

# ON A NOVEL APPROACH FOR OPTIMIZING COMPOSITE MATERIAL PANELS USING SURROGATE MODELS

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## Abstract

This paper describes an optimization procedure to design thermoplastic composite panels under axial compressive load conditions. Minimum weight is the goal. The panel design is subject to buckling constraints. The presence of the bending-twisting coupling and of particular boundary conditions does not allow an analytical solution for the critical buckling load. Surrogate models are used to approximate the buckling response of the plate in a fast and reliable way. Therefore, two surrogate models are compared to study their effectiveness in composite optimization. The first one is a *linear approximation* based on the buckling constitutive equation. The second consists in the application of the *Kriging* surrogate. Constraints given from practical blending rules are also introduced in the optimization. Discrete values of ply thicknesses is a requirement. An *ad-hoc* discrete optimization strategy is developed, which enables to handle discrete variables.

**Keywords:** thermoplastic composite panel; buckling; surrogate modeling; discrete optimization; continuous optimization.

## Introduction

One of the major challenges in the design of aeronautical structures is to reduce the total weight. Now a days, new robotized technologies to obtain thermoplastic structures are used. Tape placement is considered one of the most promising technique. It consists in placing continuous strips of unidirectional thermoplastic composite and consolidate them *in situ*. This technique increases the freedom of design in terms of ply orientation. On the other hand, it requires handling discrete sets of ply thicknesses. The high number of variables involved and the complex mechanics associated to composites, makes the optimum design difficult to achieve. Most of the time, the design of a system brings the necessity of exploring a broad design space. Often, the designer takes a decision based on his intuition and his experience and he follows conventional practical blending requirements (so called “*rules of thumb*”). To guide the designer in the solution, *structural optimization* [1] is the most promising method adopted in solving practical design problems. Still many computer simulations and experiments are necessary to validate the optimum design. The use of approximate models of the phenomenon, known also as *surrogates*, in combination with structural optimization, becomes a fast and powerful tool in the hands of the designer to improve the optimum search. The idea behind it is to create a response surface that represents the approximation of the behavior of the system, using a reasonable number of data collected from experiments or simulations, as explained in [2]. Various types of surrogates are available, such as polynomial regression, radial basis function, Kriging, neural networks, etc [3]. Particularly, this paper focuses on application of *polynomial regression* and *Kriging* surrogates.

Most optimization methods deal with continuous variables. In this paper, the authors propose a discrete variable optimization approach in the optimum setting selection, which becomes essential for obtaining a feasible optimum design from the engineering point of view. Optimization problems involving both continuous and discrete design variables are known in the literature as *Mixed Integer Non-Linear Programming (MINLP)* problems, [4].

## Problem description

An optimization strategy is developed for the design of symmetric composite panels under axial compressive load, as shown in Figure 1. Minimum weight configuration is the goal. The panel is subject to buckling constraints. Thickness and orientation of any single ply are the design variables. The panel weight can be calculated as follows:

$$W(X) = W([t_1, \dots, t_i, \dots, t_{n/2}, \theta_1, \dots, \theta_i, \dots, \theta_{n/2}]) = \rho ab 2 \sum_{i=1}^{n/2} t_i \quad (1)$$

where  $n$  is the total number of layers;  $X = [t_1, \dots, t_i, \dots, t_{n/2}, \theta_1, \dots, \theta_i, \dots, \theta_{n/2}]$  is the vector of design variables that contains the information of every  $i$ -th layer in term of thickness  $t_i$  and angle  $\theta_i$ ;  $\rho$  is the material density;  $a$  is the panel width ( $a = 135\text{mm}$ ) and  $b$  is the panel length ( $b = 450\text{mm}$ ), respectively. The thickness of any single ply has only discrete values, which are multiple of  $0.138\text{ mm}$ .

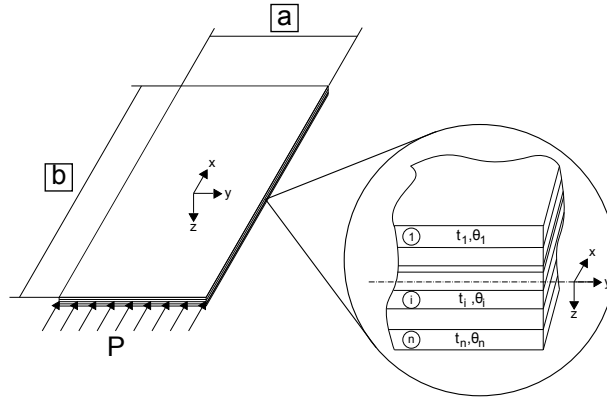


Figure 1: Composite panel subject to axial compression

The optimization problem is defined as follows:

$$\begin{array}{ll} \text{minimize} & W(X) \\ \text{subject to} & \begin{cases} P(X) \geq P_{CR} \\ X_L \leq X \leq X_U \\ \text{ply}_{nr}(\bar{\theta}) \leq 3 \end{cases} \end{array} \quad (2)$$

where  $P(X)$  corresponds to the actual critical load;  $P_{CR}$  is the required value of the buckling load;  $X_L$  and  $X_U$  are the lower and the upper bounds for the  $X$  vector, respectively. The last constraint in Equation (2) is introduced due to practical blending rules. It indicates that the allowable number of contiguous plies with the same angle  $\bar{\theta}$  is at most 3.

The panel is modeled in ANSYS [5], by 8-node SHELL99 elements with six degrees of freedom at every node. Boundary conditions are applied as follows:  $u_x$ ,  $u_z$  and  $rot_y$  are suppressed at the non-loaded transversal edge;  $u_z$  is set equal to zero at the loaded edge;  $u_y$ ,  $u_z$  and  $rot_x$  are fixed on one of the longitudinal edges;  $u_z$  and  $rot_x$  on the other one. Here,  $u$  is referred to displacement,  $rot$  to rotation, and the subscripts  $x$ ,  $y$  and  $z$  to the principal coordinates system. The selected material is a PEKK/CF composite with the following properties: Longitudinal Elastic Modulus  $E_1 = 134\text{GPa}$ ; Tangential Elastic Modulus  $E_2 = 9\text{GPa}$ ; Shear Modulus  $G_{12} = 5.3\text{GPa}$ ; Poisson coefficient  $\nu = 0.3$ ; material density  $\rho = 1600\text{kg/m}^3$ .

## Buckling Approximate Model

In composite panel design, when the structure is subject to an axial compressive load, the determination of the critical buckling load becomes essential. The buckling could cause a premature failure of the whole structure. In the *Classical Laminate Plate Theory (CLPT)* [6], the governing equation for buckling of

symmetrically laminated plates subject to an axial load is:

$$-P_x \frac{\partial^2 w}{\partial x^2} = D_{11} \frac{\partial^4 w}{\partial x^4} + 4D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26} \frac{\partial^4 w}{\partial x \partial y^3} + D_{22} \frac{\partial^4 w}{\partial y^4} \quad (3)$$

where  $w$  corresponds to the deflection along the  $z$ -axis and  $P_x$  is the in-plane load in the  $x$  direction. To evaluate the buckling load  $P$  for panels with different layouts, a surrogate model is used. The buckling load  $P$  depends on the stacking sequence. If the design space is described in terms of ply thickness and orientation,  $P(X)$  has high dimension. The dimension increases with the number of plies utilized. Fitting a surrogate model of the buckling constraint requires then a high number of data. Moreover,  $P(X)$  shows a highly non-linear dependance on  $\theta$ . From the *Classical Lamination Theory (CLT)* [7], given the stacking sequence, it is straightforward to calculate the *ABD matrix* values, while it is not easy the opposite. The  $D$  terms can be used to fit the surrogate  $P(D)$ . That is because the dimension of  $P(D)$  is fixed to 6, independently from the number of plies utilized. Moreover, the *ABD* components are linear combinations of the convex lamination parameters [8]. Therefore, the behavior of  $P$  as function of  $D$  is very smooth.

In this work the selected surrogate models are *linear regression* and *kriging*. The first choice comes from a theoretical approach. As observed in Equation (3), the value of the critical buckling load is a linear function of the  $D$  terms. This is valid only if the buckling mode-shape does not change, as plotted in Figure 2.

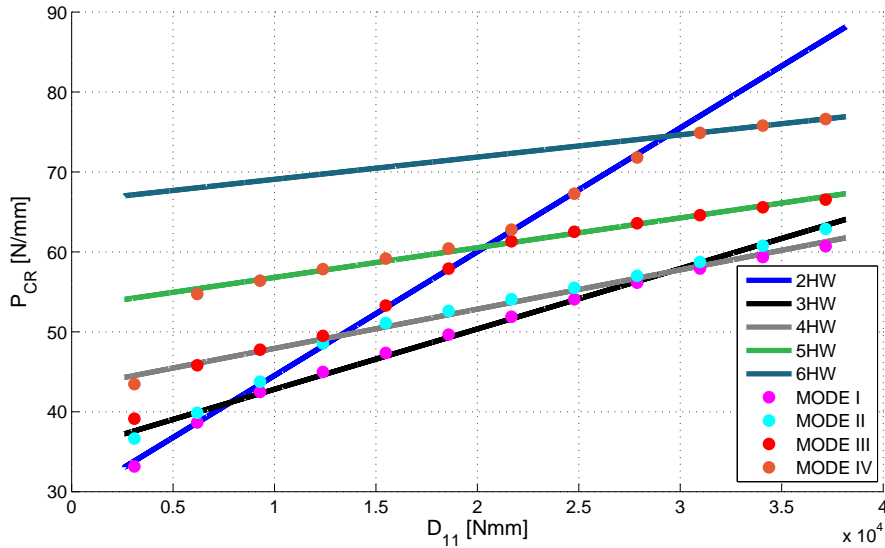


Figure 2: Influence of the  $D_{11}$  term on the critical buckling load. In the legend,  $x$ HW stands for the mode shape which has  $x$  half-waves; MODE  $X$  represents the points that belong to the response of the  $X$  mode

To evaluate the mode-shapes and to group them, the *Model Assurance Criterion (MAC)* [9] is used. The *MAC* is generally applied in the area of experimental and analytical structural dynamics to evaluate the correlation between two mode-shapes. In this case, we consider it to classify the buckling mode-shapes. The *MAC* is a scalar parameter having a value between 0 and 1. If its value is 0, the mode-shapes are not consistent; if it is 1, the response of the panels to the axial load is exactly the same. The *MAC* is defined as:

$$MAC = \frac{([\varphi(u_z, rot_x, rot_y)]^T [\psi(u_z, rot_x, rot_y)])^2}{([\varphi(u_z, rot_x, rot_y)]^T [\varphi(u_z, rot_x, rot_y)])([\psi(u_z, rot_x, rot_y)]^T [\psi(u_z, rot_x, rot_y)])} \quad (4)$$

In Equation (4), the vectors  $\varphi$  and  $\psi$  contain the information related to the displacements in the  $z$  direction and the rotations around the  $x$ - and  $y$ -axis. The total number of vectors that must be compared is  $N \times N_{BM}$ , where  $N$  is the total number of samplings and  $N_{BM}$  is the number of buckling modes used to generate the surrogate model. After generating a  $[N \times N_{BM}] \times [N \times N_{BM}]$  *MAC* matrix, all the vector combinations that

show a  $MAC$  value higher than the lower bound  $MAC_{LIMIT}$  are grouped together. Since the dimension of the  $D$ -space is 6, at least 7 points are necessary to define a *linear regression*. Therefore, all the groups that contain less than 7 points are discarded. The *linear prediction* is then computed as:

$$\hat{y}_i(X^*) = \Phi(X^*)c_i \quad (5)$$

where  $\hat{y}_i$  represents the linear prediction for  $P_{CR}$  at the point  $X^*$  for the  $i$ -th group;  $\Phi$  is the design matrix that contains the sampling points informations;  $c$  is the vector of the regression coefficients, which is computed using an *Ordinary Least Squares (OLS)* estimation of the sampling points.

The second utilized surrogate technique is *Kriging*, [2]. It is a stochastic approximation based on the correlation between the points that corresponds to the design space. A *Kriging prediction* for an unknown point of the domain has the form of:

$$\hat{y}(X^*) = F(X^*) + Z(X^*) \quad (6)$$

The left hand side represents the prediction of the *Kriging* surrogate at the point  $X^*$ . For the right hand side, it is possible to distinguish two contributions: the first part,  $F$ , represents the global trend of the surrogate, while  $Z$  describes the local deviations from the trend. Two types of *Kriging* models are utilized: *Ordinary Kriging (OK)* and *Universal Kriging (UK)*. If the prediction is computed using *OK*, the global trend is a constant value. For the case of *UK*, the  $F$  term is a linear function. The *Kriging* theory will not be argument of this work. The interested reader can obtain specific information from [2], [10] and [11].

## Optimization strategy

The design space for the optimum seeking is in the thickness and orientation dimension. However, the evaluation of the buckling constraint is in the  $D$ -space. That assures the generation of a reliable and fast surrogate of the buckling constraints. Moreover, due to design regulations, extension-shear coupling is not allowed. Therefore, the equality constraint  $A_{16} = A_{26} = 0$  is set. This equation permits to obtain an equivalent “*balanced and symmetric*“ panel. However, such equality constraint makes the optimum more difficult to achieve.

Several optimization techniques are reported in the literature. They are coventionally divided into two main groups: *gradient based* and *derivative-free* methods. In design of composites, some variables may have discrete values (e.g. layer thickness, number of layers). Moreover, the function that describes the buckling behavior is not known *a priori*. On the basis of this assumption, a combination of a gradient based method *Sequential Quadratic Programming (SQP)* and the derivative free method *Genetic Algorithm (GA)* is used in this research. A novel optimization strategy is implemented to reach the final solution in a fast and reliable way. Figure 3 shows the necessary steps to obtain the optimum solution. This process requires *pre-optimization*, *optimization* and *post-optimization*. The whole process is defined as optimum *generation*.

In the *pre-optimization* step, the approximation of the unknown constraint function is generated. Firstly, the design is filled with  $N$  sampling points using the *Latin Hypercube Sampling (LHS)* technique applied to the  $D$ -space. This technique consists of generating a population of designs which fills the design space homogeneously [2]. Every single point of the design represents a combination of the chosen variables. These points are necessary to compute the surrogate model of the buckling. In previous works [12], the surrogate was generated as explicit function of the ply thickness and for a small number of layers with fixed orientation. In this paper, the authors use the information given from the *Classical Lamination Theory (CLT)*, to build a surrogate that is not explicitly dependent on the ply parameters,  $t_i$  and  $\theta_i$ . In particular, boundary constraints are defined for ply thicknesses and orientations. Several designs are generated within the boundaries. The information is transferred to the  $D$  matrix level and the best *LHS* is selected in the  $D$ -space. The critical buckling loads that correspond to these sampling points are computed. The two different surrogate models, *linear* and *Kriging*, are generated. These surrogates are then employed in the *optimization* stage.

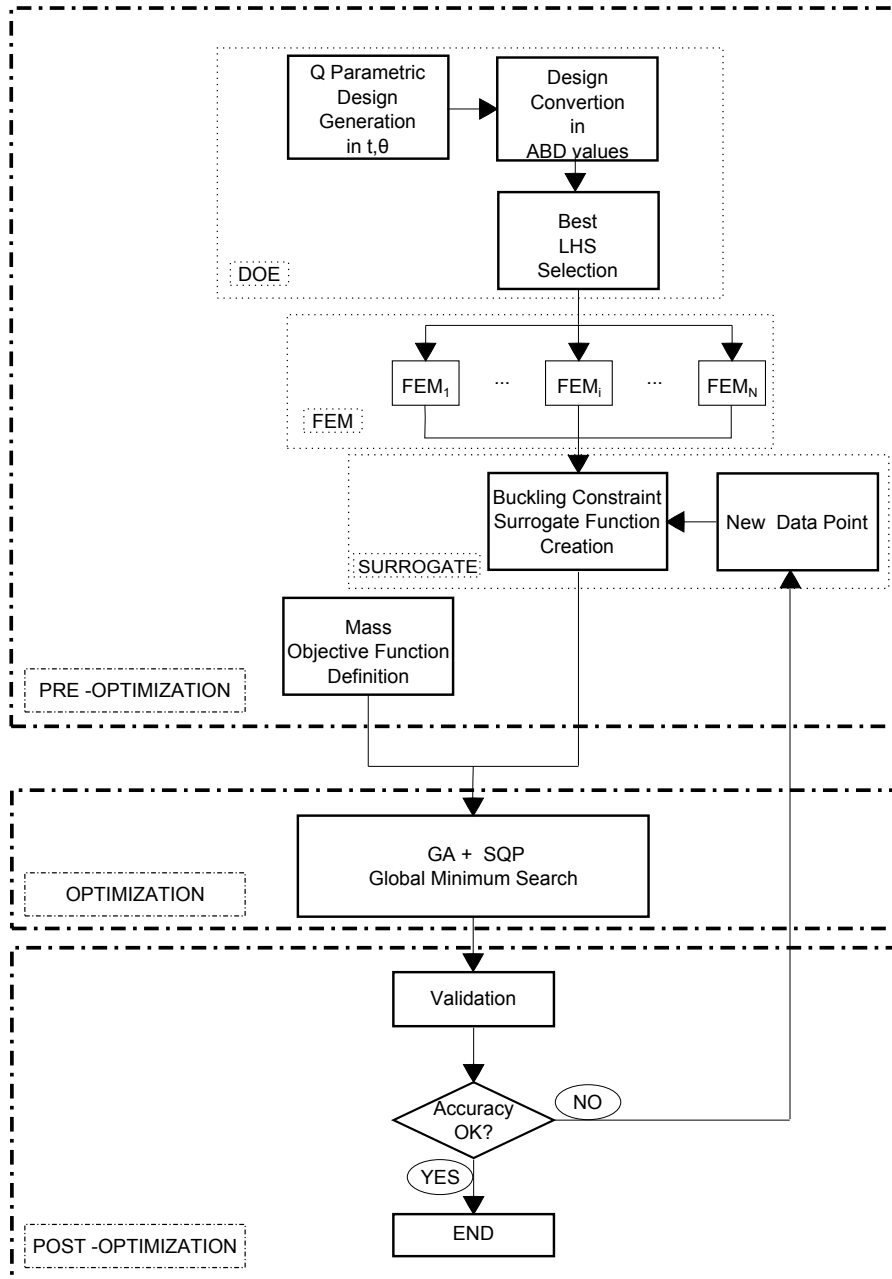


Figure 3: Optimization scheme

In the *optimization* step, the optimum set of variables is obtained. In this research, *SQP* is utilized to find the optimum set of continuous design variables. In this stage, the last constraint in Equation (2) is not considered. For the *linear approximation*, the point is accepted only when the lowest of the load predictions is equal to or higher than  $P_{CR}$ . For *Kriging*, the prediction must be at least equal to  $P_{CR}$ . Since the surrogate models are the approximations of the *FEM* model, the optimum design values computed using them needs to be validated by the *FEM* model. This is done in the *post-optimization* step.

Finally, a *post-optimization* phase is necessary to validate the result. To ensure validation, the objective is compared to previous solutions and the constraint value of the surrogate is compared with the *FEM* result. The optimization ends when, for a defined number of subsequent iterations, the weight change is equal or less than 1% and, contemporaneously, the critical value is higher than  $0.98P_{CR}$ , and when the accuracy error  $e_{PRED}$  in the last generation is less than 2%. Here,  $e_{PRED}$  is the relative error between the *FEM* and the surrogate model result. If these requirements are not satisfied, the result obtained using the surrogate model is listed among the sampling points. A new model for the buckling is generated and the optimization is performed again. To validate the optimum solution, various strategies can be considered. A first possibility consists on the following steps: using the surrogate model, a continuous solution is

found. This solution is validated. Based on the surrogate generated to find the continuous solution and using the continuous solution as starting point, the discrete search is performed. The discrete solution must be then validated. Unfortunately, this approach requires a lot of effort because two *FEM* analyses and a discrete search are involved. However, it provides two more points that can be used to improve the surrogate in the following generation. Another approach can be obtaining a discrete solution and validate it without checking the accuracy of the surrogate model for the continuous result. This approach requires only one *FEM* analysis per generation. The drawback is that the discrete optimum can be irrelevant, due to the possible bad approximation of the continuous response. Finally, we can consider only the continuous result, validate it and check its convergence. The process is repeated until convergence is obtained. After that, the discrete solution is sought and validated. If this solution does not satisfy the requirements, the information is added to the sampling set and the continuous optimization algorithm runs again. The discrete search starts only when the new continuous optimum is validated. The last strategy involves an additional *FEM* validations when compared to the previous one but it assures that the starting point for the discrete search is a verified optimum for the proposed problem. In this research, the last approach is employed.

The *discrete optimization* strategy consists in setting the continuous optimum point as a starting point for the *GA* optimization routine. The whole problem in Equation (2) is now considered. The last constraint is introduced in the problem using a penalty function which is included in the objective. The problem is divided in  $k$  branches, where  $k$  is the number of discrete parameters. In every branch, the  $i$ -th variable is first rounded-down and kept constant while the optimization is performed with  $k - 1$  continuous variables; then, the same variable is rounded-up and an analogue procedure is applied. Two results are available for which the search continues in the most promising branch. Every branch is split into  $k - 1$  new branches. A new optimization is run considering two fixed discrete values and  $k - 2$  continuous variables per branch. The process is repeated  $k$  times. The best configuration is the one that shows the minimum weight value among the possible solutions. Practically, this approach requires high computational effort. Therefore, stopping criteria are considered for not promising branches. In particular, the code generates a first discrete optimum branching one of the possible discrete candidates. This feasible solution is set as reference. All the remaining  $k(k! - 1)$  branches are then individually considered. For every discrete variable step search, if the analyzed branch does not satisfy the constraints or if the objective value is higher than the reference value, the optimum search along that branch ends. Finally, the optimum is found as the configuration with minimum weight that satisfies all the constraint.

## Results

The symmetric composite panel shown in Figure 1 is used as the test problem. The required buckling load is  $P_{CR} = 275.5N/mm$ . In order to generate the surrogates,  $N = 60$  sampling points are selected. Here  $N_{BM}$  is chosen as 4. Only the first buckling mode is used to generate the *Kriging* surrogate, while all the information is used to generate the *linear regression* model. For the convergence criteria, the number of consecutive iterations that do not show improvement is set as 5.

As represented in Figure 4, both the *Kriging* models satisfy the convergency criteria. In particular, the *UK* stops after only 8 generations; the *OK*, finds the optimum solution after 10 loops, still with a reasonable amount of *FEM* evaluations. The linear regression shows a more unstable behavior and the search ends after 18 evaluations. It is important to notice how all the solutions converge to a final weight close to 0.27kg in the continuous optimization. However, this does not correspond to a unique set of variables because the complex mechanics of composite materials permits multiple sets of angles to show the same behavior. Despite that, this approach shows the advantage of analyzing the whole design space and it gives more freedom to the angle selection. It must be emphasized that, for the *linear approximation*, the discrete solution results does not satisfy the buckling requirements. Therefore, the linear surrogate is updated with the information given from the discrete response and the whole optimization process is repeated. The second optimization terminates after 2 generations. In the end, the total number of generations for the linear approximation is 18.

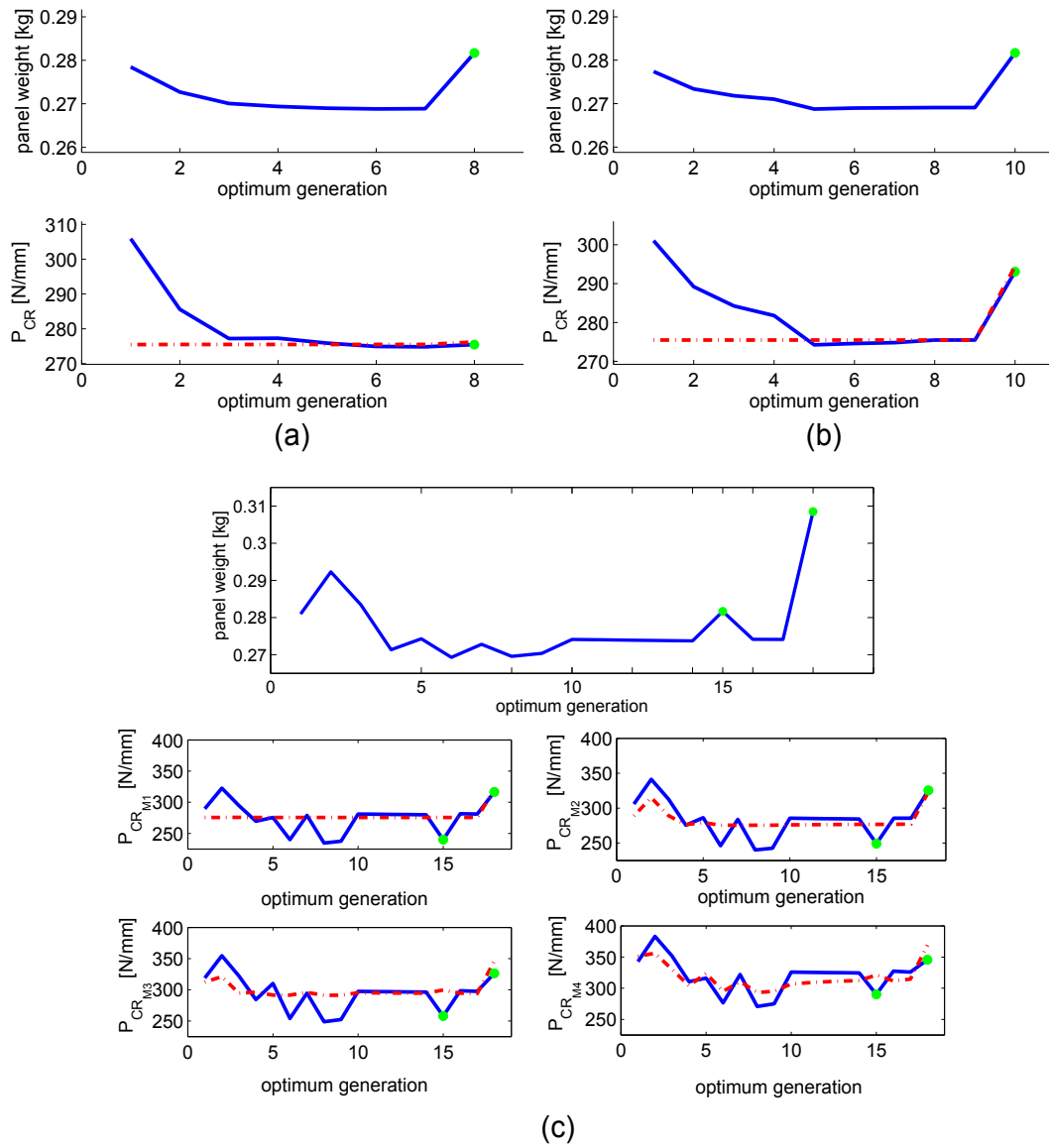


Figure 4: Objective and constraint function trends in the optimum generations for UK (a), OK, (b) and the linear regression (c). The green spot represents discrete solution. In the figures that represent the evolution of the constraints, the blue continuous line represents the actual response, the dashed line represents the surrogate

Finally, Table 1 shows the results after the discrete search. As observed from these results, UK and OK found the same weight whereas the stacking sequences are not the same. A good agreement between the approximate and actual solution is found, with an error less than the 1%. For the linear regression, it is immediately observed how the final solution diverge from the previous and ends in a final configuration that requires 2 extra plies. This can be due to the grouping of the modes. In fact, some mismatched elements are present when the panels are singularly checked in every group. Nevertheless, the approach shows promising results in the continuous environment.

Panel optimum configuration						
Surrogate type	$N_{GEN}$	Mass [Kg]	Optimum configuration	Surrogate prediction [N/mm]	FEM [N/mm]	$e_{PRED}$ [%]
UK	7	0.2817	$[51_2 / -41_2 / 1 / 7 / 1_3 / -84_{1.5}]_S$	276.21	275.18	0.38
OK	9	0.2817	$[-44_2 / 45_3 / -2 / 6 / 3 / 6 / -41 / -32_{0.5}]_S$	294.40	293.56	0.29
Linear regression*	14	0.2817	$[56_2 / 17 / -51 / 12_3 / -38_2 / -15_{1.5}]_S$	275.50	240.15	14.72
Linear regression	2	0.3085	$[30_2 / -23_3 / 24 / -55 / -30_2 / 38_2 / 30_{0.5}]_S$	320.58	316.59	1.26

\*=solution obtained after the first validation for the continuous generation

Table 1: Optimum configurations for the considered surrogate models

## Conclusions

An optimization process is proposed to perform weight optimization of composite panels subject to buckling. To reach the final configuration, a two step optimization strategy has been implemented. In the first part, a continuous solution is found. Then, a second level optimization is performed from the continuous optimum to find the discrete feasible optimum configuration.

The use of the  $D$  terms permits the generation of the surrogate as implicit function of the stacking sequence parameters  $t$  and  $\theta$ . This approach allows to obtain a simple, reliable and fast approximation of the buckling constraint. Moreover, this paper shows the possibility to perform the optimization independently from the availability of an analytical solution of the problem. Investigation on panels in which the twisting-bending coupling is present and boundary conditions different from the “*all simply supported*” or “*all clamped*” edges configuration is possible using *Kriging*. Despite the strong convergence criteria, it has been proven how the solution converges after only 8 generations for the best case (*UK*); even in the worst case (*OK*), the number of generations (10) is still acceptable.

A theoretical approach, based on the definition of the constitutive equation for buckling, has been used and results were compared with *Kriging*. The comparison shows good agreement in the continuous environment, even if the convergence rate of the *linear regression* result is low. There is no agreement in the final discrete solution between the surrogate prediction and the actual critical load. This might be due to the way of grouping the buckling responses. However, the approach shows promising results and this will be investigated further to improve the final result.

## References

- [1] R.T. Haftka and Z. Gürdal. *Elements of Structural Optimization*. Kluwer Academic Publisher, Dordrecht, 1999.
- [2] A.I.J. Forrester A. Soberster and A.J. Keane. *Engineering Design via Surrogate Modeling. A Practical Guide*. Wiley, Chichester, 2007.
- [3] N.V. Queipo R.T. Haftka W. Shyy T. Goel R. Vaidyanathan and P.K. Tucker. Surrogate-based analysis and optimizations. *Progress in Aerospace Sciences*, 41:1–28, 2005.
- [4] P. Hajela and C.J. Shih. Optimal design of laminated composites using a modified integer and discrete programming algorithm. *Computers & Structures*, 32:213–221, 1989.
- [5] ANSYS®. Release 12.0. In *Documentation for ANSYS*, 2009.
- [6] J.N. Reddy. *Mechanics of laminated composite plates and shells: theory and analysis*. CRC Press, Boca Raton, 2004.
- [7] R.F. Gibson. *Principles of composite material mechanics*. CRC Press, Boca Raton, 2007.
- [8] M.W. Bloomfield C.G. Diaconu and P.M. Weaver. On feasible regions of lamination parameters for lay-up optimization of laminated composites. *Proceedings of the Royal Society A*, 465:1123–1143, 2009.
- [9] D. Akçay Perdahcioğlu. *Optimizing the Dynamic Behavior of Structures - Using Substructuring and Surrogate Modeling*. PhD Thesis, University of Twente, 2010.
- [10] H.B. Nielsen and K.F. Thuensen. Surrogate models. In *ERCIM Copenhagen meeting*, April 2005.
- [11] H.B. Nielsen and K.F. Thuensen. Kriging and radial basis functions. In *SIAM OP05 Stockholm*, May 2005.
- [12] R. Rikards H. Abramovich J. Auzins A. Korjakins O. Ozolinsh K. Kalnins and T. Green. Surrogate models for optimum design of stiffened composite shells. *Composite Structures*, 63:243–251, 2004.