Waves in Gas-Liquid Flows

L. van Wijngaarden

§1. Introduction

At present there does not exist a general theory for dispersed two phase flow. That is to say a theory which predicts a reasonably broad range of phenomena in such flows. Apparently our physical insight in what happens in two phase flow is not developed far enough to enable us to construct satisfactory models. At best we can hope that certain model equations, tested experimentally in a particular situation, apply as well to other, similar, situations. There is definitely a lack of experimental data obtained in systematic and controlled experiments. Such experiments are impeded by the fact that both in this country and elsewhere research contracts seem to be granted primarily to experiments testing out computer codes with a large number of parameters. In such experiments there are too many variables and the results have learned us virtually nothing. In this lecture I would like to discuss two kind of waves for which sufficient experimental data are available to confirm their existence and to enable verification of theoretical reasoning. In the first case, pressure waves, a satisfactory theory is available as well. In the second case, kinematic waves or void fraction waves, a satisfactory theory has as yet not been given. In this case the lecture will emphasize on the theoretical problems which do exist here.

§2. Pressure waves in two-phase flow.

2.1. Theory for homogeneous medium.

The simplest model for a gas/liquid flow is a (fictitious) homogeneous medium with effective properties obtained from considerations regarding the properties of the involved inhomogeneities.

For example, a suspension of small air bubbles in water may be thought of as a homogeneous fluid with density ρ , given by

$$\rho = (1-\alpha)\rho_l + \alpha\rho_g \quad (2.1)$$

α being the concentration of air by volume, ρ_l and ρ_g the density of liquid and gas respectively. Such a concept makes sense when the length scale of the phenomena which we wish to consider is large with respect to the characteristic length parameters of the dispersed flow. First the typical dimension of the dispersed particles, the effective radius, R , say. Then the average distance between particles for which $n^{-1/3}$ is representative where n is the number density. With a macroscopic length scale λ we must require therefore

$$\lambda \gg R, \quad \lambda \gg n^{-1/3}. \quad (2.2)$$

Just how much λ must be larger than $n^{-1/3}$ in other words how many bubbles must be covered by the macroscopic length scale, can only be found by experiment. For spherical bubbles the void fraction α is related to the mentioned lengths by

$$\alpha = \frac{4}{3} \pi n R^3. \quad (2.3)$$

It follows from (2.1) that gas/liquid flows have, unless α is close to unity, approximately the density of the liquid. The compressibility is mainly due to the gas content because liquids are much harder to compress. If we disregard for the time being a possible difference between the velocity of the gas and that of the liquid (indicated by many workers in the field as slip) the mass of gas in a unit mass of liquid is a constant. This is expressed mathematically as

$$\frac{\alpha \rho_g}{\alpha \rho_g + (1-\alpha) \rho_l} = \text{constant} . \quad (2.4)$$

Expressing α in terms of ρ and ρ_g with this relation and introducing in (2.1) gives with

$$\frac{dp}{d\rho_g} = c_g^2 ; \frac{dp}{d\rho_l} = c_l^2 \quad (2.5)$$

for the compressibility

$$\begin{aligned} \frac{d\rho}{dp} = & \frac{\alpha^2}{c_g^2} + \frac{(1-\alpha)^2}{c_l^2} + \alpha(1-\alpha) \left\{ \frac{c_l^2 \rho_l^2 + c_g^2 \rho_g^2}{c_l^2 c_g^2 \rho_l \rho_g} \right\} \sim \frac{\alpha^2}{c_g^2} \\ & + \frac{(1-\alpha)^2}{c_l^2} + \frac{\alpha(1-\alpha) \rho_l}{\rho_g c_g^2} . \end{aligned} \quad (2.6)$$

The quantity on the left hand side of this relation is the reciprocal sound speed, squared. From the terms on the right hand side it follows that the sound speed c , is determined mainly by the third term when α is not too close to zero or unity. Neglecting the first and the second term gives

$$\begin{aligned} c^2 &= \frac{\gamma p}{\rho_l \alpha (1-\alpha)} \quad \text{for isentropic waves} \\ &= \frac{p}{\rho_l \alpha (1-\alpha)} \quad \text{for isothermal waves} . \end{aligned} \quad (2.7)$$

In figure 1 the sound speed c is given as a function of α at given pressure under isothermal behaviour. Most striking is its low value, as compared with the sound speed in both liquid and gas, at values of α off the sides $\alpha=1$ and $\alpha=0$. Relation (2.7) has been amply confirmed in experiments (see for a survey Van Wijngaarden 1972). The low speed of sound of a bubbly suspension has great practical significance. Even for a value of α as low as 1% the sound speed in a mixture of air and water is only 100m/s. Near the surface large amounts of bubbles are dispersed in the oceans which severely affect the propagation of acoustic waves. As another example we mention that the emission of sound by a turbulent liquid flow is enhanced by a factor as large as $(c_l/c)^4$ when gas is dispersed in the liquid (Crighton and Ffowcs Williams 1969). Twenty years ago acoustic properties of bubbly flows were used by the navy to

Finally we need the relation (2.3) between α , n and R and an energy equation for the bubbles. A simple one is

$$p_g R^3 = \text{constant} \quad (2.13)$$

which holds for isothermal changes, or

$$p_g R^{3\gamma} = \text{constant} \quad (2.14)$$

for isentropic behaviour, γ being the ratio of specific heats. From linearization of these equations and seeking solutions in the form of travelling waves we obtain for waves with angular frequency ω and wave number k

$$\left(\frac{k}{\omega}\right)^2 = \frac{1}{c_l^2} + \frac{1 - \omega^2/\omega_B^2 + i\delta\omega/\omega_B}{c^2\{(1 - \omega^2/\omega_B^2)^2 + \delta^2\omega^2/\omega_B^2\}}. \quad (2.15)$$

In this equation c is the sound velocity as given in (2.7) whereas δ is the logarithmic decrement associated with the attenuation. This is here due to the viscous term in (2.8). In fact we should have formulated an energy equation more realistic than (2.14). Notably the heat conduction from the gas to the liquid should have been taken into account. This leads to a net loss of energy which can be accounted for by a thermal coefficient, D_h being the thermal diffusivity of the gas phase,

$$\delta_{th} = \frac{3(\gamma-1)}{2(\omega/2D_h)^{1/2} R_0} \quad (2.16)$$

which is under most conditions larger than the value $4\nu/\omega_B R_0^2$ following from the viscous term in (2.8). The dispersion equation (2.15) has been verified in many experiments, notably Silberman (1957).

It should be noted that one bubble size only is considered. A theory valid for a distribution of bubble sizes is formally possible but hardly tractable in practice. Still the one size theory can be used to obtain data for bubble distributions, e.g. in the ocean. This is caused by the fact that the scattering cross section and the extinction cross section is largest at resonance. When sound waves are transmitted through a bubbly suspension the bubble distribution can be obtained from measurement of the attenuation under the assumption that attenuation comes

mainly from resonant bubbles. Such measurements have in this country been carried out for ocean bubble spectra by Medwin (see Medwin 1980).

2.3. Waves of finite amplitude.

There is an interesting analogy with gravity waves on a fluid of finite depth. Both such waves and the pressure waves that we have discussed in the foregoing sections display dispersion and steepening of compression waves. For gravity waves of moderate amplitude the Boussinesq equations incorporate both these phenomena. They reduce for waves travelling in one direction only to the famous Korteweg-de Vries equation. This equation holds (Van Wijngaarden 1968) also for waves of moderate amplitude in bubbly flows. When the pressure perturbation is indicated with $\tilde{p} = p - p_0$, we have, the subscript 0 referring to the undisturbed state

$$\frac{\partial \tilde{p}}{\partial t} + c_0 \frac{\partial \tilde{p}}{\partial x} + c_0 \tilde{p} \frac{\partial \tilde{p}}{\partial x} + \frac{1}{2} \frac{c_0^3}{\omega_B^2} \frac{\partial^3 \tilde{p}}{\partial x^3} = 0. \quad (2.17)$$

This equation predicts the existence of solitons in bubbly flows. These have indeed been measured by several workers, Kuznetsov (1978), Roelofsen (1981).

Figure 2. Evolution of solitons from a triangular pressure disturbance in a bubbly liquid. (From Roelofsen 1981)

In figure 2 an example from the latter reference is shown. It is clear that attenuation is significant. A better agreement with reality is therefore obtained when an attenuation term is added to (2.17), resulting in the Korteweg-de Vries-Burgers equation

$$\frac{\partial \tilde{p}}{\partial t} + c_0 \frac{\partial \tilde{p}}{\partial x} + c_0 \tilde{p} \frac{\partial \tilde{p}}{\partial x} + \frac{1}{2} \frac{c_0^3}{\omega_B^2} \frac{\partial^3 \tilde{p}}{\partial x^3} - \frac{1}{2} \frac{\delta c_0^2}{\omega_B} \frac{\partial^2 \tilde{p}}{\partial x^2} = 0. \quad (2.18)$$

This equation, apart from allowing for the attenuation of solitons, has steady solutions of shock wave type. While shock waves in single phase gases are determined by a balance between nonlinear steepening and diffusion, here it is rather the dispersion which balances the nonlinearity. These effects are represented by the third and fourth term in the above equation. Their balance gives for the thickness d of the shock wave the estimate

$$d \sim \frac{R_0}{\alpha^{1/2}} \left(\frac{p_0}{\Delta p} \right)^{1/2}, \quad (2.19)$$

Δp being the pressure jump over the shock wave. This wave is similar to the undular bore in hydraulics, steep at the front and wavy at the back. Its existence in bubbly flows has been amply verified (Noordzij 1973, Van Wijngaarden 1970, Kuznetsov et al. 1978). An example, taken from Noordzij (1973) is shown in figure 3. In connection with (2.2) it was observed that experiments should decide how large the length scale λ should be with respect to $n^{-1/3}$. In the experiments on shock waves it turned out that the theory presented here holds good even when the shock covers only a few bubbles. We have laid emphasis on bubbly flows. A theory supported by experiments is available also for waves of small amplitude in separated flow, (Matsui 1975, Morioka & Matsui 1975).

Figure 3. Pressure profile in shock wave in a bubbly liquid. The shock wave is dispersion controlled. (From Noordzij 1973)

3. Separated flows with relative velocity

The notion of a fictitious homogeneous medium becomes less easy to maintain when both phases behave differently. In the previous section we were able to account for a pressure difference by taking the pressure p in the liquid to be the external pressure as seen from the point of view of a single bubble.

Similar concepts are not so obvious when the gas phase moves at a velocity which differs from that of the surrounding liquid. It seems more plausible then to formulate equations of mass and momentum conservation for the separate phases. This has the advantage that velocities and pressures appear in a natural way in the equations of the pertinent species. The disadvantage is that there are interactions between them. These are hard to describe since their mechanisms have not been studied far enough to permit a mathematical description. Only the interaction force exerted by the isotropic part p of the stress tensor is easily incorporated. Denoting all other interactions together with I we have, indicating the velocity of the gas phase with v , as momentum equations

$$\frac{\partial}{\partial t} \rho_l (1-\alpha) u + \frac{\partial}{\partial x} \rho_l (1-\alpha) u^2 - (1-\alpha) \frac{\partial p}{\partial x} = I_l \quad (3.1)$$

$$\frac{\partial}{\partial t} \rho_g \alpha v + \frac{\partial}{\partial x} \rho_g \alpha v^2 + \alpha \frac{\partial p}{\partial x} = I_g . \quad (3.2)$$

For simplicities' sake we have assumed here that significant pressure differences like considered in the foregoing section do not occur. Nor do we consider processes as condensation or evaporation. We suppose that thermodynamic changes can be summarized by

$$p = f(\rho_g) . \quad (3.3)$$

When we add to these the two relations expressing conservation of mass:

$$\frac{\partial}{\partial t} \alpha \rho_g + \frac{\partial}{\partial x} \alpha \rho_g v = 0 , \quad (3.4)$$

$$\frac{\partial}{\partial t} (1-\alpha) \rho_l + \frac{\partial}{\partial x} (1-\alpha) \rho_l u = 0 , \quad (3.5)$$

we have five equations for the unknown quantities α , p , ρ_g , u and v . However, we have still to determine the terms indicated with I . These may depend on the particular topology of the flow and as said above their determination in a particular flow is a major problem in two-phase flow. The easiest thing to do is to ignore them completely and to put the right hand sides of (3.1) and (3.2) equal to zero. We are left then with what seems at first sight a reasonable set of equations for a first try. Unfortunately, a serious problem turns up now. Suppose we want to do, and this happens all the time for instance in calculations concerning nuclear reactor safety, a Cauchy type of problem, that is to say a problem in which some set of initial values is given and the development in space and time is asked for. It turns out that such a problem is ill posed. This follows from considering the characteristics of the equations which remain from (3.1) - (3.5) if we take $I = 0$ and specify (3.3) by

$$p/\rho_g = c_g^2 . \quad (3.6)$$

The characteristics are found by writing $\partial/\partial t = -\xi \partial/\partial x$.

For ξ we find the following equation

$$\{(\xi-u)^2 - \frac{p}{\rho_l} \frac{1-\alpha}{\alpha}\} \{(\xi-v)^2 - c_g^2\} = \frac{p c_g^2}{\rho_l} \frac{1-\alpha}{\alpha} . \quad (3.7)$$

This equation possesses two real roots (which appear to be associated with sound waves) and in addition two complex roots, which are the cause of trouble. Their effect is to make the solution depend on the initial conditions in a non-continuous way. This can also be seen in the following way. Imagine small perturbations of the form $\exp i(kx - \omega t)$ superposed on a solution of (3.1) - (3.5) (with $I=0$ and (3.3) specified by (3.6)). Then the equation which one obtains for the quantity ω/k is the same as (3.7) for ξ . This means that for real ω the complex roots give conjugate complex values for k , leading to exponential growth of the disturbance. The practical consequences of this may be not as bad as it seems because the growth rate may be small. Nevertheless one would like to avoid having complex characteristics. They are related to Helmholtz instability which becomes apparent even more when like Stuhmiller (1977) did also ρ_g is taken as a constant. In that case all characteristics become complex. One way to get rid of them is to investigate the nature of the terms indicated with I in (3.1) and (3.2) in the hope that they contain such terms as necessary to render all characteristics real. The study of possible effects to be incorporated in the right hand side of (3.1) and (3.2) is central in two phase flow research today.

Many proposals and suggestions have been made in the recent past, none of them altogether satisfactory. The terms indicated with I contain forces exerted by one phase on the other but also effects from fluctuations in the variables of the considered phase and due to the other. If we for the moment leave out the fluctuations we could formulate equations for the whole flow. Then the interaction forces drop out and as momentum equation for a bubbly flow we could use

$$\rho_l(1-\alpha) \frac{du}{dt} + \rho_l(1-\alpha)u \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x} . \quad (3.8)$$

If we follow this approach we need to complete our set of equations with a relation between the local gas velocity v and the liquid velocity u . In the theory of suspensions at low Reynolds number such a relation is provided by

Faxen's law which expresses that in a liquid velocity field $u(x)$ a particle obtains under influence of a distribution of surface forces $F(x)$ a velocity

$$U = \frac{1}{6\pi\mu R} \int F(x) dA + \frac{1}{4\pi R^2} \int u(x) dA . \quad (3.9)$$

Can we find a similar relation in a suspension which is governed by inertial effects? To answer this question we consider the flow around a spherical gas bubble. In the absence of surface active agents the flow around the bubble can to good accuracy be described by potential flow. The calculation of the viscous dissipation in the flow is relatively easy (one should be reminded here that although the divergence of the deviatoric stress is zero the stress itself and therefore the dissipation is not) and putting this equal to the relative velocity times a drag force, gives for the latter

$$D = 12\pi\mu R(v-u) . \quad (3.10)$$

The potential flow provides also the concept of virtual mass m . If the potential for the relative motion is represented by ϕ we have

$$m(v-u) = \int \phi dA \quad (3.11)$$

The virtual mass can be interpreted as the mass of liquid which has to be accelerated along with the body when its velocity is changed in time and it is immersed in a perfect liquid.

For a sphere $m = \frac{1}{2} \rho_l \tau$, where τ is the volume. Combining the idea of virtual mass with the drag force described in (3.10) gives for a massless sphere

$$\frac{d}{dt} \frac{1}{2} \rho_l \tau (v-u) = \rho_l \tau \frac{dv}{dt} - 12\pi\mu R(v-u) . \quad (3.12)$$

In fact, a time dependent term like the well-known Basset term in the relation for the drag of a rigid body should be included. Recent work by Pham Dan Tam (1981) has not resulted in a simple representation of the time dependent drag. For that reason a quasi-steady approach is supposed to be permitted here. When for $t \rightarrow -\infty$ we have $v=u$, we may integrate (3.12) to obtain

$$\bar{y} - y = 2 \int_{-\infty}^t \frac{dy}{dt'} \exp\left(\frac{t' - t}{T}\right) dt' , \quad (3.13)$$

where the time

$$T = \frac{R^2}{18\nu} \quad (3.14)$$

is introduced. This time, of order $10^{-1} - 10^{-2}$ s in a suspension with $\nu \sim 10^{-6} \text{ m}^2/\text{s}$ and $R \sim 10^{-3} \text{ m}$, is a relaxation time telling us how long it takes the liquid to adjust the velocity of a single bubble to the velocity of the liquid. For our purpose, a relation between y and $\bar{y}$ for bubble flow, (3.12) is not good enough, because we need a relation in a spatially nonuniform flow. It turns out not to be possible to obtain a simple relation like (3.12) for such a flow. For the important case in which the nonuniformity is brought about by the motion of neighbouring bodies, at an average distance D , Voinov, Voinov & Petrov (1973) showed that, leaving out viscous forces,

$$\frac{D}{Dt} (m\bar{y}) - \frac{d}{dt} (my) = \rho \tau \frac{dy}{dt} + O\left(\frac{R}{D}\right)^5 . \quad (3.15)$$

Here D/Dt is the material derivative with respect to the gas-phase and d/dt with respect to the liquid. Relation (3.15) has been discussed by several writers on two phase flow, for instance Drew et al (1979). Adding the viscous term we arrive at

$$\frac{D}{Dt} (m\bar{y}) - \frac{d}{dt} (my) = \rho \tau \frac{dy}{dt} - 12\pi\mu R(\bar{y} - y) \quad (3.16)$$

as an approximate relation to be used in bubbly flow equations. It must be emphasized that its use can only be justified by the lack of something better.

§4. Fluctuations

When (3.8) and (3.16) are used instead of two momentum equations for the separate phases and these are complemented with equations for mass conservation, we again find complex characteristics. The real ones are associated with sound waves, the nature of the complex ones is yet not clear. Recent experiments (Mercadier 1981, Bouré and Mercadier 1982, Brennen and Bernier 1982) have revealed the existence of a new type of wave. These can best be described as propagating perturbations in the void fraction α . In these

waves pressure variations are small and they travel at a speed in the neighborhood of the gas velocity v . These waves seem to be due to interactions between bubbles. Therefore, and also within the more general scope of establishing satisfactory equations for bubbly flows it is of importance to study the effects of fluctuations resulting from interactions between the bubbles. Indeed, apparently the void fraction waves just described are associated with real characteristics. The study of fluctuations is relatively new in two phase flow although formal descriptions (Ishii 1975, Nigmatulin 1978, Buyevich & Shchelchkova 1978) have been given. In the majority of these works actual interactions have not been described. A longer tradition exists in the theory of suspensions at low Reynolds number, see e.g. Batchelor (1977). In the present lecture I would like to illustrate the type of problems that are met here by looking at the stress distribution in a bubbly liquid, ignoring for convenience volume oscillations and viscous stresses. Quantities like pressure, velocity etc. fluctuate as a result of varying velocities and relative positions of the constituent inhomogeneities. We are only interested in their mean or average values. These may be volume averages, ensemble averages or surface averages. There is no time here to discuss the relative merits of these various averaging methods and we refer to the works cited above. Suppose we want to talk about the average pressure, the bulk pressure $\langle p \rangle_{\text{bulk}}$ in a bubbly suspension, where $\langle \rangle$ is understood as an ensemble average.

The ensemble consists of all the realizations of a suspension of a large number of bubbles in a volume V over which the macroscopic variables are constant. Under statistical homogeneity the ensemble average is equivalent with a volume average. Carrying this out for the pressure gives (see Van Wijngaarden 1982)

$$\langle p \rangle_{\text{bulk}} = \frac{1}{V} \int p dV = \frac{1}{V} \int_{V_\ell} p dV + \varepsilon \frac{1}{V} \int_{V_B} p dV . \quad (4.1)$$

In this expression V_ℓ denotes the part of V which is occupied by liquid. Similarly V_B is the volume of one

bubble and the summation is over all the bubbles. Making use of

$$(1-\alpha) = \frac{V_g}{V} ,$$

and introducing $\langle p \rangle_g$ as the average over the liquid phase only, we write the right hand side of (4.1) as

$$(1-\alpha) \langle p \rangle_g + \sum \frac{1}{V} \int_{V_B} p dV = \langle p \rangle_g + \sum \frac{1}{V} \int_{V_B} \{p - \langle p \rangle_g\} dV .$$

Next we introduce the quantity $\langle S \rangle$ which is the particle contribution to the bulk pressure, by

$$S = \int_{V_B} (p - \langle p \rangle_g) dV . \quad (4.2)$$

Herewith the relation for the bulk pressure $\langle p \rangle_{\text{bulk}}$ becomes

$$\langle p \rangle_{\text{bulk}} = \langle p \rangle_g + n \langle S \rangle , \quad (4.3)$$

where n is the number density, as before. The interactions between bubbles appear in the determination of S . This contains the difference between $\langle p \rangle_g$ and the actual pressure p which is the result of the motion produced by all other bubbles. In principle we would like to know the velocity potential due to the motion of all these bubbles. Since this is impossible we work along the lines of successive approximations. In the lowest approximation a bubble finds itself isolated in an infinite liquid. In the next a bubble is in interaction with just one other bubble. This is analogous to binary encounters in the kinetic theory of gases. The small parameter is the concentration α , and the first approximation gives equations accurate in α , the next produces equations accurate in α^2 .

In the lowest approximation $\langle S \rangle$ is equal to S , which is obtained in the following way. When the volume velocity for the suspension is U_0 we consider a sphere with radius R moving in an infinite liquid with this velocity, while the sphere has velocity U_g . The latter is the average velocity of the gas phase. For this motion the potential is

$$\phi = U_0 \cdot \mathbf{r} + \frac{(U_0 - U_g) \cdot R^3}{2r^3} \mathbf{r} . \quad (4.4)$$

If now the pressure at infinite distance is equal to the bulk pressure $\langle p \rangle_{\text{bulk}}$ we can calculate p in every point of the liquid by using (4.4) in Bernoulli's law. However (4.2) seems to make it necessary to know the pressure inside the bubble as well. This is however not so because the volume integral may be converted into

$$\frac{1}{3} \int (\langle p \rangle_{\ell} - p) \underline{r} \cdot d\mathbf{A} - \frac{1}{3} \int (\underline{r} \cdot \nabla p) dV .$$

For a massless sphere the volume integral is zero and the remaining surface integral can be evaluated with help of Bernoulli's theorem and (4.4). The result is

$$\langle S \rangle = S = - \frac{\pi R^3}{3} \{ (U_0 - U_g) \}^2 \quad (4.5)$$

Inserted into (4.3) this gives for $\langle p \rangle_{\text{bulk}}$,

$$\langle p \rangle_{\text{bulk}} = \langle p \rangle_{\ell} - \frac{1}{4} \alpha \rho_{\ell} \{ (U_0 - U_g) \}^2 . \quad (4.6)$$

In a similar way fluctuations contribute to the average momentum flux. These contributions are analogous to Reynolds stresses in turbulent flow. In the lowest approximation one obtains (Van Wijngaarden 1980)

$$\begin{aligned} -\langle u'_i u'_j \rangle &= L_{ij} \alpha \{ (U_0 - U_g) \}^2 \\ \text{where } L_{11} &= L_{22} = 3/20; L_{33} = 1/5; L_{12} = L_{13} = L_{23} = 0 . \end{aligned} \quad (4.7)$$

Here the fluctuating velocity components are indicated with a prime. The direction indicated with 3 coincides with the direction of U_0 . Hence accurate in α the momentum equation in one dimensional (mean) flow is

$$\begin{aligned} \rho_{\ell} (1-\alpha) \left\{ \frac{\partial U_{\ell}}{\partial t} + U_{\ell} \frac{\partial U_{\ell}}{\partial x} \right\} &= - \frac{\partial}{\partial x} \langle p \rangle_{\ell} \\ &+ \frac{1}{20} \frac{\partial}{\partial x} \{ \alpha (U_g - U_{\ell})^2 \} . \end{aligned} \quad (4.8)$$

Here U_{ℓ} is the average liquid velocity. In further approximations interactions between particles are taken into account. It is to be expected that these will contribute to the understanding of the void fraction waves mentioned earlier.

REFERENCES

- Batchelor, G. K. 1970. The stress system in a suspension of force-free particles, *J. Fluid Mech.* 41, 545.
- Batchelor, G. K. 1977. Developments in micro hydrodynamics. In 'Theoretical & Appl. Mech. (Proc. 14th IUTAM Congress, Delft 1976) Ed. W. T. Koiter, North Holland.
- Bernier, R. J. N. 1981. Unsteady two-phase flow instrumentation and measurement. Rept. no. E 200.4, Div. Eng. & Appl. Sc., Cal. Inst. of Technology (Pasadena).
- Bouere, J. A. & Mercadier, Y. 1982. Existence and properties of flow structure waves in two-phase bubbly flows. *Appl. Sc. Res.* 38, 297.
- Brennen, C. & Bernier, R. 1982. Private communication.
- Buyevich, Yu. A. & Shchelchkova, I. N. 1978. Flow of dense suspensions. *Progr. Aerosp. Sc.* 18, 121.
- Crighton, D. G. & Ffowcs Williams, J. E. 1969. Sound generation by turbulent two-phase flow. *J. Fluid Mech.* 36, 585.
- Drew, D., Cheng, L. & Lahey, R. T. Jr. 1979. The analysis of virtual mass effects in two-phase flow. *Int. J. Multiphase Flow*, 5, 233.
- Ishii, M. 1975. Thermo-fluid dynamic theory of two-phase flow. Eyrolles.
- Kuznetsow, V. V., Nakoryakov, V. E., Pokusaev, B. G. & Schreiber, I. R. 1978. Propagation of perturbations in a gas-liquid mixture. *J. Fluid Mech.* 85, 85.
- Marble, F. & Wooten, D. C. 1970. Sound attenuation in a condensing vapor. *Phys. Fluid* 13, 11.
- Marble, F. 1970. Dynamics of dusty gases. *Ann. Rev. Fluid Mech.* 2, 397.
- Minnaert, M. 1933. On musical air bubbles and the sound of running water. *Phil. Mag.* 16, 235.
- Matsui, G. 1975. Pressure wave propagation through a separated gas-liquid system in a duct. Winter Annual Meeting ASME, Houston, Texas.
- Medwin, H. 1980. Acoustical bubble spectrometry at sea. In "Cavitation and inhomogeneities in underwater acoustics". Ed. W. Lauterborn. Springer-Verlag.

- Mercadier, Y. 1981. Contribution a l'etude des propagations de perturbations de taux de vide dans les ecoulements diphasiques eau air a bulles. These Universite Grenoble.
- Morioka, S. & Matsui, G. 1975. Pressure waves propagation through a separated gas/liquid layer in a duct. J. Fluid Mech. 70, 4.
- Nigmatulin, R. I. 1979. Spatial averaging in the mechanics of heterogeneous and dispersed systems. Int. J. Multiphase Flow 5, 353.
- Noordzij, L. 1973. Shock waves in mixtures of liquid and air. Thesis Technological University Twente.
- Roelofsen, P. 1981. Solitons in liquid/bubble mixtures. Master's Thesis, Technological University Twente.
- Silberman, E. 1957. Sound velocity and attenuation in bubble mixtures measured in standing wave tubes. J. Acoust. Soc. Am. 29, 925.
- Stuhmiller, J. H. 1977. The influence of interfacial pressure forces on the character of two-phase flow model equations. Int. J. Multiphase Flow 3, 551.
- Tam, P. D. 1981. De la trainee instationnaire sur une petite bulle. Thesis University of Grenoble, France.
- Voinov, V. V., Voinov, O. V. & Petrov, A. G. 1973. Hydrodynamic interaction between bodies in a perfect incompressible fluid and their motion in nonuniform streams. Prikl. Math. Mech. 37, 680.
- Whitham, G. B. 1974. Linear and nonlinear waves. Wiley & Sons, New York.
- Wijngaarden, L. van 1968. On the equations of motion for mixtures of liquid and gas bubbles. J. Fluid Mech. 33, 465.
- Wijngaarden, L. van 1970. Propagation of shock waves in bubble liquid mixtures. Progr. Heat & Mass transfer 6, 637.
- Wijngaarden, L. van 1972. One-dimensional flow of liquids containing small gas bubbles. Ann. Rev. Fluid Mech. 4, 369.
- Wijngaarden, L. van 1980. On the mathematical modelling of two phase flows. Proc. 4th Int. Meeting on water column separation. Cagliari, Sept. 1979, Ed. M. Reali.

Wijngaarden, L. van 1982. Bubble interactions in liquid/gas flows. Appl. Sc. Res. 38, 331.

L. van Wijngaarden
Technical University Twente
Enschede, The Netherlands