An Optimum Linear Receiver for Multiple Channel Digital Transmission Systems

W. VAN ETTEN

***Abstract*—An optimum linear receiver for multiple channel digital transmission systems is developed for the minimum P_e and for the zero-forcing criterion. A multidimensional Nyquist criterion is defined together with a theorem on the optimality of a finite lenght multiple tapped delay line. Furthermore an algorithm is given to calculate the tap settings of this multiple tapped delay line. This algorithm simplifies in those cases where the noise is so small that it can be neglected. Finally as an example the transmission of binary data over a cable, consisting of four identical wires, symmetrically situated inside a cylindrical shield, is considered.**

I. INTRODUCTION

In this paper we shall investigate the transmission of digital signals over a multiple channel system, where each channel is used to transmit a data sequence. This configuration is included in the more general structure considered by Kaye and George [1]. We, however, use a technique that leads to an optimum structure for both the zero-forcing and minimum error probability criterion, instead of the minimum mean-square error criterion Kaye and George used.

Besides the intersymbol interference (ISI), interchannel interference (ICI) can be one of the major problems in such a multiple channel digital transmission system. ISI is disturbance of an output signal by symbols that originate from the corresponding input but that are shifted in time with respect to the symbol of interest. ICI is disturbance of an output signal by symbols that do not originate from the corresponding input but from input symbols that belong to neighboring channels. We introduce the name multidimensional interference (MDI) for the combined effect of ISI and ICI. Because the equalization of ISI also changes the ICI at the output, and the other way round, only a simultaneous treatment of these two phenomena can be successful in combating the overall degradation. In the following we generalize for MDI some techniques known from the ISI literature. As examples of systems where these methods can be applied, we mention multiwire cables and multichannel radio systems that make use of perpendicular polarized waves in a common frequency band.

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II. THE MULTIPLE CHANNEL COMMUNICATION MODEL

The multiple channel transmission system, to be treated in this paper, has M inputs and M outputs, where to each input j a data sequence $\sum_l a_j^l\delta(t - lT)$ is applied which we want to detect at output j. The symbols a_j^l are elements of the alphabet $\{0,1,\cdots, L - 1\}$ and are chosen equiprobable and independent of each other. In our investigations a linear, dispersive, and time-invariant channel model is assumed (Fig. 1); this means that a linear relation exists between each input and each output signal and that the output signal due to the excitation of more than one input is the sum of the individual responses to the several inputs. The relation between input j and output i is denoted by the impulse response $r_{ij}(t)$. It is assumed that the output signals are disturbed by MDI and that zero-mean white Gaussian noise is added to them. Each output is corrupted by a different noise signal $n_i(t)$.

III. THE OPTIMUM LINEAR RECEIVER

By means of an optimum linear receiver and bit by bit detection on each channel output we make an estimate of the several input sequences. The receiving filter is assumed to be linear in the sense described in the preceding section. The linear relation between input i and output n of this filter is denoted by the impulse response $h_{ni}(t)$ (see Fig. 2). The following method yields an optimum solution for the linear multiple channel receiving filter for both the zero-forcing (zero MDI) and minimum bit error probability criterion. Assuming that the several noise sample functions $n_i(t)$ are independent of each other, then the noise variance at output n of the receiving filter can be written as follows:

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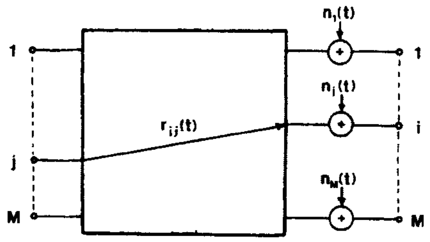

Fig. 1. Multiple channel communication model.

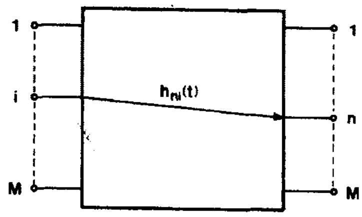

Fig. 2. Multiple linear receiving filter.

$$\sigma_n^2 = \sum_{i=1}^{M} N_i \int_0^\infty h_{ni}^2(\tau)\, d\tau \tag{1}$$

where N_i is the density of the noise spectrum of $n_i(t)$. Investigating the optimum structure of the linear receiving filter a technique is used that is presented in [4] and [5]. This means that all signal values that contribute to the possible sample values of the signal at output n are fixed. Then the noise variance σ_n^2 is minimized subject to these constraints. Defining the input vector

$$x^k \triangleq \begin{bmatrix} a_1^k \\ a_2^k \\ \cdot \\ \cdot \\ \cdot \\ a_M^k \end{bmatrix} \tag{2}$$

the constraints are found by considering the sample values of the signals at output n due to the L^M possible input vectors x^k. The latter sample values are found in the following way.

Assuming that at time $t = 0$ the vector x^k is applied to the inputs of the channel, then the response at output n of the receiving filter is given by

$$s_n^k(t) = \sum_{j=1}^{M} a_j^k \sum_{i=1}^{M} \int_0^\infty h_{ni}(\tau) r_{ij}(t-\tau)\, d\tau. \tag{3}$$

At the instant $t_s + lT$, this response has the value

$$s_n^k(t_s + lT) = \sum_{j=1}^{M} a_j^k \sum_{i=1}^{M} \int_0^\infty h_{ni}(\tau) r_{ij}(t_s + lT - \tau)\, d\tau. \tag{4}$$

In the minimization process these values for all k and l must be kept constant, therefore we have to minimize the functional

$$J_n = \sum_{i=1}^{M} N_i \int_0^\infty h_{ni}^2(\tau)\, d\tau - 2 \sum_{k=1}^{L^M} \sum_l \lambda_{nkl} \sum_{j=1}^{M} a_j^k \cdot \sum_{i=1}^{M} \int_0^\infty h_{ni}(\tau) r_{ij}(t_s + lT - \tau)\, d\tau. \tag{5}$$

Applying the calculus of variations to expression (5) yields

$$h_{ni}(t) = \frac{1}{N_i} \sum_{k=1}^{L^M} \sum_l \lambda_{nkl} \sum_{j=1}^{M} a_j^k r_{ij}(t_s + lT - t). \tag{6}$$

For the sake of simplicity we take $N_i = N$ for all i. This assumption and the assumption that the noise functions $n_i(t)$ are uncorrelated are not a restriction of the generality, as is shown in Appendix II.

With

$$c_{njl} = \frac{1}{N} \sum_{k=1}^{L^M} a_j^k \lambda_{nkl} \tag{7}$$

(6) reduces to

$$h_{ni}(t) = \sum_{j=1}^{M} \sum_l c_{njl} r_{ij}(t_s + lT - t). \tag{8}$$

The structure of the entire receiving filter follows from this equation. Each $h_{ni}(t)$ consists of a bank of matched filters, the outputs of which are added and the output signals of all $h_{ni}(t)$, which belong to the same receiving filter output n, are added again. Assuming that t_s is larger than the largest duration of all $s_n^k(t)$, then a reduction of the receiving filter is possible and Fig. 3 depicts the result for $M = 3$, for instance. For ease of notation the time axis is shifted such that $t_s = 0$. At each filter input i we see an array of filters matched to the particular responses at channel output i due to the individual excitation of the several inputs. Then all the outputs of the filters matched to the responses due to the same input are summed to form the primed outputs 1′-2′-3′. This part of the filter we call the multiple matched filter (MMF) (inputs 1-2-3 and outputs 1′-2′-3′). Each primed output is followed by a delay line with elements D giving a delay T. The rest of the receiving filter consists of M summing circuits and from each delayed primed output there is a weighted connection (with weighting coefficient c_{njl}) to each adder. This part of the filter we call the multiple tapped delay line (MTDL) (inputs 1′-2′-3′ and outputs 1″-2″-3″). The weighting coefficients c_{njl} have to be chosen such as to meet the optimization criterion. In the case of the minimum P_e criterion it is impossible to find an analytical solution for the set $\{c_{njl}\}$. By means of a steepest descent method one can find an approximation. Zero MDI offers the possibility to calculate the tap coefficients in a rather easy way as will be shown in Section V and to check the practical realization by means of the eye pattern, while the error probability P_e is of the same order of magnitude as when the minimum P_e criterion is used especially for large signal-to-noise ratios.

Considering the cascade connection of the channel, the MMF and the MTDL, the impulse responses of this overall system evaluated at the discrete instants lT are denoted by

$$F_l \triangleq \begin{bmatrix} f_{11}(lT) & f_{12}(lT) & \cdot\ \cdot & f_{1M}(lT) \\ f_{21}(lT) & f_{22}(lT) & \cdot\ \cdot & f_{2M}(lT) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ f_{M1}(lT) & f_{M2}(lT) & \cdot\ \cdot & f_{MM}(lT) \end{bmatrix} \tag{9}$$

with $f_{nj}(t)$ the response at output n of this system as result of a delta excitation on input j.

Further we define

$$F(D) \triangleq \sum_l F_l D^l \tag{10}$$

where D is the delay operator.

A measure for MDI is now defined as follows:

$$I_n \triangleq \frac{\sum_l \sum_{i=1}^{M} |f_{ni}(lT)| - |f_{nn}(0)|}{|f_{nn}(0)|} \tag{11}$$

which is the worst case distortion due to MDI on output n. The overall worst case MDI distortion is given by

$$I_0 = \max_n (I_n). \tag{12}$$

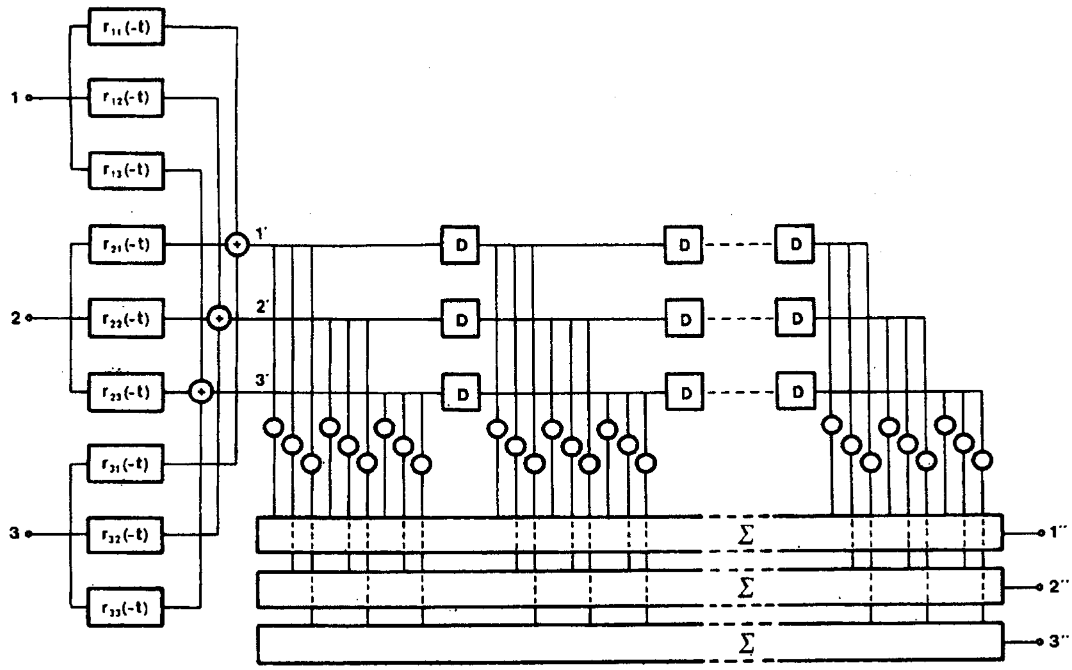

Fig. 3. Structure of the multiple linear receiving filter.

The terms "zero MDI" and "zero-forcing" are used here if $I_0 = 0$. By means of (10) and (12) we formulate a multidimensional Nyquist criterion which fits Shnidman's generalized Nyquist criterion [2].

Theorem 1: A multiple channel transmission system described by (10) satisfies the multidimensional Nyquist criterion if

$$F(D) = I \tag{13}$$

where I is the $M \times M$ identity matrix.

It will be clear from the foregoing that for a system satisfying the multidimensional Nyquist criterion the MDI will be zero.

Now consider the channel in cascade with the MMF as a multiple channel system with M inputs and M outputs. The impulse response from input j to output m of this system is called $v_{mj}(t)$ and can be written as

$$v_{mj}(t) = \sum_{i=1}^{M} r_{ij}(t) * r_{im}(-t) \tag{14}$$

where $*$ means convolution. Define

$$V_l \triangleq \begin{bmatrix} v_{11}(lT) & v_{12}(lT) & \cdots & v_{1M}(lT) \\ v_{21}(lT) & v_{22}(lT) & \cdots & v_{2M}(lT) \\ \vdots & \vdots & & \vdots \\ v_{M1}(lT) & v_{M2}(lT) & \cdots & v_{MM}(lT) \end{bmatrix} \tag{15}$$

and

$$V(D) \triangleq \sum_l V_l D^l. \tag{16}$$

The MTDL is also a multiple linear filter. For this system we define

$$C_l \triangleq \begin{bmatrix} c_{11l} & c_{12l} & \cdots & c_{1Ml} \\ c_{21l} & c_{22l} & \cdots & c_{2Ml} \\ \vdots & \vdots & & \vdots \\ c_{M1l} & c_{M2l} & \cdots & c_{MMl} \end{bmatrix} \tag{17}$$

and

$$C(D) \triangleq \sum_l C_l D^l. \tag{18}$$

From the definitions (10), (16), and (18) it follows

$$F(D) = C(D) \cdot V(D). \tag{19}$$

In Section V we shall give a procedure to calculate the tap coefficients described by $C(D)$.

IV. THE ERROR PROBABILITY OF THE EQUALIZED SYSTEM

If in a multiple channel transmission system it is possible to satisfy the multidimensional Nyquist criterion and the system has an optimum constraint receiver as described in the foregoing, the mean error probability of channel n of such a system is denoted by

$$P_{en} = 2\frac{L-1}{L} Q\left(\frac{d}{2\sigma_n}\right) \tag{20}$$

where the well-known $Q(\cdot)$ function is defined in [6, p. 82] and d is the smallest difference between two output levels. As the smallest difference between two elements of the input alphabet is taken unity and because of (13), d equals one. The noise variance at output n is calculated from (1) and (8)

$$\sigma_n^2 = \sum_m \sum_l \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{M} N c_{njm} c_{nkl} \int_{-\infty}^{\infty} r_{ik}(lT - \tau) r_{ij}(mT - \tau)\, d\tau. \tag{21}$$

For the equalized system the impulse response from input j to output n, evaluated at the instant mT, can be written as

$$f_{nj}(mT) = \sum_l \sum_{i=1}^{M} \sum_{k=1}^{M} c_{nkl} \cdot \int_{-\infty}^{\infty} r_{ik}(lT - \tau) r_{ij}(mT - \tau)\, d\tau = \delta_m \delta_{nj} \tag{22}$$

as is derived in [2]. Substituting (22) reduces (21) to the simple form

$$\sigma_n^2 = N c_{nn0} \tag{23}$$

which, if substituted in (20), gives for the error probability of channel n

$$P_{en} = 2\frac{L-1}{L} Q\left(\frac{1}{2(N c_{nn0})^{1/2}}\right). \tag{24}$$

V. THE OPTIMUM REALIZABLE MTDL

The index l of the $C(D)$ sequence runs from minus infinite to plus infinite and as a consequence the MTDL becomes infinitely long. In practice we have to make it of finite length and in this case (13) cannot be satisfied exactly. If the MTDL is of length $2K$ the optimum tap settings are given by the following theorem.

Theorem 2: If $V_0 = I$ and $\sum_l' \| V_l \| < 1$, then an upper bound of the MDI distortion I_0 is minimal for those tap setting matrices which cause $F_l = 0$, $|l| \leq K$, $l \neq 0$; where the primed summation excludes the term with $l = 0$ and the infinite norm is taken (which is the maximum over all rows of the sum of the absolute values of the components of the rows).

This theorem will be proven in Appendix I. In the special case that $\sum_l' \| F_l \|$ represents the worst case MDI I_0, this distortion itself is minimized. Under these constraints this theorem is a generalization of a theorem derived by Lucky for ISI [3, p. 138]. If $V_0 \neq I$ we can force V_0 to equal the identity matrix by placing between the MMF and the MTDL a multiple channel system with matrix D-transform V_0^{-1}. To apply the theorem all V_l matrices must then be replaced by $V_0^{-1}V_l$. In the case that $\sum_l' \| V_l \|$ represents the worst case MDI at the MMF outputs, a sufficient condition to satisfy the requirement $\sum_l' \| V_l \| < 1$ is that at none of the MMF outputs, the eye pattern is closed if $a_j^l \in \{+1, -1\}$.

The tap settings as stated in Theorem 2 are calculated as follows. Define the composite matrices

$$C \triangleq \begin{bmatrix} C_{-K} \\ C_{-K+1} \\ \cdot \\ \cdot \\ \cdot \\ C_K \end{bmatrix}, \tag{25}$$

$$V \triangleq \begin{bmatrix} V_0 & V_1 & \cdot & \cdot & V_{2K} \\ V_{-1} & V_0 & \cdot & & V_{2K-1} \\ V_{-2} & V_{-1} & V_0 & \cdot & V_{2K-2} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ V_{-2K} & V_{-2K+1} & \cdot & \cdot & V_0 \end{bmatrix} \tag{26}$$

and

$$E \triangleq \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ I \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

where 0 is the all zero matrix. To satisfy Theorem 2 we have the relation

$$C^T V = E^T. \tag{28}$$

This equation is further simplified if we look at (14), (15), and (26). It is easy to see that

$$V^T = V \tag{29}$$

so that

$$VC = E. \tag{30}$$

The solution of this equation decomposes into M times the solution of a set of linear equations, one time for each column of C as wanted vector and the corresponding column of E as known vector.

In systems where the noise does not play an important role, MDI distortion correction can directly be applied to the channel response. In this situation (29) is not true in general, but it is sometimes possible to choose $t_s < T$ giving a simplification of the expression for C_l. It is easy to see that the matrix sequence C_l starts now at $l = 0$ and runs to plus infinite. Analogous to (15) and (16) we define

$$R_l \triangleq \begin{bmatrix} r_{11}(lT) & r_{12}(lT) & \cdot & \cdot & r_{1M}(lT) \\ r_{21}(lT) & r_{22}(lT) & & & r_{2M}(lT) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ r_{M1}(lT) & r_{M2}(lT) & \cdot & \cdot & r_{MM}(lT) \end{bmatrix} \tag{31}$$

and

$$R(D) \triangleq \sum_{l=0}^{\infty} R_l D^l. \tag{32}$$

By applying Theorem 1 to this system it follows that the tap coefficients are determined by the recurrence relation

$$C_0 = R_0^{-1}$$
$$C_l = -R_0^{-1} \sum_{i=0}^{l-1} R_{l-i} C_i \qquad l \geq 1. \tag{33}$$

It will be clear that Theorem 2 is also valid now with the restriction that l has only positive values and the length of the MTDL is K. In this case the MTDL is also realizable as M shift registers with resistance matrices at the sending end. One can derive that, in doing so, the expression (33) for the C_l matrices stay unchanged.

Decision feedback is another possibility to eliminate MDI [7]. Then the MMF is followed by a "forward" MTDL and a "feedback" MTDL.

VI. AN EXAMPLE

As an example we implemented the transmission of binary data over a multiwire cable, consisting of four identical wires which are symmetrically situated within a cylindrical shield (see Fig. 4). The cable has a length of 1 km and the bit rate is taken 5 Mbit/s. In this example the length of the cable, the bit rate and the sending pulses are such that the noise can be neglected, thus the relations (33) are used for calculating the tap coefficients. We have measured the following matrices:

$$R_0 = \begin{bmatrix} 1 & 0.24 & 0.24 & 0.13 \\ 0.24 & 1 & 0.13 & 0.24 \\ 0.24 & 0.13 & 1 & 0.24 \\ 0.13 & 0.24 & 0.24 & 1 \end{bmatrix}$$

$$R_1 = 0.26I, \qquad R_2 = 0.11I, \qquad R_3 = 0.07I, \qquad R_4 = 0.04I. \tag{34}$$

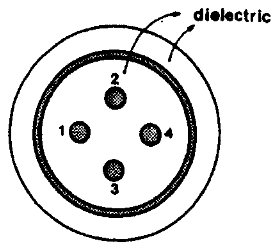

Fig. 4. Cross section of the 4-wire cable within cylindrical shield.

One can verify that $\sum_l' \| R_l R_0^{-1} \| < 1$, thus Theorem 2 can be applied. The calculated C_l matrices are

$$C_0 = \begin{bmatrix} 1 & -0.21 & -0.21 & -0.03 \\ -0.21 & 1 & -0.03 & -0.21 \\ -0.21 & -0.03 & 1 & -0.21 \\ -0.03 & -0.21 & -0.21 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.31 & 0.12 & 0.12 & -0.01 \\ 0.12 & -0.31 & -0.01 & 0.12 \\ 0.12 & -0.01 & -0.31 & 0.12 \\ -0.01 & 0.12 & 0.12 & -0.31 \end{bmatrix}. \tag{35}$$

Because of the several kinds of symmetry in both the R_l and C_l matrices, $\sum_l' \| R_l R_0^{-1} \|$ represents the worst case MDI before the MTDL. Moreover, the output matrices F_l show the same symmetry and thus $\sum_l' \| F_l \|$ represents the worst case MDI at the output, so that Theorem 2 is valid in its full consequence.

At the realization of the C_l matrices, tap coefficients equal or smaller than 3 percent are omitted because these values do not give a substantial improvement of the eye opening. All components of C_2, C_3, etc., are smaller than 3 percent, that is why they are not given at (35). Only C_0 and C_1 are realized and at these matrices the connections between a certain wire and the diagonal opposed one are omitted too. This MTDL is implemented as 4 shift registers at the sending end which are connected to the cable by means of resistance matrices forming the tap coefficients. Fig. 5 shows the eye pattern at the receiving end of the cable if all wires are excited and it is seen that the unequalized system has a fully closed eye as is calculated from (34). Fig. 6 shows the eye pattern of the system characterized by $R(D)R_0^{-1}$ which means that a multiple channel system with matrix D-transform R_0^{-1} is placed between the transmitter and the sending end of the cable. The eye pattern of this system is not closed, which shows that $\sum_l' \| R_l R_0^{-1} \| < 1$. Finally Fig. 7 shows the eye pattern of the equalized system and it appears that the multidimensional Nyquist criterion is satisfied rather well.

VII. CONCLUSIONS

It is shown that for a multiple channel transmission system both the optimum linear receiver (minimum P_e) and the optimum linear constraint receiver (minimum P_e under zero-forcing condition) have the same structure as the optimum linear receiver found by Kaye and George applying the minimum mean-square error criterion. Moreover it appears that by means of the multidimensional Nyquist criterion and the generalization for MDI of a theorem by Lucky for ISI it is rather easy to find the optimum tap settings for a finite length MTDL. The algorithm to calculate the tap settings is further simplified in the case that the noise is unimportant and the sampling instant is smaller than the bit time.

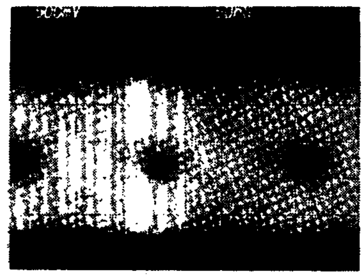

Fig. 5. Eye pattern of the unequalized system.

Fig. 6. Eye pattern of the system $R(D)R_0^{-1}$.

By means of the methods developed in this paper it is shown that MDI is the generalization of ISI.

APPENDIX I

PROOF OF THEOREM 2

Let $\{V_i\}_{i=-\infty}^{\infty}$ be given with $V_0 = I$ and let

$$M_0 = \sum_{i=-\infty}^{\infty}{}' \| V_i \| < 1. \tag{36}$$

Let

$$A = \sum_{n=-\infty}^{\infty}{}' \| \sum_{j=-N}^{N} C_j V_{n-j} \| \tag{37}$$

under the constraint

$$\sum_{j=-N}^{N} C_j V_{-j} = I. \tag{38}$$

We shall prove that a minimum for A exists and that this minimum occurs if

$$\sum_{j=-N}^{N} C_j V_{n-j} = 0, \qquad n = -N, \cdots, -1, 1, \cdots, N. \tag{39}$$

Proof: Due to (38), (37) can be written as follows:

$$A = \sum_{n=-\infty}^{\infty}{}' \| \sum_{j=-N}^{N} C_j (V_{n-j} - V_{-j} V_n) + V_n \|. \tag{40}$$

Fig. 7. Eye pattern of the equalized system.

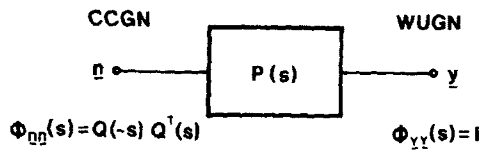

Fig. 8. Multiple noise whitening filter.

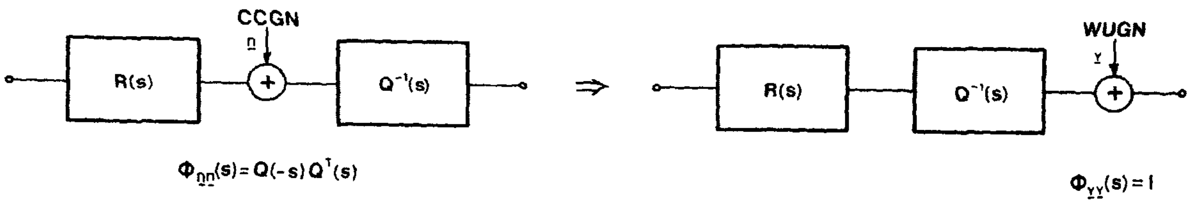

Fig. 9. System $R(s)$ disturbed by CCGN is replaced by the system $Q^{-1}(s)R(s)$ disturbed by WUGN.

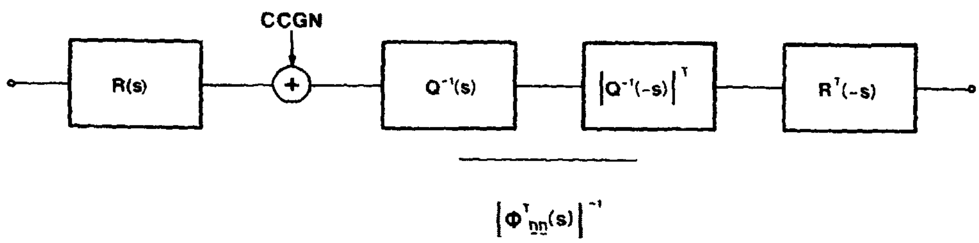

Fig. 10. Multiple channel system disturbed by CCGN in cascade with the multiple whitening matched filter.

Let A be minimal in $(C_{-N}^*,\cdots,C_N^*)$ and let its value there be A^*. Consider A in the point $(C_{-N}^*,\cdots,C_k^* + E_k,\cdots,C_N^*)$ and let its value there be $\bar{A}$. Now we must have

$$A^* \le \bar{A}. \tag{41}$$

From (40) it follows

$$\begin{aligned}\bar{A} &= \sum_{n=-\infty}^{\infty}{}' \left\| \sum_{j=-N}^{N} C_j^*(V_{n-j} - V_{-j}V_n) + V_n + E_k(V_{n-k} - V_{-k}V_n) \right\| \\ &\le A^* - \|H_k^*\| + \sum_{n=-\infty;n\ne k}^{\infty}{}' \|V_{n-k}\|\cdot\|E_k\| \\ &\quad + \sum_{n=-\infty;n\ne k}^{\infty}{}' \|V_n\|\cdot\|E_k\|\cdot\|V_{-k}\| \\ &\quad + \|H_k^* + E_k(I - V_{-k}V_k)\| \end{aligned} \tag{42}$$

where

$$H_k^* \triangleq \sum_{j=-N}^{N} C_j^* V_{k-j}. \tag{43}$$

Choose

$$E_k = -\delta H_k^*(I - V_{-k}V_k)^{-1} \qquad 0 < \delta < 1. \tag{44}$$

This is possible because the inverse of $(I - V_{-k}V_k)$ exists and besides that

$$\|(I - V_{-k}V_k)^{-1}\| \le \frac{1}{1 - \|V_{-k}\|\cdot\|V_k\|}. \tag{45}$$

By means of (44), (42) becomes

$$\begin{aligned}\bar{A} - A^* &\le \|H_k^*\| \Big[-1 + \frac{1}{1 - \|V_{-k}\|\cdot\|V_k\|} \{M_0 - \|V_{-k}\| \\ &\qquad + (M_0 - \|V_k\|)\|V_{-k}\|\} + 1 - \delta \Big] \\ &\le \frac{\delta\|H_k^*\|}{1 - \|V_{-k}\|\cdot\|V_k\|} [M_0 - \|V_{-k}\| + M_0\|V_{-k}\| \\ &\qquad - \|V_k\|\cdot\|V_{-k}\| - 1 + \|V_k\|\cdot\|V_{-k}\|] \\ &\le \frac{\delta\|H_k^*\|}{1 - \|V_{-k}\|\cdot\|V_k\|} [(M_0 - 1)(1 + \|V_{-k}\|)]. \end{aligned} \tag{46}$$

From (46) it follows that

$$\|H_k^*\| = 0, \tag{47}$$

because otherwise there is a contradiction with (41).

APPENDIX II

In this Appendix we prove that the assumptions that the noise functions $n_i(t)$ are white and uncorrelated are not a restriction of the generality; i.e., a system not satisfying these assumptions can be transformed into a system that meets these requirements. The proof starts with the remark that the spectral matrix (which is the Laplace transform of the correlation matrix) of the input noise can be factored, according to [8], in the following way:

$$\Phi_{nn}(s) = Q(-s)Q^T(s) \quad (48)$$

where s is the bilateral Laplace variable. Assume that we have a system with transfer matrix $P(s)$ such that the spectral matrix of the output noise is the identity matrix if the input spectral matrix is given by (48). Then the spectral matrix of the output y of $P(s)$ is written as follows [8]:

$$\Phi_{yy}(s) = P(-s)Q(-s)Q^T(s)P^T(s) \quad (49)$$

(see Fig. 8). From this it follows that

$$P(s) = Q^{-1}(s) \quad (50)$$

satisfies the requirement of white, uncorrelated output noise. A procedure for finding a $Q(s)$ such that both $Q(s)$ and $Q^{-1}(s)$ are stable is also given in [8]. Now we shall further investigate the MMF for colored, correlated Gaussian noise (CCGN). The several impulse responses $r_{ij}(t)$ of the multiple channel system are written in a matrix $R(t)$. From (50) it follows that the multiple channel transmission system with transfer matrix $R(s)$ disturbed by CCGN with spectral matrix $\Phi_{nn}(s)$ can be replaced by a multiple channel transmission system with transfer matrix $Q^{-1}(s)R(s)$ disturbed by white, uncorrelated, Gaussian noise (WUGN) (see Fig. 9). The MMF for this latter system is given by

$$[Q^{-1}(-s)R(-s)]^T = R^T(-s)[Q^T(-s)]^{-1}. \quad (51)$$

Note that the MMF for the system with impulse response matrix $R(t)$ disturbed by WUGN is given by $R^T(-t)$.

So that the MMF for the original system can be written as

$$R^T(-s)[Q^T(-s)]^{-1}Q^{-1}(s) = R^T(-s)[\Phi_{nn}{}^T(s)]^{-1} \quad (52)$$

(see Fig. 10). This MMF we call multiple whitening matched filter (MWMF).

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