.3	1198.3
36	97.4%	559.6	605.1	1010.3	968.6	372.4	723.1	354.4	579.3	1110.6	642.2	1138.4	1058.4	767.5	1196.3
37	97.1%	558.7	602.6	1012.6	966.8	371.9	722.5	352.4	577.5	1113.2	645.0	1137.6	1056.1	770.1	1194.4
38	96.9%	558.0	600.2	1015.3	965.1	371.5	722.1	350.4	575.8	1116.2	648.1	1136.8	1054.0	773.0	1192.8
39	96.7%	557.5	597.9	1018.4	963.6	371.2	721.7	348.6	574.1	1119.5	651.6	1136.1	1051.9	776.3	1191.2
40	96.4%	557.0	595.7	1021.9	962.1	371.1	721.5	346.8	572.5	1123.3	655.4	1135.6	1049.9	779.9	1189.8
41	96.1%	556.7	593.6	1025.7	960.8	371.1	721.4	345.1	571.1	1127.4	659.6	1135.2	1047.9	783.9	1188.5
42	95.8%	556.5	591.6	1030.0	959.6	371.1	721.3	343.5	569.6	1131.9	664.2	1134.9	1046.1	788.3	1187.3

Table 13: Expected throughput times for the Port of Rotterdam inter-terminal container flows in Zone 3

This table shows the expected utilization (U_V) of vehicles and the expected throughput times for containers $E[T_{N3,N4}]$ in Zone 3 (corresponding to a given origin-destination pair). The central dwelling point is assumed to be the midpoint between terminals N3 and N4. The values in this table are based on the flow data in Table 10 for ITT at the Port of Rotterdam, and estimated using the semi-open queueing model proposed in Section 2.

V	U_V	$E[T_{N3,N4}]$	$E[T_{N4,CD}]$	$E[T_{N3,N4}]$	$E[T_{N3,CD}]$	$E[T_{CD,N4}]$	$E[T_{CD,N3}]$
15	87.4%	1272.3	846.1	1188.5	700.9	485.6	427.6
16	85.6%	1276.0	846.0	1192.0	702.9	485.3	429.2
17	83.7%	1280.5	846.6	1196.3	705.8	485.8	431.8
18	81.7%	1285.7	847.5	1201.3	709.4	486.7	435.0
19	79.7%	1291.2	848.8	1206.7	713.5	488.0	438.9
20	77.5%	1296.9	850.2	1212.4	717.9	489.5	443.0
21	75.3%	1302.6	851.8	1218.1	722.4	491.1	447.3
22	73.1%	1308.1	853.3	1223.7	726.9	492.7	451.6
23	70.9%	1313.3	854.7	1228.9	731.2	494.2	455.7
24	68.7%	1318.0	856.0	1233.7	735.1	495.5	459.5

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Table 14: Expected throughput times for the Port of Rotterdam inter-terminal container flows in Zone 1 (with a different dwell point location)

This table shows the expected utilization (U_V) of vehicles and the expected throughput times for containers $E[T_{N1,CBS}]$ in Zone 1 (corresponding to a given origin-destination pair). The central dwelling point is assumed to be CD1 (cross-dock area of Zone 1). The values in this table are based on the flow data in Table 10 for ITT at the Port of Rotterdam, and estimated using the semi-open queueing model proposed in Section 2.

V	U_V	$E[T_{N1,CBS}]$	$E[T_{N1,CD}]$	$E[T_{CBS,N1}]$	$E[T_{CBS,N2}]$	$E[T_{CBS,CD}]$	$E[T_{N2,CBS}]$	$E[T_{N2,CD}]$	$E[T_{CD,N1}]$	$E[T_{CD,CBS}]$	$E[T_{CD,N2}]$
15	96.6%	977.5	1207.8	793.4	806.9	848.1	624.2	514.8	675.1	490.9	332.8
16	96.0%	975.6	1206.9	791.5	806.5	844.5	623.5	513.7	673.4	486.6	331.9
17	95.4%	974.4	1206.7	790.3	807.0	841.4	623.7	513.6	672.6	483.0	331.9
18	94.7%	973.7	1207.2	789.8	808.2	838.9	624.8	514.5	672.4	480.0	332.8
19	94.0%	973.6	1208.3	789.7	810.1	836.7	626.7	516.1	672.9	477.4	334.6
20	93.3%	973.9	1209.8	790.2	812.7	834.8	629.4	518.6	673.9	475.3	337.1
21	92.5%	974.6	1211.8	791.1	815.9	833.2	632.7	521.7	675.4	473.5	340.3
22	91.6%	975.7	1214.2	792.5	819.7	831.9	636.6	525.5	677.3	472.0	344.1
23	90.7%	977.1	1216.8	794.1	824.0	830.8	641.1	529.9	679.5	470.8	348.6
24	89.7%	978.8	1219.8	796.1	828.8	830.0	646.1	534.8	682.1	469.8	353.6

5 Conclusion

In this paper we present a novel modeling approach to analyze the performance of an ITT system. We believe this work is the first attempt to develop a stochastic model of the ITT problem that captures the interaction between containers and vehicles at the external matching queue. Numerical validation of our model shows that the errors for the estimated container throughput time and vehicle utilization are small, and remain low even with increasing interactions between the terminals. We analyze the ITT problem at the Port of Rotterdam using our model and show the effect of various dwell point locations on system performance.

We also develop a solution approach for the semi-open queueing network with multi-capacity resources. While the model in this paper is stylized for the analysis of the ITT system, we believe that our modeling framework for semi-open queue with bulk service has numerous other applications in analyzing vehicle-passenger transport, for instance, in warehouse and other similar material handling applications.

As part of future work, we intend to use our analytical model as input for a simulation model of the ITT system at the Port of Rotterdam. From an analytical viewpoint, exploring semi-open queues with heterogeneous capacities might be an interesting avenue for future research.

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