EFFECT OF COLLATERAL BRANCHING ON EXCITATION THRESHOLD OF DORSAL COLUMN FIBERS IN SPINAL CORD STIMULATION

Johannes J. Struijk, Jan Holsheimer, Herman B.K. Boom

Biomedical Engineering Division
Department of Electrical Engineering
University of Twente, The Netherlands

ABSTRACT

In this paper we present the effect of epidural spinal cord stimulation on myelinated dorsal column nerve fibers. An important feature of these fibers is the presence of collaterals perpendicular to the main fibers. In order to investigate the behavior of these fibers in spinal cord stimulation, we first calculated the potential field in the spinal cord, using a realistic volume conductor model. Second, this field was applied to an electrical network model of the fibers. It is concluded that both excitation threshold and blocking threshold of dorsal column nerve fibers are decreased by 30-50%, compared to thresholds of simple straight fibers.

INTRODUCTION

In spinal cord stimulation usually the electrodes are placed at the dorsal side of the spinal cord. So, it may be expected that the activated nerve fibers are mainly located in the dorsal columns. These longitudinal fibers are not simple straight fibers as in peripheral nerves. They have many collaterals [1] running into a dorso-ventral direction, almost perpendicular to the main fibers (see figure 1).

We modeled the fiber-collateral system as a McNeal-type model [2] as was proposed by Coburn [3]. Using this model, we investigated the influence of the presence of one or more collaterals on membrane polarization and on excitation and blocking stimulus thresholds.

Figure 1 Drawing of dorsal column nerve fibers with their collaterals running into the gray matter. From Cajal, 1909.

NERVE FIBER MODEL

In figure 2 the electrical network model of a nerve fiber and a collateral is shown. For all nodes at which no collaterals arise, the model is similar to McNeal's model. For these nodes the following equation holds.

$$(1.) \quad \frac{dV_n}{dt} - \frac{1}{R_a C_m} [V_{n-1} - 2V_n + V_{n+1}] + \frac{\pi d l}{C_m} i_{ion,n} = \frac{1}{R_a C_m} [V_{e,n-1} - 2V_{e,n} + V_{e,n+1}]$$

where V_n is the transmembrane potential across the membrane of node n and $V_{e,n}$ is the extracellular potential at node n . The intra axonal resistance between adjacent nodes $R_a = 4\rho_a L / \pi d^2$, and the membrane capacity of the nodal membrane $C_m = c_m \pi d l$, where ρ_a is the intra axonal resistivity, c_m is the membrane capacitance, d is the axon diameter, l is the length of a node of Ranvier, L is the internodal length which was assumed to be $100 \cdot d$. The membrane diffusion current at node n is given by $i_{ion,n}$. For the description of i_{ion} we used the equations of Chiu et al. [4] adapted to a temperature of 37°C.

For a branching node, the collateral diameter d_c , the nodal surface area k and the extracellular potential $V_{e,1}$ should be taken into account. It can be shown that the following equation is valid for a branching node.

$$(2.) \quad \frac{dV_n}{dt} - \frac{1}{k} \cdot \frac{1}{R_a C_m} [V_{n-1} - (2 + \frac{d_c}{d}) V_n + V_{n+1} + \frac{d_c}{d} V_1^c] + \frac{\pi d l}{C_m} i_{ion,n} = \frac{1}{k} \cdot \frac{1}{R_a C_m} [V_{e,n-1} - (2 + \frac{d_c}{d}) V_{e,n} + V_{e,n+1} + \frac{d_c}{d} V_{e,1}^c]$$

where V_1^c is the membrane potential at the first node of a collateral.

Figure 2 Electrical network model of a branched myelinated nerve fiber.

The geometry of the axonal bifurcation in the dorsal root ganglion fiber of the cat was described by Ha [5]. Assuming that branch points of the dorsal column fibers have a geometry similar to dorsal root ganglion fibers, except for the smaller diameter of the collateral, we estimated that the nodal membrane will be 0-50% larger than the area of an unbranched node. Assuming that the actual geometry is not important in a lumped network model, we introduce the nodal surface area factor k being the nodal area of a branching node divided by the area of an unbranched node. This is also incorporated in equation (2.).

For the nodes of the collateral fiber equation (1.) is valid again, except that V_n and $V_{e,n}$ are replaced by V_n^c and $V_{e,n}^c$.

The collaterals were oriented perpendicular to the main fiber and in parallel to each other. Fiber parameters are given in table 1.

VOLUME CONDUCTOR MODEL

The volume conductor model that was used consists of a three dimensional rectangular grid with $57 \times 57 \times 57$ grid points. The model comprises the spinal cord consisting of gray matter and white matter, the cerebrospinal fluid, the epidural space and a boundary layer representing the surrounding tissues (see figure 3, for conductivities see table 1). The center of the electrode, a monopolar voltage source, was positioned at the same level as the central node of the main fiber. Field potentials were calculated using a straightforward finite difference technique.

Figure 3 Volume conductor model of the spinal cord and surrounding tissues in a transverse section. Markers (*) indicate positions of the dorso column fibers used in the simulations.

RESULTS

Our simulations showed that the most significant parameters are the ratio of the diameters of collateral and main fiber, the dorso-ventral position of the fiber in the dorsal columns, the surface area of the branching node and, to a lesser extent, the distance between adjacent collaterals.

If a collateral is attached to node n , an additional impedance arises in the network model between node n (intracellular) and the extracellular space around the nodes of Ranvier of the collateral, thus introducing new voltage sources V_e^c into the model. The net result turned out to be an increased change of transmembrane potential (with an electrode at the level of node n). Excitation stimulus thresholds as well as blocking thresholds were 30–50% lower when a collateral was attached to the main fiber (see fig. 4).

Polarizations of nodes at the main fiber and nodes at the collateral fiber had opposite signs. This implies that an action potential that is elicited in the main fiber or at the branching node may not be propagated into the collateral because of the hyperpolarization of the collateral nodes. On the other hand, an action potential elicited at some collateral node, may be blocked at the branching node.

DISCUSSION

The threshold stimuli for excitation and blocking at branching nodes is predicted to be decreased by 30–50% as compared to unbranched nodes. The real values depend on several parameters, some of which are not well known. For the nodal surface area we only had data for cat dorsal

ganglion fibers. From these data it seems reasonable to assume the nodal area to be between 1 and 1.5 times the area of an unbranched node. Excitation threshold was found to increase by less than 15% if $k=1.5$ instead of $k=1$.

It is possible that action potentials, which are elicited at the main fiber, will be blocked in the collateral branch. However, we do not expect this phenomenon to play an important role in spinal cord stimulation because the blocking thresholds will usually be too high, because of the small diameter of the collaterals.

Figure 4 Threshold voltage as a function of electrode-fiber distance, for a fiber having 0, 1 and 11 (spaced 2 internodal lengths) collaterals. Fiber positions are as indicated in fig. 3 at the midline in the dorsal columns.

Table 1 Parameters

d	6	μm	main fiber diameter
d_c	2	μm	collateral diameter
l	1.5	μm	nodal length
L	$100 \cdot d$	mm	main fiber internodal distance
ρ_a	0.7	Ωm	intra axonal resistivity
g_m	1280	$1/(\Omega\text{m}^2)$	membrane conductivity
C_m	0.02	F/m^2	membrane capacitance
	0.23	$1/\Omega\text{m}$	conductivity of gray matter
	0.60	$1/\Omega\text{m}$	cond. of white matter (long.)
	0.083	$1/\Omega\text{m}$	cond. of white matter (transverse)
	1.7	$1/\Omega\text{m}$	cond. of cerebrospinal fluid
	0.040	$1/\Omega\text{m}$	conductivity of epidural space
	0.002	$1/\Omega\text{m}$	conductivity of surrounding layer

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Address: J.J. Struijk, Biomedical Engineering Division, Department of Electrical Engineering, University of Twente, P.O.Box 217, 7500 AE Enschede, The Netherlands. Phone:53-892691 or 53-892760.