Optical Excitation of Micro-Mechanical Resonators

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ABSTRACT

We present theoretical and experimental studies on opto-thermal excitation of bending-mode micro-mechanical resonators. The theory results in a prediction of induced bending moment (modulus and phase) as a function of the excitation frequency, geometry of the structure and material properties. Decisive roles are played by the following: absorption length of the material μ , penetration depth of a thermal wave δ and the thickness of the resonator. δ is a function of the excitation frequency while the resonance frequency depends on h . The theory results in design rules for opto-thermal resonators. It is shown that absorbing layers improve the effectivity of the opto-thermal transduction only in the case of transparent materials. Experiments agree well with theory.

INTRODUCTION

There is a growing interest in sensors based on micro machined resonators having a resonance frequency that is influenced by a measurand [1]. In order to measure the resonance frequency the structure must be excited mechanically, while the vibration amplitude must be detected at the same time. The driver and the detector can be included in a closed-loop configuration forming an oscillator which locks to the natural frequency of the resonator.

In this contribution we focus on optical excitation. This choice combines the advantages of an all-optic sensor [2] with that of the resonator. The optical driver we investigated operates via the thermal domain, so we consider the optical power

absorbed at the upper part of the resonator (a simple clamped bar, see figure 1a) inducing thermal stress. Several possibilities for optical detection have been described in literature (interferometric principles, proximity sensors etc., e.g. [3,4]). Optical detectors are not treated here.

The absorbed optical power gives rise to a thermal distribution, which in turn creates a mechanical moment, driving the beam. The simplest configuration we consider is depicted in figure 1b. We confine ourselves to the one-dimensional case, an approach that is accurate as long as the thickness of the beam is small compared to the length and width of the heated area.

It is convenient to define the thermal moment $M_T(t)$:

$$M_T(t) = 1/h \int_0^h \theta(y,t) \cdot (h/2-y) dy \quad (1)$$

with h the thickness of the beam (y -direction) and θ the temperature. We use θ to stress the fact that in the following complex numbers are used with the "real" temperature T as the real part of θ .

The mechanical moment $M_b(t)$ is directly proportional to the thermal moment [5]:

$$M_b(t) = Ebh\beta M_T(t) \quad (2)$$

with E Young's modulus, b the width of the beam (assuming heating over the full width) and β the coefficient of thermal expansion. Hence, if we have an expression for the temperature distribution $\theta(y,t)$ the problem is reduced to a purely mechanical problem, which is well known in the literature [5,6]

SINGLE LAYER GEOMETRY

In this section we treat the simple beam structure of figure 1b. We assume a sinusoidally modulated light beam, with wavelength λ incident in the y-direction perpendicular to the surface. The light intensity is given by $I_t(t) = I_m + I_0 \cos(\omega t)$, with I_m the mean intensity and I_0 the amplitude of the varying part. I_m gives rise to a background temperature distribution. Making use of an optical absorption coefficient χ the time varying part of the (local) light intensity which penetrates the material is given by:

$$I(y,t) = I_0 e^{-\chi y} \cos(\omega t) \quad (3)$$

The optical absorption length is defined as $\mu = 1/\chi$. At a wavelength of 600 nm the value of χ is about 7.10^5 m^{-1} for Silicon ($\mu = 1.5 \text{ } \mu\text{m}$). If we assume that the light absorbed by the solid is directly converted into heat, the heat source distribution can be described by:

$$q(y) = \chi I_0 e^{-\chi y} \quad (4)$$

The temperature distribution follows from the thermal diffusion equation [7]:

Figure 1. a) Experimental set-up for excitation of the beam vibration, b) clamped beam with exciting light beam.

$$\frac{\delta T}{\delta t} = \frac{k}{\rho c} \frac{\delta^2 T}{\delta y^2} + \frac{1}{\rho c} q(y,t) \quad (5)$$

with ρ the density, c the heat capacity, k the thermal conductivity of the beam material and q the time-dependent heat-source distribution. Turning to solutions in terms of complex functions, assuming $T = \text{Re}(\theta)$ and $q = \chi I_0 e^{-\chi y} e^{i\omega t}$, and taking $\delta\theta/\delta y = 0$ at $y = 0$ and $y = h$ (boundary conditions), we find after calculation:

$$\theta(y,t) = \left[e^{-\chi y} - \chi \frac{\cosh(\sigma(h-y)) - e^{-\chi h} \cosh(\sigma y)}{\sigma \sinh(\sigma h)} \right] B_3 e^{i\omega t} \quad (6)$$

with $\sigma = (1+i)/\delta$, $\delta = (2\alpha/\omega)^{1/2}$, $\alpha = k/\rho c$ the thermal diffusivity and $B_3 = \chi I_0 / [\rho c (i\omega - \alpha \chi^2)]$.

In principle the problem is now solved, but expression (6) is not very transparent. The solution is governed by three characteristic measures, the absorption length $\mu = 1/\chi$, the thermal penetration depth δ and the beam thickness h . These parameters determine the features of the temperature distribution.

Let us consider the case where $\mu < h$, which is the temperature distribution for an opaque beam and discuss what happens if δ is changed with respect to μ and h . For $\delta < \mu$ the temperature distribution is pinned to the distribution of the light, see fig. 2b. Increasing δ (which corresponds to decreasing ω , c.f. eq. (6)) leads to a phase shift of the temperature distribution with respect to the light distribution, the temperature lags behind. In fig. 2a the temperature profile lags behind. In fig. 2a the temperature distribution is shown for $\mu < \delta < h$. Increasing δ further, the temperature profile reaches the bottom of the beam. The temperature dependence on the frequency, $T_s(\omega)$ at the surface of the beam shows three regions: at low frequencies ($\delta > h > \mu$), then $T_s \sim \omega^{-1}$, in the intermediate region $h > \delta > \mu$ we have $T_s \sim \omega^{-1/2}$ and at high frequencies $h > \mu > \delta$, $T_s \sim \omega^{-1}$ again.

More important than the temperature distribution is the induced mechanical moment. $M_t(t)$ is calculated from eq. (1) using (6). The integration is straightforward but tedious and leads to the following result:

Figure 2. Time- and position-dependent temperature distribution, a) $\delta > \mu$ b) $\delta < \mu$; the relative amplitudes between a) and b) are not to scale.

$$M_t(t) = A e^{i\omega t} (B \cdot C + D) \quad (7)$$

$$\text{with: } A = \frac{I_0 h^2}{k(\gamma^2 - \xi^2)} \quad B = -\frac{\xi^2}{\gamma^2} (1 + e^{-\xi})$$

$$C = \frac{1}{2} + \frac{1}{\gamma} - \frac{2}{\gamma(1 + e^{-\gamma})} \quad D = \frac{1}{2} - \frac{1}{\xi} + \left(\frac{1}{2} + \frac{1}{\xi}\right) e^{-\xi}$$

$$\gamma = (1+i)h/\delta \quad \xi = h/\mu = h\chi.$$

In figure 3 the result of this unsatisfactory formula is depicted and it is seen that there is no characteristic change at the point where $\delta = \mu$. This means in fact, that as long as the opacity of the beam is large it is of no interest whether or not the thermal penetration depth is larger than the absorption length. The observed changes in the temperature distribution around $\delta = \mu$ are completely averaged out. It is only if the thermal penetration

Figure 3. Modulus and phase of the thermal moment $M_t(t)$ as function of the frequency for an opaque beam with a infinite small absorption length $\mu \rightarrow 0$.

length becomes larger than the thickness h that a drastic change can be noted, viz. a phase shift of $\pi/2$ and a moment becoming independent of the frequency. Exactly the same results can be obtained by thermal heating, with a heat being applied to the top of the beam [8] and we can conclude that as long as the opacity of the beam is large enough, there is no need to concentrate the absorption to a top layer. The function of a top layer is solely to reduce the reflectivity and catch as much light power as possible.

In figures 4.a and 4.b a complete overview is given of M_T as a function of h/δ and μ/h . It can be seen that for increasing μ/h , that is when the beam becomes more and more transparent, the moment decreases as can be expected, but the phase relation retains the same form.

The amplitude diagram shows four areas. The formula for $M_t(t)$ can be approximated for these areas as follows:

$$\begin{aligned} 1: M_t(t) &\approx \frac{h^2}{24k} \cdot I_0 e^{i\omega t} & 2: M_t(t) &\approx \frac{h^2 \xi^2}{144k} \cdot I_0 e^{i\omega t} \\ 3: M_t(t) &\approx \frac{h^2}{2k\gamma^2} \cdot I_0 e^{i\omega t} & 4: M_t(t) &\approx \frac{h^2 \xi^2}{12k\gamma^2} \cdot I_0 e^{i\omega t} \end{aligned} \quad (8)$$

The respective areas are indicated in figure 4. Figure 5 gives an experimental result of a transfer function of an optically excited beam, where the first resonance is chosen low enough to have $\delta > h$, so that the mechanical moment is constant and in phase with the optical intensity. The experiment confirms this.

MULTILAYER GEOMETRIES

The multilayer configuration can be modeled in full detail [5], but the resulting equations are even less transparent than those of the preceding section. Therefore we confine ourselves to the most relevant situation, where we have a transparent material as the "bulk" material. We can apply an absorbing layer to the incident side or to the opposite side.

The first configuration reduces the treatment completely into the case of a beam, with a heater deposited on top of it. The only difference is the way in which the power is produced. We have performed a number of experiments with metal coatings (NiFe, Al, Au) and have compared these with uncoated silicon beams. The results of these experiments, which in general confirm the theory presented in [5], will be presented elsewhere. We only note here that we did not find any indication

Figure 4. Modulus and phase of the thermal moment as function of the relative diffusion length δ/h and the relative absorption length μ/h . Areas indicated by A,B,C,D refer to asymptotic approximations given in eq. 8.

Figure 6 shows the result of an earlier experiment with direct heating on top of the beam and having $\delta = h$ below the first resonance. This result, which is fully representative for the optical excitation as well, shows the predicted effect of a decrease in modulus and phase of the driving moment.

Figure 7 shows the expected and the measured vibration amplitude as a function of the position of the incident light beam. The calculations were performed assuming a homogeneous intensity over a width of 2 mm., giving a total power equal to that of the fiber guided laser beam used. Our results agree with those found in the literature [9,10]. Moreover, the measured vibration amplitude is proportional to the optical power over a range of 0 - 100 mW.

Figure 5. Measured transfer function of optically excited cantilever beam, beam dimensions are $b = 1\text{ mm}$, $h = 20\text{ }\mu\text{m}$, $l = 8\text{ mm}$. Light intensity is 50 mW.

Figure 6. Measured transfer function as function of the frequency for an electro-thermally excited beam. Beam geometry, $h = 270\text{ }\mu\text{m}$, $b = 2\text{ mm}$, $l = 3.4\text{ mm}$. Resonance at $f_0 = 35\text{ kHz}$.

Figure 7. Expected and measured beam tip vibration amplitude as function of the relative position of the light beam. Beam diameter about 2.5 mm (solid curve for width of 2 mm and homogeneously intensity distribution).

of the electronic strain effects, mentioned by Stearns [11], which should show up as a phase change of the beam vibration as the light moves from the coated to the uncoated region. Pitcher [12] also refers to the electronic strain effect, but his experiments were performed at resonance, where a coating-induced change of the quality factor may be of influence. In our experiments we observe off-resonance conditions, which are not affected by changes of the Q-factor.

The configuration with an absorbing layer at the bottom side has been treated by Mallalieu [13] and Grattan [14,15], they treated a fused quartz resonator excited with a Nichrome absorbing layer. They assume that there is no temperature variation at the bottom side of the Nichrome, which results in temperature variations of $2.10^{-4} \text{ }^{\circ}\text{C}/10^4 \text{ Wm}^{-2}$. Figure 8a represents the result of our simulation in this case. However, if we assume the more realistic boundary condition of no heat flux leaving the bottom we arrive at figure 8b, with temperature variations, which are more than an order higher and a thermal moment that is even more than 70 times higher. Grattan finds a behavior which is not reproducible, which he ascribes to the varying thermal conductivity of the Nichrome layer.

Figure 8. Calculated temperature distributions in a double-layer structure according [13,14] at 8 points of time, a) zero temperature at the bottom surface of the structure, b) zero heat flow at the bottom surface.

In the model of figure 8b this has hardly any influence, so we think that their mechanical design (a clamped-clamped configuration, highly sensitive to axial stress) is responsible for the bad reproducibility of their results.

REFERENCES

- 1 R.T. Howe, Resonant microsensors, *Proc. 4th Int. Conf. Solid-State Sensors and Actuators (Transducers 1987)*, Tokyo, Japan, June 1987, pp. 843-848.
- 2 T.S.J. Lammerink and S.J. Gerritsen, Fiber-optic sensors based on resonating mechanical structures, *Proc. Int. Conf. on Fiber Optic Sensors II*, SPIE vol. 798, April 1987, The Hague, The Netherlands, pp. 86-93.
- 3 J.F. Willemin and R. Dändliker, Measuring amplitude and phase of microvibrations by heterodyne speckle interferometry, *Op. Lett.* 8 (1983) 102-104.

- 4 J. Fluitman and Th. Popma, Optical waveguide sensors, *Sensors and Actuators*, 10 (1986) 25-46.
- 5 T.S.J. Lammerink, *Optical Operation of Micro-Mechanical Resonator Sensors*, PhD. Thesis, University of Twente, Enschede, The Netherlands, 1990.
- 6 S. Timoshenko, J.M. Gere, *Mechanics of Materials*, 2nd SI edition, Van Nostrand Reinhold (UK) Co. Ltd, 1988.
- 7 A. Rosencwaig and A. Gersho, Theory of the photoacoustic effect with solids, *J. Appl. Phys.*, 47 (1976) 64-69.
- 8 T.S.J. Lammerink, M. Elwenspoek and J.H.J. Fluitman, Frequency dependence of thermal excitation of micro-mechanical resonator sensors, *Sensors and Actuators*, in press (*Proceedings Eurosensors IV, Karlsruhe, October 1990, Germany*).
- 9 M.V. Andres, K.W.H. Foulds and M.J. Tudor, Optical activation of a silicon vibrating sensor, *El. Lett.* 22 (1986) 1097-1099.
- 10 H. Wölfelschneider et al., Optically excited and interrogated micromechanical silicon cantilever structure, *Proc. Int. Conf. on Fiber Optic Sensors II, SPIE vol. 798, April 1987, The Hague, The Netherlands*, pp. 61-66.
- 11 R.G. Stearns and G.S. Kino, Effect of electronic strain on photoacoustic generation in silicon, *Appl. Phys. Lett.* 47 (1985) 1048-1050.
- 12 R.J. Pitcher, K.W.H.Foulds, J.A.Clements and J.M. Naden, Optothermal Drive of Silicon Resonators: the Influence of Surface Coatings, *Sensors and Actuators*, A21-A23 (1990) 387-390.
- 13 K.I. Mallalieu, Studies of optical driving and multiplexing of vibrating instrumentation sensors, *PhD. Thesis, University of London*, 1987.
- 14 K.T.V. Grattan, A.W. Palmer, N.D.Samaan and F. Abdullah, Mathematical Analysis of Optically Powered Quartz Resonant Structures in Sensor Applications, *J. Lightw. Technol.*, 7 (1989) 202-208.
- 15 K.T.V. Grattan, Photothermal Excitation of Resonant Structures in Fibre Optic Sensors: Application of Computer Modelling Techniques, *Sensors and Actuators*, A21-A23 (1990) 1146-1149.