Bubble dynamics and the sound emitted by cavitation

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Abstract. Noise emitted by cavitating regions in a liquid, e.g. from a cavitating ship propeller, shows a complicated spectral density distribution.

In ship hydrodynamics one is interested in the way in which various types of cavitation (bubble-, vortex-, sheet cavitation) emit sound and, most importantly, in the scaling rules for the emitted sound. The more so, now that it is possible to do model tests with ships with the full scale cavitation number.

In this lecture we review developments in this field since the Fitzpatrick & Strasberg ONR paper in 1956, in particular the gradually gained insight that clouds of cavitation bubbles, as distinct from single bubbles, contribute significantly to the emitted sound. Following this, attention is given to the question how such clouds are generated during the lifetime of a cavitation sheet.

1 Introduction

Most ship propellers cavitate and, if so, cavitating regions on an hydrofoil produce erosion and also noise. Both have their origin in the collapse (disappearance) of cavities, either bubbles or sheets. Whereas with erosion the pressure field near the collapse counts, cavitation sound has to do with the far field.

From the beginning of scientific research in ship hydrodynamics and hydro-acoustics cavitation sound has been studied. Although much valuable work has been done and nice results have been obtained, many unsolved problems remain.

The present contribution presents a brief review of what has been achieved and draws attention to some questions, yet to be answered.

2 The spectral density of cavitation sound

We shall deal with flow cavitation only and leave acoustical cavitation, as exemplified by the work of Lauterborn and co-workers (e.g. Hentschel & Lauterborn 1982), out of consideration. The flow along an hydrofoil can be characterized by a large number of dimensionless quantities such as Reynolds number, Froude number, advance coefficient etc.

Let U be a characteristic speed, for example the advance velocity of the ship or the oncoming velocity of the fluid with respect to the hydrofoil, L a characteristic length, for example the chord length of a blade, and Ω the angular velocity of the propeller. Let us further denote the kinematic viscosity of the fluid with ν , its density with ρ and the acceleration of gravity with g .

With these quantities we can define

$$\text{Reynolds number, } Re = \frac{UL}{\nu}, \quad \text{Froude number, } Fr = \frac{U^2}{gL},$$

Fig. 1. Shape of spectral density distribution of cavitation sound, from Buist 1991.

(2.1)

$$\text{Advance coefficient, } \Lambda = \frac{U}{\Omega L}.$$

A classical result of dimensional analysis is that for geometrically similar bodies the flow is similar when these dimensionless quantities are the same. When cavitation occurs more dimensionless numbers enter, in particular the cavitation number σ , defined as

$$\sigma = \frac{p_\infty - p_v}{\frac{1}{2}\rho U^2}, \quad (2.2)$$

where, apart from the quantities already mentioned, p_∞ is the ambient pressure and p_v the vapour pressure of the liquid. In steady flow cavitation can occur when somewhere the pressure p sinks below p_v . On account of Bernoulli's Law the local velocity u takes for $p = p_v$ the value

$$u = U(1 + \sigma)^{1/2}. \quad (2.3)$$

The beginning of cavitation, incipient cavitation, is in fact more complicated since size and number of available microscopic nuclei are involved. We shall here be content to mark cavitation by a local pressure below vapour pressure.

Cavitation on a hydrofoil may appear as sheet cavitation, vortex cavitation or transient bubble cavitation. We shall not deal with vortex cavitation here. Once formed, cavitating regions must eventually disappear again. With that the emission of sound is associated. The way in which this takes place is the main subject of this contribution.

The spectral density of the noise emitted from a cavitating propeller has the general shape shown in figure 1. The intensity can be formidable, up to 120 - 140dB.

An excellent review of the state of the art in the fifties was given at the ONR Symposium in Washington in 1956, by Fitzpatrick & Strasberg (1956). They

Fig. 2. Cavity volume V , zero for $|t| > \tau$, constant for $|t| < \tau$.

explain, in discussing such spectra, the trend at low frequencies f , as f^4 , and at very high frequencies, as f^{-2} . On a large time scale the overall cavity growth and disappearance can regarding the cavity volume be viewed as a 'hat function', constant for times $|t| < \tau$, and zero otherwise, see figure 2.

In the far field the cavity acts as a monopole with strength Q , proportional to $V = dV/dt$, where V is the volume of the cavity, and potential

$$\varphi \sim \frac{Q(t - r/c_l)}{4\pi r}, \quad (2.4)$$

where r denotes distance from the cavity centre and where c_l is the sound velocity in the liquid. The Fourier transform of a cavity volume as in figure 2, $F\{V(t)\}$, behaves, with transform variable f , as

$$F\{V(t)\} \sim \frac{\sin f\tau}{f}. \quad (2.5)$$

The spectral density of a monopole behaves in the far field as

$$F\{\dot{Q}\}F^*\{\dot{Q}\} \sim f^4 \frac{\sin^2 f\tau}{f^2}.$$

For frequencies $f \ll \tau^{-1}$, this increases with f as f^4 , as pointed out by Fitzpatrick & Strasberg (1956), who also explained that the f^{-2} part of the spectral density is due to the very rapid pressure variations in the final stage of bubble collapse. Then, the collapse is arrested followed by pressure waves with very strong gradients (evolving into shock waves), outgoing from the collapse centre. The Fourier transform of the signal carried by such a wave is as f^{-1} , explaining the spectral density, behaving as f^{-2} and observed indeed in all cavitation spectra at frequencies well above 10 kHz.

Bubble collapse can also serve to explain the spectral density at slightly lower frequencies f .

Rayleigh, pioneering here as in many other areas, found that the rate of change $\dot{a}$ of the radius a of an empty cavity, collapsing under an external pressure p_∞ , is

$$\dot{a}^2 = \frac{2}{3} \frac{p_\infty}{\rho} \left\{ \left(\frac{a_0}{a} \right)^3 - 1 \right\}, \quad (2.6)$$

where a_0 is the initial radius of the spherical cavity. Denoting with t_c the time needed to reduce a to zero, integration of (2.6) gives

$$\frac{a}{a_0} = \frac{5}{2} \left(\frac{2p_\infty}{3\rho a_0^2} \right)^{1/2} (t - t_c)^{0.4}. \quad (2.7)$$

With reference to (2.4) and the line in the text preceding this, the source strength Q of the collapsing bubble is $4\pi a^2 \dot{a}$, causing a pressure $\rho(\dot{a}^2 \ddot{a} + \frac{3}{2} a \dot{a}^2)$ behaving in the time domain, using (2.7), as

$$(t - t_0)^{0.8},$$

corresponding to $f^{-0.2}$ in the frequency domain and leading to a spectral density behaving as $f^{-0.4}$.

In their 1956 essay, Fitzpatrick & Strasberg attempt to interpret the spectral density of cavitation noise in terms of results of single bubble collapse, as demonstrated above. Also later on, description of cavitation noise usually was made on the basis of single bubble collapse (e.g. Blake 1986).

It is not at all obvious that such a relation exists. It implies the assumption, as has been stressed most clearly by Baiter (e.g. Baiter et al. 1982), that cavitation noise is produced by single events (bubbles collapsing) randomly distributed in time. Only in that case is the spectrum determined by the spectrum of the single event and by the numbers in which these occur.

Since 1956, experimental and diagnostical techniques have been improved and developed, permitting detailed and sophisticated measurements and observations. These have cast doubt on the validity of the assumption of randomly occurring collapses.

3 The spectral density can not be explained on the basis of collapse of single bubbles

The first investigation relevant to the subject of this section is by Arakeri & Shanmuganathan (1985). If single bubble collapse determines the spectral density, then, they reasoned, a time typical for bubble collapse should be used for representing the spectral density in a non-dimensional way. With a maximum bubble radius R_M , this typical time is the collapse time t_c , given by (see 2.7)

$$t_c \sim R_M \left(\frac{\rho}{p_\infty - p_v} \right)^{1/2},$$

or, using the definition of the cavitation number σ ,

$$t_c \sim R_M / \sigma^{1/2} U. \quad (3.1)$$

Figure 3. Non-dimensional spectral densities with hydrogen-bubble nuclei for $E = 9$ V.

$U_\infty = 8.6$ m/s	$U_\infty = 10.8$ m/s	$U_\infty = 13.6$ m/s	$U_\infty = 17.0$ m/s
○, $\sigma = 0.57$	□, $\sigma = 0.60$	▲, $\sigma = 0.57$	○, $\sigma = 0.57$
	●, 0.57	×, 0.51	
	■, 0.48		
	△, 0.45		

The conditions for the inset photograph are: $U_\infty = 10.8$ m/s, $\sigma = 0.52$.

Experimental data on cavitation noise with geometrically similar bodies were plotted by Arakeri & Shanmuganathan (1985), using t_c to scale frequency. In particular they presented, see figure 3 and figure 4, the spectral density P as

$$P = P(\sigma, ft_c). \quad (3.2)$$

Scaling frequency with t_c brings order in the data but only for small values of ft_c .

The results of Arakeri and Shanmuganathan clearly show that for $ft_c \geq 1$ it is not possible to represent measurements of cavitation sound in one curve at constant σ (the influence of Re is assumed to be negligible). Arakeri & Shanmuganathan (1985) conclude that this points at interaction between collapsing bubbles, they use the term 'interference', quite in line with the prediction by H.J. Baiter and co-workers, who advocated to consider 'clusters of single events'.

In retrospect it is hard to believe that a spectrum as shown in figure 1 can be described completely in terms of bubble collapse, because for the broad plateau in the middle of the curve, single bubble collapse analysis offers no explanation.

(from Arakeri & Shanmuganathan 1985)

Figure 4. Non-dimensional spectral densities with hydrogen-bubble nuclei for $E = 20V$.

$U_\infty = 9.5 \text{ m/s}$	$U_\infty = 11.5 \text{ m/s}$	$U_\infty = 16.6 \text{ m/s}$
●, $\sigma = 0.59$	△, $\sigma = 0.59$	□, $\sigma = 0.59$
▲, 0.52	○, 0.55	■, 0.54
○, 0.48	×, 0.41	

The conditions for the inset photograph are: $U_\infty = 10.8 \text{ m/s}$, $\sigma = 0.52$.

Experimental work relevant to our subject was done at CalTech by Ceccio & Brennen (1991). They managed to measure sound emission from individual bubbles, in contrast to the integral measurements, i.e. coming from the whole cavitating region, in most other experimental work.

Their results bring interesting facts, for example bubbles may rebound and collapse again, whereby according to Ceccio & Brennen the first collapse contributes most to the sound production. An interesting point is that they find where Rayleigh's analysis predicts a spectral density varying with f as $f^{-0.4}$ (cf. the discussion earlier in this paper), instead a variation as $f^{-0.6}$. They point out that other experiments found an almost flat spectrum in this regime. Their warning is that it is hazardous to interpret integral measurements of bubble cavitation noise in terms of noise coming from single bubbles.

The evidence of collective effects also called interference effects or interaction effects, in the literature, has generated work on the response to pressure perturbations of clouds of bubbles.

Fig. 5. A cloud of gas bubbles, with initial radius A_0 , is hit by pressure perturbation.

4 The sound emitted by clouds of collapsing bubbles

With bubble clouds we are interested in the collective effects of bubble collapses. Van Wijngaarden (1964) investigated such effects in connection with cavitation damage. Further work on the collective collapse of bubbles was done by Chahine (1982) and Mørch (1982). Parallel with this equations of motion were developed for bubbly liquids based on averaging. Biesheuvel & Van Wijngaarden (1984) formulated average equations valid at low void fractions where, with number density n , the inter bubble distance $n^{-1/3}$ is large with respect to a representative mean bubble diameter. In such averaging it is necessary that a length scale exists which is large with respect to $n^{-1/3}$ but small with respect to the distance over which significant changes of the macroscopic quantities occur. For bubble clouds this means that the radius is large with respect to $n^{-1/3}$. Void fractions in cloud cavitation are of the order of 0.1%, which is low enough to permit the use of theories such as that by Biesheuvel and Van Wijngaarden (1984).

Application of these ideas, on collective collapse of clouds, to the specific case of sound emission by bubbly clouds was independently made by d'Agostino & Brennen (1983, 1989) and Omta (1987). Following the latter, consider the hydrodynamic phenomena occurring when, (see figure 5), a spherical cloud of radius A , containing many gas bubbles at initial pressure p_0 , and embedded in pure liquid, is hit by an approaching pressure wave with a step-like profile. While inside the cloud average two-phase equations hold, the perturbations outside the cloud can be described by the linear wave equation based on a sound velocity c_l . Taking into account conditions of continuity of normal velocity and of pressure at the interface between bubbly liquid and pure liquid, Omta (1987) calculated numerically pressure perturbations, in- and outside the cloud, both for linear and for nonlinear disturbances.

In figure 6 is shown the Fourier transform of the pressure disturbance, computed

Fig. 6. Numerical calculation by Omta (1987) of far field pressure, the figure shows the Fourier transform, resulting from a pressure disturbance hitting a cloud of gas bubbles: $p_0 = 10^3 Pa$, $a_0 = 4.6 \times 10^{-4} m$, $\alpha = 3 \times 10^{-2}$.

from linearized equations, in the far field for an initial pressure p_0 of $10^3 Pa$, a volume concentration of gas in the cloud $\alpha = 3 \times 10^{-2}$ and $A = 3.2 \times 10^{-2} m$, $a_0 = 4.6 \times 10^{-4} m$. Remarkable is the peak pressure at a frequency near 100 Hz. This can be nicely interpreted with help of two-phase flow theory. The cloud consists of fluid with density

$$\rho_m = \rho(1 - \alpha), \quad (4.1)$$

and sound velocity (see e.g. Van Wijngaarden 1972)

$$c_m = \left(\frac{\gamma p_0}{\alpha \rho} \right)^{1/2}, \quad (4.2)$$

where γ is the ratio of specific heats of the content of the bubble. For the numerical values associated with figure 6, $c = 6.8 m/s$, an extremely low value compared with the sound velocity $c_l = 1500 m/s$ in water, or $340 m/s$ in air. The cloud is compressed by the oncoming pressure wave and in addition supports waves at an infinite number of eigenfrequencies. In the far field there are outgoing waves in each of these frequencies. Inside the cloud, however, there is strong attenuation of the higher frequencies due to various dissipative mechanisms of which thermal conduction is the primary one. As Omta (1987) pointed out the attenuation is, unlike the sound speed, connected with the dimensions of the individual bubbles in the cloud. The far field spectrum is dominated by the lowest eigenfrequency, f_m , say, which is of the order of the reciprocal of the time needed for a sound wave to traverse the cloud.

$$f \approx \frac{c_m}{2A}. \quad (4.3)$$

In the example of figure 6, $c/2A = 107 s^{-1}$, which is close to the peak value in figure 6. Calculations similar to those by Omta (1987) were carried out by

Fig. 7. Fourier transform of a shock wave in the centre of a cloud of collapsing hydrogen bubbles, after passage of a shock wave $A = 4.5 \times 10^{-4} m$, $\alpha = 9.5 \times 10^{-5}$, $p_0 = 10^4 Pa$. (from Buist 1991). On the right (B): without bubbles.

d'Agostino & Brennen (1989), confirming and extending Omta's (1987) results. In this connection also the work by Yoon et al. (1991) should be mentioned.

Experimental verification of Omta's (1987) computational work was made by Buist (1991) in a special shock tube designed for this purpose. In this shock tube a cloud of hydrogen bubbles generated in water at low pressure by electrolysis, is hit by a shock wave advancing in the water outside the cloud.

In figure 7 the Fourier transform is shown of the pressure perturbation which is measured at the centre of the cloud, with $A = 4.5 \times 10^{-2} m$, $\alpha = 9.5 \times 10^{-5}$ and $p_0 = 10^4 Pa$. The over all features agree with the theoretical prediction. In particular, the peak in the spectrum of the measured pressure is at a frequency close to the value given by (4.3).

Of course, under practical circumstances, bubbles contain vapour, apart from several gases gaining entrance by diffusion. Klaseboer & de Bruin (1992) have investigated the influence of condensation. They find that, unless perhaps at the very end of collapse, the vapour has its equilibrium pressure. The work by Klaseboer & de Bruin (1992) shows that the presence of vapour changes phenomena only quantitatively. Qualitatively it remains the same. At very high frequencies, single bubble collapse dominates, and there is definitely a peak at a frequency connected with cloud collapse. The velocity of sound is given by (4.2) with, for p_0 , the initial gas pressure.

In view of the results summarized in Section 3 and Section 4, one can conclude that part of the sound production of cavitation is due to clouds of bubbles rather than individual ones. The question then arises what the mechanism is by which clouds are formed and what controls their dimensions. Answers to these questions would also permit to establish scaling rules for cavitation.

Fig. 8. Partial cavitation on a two dimensional hydrofoil.

5 Cavitation sheet on thin hydrofoils

We shall discuss the questions raised at the end of the preceding section for the case of sheet cavitation on thin hydrofoils under steady external conditions.

Consider the situation of figure 8, partial cavitation on a two dimensional hydrofoil under steady incidence with a stream of constant velocity U . An interesting feature is that the continuum cavity equations of fluid mechanics and the associated conditions at the contact line have no steady solution.

In the framework of potential flow theory, the reason for this is that in S where the cavity closes the fluid velocity cannot be zero and have the free streamline value at the same time. One might think to be able to remove this anomaly by allowing for viscosity and surface tension. However, the conditions in S become more numerous (continuity of stresses) and a steady solution remains impossible.

In classical cavitation theory singular behaviour is admitted (Geurst 1959). Today, in numerical calculations, the kinematic condition at the interface between vapour and liquid is respected, but the continuity of pressure in the point of attachment is ignored. In consequence of this, the pressure violently changes in the results of numerical calculations. What happens in reality? Modern diagnostic tools as high speed cinematography and high speed video have revealed that, in line with the absence of a steady solution, partial cavitation is an unsteady phenomenon. A good description is in the book by Young (1989). Another clear description can be found in a recent paper by Le et al. (1993), who investigated partial cavities primarily in the context of measuring pressure distributions. In poster Session II of this Symposium De Lange (1993) shows some results of his work on the behaviour of partial cavities. This work is done in our group in order to learn more about the mechanism of cavitation sound.

From all these sources the following picture arises, see figure 9, taken from a report by Van der Stegen (1993). In 1 and 2 the cavity grows, reaching maximum length in 3. At this time the celebrated 'reentrant jet' appears, which was already discussed by Knapp (1970) but the mechanism of which is still unknown. The reentrant jet penetrates the cavity and leads to detachment of a larger part of the cavity. This is as a cloud of cavitation bubbles convected with the main stream and collapses further downstream, 5 and 6, in the schematical representation in figure 9. Meanwhile the cavity starts to grow again and the whole cycle is repeated.

Fig. 9. Growth and detachment of cavitation sheet, from Van der Stegen (1993).

Because of this extremely unsteady behaviour one can not speak about 'the length' of the cavity, at most one can indicate and average, or mean, length, l , say. It seems, according to Buist (1991) that numerical computations, as outlined earlier, produce reasonably well this mean length.

If this is the general picture, understanding of the way in which these cyclic events are generated, is of interest in its own right, and is of vital importance in the present context. Among others, it may provide insight in the parameters which control the dimensions of the cloud, the diameter A in the case these may be considered as spherical.

Dimensional analysis shows that, if the repetition frequency of the cycle depicted in figure 9 is $f_c s^{-1}$, there should be a dimensionless number, similar to a Strouhal number

$$S = \frac{lf_c}{U} = S(Re, \dots) \quad (5.1)$$

Both the work by De Lange (1993) and by Le et al. (1993) shows that S is probably only weakly dependent on Re and other parameters, because they find, under different Re numbers, both a value for S of about 0.3

The problem now is how to explain the sequence of events as in figure 9. Hopefully more will be known on this in the near future. Cavitation enjoys quite some attention nowadays, as the many contributions related to this subject during this Symposium testify.

As this lecture shows, cavitation sound is closely related to other aspects of cavitation phenomena, so one can hope that along with progress in the general understanding of cavitation, insight in the noise aspect will increase as well.

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