Tuning the dispersion and single/multi-modeness in a hole-assisted fiber: a finite-element study

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Abstract—Using a vectorial finite element mode solver developed earlier, we studied a hole-assisted multi-ring fiber. We report the role of the holes in tuning the waveguide dispersion and the single/multi-modeness of the particular fiber. By correctly selecting the hole's size and position, a single-mode fiber with a relatively flat and low dispersion in the S-band of the third telecom window was obtained. We also report the observation of a mode-type which has never been reported before for ordinary fibers, which we label as a “ring” mode. This mode exists as a consequence of structural symmetry breaking due to the inclusion of the microstructural holes.

I. INTRODUCTION

Fibers with inclusions of microstructural holes running parallel with the propagation axis were recently introduced [1]. This approach allows a new way to engineer the fiber through their geometrical parameters; where the size, shape, orientation, position, and arrangement of the microstructural holes are manipulated to tailor the fiber's properties. This way of engineering apparently offers more degrees of freedom to the designer. Additionally, many new properties unattainable through ordinary fibers can be obtained by this new class of fibers [1].

In this paper, we consider the so-called hole-assisted fiber (HAF) [2], i.e. an ordinary index-guided fiber with holes inclusions. Although this fiber is, in fact, not a photonic crystal fiber (PCF), it still retains the nice features of the tunability of the fiber's properties through the holes. Unlike a PCF which operates in the leaky mode regime, an HAF operates in the guided mode regime. In this paper, we will show that the microstructural holes can be used to tune both the number of guided modes and the dispersion properties of such fiber. These properties are of importance for multiwavelength fiber-optical communications.

The large varieties of possible hole-shapes and arrangements demand the use of numerical methods that can handle arbitrary cross-sectional shapes to analyze this kind of

structures. Besides, the existence of interfaces with high index-contrast between the solid material and the air holes calls for the use of the vectorial wave equation to accurately model the structure. Finite element method (FEM) is suitable for such analysis as it can handle complicated structure geometries and solve vectorial equations transparently. In this work, we used a vectorial FEM mode solver developed earlier [3], [4] to study the HAF. We should note, that although in this paper we only report results for an HAF with 4 circular holes, the model itself is well applicable for HAF with more complicated hole shapes and arrangements (see examples of structures with complicated cross-sections reported in [4]), which might be of future interest.

II. FORMULATION OF THE METHOD

The detail discussions on the formulation of the mode solver has been given elsewhere [3], but for convenience will be briefly reviewed here.

Using an H-field-based vectorial wave-equation, $\nabla \times \bar{\epsilon}_r^{-1} \nabla \times \bar{H} = k_0^2 \bar{H}$, for longitudinally-invariant structures composed of non-magnetic anisotropic materials with diagonal permittivity tensors and $\exp(j\omega t)$ time dependence of the field; it is possible to get a vectorial wave-equation expressed only in terms of the transverse components of the magnetic field as follows:

$$\begin{bmatrix} \partial_y \left[\frac{1}{n_{zz}} (\partial_x H_y - \partial_y H_x) \right] \\ -\partial_x \left[\frac{1}{n_{zz}} (\partial_x H_y - \partial_y H_x) \right] \end{bmatrix} - \begin{bmatrix} \frac{1}{n_{yy}} \partial_x (\partial_x H_x + \partial_y H_y) \\ \frac{1}{n_{xx}} \partial_y (\partial_x H_x + \partial_y H_y) \end{bmatrix} + k_0^2 n_{\text{eff}}^2 \begin{bmatrix} \frac{1}{n_{yy}} H_x \\ \frac{1}{n_{xx}} H_y \end{bmatrix} = k_0^2 \begin{bmatrix} H_x \\ H_y \end{bmatrix} \quad (1).$$

Here, the x and y denote the transverse Cartesian coordinates associated with the structure cross-section, k_0 the vacuum wavenumber, n_{eff} the modal index, H_x and H_y the x and y components of the magnetic field $\bar{H}$, while n_{xx}^2 , n_{yy}^2 , and n_{zz}^2 the non-zero entries located at the diagonal of the relative

permittivity tensor $\bar{\bar{\epsilon}}_i$ associated with the x , y , and z components of the electric field, respectively. Using the Galerkin procedure and discretizing the computational domain into triangular elements lead to the following discretized weak formulation:

$$\begin{aligned} & \sum_{\text{Boundary Element } e} \left\{ -\int_{\Gamma_e} \frac{1}{n_{zz}} w_y (\partial_x H_y - \partial_y H_x) dy - \int_{\Gamma_e} \frac{1}{n_{zz}} w_x (\partial_x H_y - \partial_y H_x) dx \right. \\ & \left. - \int_{\Gamma_e} \frac{1}{n_{yy}} w_x (\partial_x H_x + \partial_y H_y) dy + \int_{\Gamma_e} \frac{1}{n_{xx}} w_y (\partial_x H_x + \partial_y H_y) dx \right\} \\ & + \sum_{\text{Interface Element } e} \left\{ -\int_{\Gamma_{\text{int},e}} \frac{1}{n_{yy}} w_x (\partial_x H_x + \partial_y H_y) dy + \int_{\Gamma_{\text{int},e}} \frac{1}{n_{xx}} w_y (\partial_x H_x + \partial_y H_y) dx \right\} \\ & + \sum_{\text{Triangular Element } \Omega_e} \iint_{\Omega_e} \left\{ \frac{1}{n_{zz}} (\partial_x w_y - \partial_y w_x) (\partial_x H_y - \partial_y H_x) \right. \\ & \left. + \left[\partial_x \left(\frac{1}{n_{yy}} w_x \right) + \partial_y \left(\frac{1}{n_{xx}} w_y \right) \right] (\partial_x H_x + \partial_y H_y) \right. \\ & \left. + k_0^2 n_{\text{eff}}^2 \left(\frac{1}{n_{yy}} w_x H_x + \frac{1}{n_{xx}} w_y H_y \right) - k_0^2 (w_x H_x + w_y H_y) \right\} dx dy = 0 \quad (2) \end{aligned}$$

with w_x and w_y denoting the weight functions, Ω_e the area in each triangular element, $\Gamma_{\text{int},e}$ the line element at the interface between different materials, and Γ_e the line element at the computational boundaries.

Approximating the fields using quadratic nodal-based basis functions will lead to a sparse generalized matrix eigenvalue equation, which can be solved using an eigenvalue solver to obtain the eigenvalues related to the modal indices (n_{eff}) and eigenvectors associated with the transverse components of the magnetic field $[H_x, H_y]^T$ of the corresponding modes.

Fig. 1. The hole-assisted multi-ring fiber under consideration.

III. RESULTS AND DISCUSSIONS

Here, we consider a multi-ring fiber [5] with depressed core and tuned by four holes located in its outer ring as shown in Fig. 1. The refractive index of the outermost cladding n_{clad} is taken from the Sellmeier's equation of pure silica [6]

$$n_{\text{clad}}^2 = 1 + \frac{0.6961663\lambda^2}{\lambda^2 - (0.0684043)^2} + \frac{0.4079426\lambda^2}{\lambda^2 - (0.1162414)^2} + \frac{0.8974794\lambda^2}{\lambda^2 - (9.896161)^2} \quad (3)$$

with λ is wavelength in μm . In this way, the material dispersion effect is rigorously taken into account in latter

computations. The refractive indices of the rings follow from $n_i = (1 + \Delta_i)n_{\text{clad}}$, with Δ_0 , Δ_1 , Δ_2 , and Δ_3 are -0.1%, 0.45%, -0.33%, and 0.1%, respectively, and the refractive index of the holes are $n_{\text{hole}} = 1$. The size of the rings r_0 , r_1 , r_2 , and r_3 are $0.41r_1$, $3.9\mu\text{m}$, $1.9r_1$, and $3.8r_1$, respectively. The diameter (d) and position ($D = s[r_2 + r_3]/2$, where s is a measure of the hole position offset from the center of the outer ring) of the holes, as well as light wavelength are to be varied. This structure is similar to the one discussed by Yan *et al.* [7], but here we have lowered the refractive index of one of the rings (n_1) in order to obtain single mode properties as well as low dispersion parameter (D), and hence low group velocity dispersion (GVD) in the third telecom wavelength window. Our computations showed that although low GVD can be achieved for the fundamental mode of the structure proposed by Yan *et al.* [7] (where $\Delta_1 = 0.48\%$ has been used), but their fiber is not single-moded in the expected wavelength range (the third telecom window).

Fig. 2. The mode effective indices and their associated dispersion parameter for the multi-ring fiber without the microstructured air holes.

Fig. 2 shows the mode indices and their associated dispersion parameter for the multi-ring fiber without air holes. The dispersion parameter is calculated using [8]

$$Dispersion = -\frac{\lambda}{c} \frac{\partial^2}{\partial \lambda^2} [\text{Re}(n_{eff})] \quad (4)$$

numerically. We should note that since we have taken the chromatic dispersion into account using Eq. (3), this dispersion parameter is already a rigorous combination between the effect of chromatic and waveguide dispersion. As revealed by Fig. 2b, without the microstructured air holes, this structure is multi-moded within the wavelength range of interest, i.e. the third telecom window.

Fig. 3. Dispersion parameter of (a). the fundamental (HE₁₁) mode tuned by various hole diameter and position and (b). the guided modes for holes with $d=2.2\mu\text{m}$ and $s=1$ of the hole-assisted multi-ring fiber.

Fig. 3a shows the effect of introducing the four air holes to the dispersion parameter of the fundamental mode of the fiber. Enlarging the diameter of the air holes will lift the dispersion curve up, while moving the holes away from the core will flatten the curve but instead pull the curve down. This fact implies that by properly chosen hole's size and position, it is possible to obtain a fiber with flat and low dispersion properties for the mode of interest in the wavelength range of interest. Fig. 3b shows the dispersion parameter of the guided modes of the fiber for $d=2.2\mu\text{m}$ and $s=1$. For this setting, the fiber is single-moded and has a dispersion parameter of less than 4 ps/(nm·km) in the S-band of the third telecom wavelength window. A smaller dispersion parameter can be obtained by holes with $d=1.8\mu\text{m}$ and $s=1$, but for the latter setting, the fiber is not single-moded anymore in the S-band.

Fig. 4. The mode effective indices spectra of the hole-assisted multi-ring fiber with $d=2.2\mu\text{m}$ and $s=1$. By correctly chosen hole's parameters, the higher order modes cut-off just before the telecom third window as the value of their effective indices cross the value of n_{clad} .

The role of the holes to tune the single/multi-modedness of the fiber can be easily understood as follows. The introduction of air holes will reduce the average refractive index of the outer ring; therefore will reduce the number of guided modes as well. Fig. 4 shows that for $d=2.2\mu\text{m}$ and $s=1$, the higher order modes (i.e. the TE₀₁ and TM₀₁ modes) cut-off just before the third telecom window as the value of their effective indices cross the value of n_{clad} . Hence, by correctly chosen hole's parameters, the nice properties of low and flat dispersion can be achieved together with the single-mode property. The fact that this fiber works in the guided mode regime (and hence does not suffer from confinement loss as in the PCF) and the possibility to engineer its dispersion and modal properties by the arrangement of the air holes might be interesting for some applications.

(a)

(b)

(c)

Fig. 5. Mode profile of the fundamental (HE_{11}) mode of structure with air holes with $d=2.2\mu\text{m}$ and $s=1$ at $\lambda=1.2\mu\text{m}$. (a). H_x , (b). H_y , and (c). $\hat{x}H_x + \hat{y}H_y$.

(a)

(b)

(c)

Fig. 6. Mode profile of the ring mode of structure with air holes with $d=2.2\mu\text{m}$ and $s=1$ at $\lambda=1.2\mu\text{m}$. (a). H_x , (b). H_y , and (c). $\hat{x}H_x + \hat{y}H_y$.

Another interesting aspect of the inclusion of the microstructured air holes is its effect to the mode structure of the fiber. Fig. 5 shows the modal profile of the fundamental (HE_{11}) mode of the fiber for air holes with $d=2.2\mu\text{m}$ and $s=1$ at $\lambda=1.2\mu\text{m}$. We notice that by introducing the air holes, a mode with a modal field shape never reported before for ordinary fibers shows up. We label this 2-fold degenerate [9] mode as a “ring” mode since its field is confined within the outer ring as shown by Fig. 6. The mode profile of this mode looks like $q\text{-}TM_{11}$ while its degenerate pair looks like $q\text{-}TE_{11}$ of a waveguide with square core. This unusual mode is not surprising as the introduction of the air holes changes the symmetry of the structure from $C_{\infty v}$ into C_{4v} .

IV. CONCLUSIONS

We report the use of FEM vectorial mode solver to study the tuning of dispersion properties and number of modes in a hole-assisted multi-ring fiber. The model includes realistic chromatic dispersion of the materials. By choosing a proper size and position of 4 microstructural holes in the cladding, we obtained a relatively flat and low dispersion single-mode fiber in the S-band of the third telecom window of fiber optic communications. Due to the symmetry breaking by the inclusion of the holes, at lower wavelength, we also observed a mode-type which is unusual for an ordinary optical fiber. We term this mode as a “ring” mode.

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