Nonlinear Interaction of Nearshore Morphology

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Abstract

An idealized cross-shore transport model was used to investigate the interaction between equilibrium beach profiles and bathymetric perturbations that were triggered by an infinitesimally small change in the incident wave height. A low tide terrace shape characterized the computed equilibrium profiles. This shape resulted from the assumed, nonlinear dependence of the equilibrium slope on the equilibrium value of the relative wave height (the latter was defined as the ratio of the rms wave height and water depth). We found that a small modification of the incident wave conditions resulted in the formation of bar-like perturbations on the equilibrium profile's terrace edge. Strong transport gradients associated with changes in the location of initial wave breaking caused the localization of the bar-like perturbations. This analysis did not resolve the potential for the bar-like perturbations to grow as free instabilities.

1. Introduction

Typically, nearshore morphology can be decomposed into a small number of morphologic elements, such as the large-scale nearshore slope (i.e., slope averaged over the entire range of the surfzone), and one or more surfzone sandbars. In general, we would like the different morphologic elements to be distinguished by their different spatial and temporal scales. Such a separation can be applied to cross-shore beach profiles. For instance, profile variations about a smooth, long term mean may be decomposed into empirical orthogonal functions [Winant *et al.*, 1975; Wijnberg and Terwindt, 1995], or locally sinusoidal functions [Pruszek and Rozynski, 1998], or local Gaussian-shaped functions [Plant *et al.*, in press-a]. Once a particular element has been extracted, its behavior can be described independently of all the other functions.

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If nearshore sediment transport rates were a linear function of depth, a particular morphologic element would not affect another element. However, because sediment transport in the nearshore environment is a nonlinear function of water depth, morphologic elements characterized by different scales must interact [Whitham, 1974]. One interaction of interest occurs between a steady state morphology (i.e., an equilibrium profile) and some sort of time-varying perturbation. If the external forcing (such as incident wave height) is fixed then this situation is the starting point for a stability analysis. Rather than performing a stability analysis of a system that experiences constant forcing, we will explore the response of a morphologic perturbation that has been forced by changes in the incident wave height. This situation corresponds to natural beach systems, which experience constant changes in the forcing.

A simplified, process-based, sediment transport model is used in our analysis. The model is restricted to cross-shore hydrodynamic and sediment transport processes (Section 2). The first step in the analysis of the morphologic behavior is calculation of equilibrium profiles (Section 3.1). Then, small wave height and bathymetric perturbations are introduced, allowing us to analyze a linearized system (Section 3.2). The response of the perturbations to small variations in offshore wave height is investigated. The implications of our analysis results are discussed in Section 4, and the conclusions are presented in Section 5.

2. Theory

2.1. Hydrodynamic model

A primary objective of this study is to understand the effects that nonlinear processes have on nearshore morphology. Wave shoaling and breaking is clearly a relevant process to consider. Because the time scale for morphologic response is expected to be much longer than the inherent wave time scale, we assume that wave processes are adequately described by their wave-averaged statistics. These statistics include, for example, the rms wave height, which we consider to be of primary importance. For an arbitrary bathymetry, $h(x)$, the rms wave height profile, $H(x)$, can be computed using the following wave transformation model, which is valid for shallow water waves [Thornton and Guza, 1983]:

$$\frac{\partial H}{\partial x} = -\frac{\gamma}{4} \frac{\partial h}{\partial x} + \frac{3\sqrt{\pi}}{4\gamma_s^2} \frac{f_p B \sqrt{h}}{\sqrt{g}} \left\{ \left[1 + \left(\gamma^2 / \gamma_s^2 \right) \right]^{-5/2} - 1 \right\} \gamma^4, \quad (1)$$

where γ is the local ratio of H/h (i.e., the relative wave height), γ_s is a model parameter related to the saturation value of γ , f_p is the peak incident wave frequency, and B is an adjustable parameter. The wave height profile may be computed for arbitrary offshore wave heights, $H(x=0) = H_0$. We will experiment with variable wave conditions, fixing $f_p=0.1$ Hz, $B=1$, and $\gamma_s=0.5$.

2.2. Sediment transport model

The near-bed currents that are associated with the incident wave field drive sediment transport. Both the wave orbital velocities and near-bed, seaward-directed mean flow (undertow) are clearly relevant to the large-scale cross-shore transport [Gallagher *et al.*, 1998]. Given an underlying bathymetry and the rms wave height, the magnitude of near bed currents may be computed approximately using linear wave theory (orbital velocities) and second order theory (undertow) [e.g., Masselink and Black, 1995]. Onshore transport is associated with wave and sediment transport nonlinearity, such as wave skewness [e.g., Stive, 1986].

What is important to beach profile evolution is the relative strengths and cross-shore structure of the processes driving onshore and offshore transport. A particular transport formulation that captures the role of these processes has been described recently by Plant *et al.* [in press-b], which we adopt because of its simplicity. The time-averaged sediment transport, $\bar{Q}$, is assumed to be of the form

$$\bar{Q}(x,t) = q(x,t) r(x,t), \quad (2a)$$

which has been used by others [e.g., Horikawa, 1981]. The function $q(x,t)$ on the right hand side of equation 2a describes the potential magnitude of sediment transport driven by the waves (this is also known as wave stirring [Roelvink and Stive, 1989]) and is based on Bagnold's [Bagnold, 1963] bedload formulation applied to oscillatory currents, which yields

$$q = c_2 \rho \sqrt{g} \frac{H_{rms}^3}{h^{3/2}}, \quad (2b)$$

Figure 1. Relative contribution of onshore and offshore transport mechanisms (with zero slope and $p=1$). γ_{max} is the maximum value of γ (ratio H/h) that is achieved by the system (Note that γ_0 is a parameter in the wave transformation model, which is not necessarily the saturation value of the system.)

where c_2 is a non-dimensional constant (which is irrelevant to this analysis) and ρ is the density of water.

The function $r(x,t)$ on the right hand side of equation 2a, which describes the relative importance of mechanisms that could rectify the wave "stirring" transport in (2b), has the form

$$r = r_0 \frac{\partial h}{\partial x} + (\gamma/\gamma_0)^p [1 - \gamma/\gamma_0]. \quad (2c)$$

The first term on the right hand side of 2c describes the importance of down slope transport, and it is controlled by the parameter r_0 . The next term describes the relative importance of offshore transport due to an undertow and other nonlinear affects, such as wave velocity skewness. The assumption made in equation 2c is that this balance is a nonlinear function (p controls the degree of nonlinearity) of the

Figure 2. Equilibrium profiles of relative wave height (scaled by value at saturation) and bathymetry. Boundary conditions were $h(x=0) = 1.5/(0.05 L_s)$ and $\gamma(x=0) = \{0.05, 0.075, 0.1, 0.15\} \gamma_0$. Cross-shore distance is scaled by $L_s = g T^2/(2 \pi)$, and depths are scaled by $0.05 L_s$. A wave period, T , of 10 s was used for all runs. (Thus, a scaled depth of 1 ~ 8 m and a scaled distance of 1 ~ 150 m.)

relative wave height, γ . Figure 1 shows the form of r . Onshore (i.e., positive) transport prevails when the relative wave height is well below the saturation level. Undertow dominates when wave breaking intensifies (and this effect is controlled by the parameter γ_0). Plant *et al.* [in press-b] showed that this parameterization was consistent with field observations of sediment transport on the Dutch coast (with $p=1$).

3. Results

3.1. Equilibrium profile

An often-made assumption about nearshore profile response is that the profile will approach equilibrium if the incident wave conditions are steady. Equilibrium results when transport gradients vanish. In our formulation, transport vanishes at the shore, where the wave height is zero. Thus, equilibrium results when the transport is zero everywhere, requiring that $r=0$ for all x . Then, the equilibrium wave height and

Figure 3. Dependence of equilibrium profile shape on the downslope transport, r_0 , (with $p=1$ and $\gamma_0=0.5$) and dependence on the nonlinearity parameter, p , (with $r_0=2.5$ and $\gamma_0=0.5$). For all cases, the relative wave height at the seaward boundary was $0.1\gamma_0$.

beach profiles solve the system

$$\left[\begin{array}{l} \text{equation (1)} \\ \frac{\partial h}{\partial x} = -\frac{1}{r_0} (\gamma/\gamma_0)^p [1 - \gamma/\gamma_0] \end{array} \right] \quad (3)$$

This system can be solved if the wave height and water depth are specified on the seaward boundary.

Equilibrium profiles were estimated numerically for a range of offshore wave heights (Figure 2). The parameter values used were $p=1$, $r_0=2.5$, and $\gamma_0=0.5$. Results are presented in non-dimensional variables as described in the figure. The equilibrium profiles are characterized by a concave up profile that meets a nearly planar profile near shore. Seaward of the break-point ($\gamma/\gamma_s < 1$), the profile takes on a familiar concave shape [Bowen, 1980]. As waves shoal over the equilibrium profile, the relative wave height increases and so does the equilibrium slope, reflecting the assumed increasing ability of wave nonlinearity to drive onshore transport. However, the quadratic dependence of the transport on the relative wave height results in flattening of the equilibrium slope near the break point ($\gamma/\gamma_s \sim 1$), reflecting the assumed importance of an increased undertow. Ultimately, the waves break and the relative wave height saturates (i.e., γ is constant). The corresponding equilibrium slope is also constant, yielding a familiar "low tide terrace" type profile [Inman *et al.*, 1993; Komar and McDougal, 1994; Larson *et al.*, 1999]. As the offshore wave height is varied, the terrace edge tracks the seaward movement of the deepening break point (Figure 2, arrows).

Figure 3 shows how variation in the nonlinearity parameter (p) and the down slope transport parameter (r_0) change the form of the equilibrium profile. Increasing the nonlinearity leads to a profile with increased upward concavity in the offshore zone. Increasing the down slope transport parameter reduces the maximum slope of the equilibrium profile.

3.2. Initial response to incident wave height perturbation

How does a profile that is initially at equilibrium respond to changes in incident wave conditions? To explore this question, we investigate the response of a particular equilibrium state to an infinitesimally small increase of the incident wave height. The hydrodynamic and sediment transport system described by equations 1 and 2 was linearized about a particular equilibrium state ($\gamma(x=0) = 0.15\gamma_s$). The linearized variables were defined as the depth relative to the equilibrium depth (η) and the relative wave height relative to the equilibrium relative wave height (α):

$$\begin{aligned} h &= h_{eq} [1 + \eta(x, \tau)] \\ \gamma &= \gamma_{eq} [1 + \alpha(x, \tau)] \end{aligned} \quad \eta, \alpha \ll 1 \quad (4)$$

These definitions were inserted into equations 1 and 2. Using sediment continuity, a system of equations including one for bed evolution was derived:

$$\begin{aligned} \frac{\partial}{\partial t} \eta &= f_1[\eta, \alpha : h_{eq}, \gamma_{eq}] \\ 0 &= f_2[\eta, \alpha : h_{eq}, \gamma_{eq}] \end{aligned} \quad (7)$$

where f_1 describes the linear response of the depth profile given an equilibrium state (h_{eq}, γ_{eq}) and a perturbation to either the relative wave height, the beach profile, or both. f_2 describes the corresponding relative wave height perturbation. Because we assume that the waves evolve on a time scale that is very short compared to a bathymetric evolution time scale, α is always in equilibrium with η and α contains no time-dependence other than that related to changes in η .

Figure 4. Relative wave height and bathymetric perturbations (α and η , respectively) are responses to a small wave height perturbation imposed at seaward boundary (arrow). The two lower panels show the sum of the arbitrarily scaled perturbations and corresponding equilibrium profiles. Model parameters were $p=1$, $r_0=2.5$, $\gamma_0=0.5$, and $\gamma(x=0)=0.1\gamma_0$.

A small perturbation was applied to the offshore, incident wave field, and the system in equation 7 was solved to find the initial response of the bed (Figure 4). Since the system in equation 7 is linear in η and α , the magnitudes of the perturbations are arbitrary. Therefore, we apply $\alpha(x=0)=1$ as the wave height perturbation at the seaward boundary (Figure 4, 1st panel). We assume that, initially, $\eta(x, t=0)=0$, and compute first $\alpha(x, t=0)$ and then $\frac{\partial}{\partial t}\eta(x, t=0)$. The depth perturbation response rate is then multiplied by an arbitrary time scale to show the magnitude of bed response (Figure 4, 2nd and 4th panels).

Morphologic response (Figure 4, 2nd panel) is seen only near the break point, corresponding to strong gradients in the wave height perturbation (Figure 4, 1st panel). We note that the cross-shore scale of the morphologic perturbation is similar to that of sandbars. (The non-dimensional cross-shore scale may be multiplied by $L_s \sim 150$ m, in order to convert to dimensional variables). In the case of an increased wave height, a bar crest is located just seaward of the terrace edge. Because of the linearity of equation 7, a small decrease in wave height would reverse the sign of the perturbation, placing the bar crest just landward of the terrace edge.

4. Discussion

The goal of this analysis was to investigate the interaction between two morphologic elements. The first element was an equilibrium profile and the second element was a perturbation that was superimposed on the equilibrium profile. The perturbation was the response to a modification in the incident wave conditions. The equilibrium profile described the morphology across the entire expanse of an idealized surfzone. As we have shown, perturbations were focused on a narrow region of the surf zone. However, because the equilibrium profile contained a short-scale feature (i.e., the terrace edge), the two morphologic components were not clearly separable based on spatial scale.

The two morphologic elements were, however, separated dynamically, since the equilibrium profile was by definition a steady state feature. The equilibrium state contributed to the evolution of a perturbation, but not vice versa. We explored this 1-way interaction by allowing small perturbations in the relative wave height and bathymetry. A small increase of the offshore wave height resulted in morphologic response that was focused on the break point region of the equilibrium profile. The focussing of the perturbation on the terrace edge was due to a corresponding focussing of perturbations in the wave breaking. Offshore of the point of initial wave breaking, a small wave height perturbation is unchanged from its initial (offshore) value, producing no transport gradients and no profile modifications. Onshore of the point of initial wave breaking, the relative wave height saturates and any wave height perturbations are rapidly damped as the wave height returns to the saturation value (Figure 4, 1st panel) – again producing no transport gradients and no profile modifications. Strong gradients in the in the relative wave height perturbation are

found in the region of the terrace edge, and these gradient drive profile modifications.

At this point we note an interesting paradox associated with the perturbation results. The bathymetric perturbation that was associated with a perturbation in the incident wave height vanishes at both onshore and offshore boundaries. Also, the bathymetric perturbation conserves mass. However, the equilibrium profiles that are associated with different incident wave conditions do not conserve mass (Figure 2). It is not clear how the initial bathymetric perturbation might lead to a profile adjustment towards a new equilibrium. Alternatively, the perturbations could grow into well-defined sandbars, such as are observed on many sandy coasts. It is possible that the formation of sandbars may be the result of the interaction between a near-equilibrium, low-tide terrace type morphology and a small (or even large) perturbations to incident wave conditions.

5. Conclusions

Analysis of an idealized cross-shore transport model showed that a low tide terrace shape is characteristic of theoretically derived equilibrium beach profiles. In our idealized model, this result is due to the explicit dependence of the equilibrium slope on the equilibrium value of the relative wave height (i.e.,

$$\frac{\partial}{\partial x} h_{eq} = \text{funct.} \left[\frac{H_{eq}}{h_{eq}} \right].$$

The terrace slope is milder than the maximum profile slope because of the assumed quadratic relationship between slope and relative wave height. The specific shape of the equilibrium profile depend on 3 model parameters (p , γ_s , r_0), governing the transport nonlinearity, the sensitivity to wave breaking, and the relative importance of down-slope transport.

The importance of the equilibrium profile in this analysis did not focus on the possibility of using it to predict the shape of naturally occurring nearshore profiles. Instead, we used the equilibrium profile as a starting point for an investigation into the inherent dynamics of nearshore morphology. We focused further on the morphologic effects of constantly changing wave conditions. We found that a small modification (i.e., a linearized perturbation) of the incident wave conditions focused the evolution of bar-like perturbations on the equilibrium profile's terrace edge. Strong transport gradients associated with changes in the location of initial wave breaking caused the localization of the bar-like perturbations.

Finally, profiles observed in nature constantly experience changes in offshore wave conditions. Therefore, even if the beach profile was "near" its equilibrium shape, the changes in wave conditions force perturbations on the parts of the profile where waves initially break. A reasonable hypothesis that extends from our analysis is that sandbar generation may result from the interaction between near-equilibrium profiles and instabilities that are triggered by changing wave conditions.

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