Interactive 3D Simulation of Escher-like Impossible Worlds

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Abstract

Maurits Cornelis Escher (1898-1972) is a famous Dutch artist known – amongst others – for his prints of impossible figures and impossible worlds. Many of his works illustrate mathematical and geometrical concepts such as perspective and limits. Works by Escher have motivated scientists over the years to discover the mathematical foundations of his work, ultimately leading to applications that are able to model and render scenes similar to the ones created by Escher. Presented is an application that is capable of displaying a special class of impossible worlds that have been created by the artist. The software displays worlds that appear physically correct, but are connected in an impossible manner, similar to Escher's *Another World II* or *Relativity*. Portal rendering is employed to create real-time interactive visualizations of such scenes, which can be freely explored by the user.

1 Introduction

Over the ages artists have employed many different techniques to make people experience worlds that differ tremendously from what we consider possible in the "real" world. One might for example tap into a person's imagination by describing such worlds in stories, others have resorted to more graphical approaches and have created visual representations of impossible worlds. More recent developments have permitted artists to combine the story and graphical aspects by animating impossible worlds in, for example, movies. One of the artists who is most famous for his stories that incorporate worlds that are impossible in every sense of the word is Lewis Caroll: one clearly cannot enter a rabbit's hole in order to find a hall there that has a door, which in turn leads to a garden, as described in the work *Alice's Adventures in Wonderland* published in 1865. The sequel even describes Alice stepping into the impossible world that lies "behind" a mirror. One of the most famous artists to have created visual representations of impossible worlds is the Dutch artist M.C. Escher, who has inspired many other artists over the years to create similar works.

Amongst the impossible worlds created by Escher an interesting class of worlds exists, that at first glance look like they could exist in reality, but when looking closer appear to have some elements that are incorrect. Escher's *Another World II* (Figure 1) is an example of such a structure. The views through each of the arches appear to be correct, but they are combined in the image in an impossible way. The image suggests that each arch leads to the same landscape from a different point of view, but in reality one will never be able to see this combination of views simultaneously. Another example that plays with perspective in a similar manner is depicted in Figure 2. Escher's *Relativity* shows three different worlds that exist inside the same structure in a slightly different manner when compared to *Another World II*. The geometry appears plausible, however the rules of physics seem to apply differently to each world. Escher also frequently uses (semi-)

Figure 1: M.C. Escher's Another World II ©2008 The M.C. Escher Company – Baarn – The Netherlands. `www.mcescher.com`

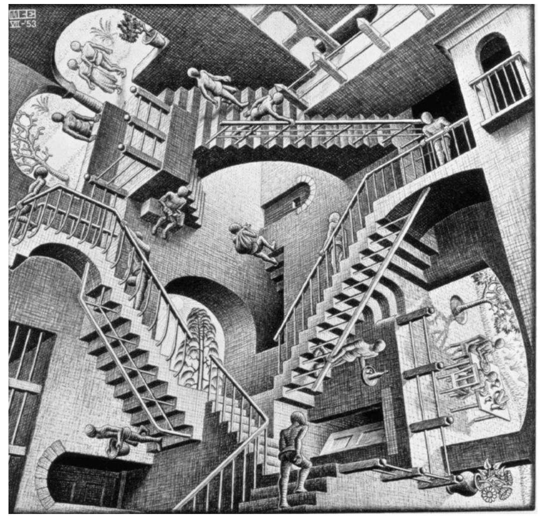

Figure 2: M.C. Escher's Relativity ©2008 The M.C. Escher Company – Baarn – The Netherlands. `www.mcescher.com`

reflective surfaces as a boundary between two worlds, usually these worlds are connected, suggesting that there is a way to enter the mirror universe – *Magic Mirror* (1946). Inspired by the impossible worlds created by Escher we present an application that allows users to explore such impossible worlds in an interactive manner. Using a special rendering trick (see Section 2) and geometry that is specifically tailored for this purpose, real-time rendering of impossible worlds is accomplished. Using this software people are finally able to give in to the urge to freely explore the space depicted by Escher.

The paradoxical and mathematical nature of some of Escher's works has stimulated many researchers and artists to create, both interactive and non-interactive, computer generated renderings of similar scenes in 3D. Many of these feature impossible figures similar to the *Penrose Triangle* [PR58], this includes visualizations of the famous Escher prints *Waterfall* and *Endless Staircase*. Such structures can maintain the illusion only when displayed from a very specific viewpoint. Mathematical theories have been developed that are used to model and render such scenes [SDG99], others have even presented methods for animating such objects while maintaining the illusion [KK99]. Other Escher works that have been transferred to the digital medium include some of his works featuring tessellations and tilings of closed figures. Presented by [KS00] for example is a method for the algorithmic creation of Escher-like tilings, while [YS01] present a method to apply similar tilings to spheres. In 1986 Ned Greene created an animation [CEPT86] that sends the viewer on a trip through *Another World II*. In this animation the camera moved along a fixed path, however it did give a glimpse of what Escher's world might look like beyond what is visible in the famous print. The work presented in this article goes even further by letting the user freely explore the impossible world. Other Escher works have inspired many artists to create digital renderings and animations [Ale09].

Section 2 gives an insight into the technology used for visualization, while Section 3 shows how this technology is implemented. Finally Section 4 delves into several possibilities of creating interesting impossible worlds in future work.

2 Portal rendering

Rendering impossible scenes is a challenging task. Most classical rendering techniques work under the assumption that geometry is physically correct and therefore cannot be used without modification when two worlds are connected in an inconsistent, non-euclidean, way. Fortunately, similar to what the original artist did on paper, the effect can easily be “faked” by giving the viewer the impression that two worlds are connected as seen on the screen even though in the physical model of the scene they are not. An adaptation of a technique called *portal rendering* is used for this purpose, which is described in the following sections.

2.1 Portals and hidden surface removal

One of the major concerns in real-time rendering has always been the problem of *hidden surface removal*. The complexity of the geometry often exceeds the processing capabilities of computer hardware by far, making it impossible to process the scene's geometry in a “brute-force” approach. Fortunately, in most cases only a relatively small subset of the geometry would be visible from a single point of view, especially when dealing with scenes that are situated indoors. Therefore it pays off to only select and process the geometry that can be seen from the current point of view, this process is called *hidden surface removal* (HSR). Many different techniques have been proposed and successfully applied in the past, ranging from simple back-face culling where only those surfaces are selected that face the viewer to more complicated methods that involve complicated spatial subdivision schemes. Often multiple HSR techniques are combined in a chain where each separate technique is used to further reduce the set of visible surfaces. While processing power increases rapidly, especially given the rise of dedicated graphics hardware, so does the complexity of the scenes that are to be displayed. Therefore effective hidden surface removal will play an important role in computer graphics for years to come. Most of these HSR techniques and the algorithms used to implement them are however beyond the scope of this article and we refer to [WP01] for a detailed discussion of the most commonly used methods.

The application presented in this article uses an adaptation of a HSR technique called *Portal Rendering* to connect worlds or parts of worlds in an impossible manner. Portal rendering is a spatial subdivision scheme that can operate very efficiently on geometry that is specifically designed to be used with this subdivision scheme. In portal rendering the scene – also referred to as the *world* – is divided into cells, which are in turn connected using portals. Figure 3 shows a top down view of an example scene that contains five cells – numbered one through five – connected by four portals, indicated by the dashed lines. The key to the effectiveness of the portal rendering algorithm lies in the placement of the portals. The algorithm that determines visible geometry uses the portals that connect cells to determine the cells that are visible from the current viewpoint. The algorithm starts by determining the cell that contains the viewpoint, this cell is always considered visible. For each portal that connects this cell to another cell, the following actions are performed:

- The portal is intersected with the current view frustum, if it does not at least partially intersects it is discarded and the algorithm proceeds with the next portal.
- The edges of the visible portal fragment are extruded away from the current point of view by casting rays from the viewpoint through its vertices. The resulting volume is used as the new view frustum.
- The algorithm recursively performs the same operation on the cell behind the portal using the newly created view frustum.

Naturally measures should be taken to prevent the algorithm from entering a portal that has already been considered once, to prevent it from continuously looping between two neighbouring cells. This operation terminates when a cell is encountered that contains no visible portals. Figure 4 shows the same five cells and includes a view frustum originating at a point of view in cell 1. The gray area shows the visible area of the scene that can be determined by updating the viewing frustum based on the portal's geometry, the dotted lines indicate the original unconstrained viewing frustum. Notice that based on the unconstrained frustum cells 4 and 5 would be considered visible, while by updating the frustum they are culled away from the visible scene. In contrast to other spatial subdivision schemes that operate automatically on any arbitrary geometry, portal rendering works most effectively when the portals are manually and explicitly placed by the person creating the geometry. This implies that this person must be aware of the properties of the HSR algorithm. This fact is exploited to create interesting and impossible scenes.

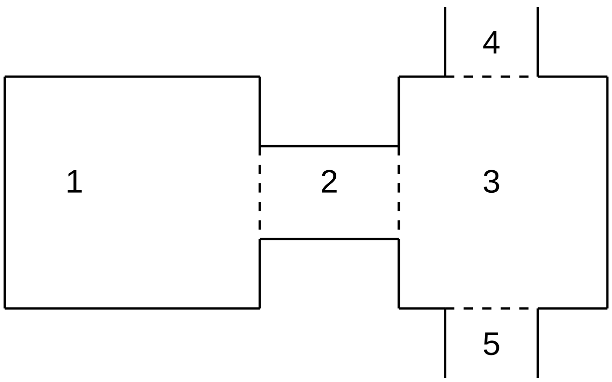

Figure 3: An example scene consisting of five cells and four portals connecting those cells

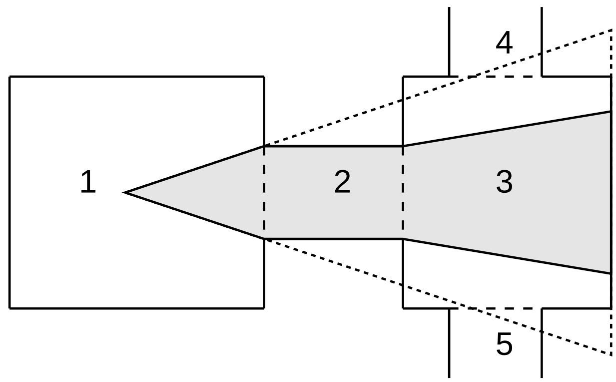

Figure 4: An example scene consisting of five cells and four portals connecting those cells, displaying a view frustum

2.2 Portals to create impossible worlds

Basic portal rendering as it is described in section 2.1 does not handle impossible geometry where worlds are connected in an impossible manner. However, when one considers each cell to be a separate "world" and by slightly extending the technique portal rendering can be used to allow users to interactively explore impossible scenes like the one created by M.C. Escher.

Each portal can be thought of as having two sides (even though the portal itself has a thickness that is infinitely small). Each side of the portal is part of a cell and connects the portal to the other side, which is in turn a part of the connecting cell. When the technique is used mainly for hidden surface removal both portal sides are equal and are often modelled as one, however when both sides are separated they do not have to occupy the same position in space, see Figure 5. Cells 1 and 2 are connected using a portal as usual, however both sides of the portal have a different position and orientation in space. This connection would not be possible in the real world because the geometry

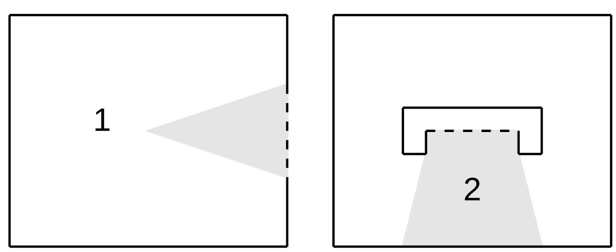

Figure 5: An example scene consisting of two cells which are connected in an impossible way using a single portal

that is behind the portal in cell 2 would overlap the contents of cell 1. The rendering algorithm needs to be changed in such a way that all geometry behind the portal is transformed to make it appear as if both sides of the portal are physically connected. Extra measures should be taken to ensure that only those parts of the geometry that can be seen through a portal are actually visible. This task is common practice using modern 3D graphics hardware, which has the ability to mask out portions of the screen that should not be rendered to.

Each side of a portal is assigned three vectors that together form an orthonormal matrix P_i, where $1 \leq i \leq 2$ for side i, which define a local coordinate system on that side of the portal. Two of those vectors lie on the plane of the portal and the third is perpendicular to the portal's plane. A transformation matrix R is calculated as $R = P_1^{-1} P_2$, which represents the rotation from side 1 to side 2. Combined with a translation vector $T = C_2 - C_1$, where C_i is defined as the center of portal side i, this transformation can be used to transform any geometry behind the portal as though the two sides of the portal have the same position and orientation. The same transformation should be applied to the position and orientation of the viewer when the view point passes through a portal in order to allow the viewer to interactively explore the scene.

3 Implementation details

The software that is used to visualize the impossible worlds has been created using the Java programming language. Visualization of three-dimensional geometry was performed using the OpenGL library, which provides access to 3D graphics hardware, resulting in smooth and real-time animations. Specifications of the geometry that is to be displayed, including the location and orientation of each portal side, is read from files. Therefore the use of the software is not limited to the scenes that are presented in this article.

3.1 Visualization of Escher's Another World II

The first scene that was visualized using the application was based on Escher's *Another World II* [RO07] – Figure 1. Escher created the woodcut in 1947 to demonstrate the relativity of vanishing points [Ern76]. The picture only has a single vanishing point, however when looking through each of the arches this vanishing point has a different function. This acts as the main reason for the paradoxical nature of the print.

Just looking at the image raises some interesting questions: *Does each of the arches show the same lunar landscape?, What does the outside of the cube the viewer is in look like?* and most importantly *What would happen if one were to step through one of the arches?* Because the landscape looks similar through each of the arches, there is assumed to be only one landscape in the scene. This landscape is displayed through each of the arches. Three cube-shaped structures are positioned on the landscape, each having two arches that lead to the cube that is central in Escher's print. The landscape is connected to the cube by placing portal sides that coincide with the arches. The viewer is now able to leave the central cube through one of its arches and enter the cube again through another. Because the perspective through each of the arches is different the viewer will not be able to distinguish up from down or left from right when entering the central cube only to regain his or her sense of direction when stepping out onto the landscape.

Figures 6 and 7 show two final renderings of the scene based on *Another World II* from the inside and outside of the central cube respectively.

3.2 Visualization of Escher's Relativity

Another print that illustrates Escher's creativity with vanishing points is *Relativity* – Figure 2. It shows how three different worlds coexist in an impossible manner within the same structure. While both prints express the same ideas about the relativity of vanishing points, the print is different from *Another World* because it represents geometry that could be created in reality. It is not until one inspects the figures that walk through the scene before noticing that there must be in fact three different gravity directions in the picture, because the ground planes of the figures are oriented differently.

One is only able to truly appreciate a scene similar to *Relativity* in an interactive simulation when presented with the ability to walk through the scene similar to the figures in Escher's print. However the figures in the print do not seem to have the ability to step from one of the worlds into the other, something that an inquisitive user of the application might want to attempt. Portals are used once again to allow the user to navigate between the different worlds, portal sides are placed such that they coincide with some of the doors that are visible in Escher's original print. Users of the software are given the ability to walk, by subjecting them to gravity, which was not present in the simulation of *Another World II*. The gravity direction is variable and changes as the user steps through a portal. When two connected portal sides are oriented differently the gravity vector is modified accordingly using the transformation matrix R – see Section 2.2.

Figures 8 and 9 show two renderings of a scene based on *Relativity* from a perspective similar to the original print and from a novel viewpoint taken from one of the other "worlds".

4 Discussion

By applying portal rendering techniques people are able to interactively explore impossible worlds similar to the ones created by Escher. The simulation is effective in translating the paradoxical scenarios to a digital medium and shows that allowing the user to manoeuvre through the scene adds to the sense of confusion, especially when crossing a portal into the next "world". People using the software appear eager to explore the familiar scenes on a quest to find a logical explanation for the paradox only to find that the geometry remains impossible in the world behind a portal. The scene based on *Another World II* appears hard to navigate for most people, even those that have a background in computer graphics, mainly due to the fact that there are no valid reference points available when the scene is viewed from inside the cube. Visual aids could be introduced in the future to aid in navigation, such as assigning different colours or textures to different arches in the cube, although one might argue that this contradicts the ideas the original artist wished to express. The geometry differs from what can be observed in the prints created by Escher, most notably the fact that all arches in the *Another World II* scene are of the same size. The original artist could not use all six sides of the cube and therefore had to combine two arches in a single side. An interactive model of such a scene does not have this limitation because the viewer is free to look around inside the cube. The scene based on *Relativity* lacks a virtual representation of the figures that appear to inhabit the three worlds in the original print. These figures add to the belief that all three worlds can be inhabited simultaneously, a property which is partially lost in the interactive simulation. Therefore a future revision should include similar figures, which might even be animated to add extra realism.

Presented is a generic framework that supports navigation through scenes that use portals as a means to connect portions of this world in an impossible manner. The framework could be applied to many more scenarios, including impossible worlds created by other artists. The geometry need only adhere to the requirements outlined in previous sections – especially with regard to portal shape and placement. A mix of “indoor” and “outdoor” scenes is supported, yet portals are traditionally associated with indoor scenes only. By correctly placing the portals they can be used as a mechanism to connect an indoor portion of the scene to an outside world. The technology also has great potential for use in computer games to create interesting gameplay elements. This fact is proved by the popularity of games such as *Prey* (2006, Take-Two Interactive Software) and the hit game *Portal* (2007, Valve Corporation). Both games have been partially designed around similar techniques.

In the future experiments could be performed using non-planar portals. Imagine a lens-shaped portal that distorts the geometry behind it based on the curvature of the portal. When stepping through the portal, the viewer could end up in the transformed “world” where all geometry seems to converge towards a single focal point. The worlds that are connected using portals could also be segmented in time as well as in space, imagine looking back into the same room in a different time. The difference in time could be a few seconds, but also entire seasons or even eras. This property could be explored even further when an avatar is displayed representing the viewer. Such applications could be created for entertainment purposes, however educational software could also be created that illustrate certain mathematical concepts.

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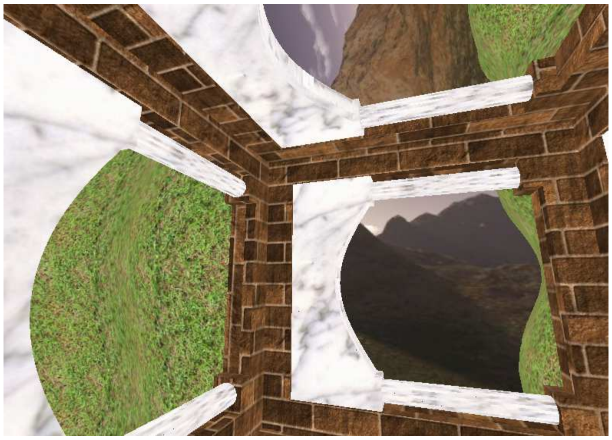

Figure 6: Rendering of the virtual Another World (inside view)

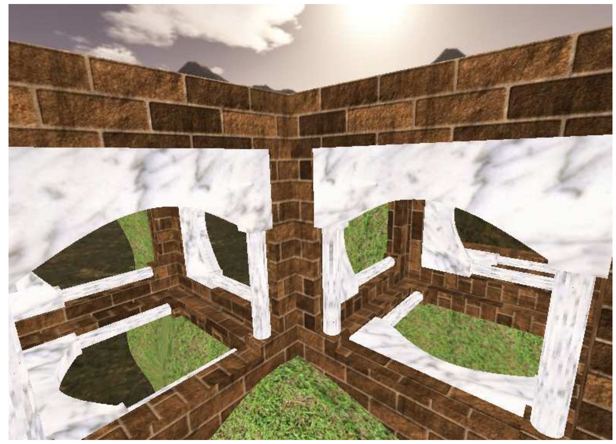

Figure 7: Rendering of the virtual Another World (outside view)

Figure 8: A scene similar to Escher's Relativity from a familiar viewpoint

Figure 9: A scene similar to Escher's Relativity from an unfamiliar viewpoint