An analysis of the lifetime of OLSR networks

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Abstract

The Optimized Link State Routing (OLSR) protocol is a well-known route discovery protocol for ad-hoc networks. OLSR optimizes the flooding of link state information through the network using multipoint relays (MPRs). Only nodes selected as MPRs are responsible for forwarding control traffic. Many research papers aim to optimize the selection of MPRs with a specific purpose in mind: e.g., to minimize their number, to keep paths with high Quality of Service or to maximize the network lifetime (the time until the first node runs out of energy). In such analyzes often the effects of the network structure on the MPR selection are not taken into account. In this paper we show that the structure of the network can have a large impact on the MPR selection. In highly regular structures (such as grids) there is even no variation in the MPR sets that result from various MPR selection mechanisms. Furthermore, we study the influence of the network structure on the network lifetime problem in a setting where at regular intervals messages are broadcasted using MPRs. We introduce the 'maximum forcedness ratio', as a key parameter of the network to describe how much variation there is in the lifetime results of various MPR selection heuristics. Although we focus our attention to OLSR, being a widely implemented protocol, on a more abstract level our results describe the structure of connected sets dominating the 2-hop neighborhood of a node.

Key words: network lifetime, multipoint relay selection, Optimized Link State Routing (OLSR), dominating set

1 Introduction

The Optimized Link State Routing (OLSR) protocol is a well-known and often implemented MANET (Mobile Ad-Hoc Network) route discovery protocol. OLSR optimizes the flooding of link state information through the network by using multipoint relays (MPRs). Only nodes selected as MPRs are responsible for forwarding control traffic. Besides the mechanism for flooding *control* traffic like OLSR, there exist also flooding mechanisms for *data traffic*, like the Simplified Multicast Forwarding (SMF) protocol [6] that can work together with MPR selection algorithms (like the one in OLSR). An improved version of SMF is BMF [15]. Both mechanisms flood IP Multicast traffic over an OLSR network, where the MPRs identified by OLSR are used to optimize the flooding.

In this paper we present a graph related study investigating the impact of the network structure on the MPR selection. A main question in this context is how to measure the impact of the structure of the graph on the MPR selection. We show that for highly regular network structures (such as e.g. grids) MPR selection algorithms only can have a marginal influence on the choice of MPR nodes since many nodes are fixed as MPR nodes in the sense that every MPR selection algorithm has to choose these nodes. To demonstrate the influence of the fixed nodes, we consider the maximization of the network lifetime (the time at which the first communication fails due to depletion of battery-resources). We introduce a key parameter of networks (maximum forcedness ratio) and show how the network lifetime under different MPR selection algorithms depends on this parameter. Although we focus our attention to OLSR, being a widely implemented protocol, on a more abstract level our results describe the structure of connected dominating sets covering the 2-hop neighborhood of a node.

This paper consists of two main parts. In Section 3 we provide structural results on MPR-sets that are independent of the selection algorithm that is used. In Section 4 and Section 5 we apply this theory to grid graphs showing that for 'central nodes' in grid graphs all MPR selection algorithms yield the same sets. In Section 6 the network lifetime is addressed, showing that a new graph parameter, the 'maximum forcedness ratio' is strongly related to the degree in which the structure of the graph allows improvement of the network lifetime by a better MPR selection heuristic. For graphs with a maximum forcedness ratio close to 1, the concrete MPR selection heuristic has little impact on the resulting network lifetime.

2 Related work

2.1 MPR selection

The classical MPR selection problem is to find for a given node a set of MPRs of minimum size that covers the whole 2-hop neighborhood. Selecting the MPR-set of minimal cardinality has been proven to be NP-complete ([17], [14]).

In practice, heuristic algorithms are used to select MPR-sets. To set up MPR sets in a network, different MPR selection algorithms exist. In these algorithms each node (the selector nodes) independently chooses its MPR-set. These sets then act as relay nodes for messages sent by the selector node, and, thus can organize the broadcast communication in a network. The existing approaches mostly aim to optimize the selection of MPRs with a specific purpose in mind: e.g., to minimize their number (as was the objective in the original specification ([5]), or to improve QoS (see [1]). In [12] other purposes are presented: to reduce the number of collisions, minimize the overlap between MPRs or maximize the global bandwidth.

The heuristics mentioned above have a structure that can be divided into three steps and use an incremental approach to compute an MPR-set. The first step always consists of selecting neighboring nodes as MPR that cover nodes in the 2-hop neighborhood that cannot be covered by other neighboring nodes. The second step extends this set in order to ensure that the complete 2-hop neighborhood is covered and in the last step it is investigated if some of the current selected nodes can be dropped without violating the requested properties of an MPR set. In [2] an interesting probabilistic analysis of the influence of the first step is given. The authors conclude that almost 75 % of the relay nodes are selected by the first step of the heuristics. In this paper we show that for a specific class of graphs *all* MPRs are selected in the first step of the algorithm. In [8] MPR selection algorithms in a specific probabilistic setting are analyzed. In this setting the edges in the graph have a weight, which represents the probability of successful transmission over that edge. For this probabilistic edge model the MPR selection heuristics are more complicated than the three step model.

2.2 Network lifetime problem

The network lifetime is an important parameter for battery-operated networks. Examples for such networks are personal area networks that are used in emergency situations. Such networks are deployed in regions where it is impractical to recharge/replace the battery of a node. This limited battery capacity of nodes participating in a MANET is a topic of a wide variety of literature on problems related to energy-efficiency. Many algorithms have been developed addressing the Network Lifetime Problem in general networks. From them, the following approaches are closely related to the topic of this paper:

(a) *maximization of network lifetime for broadcast traffic.* Kang and Poovendran [9] present an algorithm that maximizes the static network lifetime. Low and Goh [11] consider the problem of maximizing the minimum residual energy that remains after a broadcast transmission from a source. Park and Sahni [13] present an alternative heuristic for determining a tree with maximum 'critical energy' (minimum residual energy). These references form a small collection of approaches in this area. Note, that all approaches above assume that the transmission originates from a single source and that none of the approaches provides a specific discussion of the impact on MPR selection.

(b) *minimization of total energy consumption for broadcast traffic.* The problem of minimizing the total energy consumption for broadcast has been widely studied. The relation with the lifetime problem is that each broadcast reduces the sum of all battery capacities in the network with the total energy required for that broadcast. Liang [10] and Cagalj et al. [3] have proven independently that the minimum-energy broadcast problem with the objective of minimizing the total transmitted power is NP-hard. One of the first algorithms on broadcasting in wireless network with usage of the wireless multicast advantage is the Broadcast Incremental Power algorithm (BIP) [19], with its variants [20].

(c) *Extension of network lifetime by topology control.* The idea behind topology control is to reduce the number of connections in a network, to get a subnetwork with some given desired properties. This reduction can be realized by lowering the transmission power at certain nodes. The main issue is to find a topology with less connections and consequently less transmit power. The distributed algorithm XTC [18] is an algorithm that provides such a reduction. Calinescu [4] studies an approach where the lifetime of the network is maximized taking into account the energy cost to maintain the topology. Closest to the problem studied in this paper is [7], where adjustments are made to the MPR selection algorithm to increase the network lifetime.

3 The structure of MPR sets: forced sets and fixed nodes

In this section we derive properties of multipoint relay (MPR) sets. A MPR set of a node is defined as a subset of its neighbors which cover the complete 2-hop neighborhood of that node, i.e. if all vertices of an MPR set of a vertex v forward a message received by v, the complete 2-hop neighborhood of v receives that message.

More formally, let $G = (V, E)$ be a connected graph (throughout this paper we assume bi-directional links), and let $N^k(u)$ denote the strict k-hop neighborhood of u, i.e., the set of nodes for which the shortest path to u has exactly k edges. A subset $M(u) \subset N^1(u)$ is called an MPR-set if $M(u)$ *dominates*

$N^2(u)$, i.e., each node in $N^2(u)$ has a neighbor in $M(u)$). Furthermore, for a given MPR-set $M(u)$, we call nodes from this set an MPR of node u. Finally, we denote the set of all possible MPR-sets of u by $MPR(u)$.

To avoid circulating messages, MPRs only react on the first instance of a message. If this first instance is received by a neighboring node for which the given node is an MPR, the message is retransmitted, otherwise it is ignored. Further instances of the same message are ignored independently of the sender of this message. This is called 'duplicate message detection'. To be able to implement this process, every node maintains a duplicate set, in which all received messages are listed. This set is used to check if an incoming message already has been processed. Consequently, the following is possible: (1) a node receives a message from a node for which it is not an MPR; (2) later it receives the same message from a node for which it is an MPR. Both messages will not be retransmitted: in case (1) because the node was not an MPR, in case (2) because the message is in the duplicate set. However, it is still easy to see that broadcasting via MPR's in the above sense reaches all possible nodes in the network.

The existing MPR selection algorithms differ in the selection process of the sets $M(u)$, $u \in V$. Our aim in this section is to analyze how far the chosen MPR sets can differ. More precisely, we are interested in the subset of nodes of a neighborhood $N^1(u)$ which belong to every possible MPR-set of u. We denote this set by $F^1(u)$ and call it the forced set of node u. Note, that nodes in $F^1(u)$ are chosen as MPR for node u by every MPR selection algorithm.

To simplify arguments, we also introduce the inverse notion of a forced MPR-set. For a given node $u \in V$, the set of nodes that force u to be MPR, is defined as $F^{-1}(u) = \{v | u \in F^1(v)\}$. Clearly, both definitions are related by: $v \in F^1(u) \Leftrightarrow u \in F^{-1}(v)$. In the remainder of this paper, when it is not of any interest to the situation at hand, we simply state that v is an MPR and omit the name of the selector node.

The following lemma gives a characterization of $F^{-1}(u)$ in terms of properties of the graph G.

Lemma 1 *$v \in F^{-1}(u)$ if and only if there exists a node $v^* \in N(u)$ such that there a unique 2-hop path from v to v^*, being the path $v - u - v^*$. In this case, node v^* belongs also to the set $F^{-1}(u)$.*

Proof. ($\Leftarrow$) Suppose that there exists a node $v^* \in N(u)$ such that there is a unique 2-hop path from v to v^*, being the path $v - u - v^*$. Then any subset of $N^1(v)$ that dominates $N^2(v)$ must contain u in order to dominate v^*. So $v \in F^{-1}(u)$, and by symmetry also $v^* \in F^{-1}(u)$.
($\Rightarrow$) Let $v \in F^{-1}(u)$ and suppose that every node $v^* \in N^2(v)$ can be reached via a 2-hop path $v - w - v^*$ with $w \neq u$. In this case $N^1(v) \backslash u$ is a possible MPR-set of v, which contradicts the fact that u is forced to be MPR. Consequently, there exists a $v^* \in N^2(v)$ for which the only 2-hop path between v and v^* is $v - u - v^*$. Since we have bi-directional links, $v^* \in N^2(v)$ means that also $v \in N^2(v^*)$. If we combine this with the fact that u is the only node that connects to both v and v^*, u has to be MPR for v^* in each possible MPR selection. Thus, $v^* \in F^{-1}(u)$. □

An extreme case occurs, when all neighbors of a node u force u to be MPR, i.e., $F^{-1}(u) = N(u)$. In such a case we call u a *fixed MPR*. An example of a fixed MPR is e.g. the center node in a star topology. In Figure 1 and Table 1 we present an example graph with its forced and fixed nodes. In this example there is only one fixed node, namely node u_4.

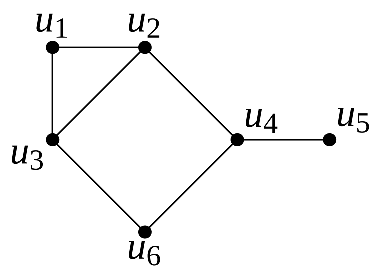


Fig. 1. An example network with forced and fixed nodes.

Node u	$F^1(u)$	$F^{-1}(u)$
u_1	$\{u_2, u_3\}$	$\emptyset$
u_2	$\{u_4\}$	$\{u_1, u_4\}$
u_3	$\emptyset$	$\{u_6\}$
u_4	$\{u_2\}$	$\{u_2, u_5, u_6\}$
u_5	$\{u_4\}$	$\emptyset$
u_6	$\{u_3, u_4\}$	$\emptyset$

Table 1

$F^1(u)$ and $F^{-1}(u)$ for the example network in Figure 1.

Fixed nodes have an important impact on the network lifetime. The following proposition states that fixed nodes provide an upper bound to the network lifetime independent of which nodes initiate the broadcasts. In the setting of this paper, we assume that different broadcasts do not interfere in time (a new broadcasts does not start before the previous is finished). Therefore, the network lifetime can be expressed in the number of messages that can been broadcasted until the first node runs out of energy.

To formulate the proposition, we introduce the notion of the Network Lifetime

$NLT(G)$ of a graph G. This value denotes the maximum number of messages which can be broadcasted within the network represented by G. Note, that in this definition the nodes which broadcast the messages may be chosen in such a way that a maximal lifetime is achieved.

Proposition 2 *Let $G = (V, E)$ be a connected graph with a set $S \neq \emptyset$ of fixed MPRs. Furthermore, let the initial battery capacity of a node u be denoted by $E(u)$ and the battery cost per transmission of a message in node $u \in V$ be $C(u)$. Then,*

$$NLT(G) \leq \min_{u \in S} \left\lfloor \frac{E(u)}{C(u)} \right\rfloor . \tag{1}$$

Proof. Let u be a fixed MPR in a connected network. Then u has to transmit each broadcast message in the network if MPR flooding is used for communication. To see this, we distinguish two cases. If u is the source, it obviously transmits the message. If u is not the source, the message reaches node u via one of its neighbors. Since $F^{-1}(u) = N(u)$, u is MPR for every neighbor. Therefore, the first message that arrives at u is being relayed and the (possibly) next duplicate messages are ignored. So, every broadcast message reduces the battery of a node $u \in S$ exactly once with $C(u)$. This immediately gives the bound stated in the proposition. □

Note, that we need the fact that u is MPR for all its neighbors to ensure that it relays all message it receives. If node u would not be MPR for some neighbor v and u receives some message first from this node v, then u would not relay this message due to the duplicate message detection property.

The previous proposition provides an upper bound to the network lifetime. In general, the lifetime of the network may be even smaller, if a non-fixed MPR, say w, exists with a low ratio $\frac{E(w)}{C(w)}$. However, if we assume that this is not the case, the given bound is tight as we show below.

Corollary 3 *Let $G = (V, E)$ be a connected graph with a set $S \neq \emptyset$ of fixed MPRs. If*

$$\min_{w \in V} \left\lfloor \frac{E(w)}{C(w)} \right\rfloor \geq \min_{u \in S} \left\lfloor \frac{E(u)}{C(u)} \right\rfloor , \tag{2}$$

then $NLT(G) = \min_{u \in S} \left\lfloor \frac{E(u)}{C(u)} \right\rfloor$.

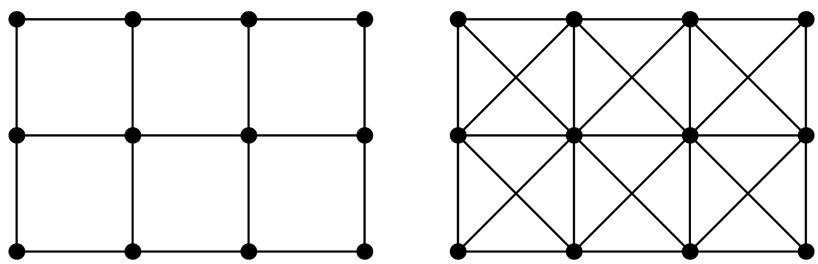
Proof. By the duplicate message detection property, a single message will reduce the battery capacity of a node w by at most $C(w)$. Therefore node w can not run out of battery before $\frac{E(w)}{C(w)}$ broadcasts. Thus, the inequality (2) guarantees that the bound in Proposition 2 is tight. □

4 The structure of MPR sets in grid graphs

In this section, we apply the results of the previous section to grid structures. Grid structures are characterized by their regular structure and turn out to have restricted possibilities to vary their MPR sets. Even though grid graphs in their pure form hardly occur in practical settings, a lot of real networks have grid-like structures or sub structures and, as a consequence, the results derived for pure grid graphs occur in some 'weaker' sense also in these networks.

Formally, we denote by $G_{m \times n}(r)$ a graph with $m \cdot n$ nodes on grid points of a $m \times n$ grid. We assume the horizontal and vertical distance between neighboring grid points to be 1. Furthermore, two nodes u and v are connected by an edge in $G_{m \times n}(r)$ if and only if $d(u, v) \leq r$, where $d(u, v)$ denotes the

Euclidean distance between the nodes u and v. Note, that n and m determine the vertex set and r determines the edge set of the graph $G_{m\times n}(r)$. As examples (see Figure 2) we consider the graphs for $r = 1$ and $r = \sqrt{2}$. $G_{m\times n}(1)$ has only horizontal and vertical edges and $G_{m\times n}(\sqrt{2})$ has also edges between diagonal neighboring grid points. Note, that $G_{m\times n}(1)$ is what normally is considered as a grid graph.



(a) The graph $G_{3\times 4}(1)$. (b) The graph $G_{3\times 4}(\sqrt{2})$.

Fig. 2. Two graphs on grids.

The grid structure of a graph induces two important properties, which are the basic elements for the proof of the theorem presented below.

- Translation Property

  If u, v are grid points and $\overrightarrow{a}$ is a vector such that $u + \overrightarrow{a}$ is a grid point, then $v + \overrightarrow{a}$ is also a grid point.
- Symmetry Property

  If v is a grid point, then the point obtained by mirroring v through another grid point is again a grid point.

Note, that for the two mentioned properties we assume that the grid structure is large enough; i.e. that the translated or mirrored point is still within the grid. Also for the following results, we are not interested in the specific issues at the border of the graph, but we concentrate on central nodes. A node

$v \in G_{m\times n}(r)$ is called a *j-hop central node, with* $j \in \{1,2\}$, if v is located in the $(m-2j\lfloor r\rfloor)\times(n-2j\lfloor r\rfloor)$ *subgrid* that is created by removing $j\lfloor r\rfloor$ grid rows from the upper and lower side of the grid and $j\lfloor r\rfloor$ grid columns from the left and right side of the grid. Nodes in $G_{m\times n}(r)$ that are not $j-hop$ central nodes, are called *border nodes*. Note that the $j-$hop neighborhood of a $j-$hop central node v is point symmetrical in v, $j=1,2$.

Using the above terminology, we now apply the results of the previous section to grid graphs. The following theorem states the rather surprising fact that, for a 2-hop central node u in a grid graph we have $F^{-1}(u)=F^{1}(u)$ and that $F^{1}(u)$ is an MPR-set of u. As can be seen from Figure 1, this is in general not true.

To keep the paper concise, we only present a sketch of the proof of Theorem 4. A full proof can be found in [16].

Theorem 4 *For every 2-hop central node* $u \in G_{m\times n}(r)$ *with* $r \geq 1$*, the set* $F^{-1}(u)$ *is an MPR-set of* u*, i.e.* $F^{-1}(u) \in MPR(u)$.

Proof. *(Sketch).* Let u be a 2-hop central node in $G_{m\times n}(r)$ with $r \geq 1$.
(1) Then $v \in F^{-1}(u)$ if and only if the mirror image $\overline{v}^{u}$ (obtained by point mirroring v in u) is the only node in $N(u)$ with a unique 2-hop path to v.
(2) Using this, it follows from the translation property that $v \in F^{-1}(u)$ if and only if $u \in F^{-1}(v)$, hence $F^{-1}(u)=F^{1}(u)$.
(3) Next a geometrical characterization of $F^{-1}(u)$ is given: $v \in N(u)$ is in $F^{-1}(u)$ if and only if v is an extreme point of the convex hull $C(u)$ of $N(u)$; i.e. the smallest convex space in which all nodes in $N(u)$ are located (a point $x \in C$ is called an *extreme point* if it is not an interior point of any line segment in C).

(4) Let u be a 2-hop central node in $G_{m \times n}(r)$ with $r \geq 1$. Then for every node $w \in N^2(u)$ it holds that $w \in C_2(u)$. Here, $C_2(u)$ is the convex hull multiplied by a factor of 2: $C_2(u) = \{2x \| x \in C(u)\}$.
(5) The proof is completed by showing: all grid points in $C_2(u)$ are the grid points defined by $\bigcup_{f \in F^{-1}(u)} N(f)$. This can be proven by a simple geometric argument. □

5 MPR selection algorithms

The analysis in the previous section forms the fundament for our main result on MPR selection algorithms for grid graphs. Before presenting the theorem we discuss the structure of MPR selection algorithms. MPR-selection algorithms are localized algorithms, where each node $u \in V$ selects an MPR-set $M(u)$, independently from the other nodes. Most MPR selection algorithms use the following structure to calculate an MPR-set $M(u)$ for node u:

(1) Start with an empty MPR-set of node u, and add nodes of $N(u)$ that are the only neighbor of a node in $N^2(u)$. So, after this step $M(u) = F^1(u)$.
(2) While there are still uncovered nodes in $N^2(u)$, select the nodes from $N(u)$ that cover at least one uncovered node and provide the highest revenue (the definition of revenue depends on the MPR selection algorithm).
(3) Optimize the MPR-set by attempting to remove a node from $M(u)$ and checking if $N^2(u)$ is still dominated. If this is the case, the node is removed from $M(u)$. Nodes are removed in the order 'lowest revenue first'.

For every MPR selection algorithm, the MPR-set of node u contains at least the forced set of node u. Consequently, Step 1 of the three-step algorithmic has to be part of each MPR selection algorithm. A procedure to optimize the selected MPR-set, as in Step 3, can also be expected to be part of an MPR selection algorithm. However, there is some variation possible in the order the nodes are considered for discarding. Between the initial step, Step 1, and the final step, Step 3, there has to be an intermediate step, Step 2, that selects non-forced MPRs according to some optimization criteria. These criteria vary per MPR selection algorithm and, therefore, mainly this step characterizes the MPR selection algorithm. For example, instead of adding nodes with maximum coverage, one can add nodes from $N(u)$ in the order 'most energy first'.

Based on the above considerations, we present our main theorem of the study on MPR selection in graphs $G_{m\times n}(r)$. It states that the MPR-set for a 2-hop central node in a grid graph is equal to its forced set.

Theorem 5 *For every 2-hop central node $u \in G_{m\times n}(r)$ with $r \geq 1$, the set $F^1(u)$ is selected as MPR-set of node u if a three-step MPR selection algorithm is used.*

Proof. Since $F^1(u) = F^{-1}(u)$ (see the proof of Theorem 4) and since $F^{-1}(u)$ dominates $N^2(u)$ (Theorem 4), after Step 1 the set $M(u)$ dominates $N^2(u)$. Thus, Step 2 of the general three step MPR selection algorithm is never processed. (Step 2 of the algorithm is only processed if there are uncovered nodes in $N^2(u)$). So there are no nodes added to the MPR-set and therefore $F^1(u)$ is selected as MPR-set of u. Step 3 can not remove any of the nodes in the MPR-set as they are all forced to be MPR. $\square$

This theorem implies that in the case of graphs of type $G_{m\times n}(r)$, MPR selection algorithms do not influence the lifetime of 2-hop central nodes. Since the above results mainly are a consequence of the regular structure (translation, symmetry) of the graphs, we may expect similar results for all graphs with regular (sub) structures.

6 Network Lifetime Simulations

In this section we describe simulation results for MPR flooding. First, we describe how the theoretical results derived for grid graphs, are supported by simulations. Afterwards, we complement the analysis on grid graphs by concentrating on random graphs. We define a graph parameter called forcedness and investigate to which extent it influences the network lifetime. For the simulations we use three MPR selection algorithms: MinCar, MaxWill and MaxWillMinForced. These algorithms are described in Table 2. (For the 'revenue' we refer back to Section 5.)

6.1 Grid graphs

In [16] simulations are described to verify the results for grid graphs presented in this paper. The simulations consider grid graphs $G_{m\times m}(r)$ with $m = 8, \ldots, 15$ and $r = 1, 2, 3$. For the MPR selection algorithms: MinCar, MaxWill and MaxWillMinForced the network lifetime is determined. These simulations show that: (1) on the torus, for grid graphs $G_{m\times m}(r)$ with r 'small' with respect to m ($r < m/4$), there is no difference in lifetime for the three considered MPR selection algorithms; (2) on the plane, for grid graphs $G_{m\times m}(r)$, with $r > 1$ there are minor differences between the three algorithms, but these can be explained from border effects: not all nodes are '2-hop central nodes;

MinCar	Revenue is defined as 'the number of uncovered nodes from $N^2(u)$ that are covered'. This leads to the well-known algorithm minimizing the cardinality of $MPR(u)$.
MaxWill	Revenue is defined as 'willingness'. OLSR has eight values available for the willingness (from 0 ("will never") to 7 ("will always")). This MPR selection algorithm selects first the nodes with the highest willingness. When willingness indicates the remaining energy of the node, this algorithm attempts to maximize the minimum energy of the network by saving energy of nodes with a low remaining energy. In this paper we assume 'willingness' to be equal to the residual energy, to avoid rounding effects.
MaxWillMinForced	In this variant revenue is defined as $s(v) = E(v)(1 - f(v))$. The term $E(v)$ denotes the residual energy of a node v. This additional element aims to further improve the network lifetime. It provides a look-ahead on the residual energy in the future, as it describes the expectation that a node will consume much energy as it is forced to be MPR by many nodes.

Table 2

The MPR selection algorithms considered in this paper

(3) When $r = 1$ for both torus and plane all algorithms give identical results.

In Section 3 we have shown that fixed nodes provide an upper bound to the network lifetime, independent of the MPR selection algorithms. We therefore are interested in the impact of 'almost' fixed nodes on the network lifetime performances of different MPR selection algorithms. To that order, we introduce the *Forcedness Ratio* to define 'how fixed' a node is. For a node u the Forcedness Ratio $f(u)$ is defined as

$$f(u) = \frac{|F^{-1}(u)|}{|N^1(u)|}.$$

Obviously, we have $0 \leq f(u) \leq 1$ and $f(u) = 1$ if and only if node u is fixed. If $f(u)$ is close to 1, node u is forced by many of its neighbors, so for many neighbors of u the node u has to be an MPR, irrespective of the MPR selection algorithm. We define $MFR := \max_{u \in V} f(u)$. Evidently, $MFR = 1$ if there is a fixed node in the network. Consequently, when MFR is close to 1, we expect the MPR selection algorithms too have little influence on some of the MPR-sets and, therefore, on the network lifetime.

To analyze the relation between the MFR and the network lifetime realized by different MPR selection algorithms, we generated a set of networks with a wide range of MFRs. Based on preliminary simulations, we choose to place 150 nodes in a square field of 1000×1000 units, while selecting for each simulation a transmission range from the set $[200, 250, 300, 350]$ and assigning this range to all nodes. For every transmission range we created 200 networks, resulting in 800 networks in total.

In each of these networks we initiated broadcast messages according to the same sendpattern. The messages are broadcasted by MPR flooding, where the MPRs are selected by the different algorithms. We discuss them separately.

6.2.1 MaxWill versus MinCar

The results of the simulations are presented in Figure 3. The Performance Ratio in this graph is defined as the network lifetime using MaxWill divided by the network lifetime using MinCar.

Analyzing the graph we see that when the Maximum Forcedness Ratio approaches 1 the difference in network lifetimes become smaller. This can also be concluded from Table 3 in which the mean and the standard deviation of the Performance Ratio is listed per intervals of the Maximum Forcedness Ratio. The results support the expected effect that the 'more fixed' the 'most fixed' node is, the smaller the performance differences are. If the Maximum Forcedness Ratio is smaller, also bigger performance differences occur. The graph also shows that in almost every simulation the network lifetime of MPR flooding using MaxWill is larger than the network lifetime of MPR flooding using MinCar.

	Maximum Forcedness Ratio Intervals							
	$[0.2, 0.3)$	$[0.3, 0.4)$	$[0.4, 0.5)$	$[0.5, 0.6)$	$[0.6, 0.7)$	$[0.7, 0.8)$	$[0.8, 0.9)$	$[0.9, 1)$
Mean	1.89	1.71	1.52	1.29	1.19	1.10	1.05	1.02
SD	0.32	0.39	0.33	0.22	0.15	0.11	0.06	0.03

Table 3

The mean and standard deviation of the Performance Ratio concerning MaxWill and MinCar per interval of the Maximum Forcedness Ratios.

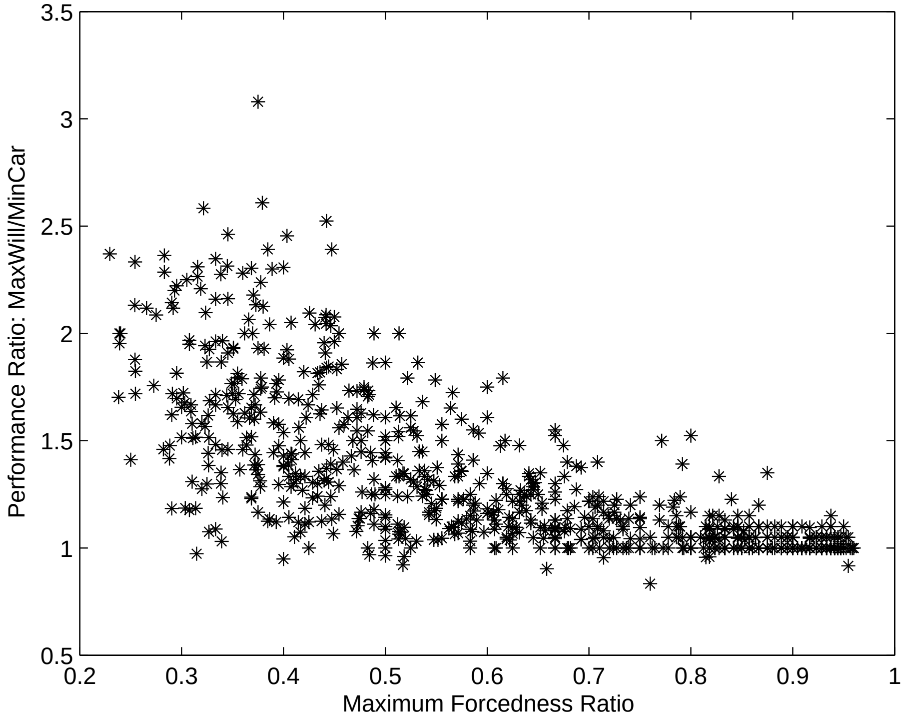


Fig. 3. The effect of the Maximum Forcedness Ratio on the performance differences using MPR flooding with MPRs selected by the MaxWill algorithm and the MinCar algorithm.

6.2.2 MaxWillMinForced versus MinCar

The relation between the performances of MPR flooding using MaxWillMinForced and MinCar to select MPRs are shown in Figure 4 and Table 4. In the graph the Performance Ratio is defined as the network lifetime using the MaxWillMinForced MPR selection algorithm divided by the network lifetime using the MinCar algorithm. There are some similarities between this graph and the corresponding table and the ones discussed in the previous section, which is not surprising, as the MaxWillMinForced algorithm is based on MaxWill and only adds a sort of look-ahead for the energy consumption. So, the performances vary less if the Maximum Forcedness Ratio approaches one and the MaxWillMinForced algorithm is almost in every situation better than the MinCar.

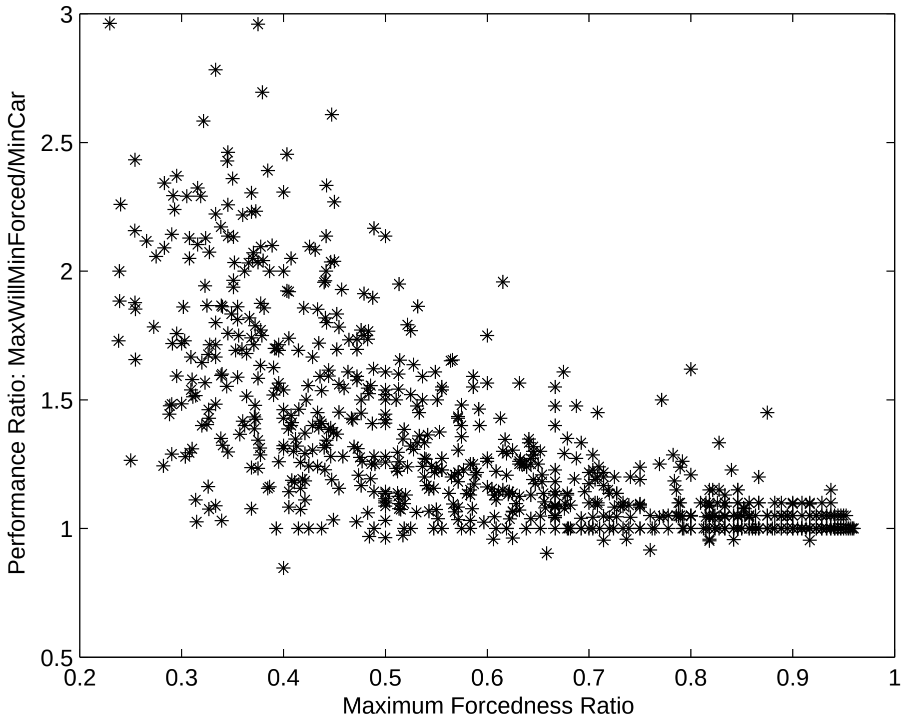


Fig. 4. The effect of the Maximum Forcedness Ratio on the performance differences using MPR flooding with MPRs selected by the MaxWillMinForced algorithm and the MinCar algorithm.

	Maximum Forcedness Ratio Intervals							
	$[0.2, 0.3)$	$[0.3, 0.4)$	$[0.4, 0.5)$	$[0.5, 0.6)$	$[0.6, 0.7)$	$[0.7, 0.8)$	$[0.8, 0.9)$	$[0.9, 1)$
Mean	1.91	1.74	1.51	1.29	1.18	1.10	1.05	1.01
SD	0.41	0.40	0.34	0.22	0.17	0.11	0.07	0.03

Table 4

The mean and standard deviation of the Performance Ratio concerning MaxWillMinForced and MinCar per interval of the Maximum Forcedness Ratios.

6.2.3 MaxWillMinForced versus MaxWill

The performance comparison between the MaxWillMinForced and MaxWill MPR selection algorithms does not yield a clear winner. The absence of a winner is underlined by the mean of the Performance Ratio, which equals 1.0023. When we compare MaxWillMinForced with MaxWill we see that for 424 of the 800 simulations the selection algorithms lead to exactly the same network lifetime. MaxWillMinForced beats MaxWill in 203 simulations, but on the other hand, MaxWill beats MaxWillMinForced in 173 simulations.

7 Conclusions

We presented an analysis of MPR flooding by looking separately at MPRs, MPR flooding and MPR selection. By this, we are able to point out the effects of the specific elements of MPR flooding. Our conclusions are that for general graphs fixed nodes provide an upper bound to the network lifetime, independent of the MPR selection algorithm.

For grid graphs all MPR selection algorithms provide the same MPR-set for 2-hop central nodes, namely the set of forced nodes that is selected in the first step of a three step MPR selection algorithm. Since this result is a consequence of the regular structure (translation, symmetry) of the graphs, we may expect similar results for all graphs with regular (sub) structures.

For random graphs, the maximum forcedness ratio parameter, that we introduce in this paper, seems a good descriptor of the degree in which MPR selection algorithms yield network lifetimes. In random graphs with an MFR close to 1 there is less difference between MPR selection algorithms than in graphs with MFR close to 0. Therefore, it is not worthwhile to investigate new MPR selection algorithms if the networks under consideration have an MFR close to 1. More generally, all computational studies on MPR selection should take into account the MFR to ensure that no wrong conclusions are drawn from the achieved results.

As a byproduct of the simulations we got some insight in the effectiveness of some MPR selection algorithms for random networks. MPR flooding with MPR-sets selected by MaxWill or MaxWillMinForced leads to a significantly longer network lifetime compared to MPR flooding with the MPR-sets selected by MinCar. However, the comparison between MaxWill and MaxWillMinForced does not yield a clear winner, even if the networks have a low MFR. Combined with the fact that MaxWillMinForced is more difficult to implement, MaxWill therefore seems a good choice to implement in OLSR networks where lifetime of the network is important.

While in this paper we focus on mechanisms for broadcasting traffic with a homogeneous traffic load throughout the network, in future research it may be interesting to verify if similar conclusions hold for heterogeneous unicast traffic.

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