X International Conference on Computational Plasticity
COMPLAS X
E. Oñate and D.R.J. Owen (Eds)

ON THE USE OF LOCAL MAX-ENT SHAPE FUNCTIONS FOR THE SIMULATION OF FORMING PROCESSES

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Key words: local max-ent, meshless methods, elasto-plasticity, metal-forming

Summary. In this work we review the opportunities given by the use of local maximum-entropy approximants (LME) for the simulation of forming processes. This approximation can be considered as a meshless approximation scheme, and thus presents some appealing features for the numerical simulation of forming processes in a Galerkin framework.

Especially the behavior of these shape functions at the boundary is interesting. At nodes on the boundary, the functions possess a weak Kronecker-delta property, hence simplifying the prescription of boundary conditions. Shape functions at the boundary do not overlap internal nodes, nor do internal shape functions overlap nodes at the boundary. Boundary integrals can be computed easily and efficiently compared to for instance moving least-squares approximations. Furthermore, LME shapes also present a controllable degree of smoothness.

To test the performance of the LME shapes, an elastic and a elasto-plastic problem was analyzed. The results were compared with a meshless method based on a moving least-squares approximation.

1 INTRODUCTION

The main component of a meshless or finite element method is a set of shape functions to parameterize the displacement field. This can be represented as:

$$\mathbf{u}_{\mathrm{h}}(\mathbf{x}) = \sum_{i=1}^{N} \phi_i(\mathbf{x})\mathbf{u}_i \tag{1}$$

where N is the number of nodes. Two essential properties of the shape functions $\boldsymbol{\phi}$ are a partition of unity:

$$\sum_{i=1}^{N} \phi_i(\mathbf{x}) = 1 \tag{2}$$

and linear reproducibility:

$$\sum_{i=1}^{N} \phi_i(\mathbf{x})\mathbf{x}_i = \mathbf{x} \tag{3}$$

Local maximum-entropy shape functions were introduced by Arroyo and Ortiz [1]. The essence is that the shape functions are constructed by using information-theoretic principles. A potential containing both an entropy term, as well as a potential expressing the locality, is minimized with respect to the shapes functions $\boldsymbol{\phi}$. This potential is stated as:

$$\Pi_{\text{lme}} = \beta U(\mathbf{x}, \boldsymbol{\phi}) - H(\boldsymbol{\phi}) \tag{4}$$

with the Shannon entropy H defined as:

$$H(\boldsymbol{\phi}) = \sum_{i=1}^{N} \phi_i \ln(\phi_i) \tag{5}$$

and the locality:

$$U(\mathbf{x}, \boldsymbol{\phi}) = \sum_{i=1}^{N} \phi_i \|\mathbf{x} - \mathbf{x}_i\|^2 \tag{6}$$

Since both goals H and U are conflicting, a parameter β was introduced to control the weight of one goal relative to the other. By setting this parameter either local or diffuse shape functions can be obtained. The minimization of Equation (4) is constrained by Equations (2) and (3).

2 RESULTS

2.1 ELASTICITY

In the following test, the numerical integration of the LME functions is investigated. Finding the local maximum-entropy shape function at a location requires an iterative procedure. The residual at which to stop the iterations has to be chosen. A test is performed to investigate the influence of this residual on the integration scheme. A patch test with a prescribed constant traction is chosen as problem. A Delaunay triangulation is made of the cloud of nodes. In each triangle an integration rule is defined. A moving least-squares (MLS) shape function is included in the analysis for comparison. Figure 1 shows the results, with on the horizontal axis the number of integration points in a triangle and on the vertical axis the error e_{u} which is defined as:

$$e_{\text{u}} = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \|\mathbf{u}_{\text{h}}(\mathbf{x}_i) - \mathbf{u}_{\text{exact}}(\mathbf{x}_i)\|^2} \tag{7}$$

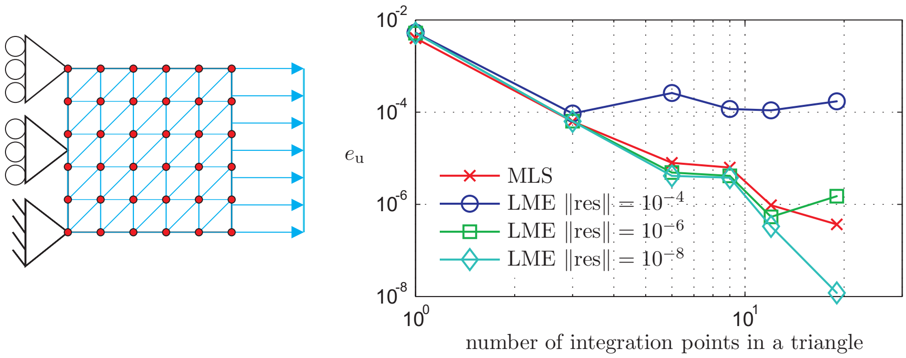


Figure 1: model and results at the patch test for the two shapes

It can be seen that for a LME shape with a tolerance of 10^{-4} results are not improving by adding more than three integration points. For a LME shape with a tolerance of 10^{-6}, twelve integration points are sufficient. The integration rule chosen, should be in correspondence with the tolerance of the LME shape function.

The times required to build the stiffness matrix for a MLS shape and a LME shape with a tolerance of 10^{-4} are given in Table 1. Though these numbers are depending on the computer used and the implementation, it is showing that MLS and MLE shapes have approximately the same computational price.

ϕ	MLS	LME
time (sec.)	6.65	6.71

Table 1: computational times for building the stiffness matrix

2.2 PLASTICITY

For this section an elasto-plastic material model is used with a Von-Mises yield surface and a power-law hardening rule. In order to circumvent the problem of volumetric locking, the stabilized conforming nodal integration (SCNI) scheme as proposed by Chen *et al.* is used [2]. As a problem we take a hinge and bend it plastically. Figure 2 shows the geometry. The concept of α-shapes provides us a correct tesselation of the non-convex region. Again the two shape functions, MLS and LME, are used. The domain of influence is approximately similar for both shapes. Figure 3 shows the stress σ_{xx} for the two different simulations. It can be seen that the results with the LME shapes are very similar to the MLS results. One of the main benefits now of the LME shapes is that the displacement at left-hand side of the hinge can be prescribed directly without resorting to penalty methods or Lagrangian multipliers. Similarly, the prescription of the traction on the right-hand side, requires only a neighbor search on the boundary and not in the domain.

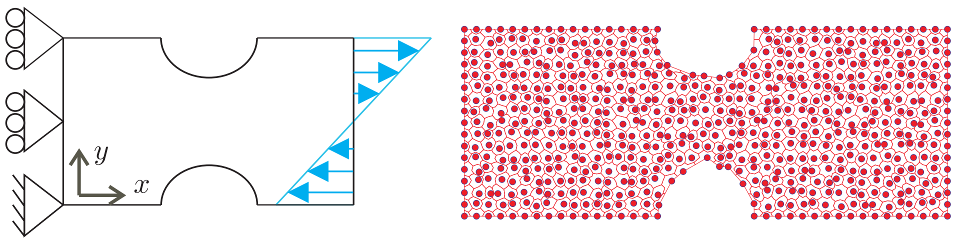
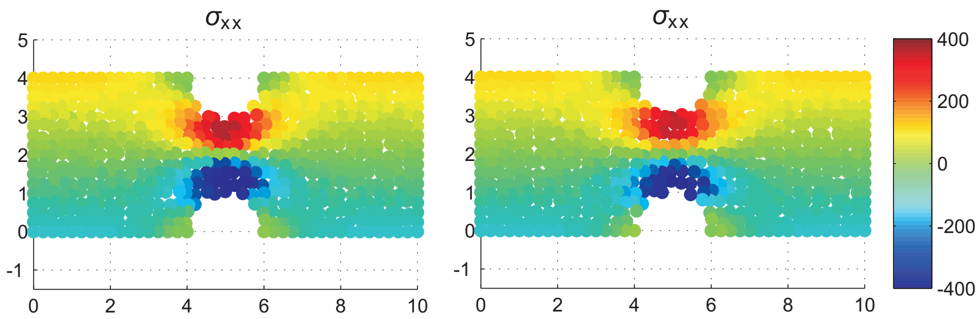


Figure 2: geometry and model of the hinge



(a) MLS

(b) LME

Figure 3: stress plots after plastic deformation for the two shape functions

3 CONCLUSIONS

Good results were obtained with the LME approximation in plasticity. The performance of the approximation is similar to that of MLS though on the boundary the LME functions are proving to be more convenient. Currently, a higher-order max-ent scheme is being developed by the authors.

REFERENCES

[1] M. Arroyo and M. Ortiz. Local maximum-entropy approximation schemes: a seamless bridge between finite elements and meshfree methods. *Int. J. Num. Meth. Engng.*, **65**, 2167–2202, (2006).

[2] J.S. Chen, C.T. Wux, S. Yoon and Y. You. A stabilized conforming nodal integration for Galerkin mesh-free methods. *Int. J. Num. Meth. Engng.*, **50**, 435–466, (2001).