Content Based Routing in networks with time-fluctuating request rates

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Abstract

In large-scale distributed applications, a loosely-coupled event-based style of communication as in publish/subscribe systems eases the integration of autonomous, heterogeneous components. In a publish/subscribe system, content based routing - where routing is based on the content of the messages - is an alternative to address-based delivery. In this paper we compare the efficiency of two content-based routing algorithms: the flooding scheme and the more sophisticated identity-based routing scheme. Our analytical approach is based on continuous time Markov Chains and extends the steady state approach by Jaeger and Mühl [8] to systems with time-fluctuating parameters. We obtain explicit closed form solutions for the time-dependent distribution of the number of active clients, taking into account the use of advertisements and roaming clients. The results allow us to investigate, for example, the switching point between optimality of flooding and identity-based routing.

Key words: content based routing, publish-subscribe mechanisms, transient behavior

1 Introduction

1.1 Motivation

In many computer networks, independently created applications have to be integrated into a complex information system. Especially in large-scale distributed applications, a loosely-coupled event-based style of communication has many advantages. It eases the integration of autonomous, heterogenous entities.

In publish/subscribe systems individual processing entities, called clients, can publish information without specifying a particular destination. Similarly,

clients can express their interest in receiving certain types of information by subscribing. The messages are asynchronously exchanged over the network. The *publishers* are responsible for the input of information by publishing *notifications* to the network. The *subscribers* subscribe to information they are interested in by issuing *subscriptions*.

Content based routing is used in publish-subscribe systems to distribute the messages. This service bases its routing on the content of the messages (as opposed to the addresses of the messages). In this paper, we contribute to the analysis of scalability of content based routing. Two common performance measures related to the scalability of content based routing are the *sizes of the routing table* and the *amount of messages* flowing through the network. Our focus is on the latter. We compare the total amount of messages (subscriptions and notifications) flowing through the network under two content based routing algorithms: *flooding* and *identity-based routing*. The reason for this is that by analyzing these algorithms, we get indications on how much control traffic may be added in order to reduce the amount of data traffic. This is mainly important in distributed systems where bandwidth is a limitation (due to e.g. wireless connections).

In Section 2 of this paper we describe content based routing algorithms in more detail. In Section 3 we present our main contribution: a time-dependent analysis of the message traffic under content based routing algorithms. In Section 4 we present a number of possible model extensions. Section 5 presents an example comparing the amount of message traffic in both a transient and a steady state setting. Section 6 discusses the impact of local interest. Section 7 presents the conclusions of this work.

1.2 Statement of contribution

Jaeger and Mühl [8] present an analytical approach to analyze a content based routing system. Their work is based on a stochastic approach involving continuous time birth-death Markov chains. The main contribution of the present paper is the extension to allow the rate at which subscribers initiate content requests to depend on time (e.g. due to time of day effects). The importance of such a time dependent analysis - as opposed to steady state distributions - is discussed below. As an additional contribution, we describe additional model features, such as finite numbers of clients, roaming clients, locations of the publishers in the network and the use of advertisements.

Our model falls in the class of networks of infinite server queues. For such networks the limiting distribution of the number of customers in the queues is known to have a so-called multivariate Poisson distribution. Surprisingly,

for content based routing, performance measures such as the amount of toggle traffic, and the traffic due to notifications and subscriptions, can be shown to depend only on the load and arrival rate parameters of this time dependent distribution. This allows us to evoke the equilibrium results of Jaeger and Mühl [8] to also obtain explicit results for the performance measures of this paper in the time dependent case.

In stark contrast with the seemingly direct correspondence of our results with those of Jaeger and Mühl, we stress that the reasoning for obtaining explicit results for the time dependent performance measures is not based on equilibrium methods since regenerative and insensitivity arguments may not be applied to the time dependent case. Instead, we use a new method based on functional equivalence. An example based on our model shows that the traffic generated in the time dependent and in the steady state case can differ considerably. The main cause for this is the 'toggle traffic' that is generated when the number of active subscribers at an end-broker toggles between 0 and 1.

1.3 Example: the added value of a time-varying analysis

Why is time dependent behavior important? The approach presented in this paper allows for the fact that the rate at which subscribers initiate content requests depends on time (e.g. due to time of day effects). One may wonder why this is important: one could imagine performing various equilibrium-based analysis based on various regimes to account for the effects of the time of the day. Below we show that -depending on the system parameters- time-dependent analyzes much better capture the behavior of a queuing system than an approximation based on several equilibrium analysis could do. To that order consider the following simple example. Suppose a time dependent queuing system consist of a single queue with a Poisson arrival rate (denoting the number of customers arriving per time unit) that oscillates between 0 and 1 as given by: $\lambda(t) = 1$ for $t \in [3i, 3(i+1)]$, $i = 0, 2, 4$, and $\lambda(t) = 0$, for $t \in [3i, 3(i+1)]$, $i = 1, 3, 5$. Here, the periods where $\lambda(t) = 1$ represent the busy periods, the periods where $\lambda(t) = 0$ represent the quiet periods, when no new customers are entering the system. The exponential completion rate (denoting the number of customers served per time unit) is a constant $\mu = 1$.

Now one could argue that in order to account for the time dependent arrival rate, it could be sufficient to consider different equilibrium analyzes with different values of the parameters. Such a method (we call it here 'the approximation method'), would lead to the following time dependent system load $\rho(t) = \frac{\lambda}{\mu} = 1$, when $\lambda(t) = 1$ and $\rho(t) = 0$ otherwise.

A time varying (or transient analysis) using the methods in [10], shows that

in this case the time dependent system load is composed of functions $\rho_i(t)$ for $t \in [3i, 3(i+1)]$, and $i = 0, 1, 2, \dots$. The time dependent system load for the periods $t \in [3i, 3(i+1)]$, are then given as:

$$\rho_i(t) = 1 + e^{-t}(\rho_{0,i} - 1) \text{ when } i \in \{0, 2, 4\}$$

and

$$\rho_{i+1}(t) = \rho_{0,i+1}e^{-t} \text{ when } i \in \{1, 3, 5\}.$$

Here $\rho_{0,i}$ denotes the initial value for a new period $[3i, 3(i+1)]$ which is recursively defined as $\rho_{0,i} = \rho_{i-1}(3(i+1))$ for $i = 1, 2, \dots$ and $\rho_{0,0} = 0$. (So we start with a system with no customers).

Fig. 1. Time varying analysis vs Equilibrium analysis: a simple example

Both the approximation method and the exact method are displayed in Figure 1.3 (a) System Load versus time. Here the dashed lines indicate the system load ρ as calculated by the approximation method. The bold lines indicate the system load $\rho(t)$ as calculated by a transient analysis. The figure makes clear that the time varying and the described approximation method yield different results. For example, assume that for dimensioning purposes it is important to have insight in the fraction of time that the system load exceeds the value 0.8 (which is called: a high load period). The approximative method would conclude that for this system is half the time in a high load period. The time dependent analysis shows that the fraction of time that system is in a high load period is much less: only about one quarter of the time. Similarly, from the approximative analysis we would conclude that the system is only half the time having a strictly positive probability for at least one customer. However, the time dependent analysis shows that the system has all the time a strictly positive probability for at least one customer.

Note however, that increasing the length of the intervals for which $\lambda(t) = 1$, or increasing the value of μ , will lead to a much better approximation by the

approximative analysis. The example therefore demonstrates that the a time dependent analysis can be a much more accurate method for modelling the time dependent effects in the arrival process. Depending on the system parameters, a time dependent analysis cannot be easily approximated by equilibrium analysis.

2 Content based routing algorithms, Model, and Literature

2.1 Content based routing algorithms

The simplest content-based routing algorithm is *flooding*. In this method, a subscribers interest is saved locally i.e., in the routing table of the broker to which the client is attached (the local broker), and is not forwarded into the network. When a publisher sends a notification to its local broker, this notification is sent to all brokers in the network. When a notification is received by a local broker, the local broker determines to which attached subscribers the notification has to be forwarded.

A more sophisticated content-based routing algorithm is *identity-based routing*. Here, subscriptions are forwarded between the brokers and saved in the routing tables at the brokers. The routing tables at the brokers are set up so that notifications are sent only to local brokers that have subscribers attached. To reduce the amount of traffic in the network, a subscription is not forwarded over a link if previously the same subscription (of another subscriber) was forwarded over that link, and an unsubscription is not forwarded over a link, if there is still another subscription on the same content active. The routing table of a broker contains those brokers that require a notification as determined by the subscriptions/unsubscriptions these brokers have sent.

Under flooding, there is no subscription-traffic, but a lot of notification-traffic, whereas under identity based routing there usually will be less notification-traffic at the cost of additional subscription traffic. In this paper, we present a general model that enables us to make the trade-off with respect to total network traffic between flooding and identity-based routing. Intuitively, variants of identity based routing will exhibit roughly the same behavior compared to identity based routing (though it would be interesting to investigate this further). Alternative content based routing algorithms, are *simple routing*, where each subscription is treated independently from the other subscriptions, *covering-based routing* (where it is exploited that subscriptions can cover other subscription on a smaller part of that information) and *filter merging* (a technique that combines filters into a covering filter to reduce the number of propagated filters and thus the size of distributed state).

2.2 Literature

A of research is focused on the design of a properly working publish/subscribe system in practice. Several research groups designed their own prototypes, created in their own environment and networked applications. Some example prototypes are the SIENA system (Carzaniga, Rosenblum and Wolf [3]) (where simulation results on covering based routing are presented), the ELVIN system (Segall and Arnold [15]) (focusing on implementation aspects) and the REBECA system (Mühl [11]) (analyzing characteristics of a large set of content based routing algorithms). An extensive overview of a publish/subscribe system can be obtained from Mühl [11] and Eugster et al. [6]. More recent work focuses on supporting mobility. For example, Fiege et al. [7] propose a scheme which supports mobility in the REBECA system. Mühl et al. [12] evaluated the message traffic of several routing algorithms in a working prototype. In [4] it is argued that Content-Based Routing (CBR) has the potential of becoming the technology to address both global service retrieval and large-scale asynchronous interaction in service oriented architectures, with a single technology and a single routing infrastructure. In [18] filter merging is investigated.

We are interested in an analytical performance analysis of content-based routing. Baldoni et al. [1] present a formal modelling framework. Their approach is based on the correctness of the system when it is evolving in time. Another approach for modelling a fault tolerant publish/subscribe system is proposed by Mühl et al. [13]. They define a self-stabilizing algorithm that enables the system to automatically recover from faults and is implementable in highly dynamical environments. Tarkoma [17] investigates the cost and safety of handoff protocols for subscribers and publishers in mobility aware content-based routing systems. Note that this work also relates to peer to peer systems. In such distribution systems, a session consists of a peer joining the system when a user starts the application and leaves the system when the user exits the application. In literature research has been done on the interarrival distribution of sessions and on the session length distribution. Prior simulation and analysis studies have typically assumed both distributions to be exponential e.g. see [14]. In [16], however, an analysis of the session length distribution is provided, concluding that this is more accurately modelled by a Weibull distribution.

There is a large body of theoretical literature studying queuing networks in equilibrium. For networks with other than infinite server queues there seem to be very few analytical results concerning transient behavior. The relevant theory on Markov chains and transient behavior is largely developed in [10], [5] and [2]. The last reference provides a short overview of further material on transient behavior of open networks.

2.3 Methodology

In the remainder of this paper, we present an analytical approach towards performance evaluation of content-based routing. Our model is based on queueing networks with time-varying parameters. In this model, a heterogeneous population of publishers (distributed over the system) serves a heterogeneous population of subscribers. The subscribers initiate requests according to an arrival process that fluctuates over time. Subscribers remain active for a random amount of time that may also fluctuate over time. In addition, subscribers may roam among the brokers, that is change their attachment from one broker to another, due to e.g. mobility. Our model explicitly takes into account the influence of the time varying subscription process on the amount of data and control traffic, and takes into account advertisements.

3 Main results

In this section we provide a time-dependent analysis of the amount of traffic generated under identity based routing and flooding. To that order, consider an m -ary tree with k levels where one publisher is active at the root broker and possible subscribers are active at the end-brokers. The m denotes the number of children of each node and the k denotes the depth of the tree. The choice for using this tree-shaped network is motivated by the fact that many underlying multicast routing protocols are based on creating an underlying logical structure that resembles a tree. An m -ary tree is chosen as the simple representant of such a tree. Figure 1 shows this structure where $m = 2$ and $k = 4$. We assume subscriptions only have to be propagated towards the root (there is a prior knowledge where the publisher is located). The nodes without clients are the routing brokers. The number of brokers on the i -th level equals m^i . Hence, the total number of end-brokers is $N = m^{k-1}$. The brokers $B_{i,j}$ are numbered as indicated in the figure. We refer to the end-brokers at level $k - 1$ (these are: $B_{k-1,j}$, $j = 1, 2, \dots, N$), as the end-brokers $1, 2, \dots, N$ (numbered from left to right). Let l denote the number of links in the network). For an m -ary tree with k levels: $l = \sum_{i=0}^{k-1} m^i - 1 = \frac{m^k - m}{m - 1}$. The random variable $X_j(t)$ denotes the number of active subscribers at end-broker j , $j = 1, \dots, N$ at time t with state space $S = \{0, 1, 2, 3, \dots\}$. Let $\mathbf{X}(t) = (X_1(t), \dots, X_N(t))$ and let $\mathbf{n} = (n_1, \dots, n_N)$. We assume the system starts empty, i.e., $\mathbf{X}(0) = \mathbf{0}$. Consider the stochastic process in *state* $\mathbf{n}$, i.e., under the condition that $\mathbf{X}(t) = \mathbf{n}$. (Figure 3 displays the process in state $\mathbf{n} = (2, 0, 1, 3, 5, 1, 0, 0)$). Given a state, we call a link or a broker *active* if it is on the path from the root to an end-broker j with $n_j > 0$. An end-broker j is called active if $n_j > 0$. Let $L(\mathbf{n})$ denote the number of links that are active in state $\mathbf{n}$. (In Figure 3 we have $L(2, 0, 1, 3, 5, 1, 0, 0) = 10$. The number of active

brokers is 11.) With $y(t)$ we denote the average number of active links at time t , averaged over all instances.

Fig. 2. Example of m -ary tree with k levels, with $m = 2$ and $k = 4$ and broker numbering

We assume the state of a broker j at level i equals either 0 or 1 according to: $B_{i,j} = 1$ if $B_{i,j}$ is active, and $B_{i,j} = 0$, otherwise. As the number of vertices with $B_{i,j} = 1$ induces a subtree of the m -ary tree, for each state $\mathbf{n}$, the number of active links $L(\mathbf{n})$ is:

$$L(\mathbf{n}) = \sum_{i=0}^{k-1} \sum_{j=1}^{m^i} B_{i,j} - 1 \quad (1)$$

The publisher frequently publishes notifications according to an exponentially distributed inter-arrival time between consecutive notifications with an expected length of ω^{-1} . We assume that subscribers independently arrive at an end-broker and become active for a while until leaving again. When a subscriber becomes active, it sends a subscription on information. We assume each notification message contains the same type of content. Analogously we have one subscription-message, which indicates whether or not there is interest for the notification.

Active subscribers will frequently update their subscription by sending a re-subscription each period r . Then, the local broker of the subscriber will re-new the routing entry by removing the old subscription and replacing it with the re-subscription. Next it forwards the re-subscription to its neighbor brokers which will perform the same tasks. In this manner all routing entries in the network for this subscription are refreshed. When a subscriber switches to the inactive state an unsubscription is sent. The size of a message is 1 unit and

the cost for sending one message over one link equals 1 unit of traffic. A final assumption is that messages sent over the links between subscriber and the brokers they are attached to is free, because we are only interested in the traffic between the brokers. Furthermore, we assume that there is no delay when messages where propagated over the links.

We consider a non-homogeneous Poisson arrival rate of subscriber $\lambda_j(t)$ at end-broker j which depends on the time t . The time a subscriber is active at time t is exponentially distributed with completion rate $\mu_j(t)^{-1}$ at time t .

The message rate by *flooding* is given by $A_f(t) = \omega l$. Thus, the total traffic over a time interval T follows as:

$$A_f^T = \int_T A_f(t) dt = T\omega l. \quad (2)$$

In order to calculate the traffic in the system for *identity-based routing*, we distinguish two types of message flows through the network. We will call these the *toggle traffic rate* $A_t(t)$ and *state dependent traffic rate* $A_{sdt}(t)$, at time t . Toggle traffic can only originate when new subscribers arrive or when subscribers leave: this traffic is caused by subscriptions and unsubscriptions. The state dependent traffic consists of the notifications and re-subscriptions that are flowing through the network in a specific state of the system. Notice that in a certain state $\mathbf{n}$ the state dependent traffic is easily be calculated as $(\omega + \frac{1}{r}) L(n)$. The difficulty lies in determining the number of active links $y(t)$ averaged over all states. For the steady state, our main result provides an explicit expression for this. For the toggle traffic, the important issue is the number of links affected by a toggle. For example, considering Figure 3, when the number of subscribers of end broker 7, $B_{3,7}$ increases from 0 to 1, a subscription message is carried over 2 links. The difficulty in determining the toggle traffic rate lies in determining the number of links that carry new subscription or unsubscription messages. For the toggle traffic, our main result provides an explicit expression for the number of links affected by a toggle.

Theorem 1 *The message rate at time t for identity based routing is $A_{ibr}(t) = A_{sdt}(t) + A_t(t)$, where*

$$A_{sdt}(t) = \left(\omega + \frac{1}{r} \right) y(t), \quad (3)$$

where $y(t)$ is defined as:

$$y(t) = \left(l - \sum_{h=0}^{k-2} \sum_{j=1}^{m^{k-1-h}} \prod_{i=m^h(j-1)+1}^{jm^h} p_{0,i}(t) \right), \quad (4)$$

and

$$A_t(t) = 2 \sum_{j=1}^N \left(1 + \sum_{h=1}^{k-2} \prod_{i=m^h(\lceil \frac{j}{m^h} \rceil - 1) + 1}^{j-1} p_{0,i}(t) \prod_{i=i+1}^{m^h \lceil \frac{j}{m^h} \rceil} p_{0,i}(t) \right) \lambda_j(t) p_{0,j}(t), \quad (5)$$

where $p_{0,i}(t) = P(X_i(t) = 0)$, $i = 1, \dots, N$.

The total amount of traffic which is generated in the network in a time-interval of length T is found as:

$$A_{ibr}^T = \int_T A_{ibr}(t) dt. \quad (6)$$

We prove Theorem 1 by means of the following lemma's.

Lemma 2 *For this system, with Poisson arrival rate $\lambda_j(t)$ and exponential completion rate $\mu_j(t)$ at end-broker j , the time-dependent multivariate distribution is expressed by:*

$$P(\mathbf{X}(t) = \mathbf{n}) = \prod_{j=1}^N \frac{\rho_j(t)^{n_j}}{n_j!} e^{-\rho_j(t)} \quad (n_j \in S, t \geq 0) \quad (7)$$

Where $\rho_j(t)$ is given by the following differential equation:

$$\frac{d\rho_j(t)}{dt} = \lambda_j(t) - \mu_j(t)\rho_j(t), \quad j = 1, \dots, N \quad t \geq 0; \quad (8)$$

$$\rho_j(0) = 0, \quad j = 1, \dots, N \quad (9)$$

Proof. Notice that the stochastic process $\mathbf{X}(t) = (X_1(t), \dots, X_N(t))$ corresponds to a network of independent infinite server queues $M_t/M_t/\infty$, with arrival rates $\lambda_j(t)$ and completion rates $\mu_j(t)^{-1}$, at time t . Now the lemma follows from [10].

Note that (7) implies that $\rho_i(t)$ is the expected number of subscribers at end broker i at time t . Moreover, $p_{0,i}(t) = e^{-\rho_i(t)}$. The following theorem provides an expression for the total traffic as an expectation with respect to the time dependent distribution (7).

Theorem 3 *The total message traffic rate for identity based routing equals $A_{ibr}(t) = A_{sdt}(t) + A_t(t)$, where*

$$A_{sdt}(t) = \left(\omega + \frac{1}{r} \right) \sum_{\mathbf{n}} L(\mathbf{n}) P(\mathbf{X}(t) = \mathbf{n}), \quad (10)$$

and

$$A_t(t) = \sum_{j=1}^N \left[\lambda_j(t) \sum_{(\mathbf{n}_{j,0}, \mathbf{n}_{j,1})} b(\mathbf{n}_{j,0}, \mathbf{n}_{j,1}) P(\mathbf{X}(t) = \mathbf{n}_{j,0}) + \mu_j(t) \sum_{(\mathbf{n}_{j,1}, \mathbf{n}_{j,0})} b(\mathbf{n}_{j,1}, \mathbf{n}_{j,0}) P(\mathbf{X}(t) = \mathbf{n}_{j,1}) \right], \quad (11)$$

where $b(\mathbf{n}, \mathbf{n}') = L(\mathbf{n}) - L(\mathbf{n}')$ and the vectors $\mathbf{n}_{j,0} = (n_1, \dots, n_{j-1}, 0, n_{j+1}, \dots, n_N)$ and $\mathbf{n}_{j,1} = (n_1, \dots, n_{j-1}, 1, n_{j+1}, \dots, n_N)$ indicate all possible states of the system with the state at end-broker j being resp. 0 and 1.

Proof. Let $a : S^N \rightarrow \mathbb{N}$ be a function indicating the amount of messages in the network per time unit in state $\mathbf{n}$. As notifications are sent according to an average rate ω and re-subscriptions according a rate $1/r$ over all active links in that state, we find,

$$A_{sdt}(t) = \sum_{\mathbf{n}} a(\mathbf{n}) P(\mathbf{X}(t) = \mathbf{n}) = \sum_{\mathbf{n}} (\omega + 1/r) L(\mathbf{n}) P(\mathbf{X}(t) = \mathbf{n}). \quad (12)$$

The toggle traffic rate at time t , is the expected rate for $\mathbf{X}(t)$ of traffic originating from transitions between states $\mathbf{n}$ and $\mathbf{n}'$. Let the function $b : (S^N, S^N) \rightarrow \mathbb{N}$ be amount of messages in the network caused by a transition between $\mathbf{n}$ and $\mathbf{n}'$. The toggle traffic rate is:

$$A_t(t) = \sum_{(\mathbf{n}, \mathbf{n}')} b(\mathbf{n}, \mathbf{n}') P(\mathbf{X}(t) = \mathbf{n}) q(\mathbf{n}, \mathbf{n}'), \quad (13)$$

where $q(\mathbf{n}, \mathbf{n}')$ is the transition rate from state $\mathbf{n}$ to $\mathbf{n}'$. For our model, the transition rates $q(\mathbf{n}, \mathbf{n}')$ are given by: $q(\mathbf{n}, \mathbf{n} + \mathbf{e}_j) = \lambda_j(t)$, corresponding to an arrival at end-broker j , and $q(\mathbf{n}, \mathbf{n} - \mathbf{e}_j) = \mu_j(t)$, corresponding to a departure from end-broker j , where $\mathbf{e}_j$ equals the j -th unit vector of length N with value 1 on place j and value 0 elsewhere. Other transitions cannot occur.

To derive the toggle traffic rate we need to express the function $b(\mathbf{n}, \mathbf{n}')$. First observe that a toggle between states only affects the amount of traffic when there is a transition between states 0 and 1 at an end-broker. Transitions between higher states (e.g. 1 and 2) will not add toggle traffic due to the identity based routing algorithm. For this reason, a subscription needs to be forwarded over a link if and only if that link is goes from inactive to active (and an unsubscription if and only if that link goes from active to inactive). Therefore, the number of links an (un)subscription has to be forwarded on toggle between states $\mathbf{n}$ and $\mathbf{n}'$, is expressed by:

$$b(\mathbf{n}, \mathbf{n}') = |L(\mathbf{n}) - L(\mathbf{n}')| \quad (14)$$

Thus, the toggle traffic rate $A_t(t)$ is given by (11), as the sum over all possible transitions at all end-brokers j , where the first expression indicates the subscription traffic caused by an arrival of a subscriber at a 0-state of end-broker j and the second expression indicates all subscription traffic when the last subscriber leaves end-broker j (for all possible transitions).

The recursive expression (1) and Theorem 3 yield the traffic rate for the identity based routing : $A_{ibr}(t) = A_{sdt}(t) + A_t(t)$. We can derive this rate for various values of the parameters $\lambda_j(t)$ and $\mu_j(t)$, describing the subscriber behavior, and for each m -ary tree network with k levels. However, the equilibrium results of Jaeger and Mühl [8] allow us also to obtain an analytical expression for $A_{ibr}(t)$. By straightforward modification of their proof, a generalization to the case where λ_i, μ_i , $i = 1, \dots, N$ may all be different is obtained. Without further proof we state:

Lemma 4 ([8]) *Consider the network with $\lambda_i(t) = \lambda_i$, $\mu_i(t) = \mu_i$, $i = 1, \dots, N$, independent of t , and assume that the system is in equilibrium. The state dependent traffic rate equals:*

$$A_{sdt} = \left(\omega + \frac{1}{r} \right) \left[l - \sum_{h=0}^{k-2} \sum_{j=1}^{m^{k-1-h}} \prod_{i=m^h(j-1)+1}^{jm^h} p_{0,i} \right], \quad (15)$$

Note that according to (7) and (9) it follows that $p_{0,i} = e^{-\rho_i}$, and $\rho_i = \lambda_i/\mu_i$, $i = 1, \dots, N$.

The proof of the next lemma is included for completeness and to clarify the proof of the similar lemma provided in [8], by explicitly pointing out the insensitivity and the regenerative arguments.

Lemma 5 *Consider the network with $\lambda_i(t) = \lambda_i$, $\mu_i(t) = \mu_i$, $i = 1, \dots, N$, independent of t , and assume that the system is in equilibrium. The toggle traffic rate equals:*

$$A_t = 2 \sum_{j=1}^N \left(1 + \sum_{h=1}^{k-2} \prod_{i=m^h(\lceil \frac{j}{m^h} \rceil - 1) + 1}^{j-1} p_{0,i} \prod_{k=i+1}^{m^h \lceil \frac{j}{m^h} \rceil} p_{0,k} \right) \lambda_j p_{0,j} \quad (16)$$

Proof. Toggle traffic occurs when an end-broker switches its state between 0 and 1, only. The amount of messages a toggle causes depends on the states of the routing brokers. If a new subscription arrives at an end-broker that is the only active subscription in the complete system, it will be forwarded from end-broker towards the root-broker: a path of length $k - 1$. However, if there are already other active subscribers, it is possible that a routing broker on this path already has its routing table occupied from another active subscriber. In

this situation, the new subscription is not forwarded due to the identity-based routing algorithm. Counting the toggle traffic rate comes down to counting the expected path length an (un)subscription has to travel in the network and determining the expected toggle rate per end-broker.

To obtain the expected path-length an (un)subscription needs to travel, observe that upon a transition between states 0 and 1 at an end-broker, the message will always be forwarded over the link between the end-broker and its parent, contributing 1 unit of traffic. Now, the parent broker which receives this message, will only forward this message to its parent broker if there is no other active subscriber in its subtree. This is determined as the fraction of time each link towards the root is not occupied with forwarding messages (i.e. the routing tables are not occupied). Carefully counting the expected path length of a toggle message d_j for end-broker j yields (see [8])

$$d_j = 1 + \sum_{h=1}^{k-2} \prod_{i=m^h(\lceil \frac{j}{m^h} \rceil - 1) + 1}^{j-1} p_{0,i} \prod_{k=i+1}^{m^h \lceil \frac{j}{m^h} \rceil} p_{0,k}. \quad (17)$$

The expected toggle traffic rate for end broker j is obtained as follows. The average time end-broker j remains in the zero-state (state 0) is λ_j^{-1} . Let the random variable G_j denote the time that end-broker j remains in the non-zero-state (i.e., in all other states 1, 2, 3...) before it returns to the zero-state, and let $\mu_j'^{-1}$ denote its mean. Observe that the stochastic process that alternates between zero-state and the non-zero-state is insensitive to the distribution of the time the process spends in the non-zero-state, see e.g. [5]. Therefore, to obtain the probability this process is in the zero-state, we may analyze the process as if the time it spends in the non-zero-state is exponentially distributed with the same mean. Furthermore, notice that the fraction of time the original process spends in the zero-state equals that fraction for the process that alternates between 0 and 1. Let $p'_{1,j}$ denote the probability this process is in the non-zero-state. The global balance equations for this process are $\lambda_j p_{0,j} = \mu_j' p'_{1,j}$, from which we obtain using $p_{0,j} + p'_{1,j} = 1$, that

$$\mu_j' = \lambda_j \frac{p_{0,j}}{1 - p_{0,j}} \quad (18)$$

As the process that alternates between the zero-state and the non-zero-state is a regenerative process, that regenerates each time it moves from the zero-state to the non-zero-state, and toggles twice in this regeneration cycle, we have that the process will toggle on average twice in the the time period of

length $\lambda_j^{-1} + \mu_j'^{-1}$. So the expected toggle rate $t_{r,j}$ at end-broker j equals:

$$t_{r,j} = \frac{2}{\lambda_j^{-1} + \mu_j'^{-1}} = \frac{2}{\lambda_j^{-1} + \left(\lambda_j \frac{p_{0,j}}{1-p_{0,j}}\right)^{-1}} = 2\lambda_j p_{0,j}.$$

For end-broker j the expected toggle traffic rate thus equals $d_j t_{r,j}$ which completes the proof.

Proof of Theorem 1.

Theorem 3 provides an expression for the traffic rate: $A_{ibr}(t) = \sum_{\mathbf{n}} a(\mathbf{n})P(\mathbf{X}(t) = \mathbf{n}) + \sum_{(\mathbf{n}, \mathbf{n}')} b(\mathbf{n}, \mathbf{n}')P(\mathbf{X}(t) = \mathbf{n})q(\mathbf{n}, \mathbf{n}')$. This expression is also valid under the equilibrium conditions of Lemma 4. Moreover, the functional form of the time dependent distribution as a multivariate Poisson distribution with parameters $\rho_i(t)$, $i = 1, \dots, N$, equals that in equilibrium with ρ_i , $i = 1, \dots, N$. Observe that the expressions obtained in equilibrium in Lemma 4 and Lemma 5 for $\sum_{\mathbf{n}} a(\mathbf{n})P(\mathbf{X} = \mathbf{n})$ resp $\sum_{(\mathbf{n}, \mathbf{n}')} b(\mathbf{n}, \mathbf{n}')P(\mathbf{X} = \mathbf{n})q(\mathbf{n}, \mathbf{n}')$ depend only on the λ_i and ρ_i , $i = 1, \dots, N$ (via $p_{0,i}$). Thus, we explicitly evaluated also the sums $\sum_{\mathbf{n}} a(\mathbf{n})P(\mathbf{X}(t) = \mathbf{n})$ and $\sum_{(\mathbf{n}, \mathbf{n}')} b(\mathbf{n}, \mathbf{n}')P(\mathbf{X}(t) = \mathbf{n})q(\mathbf{n}, \mathbf{n}')$, that have the same functional form as the expressions in Lemma 4 and Lemma 5 with ρ_i replaced by $\rho_i(t)$, $i = 1, \dots, N$, and λ_i replaced by $\lambda_i(t)$. This completes the proof.

Remark 1. Notice that the proof of our Theorem builds upon the results of [8]. However, the arguments used in [8] cannot be directly applied in a time-dependent setting. In particular, the insensitivity and regenerative arguments in the proof of Lemma 5, are not valid for processes with time dependent transition rates. (Insensitivity: the argument that the stochastic process that alternates between zero-state and the non-zero-state is insensitive to the distribution of the time the process spends in the non-zero-state. Regenerative: the argument leading to the conclusion that the stochastic process will toggle on average twice in the time period of length $\lambda_j^{-1} + \mu_j'^{-1}$.) It is the observation that the functional form of the state distribution $P(\mathbf{X}(t) = \mathbf{n})$ is multivariate Poisson, just like the equilibrium distribution, and the observation that the expressions obtained [8] for the equilibrium traffic rates depends only on the parameters λ_i , ρ_i , $i = 1, \dots, N$, of that multivariate Poisson distribution, that allows us to exploit these results to obtain the time dependent traffic rates.

Remark 2. Note that the equilibrium results of [8] do not require the completion rates to be exponentially distributed, since in equilibrium the state distribution $P(\mathbf{X} = \mathbf{n})$ is insensitive to the distribution of that rate. Further note that our results for $P(\mathbf{X}(t) = \mathbf{n})$ readily generalize to general subscription times too, see e.g. [10].

4 Extensions

Until now we assumed one publisher always being active at the root and various subscribers being active at the end-brokers. In this section we will extend this model to a more general setting.

4.1 Finite population; roaming clients

Assume a finite population of N_i clients at end-broker i , that independently switch between an exponentially distributed off-period with time varying intensity $\lambda_i(t)$ and an exponentially distributed on-period with time varying intensity $\mu_i(t)$. Now the time-dependent distribution for the number of active subscribers at time t is given by:

$$P(X_i(t) = n_i) = \binom{N_i}{n_i} \rho_i(t)^{n_i} (1 - \rho_i(t))^{N_i - n_i} \quad (19)$$

With $\rho_i(t)$ from: $\frac{d\rho_i(t)}{dt} = \lambda_i(t)(1 - \rho_i(t)) - \mu_i(t)\rho_i(t)$.

Using $p_{0,i}(t) = P(X_i(t) = 0)$ from (19) in (4), we find the amount of traffic that originates in a time-interval of length T , when there is a finite population at each single end-broker.

A further extension, both for finite and infinite populations of clients, is the ability for clients to roam between brokers while they are active. This phenomenon appears in wireless networks, where clients physically move around in an area and change their connections as they are becoming in reach of different base stations (which in our model are represented by the brokers). To this end, let $r_{ij}(t)$ denote the rate at which clients migrate from broker i to broker j at time t . For the case of infinite population, the time-dependent distribution of the number of active clients remains (7), where $\rho_i(t)$ now solves, for $i = 1, \dots, N$,

$$\frac{d\rho_i(t)}{dt} = \lambda_i(t) + \sum_j \rho_j(t)r_{ji}(t) - \mu_i(t)\rho_i(t) - \sum_j \rho_i(t)r_{ij}(t)$$

and similar for a finite population of clients.

4.2 Publisher and subscriber locations

We also derive expressions for the traffic rate for identity based routing in a tree, where publishers and subscribers are located at *any* broker (as opposed to publishers at the root and subscribers at the end-nodes). Suppose we have a tree with N brokers numbered from 1 to N . Let $l = N - 1$ denote the number of links in the tree. Let H_j and B_j be resp. the number of publishers and the number of possible subscribers at broker j . For all publishers $k \in \{1, \dots, H_j\}$, for all brokers $j \in \{1, \dots, N\}$, we assume each publisher is sending distinct notifications according to an exponentially distributed inter-arrival time ω_{jk} for a publisher k at broker j . So we have $Z = \sum_{j=1}^N H_j$ different notifications.

A subscriber at broker i subscribes to notifications of publisher k at broker j according to a Poisson process with arrival rate $\lambda_{i,jk}(t)$ and completion rate $\mu_{i,jk}(t)$, at time t . While on, the subscriber indicates its interests by sending (re)-subscriptions according to a refresh period r_{jk} . Define the random variable $X_{i,jk} \in \mathbb{N}_0$ as the number of active subscribers at broker i on publisher k at broker j . For the general network setting, let the expression $x_{jk}(t)$ define the expected time that broker $i \in \{1, \dots, j-1, j+1, \dots, N\}$ has a routing entry for a subscription on publisher k at broker j . Let the expression $d_{i,jk}$ define the expected length of the path from broker i to publisher k at broker j . We obtain,

Theorem 6 *For any tree with N brokers, the message traffic rate for identity-based routing is:*

$$A_{ibr} = \sum_{j=1}^N \sum_{k=1}^{H_j} \left(\omega_{jk} + \frac{1}{r_{jk}} \right) x_{jk}(t) + \sum_{j=1}^N \sum_{k=1}^{H_j} \sum_{i=1}^N d_{i,jk} t_{r,i,jk}(t), \quad (20)$$

where $p_{i,jk}(t) = P(X_{i,jk}(t) = 0)$ denotes the probability for staying in state 0, $t_{r,i,jk}(t) = 2\lambda_{i,jk}p_{i,jk}(t)$ denotes the toggle rate for subscribers at a broker i on a publisher k at node j and $x_{i,jk}(t)$ can be expressed by analogy to (4).

Proof. By summation over all publishers.

4.3 Advertisements

The previous model can be further extended by assuming that publishers are now acting as on/off sources: while on, they send notifications into the network according to the rate ω_{jk} and while off, they are silent. When a publisher is (or becomes) inactive we assume that there is no traffic (subscriptions are blocked immediately). At the moment the publisher switches its state, it floods

an advertisement message to indicate its new state. Secondly, we assume re-advertisements are sent according to a refresh period r_{a,j^k} while publisher k at broker j is active.

Let the activity state of publisher k at broker j be given by a (deterministic) function $b_{j^k}(t) \in \{0, 1\}$ (with $b_{j^k}(t) = 1$ when the publisher j is active, and $b_{j^k}(t) = 0$ otherwise). So for the flooding model we obtain: $A_f(t) = l \sum_{j=1}^N \sum_{k=1}^{H_j} \omega_{j^k} b_{j^k}(t)$. Let r_{s,j^k} denote the refresh period of subscriptions (which was r_{j^k} previously), similarly, let r_{a,j^k} denote the refresh period of advertisements. We denote $p_{X_n,i,j^k}(t) = P(X_{i,j^k}(t) = n)$ for the probability that there are n active subscribers to an active publisher i at broker k . So $P(X_{i,j^k}(t) = 0) = 1$ if $b_{j^k}(t) = 0$.

Theorem 7 *For this model, the steady state traffic $A_{sdt}(t)$ is given by:*

$$A_{sdt}(t) = \sum_{j=1}^N \sum_{k=1}^{H_j} \left[\left(\omega_{j^k} + \frac{1}{r_{s,j^k}} \right) x_{j^k}(t) + \frac{l}{r_{a,j^k}} b_{j^k}(t) \right] \quad (21)$$

The total toggle traffic caused by publishers $A_{p,t}(t)$ during the time interval $[0, t]$, is given by:

$$A_{p,t}(t) = l \sum_{j=1}^N \sum_{k=1}^{H_j} \nu_{i,j^k}(t), \quad (22)$$

where $\nu_{i,j^k}(t)$ denotes the number of state transitions of publisher k at node j in $[0, t]$.

Proof. The contribution of notification and subscription traffic is only when the publisher is active. Considering the state dependent traffic first, we need to add the traffic contribution by the use of re-advertisements during the active state of the publisher. These are flooded over all links l with refresh rate $1/r_{a,j^k}$, leading to (21). We also need to take into account the toggle traffic generated by publishers. As these messages are flooded, each toggle ‘costs’ l links of traffic. So over a time interval $[0, t]$ the toggle traffic is as described by (22). The toggle traffic for subscribers is analogous to (17).

By integration over each possible realization, this result can be generalized to publishers that change from active to inactive according to a stochastic process.

5 Example: Influence of a varying arrival rate

Consider the situation with an infinite population and similar processes at each end-broker (a differentiable version of the example in Section 1.3). For the non-homogeneous arrival process we use: $\lambda(t) = c + b \sin at$ with $0 \leq b \leq c$. For the service time we use: $\mu(t) = \mu$. Using Laplace integration of (7) we find: $\rho(t) = \frac{c}{\mu} + \left(\rho(0) + \frac{ba}{a^2 + \mu^2} - \frac{c}{\mu}\right) e^{-\mu t} + \left(\frac{b\mu}{a^2 + \mu^2}\right) \sin at - \left(\frac{ba}{a^2 + \mu^2}\right) \cos at$

With Theorem 1, (6) and (7) we calculate the total traffic for identity-based routing. For examining the influence of a varying arrival rate, note that when $b = 0$ and $\rho(0) = \frac{c}{\mu}$, the time-dependent distribution equals the steady state distribution with $\lambda = c$. In an exemplary setting, we compare the homogeneous arrival process (by taking $b = 0$) with the non-homogeneous arrival process (by taking $b = c$) for different values of the average arrival rate c and different values for the frequency a . For the remaining parameters we use $T = 25$, $k = 5$, $m = 3$, $\omega = 1$, $r = 1$, $\mu = 5$ and $\rho(0) = \frac{c}{\mu}$. Figure 3 shows the traffic generated by the identity based routing algorithm for the non-homogeneous arrival process minus that of the homogeneous arrival process for varying a and c .

Fig. 3. Influence on A_{ibr} for non-homogeneous arrival process compared with homogeneous arrival process for different values of c and a

At points where the curve is below zero, the non-homogeneous arrival process generates less traffic. The figure shows that the identity based routing algorithm for the non-homogeneous arrival process generates much less traffic for low a and low c . This is explained by the toggle traffic. When c is low, there will be more toggle traffic in the homogeneous case due to the fact that this system then has a continuously low load. In the non-homogeneous case

there will be periods where the load is relatively high, and periods where it is very low. Apparently, this leads to less toggle traffic. When c increases, the opposite effect occurs. In case of the homogeneous arrival process there will be very little toggle traffic. However, the non-homogeneous arrival process still generates toggle traffic, due to the fact that there are time intervals where the arrival process is close to 0. Considering a , when a grows (a faster fluctuating arrival rate) we see that the non-homogeneous arrival process generates more traffic than the homogeneous arrival process, explained by the toggle traffic. However, at a certain point there is a maximum reached (at higher values for c) and for larger a the differences between both processes converge to 0. This effect can be derived from the function $\rho(t)$: when we take the limit $a \rightarrow \infty$, this function converges to c/μ , i.e. exactly the parameters for the homogeneous arrival process. Overall, the example shows that for certain regions of the parameters, the traffic of the identity based routing algorithm is considerably different in the transient case when compared to the homogeneous case. This can largely be explained by the toggle traffic: the extra traffic that is generated when the number of subscribers at an end broker changes between 0 and 1.

6 Example: impact of local interest

We consider stationary traffic in an m -ary tree network for an example with one publisher, publishing at rate ω in a tree rooted at $(0, 1)$ with three levels (see Figure 4).

Fig. 4. The content based routing network

As the case with a root publisher has been addressed e.g. in [8], we investigate the effect of a non-root publisher. For that reason, we assume the publisher is located at $(3, 1)$.

Fig. 5. Flooding compared with identity-based routing for different uniform and local behavior. (The y-axis indicates the traffic amount in messages per minute. Aibr denotes the amount of messages generated by identity based routing. Af denotes the amount of messages generated by flooding).

To investigate the impact of local interests, we assume three distinct types of behavior among subscribers, based on the distance from the publisher. We define three groups as: *Local* (L): The members located in the subtree rooted at $(2, 1)$, with arrival rate is λ . *Average* (A): the non-local members located in the tree rooted at $(1, 1)$ with arrival rate $\frac{\lambda}{q}$ and *Far* (F): all remaining subscribers with arrival rate $\frac{\lambda}{q^2}$. By varying parameter q we are able to express a stronger local interest. For fixed $\mu = 1$, $\omega = 6$ and $r = 1/6$ we compared A_f with A_{ibr} for $q = 1$ (uniform), $q = 10$ and $q = 100$. Figure 5 plots the traffic against Ψ . Here, Ψ indicates the total expected number of active subscribers in the network, which can be calculated from (19). The intervals correspond to an area around the break even points (where A_{ibr} crosses A_f). When q is higher, identity-based routing improves in comparison with flooding. This corresponds to the intuition that identity based routing is suitable when there is more local interest in the content.

7 Conclusions

In this paper we compared the efficiency of two content-based routing algorithms: the flooding scheme and the more sophisticated identity-based routing scheme. Our analytical approach is based on continuous time Markov Chains and extends the steady state approach by Jaeger and Mühl [8] to systems with time-fluctuating parameters. This more accurately models reality as often the rate at which subscribers initiate content requests depends on time (e.g. due to time of day effects). We obtain explicit closed form solutions for the time-dependent distribution of the number of active clients, taking into account the use of advertisements and roaming clients. The most important parameters of the model include the arrival rate of subscribers λ , the departure rate μ , the notification rate ω , the re-subscription period r , the number of brokers N . In an exemplary setting, we compared the traffic generated by the identity based routing algorithm for a nonhomogeneous arrival process to a homogeneous arrival process with the same load. In this setting, identity based routing with a non-homogeneous arrival process yields less traffic for lower arrival rates, but more for higher arrival rates. This is caused by the impact of the toggle traffic. Toggle traffic is also the reason that when the arrival process heavily fluctuates in time, the non-homogeneous arrival process will generate more traffic.

In an exemplary setting we found that when more than 12% of the subscribers are active, flooding is preferred, when less than 12%, identity-based routing is preferred. By varying the parameters, ω , μ , r and the distribution of the interest of subscribers (i.e. locally more interest than farther away from the publisher) this break-even-point will become higher when: the notification ω rate is higher (this has a strong impact), the period of being active as subscriber becomes longer (so μ lower) (this has little impact), the re-subscription periods r becomes longer (this has strong impact), and when locally the interest is higher than further away in the network. This corresponds to the intuition that identity based routing is suitable when there's a nonuniform interest in the content. It would be interesting to derive the same models for other forms of content based routing like covering and filter merging. Other future work includes the generalization of the advertising model to a model with random activity states for publishers.

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8 Appendix: Table with main variables

Variable	Description	Type
m	number of children at each node	D
k	number of levels in the tree	D
N	total number of end-brokers	D
$B_{i,j}$	state of broker	R
l	number of links in the network	D
$y(t)$	expected number of active links at time t	D
ω	notification rate	D
r	re-subscription rate	D
$\lambda_j(t)$	arrival rate of subscribers at end broker j	D
$\mu_j(t)^{-1}$	completion rate of subscribers at end broker j	D
$X_j(t)$	number of active subscribers at end broker j , $\mathbf{X}(t)$ denotes the vector $(X_1(t), \dots, X_N(t))$	R
S	possible set of values for $X_j(t)$	D
$\mathbf{n}$	vector $(n_1, \dots, n_N)$ denoting state of the system	D
$p_{0,i}(t)$	$P(X_i(t) = 0)$	D
$\rho_j(t)$	momentary load at end-broker j	D
$L(n)$	number of active links in state $\mathbf{n}$	D
$b(n, n')$	number of (un)subscription messages caused by a change between states $\mathbf{n}$ and $\mathbf{n}'$	D
$q(n, n')$	transition rate between n and n'	D

Table 1

Main variables

R=Random Variable. D = Deterministic Variable