Three Notes on Controlled Hyper-Algebraic and Dhyper-Algebraic Extensions*

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Abstract — (1) Regular control does not increase the generating power of 1-restricted [d]K-iteration grammars provided that $K \supseteq$ SYMBOL, and K is closed under isomorphism and under union with SYMBOL-languages.

(2) Let Γ be a prequasoid closed under the regular operations. If K is a prequasoid [pseudoid], then $H(\Gamma) \subseteq H(\Gamma, K)$ [$\eta(\Gamma) \subseteq \eta(\Gamma, K)$]. In particular we have $H(\Gamma) \subseteq$ (Γ)ETOL and $\eta(\Gamma) \subseteq$ (Γ)EDTOL.

(3) Under weak conditions on Γ and K, the decidability of the emptiness problem for Γ and K implies the decidability of the emptiness problem and the membership problem for the families $\eta(\Gamma, K)$ and $\eta(K)$.

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1 Regularly Controlled 1-Restricted Hyper-Algebraic and Dhyper-Algebraic Extensions

In [1, 3] we showed that regular control does not increase the generating capacity of hyper-algebraic and dhyper-algebraic extensions. In the proof the number of substitutions is however increased with 1 [1, 3] and since we can only reduce the number of substitutions to 2 in general [1, 3], the argument cannot be applied in the 1-restricted case, i.e., to iteration [d-iteration] grammars containing only one substitution [d-substitution].

In this note we show that for 1-restricted hyper-algebraic and dhyper-algebraic extensions, regular control does also not provide any additional generating power; thus yielding a similar result as in the unrestricted case.

Remember that a family K of languages is called *α-simple* [2] if K includes SYMBOL and if K is closed under isomorphism ("renaming of symbols") and under union with SYMBOL-languages. For all unexplained terminology we refer to [1, 2, 3] and the references mentioned there.

Theorem. *If K is α-simple, then*

(i) $H_1(\mathrm{REG}, K) = H_1(K)$,

(ii) $\eta_1(\mathrm{REG}, K) = \eta_1(K)$.

Proof. (i) Obviously, $H_1(K) \subseteq H_1(\mathrm{REG}, K)$.

Conversely, let $G = (V, \Sigma, \tau, M, S)$ be a (REG, K)-iteration grammar with a single K-substitution τ. Let $(Q, \{\tau\}, \delta, q_0, Q_F)$ be a complete deterministic finite-state acceptor for M, where Q is the set of states, $\{\tau\}$ is the input alphabet, $\delta : Q \times \{\tau\} \to Q$ is the transition function, $q_0 \in Q$ is the initial state, and $Q_F \subseteq Q$ is the set of final states.

Consider the K-iteration grammar $G_0 = (V_0, \Sigma, \tau_0, S_0)$, where $V_0 = V \times Q \cup \Sigma \cup \{F\}$, $S_0 = [S, q_0]$, and F is a rejection symbol. Distinguish the following cases.

Case 1: M is finite.

We may assume that each state in Q is non-recurrent, i.e., it is impossible to visit a state more than one time. Then we define for α in V and q in Q,

$$\tau_0([\alpha, q]) = \{[\alpha_1, q'] \cdots [\alpha_n, q'] \mid n \geq 0, \alpha_1 \cdots \alpha_n \in \tau(\alpha); \delta(q, \tau) = q'; q \in Q\} \cup T_{\alpha,q} \cup \{F\},$$

and for $\alpha \in \Sigma \cup \{F\}$,

$$\tau_0(\alpha) = \{F\},$$

with $T_{\alpha,q} = \{\alpha\}$ if $q \in Q_F$ and $\alpha \in \Sigma$ and $T_{\alpha,q} = \varnothing$ otherwise.

Clearly, we have $L(G_0) = L(G)$, and hence $H_1(\mathrm{REG}, K) \subseteq H_1(K)$.

Case 2: M is infinite.

In this case the substitution τ_0 is defined as follows: for $\alpha \in V$ and $q \in Q$,

$$\tau_0([\alpha, q]) = \{[\alpha_1, q'] \cdots [\alpha_n, q'] \mid n \geq 0, \alpha_1 \cdots \alpha_n \in \tau(\alpha); \delta(q, \tau) = q'\} \cup T_{\alpha,q} \cup \{F\},$$

and for $\alpha \in \Sigma \cup \{F\}$,

$$\tau_0(\alpha) = \{F\},$$

where $T_{\alpha,q}$ is as in Case 1.

It is easy to prove that $L(G_0) \cap \Sigma^+ = L(G) \cap \Sigma^+$. As usual the only possible difficulty is caused by the empty word.

Assume $\lambda \in L(G)$, i.e., there exists $n \geq 1$ with $\lambda \in \tau^n(S)$ and $\tau^n \in M$. By the construction of G_0, we have $\lambda \in \tau_0^n(S_0)$.

On the other hand we have to show that $\lambda \in L(G_0)$ only if $\lambda \in L(G)$. Suppose $\lambda \in L(G_0)$, then two possibilities occur.

(α) There exists a derivation $S_0 \Rightarrow w_1 \Rightarrow \cdots \Rightarrow w_n = \lambda$ according to G_0 such that $w_i \neq \lambda$ $(1 \leq i \leq n-1)$ and $\tau^n \in M$. From the construction of G_0 it follows that $\lambda \in L(G)$.

(β) There exists a derivation $S_0 \Rightarrow w_1 \Rightarrow \cdots \Rightarrow w_n = \lambda$ according to G_0 such that $w_i \neq \lambda$ $(1 \leq i \leq n-1)$ and $\tau^n \notin M$. Since M is infinite, there is a control word $\tau^p \in M$ with $p > n$. Then $\lambda \in \tau^p(S)$ and hence $\lambda \in L(G)$ as $\tau^{p-n}(\lambda) = \{\lambda\}$. Note that in the subderivation $w_n \Rightarrow \cdots \Rightarrow w_p = \lambda$, $w_i = \lambda$ $(n \leq i \leq p)$ no state from Q is attached to λ and so we lost the remaining part of the control word. This loss is however quite immaterial since $\tau_0^{p-n}(\lambda)$ can only yield λ.

So we have $L(G_0) = L(G)$ and consequently $H_1(\mathrm{REG}, K) \subseteq H_1(K)$,

(ii) Exactly the same construction can be applied in the deterministic case. $\square$

2 A Remark on Controlled Hyper-Algebraic and Dhyper-Algebraic Extensions

In [1] we showed that under weak assumptions on the families Γ and K, the Γ-controlled hyper-algebraic extension $H(\Gamma, K)$ of K is a full hyper-AFL including the families K, Γ and $H(K)$. Similarly, in the deterministic case [3] we have that under weak conditions on Γ and K the Γ-controlled dhyper-algebraic extension $\eta(\Gamma, K)$ of K is a full dhyper-QAFL including the families K, Γ and $\eta(K)$.

We now prove that under an additional assumption on Γ the family $H(\Gamma, K)$ [$\eta(\Gamma, K)$] also includes $H(\Gamma)$ [$\eta(\Gamma)$, respectively]. (This result has been inspired by a remark in [4].)

Remember that a *full* [FIN, REG]-*structure* is a quasoid closed under the regular operations (union, concatenation and Kleene $\star$).

Theorem. *Let* Γ *be a full* [FIN, REG]*-structure.*

(i) *If* K *is a prequasoid, then* $H(\Gamma) \subseteq H(\Gamma, K)$.

(ii) *If* K *is a pseudoid, then* $\eta(\Gamma) \subseteq \eta(\Gamma, K)$.

Proof. Let $M \subseteq \Sigma^\star$ be a language in Γ and let a and b be symbols not in Σ. Both $\{a\}$ and $\{b\}$ are in Γ, because Γ is a prequasoid and therefore REG $\subseteq \Gamma$. Since Γ is closed under concatenation, we have $aMb \in \Gamma$, i.e., Γ is closed under full marking.

(i) The main result in [1] implies that $H(\Gamma, K)$ is a full hyper-AFL including Γ. Since Γ is a prequasoid, $H(\Gamma)$ is the smallest full hyper-AFL including Γ [1, 2]. Hence $H(\Gamma) \subseteq H(\Gamma, K)$.

(ii) From [3] it follows that $\eta(\Gamma, K)$ is a full dhyper-QAFL. Similar to Lemma 4.1 in [1] it is easy to show that $\eta(\Gamma, K)$ includes Γ. As Γ is a pseudoid, $\eta(\Gamma)$ is the smallest full dhyper-QAFL including Γ [3]. Hence $\eta(\Gamma) \subseteq \eta(\Gamma, K)$. □

Corollary. [4] *Let* Γ *be a full* [FIN, REG]*-structure. Then*

- $H(\Gamma) \subseteq H(\Gamma, \mathrm{FIN}) = (\Gamma)\mathrm{ETOL}$,
- $\eta(\Gamma) \subseteq \eta(\Gamma, \mathrm{ONE}) = (\Gamma)\mathrm{EDTOL}$. □

3 Decision Problems for the Families $\eta(\Gamma, K)$ and $\eta(K)$

In the following we restrict our attention to effective closure properties only; cf. [1], §7.

Since [1] Lemma 7.1 (i.e., [5] Theorem 1) can easily be proved for dK-iteration grammars [3] too, we obtain by an argument almost identical to the non-deterministic case (cf. [1], §7) the following decidability results.

Theorem.
(1) *Let Γ and K be closed under intersection with regular languages and let the emptiness problem be decidable in Γ and in K. Then the emptiness problem is decidable for languages in $\eta(\Gamma, K)$.*

(2) *Let K be a pseudoid and let Γ be closed under full marking and intersection with regular languages. If the emptiness problem is decidable in Γ and in K, then the membership problem is decidable in $\eta(\Gamma.K)$.* □

Corollary.
(1) *If K is closed under intersection with regular languages and if the emptiness problem is decidable for languages in K, then the emptiness problem is also decidable in $\eta(K)$.*

(2) *Let K be a pseudoid and let the emptiness problem be decidable in K. Then the membership problem is decidable in $\eta(K)$.* □

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Note

The original typescript of this report consists of 7 pages; the present LaTeX version reduced this number to 6.