Calibration-Free Signal-Strength Localization using Product-Moment Correlation

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Abstract—Localization, a process of determining the position of a blind node, can be used in various applications. Signal-strength localization provides a low-cost and low-power solution to positioning. Signal-strength positioning approaches using fingerprinting or calibrated approaches require a time-consuming calibration phase. Existing self-calibrating approaches, which do not require a priori calibration, use a least-squares fitting model to determine both the position of the blind node as well as the optimal environmental parameters.

In this paper, we propose an approach using the Product-Moment correlation between the measured signal strength and the estimated signal strengths. Such approach does not require estimation of the environmental parameters or prior calibration and outperforms existing self-calibrating least-squares approaches. We compare our approach to existing least-squares calibration-free positioning approaches. Moreover, we look at the Cramer-Rao Bound (CRB) of signal-strength localization and using simulations we show that the product-moment correlation outperforms least-squares approaches and follows the CRB closely.

Simulation and evaluation using a real-world experiment dataset show the product-moment approach significantly outperforms least-squares approaches. The product-moment approach follows the CRB much more closely and achieves up to twice more accurate positions in certain scenarios. When the error ratio increases and the number of reference positions stays fixed at 6, the product-moment approach scores 20% more accurate positions. In the cooperative localization scenario, the product-moment correlation performs 40% better.

Keywords-signal-strength localization; indoor positioning; calibration-free; product-moment correlation;

I. INTRODUCTION

Localization refers to a process of identifying the relative or absolute position of blind nodes. Localization applications range from tracking large objects such as containers, ships and planes down to small battery-powered sensors in Wireless Sensor Networks (WSN). Although the Global Positioning System (GPS) provides a worldwide infrastructure for positioning devices outdoors, no such infrastructure is available indoors. Moreover, the price, energy consumption and accuracy of the GPS limit the application in outdoor environments.

Localization can be performed in both cooperative and non-cooperative approaches. The difference between cooperative and non-cooperative localizations is illustrated in Figure 1. In the case of non-cooperative localization, ranging measurements between reference (anchor) nodes and a single blind node are used to determine the location of the blind node.

Figure 1: Example of cooperative and non-cooperative localization. Cooperative localization uses all pairwise measurements available, whereas non-cooperative localization uses only measurements between reference nodes and blind nodes.

In cooperative localization, all pairwise measurements are used to determine the locations of all blind nodes.

Many approaches were proposed to determine the position of an object, for instance, Time of Flight (ToF), Angle of Arrival (AoA) and signal-strength based approaches. Although ToF and AoA provide good performance in Line-of-Sight (LoS) environments, they perform poorly in Non-Line-of-Sight (NLoS) environments [1]. Moreover, ToF and AoA is generally not available in radio platforms, limiting the application of this technique to specialized hardware.

Signal-strength localization offers a low-cost approach to positioning and Received Signal Strength (RSS) information is available on many low-cost radios. To calculate a position based on the signal-strength measurement, however, is not straight-forward. Many localization approaches have been proposed, such as fingerprinting approaches [2], calibration-based and calibration-free approaches. Although fingerprinting and calibration-based approaches generally provide good accuracy, they require an initial calibration phase. Fingerprinting requires building a database of signal-strength fingerprints of the environment, whereas calibration-based approaches determine the parameters of the propagation model of the environment. This initial calibration is very laborious but its result generally stays valid only for a short duration because environments change, doors open and close, and broken beacons need to be replaced.

Calibration-free approaches overcome the problem of requiring an initial calibration phase before positioning can be used. These approaches determine the propagation

model parameters at run-time. Examples of such approaches include [3] [4] [5] and [6]. All these approaches use the Log-Normal Shadowing Model (LNSM) and determining both the transmission power, the path loss exponent and the position of the device by applying a least-squares fitting. Such approaches, however, require an initial estimate of the channel parameters. Moreover, least-squares approaches are generally not resilient against measurement outliers.

In this paper, we propose a calibration-free signal-strength positioning approach based on the product-moment correlation. Such approach requires an assumption on the channel propagation model but does not require an estimate of the channel parameters. Moreover, the product-moment approach is more resilient towards outliers and outperforms the least-squares approaches as we will demonstrate with simulations and an experiment dataset evaluation. We evaluate these localization approaches in both non-cooperative and cooperative approaches.

The outline of this paper is as follows. After discussing related work in Section II, we define the problem statement of non-cooperative and cooperative localization in Section III. In Section IV we describe the proposed approach and compare it with existing calibration-free signal-strength positioning approaches. Moreover, we look at the CRB which gives an indication of the bound of localization accuracy. In Section V, we evaluate our product-moment localization approach and existing least-squares approaches and compare their performance with the CRB. In Section VI, we evaluate the approaches using experimental data.

II. RELATED WORK

Among many proposed approaches for signal-strength localization [7] [8] and [9], the fingerprinting approaches including [10] [11] are popular but require an offline phase for building the fingerprint database. Building this database using a ground-truth measurement is very cumbersome. Approaches such as WiFi-SLAM [12] are able to speed up the process of building these maps, however, still require an offline phase for building fingerprint maps. Moreover, environments may change as objects may be moved and beacons need to be replaced when broken, thus, the fingerprinting needs to be done periodically. The advantage of fingerprinting localization is that it has a general assumption about the propagation model of the Radio Frequency (RF) signal.

Proximity localization [13], connectivity localization [14] and sequence-based localization approaches [15] [16] are type of approaches that have little assumption about the signal propagation. In the case of connectivity and proximity localization, the localization approach only considers whether two nodes are within range, whereas in the case of sequence-based localization, the order of the received beacons is assumed to have a relation with the distance-based order of the nodes. Because a very general assumption about the propagation is made, these approaches are very resilient

against ranging errors but generally have low accuracy and precision.

The RF signal is generally considered to follow the LNSM in indoor environments [17]. Many existing approaches use the LNSM, which defines the decay of the signal over a distance as follows:

$$P_d = P_{d_0} - 10n_p \log_{10}\left(\frac{d}{d_0}\right) + \mathcal{N}(0, \sigma^2) \quad (1)$$

The LNSM defines the received signal strength (P_d) as a function of the distance (d) and two environmental parameters P_{d_0} and n_p . P_{d_0} is the transmission power of the reference transmitter. n_p is the path loss exponent and defines the signal decay over distance. Finally, the received signal strength has an error that is normal distributed with zero mean and a standard deviation of σ , and is considered independent from the distance. In [17] the ratio $\frac{\sigma}{n_p}$ is used to make the error independent from the n_p path-loss parameter.

To calculate the distance using the received signal strength, the parameters P_{d_0} and N of the LNSM needs to be known or calculated. Approaches such as [3] [4] [5] and [6] use a Maximum-Likelihood Estimator (MLE) and apply a least-squares fitting technique to calculate the position of the blind node as well as estimate the optimal LNSM model parameters. Some of these approaches only estimate the path-loss exponent (n_p), whereas others estimate both the path-loss exponent (n_p) as well as the transmission power (P_{d_0}).

While the product-moment correlation has been applied to calculate the correlation of fingerprints in fingerprinting approaches [18], it has never been applied in an optimization approach. In this paper, we show that an approach based on optimizing the product-moment correlation of the estimated positions given the measured signal strengths, which does not require a priori calibration, will outperform an approach based on estimating the environmental parameters and position using a least-squares fitting.

III. PROBLEM STATEMENT

In this section, we define the problem of localization. We consider a network including M reference (anchor) nodes that have known location information and N blind nodes that have unknown location information. The nodes are deployed in 2-dimensional space. Let $\mathbf{S} = \{\mathbf{s}_i : i = 1 \dots N\}$ denote the actual vector positions of the blind nodes and $\mathbf{A} = \{\mathbf{s}_i : i = N+1 \dots N+M\}$ the vector positions of the reference (anchor) nodes. The pairwise RSSI measurements $\{P_{i,j} : i, j = 1 \dots N\}$ are given or measured by the node radio hardware. Let $\mathbf{H}_i$ denote the set of nodes in range of the node i . Clearly, $i \notin \mathbf{H}_i$ and $\mathbf{H}_i \subset \mathbf{S} \cup \mathbf{A}$. The localization problem is to estimate the vector positions of the blind nodes, given the vector positions of the reference nodes $\{\mathbf{s}_j\} \in \mathbf{H}_i$ and the measurements $\{P_{i,j} : j \in \mathbf{H}_i, j \neq i\}$. In a 2-dimensional space we have $\mathbf{s}_i = (x_i, y_i)^T$ and $\theta_i = (\hat{x}_i, \hat{y}_i)^T$ are the true and estimated positions of node i , respectively.

Without loss of generality, we assume that all measurements $\{P_{i,j} : j \in \mathbf{H}_i, j \neq i\}$ between a blind node i and its neighbouring nodes $\{j\} \in \mathbf{H}_i$ are available. In the case of non-cooperative localization, the set $\{j\} \in \mathbf{H}_i$ is referred to only the reference nodes. On the other hand, the set $\{j\} \in \mathbf{H}_i$ includes reference nodes and other blind nodes around node i in the case of cooperative localization. We also assume that $P_{i,j}$ is log-normal, thus the random variable $P_{i,j}$ is Gaussian. Finally, we assume that $\{P_{i,j}\}$ are statistically independent. This assumption can be somewhat oversimplified in practical environments, but it is necessary for analysis.

A. Measurement Statistical Model

As we assume that the random variable $P_{i,j}$ is log-normal, which is empirically proven in other work such as [17], the $P_{i,j}$ is Gaussian caused by the shadowing of the radio having the variance σ^2 ,

$$P_{i,j} \sim \mathcal{N}(\bar{P}_{i,j}, \sigma^2), \quad (2)$$

where

$$\bar{P}_{i,j} = P_{d_0} - 10n_p \log_{10} \left(\frac{d_{i,j}}{d_0} \right) \quad (3)$$

is the mean power in dBm of the Gaussian distribution and P_{d_0} is the received power at a reference distance d_0 in free space with the path loss formula described in [19]. For well-known environments, the path loss n_p might be estimated from prior calibration. For unknown environments, the path loss n_p can be handled as a “nuisance” parameter. In addition, d_0 is typically set to 1 meter.

IV. METHODOLOGY

In this section, we address the methodology to calculate the position of a blind node given the measured signal strengths using a least-squares fitting approach as well as the product-moment fitting approach. We first consider a non-cooperative localization approach and then the cooperative localization approach. Both the least-squares as well as the product-moment approach can be used with an iterative minimization approach to estimate the optimal parameters. Both approaches differ only in the optimization criteria to be calculated and time complexity of the optimization algorithm is the same for both approaches. Finally, we show how the CRB is derived for cooperative and non-cooperative localization. We derive the CRB in such a way to ease an implementation.

A. Non-Cooperative localization

In a non-cooperative signal-strength localization approach, the estimated position of blind node i , $(\hat{x}_i, \hat{y}_i)^T$, is determined using signal-strength measurements to a set of reference nodes with known positions. The positions of reference nodes $\{s_j\}$ are defined as $\{(x_j, y_j)^T\}$ and the measurement between blind node i and the reference nodes are $\{P_{i,j} : j \in \mathbf{H}_i \cap \mathbf{A}, j \neq i\}$.

1) *Least-Squares*: To estimate now the position of a blind node i , approaches such as [3] [4] [5] and [6] minimize the following least-squares sum:

$$\min_{\theta} \sum_{j \in \mathbf{H}_i \cap \mathbf{A}} (P_{i,j} - \hat{P}_{i,j})^2, \quad (4)$$

where $\hat{P}_{i,j}$ is defined as:

$$\begin{aligned} \hat{P}_{i,j} &= \hat{P}_{d_0} - 10\hat{n}_p \log_{10} \left(\frac{\hat{d}_{i,j}}{d_0} \right), \\ \hat{d}_{i,j} &= \sqrt{(x_j - \hat{x}_i)^2 + (y_j - \hat{y}_i)^2}. \end{aligned} \quad (5)$$

This results in the estimated power based on the distance to the reference node ($\hat{P}_{i,j}$) to be brought as close as possible to the measured power ($P_{i,j}$). This can be solved using an iterative minimization algorithm such as Gauss-Newton or Levenberg-Marquardt. The set of parameters to be minimized (θ) should at least contains the $\hat{x}$ and $\hat{y}$ position of the blind node. If the set of parameters contains only $\hat{x}$ and $\hat{y}$, the environmental parameters (P_{d_0} and n_p) need to be calibrated a priori. A complete calibration-free approach, however, estimates both $\hat{x}_i$ and $\hat{y}_i$, as well as the environmental parameters $\hat{P}_{d_0}$ and $\hat{n}_p$.

Minimization of this sum of squares is shown to perform well in realistic environments. The initial estimation of $\hat{P}_{d_0}$ and $\hat{n}_p$, however, can significantly influence the performance of the final calculated position. An approach independent from the environmental parameters is preferred. In our simulations presented in Section V, we take the center of all reference positions as an initial estimate of $\hat{x}_i$ and $\hat{y}_i$ and calculate the optimal estimated for $\hat{P}_{d_0}$ and $\hat{n}_p$ using an Ordinary Least-Squares (OLS) fitting.

2) *Product-Moment*: The product-moment correlation calculates the linear correlation between variables X and Y . The product-moment correlation is defined as follows:

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \quad (6)$$

The product-moment correlation ($\rho_{X,Y}$) is a number within -1 (indicating a reverse linear correlation) and 1 (indicating linear correlation between the two sets). In the product-moment correlation approach we optimize the correlation between the measured values ($P_1..P_N \in X$) and the estimated powers ($\hat{P}_1..\hat{P}_N \in Y$) as shown in Figure 2. The LNSM is linear in the environmental parameters (n_p and P_{d_0}) and therefore a linear correlation can be calculated without estimating the actual values of the environmental parameters when a position and distance is estimated. The position of blind node i is estimated by minimizing the following equation:

$$\min_{\hat{x}_i, \hat{y}_i} \left(\frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} - 1 \right)^2 \quad (7)$$

Figure 2: Example of calculating the linear correlation between the measurement power and the estimated powers, derived from the estimated position of the blind node. Because the product-moment correlation only calculates the linear correlation, the n_p and P_{d0} do not need to be estimated to calculate the correlation. By changing the estimated coordinates and optimizing the correlation towards 1, the best possible estimated position can be calculated.

and the estimated powers $\hat{P}_1 \dots \hat{P}_N \in Y$ are calculated as follows:

$$\hat{P}_{i,j} = -1 \cdot \log_{10}\left(\frac{\hat{d}_{i,j}}{d_0}\right), \quad (8)$$

$$\hat{d}_{i,j} = \sqrt{(x_j - \hat{x}_i)^2 + (y_j - \hat{y}_i)^2}.$$

The environmental parameters (P_{d0} and n_p) in this approach are irrelevant because the product-moment calculates the linear correlation between Equation 8 and the measured signals, i.e. the linear correlation between the LNSM and the measured signal strengths.

For the initial estimate of the $\hat{x}_i$ and $\hat{y}_i$, we take the center of the reference positions. Our proposed product-moment correlation minimization approach does not require an initial estimation of any environmental parameters.

B. Cooperative localization

In cooperative localization, all pairwise measurements between nodes are assumed to be available. These measurements can be used to improve the accuracy of the positioning of nodes.

1) *Least-Squares*: In the case of the least-squares approach, the positions are estimated by minimizing the difference between the estimated received signal following the LNSM and the measured signal strength. To do so, we minimize the following sum:

$$\min_{\theta} \sum_{\substack{i \in \mathbf{S} \\ j \in \mathbf{H}_j \cap \mathbf{S} \cap \mathbf{A} \\ i \neq j}} (P_{i,j} - \hat{P}_{i,j})^2, \quad (9)$$

where $\hat{P}_{i,j}$ is defined as:

$$\hat{P}_{i,j} = \hat{P}_{d0} - 10\hat{n}_p \log_{10}\left(\frac{\hat{d}_{i,j}}{d_0}\right), \quad (10)$$

$$\hat{d}_{i,j} = \sqrt{(\hat{x}_i - \hat{x}_j)^2 + (\hat{y}_i - \hat{y}_j)^2}.$$

Differently from the non-cooperative localization, the positions in the distance estimation are now all estimated. Not all positions need to be estimated. In the simulations performed in Section V, we assume a number of nodes have known reference positions. We calculate the initial positions for the cooperative localization using the non-cooperative approach, given the measurements between the nodes and the reference nodes. The initial environmental parameters ($\hat{P}_{d0}$ and $\hat{n}_p$) are again estimated using an OLS fitting given the initial estimated positions and all available measurements.

2) *Product-Moment*: Our proposed product-moment localization approach uses Equation 6 to optimize the correlation between the measured powers in set ($P_1 \dots P_N \in X$) and the estimated signal strengths in set ($\hat{P}_1 \dots \hat{P}_N \in Y$). However, in the cooperative approach, much more measurements are available. In addition, the estimated powers $\hat{P}_1 \dots \hat{P}_N \in Y$ are calculated as follows:

$$\hat{P}_{i,j} = -1 \cdot \log_{10}\left(\frac{\hat{d}_{i,j}}{d_0}\right), \quad (11)$$

where

$$\hat{d}_{i,j} = \begin{cases} \sqrt{(\hat{x}_i - x_j)^2 + (\hat{y}_i - y_j)^2} & \text{if } j \in \mathbf{A}, j \neq i \\ \sqrt{(\hat{x}_i - \hat{x}_j)^2 + (\hat{y}_i - \hat{y}_j)^2} & \text{if } j \in \mathbf{S}, j \neq i \end{cases} \quad (12)$$

Similar to the least-squares approach, the distance functions contain the estimated coordinates of node i ($\hat{x}_i$ and $\hat{y}_i$) and node j (x_j and y_j). We calculate the powers and distance between all pair of nodes where $i \neq j$. For the initial estimated of the positions of the nodes, we use the non-cooperative product-moment localization approach given the available reference positions.

C. Cramer-Rao bound for Received Signal Strength

To make an estimation of the bound of the localization performance, the CRB was derived. The CRB for RSS measurements was derived from the Fisher Information Matrix (FIM) in previous work [17] for cooperative approaches. In this section, we briefly derive the CRB for both non-cooperative and cooperative localization schemes in such a way that makes it easier to be implemented. The conditional probability density function (pdf) for $P_{i,j}$ given Θ is given by:

$$f(\mathbf{P}|\Theta) = \prod \frac{10}{P_{i,j} \log 10 \sqrt{2\pi}\sigma} \exp \left[-\frac{b}{8} \left(\log \frac{d_{i,j}^2}{\hat{d}_{i,j}^2} \right) \right], \quad (13)$$

where $b = \left(\frac{10n_p}{\sigma \log 10} \right)^2$ and $\hat{d}_{i,j} = d_0 \left(\frac{P_0}{P_{i,j}} \right)^{\frac{1}{n_p}}$.

To derive the FIM, we define $l_{i,j} = \log f(P_{i,j}|\theta)$ as the log-likelihood function. We have

$$l_{i,j} = \log \frac{10}{P_{i,j} \log 10 \sqrt{2\pi}\sigma} - \frac{b}{8} \left(\log \frac{d_{i,j}^2}{\hat{d}_{i,j}^2} \right). \quad (14)$$

If $l_{i,j} = \log f(P_{i,j}|\theta)$ is twice differentiable with respect to θ , then the FIM is computed as

$$J = -E \left[\frac{\partial^2 \log f(\mathbf{P}|\Theta)}{\partial \theta_k^2} \middle| \Theta \right] \quad (15)$$

Through steps of the second order partial derivatives, we obtain the FIM elements for the non-cooperative and cooperative localization schemes.

1) *CRB for Non-cooperative Localization:* Every arbitrary i^{th} blind node has its own FIM, which is $J_{2 \times 2}$

$$\begin{aligned} J_{1,1}(i) &= b \sum_{j \in \mathbf{H}_i} \frac{(x_i - x_j)^2}{d_{ki}^4}, \\ J_{2,2}(i) &= b \sum_{j \in \mathbf{H}_i} \frac{(y_i - y_j)^2}{d_{ki}^4}, \\ J_{1,2}(i) &= J_{2,1}(i) = b \sum_{j \in \mathbf{H}_i} \frac{(x_i - x_j)(y_i - y_j)}{d_{ki}^4}, \end{aligned} \quad (16)$$

By the definition, the CRB for the localization error of the i^{th} blind node can be computed by the sum of the trace of the inverse matrix of the FIM $J(i)$, denoted by $J^{-1}(i)$,

$$E[(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2] \geq J_{1,1}^{-1}(i) + J_{2,2}^{-1}(i). \quad (17)$$

2) *CRB for Cooperative Localization:* Since in cooperative localization a blind node i uses not only location information of reference nodes but also the estimated location of blind nodes in its neighbourhood, all blind nodes have a shared FIM of which dimension is $2n \times 2n$, denoted by $J_{2n \times 2n}$. Note that J can be expressed in n^2 block matrices of which dimension is 2×2 .

The elements of the diagonal sub-matrices, $i = j$, are given by

$$\begin{aligned} J_{2i-1,2i-1} &= b \sum_{k \in \mathbf{H}_i} \frac{(x_i - x_k)^2}{d_{ik}^4}, \\ J_{2i,2i} &= b \sum_{k \in \mathbf{H}_i} \frac{(y_i - y_k)^2}{d_{ik}^4}, \\ J_{2i-1,2i} &= J_{2i,2i-1} = b \sum_{k \in \mathbf{H}_i} \frac{(x_i - x_k)(y_i - y_k)}{d_{ik}^4}. \end{aligned} \quad (18)$$

The elements of non-diagonal blocks, $i \neq j$, are given by

$$\begin{aligned} J_{2i-1,2j-1} &= J_{2j-1,2i-1} = -b \mathbf{I}_{\mathbf{H}_i}(j) \frac{(x_i - x_j)^2}{d_{i,j}^4}, \\ J_{2i,2j} &= J_{2j,2i} = -b \mathbf{I}_{\mathbf{H}_i}(j) \frac{(y_i - y_j)^2}{d_{i,j}^4}, \\ J_{2i-1,2j} &= J_{2j,2i-1} = J_{2i,2j-1} = J_{2j-1,2i} \\ &= -b \mathbf{I}_{\mathbf{H}_i}(j) \frac{(x_i - x_j)(y_i - y_j)}{d_{i,j}^4}, \end{aligned} \quad (19)$$

where $\mathbf{I}_{\mathbf{H}_i}(j)$ is the indicator function that allows us to include the information only if node i made a measurement with node j , $\mathbf{I}_{\mathbf{H}_i}(j) = 1$ if $j \in \mathbf{H}_i$ or 0 if not.

The CRB for cooperative localization error of the i^{th} blind node can be computed by the sum of the trace of the i^{th} block in the inverse matrix of J

$$E[(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2] \geq J_{2i-1,2i-1}^{-1} + J_{2i,2i}^{-1}. \quad (20)$$

Note that $P_{i,j}$ is assumed to be random variables when deriving the CRB, the CRB gives a lower bound for an unbiased estimators. Note that errors caused by obstacles such as building infrastructures and furnitures in the measuring environment may be large constants, which violate the zero-mean Gaussian assumption of the CRB [20]. Therefore, the variance of errors of an estimator can be lower than the bound presented by the CRB if the estimator can overwhelm the effects of obstacles. Also, when (box) constraints are applied to estimator, the estimator becomes biased and may outperform the CRB. Nonetheless, the CRB is a practical approach to estimate the expected performance of a localization system.

V. SIMULATION

We evaluate the different localization approaches and compare them to the CRB. To compare the approaches, we conduct simulations with increasing ranging errors and with increasing number of reference nodes available in the network. We evaluate the performance of both the non-cooperative and the cooperative localization. All simulations and evaluations were performed with the Matlab tool.

A. Non-Cooperative localization

To evaluate the different non-cooperative approaches, we simulated the localization approaches with various ranges of error and the number of reference nodes. In a simulated environment of 100 m $\times$ 100 m we deploy 6..20 reference nodes and calculate the accuracy of the estimated position for 5000 random positions using the two approaches and the CRB. The calculated positions are constrained to the deployment area of 100 m $\times$ 100 m. All nodes within the deployment area are connected, the communication range is assumed to be larger than $\sqrt{2 \cdot 100^2} \approx 141$ m.

For the least-squares self-calibration approach we take the center of the positioning area (50,50) as the initial estimate

(a) Results of simulation in a non-cooperative approach with number of reference nodes changing from 6 to 20 nodes, 5000 blind nodes are positioned randomly in an area of 100 m $\times$ 100 m. All simulations were performed with a relation $\frac{\sigma}{n_p} = 1.7$.

(b) Results of simulation in a non-cooperative approach with 6 reference nodes and relation $\frac{\sigma}{n_p}$ changing from 1.5 to 2.9.

Figure 3: Results of the simulation in a non-cooperative localization approach. The Root-Mean Squares (RMS) error of resulting position is calculated for 5000 simulations.

Figure 4: Results of simulation with averaging of the resulting positions from 1 to 20 trials. The RMS is calculated from 1000 simulations. When averaging the calculation of two positions, both approaches perform better than the CRB.

of the position. At this position, we calculate the initial environmental parameters ($\hat{P}_{d_0}$ and $\hat{n}_p$) using a linear least squares approach. Equation 4 is then optimized using the fsolve in Matlab, where both the position of the blind node ($\hat{x}$ and $\hat{y}$) and the optimal calibration parameters ($\hat{P}_{d_0}$ and $\hat{n}_p$) are estimated simultaneously.

For the product-moment correlation approach, we also take the center of the positioning area (50, 50) as the initial estimate of the position. Note that the product-moment correlation does not require an initial estimate of the calibration parameters. Next, equation 7 is minimized using the fsolve in Matlab, the optimal $\hat{x}$ and $\hat{y}$ are estimated.

We perform this simulation with different numbers of reference nodes, ranging from 5 to 20 reference nodes. The $\frac{\sigma}{n_p}$ relation is kept at 1.7 for this simulation. The results are

shown in Figure 3a.

The results show that the product-moment approach follows the CRB quite well, whereas the localization accuracy of the least-squares approach improves very little when the number of reference nodes increases. The improvement of the product-moment correlation approach over the least-squares approach ranges from 9% to 104%. In the case of 20, reference nodes the product-moment approach is twice as accurate as the least-squares approach.

We also perform another simulation with a fixed number of reference nodes and the ratio $\frac{\sigma}{n_p}$ varying from 1.5 to 2.9. Again, the simulation is repeated for 5000 random positions in an area of 100 m $\times$ 100 m. The results of this simulation are shown in Figure 3b.

The results show that the product-moment approach performs significantly better than the least-squares approach, on average 20% more accurate. The product-moment approach comes very close to the CRB. It performs on average 3% better than the CRB. Once the error in the measurements increases up to a certain value, such as when $\frac{\sigma}{n_p} = 2.1$, the product-moment approach starts outperforming the CRB. The CRB assumes the estimator is unbiased, however because the calculated positions are constrained to the localization area, the estimator is biased and can outperform the CRB.

We also evaluate the accuracy of the positioning by rerunning every position up to 20 times and taking the average position of multiple runs. This simulation is performed with 6 reference nodes and the error ratio was set to 1.7. The position is calculated by averaging between 1 and 20 simulation runs. The results are shown in Figure 4. On average the product-moment approach performs 28% better than the least-squares approach. Both approaches perform better than the CRB when averaging two calculated positions.

(a) Results of simulation in cooperative network with 6 randomly deployed reference nodes and number of blind nodes ranging from 14 to 54 randomly deployed in an area of $100\text{ m} \times 100\text{ m}$. Ratio $\frac{\sigma}{n_p}$ is kept fixed at 1.7.

(b) Results of the simulation of the cooperative localization approach with 6 randomly deployed reference nodes, 34 randomly deployed blind nodes and the ratio $\frac{\sigma}{n_p}$ is increased from 1.5 up to 2.9.

Figure 5: Results of the simulation in a cooperative localization approach. In this scenario all pairwise measurements are available.

B. Cooperative localization

For the cooperative approach, we simulate the localization in a network of $100\text{ m} \times 100\text{ m}$ with fixed 6 reference nodes. We evaluate the results with increasing number of nodes in the network and with increasing error ratio.

In the scenario of increasing number of nodes, we keep the ratio $\frac{\sigma}{n_p}$ fixed at 1.7. The number of blind nodes in the network varies from 14 to 54 nodes. Simulation is repeated for 100 different random deployments of the blind nodes and reference nodes. In the scenario of increasing error ratio, we keep the number of nodes constant, 34 blind nodes and 6 reference nodes, and vary the error ratio from 1.5 to 2.9. The results from both scenarios are shown in Figure 5.

The results show that when increasing the number of nodes the product-moment approach performs on average 40% better than the least-squares approach and performs 122% worse than the CRB. With increasing error-rate, the product-moment approach performs about 32% better than the least-squares approach and 104% worse than the CRB.

VI. REAL-WORLD EXPERIMENTS

To evaluate the performance of the localization approach in a more realistic setup, we have used the dataset from the sequence-based localization paper [15]. This experiment was performed using MICA 2 motes in an outdoor environment at a parking lot. Within the environment 11 MICA 2 motes were placed randomly on the ground. All the nodes were placed in line of sight and were programmed to broadcast a single packet. The measured signal strengths were recorded on the device EEPROM and were read out once the experiment was finished.

We use this dataset to compare our product-moment correlation approach with a least-squares calibration-free approach and, because we use the same dataset as the

sequence-based localization evaluation, we can also compare to the sequence-based localization [15]. The evaluation was run with different number of reference nodes, similar to how the simulation was performed in [15]. Because the dataset has all pairwise measurements between nodes, it is possible to additionally evaluate the cooperative localization approaches.

The results shown in Figure 6 show the product-moment correlation approach is twice as accurate as the least-squares approach. In this scenario, the least-squares approach performs even worse than the proximity approach. The sequence-based approach and the product-moment correlation approach perform very similarly. With a small number of reference nodes (5 or 7), the product-moment correlation approach performs better, whereas with a higher number of reference nodes (9 or 10), the sequence-based localization outperforms the product-moment approach.

The sequence-based localization and product-moment correlation are actually two approaches that are very closely related to each other. The sequence-based localization uses the Spearmans correlation index, which is the product-moment perform on the indices (or order numbers) of the reference nodes. The product-moment correlation calculates directly on the signal-strength measurements itself. It is surprising to see how well the sequence-based approach performs, albeit it uses only the order of the reference nodes. However, the sequence-based localization approach requires calculating the arrangement of lines of the reference node deployment, which is a computational intensive operation, and cannot be applied in an cooperative approach. Conversely, our product-moment correlation approach provides a much simpler approach for the same accuracy and provides improved accuracy when a cooperative approach is applied.

Because all pairwise measurements are available in the dataset, it is possible to optimize the calculated positions

Figure 6: Results of the localization run on the experiment dataset, shown are the mean positioning errors in meters. 11 nodes were deployed in an area of $11\text{ m} \times 9\text{ m}$ and all pairwise signal-strengths were measured. Evaluated were the least-squares (LS) and product-moment approach (PM). Results of the proximity (Prox) and sequence-based localization (Seq) were taken from [15]. Results of the cooperative localization approaches are labels PM Coop and LS Coop.

further using a cooperative approach. We use the non-cooperative positions as the initial estimates for the nodes and perform further optimization using the cooperative approach. Results are shown in Figure 6. The results show that in the cooperative scheme the product-moment approach outperforms the least-squares approach by 30%.

VII. CONCLUSION

Localization has been used in many different applications. Although the GPS provides a worldwide positioning service outdoors, indoor positioning is still a challenge. Signal-strength localization provides a low-cost solution that can be applied on many different radio platforms. Many signal-strength localization approaches such as fingerprinting localization and many LNSM based approaches require a calibration or training phase. This calibration process is time-consuming, albeit some approaches can speed up the calibration, and may need to be repeated when the environment changes.

Calibration-free localization methods exist, they use a least-squares fitting model to estimate both the coordinates and the environmental parameters. In this paper, we have shown an alternative approach to estimate the position using the product-moment correlation which estimates the correlation between the estimated position and the measured signal strengths. Such approach eliminates the estimation of environmental parameters. Moreover, through simulations and evaluation using a localization dataset we have shown the product-moment approach outperforms the least-squares approach.

We have shown how the product-moment correlation

approach and the least-squares approach can be used to estimate the positions of the blind nodes in both non-cooperative and cooperative networks. In the case of the least-squares approach, the position of the blind node as well as the environmental parameters are estimated. In the case of the product-moment approach, the correlation of estimated signal strengths following the LNSM with the measured signal strength is optimized to estimate the position of the node without requiring the environmental parameters.

We have shown how the CRB can be derived and performed simulations with both the non-cooperative and the cooperative localization approaches. When increasing the number of reference nodes in the non-cooperative network, the product-moment correlation approach performs up to 104% better. In the case of 6 reference nodes, the product-moment approach performs 20% more accurate when increasing the error ratio. In the case of the cooperative approach, overall, the product-moment approach performs 40% better than the least-squares approach.

Results of using a real-world experiment dataset show that the product-moment correlation approach performs twice as accurate as the least-squares fitting approach. The product-moment correlation approach not only performs very similarly to the sequence-based localization, but also is much simpler to calculate the position. For the cooperative localization approach, the product-moment approach outperforms the least-squares approach by 25%.

Traditionally, the least-squares fitting is used to perform calibration-free signal-strength positioning. In this paper, we have shown the product-moment correlation is a better approach. Not only does this approach require no initial estimate of the environmental parameters but also it outperforms the least-squares fitting significantly. At the moment, we are applying this localization technique in actual networks and in future work we would like to further evaluate the performance of the product-moment correlation approach in an actual deployment of a signal-strength localization network.

ACKNOWLEDGMENT

This work is supported by the Lost & Found project within the Dutch Commit program.

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