An Analytical Model for Beaconing in VANETs

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Abstract—**IEEE 802.11 CSMA/CA is generally considered to be well-understood, and many detailed models are available. However, most models focus on Unicast in small-scale W-LAN scenarios. When modelling beaconing in VANETs, the Broadcast nature and the (potentially) large number of nodes cause phenomena specific to large-scale broadcast scenarios not captured in present models of the 802.11 DCF. In a VANET scenario, transmissions from coordinated nodes are performed in so-called Streaks, without intermediate backoff counter decrement. We adapt the model by Engelstad and Østerbø and provide several improvements specific to VANET beaconing. The resulting analytical model is shown to have good fit with simulation results.**

I. Introduction

A Vehicular Ad hoc Network (VANET) is a very dynamic environment from both network and road traffic point of view. Various applications have been developed over the years (e.g. see [1]) based on the notion of vehicles exchanging Cooperative Awareness Messages (CAM) or *beacons*, to increase traffic efficiency and safety or to provide infotainment. Especially the efficiency and safety applications can come with steep requirements with respect to maximum allowable delay and success probability.

One of the reasons why standardisation is moving towards the adoption of IEEE 802.11p [2] (or its European counterpart ETSI-G5 [3]) for this purpose is that the behaviour of the 802.11 family is well-understood. However, present Unicast 802.11 models are not directly suited for the VANET scenario.

Beacons are transmitted using the CSMA/CA Broadcast method, hence it is vital to correctly model the countdown and blocking behaviour of the MAC layer. Such detail was not present in our simple model in [4]. When designing a model for beaconing in VANETs, the model by Engelstad and Østerbø [5] provides a good starting point because it covers the entire saturation spectrum. Even though [5] claims to model EDCA, its backoff counter decrement behaviour is that of the DCF. When removing the CW and AIFS differentiation, an accurate model of the DCF remains.

In this paper, we adapt their model for a broadcast channel and introduce the concepts of Collision Multiplicity and Streak length to calculate a better approximation of the expected service time and of the blocking probability in a broadcast channel. This is necessary since we have found that when the channel becomes saturated, transmissions are performed consecutively, without intermediate empty slot. For the same reason, transmissions cannot be considered independent of previous and subsequent transmissions, as is widely done in the related work [5]–[10].

Sec. II describes related work. Sec. III describes the 802.11p beaconing system, and Sec. IV describes our modeling and analysis approach. A comparison with simulation results is provided in Sec. V. Sec. VI concludes.

II. Related Work

There is a long history of 802.11 models (e.g. [5]–[9]). Bianchi [7] models the 802.11 Distributed Coordination Function (DCF) Basic Access and Request-to-Send/Clear-to-Send (RTS/CTS) under saturation conditions, with a focus on obtaining throughput estimates for Unicast transmissions. The DCF is modelled as a Discrete Time Markov Chain (DTMC), where time is discretised into either empty or busy slots, the latter of which are subdivided into success or collision. However the model in [7] does not consider backoff counter (bc) blocking and hence decrements even in busy slots.

Proper bc decrementing behaviour is added by Ziouva and Antonakopoulos [8] by adding self-loops to the backoff states. Xiao [9] later adds traffic class priority differentiation (by means of Contention Window (CW) size) and a finite retransmission limit which was not present in earlier models.

Engelstad and Østerbø [5] add AIFS differentiation to Xiao's model, and make the model suitable for the entire saturation range by adding post-backoff behaviour which depends on the state of the transmission queue. This model allows throughput and delay estimates, Later, and estimate for the queueing delay was derived using a Z-transform [10]. These models all consider Unicast transmissions in typical W-LAN situations, where stations are clustered around an Access Point. Furthermore, the traffic regime in these models is a small number of nodes with high supplied load per station.

In [11], Vinel *et al.* address the trade-off between generation rate and network performance using deterministic arrivals, assuming a node always performs backoff prior to transmission. In [12] a Markov Chain is used to model the number of *active* stations and the number of transmissions in a slot under the assumption of a Bernoulli arrival process. An important assumption in [12] is that, when a new beacon arrives in the MAC queue and finds it non-empty, it replaces the current contents. Ma and Chen [13] provide a model for broadcast in VANETs including the presence of hidden terminals. Their model always performs backoff, exhibiting saturation behaviour.

III. System Description

A beaconing system for Cooperative Awareness broadcasts small status messages on the Control Channel (CCH) called *beacons* using IEEE 802.11p. It differs from a Unicast WLAN situation in that the number of nodes can potentially be much higher, but the supplied load per node is lower. Beacons are generated at 10 Hz [1], which has been shown to be sufficient to run a real-time control system such as a CACC system [14].

The IEEE 802.11p MAC layer uses Carrier-Sense Multiple Access with Collision Avoidance (CSMA/CA) [15]. Beacons share the CCH with emergency event messages, which are also broadcast. These event messages should be scarce; hence for simplicity in this paper we assume only Cooperative Awareness Messages on the CCH. Given the broadcast nature of these messages, we focus on the Broadcast part of 802.11p, only. Moreover, the current paper only considers nodes that are in each other's carrier sense range, so-called coordinated nodes. The impact of possible hidden terminals is left to future work. Additionally, we do not consider single-radio multi-channel operations proposed by IEEE 1609.4 [16], instead we assume, that all nodes reside on the same channel with one radio at all times.

Prior to transmission, a node must ensure that at least the duration of Inter-Frame Spacing (IFS) has passed in which the channel was idle. If the channel is found to be idle, transmission is allowed immediately. If the channel is found to be busy, a backoff procedure (BO) is performed by drawing a random backoff counter bc according to a uniform distribution from $[0, W-1]$, where W is the size of the Contention Window (CW). The bc is decremented for every slot σ in which the channel is perceived idle. When the bc reaches zero, a node transmits.

After transmission, if the queue is empty the node performs a post-backoff (PBO), again choosing a bc and decrementing it in each empty slot. If, however, the queue is non-empty after a transmission the node performs a new backoff for medium access, also drawing a bc and subsequently counting down in each slot σ the medium is idle.

While in BO or PBO, the bc is frozen if the channel is busy due to a transmission from an other node. If a node has no packet to transmit and has completed PBO, it is IDLE.

A collision between coordinated nodes can occur because i) two or more nodes simultaneously perform carrier sensing, and find the medium idle, or ii) because their bc expires simultaneously. The number of nodes involved in a collision is called *Collision Multiplicity* (CM), where a CM of 1 indicates a successful transmission.

The DCF's CSMA/CA is a random access mechanism which is intended to function without central coordinator or synchronisation between nodes. However, as the medium goes into saturation, nodes can become synchronised, as follows. Under channel saturation, multiple transmissions can follow each other directly without intermediate empty slots. Such a series is in the following denoted as a *Streak*, and illustrated in Fig. 1. Discretising time to slots, what happens on the channel is the following:
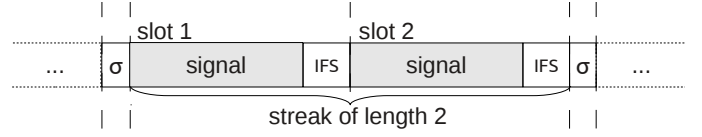

Fig. 1.   A Streak: Consecutive transmissions, no intermediate empty slots.

1) The medium is idle for an empty slot, indicated by σ in Fig. 1. All nodes in contention decrement their bc.
2) Nodes with $bc = 0$ transmit in slot 1, possibly causing a collision.
3) In the next slot, nodes with $bc = 1$ in slot 1 are *not* allowed to transmit, they require an empty slot to decrement their bc. However, there are two groups of nodes which possibly *can* transmit:

$\mathbb{G}_{TX}$ A node has performed a transmission in slot 1. It finds its queue non-empty and chooses $bc = 0$ with probability $\frac{1}{W}$.

$\mathbb{G}_{IDLE}$ A node was IDLE in slot 1, its queue was empty. Sometime during slot 1, a packet arrives in the queue. The node senses the medium busy and chooses $bc = 0$ with probability $\frac{1}{W}$. In the next slot (after the IFS) it will transmit.

This process can repeat, and is more likely to repeat for larger number of nodes. Note that the size of the CW directly influences the Streak Length, because a larger CW reduces the probability of choosing a bc of zero. Note that the backoff behaviour is slightly different in the EDCA, which we intend to model in follow-up work. We expect that both Streak Length and Collision Multiplicity behaviour will then be different.

IV. Beacon Model

We extend the model in [5], [10] for the traffic regime as found in a typical VANET beaconing scenario: 1) transmissions are Broadcast, 2) the number of nodes can become large, and 3) the supplied load per station is small. We consider a system of n coordinated nodes, which are within each other's symmetric carrier sense range.

Fig. 2 illustrates the high-level model of a single node, in which the detailed MAC model is embedded. Interactions with other nodes is modelled in the MAC and receiver components. Beacons are generated at the network layer at rate λ_g and contain application data from which we abstract in this work.
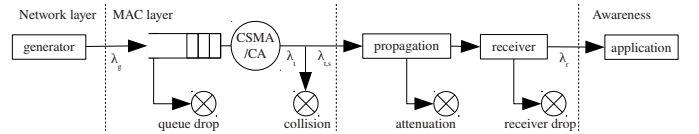

Fig. 2.   The Beacon Model of a single node

Beacons enter the transmission queue and will then be served by the 802.11p CSMA/CA MAC. After propagation through the channel, beacons arrive at the receiver. Not all generated beacons will be received successfully, due to 1)

queue drops in case of a full queue[1], 2) collision loss with coordinated nodes, 3) propagation loss due to attenuation[2], and finally 4) receiver drop because of interference with hidden nodes. Ultimately, the resulting receive rate, λ_r, and the delay are important measures of interest, since they determine the quality of the ITS application.

Typically, the load generated per node by beaconing is low, making queue drops negligible because queue-build-up does not occur. Though attenuation and collisions with hidden terminals are surely limiting factors in the performance of a beaconing system [18], the focus of this paper is on the MAC layer, and its ability to coordinate transmissions. The high-level model allows adding more detail later on.

The MAC is modelled as a queueing station; and includes the transmission queue and the medium access "server" which is modelled as a DTMC. In order to apply this modelling technique, an infinite queue and Poisson arrivals are required. Frames arrive at the infinite FIFO queue with rate λ_g. The server takes a frame from the queue and then serves it: after contention the frame will be transmitted. Because the server contains loss (by means of a collision with one or more other frames) the resulting "successful transmission rate" $\lambda_{t,s}$ may be lower than λ_g. Unlike in Unicast, where frames are dropped when the maximum number of retries has been reached, frames are never dropped for this reason in Broadcast mode.

A. MAC Layer Model

The IEEE 802.11p CSMA/CA is modelled as a DTMC that is a modified version of the one presented in [5], [10]. Time is discretised into generic slots. A slot is either idle or busy; a busy slot is either successful or a collision. In reality, idle, collision or success slots have different durations.

Fig. 3 shows the resulting DTMC model for a beaconing node. We implicitly assume a node is always listening when it is not transmitting[3].

The original model [5] describes the IEEE 802.11e MAC for both non-saturation and saturation circumstances. In the saturation case, a node always finds a packet in its queue to transmit. After transmission of one frame it will immediately start contention for the next frame. In the non-saturation case, it may happen that a node finds the queue empty after transmission. It will perform PBO and subsequently remain IDLE, until a packet enters the queue, or slide from PBO into BO when a packet arrives during PBO.

The model abstracts from the number of packets in the queue, it only considers the queue empty or non-empty.

While the model in [5] allows for traffic within different Access Categories, this work focusses on single-class beacon-ing and does not include traffic of other classes. Furthermore, IEEE 802.11e Unicast, as modelled in [5], considers multiple retries and CW increase per retry. For a beaconing system using 802.11p, we consider only Broadcast transmissions. Hence, there are no retransmissions and no CW increments.

Explicit differences from [5], [10] are presented in the following equations. In Eq. (13), q is adapted to more accurately model the packet arrival probability while IDLE. In Eq. (16), the bc blocking probability while in PBO is improved. The service time of the MAC is obtained through Eq. (18), which depends on the Streak Length in Eq. (25) and the Collision Multiplicity in Eq. (20).

The state space S of the DTMC consists of a finite set of states $S = \{s_{j,k} | j \in \{0,1\} \wedge k \in \{0, \ldots, W-1\}\}$, where $j = 0$ holds for a node that is currently not in the process of accessing the medium (it is either in PBO or IDLE) and $j = 1$ means that the node is contending for medium access (BO), or actually transmitting.

Parameter $k \in \{0 \ldots W-1\}$ denotes the current bc value, which is randomly choosen according to a uniform distribution when 1) a station takes a packet from the queue and starts its medium access attempt and it finds the medium busy, or 2) when a station starts PBO.

After transmission, with probability ρ the station finds another packet in its queue and will perform a new BO for medium access. With probability $1-\rho$ the queue is empty and the node will enter PBO.

While in PBO, the bc is decremented for every empty timeslot, until the system reaches $s_{0,0}$. If a transmission by an other node is overheard (with probability $p^\star$) the bc is frozen. Countdown resumes when the channel turns idle again, with probability $1-p^\star$.

During PBO, with probability $q^\star$ a frame enters the transmission queue. The bc countdown will continue, in order to access the medium. This is modelled by the diagonal transitions. While in BO (states $s_{1,k}$ for $k \in \{1 \ldots W-1\}$), new arrivals in the transmission queue are not considered, because the station already has a packet in the queue (the one currently in contention). The number of packets currently in the queue is not explicitly modelled. Note, however, that ρ accounts for arrivals during the service time.

When a node reaches $s_{0,0}$, which represents an IDLE node, it receives a packet in its transmission queue with probability q or remains IDLE with probability $1-q$. A node perceives the channel busy with probability p, and hence will perform a BO with probability qp, or a direct transmission with $q(1-p)$. In the latter case the carrier-sensing found the channel idle and transmission is allowed immediately.

B. State distribution

Let $b_{0,k}$ and $b_{1,k}$ denote the stationary probability of being in states $s_{0,k}$ and $s_{1,k}$ respectively for the DTMC shown in Fig. 3. By working recursively from right to left, the following expression for the nodes currently in PBO can be derived:
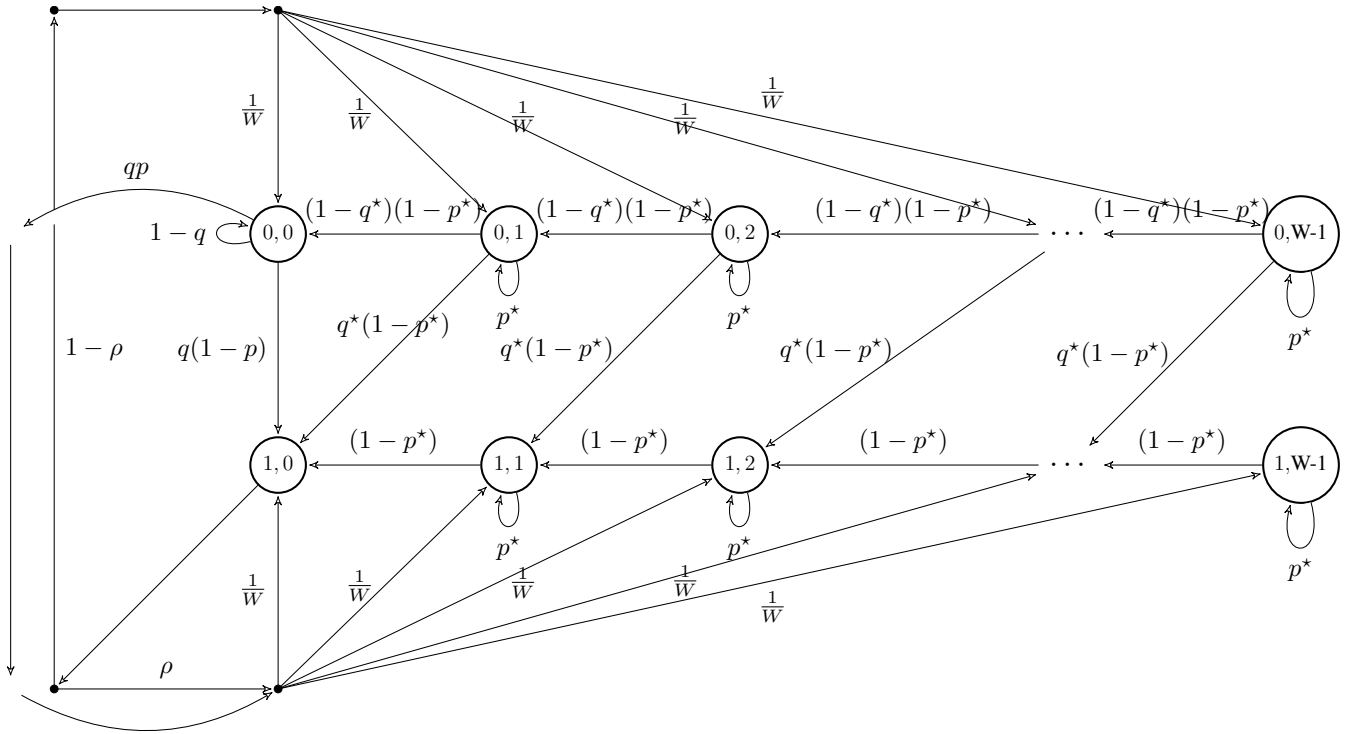
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[1]This modelling technique requires an infinite queue, but since $\frac{1}{\lambda_g} >$ Service Time $\mathbb{E}[S]$, queueing loss will be negligible for a finite queue. Note that it makes sense to drop old beacons if a new arrival finds the queue non-empty since it carries updated information [12], [17]

[2]which could be modelled with simple Bit-Error Rate (BER) equations.

[3]This assumption is quite vital to the concept of Cooperative Awareness – not only does a node need to transmit its own beacons, it must also receive those sent by others.

Fig. 3. The Markov chain of the transmission process of a beaconing node

$$b_{0,k} = \frac{(1-\rho)b_{1,0}}{W(1-p^\star)} \frac{1-(1-q^\star)^{W-k}}{q^\star}; \text{ for } k = 1, \ldots, W-1.$$ (1)

The steady-state probability for a node that has completed PBO and is IDLE is given by:

$$b_{0,0} = \frac{(1-\rho)b_{1,0}}{Wq} \frac{1-(1-q^\star)^W}{q^\star}.$$ (2)

The steady-state probability for a node that is in BO to get access to the medium is given by:

$$b_{1,k} = \frac{(1-\rho)b_{1,0}}{W(1-p^\star)} \left((W-k) \right.$$

$$\left(1 + \frac{\rho}{1-\rho} + \frac{p}{W} \frac{1-(1-q^\star)^W}{q^\star} \right) -$$

$$\left. \frac{1-(1-q^\star)^W}{q^\star} \right); \text{ for } k = 1, \ldots, W-1.$$ (3)

We can now find $b_{1,0}$ by normalising the above three expressions. The result expresses the probability that a node has completed BO and will transmit the frame in a generic slot, with probability τ equal to the probability of being in $s_{1,0}$:

$$\frac{1}{\tau} = 1 + \frac{(W-1)}{2(1-p^\star)} +$$ (4)

$$\frac{1-\rho}{q} \frac{1-(1-q^\star)^W}{Wq^\star} \left(1 + \frac{(W-1)qp}{2(1-p^\star)} \right).$$

Like in [5], the first two terms dominate the saturation case, the second part the non-saturated case. The factor $(1-\rho)$ represents the probability that the queue is empty after a transmission. In this case, the system will enter PBO which is modelled by the geometric sum $\frac{1-(1-q^\star)^W}{Wq^\star}$. This expresses the probability that the queue remains empty during the PBO and we finally reach state $s_{0,0}$ instead of moving to the regular BO procedure that is represented by states $s_{1,0}, \ldots, s_{1,W-1}$.

The last factor in Eq. (4) is quite vital, as it models 802.11's Carrier Sensing in state $s_{0,0}$; whether or not to perform a BO prior to transmission, or directly transmit.

We now have an expression for τ, the probability that a node is transmitting in a generic slot. The system can be solved numerically using fixed-point iteration, solving the non-linear equations for τ and p. In the following, τ is used to derive subsequent measures of interest.

C. Model variables

The expressions for p_b, p, and p_s are based on an independence assumption for the system to be Markovian. Let n be the number of coordinated nodes in the system. The probability that the medium is busy in a generic slot, p_b, is the probability that *at least* one node transmits in a slot:

$$p_b = 1 - (1-\tau)^n.$$ (5)

When a node is sensing the channel, it encounters a busy slot with probability p:

$$p = 1 - (1-\tau)^{n-1}.$$ (6)

The probability that a generic slot contains a successful transmission p_s can be found as follows:

$$p_s = n\tau(1 - \tau)^{n-1}. \tag{7}$$

The probability that a slot contains a collision is given by $p_c = p_b - p_s$, and the probability that a generic slot is an empty slot by $p_e = 1 - p_b$.

Let T_e be the duration of an empty slot, T_s the duration of a successful transmission and T_c the duration of a collision. These durations depend on the MAC parameters (see Table I):

$$T_e = \texttt{aSlotTime}, \tag{8}$$

$$T_s = \texttt{T}_{\texttt{phy}} + \texttt{T}_{\texttt{mac}} + \texttt{b}/\texttt{B} + \texttt{T}_{\texttt{prop}} + \texttt{DIFS}, \tag{9}$$

$$T_c = \texttt{T}_{\texttt{phy}} + \texttt{T}_{\texttt{mac}} + \texttt{b}/\texttt{B} + \texttt{T}_{\texttt{prop}} + \texttt{EIFS}. \tag{10}$$

T_s and T_c consist of a signal part and a guard space part. The former depends on physical- and MAC-layer headers and the payload b which is transmitted at data rate B. The latter is the Inter Frame Spacing (IFS), which is defined by the standard to separate contiguous signals on the channel. The Extended IFS (EIFS) is defined as a courtesy feature towards Unicast transmissions: the intended receiver must have an opportunity to return an Acknowledgment frame, even if other nodes were not able to successfully decode the frame. EIFS is defined as $\texttt{SIFS} + \texttt{T}_{\texttt{phy}} + \texttt{T}_{\texttt{ack}} + \texttt{DIFS}$. The EIFS is used after an *observed* collision; this excludes nodes whose transmission is part of the collision. As a result, not all nodes use the EIFS in response to a collision. This behaviour is not reflected in the model and may introduce minor inaccuracy.

Now we can express the duration of the mean busy slot as:

$$T_b = \frac{p_s}{p_b}T_s + \left(1 - \frac{p_s}{p_b}\right)T_c, \tag{11}$$

and the mean slot duration as:

$$\mathbb{E}[T] = p_e T_e + p_s T_s + p_c T_c. \tag{12}$$

D. Modelling Packet Arrivals

A generic slot in the model is composed of empty and busy slots. The busy slots are divided into successful and collision slots, each with their effective duration (see Eqs. (8)—(10)). The probability of receiving an arrival depends on the type (and hence duration) of a slot in which an arrival can occur.

1) Arrivals while in IDLE: When a node is IDLE, the probability that a packet arrival occurs in a generic slot, q, is given by a weighted Poisson arrival process:

$$q = 1 - \left(p_s^\star e^{-\lambda_g T_s} + (1 - p_b^\star)e^{-\lambda_g T_e} + (p_b^\star - p_s^\star)e^{-\lambda_g T_c}\right), \tag{13}$$

where $p_s^\star = (n-1)\tau(1-\tau)^{n-2}$ is the probability that this node observes a slot containing a successful transmission and $p_b^\star = 1 - (1-\tau)^{n-1}$. The similarity with p is no coincidence, as both model a node observing a system of $n-1$ other nodes. The difference with [5] is that Eq. (13) considers a system of $n-1$ nodes, whereas the expression for q in [5] considers n nodes. Our expression in Eq. (13) is more accurate because it models arrivals based on the state of the $n-1$ other nodes, given that the node under consideration is IDLE.

Parameter	Value	Parameter	Value
data rate B	3Mbit[4]	payload b	3200 bits[5]
aSlotTime σ	16μs[6]	DIFS	64μs
CW	16	$\texttt{T}_{\texttt{phy}}$	40μs
$\texttt{T}_{\texttt{mac}}$	53μs[7]	$\texttt{T}_{\texttt{ack}}$	112μs

TABLE I
PARAMETERS USED IN THE EXPERIMENTS

2) Arrivals during a busy slot: The probability of getting an arrival during a busy slot is given by:

$$q_b = 1 - \left(\frac{p_s}{p_b}e^{-\lambda_g T_s} + \left(1 - \frac{p_s}{p_b}\right)e^{-\lambda_g T_c}\right). \tag{14}$$

3) Arrivals while in PBO: The probability $q^\star$ of receiving an arrival while in a PBO state can be derived by dividing the probability that there is no arrival during an empty slot by the probability that there is no arrival during a busy slot, in [10]:

$$q^\star = 1 - \frac{(1 - p^\star)e^{-\lambda_g T_e}}{1 - p^\star\left(\frac{p_s}{p_b}e^{-\lambda_g T_s} + \left(1 - \frac{p_s}{p_b}\right)e^{-\lambda_g T_c}\right)}. \tag{15}$$

The $p^\star$ present in both numerator and denominator already relate to a system of $n-1$ nodes. Like in [5], $p^\star$ is the probability of observing the channel busy during (P)BO. We do not model the AIFS differentiation because of the single-AC assumption. However, the expression provided in [5], $p^\star = p$ without AIFS differentiation, does not yield proper results for large n. The reason for this is that transmissions occur in Streaks with increasing number of nodes, and are not independent of each other. A better approximation based on the Streak Length is needed.

While in $s_{j,k}$ for $k = 1 \dots W - 1$ the progress towards $k - 1$ could be seen as a series of Bernoulli trials, where success means to decrement the bc and failure to self-loop. In Eq. (25) we find $\frac{p}{1-p'}$ to be a good estimator of the Streak Length. Let $p^\star$ be the probability of staying in $s_{j,k}$. Then it should be similarly distributed:

$$\frac{p\star}{1 - p\star} = \frac{p}{1 - p'} \Rightarrow \qquad p^\star = \frac{p}{(1 - p') + p} \tag{16}$$

E. Service Time

We can now derive an expression for the service time of the medium access server. This is the sum of the time it takes to transmit a frame (including the IFS) and the time spent in contention. Recall that whether or not to perform BO depends on the state of the channel.

The probability that a slot is busy is expressed as p, see Eq. (6). However, given that arrivals can happen at any moment in time (and not only on slot boundaries) we need to find the observed real-time medium busy fraction (MBF) by

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[4]Default bitrate is 6Mbit, we use 3Mbit for greater robustness.

[5]400 bytes are sufficient for an EIVP CAM plus security fields [19].

[6]duration of aAirPropagation is assumed 4μs to account for the increased transmission range, yielding σ=16μs opposed to default of 13μs.

[7]duration of 160 bits at 3Mbit.

multiplying the probability of encountering a busy slot with the duration of such a slot, and divide by the duration of a generic slot:

$$\text{MBF} = \frac{pT_b}{\mathbb{E}[T]}. \tag{17}$$

The expected service time ($\mathbb{E}[S]$, Eq. (18)) can be obtained as follows. Transmission of a message, including the IFS, has a duration of T_b. A station has to perform BO with probability MBF. In this case, the event which caused it to do BO has a mean remaining duration of $\frac{T_b}{2}$. The station has to count down empty slots, and for every bc decrement there is a probability of finding the medium busy, freezing the bc. The duration of this freeze is the expected Streak Length $\mathbb{E}[L]$, see Eq. (25).

$$\mathbb{E}[S] = T_b + \text{MBF}\left(\frac{T_b}{2} + \frac{(W-1)}{2}\big(T_e + T_b\mathbb{E}[L]\big)\right). \tag{18}$$

Then Little's Law can be used to obtain ρ:

$$\rho = \lambda_g\mathbb{E}[S]. \tag{19}$$

[5] considers the PBO as part of the service time, a correction to ρ is required for the non-saturated regime. Because of our definition of $\mathbb{E}[S]$ we can directly use Little's Law.

1) Collision Multiplicity: The CM expresses the number of nodes involved in a collision, because they are simultaneously transmitting in the same slot.

Observing $n-1$ other nodes from a certain node, we express the mean CM in slot s of a given Streak as:

$$\mathbb{E}[\text{CM}_s] = \sum_{i=1}^{n-1} i \cdot \binom{n-1}{i}\tau_s^i(1-\tau_s)^{n-1-i} \cdot \frac{1}{1-(1-\tau_s)^{n-1}}$$
$$= \frac{(n-1)\tau_s}{1-(1-\tau_s)^{n-1}}. \tag{20}$$

The fraction in Eq. (20) ensures that we only consider busy slots. Since $\mathbb{E}[\text{CM}_1]$ relates to the first slot in a Streak, the previous slot was empty. We obtain a special τ for the first slot in a Streak, which we indicate as τ_1. This is the probability that a slot is busy, given that the previous slot was empty. By dividing all paths leading to $s_{1,0}$ by the probability of *not* transmitting, we obtain:

$$\tau_1 = \frac{b_{1,1} + b_{0,1}q^\star + b_{0,0}q}{1 - b_{1,0}}. \tag{21}$$

2) Streak Length: In [5] and [10], the share of slots where the countdown process is being blocked is expressed as $\frac{p}{1-p}$ under the assumption that a transmission is independent of previous or subsequent transmissions.

Simulation results indicate that the assumption of independence does not hold for large values of n, where the geometric distribution shows a particularly heavy tail compared to findings in the simulator. This is because of an increase in CM: a very large number of collisions occurs in the first slot on a Streak, causing many nodes to perform their service in parallel

and lowering the effective load in the rest of the slots. This effect is not visible in [5], [10], and [9] because these works do not consider n large enough to show this effect, nor is it present in the models in [11] and [12].

The denominator of the share of slots where bc blocking occurs in [5] cannot be found as the mean of the Geometric distribution because of its dependency on what happened in the preceding slot.

Hence, let p' be the probability that a slot is observed busy, given that the previous slot was busy, then this situation refers to the second or later slots in a Streak (see also Fig. 1). We express p' as 1 minus the probability that no nodes wil transmit in this situation:

$$p' = 1 - (1 - \Psi_{TX})(1 - \Psi_{IDLE}). \tag{22}$$

In these slots, a transmission can come from two groups of nodes: $\mathbb{G}_{TX}$ and $\mathbb{G}_{IDLE}$ as described in Sec. III. The probability that at least one node from these groups will transmit can be obtained as follows:

$$\Psi_{TX} = \mathbb{E}[CM_1]\rho\frac{1}{W} \tag{23}$$
$$\Psi_{IDLE} = (n-1)b_{0,0}\frac{q_b}{W}. \tag{24}$$

In Eq. (23) we multiply the number of nodes which have performed a transmission in slot 1 with the probability of finding the queue non-empty, and then choosing $bc = 0$. In Eq. (24) we do the same: multiply the number of nodes currently IDLE with the probability of receiving an arrival during a busy slot, *and* choosing $bc = 0$. We define the number of consecutive blocked slots as the mean Streak Length:

$$\mathbb{E}[L] = \frac{p}{1-p'}. \tag{25}$$

V. VALIDATION

The analytical results obtained from the DTMC model are compared to simulation experiments performed using OMNeT++[8] 4.2.2 and a modified version of MiXiM[9] 2.1 Mac80211 to comply with the 802.11p DCF [15]. The parameters are equivalent to those in Table I. To achieve valid comparison, a Unit Disc propagation model was designed and all nodes are in each other's range to adhere to the same assumptions of the analytical model and isolate MAC-layer behaviour. Using the method of independent replications, ten simulation runs per datapoint were performed. After a warmup period sufficiently long to reach steady-state behaviour the experiments simulate 250s of beaconing. The results have small 95% confidence intervals, as displayed in the graphs.

A. Medium Busy Fraction

The MBF in Eq. (17) is compared to simulation results in Fig. 4 for varied n, evaluating non-saturation to saturation. A busy slot in the model consists of a signal part and an IFS part, plotted as the top line in Fig. 4. In the simulator, the fraction
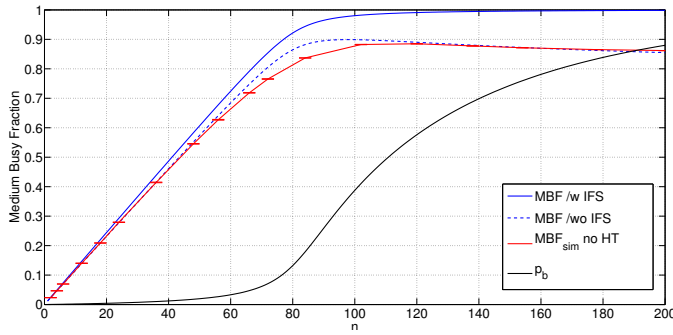
[8]http://www.omnetpp.org [9]http://mixim.sourceforge.net

Fig. 4. Medium Busy Fraction: Analysis and simulation

Fig. 5. Service Time, analysis and simulation

of time there was signal on the medium was measured, i.e., without the IFS. By subtracting the IFS part from the MBF results, we derive a comparable analytical measure, which matches reasonably well with simulation results.

Note the inaccuracy in the results in the semi-saturation part between n=60 and 100. This was also observed in [5] and [10] (albeit for different values for n because the generated load differs). The numerical solution has difficulty converging in this area, causing the number of iterations for convergence to peak. This inaccuracy is present in all metrics derived from the model. We verified that the fixed-point iteration converges indeed to the single fixed point, hence the inaccuracy stems from the model and not from the numerical solution method. The inaccuracy is present in the area where the variance of $\mathbb{E}[T]$ is largest, leading to the conclusion that the inaccuracy is introduced by the mapping of time slots to generic slots in order for the system to be Markovian.

From Fig. 4 it becomes clear that roughly 10% of the available channel resources are used by the IFS. Since a beacon channel consists solely of Broadcast transmissions, it would improve performance to remove the concept of EIFS in this case. This feature makes transmissions more Unicast-friendly, but is a waste of channel resources in a pure broadcast system. The waste is exacerbated by the small packet size and the relatively large IFS.

B. Service Time and Delay

Vinel *et al.* [12] reported that delay requirements of ITS applications are more easily met than the reliability require-ments. A simulation study in our earlier work confirmed this [20]. Fig. 5 shows that the expected Service Time $\mathbb{E}[S]$ in Eq. (18) and the measured service time in the simulator match reasonably well, except for the inaccuracy in semi-saturation.

The average end-to-end delay in the simulator is measured only for successfully received frames. It is plotted as the dashed line in Fig. 5. The average end-to-end delay actually declines with the number of nodes increasing beyond the point where the channel is saturated, as also observed in [17]. The explanation for this is that the first slot in a Streak has a high average CM. Subsequent slots in a Streak all have lower CM, because of the smaller probability that a node will transmit in these subsequent slots. Furthermore, transmissions in the first slot in a Streak often go through contention, whereas
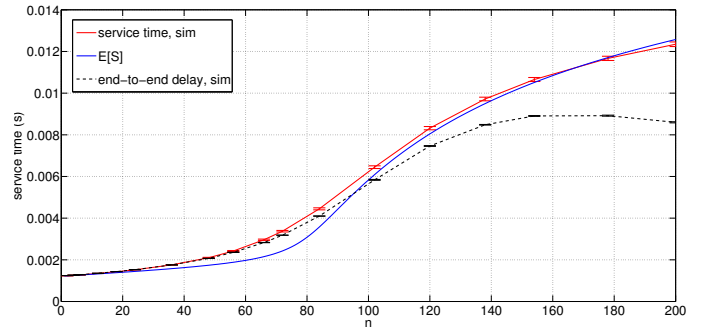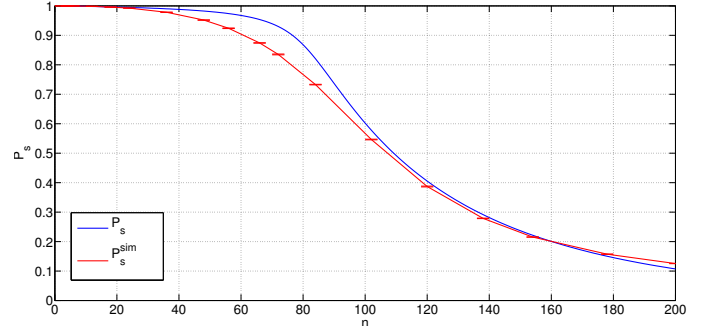

Fig. 6. Reception Probability: Analysis and simulation

transmissions in later slots in a Streak incur little delay because they are directly transmitted as soon as the medium turns idle (without intermediate T_e). The result is a lower mean delay of successfully received information with increasing n. The resulting conclusion is that the end-to-end delay of received information cannot be directly derived from the service time, because the different CMs in a Streak provide a bias. It is possible to derive an estimate for the end-to-end delay based on CM and the Streak concept. This is left as future work.

Another important observation from Fig. 5 is that the system will maintain progress and does not contend for channel access indefinitely because $\mathbb{E}[S]$ does not go to infinity. Eventually a Streak will be over and many nodes will perform their service in parallel (causing a collision but safeguarding the progress).

C. Reception Probability

The successful reception rate is an important performance metric in a VANET. We can express the probability that no other node transmits, given that this node transmits, as:

$$P_s = (1 - \tau)^{n-1} \tag{26}$$

Fig. 6 shows the typical CSMA/CA curve which remains high with increasing number of nodes (resulting in increasing load on the channel, almost linear growth in Fig. 4), then starts to drop as the channel reaches saturation and collisions become more prominent. Here too, the inaccuracy in the semi-saturated area is visible.

D. Throughput

A classic metric in IEEE 802.11 modelling is throughput. The throughput, in received beacons per second, can be
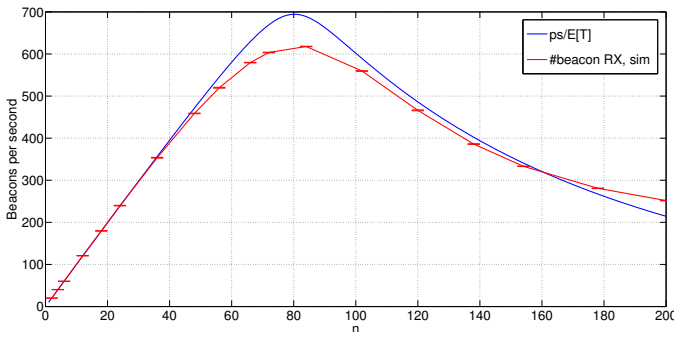
Fig. 7. Throughput, analysis and simulation

derived as follows:

$$X = \frac{p_s}{\mathbb{E}[T]}. \tag{27}$$

X is plotted in Fig. 7 against the average number of beacons received per second, as measured in the simulator.

Although (just like the throughput plots in [5] and [10]) the inaccuracy in the semi-saturated area is clearly visible, an important observation is that the peaks occur at the same value of n (albeit at different magnitude). This indicates that the point where the channel becomes saturated is accurately modelled. This is a useful input for the design of adaptive congestion control mechanisms; since beyond this point performance degrades.

VI. CONCLUSIONS

In a VANET, the large number of nodes and the low supplied load per node result in a traffic regime for which the assumption of independence among transmissions does not provide good estimates of the duration of bc blocking. In this paper, a model is presented to provide a better estimate of DCF performance using the concepts of Collision Multiplicity and Streak Length. Furthermore, we find that whether a transmission is performed in the first or subsequent slots in a Streak impacts its reception probability and delay.

Analytical results are obtained from the DTMC and validated against simulations. We observe that the probability of successful reception and service time match well. Additionally, the saturation point of the channel can accurately be predicted, which is an important input in the development of congestion control mechanisms.

Future work includes a comparison between the DCF and EDCA for beaconing in IEEE 802.11p VANETs. The bc decrement rules of the EDCA are expected to affect the Streak Length and Collision Multiplicity behaviour.

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