A metamodel based optimisation algorithm for metal forming processes

M.H.A. Bonte — A.H. van den Boogaard
J. Huétink

University of Twente, Faculty of Engineering Technology
P.O. Box 217
7500 AE Enschede, The Netherlands
m.h.a.bonte@utwente.nl; a.h.vandenboogaard@utwente.nl; j.huetink@utwente.nl

ABSTRACT. *Cost saving and product improvement have always been important goals in the metal forming industry. To achieve these goals, metal forming processes need to be optimised. During the last decades, simulation software based on the Finite Element Method (FEM) has significantly contributed to designing feasible processes more easily. More recently, the possibility of coupling FEM to mathematical optimisation algorithms is offering a very promising opportunity to design* optimal *metal forming processes instead of only* feasible *ones. However, which optimisation algorithm to use is still not clear.*

In this paper, an optimisation algorithm based on metamodelling techniques is proposed for optimising metal forming processes. The algorithm incorporates nonlinear FEM simulations which can be very time consuming to execute. As an illustration of its capabilities, the proposed algorithm is applied to optimise the internal pressure and axial feeding load paths of a hydroforming process. The product formed by the optimised process outperforms products produced by other, arbitrarily selected load paths. These results indicate the high potential of the proposed algorithm for optimising metal forming processes using time consuming FEM simulations.

KEYWORDS: *Optimisation, Metal forming, Finite Element Method, Metamodelling, Hydroforming*

1. Introduction

During the last decades, Finite Element (FEM) simulations of metal forming processes have become important tools for designing feasible production processes. In more recent years, several authors recognised the potential of coupling FEM simulations to mathematical optimisation algorithms to design *optimal* metal forming processes instead of only *feasible* ones.

The basic concept of mathematical optimisation is presented in Figure 1. Basically, it consists of two major phases: the *modelling* and the *solving* of the optimisation problem. The modelling phase consists of:

1. Selecting a number of design variables the user is allowed to adapt
2. Choosing an objective function, i.e. the optimisation aim
3. Taking into account possible constraints

These three items are closely related to each other as depicted in Figure 1. Both the objective function and the constraints should be quantified by the design variables. The objective function and constraints are also related to each other in the sense that they are often exchangeable. Consider for example that we would like to make a metal formed product and two relevant properties are the product quality and the costs. Then two approaches can be followed: either the quality is maximised while putting a certain limit on the allowed production costs, or the costs could be minimised while ensuring a certain minimum level of the product quality. In the former case, the quality is clearly the optimisation objective and the costs are constraints, whereas it is just the other way around in the latter case.

Next to the modelling phase, mathematical optimisation's second phase is solving the optimisation problem. This comprises applying an optimisation algorithm to the modelled optimisation problem. The arrows between the modelling and the solving parts in Figure 1 denote that both phases cannot be seen separately from each other. One should select the right optimisation algorithm for a certain modelled optimisation problem and one should model the optimisation problem cleverly to adjust it to the optimisation algorithm one is planning to apply. If the optimisation model does not match the algorithm, it is likely that the optimisation problem is not solved efficiently or cannot be solved at all [Pap00].

This paper focuses on the solving part of optimisation problems in metal forming using time consuming nonlinear FEM simulations. One simulation can easily take hours or even days to execute. It is important to keep this fact in mind when selecting a suitable optimisation algorithm for metal forming processes.

One way of optimising metal forming processes is using classical iterative optimisation algorithms (Conjugate gradient, BFGS, etc.), where each function evaluation means running a FEM calculation, see e.g. [Kle03, Lin03, Nac01]. As mentioned

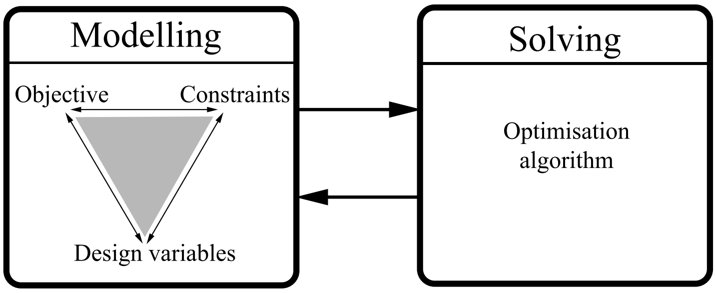


Figure 1. *The basic concept of mathematical optimisation: modelling and solving*

above, in case of metal forming these FEM calculations can be extremely time consuming and need to be sequentially evaluated. Furthermore, many classical algorithms require sensitivities, of which the efficient calculation is not straightforward for FEM simulations. A third difficulty concerning iterative algorithms is the risk to be trapped in local optima.

Alternatively, several authors have tried to overcome these disadvantages by applying genetic or evolutionary optimisation algorithms, see e.g. [Cas04, Fou05, Sch04]. Genetic and evolutionary algorithms look promising because of their tendency to find the global optimum and the possibility for parallel computing. However, the rather large number of function evaluations that is expected to be necessary using these algorithms is regarded as a serious drawback [Emm02].

Yet another way of optimisation in combination with expensive function evaluations is using approximate optimisation algorithms, of which Response Surface Methodology (RSM) is a well-known representative. RSM is based on fitting a low order polynomial metamodel through response points, which are obtained by running FEM calculations for carefully chosen design variable settings and finally optimising this metamodel [Mye02]. Metamodels are sometimes also referred to as Response Surface models or surrogate models. Allowing for parallel computing and lacking the necessity for sensitivities, RSM is appealing to many authors in the field of metal forming, see e.g. [Jan02, Jan05, Nac04].

Although the practical effectiveness of RSM has been frequently demonstrated, statisticians claim that RSM, being developed for stochastic physical experiments, is theoretically not applicable to deterministic computer experiments such as FEM: running a simulation twice with exactly the same input will generally result in exactly the same answer. They propose the field of “Design and Analysis of Computer Experiments” or DACE instead [Sac89a, Sac89b, San03]. DACE is similar to RSM, but interpolates a metamodel through the response points. Allowing for no error at the response points, interpolation better suits the deterministic nature of computer experiments. However, DACE is rarely used for metal forming problems, probably due to its complex statistical nature and the lack of readily available software [San03].

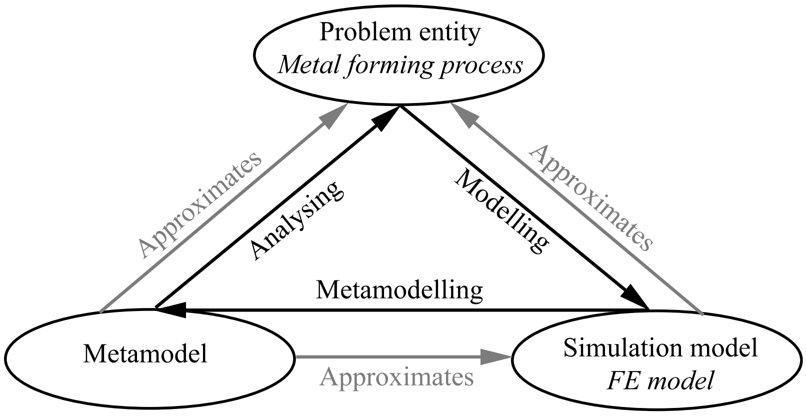


Figure 2. *The principle of metamodelling*

In this paper an optimisation algorithm incorporating both RSM and DACE metamodelling techniques is proposed for metal forming. Section 2 introduces the basic concept of metamodelling and provides a more detailed description of RSM and DACE. The proposed optimisation algorithm is presented in Section 3 and the applicability to metal forming is demonstrated in Section 4 where it is applied to the optimisation of a hydroforming process. Conclusions are presented in Section 5.

2. Metamodelling

The principle of metamodelling is presented in Figure 2 [Kle00]. The basic idea is to evaluate a certain problem entity, in our case a metal forming process. This problem entity can be modelled by some sort of a simulation model. For metal forming, this simulation model is usually a nonlinear Finite Element Model (FEM). These nonlinear FEM calculations are very time consuming to evaluate. Therefore, a *metamodel* or a *model from a model* [Sim01] is made, which can be quickly evaluated. An accurate metamodel should be valid with respect to both the Finite Element Model and the metal forming process and if it is, it forms a very useful substitute for both the process and the FE model.

Kleijnen and Sargent distinguish four goals that can be served by metamodelling [Kle00]: (i) Understanding the problem entity, (ii) Predicting values of the output or response variable, (iii) Optimisation and (iv) Verification and Validation of prior qualitative knowledge on the simulation model with respect to the problem entity. For the optimisation of metal forming processes, the primary interest lies in optimisation as a metamodelling goal. However, the other goals additionally come at no or low computational costs, which is seen as a major advantage of using metamodelling techniques for optimisation purposes.

In the next sections, two metamodelling techniques, Response Surface Methodology and Design and Analysis of Computer Experiments or Kriging, are shortly introduced. Prior to fitting the metamodels, a Design Of Experiments (DOE) strategy carefully selects a number of design variable settings for which FEM simulations are being run. These simulations provide a number of response measurements.

2.1. *Response Surface Methodology (RSM)*

Starting with RSM, the response measurements **y** are presented as the sum of a lower order polynomial metamodel and a random error term ε [Mye02]:

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon \qquad [1]$$

where **X** is the design matrix containing the experimental design points and β are the regression coefficients obtained by least squares regression:

$$\hat{\beta} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y} \qquad [2]$$

Although Equation 1 seems to be a linear relation of the design variables, the design matrix **X** can also incorporate terms that are nonlinear with respect to the design variables. Equation 1 should, however, be linear with respect to the regression coefficients [Mye02], which is clearly the case.

The metamodel is

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\beta} \qquad [3]$$

Four possible shapes are commonly applied. They are in ascending complexity:

- linear
- linear + interaction
- pure quadratic or elliptic
- (full) quadratic

A second order RSM metamodel dependent on one design variable is shown in Figure 3(a). The cross marks represent the response measurements.

2.2. *Design and Analysis of Computer Experiments (DACE)*

DACE was proposed by Sacks et al. [Sac89a, Sac89b] to fit metamodels using deterministic computer experiments. *Kriging* is used to interpolate between the response measurements. Using Kriging, the random error term ε in Equation 1 is replaced by

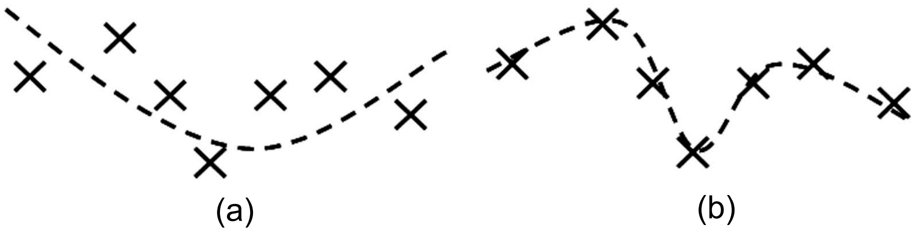


Figure 3. *Metamodels based on (a) RSM and (b) DACE*

a Gaussian random function $Z(\mathbf{x})$, which forces the metamodel to exactly go through the measurement points:

$$\hat{\mathbf{y}} = \mathbf{X}\beta + Z(\mathbf{x}) \qquad [4]$$

The first part of Equation 4 covers the global trend of the metamodel. The Gaussian random function Z, which accounts for the local deviation of the data from the trend function, has zero mean, variance σ_z^2 and covariance

$$\mathrm{cov}\left(Z(x_1), Z(x_2)\right) = \sigma_z^2 R\left(x_1 - x_2\right) \qquad [5]$$

where R is the correlation function and x_1 and x_2 are two locations, which are determined by the design variable settings at these locations. For the proposed algorithm, a Gaussian exponential correlation function is adopted:

$$R(\vartheta, x_1, x_2) = \exp^{-\vartheta(x_1 - x_2)^2} \qquad [6]$$

As opposed to other possibilities for the correlation function like e.g. cubic splines and ordinary exponential functions, see e.g. [Koe96, Lop02b, Lop02a, San03], Gaussian exponential functions are intuitively attractive because they are infinitely differentiable. Moreover, Gaussian exponential functions are frequently used in literature [San03] and have been found to give accurate results [Lop02a].

Assume k design variables are present. Then the total correlation function R depends on the k one-dimensional correlation functions R_j as follows [Sac89a]:

$$R\left(x_1 - x_2\right) = \prod_{j=1}^{k} R_j\left(x_{1j} - x_{2j}\right) \qquad [7]$$

This implies that it is assumed that there is no relation between the different dimensions. Adopting the Gaussian correlation function introduced in Equation 6, the total correlation function becomes:

$$R\left(x_1 - x_2\right) = \prod_{j=1}^{k} \exp^{-\vartheta_j (x_{1j} - x_{2j})^2} \qquad [8]$$

Thus, one ϑ is present for each design variable (each dimension).

Figure 3(b) presents a Kriging interpolation metamodel, which is fitted through the same response measurements as the RSM metamodel in Figure 3(a). Note the differences: the Kriging metamodel's shape is more complex than the second order polynomial shape of the RSM metamodel. However, the Kriging metamodel interpolates through the response points, whereas the RSM metamodel allows for random error. As was stated in the introduction, it is argued whether allowing for random error is appropriate in case of deterministic computer experiments such as FEM.

3. A metamodel based optimisation algorithm for metal forming

The proposed metamodel based optimisation algorithm for the optimisation of metal forming processes using time consuming FEM simulations is presented in Figure 4. Several steps mentioned in the figure are explained in the Sections 3.1 through 3.4. Section 3.5 contains a few words on the implementation of the algorithm.

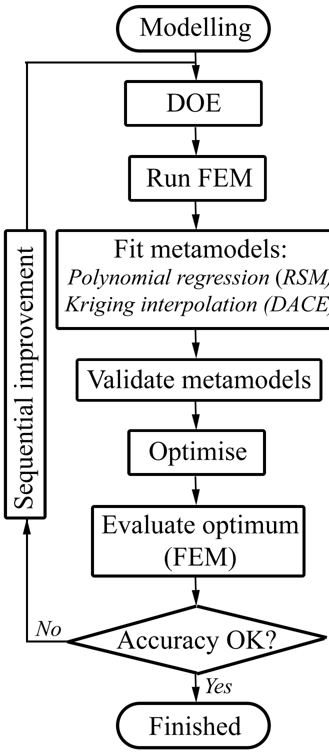


Figure 4. *A metamodel based optimisation algorithm for metal forming processes*

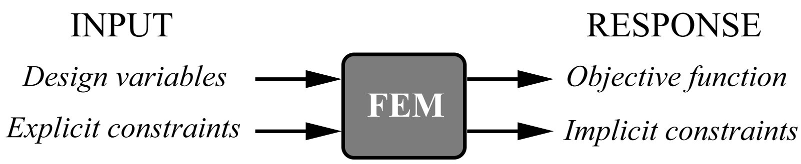


Figure 5. *FEM as an input–throughput–response model*

3.1. *Modelling*

The first step is to start with modelling the optimisation problem, i.e. quantifying objective function and constraints and selecting the design variables. Regarding the constraints, a distinction is made between explicit and implicit constraints. To explain the difference, running a FEM simulation can be seen as an input-throughput-response model such as the one depicted in Figure 5. Certain quantities are known beforehand: there is no necessity to run a FEM calculation for evaluating them. The design variables are clear examples of these quantities and there can also be constraints that explicitly depend on the design variables. These constraints are called explicit constraints. In case of metal forming explicit constraints are related to the undeformed product, e.g. constraints on the initial shape of a blank.

Quantities that depend on the response require a FEM simulation for evaluating them: they implicitly depend on the design variables. The objective function is generally such an implicit quantity and it is also possible to have implicit constraints. For metal forming, implicit constraints are related to the deformed product, e.g. excessive thinning is not allowed to exceed a specified limit.

It is stressed again that the modelling of an optimisation problem is formally not part of an optimisation algorithm: the algorithm is solely a mean for solving the optimisation model. Clever modelling and solving are both crucial for mathematically optimising a problem as was already emphasised in the introduction.

3.2. *Design Of Experiments (DOE)*

When the optimisation problem has been modelled, Figure 4 shows that the first step of the algorithm is to carefully select a number of design sites by a Design Of Experiments (DOE) strategy. A spacefilling Latin Hypercube Design (LHD) is a good and popular DOE strategy for constructing metamodels from deterministic computer experiments [McK79, San03] and has been selected for the optimisation of metal forming processes.

However, when a metamodel is used for optimisation, it is important that the metamodel gives accurate results in the neighbourhood of the optimum. Often, this optimum will be constrained, i.e. lies on the boundary of the design space. Therefore, an accurate prediction is needed on the boundary, which implies performing measure-

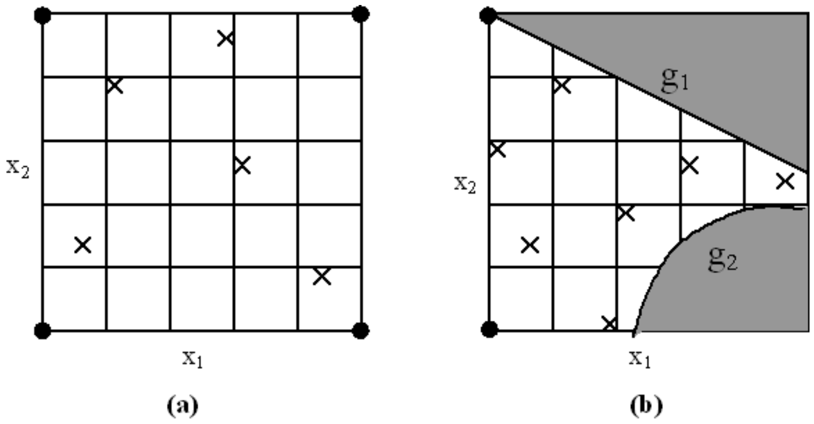


Figure 6. *(a) LHD + full factorial design (b) LHD + full factorial design including explicit constraints*

ments on that boundary. An LHD will generally provide design points in the interior of the design space and not on the boundary. To compensate for this lack of points on the boundary, the LHD is combined with a full factorial design, which puts DOE points right in the corners of the design space. This method was also proposed by Van Beers et al. [vB04] and Kleijnen et al. [Kle04]. Figure 6(a) presents the LHD modified with a full factorial design for a two dimensional rectangular design space.

Unfortunately, the design space will often not be rectangular when explicit constraints are present. In this case, the proposed algorithm will:

1. check which points of the LHD + full factorial design are non-feasible
2. skip the non-feasible points
3. replace the non-feasible points with new points
4. repeat the above procedure until all points are feasible

Replacing the non-feasible points is also done in a spacefilling way by selecting a large number of sets of additional design points. The new set of points is the one for which the minimum point to point distance is maximised. This so-called *maximin* criterion is used for both the initial DOE and for the case when the user wants to generate additional experimental design points, for example for improving the accuracy of the metamodels. The final DOE strategy incorporated in the proposed optimisation algorithm is presented in Figure 6(b) for two design variables (x_1 and x_2) and two explicit constraints (g_1 and g_2).

3.3. *Running the FEM simulations and fitting the metamodels*

Subsequently, using the settings indicated by the DOE strategy, a number of FEM calculations is run on parallel processors and the response points (objective function and implicit constraint values) are obtained. Following Figure 4, the next step is to fit for each response seven metamodels:

1. A linear polynomial using RSM
2. A linear + interaction polynomial using RSM
3. A pure quadratic or elliptic polynomial using RSM
4. A full quadratic polynomial using RSM
5. A Kriging interpolation metamodel with a 0^{th} order polynomial as a trend function
6. A Kriging interpolation metamodel with a 1^{st} order polynomial as a trend function
7. A Kriging interpolation metamodel with a 2^{nd} order polynomial as a trend function

3.4. *Validation and optimisation*

Metamodel validation based on cross validation (see e.g. [Mar03]) is used to select the best metamodel for the observed response. Using cross validation, one leaves out one, say the i^{th}, of the response measurements and fits the metamodel through the remaining response measurements. The difference between the real value y_i and the value predicted by the metamodel at this location $\hat{y}_{-i}$ is a measure for the accuracy of the metamodel. One can repeat this procedure for all say n measurement points and calculate the cross validation Root Mean Squared Error (RMSE_{CV}):

$$\text{RMSE}_{\text{CV}} = \sqrt{\sum_{i=1}^{n} \frac{(y_i - \hat{y}_{-i})^2}{n}} \qquad [9]$$

As RMSE_{CV} approaches 0, the metamodel becomes more and more accurate. Cross validation can also be visualised in a cross validation plot. An example of such a plot is presented in Figure 7. If the measurements follow the line $x = y$, the metamodel fits the data well.

For each response (objective function and implicit constraints) the metamodel outperforming the other six metamodels is selected. These best metamodels for objective function and implicit constraints are added to the explicit constraints in the optimisation model, which is subsequently optimised using a standard Sequential Quadratic

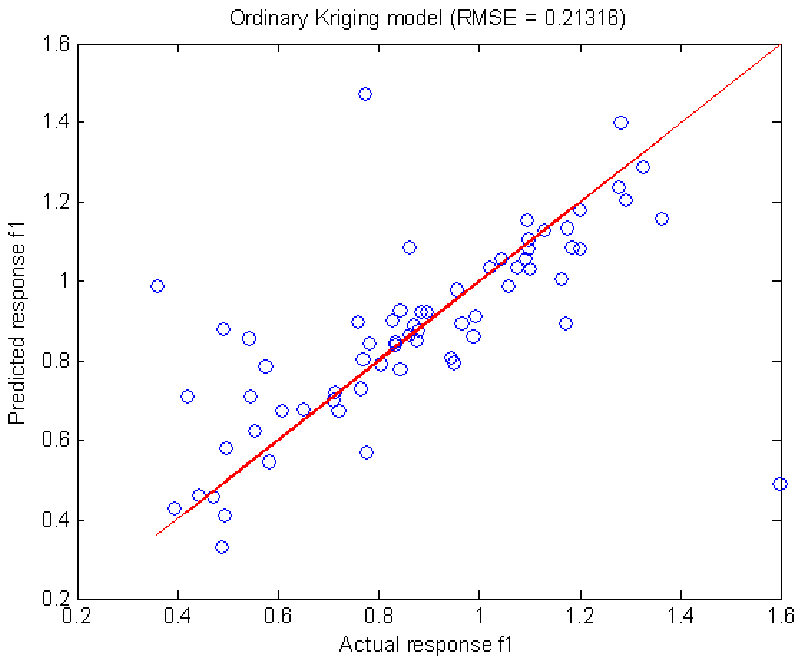


Figure 7. *A cross validation plot*

Programming (SQP) algorithm, see for example [Haf92]. In case constraints or Kriging metamodels are present in the final optimisation problem, there is a risk of ending up in a local optimum. This problem is overcome by initialising the SQP algorithm at multiple locations. This implies performing many function evaluations, but this is hardly a problem since both RSM and DACE metamodels, being explicit mathematical functions, can be evaluated thousands of times within a second. The DOE points are used as initial locations for the SQP algorithm.

The obtained approximate optimum is finally checked by running one last FEM calculation with the approximated optimal settings of the design variables. The difference between the approximate objective function value and the real value of the objective function calculated by the last FEM run is a measure for the accuracy of the obtained optimum. If the user is not satisfied with this accuracy, the algorithm allows for sequential improvement (e.g. zooming near the optimum) and repeating the procedure presented above until one is satisfied with the accuracy. Hence the proposed algorithm incorporates all the advantages of sequential approximate optimisation algorithms.

3.5. *Implementation*

The optimisation algorithm presented in Figure 4 and the previous sections was implemented in MATLAB and can be used in combination with any Finite Element code. For the fitting of the DACE/Kriging metamodels, use was made of the MATLAB Kriging toolbox implemented by Lophaven, Nielsen and Søndergaard [Nie, Lop02b, Lop02a].

4. A metal forming application

The optimisation algorithm introduced in the previous section is applied to a simple hydroforming process. The product to be hydroformed is presented in Figure 8(a). Figure 8(b) presents the dimensions.

For metal forming, several groups of design variables can be distinguished:

1. Geometrical parameters:
   (a) Final product geometry
   (b) Initial workpiece geometry
   (c) Tool geometry
2. Material parameters
3. Process parameters

The group of geometrical parameters is divided further into variables belonging to the final, deformed product, e.g. product radii, thicknesses, etc., variables related to the initial, underformed workpiece (blank shape, blank thickness, etc. and variables related to the tool geometry (drawbeads, tool radii and so on). Examples of material parameters are strain hardening coefficients, the initial yield stress or simply several discrete materials in itself. The group of process parameters includes process forces, pressures, tool displacements, friction coefficient, process temperature, etc.

For the simple metal forming example considered here, we are interested in optimising the time variation of the internal pressure p and axial feeding u. These are typically process parameters for the hydroforming process. A typical time dependent load path for hydroforming is shown in Figure 8(c). The velocity of pressure increase α is set to $10\,\mathrm{MPa/s}$ during a total hydroforming time of $10\,\mathrm{s}$. Hence, three design variables remain: the time when axial feeding starts t_1, the time when axial feeding stops t_2 and the total amount of axial feeding $u_{\max}$.

As an optimisation objective, it was chosen to minimise deviations from the initial tube wall thickness. One implicit and one explicit constraint were formulated. The implicit constraint ensures that the final product fills out the die nicely, the explicit constraint makes sure that the time when axial feeding stops is larger than the time

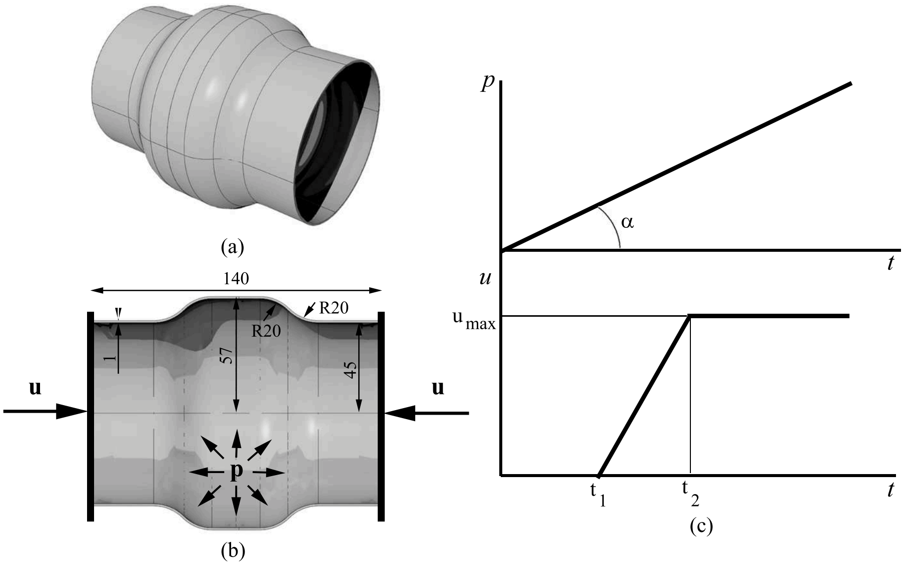


Figure 8. *(a) A simple hydroformed product; (b) Dimensions; (c) Typical load paths for hydroforming*

when it starts. Convergence problems of the FEM simulations have been encountered when t_2 approaches t_1 and the amount of axial feeding is high (large $u_{\max}$). Methods to handle non converged simulations are lacking and is a field of open research. An extra explicit constraint has been formulated to overcome the convergence problems. The second explicit constraint makes the first explicit constraint redundant, as one can see in the model of the optimisation problem:

$$
\begin{aligned}
\min f\left(t_1, t_2, u_{\max}\right) &= \left\| \frac{h - h_0}{h_0} \right\|_2 \\
\text{s.t.} \qquad g_{\text{impl}} &= V \leq 0 \\
g_{\text{expl1}} &= t_1 - t_2 \leq 0 \\
g_{\text{expl2}} &= u_{\max} - 9\left(t_2 - t_1\right) \leq 0 \qquad [10] \\
0\,\text{s} &\leq t_1 \leq 5\,\text{s}
\end{aligned}
$$

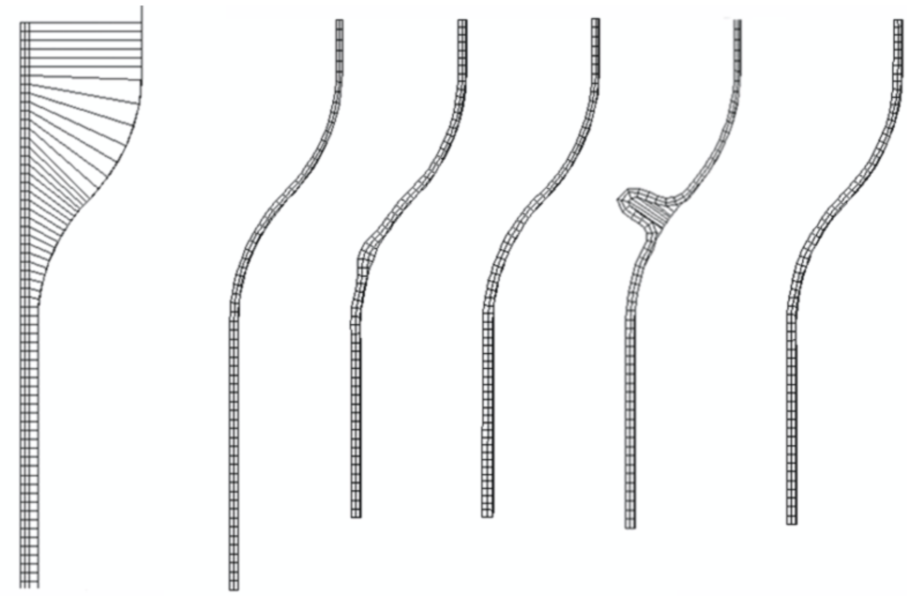


Figure 9. *(a) FE model of the initial tube; (b-e) Final product formed with several arbitrary selected load paths; (f) Final product formed with optimised load paths*

$$3\,\mathrm{s} \leq t_2 \leq 10\,\mathrm{s}$$

$$0\,\mathrm{mm} \leq u_{\mathrm{max}} \leq 9\,\mathrm{mm}$$

where h is the final wall thickness in the hydroformed product, h_0 is the wall thickness of the initial tube and V is the volume between the final product and the die. If this volume is larger than zero, there is a gap between the final product and the die and the final shape of the product is not satisfactory.

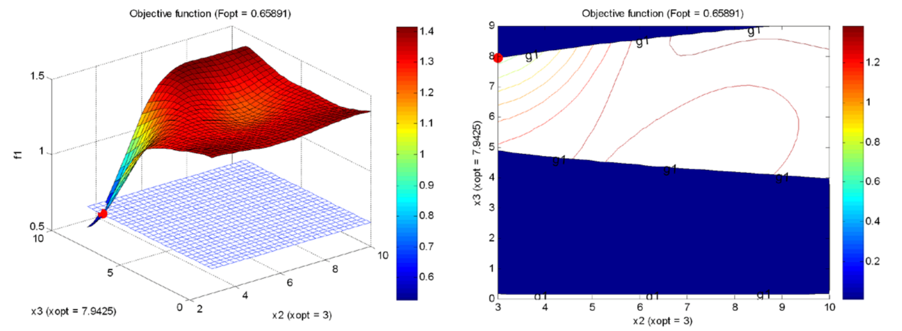
The 2D FE model of the axisymmetric part is presented in Figure 9(a). The contact between the product and the die is modelled by contact elements using a penalty formulation. The calculations were performed on 16 parallel processors, which limited the total time to the run time of one calculation, i.e. a couple of minutes for the 2D FE model we are considering. Before the optimisation algorithm is applied, we will first arbitrarily select some combinations of the design variables and investigate the effect on the final product, the objective function and the implicit constraint. Subsequently, the algorithm is applied and the optimised results are compared to the results obtained with the arbitrarily selected load paths.

Product	$t_1(\mathrm{s})$	$t_2(\mathrm{s})$	$u_{\max}(\mathrm{mm})$	f	g_{impl}
(a)	–	–	–	0	–
(b)	0	0	0	1.39	-0.29
(c)	0	3	9	0.52	1.79
(d)	0	10	9	1.42	-0.34
(e)	4.8	6.2	7.7	1.37	32.64
(f)	0	2.5	8.3	0.37	-0.47

Table 1. *Design variable settings and response values*

Figures 9(b) through (e) present the final products deformed with the arbitrarily selected load paths. The design variable settings for t_1, t_2 and $u_{\max}$ and the response values for the objective function f and the implicit constraint g_{impl} are presented in Table 1. Note that product (a) is the initial undeformed product, which is seen as the product with the perfect wall thickness distribution by the objective function quantified in Equation 10. For the perfect product, the objective function equals 0. Also note that products (c) and (e) do not satisfy the implicit constraint g_{impl}, which can also clearly be seen from Figures 9(c) and (e).

The metamodel based optimisation algorithm presented in Section 3 is now applied to optimise the wall thickness distribution. The DOE strategy introduced in Section 3.2. was applied to generate 16 initial design variable settings for which 16 FEM calculations were performed with the in-house code DiekA. Subsequently, the four Response Surfaces and three Kriging metamodels were fitted for both responses (the objective function and the implicit constraint). Based on cross validation, a 0^{th} order Kriging metamodel appeared to be the most accurate metamodel for the objective function. In a similar way, a 1^{st} order Kriging metamodel was identified to be most accurate for the implicit constraint. Both were included in the optimisation problem,



(a) (b)

Figure 10. *(a) Metamodel of the objective function after batch 1; (b) Contour plot of the objective function after batch 1*

# FEM	$t_1(\text{s})$	$t_2(\text{s})$	$u_{\max}$	f_{opt}	f_{actual}
16	0	3.0	7.9	0.66	0.47
32	0	1.5	7.9	0.37	—
48	0.8	2.5	9.0	-0.06	0.55
64	0	3.0	7.7	0.31	0.50
80	0	3.1	8.2	0.37	0.49
96	0	2.5	8.3	0.30	0.37

Table 2. *Optima of the DoC after the 6 batches of 16 FEM calculations each*

which was subsequently solved using the multistart SQP algorithm described in Section 3.4.

Several local optima were observed; the global optimum was located at $(t_1, t_2, u_{\max}) = (0, 3, 7.9)$ and the corresponding objective function value was observed to be 0.66. Figure 10(a) shows the approximate optimum located on the metamodel of the objective function. The metamodel is depicted dependent on $x_2 = t_2$ and $x_3 = u_{\max}$ at the constant, optimal level of $t_1 = 0$. Figure 10(b) presents a contour plot of the objective function and the constraints, where one can easily observe that the optimum is constrained by the implicit constraint and the box constraint $t_2 \geq 3$.

To validate the optimum, a FEM calculation was performed with the optimal design variable settings. The actual objective function value was found to be 0.47. The large difference between the approximate and the actual objective function values at the approximate optimum motivates to sequentially improve the results. Making use of the metamodel visualised in Figure 10, it was decided to zoom in near the optimum and to relax the box constraint $t_2 \geq 3$: it was replaced by the new box constraint $t_2 \geq 2.5$.

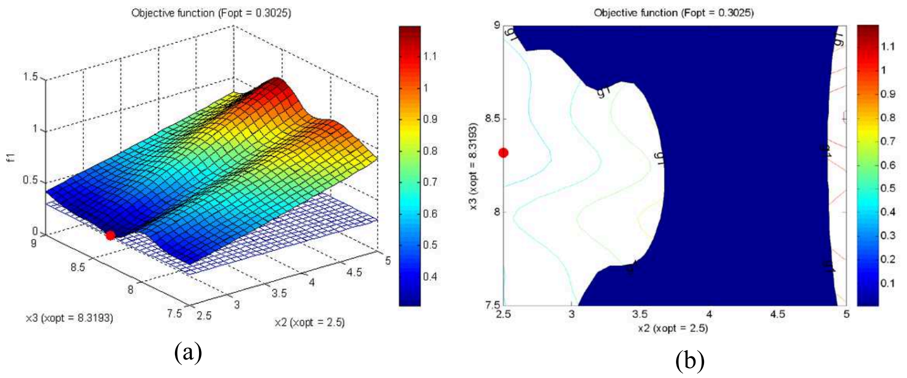


Figure 11. *(a) Metamodel of the objective function after batch 6; (b) Contour plot of the objective function after batch 6*

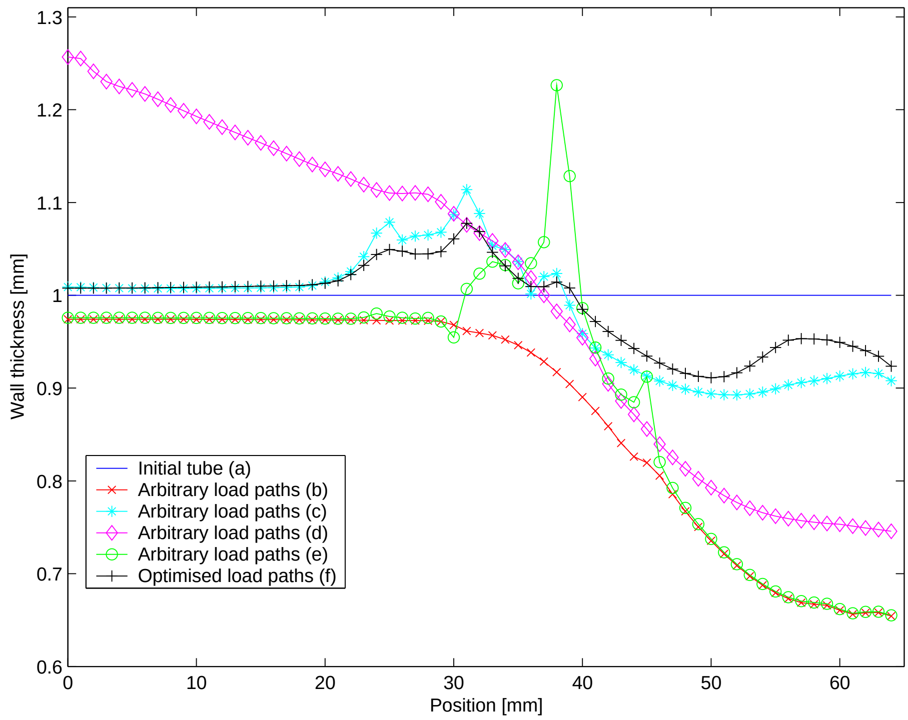


Figure 12. *Wall thickness distribution of several hydroformed products*

In total, six batches of each 16 FEM calculations were performed. Each time, the design space was reduced and/or shifted with the aim to fit accurate metamodels in the vicinity of the optimum. The results of all batches are presented in Table 2.

Figure 11 presents the metamodel of the objective function as well as a contour plot of the objective function after having run the sixth batch of simulations. The obtained optimum $(t_1, t_2, u_{\max}) = (0, 2.5, 8.3)$ is constrained by the box constraint $t_2 \geq 2.5$. A final 97$^{\text{th}}$ FEM calculation was performed with the optimised design variable settings. This calculation resulted in a real objective function value of 0.37, which is fairly close to the approximate objective function value of 0.30. Although there is still a small difference between the approximate and the actual value of the objective function, it was decided to be satisfied with the approximate optimum obtained by the metamodel based optimisation algorithm.

The optimised settings found by the proposed optimisation algorithm are presented in Table 1 as product (f). The final shape of this product is shown in Figure 9(f). Figure 12 shows the wall thickness throughout the final product for all load paths. It can be concluded from the Figures 9 and 12 and Table 1 that the product deformed with the optimised load paths outperforms the other products formed with arbitrary settings, which demonstrates the good applicability of the proposed algorithm to metal forming.

5. Conclusions

An optimisation algorithm based on metamodelling techniques is proposed for the optimisation of metal forming using time consuming FEM calculations. It uses both Response Surface Methodology and DACE (or Kriging) as metamodelling techniques. As a Design Of Experiments strategy, a combination of a maximin spacefilling Latin Hypercubes Design with a full factorial design was implemented, which takes into account explicit constraints. Additionally, the algorithm incorporates cross validation as a metamodel validation technique and uses a Sequential Quadratic Programming algorithm for metamodel optimisation. To overcome the problem of ending up in a local optimum, the SQP algorithm is initialised from every DOE point, which is very time efficient since evaluating the metamodels can be done within a fraction of a second. The proposed algorithm allows for sequential improvement of the metamodels to obtain a more accurate optimum.

As an example case, the optimisation algorithm was applied to obtain the optimised internal pressure and axial feeding load paths to minimise wall thickness variations in a simple hydroformed product. The product formed with optimised load paths outperforms several products formed with arbitrarily chosen load paths, which demonstrates the good applicability of metamodelling techniques to optimise metal forming processes.

Acknowledgements

This work is conducted within the framework of project MC1.03162, Optimisation of Forming Processes, which is part of the research programme of the Netherlands Institute for Metals Research (NIMR). The NIMR and its industrial partners are gratefully acknowledged for their support and useful input.

6. References

[Cas04] CASTRO, C., ANTÓNIO, C., SOUSA, L., "Optimisation of shape and process parameters in metal forging using genetic algorithms", *International Journal of Materials Processing Technology*, Vol. 146, 356–364, 2004.

[Emm02] EMMERICH, M., GIOTIS, A., ÖZDEMIR, M., BÄCK, T., GIANNAKOGLOU, K., "Metamodel-assisted evolution strategies", in *Proceedings of the International Conference on Parallel Problem Solving from Nature*, 2002.

[Fou05] FOURMENT, L., DO, T., HABBAL, A., BOUZAIANE, M., "Gradient, non-gradient and hybrid algorithms for optimizing 2D and 3D forging sequences", in *Proceedings of ESAFORM*, Cluj-Napoca, Romania, 2005.

[Haf92] HAFTKA, R., GÜRDAL, Z., *Elements of structural optimization*, Kluwer academic publishers, Dordrecht, Netherlands, 3rd ed., 1992, ISBN 0-7923-1504-9.

[Jan02] JANSSON, T., *Optimization of Sheet Metal Forming Processes*, Licentiate thesis, Universitet Linköping, Linköping, Sweden, 2002.

[Jan05] JANSSON, T., ANDERSSON, A., NILSSON, L., “Optimization of draw-in for an automotive sheet metal part – an evaluation using surrogate models and response surfaces”, *Journal of Materials Processing Technology*, Vol. 159, 426–234, 2005.

[Kle00] KLEIJNEN, J., SARGENT, R., “A methodology for fitting and validating metamodels in simulation”, *European Journal of Operational Research*, Vol. 120, 14–29, 2000.

[Kle03] KLEINERMANN, J. P., PONTHOT, J. P., “Parameter identification and shape/process optimization in metal forming simulation”, *Journal of Materials Processing Technology*, Vol. 139(1-3), 521–526, 2003.

[Kle04] KLEIJNEN, J., VAN BEERS, W., “Application-driven sequential designs for simulation experiments: Kriging metamodelling”, *European Journal of Operational Research*, submitted in 2004.

[Koe96] KOEHLER, J., OWEN, A., *Handbook of Statistics*, chap. Computer Experiments, 261–308, Elsevier Science, New York, USA, 1996.

[Lin03] LIN, Z., JUCHEN, X., XINYUN, W., GUOAN, H., “Optimization of die profile for improving die life in the hot extrusion process”, *Journal of Materials Processing Technology*, Vol. 142(3), 659–664, 2003.

[Lop02a] LOPHAVEN, S., NIELSEN, H., SØNDERGAARD, J., “Aspects of the MATLAB Toolbox DACE”, Technical Report IMM-REP-2002-13, Technical University of Denmark - Department of Informatics and Mathematical Modelling, Lyngby, Denmark, 2002.

[Lop02b] LOPHAVEN, S., NIELSEN, H., SØNDERGAARD, J., “DACE - A MATLAB Kriging Toolbox”, Technical Report IMM-TR-2002-12, Technical University of Denmark - Department of Informatics and Mathematical Modelling, Lyngby, Denmark, 2002.

[Mar03] MARTIN, J., SIMPSON, T., “A study on the use of Kriging models to approximate deterministic computer models”, in *Proceedings of the ASME Design Engineering Technical Conferences DETC*, 2003.

[McK79] MCKAY, M., BECKMAN, R., CONOVER, W., “A comparison of three methods for selecting values of input variables in the analysis of output from a computer code”, *Technometrics*, Vol. 21, 239–245, 1979.

[Mye02] MYERS, R., MONTGOMERY, D., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, John Wiley and Sons, Inc., New York, USA, 2nd ed., 2002, ISBN 0-471-41255-4.

[Nac01] NACEUR, H., GUO, Y. Q., BATOZ, J. L., KNOPF-LENOIR, C., “Optimization of drawbead restraining forces and drawbead design in sheet metal forming process”, *International Journal of Mechanical Sciences*, Vol. 43(10), 2407–2434, 2001.

[Nac04] NACEUR, H., BEN-ELECHI, S., KNOPF-LENOIR, C., BATOZ, J., “Response surface methodology for the design of sheet metal forming parameters to control springback effects using the inverse approach”, in *Proceedings of NUMIFORM*, Columbus OH, USA, 2004.

[Nie] NIELSEN, H., “DACE, A MATLAB Kriging toolbox”, http://www.imm.dtu.dk/hbn/dace/.

[Pap00] PAPALAMBROS, P. Y., WILDE, D. J., *Principles of optimal design*, Cambridge University Press, New York, USA, 2000, ISBN 0-521-62727.

[Sac89a] SACKS, J., SCHILLER, S., WELCH, W., “Design for computer experiments”, *Technometrics*, Vol. 31, 41–47, 1989.

[Sac89b] SACKS, J., WELCH, W., MITCHELL, T., WYNN, H., “Design and analysis of computer experiments”, *Statistical Science*, Vol. 4, 409–423, 1989.

[San03] SANTNER, T., WILLIAMS, B., NOTZ, W., *The Design and Analysis of Computer Experiments*, Springer-Verlag, New York, USA, 2003, ISBN 0-387-95420-1.

[Sch04] SCHENK, O., HILLMANN, M., “Optimal design of metal forming die surfaces with evolution strategies”, *Computers and Structures*, Vol. 82, 1695–1705, 2004.

[Sim01] SIMPSON, T., PEPLINSKI, J., KOCH, P., ALLEN, J., “Metamodels for computer-based engineering design: Survey and recommendations”, *Engineering with Computers*, Vol. 17, 129–150, 2001.

[vB04] VAN BEERS, W., KLEIJNEN, J., “Kriging interpolation in simulation: A survey”, in R. Ingalls, M. Rossetti, J. Smith, B. Peters, eds., *Proceedings of the 2004 Winter Simulation Conference*, 2004.

Received: September 2005
Revised: January 2006