A level-set-based strategy for thickness optimization of blended composite structures

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ABSTRACT

An approach is presented for the thickness optimization of stiffened composite skins, which guarantees the continuity (blending) of plies over all individual panels. To fulfill design guidelines with respect to symmetry, covering ply, disorientation, percentage rule, balance, and contiguity of the layup, first a stacking sequence table is generated. Next, a level-set gradient-based method is introduced for the global optimization of the location of ply drops. The method aims at turning the discrete optimization associated with the integer number of plies into a continuous problem. It gives the optimum thickness distribution over the structure in relation to a specific stacking sequence table. The developed method is verified by application to the well-known 18-panel Horseshoe Problem. Subsequently, the proposed method is applied to the optimization of a composite stiffened skin of a wing torsion box. The problem objective is mass minimization and the constraint is local buckling.

1. Introduction

The application of composite structures in aerospace industry has noticeably increased in recent years. Designing such structures with respect to necessary manufacturing and design guidelines with minimum mass involves a large number of design variables and constraints. This results in a highly challenging optimization problem [1–3]. To meet a specified strength with minimum mass, a laminate design procedure may require determination of the total number of plies and the sequence of fiber orientation angles as design variables. Also, several strength related guidelines have to be satisfied. These guidelines are discussed in detail in [4,5]. Moreover, manufacturability of the designed structure must be guaranteed. In a large scale structure, different regions may be subject to different loads. In an optimized design a laminate thickness may vary all-over the structure depending on the distributed loads. Also, for large scale composite structures, such as an aircraft wing or fuselage, stiffeners are added to enhance structural performance in carrying compressive and tensile loads. The stiffeners divide the structure into smaller panels. To ensure manufacturability of the composite skin it is crucial for the plies to be continuous among adjacent panels while the laminate thickness varies. Continuity of plies in adjacent panels, which is commonly referred to as blending [6], is a particularly difficult constraint to deal with [3]. Blending has to be satisfied in addition to the earlier mentioned strength related

guidelines. Designing panels individually using local loads results in designs with significant manufacturing difficulties. The reason is that the resulting stacking sequences of laminates in adjacent panels may differ considerably [7]. Therefore, various methods have been proposed to address the global optimization of composite skins taking blending into account, see e.g. [8–11]. Liu and Haftka [12] introduced blending as a constraint to the optimization problem. They proposed a two-level algorithm. At the global level optimization, the continuous number of each ply in each panel is obtained. At the local level, those continuous numbers are rounded such that a genetic algorithm (GA) can prescribe blended stacking sequences for each panel. The rounding of the ply stack numbers, however, can cause internal panel load redistribution and subsequently causes constraints such as strain or buckling to be violated [12]. Adams et al. [8] introduced the ‘edit distance’ method in combination with a GA. In this study, a set of panel populations evolves to converge to a globally blended solution. In their research, the “edit distance” is a blending indicator of the stacking sequences in adjacent panels. Designs with a higher degree of blending are rewarded via a fitness function. However, a large number of constraints was required to enforce blending. This made the optimization problem very difficult to be solved with any optimization algorithm [7]. In their later works [7,13], Adams et al. introduced the concept of the guide-based blending in which the thickest stacking sequence is called the guide laminate and the stacking sequence of the laminates in all panels is obtained by

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dropping plies from the guide laminate. The method ensures the continuity of plies across neighboring panels without adding extra blending constraint. However, being restricted to a certain trend of dropping plies, which results in either outwardly or inwardly blended laminates, it ignores parts of the feasible region in the design space [1]. Zehnder and Ermanni [14,15] introduced the patch concept for a globally blended design. In their work, instead of panel wise optimization of each laminate, the structure is treated as several globally extended layers that will be assembled together. The globally extended layers are called patches and are characterized by their geometry, material, and material orientation. Studying the structure globally has an advantage over panel wise optimization. A change of the stiffness of a panel also changes the local load distribution over the panels [16]. This invalidates the panel wise optimized design in terms of global constraint satisfaction and raises the need for a multi-level procedure. The patch concept is also attractive as the globally blended design can be obtained without narrowing the feasible region of the design space as happens when the ply drop is restricted to a specific pattern. Nevertheless, the patch concept suggests a large number of design variables which makes the optimization problem difficult to be solved [17].

Density-based topology optimization was originally introduced to solve for a two-phase solid-void problem [18,19]. The interpolation schemes such as SIMP (Solid Isotropic Material with Penalization) [19,20] (see [21,22] for recent advancements in the SIMP method) and RAMP (Rational Approximation of Material Properties) [23] are used to relax the discrete nature of the solid-void optimization problem. The interpolation schemes were subsequently extended to account for multiple phases in a design domain, see e.g. [24–27]. The application of topology optimization to laminated composites considering manufacturing constraints is investigated in Sørensen and Lund [28], Sørensen et al. [29], Sørensen and Stolpe [30], and Lund [31]. Recently, Allaire and Delgado [3] investigated the optimal design of a composite laminate by introducing the shape and the topology of each ply as design variables in addition to the fiber orientation angles and the sequence of the stack. Their proposed algorithm is, however, unable to prescribe a blended design [3].

A convenient aid in globally blended design optimization is the notion of the stacking sequence table (SST) introduced by Carpentier et al. [32]. The SST is a reference table for the stacking sequence of laminates with different thicknesses where a thicker laminate is obtained by adding plies to a thinner one resulting in admissible stacking sequences. The SST idea using a GA and the concept of the sequence of ply drops are investigated in [1,2,17], respectively. Meddaikar et al. [2] combined a GA for SST optimization with a structural and a load approximation scheme. In their work, the approximation scheme is shown to be efficient in terms of computation cost.

The idea of the SST is shown to be practical to obtain optimized blended designs [2,17]. However, the ply drop locations in a fully GA-based final design are restricted to pre-specified locations. The reason is that the discrete nature of a GA does not allow for continuous ply drop locations. Irisarri et al. [33] have recently carried out a study to develop an optimization algorithm based on SST in which the location of the ply drop is not pre-specified but obtained using topology optimization.

The majority of the algorithms dedicated to global optimization of blended stiffened composite structures are fully or partially dependent on evolutionary algorithms [1,7,12,17,34–37], typically the GAs, to deal with the discrete nature of the variables in designing a composite laminate (the use of evolutionary algorithms for the optimization of stacking sequences is reviewed in [6,38]). However, a GA is generally more time consuming than a gradient-based algorithm as there is a large number of designs to be analyzed [2]. Furthermore, to avoid narrowing the feasible region of the design space, ply drop locations do not have to be pre-specified. The ply drop location can be a continuous variable which suggest a gradient-based optimization algorithm. For the aforementioned reasons, the current research aims at investigating a problem parametrization that is suitable for the application of a

gradient-based optimization algorithm. The proposed method is capable of generating a globally blended design at a limited computation cost while all other manufacturing guidelines are also satisfied.

The proposed approach separates the optimization of the stacking sequences from the optimization of the thickness distribution. A method has been developed by the authors to generate laminates with desired stacking sequences with respect to the optimization problem [39].

The present research is mainly focused on optimizing the thickness distribution with a given (fixed) set of stacking sequences. To this end, first, an SST is generated based on an estimation about the optimized stiffness and thickness distribution over the structure. The laminates in an SST satisfy symmetry, covering ply, disorientation, percentage rule, balance, and contiguity of the layup. Next, a novel level-set gradient-based method is introduced for the global optimization of the locations of the ply drops. A single function delineates the span of multiple levels where each level represents a ply in a stiffened composite skin. This stands in contrast to the studies where multiple level-set functions are used to represent multiple material phases [40,41]. The proposed method aims at turning the discrete optimization problem associated with the integer number of plies into a continuous problem. This is done through the way the problem is parametrized; the design variables are never rounded in this approach. The level-set function gives the optimum thickness distribution over the structure for a specific SST. The idea of combining an SST with an optimization algorithm to obtain the span of each ply is close to that introduced in [33]. However, the presented level-set method benefits from a straightforward procedure compared to rather complex topology optimization technique used in [33]. This allows convenient application of the method to the optimization of large scale structures. The developed method is verified by its successful application to the well-known horseshoe panel optimization problem studied in [1,7,11,17,34–37]. To investigate the performance of the method in dealing with a real problem, the proposed method is then applied to the layup optimization of a composite skin of a wing. Local buckling is considered as the constraint of the problem and a standard finite element package is used to calculate buckling factors.

2. Generating a stacking sequence table (SST)

An SST is a reference table for the stacking sequences of laminates with different thicknesses. Each column of the SST represents the stacking sequence of a certain number of plies. To guarantee the blending of a design, a thicker stacking sequence can only be obtained through adding plies to a thinner one [1]. To keep the design blended during the optimization process, the laminates across the structure are only allowed to be selected from the SST. Industrial requirements impose that the ply angles should be selected from a limited set [17,42]. In the present study, the ply orientations are limited to the set $\{0^\circ, \pm 45^\circ, \text{ and } 90^\circ\}$ of angles [42,43]. Every laminate in an SST may be required to satisfy a number of strength related guidelines. In the present study, the following conventional guidelines proposed and applied in [2,17,33,42,43], are imposed in an SST:

- *Symmetry*, the laminate should be symmetric with respect to its center line.
- *Covering plies*, the outermost ply has to have the orientation of either $+45^\circ$ or -45° .
- *Disorientation*, the maximum orientation difference of two adjacent plies is 45° .
- *Percentage rule*, the number of plies of a certain orientation has to be at least 10% of the total number of plies in a laminate.
- *Balance*, the total number of plies with $+45^\circ$ orientation in a laminate is equal to the total number of plies with -45° orientation.
- *Contiguity*, not more than 4 successive plies with a same orientation are allowed to stack together.

In general, imposing the aforementioned guidelines has a large

Fig. 1. The flowchart of the 2-step procedure of generating an SST.

influence on the complexity, computation cost, and the quality of the optimum design of the optimization problem (see e.g. [44,45]). The proposed method to generate an SST can be simply adapted to ignore or relax any of the aforementioned strength related guidelines.

A 2-step method was presented by the authors to generate an SST where the fiber orientations have to be selected from a limited set of angles [39]. As generating the stacking sequences is a key part of the optimization problem of composite structures, the (modified) 2-step approach is described in the following.

2.1. Step 1: obtaining the optimized stiffness and thickness distribution (idealized design)

To generate an SST, first the optimized stiffness and thickness distribution are estimated [33,42,46–48]. To this end, lamination parameters [49,50], polar parameters [47,48], or the smeared stiffness

method [42] can be used. The smeared stiffness method requires fewest parameters and is computationally least expensive. Therefore, it has been selected to obtain an estimate of the optimized stiffness and thickness distribution (idealized design [33]).

The smeared stiffness method is used to estimate the extensional (**A**) and the bending (**D**) stiffness matrices without knowing the stacking sequence of a laminate. This is achieved by assuming a homogeneous section for the layouts [42]. According to the classical laminate theory [51], the matrix **A** is defined as:

$$\mathbf{A} = h \left(\sum_{k=1}^N \frac{(\bar{Q}_{ij})_k}{N} \right), \quad i = j = 1, 2, 6 \quad (1)$$

where N and h represent the total number of plies and the total thickness of the laminate, respectively. $\bar{Q}_{ij}$ represents the transformed plane stiffness [51].

	32	30	26	24	22	18	16	14	10	8
ply index										
1	45	45	45	45	45	45	45	45	45	45
2	90	90	90	90	90	90	90	90	90	90
3	-45	-45	-45	-45	-45					
4	-45	-45	-45	-45	-45	-45	-45	-45		
5	90	90	90	90	90	90				
6	-45	-45								
7	90									
8	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0						
11	45	45	45	45	45	45	45	45		
12	0	0	0	0	0	0	0			
13	0	0	0	0	0	0	0	0	0	
14	45	45								
15	45	45	45	45	45					
16	0	0	0							

Fig. 2. A generated SST using the 2-step method. Fields marked red indicate dropped plies. Due to symmetry, only the stacking sequences of half-laminates are shown. This SST is generated for the second example of Section 4.

Using the material homogeneity assumption [42], the bending stiffness matrix (**D**) can be obtained through:

$$\mathbf{D} \approx \mathbf{A}h^2/12 \quad (2)$$

Liu et al. [42] proposed solving the following optimization problem to obtain an estimation about the thickness and the stiffness distribution over the structure:

$$\min f = \sum_{j=1}^{N_p} (n_0^j + n_{45}^j + n_{90}^j) S_j t \quad (3)$$

where N_p is the total number of panels, S_j is the area of panel j , and t is the ply thickness. The number of plies of each orientation in each panel (or a set of panels), n_0^j , n_{45}^j , n_{90}^j , $j = 1, \dots, N_p$ are the design variables of this optimization problem. Due to the assumed balance guideline, the number of $+45^\circ$ plies has to be equal to the number of -45° plies. Therefore, n_{45}^j is defined to be the sum of $+45^\circ$ and -45° plies.

Here, the constraints of the optimization problem in Eq. (3) are defined as follows:

$$\begin{aligned} g_i &\leq 0 \quad i = 1 \text{ to } N_c \\ \frac{n_0^j}{N^j} &\geq 0.1 \quad j = 1 \text{ to } N_p \\ \frac{n_{45}^j}{N^j} &\geq 0.2 \quad j = 1 \text{ to } N_p \\ \frac{n_{90}^j}{N^j} &\geq 0.1 \quad j = 1 \text{ to } N_p \end{aligned} \quad (4)$$

where g_i is a buckling constraint (defined in Section 3.3), N_c represents the number of required buckling constraints, and $N^j = n_0^j + n_{45}^j + n_{90}^j$. As it can be seen in Eq. (4), it is required that the percentage of the plies of each orientation is $\geq 10\%$ in each panel.

Solving the optimization problem defined in Eq. (3) gives an estimation about the 'idealized' stiffness and thickness distribution. The output of this step is a table called SST-data. In an SST-data table, each thickness value appears once and a unique **AD** stiffness vector is assigned to every (rounded to the nearest integer) thickness value. Some panels (or sets of panels) may obtain the same thickness value while having different values of n_0^j , n_{45}^j , and n_{90}^j . This means that, for a laminate with a certain thickness, different stiffness values are required in different panels (regions) of the structure. This suggests the existence of multiple SST-data tables.

2.2. Step 2: fitting the stacking sequences

In the second step, the SST-data table(s) obtained in step 1, is used to generate the stacking sequence of laminates with different thickness values. In the procedure of generating the stacking sequences, all required laminate design guidelines have to be satisfied.

All valid laminates (laminates that satisfy the strength related guidelines) with the thickness equal to that of the thinnest laminate obtained in the step 1 are generated (see Fig. 1 for details on this procedure).

Among the set of the valid thinnest laminates, the one which has the closest stiffness values to those estimated for the same thickness laminate in step 1, is selected. A Root Mean Square Error (RMSE) of the components of the **A** and the **D** matrices was used to identify the laminate with the desired stiffness values.

To generate a thicker laminate, a ply (or two plies in case the fiber orientations are -45° and $+45^\circ$) has to be added to a thinner laminate. All valid laminates, according to the required guidelines, are generated. From the set of the newly built laminates the one with stiffness values closest to those of the laminate with the same thickness in the SST-data table is selected. This procedure continues until a laminate with the thickness equal to that of the thickest laminate in the SST-data table is reached.

It is possible that as a result of adding a ply to a thinner laminate, the thickness of the newly built laminates does not exist among the thickness values in the SST-data table. In this case, the stiffness values of the laminate with the closest thickness values in the SST-data table are (linearly) interpolated and used to select the fittest laminate.

Here, the SST was generated by successively adding plies to the thinnest laminate. Alternatively, the thickest laminate could have been generated from which plies had to be dropped successively. However, as the number of valid thinnest laminates is smaller than that of thickest laminates, it is cheaper to start with the thinnest laminate. The advantage of fitting an SST the way discussed is that it can be generated quite cheaply.

In case multiple SST-data tables are obtained from step 1, multiple SSTs (corresponding to each SST-data table) can be generated quickly using the proposed fitting method. To have an indication about the performance of the k th SST, using the idealized thickness distribution, the structure has to be covered with the stacking sequences of the k th SST. For this structure, the following expression has to be calculated:

(a) Level-set function covering a 1D structure.

(b) Thickness distribution specified by the level-set function.

Fig. 3. Determining thickness distribution using a level-set function.

$$\eta_i^k = w_i^T K_B^k w_i, \quad i = 1 \text{ to } N_c \quad (5)$$

where w_i represents an eigenvector obtained as a result of solving the optimization problem defined in Eq. (3). K_B^k represents the bending stiffness of the structure when it is covered with the stacking sequences of the k th SST and the idealized thickness distribution is applied. The SST which maximizes the minimal η_i^k is selected as the best generated SST.

Laminates with the same thickness but in different SSTs are required to satisfy the percentage rule guideline. Therefore, the number of plies of each orientation angle is almost equal for the laminates with the same thickness value. This means that, the in-plane stiffness for equally thick laminates in different SSTs is almost similar. Thus it can be assumed that the load redistribution is negligible as a result of using the same thickness distribution but stacking sequences from different SSTs.

The advantage of using the quality indicator defined in Eq. (5) is

that different SSTs can be evaluated without performing a finite element analysis. This results in a cheap SST evaluation.

Fig. 1 shows the flowchart of the 2-step procedure of generating an SST.

As mentioned before, some of the aforementioned strength related guidelines may not be required. As the major part of generating an SST concerns searching for laminates that satisfy the required guidelines (see Fig. 1), the search criteria can be easily adapted to only select the required laminates.

An SST, in general, can include symmetric laminates with even and odd number of plies. However, only symmetric laminates with even number of plies are used.

An SST, as an example, is shown in Fig. 2. Fields marked red indicate dropped plies and ply indices (first column from left) are in the ascending order from the outer most ply towards the laminates center.

3. Level-set-based thickness optimization

Using an SST, the optimization problem will be simplified to determine the optimum value of the thickness (from the SST) that has to be placed in each region of the structure. The boundaries of the stacks from the SST have to be determined and as long as laminates are selected from the SST, it is guaranteed that all guidelines are satisfied and also the final design is blended. Nevertheless, working with an SST demands dealing with discrete variables which cannot be done in the framework of gradient-based algorithms. In the following we introduce an efficient level-set method which turns the discrete nature of the above optimization problem into a continuous problem.

3.1. The proposed level-set method

An auxiliary level-set function is used to determine the boundary of each stacking sequence of an SST. The level-set function (Φ) is discretized by interpolation among a limited set of N_ϕ design nodes X_ϕ^n distributed over the structure. The values $\phi^n = \Phi(X_\phi^n)$ are the variables in the optimization. The value of Φ at any location X can then be found by the interpolation

$$\Phi(X) = \sum_{n=1}^{N_\phi} \Psi^n(X) \phi^n \quad (6)$$

where $\Psi^n(X)$ are interpolation functions. Here, bilinear interpolation functions on quadrilateral domains, spanning multiple finite elements are used. The discretization of Φ is independent of the finite element discretization. The same Φ can be used on models of the same structure with different mesh sizes, e.g. a coarse model for stiffness and a fine model for stress evaluation.

In the proposed method, the level-set function is used to select a ply stack from the SST. The number of plies N_{plies} of the laminate at point X is determined through the level-set function by:

$$N_{plies}(X) = \{\max LV_i | LV_i \leq \Phi(X), (LV_i \in LV)\} \quad (7)$$

where LV represents the set of all ply stack values in the SST. According to Eq. (7) the highest ply stack value below the level-set function at point X gives the number of plies of the laminate covering point X . Fig. 3a shows a level-set function covering a 1D structure with length L , where the ply stack thicknesses are selected from the SST in Fig. 2. According to the SST, LV is: {8, 10, 14, ..., 32} plies. Fig. 3b shows the resulting thickness distribution over the structure.

Each finite element node will be assigned a level-set function value based on interpolation from the design node values. Within an element the interpolation of the value of the element nodes specifies the level-set function. Figure 4 schematically shows this procedure in three steps. The element obtains area weighted stiffness data based on the area that each stacking sequence covers in the element.

Using the introduced method, the discrete nature of the SST-based

Fig. 4. A 3-step interpolation procedure towards defining a level-set function given the design node values.

optimization problem is turned into a continuous one as the design variables change continuously. The “max” function used in Eq. (7) determines the location of the ply drop. It is the continuous change of the location of the ply drop with respect to the (continuous) change of a design variable which is differentiable and gives sensitivities. Figure 5 schematically shows the continuous change of the ply drop location (thickness distribution) as a result of continuous change of design variables. Continuous change of the design variables allows using any linear or quadratic programming algorithm to solve the optimization problem. In the present study, the ‘constrained steepest-descent’ algorithm [52] is used. Figure 6 schematically shows the optimization framework.

3.2. Optimization objective

In the present study, the optimization objective is mass

minimization. As the level-set function specifies the thickness distribution all over the structure, it can be concluded that the structure’s mass and stiffness can be implicitly derived with a given level-set function by determining the number of plies covering different regions of the structure as discussed earlier. Moreover, since the ply thickness is directly related to the value of the level-set function, a good approximation of the mass W can be obtained by integration of the level-set function over the structural domain Ω :

$$W \approx \rho \int \Phi(X) d\Omega = \rho \sum_{n=1}^{N_\phi} A^n \phi^n \quad (8)$$

where ρ is the ply density and A^n is connected to the area belonging to design node n . This means that the objective function is taken as linear in the design variables.

(a) The $\Phi^A \Phi^B$ line represents a level-set function. The resulting thickness distribution is shown with the black solid line. X_0 indicates the ply drop location. (b) As a result of the continuous change of the design variable Φ_0^A towards Φ_1^A , the ply drop location changes continuously from X_0 towards X_1 .

Fig. 5. Continuous change of the thickness distribution as a result of continuous change of design variables.

Fig. 6. Flowchart of the optimization procedure. The dashed line represents the internal loop for the numerical sensitivity analysis in each iteration of the optimization problem.

Fig. 7. 18-panel horseshoe configuration, all loads in lbf/in ($\times 175.1$ for N/m).

3.3. Constraint definition

In this section we discuss how to minimize the structural mass while subjected to local buckling constraints. The following eigenvalue equation gives the buckling loads [53]:

$$(K_B - \lambda_i K_G) \cdot w_i = 0 \quad (9)$$

where K_B is the global bending stiffness matrix, K_G is the global stress stiffness matrix, and the eigenvalue λ_i is the i^{th} load multiplier or the buckling factor. The solution of Eq. (9) gives a number of eigenvalues λ equal to the rank of K_G . The vector w_i is the mode shape corresponding

to the i^{th} buckling factor. The constraints of the problem are defined as [12]:

$$g_i: 1 - \lambda_i \leq 0 \quad i = 1 \text{ to } N_c \quad (10)$$

3.4. Sensitivity analysis

As the objective function (Eq. (8)) is a linear function of design variables, the sensitivity of the objective function with respect to the design variable ϕ^i can be simply obtained through:

$$\frac{dW}{d\phi^i} = \rho A^i \quad (11)$$

To obtain the sensitivity of constraints, numerical forward difference scheme is used. At a given design, there is a set of N_c buckling factors. Each corresponds to a constraint. The sensitivity of a buckling constraint in (3) can be translated to the sensitivity of the buckling factors. As a result of perturbing a design variable, a new set of N_c buckling factors will be obtained through a finite element analysis. The sensitivity of each buckling factor λ_i to the j^{th} design variable is calculated through:

$$\frac{d\lambda_i}{d\phi^j} = \frac{\lambda_i(\Phi + \delta\phi^j) - \lambda_i(\Phi)}{\delta\phi^j} \quad (12)$$

3.5. Switching of the mode shapes

The order of the buckling mode shapes may change as the design of the structure changes. During the numerical sensitivity analysis, it is crucial that in (12) the two buckling factors being subtracted from each other, belong to the same mode shape. Therefore, when a new set of buckling factors is obtained after perturbing a design variable, their corresponding mode shapes must be extracted too. As mentioned earlier, eigenvectors in (3) represent the buckling mode shapes corresponding to each buckling factor. The eigenvectors include the values of the out of plane translational and rotational degrees of freedom. For two eigenvectors w_i and w_j , obtained from two different buckling analyses, we can calculate $MAC_{ij} = \frac{|w_i^T \cdot w_j|}{\|w_i\| \|w_j\|}$. If $MAC_{ij} \approx 1$, the eigenvectors are correlated; otherwise, w_i and w_j represent different mode shapes. Detailed information on tracking mode shapes can be found in [54].

In the present study, the linearized 'constrained steepest-descent' algorithm [52] is used to solve the optimization problem. If,

	48	46	44	42	40	36	34	32	30	28	24	22	20	18	16	12	10	8
ply index																		
1	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
2	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
3	90																	
4	-45	-45	-45	-45	-45													
5	90	90	90	90	90	90												
6	90	90																
7	45	45	45	45	45	45	45	45	45	45								
8	0	0	0															
9	0	0	0	0														
10	45	45	45	45	45													
11	90	90	90	90	90	90	90											
12	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
13	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45								
14	0	0	0	0	0	0	0	0	0									
15	0	0	0	0	0	0	0	0	0	0								
16	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45			
17	90	90	90	90	90	90	90	90	90	90	90	90	90	90				
18	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90			
19	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45		
23	90	90	90	90	90	90	90	90	90	90	90							
24	90	90	90	90	90	90	90	90	90	90	90	90						

Fig. 8. The SST generated for the Horseshoe Problem.

Fig. 9. The distribution and the numbering of 12 design nodes.

Table 1

The initial design used for the level-set-based thickness optimization for the Horseshoe Problem. The distribution of the design variables is shown in Fig. 9

Design variable number	1	2	3	4	5	6	7	8	9	10	11	12
Initial value (plies)	38	25	20	44	40	26	42	38	28	35	32	24

alternatively, a non-linear programming algorithm (e.g. sequential quadratic programming (SQP) [52]) is used, the mode shapes also have to be tracked between the iterations of the optimization problem. The reason is that the second derivative information is required in these algorithms. The second derivative information is usually approximated using the first derivative information of the iterations of the optimization problem [52]. Therefore, it is crucial to make sure that the first derivative information of the buckling constraints also belong to the same mode shape.

A linearized optimization algorithm (although less efficient compared to e.g. an SQP algorithm) does not require the second derivative

information and thus is selected for the present study.

3.6. Mesh density

A reliable buckling analysis requires sufficient density of the finite element mesh. To obtain a proper element size, a target structure has to be subjected to buckling analysis, each time with a finer mesh size until the difference in the critical buckling factor as the result of remeshing is negligible. A mesh study is performed for the examples presented in Section 4.

4. Results and discussion

In this section two examples are presented to show the performance of the proposed level-set method. In the first example it is applied to the well-known 18-panel problem in a horseshoe configuration as shown in Fig. 7. This problem was first proposed by Soremekun et al. [11] and subsequently studied in [1,7,17,34–37].

In the Horseshoe Problem, the load redistribution is ignored. This results in a too simplified problem. To examine the capability of the method in dealing with real problems, in the second example the proposed method is applied to the optimization of a multi-panel composite skin of a torsion box.

Four-node, quadrilateral shell elements with 6 degrees of freedom per node are used for the finite element analysis in both examples. As mentioned earlier, to form the level set function Φ , as defined in Eq. (6), bilinear interpolation functions are used.

4.1. Example 1, Horseshoe Problem

The optimization objective in this problem is mass minimization of the whole structure without individual panel failure under buckling. The final solution in this problem has to be blended. Here, the formerly mentioned set of $\{0^\circ, \pm 45^\circ, \text{ and } 90^\circ\}$ is used as fiber orientations. For the construction of each ply a Graphite/Epoxy IM7/8552 material is used where $E_1 = 141$ GPa (20.5 MSi), $E_2 = 9.03$ GPa (1.31 MSi), $G_{12} = 4.27$ GPa (0.62 MSi), $\nu_{12} = 0.32$, and ply thickness is 0.191 mm (0.0075 in.). The optimization problem is formulated as follows:

Fig. 10. Contour plot of the thickness distribution for the optimization problem with 12 design nodes.

Table 2
Average number of plies and the obtained buckling factors for each panel at the optimum design with 12 design nodes.

Panel	Buckling factor	Average number of plies
1	1.035	36.73
2	1.060	32.87
3	1.141	22.08
4	1.110	19.53
5	1.037	16.77
6	4.064	35.76
7	3.860	30.54
8	1.000	25.23
9	1.034	41.75
10	1.000	38.74
11	1.012	33.13
12	1.211	32.54
13	2.661	30.13
14	3.625	27.67
15	1.007	25.14
16	1.004	31.40
17	2.456	26.16
18	1.028	22.57

Fig. 11. The distribution and the numbering of 30 design nodes. The bold labels are related to the design nodes added to the configuration shown in Fig. 9.

$$\begin{aligned} \min \quad & W(\phi^N) \\ \text{s. t.} \quad & g_{ij} \leq 0 \quad i = 1 \text{ to } 5 \quad \text{and} \quad j = 1 \text{ to } 18 \\ & \phi_{\min} \leq \phi^n \leq \phi_{\max} \end{aligned} \quad (13)$$

where W represents the overall mass of the structure as a function of design node vector ϕ^N . $\phi_{\min}$ and $\phi_{\max}$ denote lower bound and upper bound for the values at the design nodes, respectively. $\phi_{\min}$ and $\phi_{\max}$ has to be determined based on the idealized design. g_{ij} represents a constraint on the i th buckling factor of the j th panel. As it can be seen in Eq. (13), five mode shapes are considered as buckling constraints for each panel. Thus the total number of 90 buckling constraints is imposed to this problem.

The SST shown in Fig. 8 is used for the thickness optimization of the Horseshoe Problem. A detailed description on generating this SST, as an example, is provided in A.

For the optimization of the thickness distribution, 12 design nodes are distributed as shown in Fig. 9. As each panel may have a varying thickness, finite element analysis is performed to calculate the buckling factors for each panel. To have a valid mesh size, a mesh study, as described in Section 3.6, is performed on a typical panel. Based on the

mesh study, shell elements with mesh size of 0.0254×0.0254 m (1×1 in.) are adopted for the simulations. According to the generated SST, shown in Fig. 8, the thickness value of the thickest laminate is $0.191 \times 48 = 9.17$ mm. As this value is small compared to the smallest panel dimension (305 mm), laminates are considered to be thin and the kinematics of Kirchhoff theory is used for the shell elements. The entire Horseshoe Problem is discretized into 5472 shell elements and the total number of 5701 nodes.

As described before, the values of the design variables prescribe a thickness distribution over the structure. As for the initial design, it is reasonable to assign values to design variables such that the resulting thickness distribution is close to the idealized thickness distribution. Table 1 shows the initial design used for the level-set-based thickness optimization (refer to A for the idealized design information related to the Horseshoe Problem).

Fig. 10 shows the contour plot of thickness distributions at the point of optimum. The optimization problem converges in 4 iterations where each iteration takes about 15 min on a regular PC (CPU: 2.6 GHz, RAM: 8 GB).

Fig. 12. Contour plot of the thickness distribution for the optimization problem with 30 design nodes.

Fig. 13. Mass evolution for the optimization of both 12 and 30 design nodes. The final design of the optimization with 12 design nodes is used as the initial design for optimization with 30 design nodes.

Table 2 shows the average number of plies in each panel. As the thickness distribution varies over the surface of each panel, to show the final result, an average thickness is used to have an indication of the number of plies in each panel. The overall weight of the structure in this case is 32.64 kg. Buckling factors obtained for each panel at the point of optimum are also shown in Table 2. As it can be seen in the table, panels 6, 7, 13, and 14 have eigenvalues much higher than their critical value. The reason is that the number of design nodes shown in Fig. 9 is not sufficient to prescribe thickness distributions such that all panels become critical with respect to buckling. For example, the thicknesses of panels 13–18 are all interpolated through only 4 design nodes (design nodes 8, 9, 11, and 12 in Fig. 9). As panels 15 and 16 have eigenvalues very close to their critical value, the mentioned design nodes do not allow for thinner laminates. Therefore, some panels inevitably remain thicker than their potential optimum thickness.

Table 3
Average number of plies and the obtained eigenvalues for each panel at the optimum design with 30 design nodes.

Panel	Buckling factor	Average number of plies	Number of plies Yang et al. [1]
1	1.015	36.61	34
2	1.006	32.43	28
3	1.033	21.14	22
4	1.001	18.48	20
5	1.092	16.82	16
6	1.448	26.04	22
7	1.014	19.18	20
8	1.000	26.21	26
9	1.007	41.22	38
10	1.002	38.71	36
11	1.011	33.12	30
12	1.017	31.61	28
13	1.413	24.45	22
14	1.006	18.82	20
15	1.004	25.70	26
16	1.009	32.39	32
17	1.378	21.35	20
18	1.001	22.29	24

To overcome this, more design nodes are added as shown in Fig. 11. As it can be seen in the figure, the design nodes do not have to be placed on panels corners. The design nodes can be placed wherever a more detailed design is required. Figure 12 shows the contour plot of the optimized thickness distribution with 30 design nodes. The same trend of thickness distribution (left side of the horseshoe with thicker laminates than the right side) shown in Fig. 10 is observed in Fig. 12 as well. However, due to the increased number of design nodes, the ply drop locations are more freely prescribed over the surface of each panel.

It is interesting to mention that the optimum design of the problem with 12 design nodes is used as the initial design of the optimization problem with 30 design nodes. As the initial design is already reasonably close to the optimum point, it takes only 4 iterations until the optimization problem with 30 design nodes converged. The final mass of the structure in this case is 30.60 kg. Compared to the problem with 12 design nodes, a further reduction of 2.04 kg in mass is obtained by increasing the number of design nodes. As indicated by the results, a lighter design may require more design nodes. A problem with more

Fig. 14. Finite element model of a wing torsion box. The torsion box is subjected to a load case and has been analyzed with a reasonably coarse mesh.

Fig. 15. The geometry, loads, and supports of the torsion box skin.

Fig. 16. Design nodes' locations are shown with pointed dots.

Fig. 17. Contour plot of the thickness distribution of the optimized skin, the stacking sequences can be read from the SST in Fig. 2.

Table 4

The first 9 eigenvalues of the optimized structure.

Eigenvalues 1 to 3	Eigenvalues 4 to 6	Eigenvalues 7 to 9
1.0043	1.0661	1.1719
1.0358	1.0962	1.1897
1.0489	1.1360	1.2031

design nodes is computationally more expensive because of more required sensitivity calculations. However, here it is shown that the optimization problem can start with a reasonably small number of design nodes until the initial design is improved towards the optimum point in that design space. Then design nodes can be added locally wherever a more detailed design is required. As a result, only a small part of the optimization problem proceeds with a relatively high number of design nodes. This reduces the overall computation time of the optimization problem. Figure 13 shows the mass evolution for the optimization of

both 12 and 30 design nodes in a single graph.

Table 3 shows the average ply number and the buckling factor of each panel after optimization with 30 design nodes. The average number of plies in each panel is compared to those reported in [1]. As it can be seen in Table 3, the obtained ply number for each panel is in a good agreement with those reported in literature.

The final weight of the structure obtained using the proposed method is higher than that reported in [1,35]. This is due to two reasons. Firstly, in [1,35] there are more options for ply orientations than used in the current study. More options for ply orientations result in a more optimal placement of fibers which may finally cause a lower number of plies to provide sufficient stiffness in a specific region of the structure. Secondly, the reported results in [1,35] are only valid for symmetric and balanced laminates while the presented results in the current study are valid for all design guidelines of laminates mentioned in Section 2. Naturally, as more design guidelines are added to the optimization problem, a heavier feasible design is obtained. For example, in [1] the reported mass for only symmetric laminate is

Fig. 18. Improvement of the eigenvalues from the initial design towards their critical value at the point of optimum.

Fig. 19. Mass history towards the minimized value.

28.44 kg. But, this value increases to 28.82 when balance is added.

Regarding the result comparison, it is important to mention that the results reported in [1,35] are obtained for panels with constant laminate thickness. However, using the proposed method, the laminate thickness can vary in each panel. As the purpose of the result comparison is to investigate the validity of the proposed method, an area weighted average of thicknesses in each panel is presented.

4.2. Example 2, torsion box skin

The stiffened skin of a wing torsion box (Fig. 14) is the target structure for optimization in this example. The wing torsion box is subjected to a load case, which has been analyzed with a reasonably coarse mesh (Fig. 14). From this analysis the free body diagram of the skin has been isolated and the loads applied on it have been extracted. Figure 15 shows the loads applied to the skin. In the figure, arrows parallel to each edge represent shear loads in the direction of the arrow. The skin is simply supported on the right edge and the out of plane translational degree of freedom for all edges and stiffeners is set to be zero. To obtain proper values for local buckling, the finite element model has been modified such that each (slightly curved) panel is modeled as flat. To obtain a proper mesh density for buckling analysis, a panel between two ribs and two stringers was studied as described in Section 3.6. A typical panel is discretized into 22 by 6 elements. The entire structure is discretized into 13311 shell elements with 13624 nodes. ABAQUS 6.12 finite element package is used for analysis. Following the procedure shown in Fig. 1, the SST shown in Fig. 2 is obtained. According to the SST, the LV set is {8, 10, 14, 16, ...} plies, where the ply thickness is equal to 0.13 mm. According to the generated SST

(shown in Fig. 2), the thickness value of the thickest laminate is $0.13 \times 32 = 4.16$ mm. As this value is small compared to the smaller dimension of a typical panel (143 mm), laminates are considered to be thin and the kinematics of Kirchhoff theory is used for the shell elements. For each individual element individual values of the **A** (extensional stiffness) and the **D** (bending stiffness) matrices can be specified according to the classical laminated plate theory. To optimize the structure, 9 nodes have been appointed to be the design nodes of the problem with locations as can be seen in Fig. 16. Thirty eigenvalues are considered as constraints of the problem. The initial values of the design variables are given such that the resulting level-set function prescribes a laminate with constant thickness equal to 24 plies all-over the structure. The purpose for this choice is to investigate the capability of the proposed method in thickness optimization when the designer has no clue about the optimized configuration.

Figure 17 shows the contour plot of the thickness distribution of the optimized structure. As it can be seen in the figure, the laminates are thicker in the aft-root region of the structure. The front-root corner of the structure is covered with a relatively thin laminate. This can be expected because according to Fig. 15, for this load case the front-root corner of the structure is under less compression thus is less critical for buckling.

In a well-posed optimization problem it is expected to have active constraints not more than the number of design variables. Thirty eigenvalues are considered as constraints of the optimization problem. Table 4 shows the first 9 eigenvalues at the point of optimum.

To show the convergence speed of the algorithm, evolution of the first 30 eigenvalues at each iteration is shown in Fig. 18. As the design improves towards the optimum point, the structure becomes more

Fig. 20. The first five buckling mode shapes together with the 30th one where the structure is covered with the optimized configuration and is subjected to loads as shown in Fig. 15.

Fig. 21. Updated design nodes' locations are shown with pointed dots.

critical with respect to buckling. As it can be seen in Fig. 18, it takes only 4 iterations until eigenvalues of the problem become very close to 1. Figure 19 shows the history of the mass converging to the minimized value. The computation time for each iteration of the optimization process on a regular PC is about 40 min and the convergence to the lightest design takes about 450 min. Here, the convergence criterion is defined as $\|d^{(k)}\| \leq \epsilon_1$ and the maximum constraint violation $V_k \leq \epsilon_2$, where $\|d^{(k)}\|$ denotes the norm of the search direction at iteration k and ϵ_1 and ϵ_2 are two small numbers larger than zero [52].

According to Fig. 15, the aft-root corner of the structure is under compression from both the spar and the ribs, thus is covered with the highest number of plies relative to the other regions of the skin. This

means that the aft-root corner was critical for buckling before optimization. For the optimum design, however, this region is no longer critical for buckling. This can be verified in Fig. 20 where the first five buckling mode shapes together with the 30th one are shown. As it can be seen in the figure, the critical modes occur in regions other than the aft-root corner of the skin.

Only 9 design nodes are used for the design shown in Fig. 17. However, as described in example 1, the proposed method is flexible to add more design nodes to obtain a more detailed design which results in a lower mass. To verify this, 3 more design nodes are added to the optimization problem as can be seen in Fig. 21.

Figure 22 shows the contour plot of the thickness distribution of the

Fig. 22. Contour plot of the thickness distribution of the optimized skin using 12 design nodes, the stacking sequences can be read from the SST in Fig. 2.

new optimum design. As more design nodes are added, the optimization program has more freedom to minimize the mass while local buckling is prevented. This can be seen in Fig. 23 where the mass histories of the optimizations with 9 and 12 design nodes are compared.

As described in example 1, an interesting feature of the proposed method is that the optimization can start with reasonably few design nodes and continue until a design close to the optimum is reached. Then, the design obtained with few design nodes is used as the initial design of the optimization problem with more design nodes to obtain a more detailed design resulting in a lower mass. This can be seen in Fig. 24 where the design with 9 design nodes (Fig. 16) after 5 iterations is used as the initial design of the optimization with 3 additional design nodes (Fig. 21). Table 5 shows the first 12 eigenvalues at the point of optimum.

As the optimization can partly proceed using less design nodes, the overall convergence time is reduced compared to the case where the optimization starts with 12 design nodes.

5. Conclusion and outlook

Optimization of large scale stiffened structures with the goal of mass minimization under local buckling constraint is addressed in the current research. The proposed method separates the optimization of the stacking sequences from the optimization of the thickness distribution. A stacking sequence table (SST) is generated first. A gradient-based optimization is performed to obtain an estimate about the optimal stiffness and thickness distribution over the structure. Using this information optimized stacking sequences that satisfy blending and other required laminate design guidelines were generated. In particular, symmetry, covering ply, disorientation, percentage rule, balance, and contiguity guidelines are addressed in this study.

Next, an auxiliary level-set function is introduced to specify the boundaries of the laminates with different thicknesses from the obtained SST over the structure. The value of the level-set function specifies the boundaries of regions with equal thickness over the structure.

As long as the laminates covering the structure are selected from the SST, the blending of the design is guaranteed and the required laminate design guidelines are fulfilled without adding extra constraints to the optimization problem.

Separating the optimization problem is in general cheaper compared to when the stacking sequences and the thickness distribution are optimized simultaneously where blending as well as other laminate design guidelines are added to the structural response (e.g buckling) as constraints of the optimization problem.

The proposed method is efficient as it is straightforward, fast, and compatible with any standard finite element package. The proposed level-set method allows continuous change of the location of the ply drop over the structure using a straightforward approach. As the number of design nodes is independent from the number of finite element nodes, no matter how dense the mesh is, the selection of a very limited number of design nodes makes a large finite element model to be optimized cheaply. Since the finite element model remains unchanged during the optimization process, the generated program (as shown in Fig. 6) only updates the input file of a finite element program with element stiffness values; therefore, it can be easily connected to any commercial finite element package.

The proposed level-set method is flexible in terms of the number and the location of design nodes. If in a region of the structure more details are required to be captured, more design nodes can be added locally.

The computation time in the proposed method is directly related to number of design nodes in the level-set-based thickness optimization problem. As described in Section 4, the overall convergence time of a problem can be decreased as part of the optimization procedure can be performed with a smaller number of design nodes.

The choice of the initial design of the level-set-based optimization may result in the final design of the problem to be trapped in a local minimum. As for the initial design, it was suggested to assign values to design variables such that the resulted (area weighted average) thickness distribution is close to the idealized thickness distribution.

Fig. 23. Mass history comparison of the optimization using 9 design nodes with that using 12 design nodes.

Fig. 24. Mass history comparison of the optimization problem with 9 design nodes with that using 12 design nodes. The design of the optimization with 9 design nodes after 5 iterations is used as the initial design of the optimization problem with 12 design nodes.

Table 5

The first 12 eigenvalues of the optimized structure with 12 design nodes.

Eigenvalues 1 to 4	Eigenvalues 5 to 8	Eigenvalues 9 to 12
1.0000	1.0791	1.1224
1.0096	1.0848	1.1235
1.0554	1.0960	1.1314
1.0709	1.0963	1.1384

In the procedure of generating an SST, a gradient-based optimization algorithm was used. Thanks to the proposed level-set parametrization, the discrete thickness optimization problem was also

solved using a gradient-based algorithm. Solving the entire optimization problem using a gradient-based algorithm is in general faster and less expensive compared to the application of a genetic algorithm.

As an SST determines the stacking sequence of the laminates, the optimum design of the structure is strongly dependent on the generated SST. Using a method more accurate than the smeared stiffness to obtain an idealized design is the subject of future research.

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Appendix A. SST generation for the Horseshoe Problem

According to the step 1 procedure shown in Fig. 1, the optimization problem defined in Eq. (3) was solved using the MATLAB optimization toolbox. As the laminate thickness is constant in each panel and shear loads are excluded, the following equation [1,17] is used to calculate the buckling load in each panel during the procedure of optimization problem in Eq. (3).

$$\lambda(m, n) = \frac{\pi^2 [D_{11}(m/a)^4 + 2(D_{12} + 2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4]}{(m/a)^2 N_x + (n/b)^2 N_y} \quad (\text{A.1})$$

where m and n are the number of half-waves in x and y directions, respectively. Here, $m = 1, 2$ and $n = 1, 2$ are considered. The dimensions of the panel are a and b in the x and y directions, respectively. N_x and N_y are the in-plane loads along the x and the y directions, respectively. D_{11} , D_{12} , D_{22} , and D_{66} are the components of the bending stiffness matrix.

The sensitivity of the objective function was calculated according to:

$$\frac{df}{dn_r^j} = S_{jt}, \quad j = 1 \text{ to } N_p, \quad r = 0, 45, 90 \quad (\text{A.2})$$

Table A.6

The result of the optimization problem defined in Eq. (3). The obtained (rounded) thickness values of the idealized design are compared with those reported as the optimized solution in [1].

Panel	n_0	n_{45}	n_{90}	Conceptual number of plies (rounded) $2(n_0 + n_{45} + n_{90})$	Number of plies Yang et al. [1]
1	1.65	13.21	1.65	34	34
2	1.41	11.29	1.41	28	28
3	1.04	2.08	7.27	20	22
4	1	2	6.19	18	20
5	1	2	4.84	16	16
6	1.07	2.15	7.53	22	22
7	1	2	6.29	18	20
8	1.22	2.45	8.58	24	26
9	1.91	15.30	1.91	38	38
10	1.76	14.10	1.76	36	36
11	1.48	11.87	1.48	30	30
12	1.41	11.33	1.41	28	28
13	1.06	2.12	7.42	22	22
14	1	2	6.02	18	20
15	1.23	2.47	8.67	24	26
16	1.5	3.01	10.55	30	32
17	1	2	6.24	18	20
18	1.11	2.22	7.78	22	24

	48	46	44	42	40	36	34	32	30	28	24	22	20	18	16	12	10	8
ply index																		
1	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
2	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
3	90																	
4	-45	-45	-45	-45	-45													
5	90	90	90	90	90	90												
6	90	90																
7	45	45	45	45	45	45	45	45	45	45								
8	0	0	0															
9	0	0	0	0														
10	45	45	45	45	45													
11	90	90	90	90	90	90	90											
12	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
13	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45								
14	0	0	0	0	0	0	0	0										
15	0	0	0	0	0	0	0	0	0									
16	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45			
17	90	90	90	90	90	90	90	90	90	90	90	90	90	90				
18	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90			
19	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
23	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
24	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90

(a) SST-A

	48	46	44	42	40	36	34	32	30	28	24	22	20	18	16	12	10	8
ply index																		
1	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
2	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
3	90																	
4	-45	-45	-45	-45	-45													
5	90	90	90	90	90	90	90	90	90									
6	90	90																
7	45	45	45	45	45	45	45	45	45	45								
8	0	0	0															
9	0	0	0	0														
10	45	45	45	45	45													
11	90	90	90	90	90	90												
12	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
13	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45								
14	0	0	0	0	0	0	0	0										
15	0	0	0	0	0	0	0	0	0									
16	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45			
17	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90			
18	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90			
19	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
23	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
24	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90

(b) SST-B

Fig. A.25. Two different SSTs resulting from the idealized design where fields marked red indicate dropped plies. Ply indices (first column from left) are in the ascending order from the outer most ply towards the laminates center.

Table A.7

The result of the buckling analysis for panels 1, 11, and 16 using SST-A and SST-B.

Panel	λ_{cr} using SST-A	λ_{cr} using SST-B
1	0.85	0.83
11	0.84	0.77
16	0.91	0.99
Min	0.84	0.77

where n_r^j represents the design variable related to the ply with fiber orientation r , for the j th panel. S_j and t represent the area of the j th panel and the ply thickness, respectively. N_p represents the total number of panels.

Considering Eq. (2), the sensitivity of a buckling factor to a design variable was analytically calculated as follows:

$$\frac{d\lambda(m, n)}{dn_r^j} = \frac{d\lambda(m, n)}{dD} \left(\frac{dD}{dA} \frac{dA}{dn_r^j} + \frac{dD}{dh^j} \frac{dh^j}{dn_r^j} \right) \quad j=1 \text{ to } N_p, \quad r=0, 45, 90 \quad (\text{A.3})$$

where h^j represents the thickness of the laminate in the j th panel: $h^j = 2t(n_0^j + n_{45}^j + n_{90}^j)$. As each design variable represents the number of its corresponding plies in a half laminate, the sum of the design variables in each panel is multiplied by 2.

Table A.6 shows the result of the optimization problem defined in Eq. (3). To have an indication of the quality of the idealized design, the obtained (rounded to the nearest integer) thickness values are compared with those reported as the optimized solution in [11].

As it can be seen in Table A.6, the thickness value for each panel in the idealized design is in a good agreement with those reported in [11]. According to Table A.6, only panels 11 and 16 have the same thickness value, while having considerably different values of design variables. The majority of the plies in panel 11 have $\pm 45^\circ$ fiber orientation while panel 16 mainly consists of plies with 90° fiber orientation. Thus using the idealized design, 2 different SSTs (called SST-A and SST-B) can be generated. Fig. A.25 shows the two SSTs resulting from the idealized design. Only the stacking sequences of half laminates are shown.

The idealized design gives a constant thickness value per panel. Using the proposed level-set method, however, each panel may include various thickness values. The area weighted average of various thicknesses in each panel is expected to be close to the idealized thickness value obtained for each panel (see Section 4.1). Therefore, as a result of the level-set-based thickness optimization, laminates thicker and thinner than the idealized laminate are also expected in each panel. For this reason, the SSTs shown in Fig. A.25 include thinner and thicker laminates compared to those obtained in the idealized design (see Table A.6) to avoid unnecessarily restricting the design variables [39]. The AD stiffness values of the thickest and the thinnest laminates in the idealized design were extrapolated to have an estimate of the stiffness values of laminates that are not suggested by the idealized design, but do exist in the SST. The extrapolated stiffness values were used in the selection procedure of the fittest laminate as discussed in Section 2.

For the Horseshoe Problem, Eq. (A.1) can be used to easily calculate the buckling factors. To evaluate the performance of the SSTs shown in Fig. A.25, Eq. (A.1) is used to directly calculate the buckling factors instead of using the quality indicator defined in Eq. (5). Using the idealized thickness distribution (shown in Table A.6), the structure is subject to buckling analysis. First, stacking sequences are selected from SST-A (see Fig. A.25), and then the stacking sequences in SST-B are used. As it can be seen in Fig. A.25, the stacking sequences of laminates with 30, 32, and 34 plies are different between SST-A and SST-B. Therefore, only the stacking sequence of panels 1, 11, and 16 differs as a result of using the two different SSTs. Thus here, only these panels are subjected to buckling analysis (considering the fact that load redistribution is ignored in the Horseshoe Problem). Table A.7 shows the result of the buckling analysis for panels 1, 11, and 16 using SST-A and SST-B.

As the minimum critical buckling factor using SST-A is larger than the one obtained using SST-B, SST-A in Fig. A.25 is selected to be used for thickness optimization.

References

- [1] Yang J, Song B, Zhong X, Jin P. Optimal design of blended composite laminate structures using ply drop sequence. *Compos Struct* 2016;135:30–7. <https://doi.org/10.1016/j.compstruct.2015.08.101>.
- [2] Meddaikar YM, Irisarri FX, Abdalla MM. Laminate optimization of blended composite structures using a modified shepard's method and stacking sequence tables. *Struct Multi Optim* 2017;55(2):535–46. <https://doi.org/10.1007/s00158-016-1508-0>.
- [3] Allaire G, Delgado G. Stacking sequence and shape optimization of laminated composite plates via a level-set method. *J Mech Phys Solids* 2016;97:168–96. <https://doi.org/10.1016/j.jmps.2016.06.014>.
- [4] Military Handbook – MIL-HDBK-17-3F: Composite Materials Handbook, Volume 3 - Polymer Matrix Composites Materials Usage, Design, and Analysis, U.S. Department of Defense; 2002.
- [5] Bailie J, Ley R, Pasricha A. A summary and review of composite laminate design guidelines. Technical report NASA, NAS1-19347, Northrop Grumman-Military Aircraft Systems Division; 1997.
- [6] Ghiassi H, Fayazbakhsh K, Pasini D, Lessard L. Optimum stacking sequence design of composite materials Part 2: variable stiffness design. *Compos Struct* 2010;93(1):1–13. <https://doi.org/10.1016/j.compstruct.2010.06.001>.
- [7] Adams DB, Watson LT, Gürdal Z, Anderson-Cook CM. Genetic algorithm optimization and blending of composite laminates by locally reducing laminate thickness. *Adv Eng Softw* 2004;35(1):35–43. <https://doi.org/10.1016/j.advengsoft.2003.09.001>.
- [8] Adams DB, Watson LT, Gürdal Z. Optimization and blending of composite laminates using genetic algorithms with migration. *Mech Adv Mater Struct* 2003;10:183–203.
- [9] Kristinsdottir BP, Zabinsky ZB, Tuttle ME, Neogi S. Optimal design of large composite panels with varying loads. *Compos Struct* 2001;51:93–102.
- [10] McMahon MT, Watson LT. A distributed genetic algorithm with migration for the design of composite laminate structures. *Parallel Algorithms App* 2000;14:329–62.
- [11] Soremekun G, Gürdal Z, Kassapoglou C, Toni D. Stacking sequence blending of multiple composite laminates using genetic algorithms. *Compos Struct* 2002;56(1):53–62. [https://doi.org/10.1016/S0263-8223\(01\)00185-4](https://doi.org/10.1016/S0263-8223(01)00185-4).
- [12] Liu B, Haftka R. Composite wing structural design optimization with continuity constraints. In: 19th AIAA applied aerodynamics conference; 2001.
- [13] Adams DB, Watson LT, Seresta O, Gürdal Z. Global/local iteration for blended composite laminate panel structure optimization subproblems. *Mech Adv Mater Struct* 2007;14(2):139–50. <https://doi.org/10.1080/15376490600719212>.
- [14] Zehnder N, Ermanni P. A methodology for the global optimization of laminated composite structures. *Compos Struct* 2006;72(3):311–20. <https://doi.org/10.1016/j.compstruct.2005.01.021>.
- [15] Zehnder N, Ermanni P. Optimizing the shape and placement of patches of reinforcement fibers. *Compos Struct* 2007;77(1):1–9. <https://doi.org/10.1016/j.compstruct.2005.05.011>.
- [16] Starnes Jr. JH, Haftka RT. Preliminary design of composite wings for buckling, strength, and displacement constraints. *J Aircraft* 1979;16(8):564–70. <https://doi.org/10.2514/3.58565>.
- [17] Irisarri FX, Lasseigne A, Leroy FH, Le Riche R. Optimal design of laminated composite structures with ply drops using stacking sequence tables. *Compos Struct* 2014;107:559–69. <https://doi.org/10.1016/j.compstruct.2013.08.030>.
- [18] Bendsøe MP, Kikuchi N. Generating optimal topologies in structural design using a homogenization method. *Comput Methods Appl Mech Eng* 1988;71(2):197–224.

- [https://doi.org/10.1016/0045-7825\(88\)90086-2](https://doi.org/10.1016/0045-7825(88)90086-2).
- [19] Bendsoe MP. Optimal shape design as a material distribution problem. *Struct Optim* 1989;1(4):193–202. <https://doi.org/10.1007/BF01650949>.
 - [20] Rozvany GIN, Zhou M, Birker T. Generalized shape optimization without homogenization. *Struct Optim* 1992;4(3):250–2. <https://doi.org/10.1007/BF01742754>.
 - [21] Costa G, Montemurro M, Pailhès J. A 2D topology optimisation algorithm in NURBS framework with geometric constraints. *Int J Mech Mater Des*. doi:10.1007/s10999-017-9396-z.
 - [22] Qian X. Topology optimization in B-spline space. *Comput Methods Appl Mech Eng* 2013;265:15–35. <https://doi.org/10.1016/j.cma.2013.06.001>.
 - [23] Stolpe M, Svanberg K. An alternative interpolation scheme for minimum compliance topology optimization. *Struct Multi Optim* 2001;22(2):116–24. <https://doi.org/10.1007/s001580100129>.
 - [24] Sigmund O, Torquato S. Design of materials with extreme thermal expansion using a three-phase topology optimization method. *J Mech Phys Solids* 1997;45(6):1037–67. [https://doi.org/10.1016/S0022-5096\(96\)00114-7](https://doi.org/10.1016/S0022-5096(96)00114-7).
 - [25] Gibiansky LV, Sigmund O. Multiphase composites with extremal bulk modulus. *J Mech Phys Solids* 2000;48(3):461–98. [https://doi.org/10.1016/S0022-5096\(99\)00043-5](https://doi.org/10.1016/S0022-5096(99)00043-5).
 - [26] Thomsen J. Topology optimization of structures composed of one or two materials. *Struct Optim* 1992;5(1):108–15. <https://doi.org/10.1007/BF01744703>.
 - [27] Hvejsel CF, Lund E. Material interpolation schemes for unified topology and multi-material optimization. *Struct Multi Optim* 2011;43(6):811–25. <https://doi.org/10.1007/s00158-011-0625-z>.
 - [28] Sørensen SN, Lund E. Topology and thickness optimization of laminated composites including manufacturing constraints. *Struct Multi Optim* 2013;48(2):249–65. <https://doi.org/10.1007/s00158-013-0904-y>.
 - [29] Sørensen SN, Sørensen R, Lund E. DMTO – a method for discrete material and thickness optimization of laminated composite structures. *Struct Multi Optim* 2014;50(1):25–47. <https://doi.org/10.1007/s00158-014-1047-5>.
 - [30] Sørensen SN, Stolpe M. Global blending optimization of laminated composites with discrete material candidate selection and thickness variation. *Struct Multi Optim* 2015;52(1):137–55. <https://doi.org/10.1007/s00158-015-1225-0>.
 - [31] Lund E. Discrete material and thickness optimization of laminated composite structures including failure criteria. *Struct Multi Optim*. doi:10.1007/s00158-017-1866-2.
 - [32] Carpentier A, Michel L, Grihon S, Barrau J-J. Buckling optimization of composite panels via lay-up tables. In: 3rd European conference on computational mechanics solids, structures and coupled problems in engineering. Lisbon, Portugal; 2006.
 - [33] Irisarri FX, Peeters DMJ, Abdalla MM. Optimisation of ply drop order in variable stiffness laminates. *Compos Struct* 2016;152:791–9. <https://doi.org/10.1016/j.compstruct.2016.05.076>.
 - [34] IJsselmuiden ST, Abdalla MM, Seresta O, Gürdal Z. Multi-step blended stacking sequence design of panel assemblies with buckling constraints. *Compos Part B: Eng* 2009;40(4):329–36. <https://doi.org/10.1016/j.compositesb.2008.12.002>.
 - [35] Seresta O, Abdalla MM, Gürdal Z. A genetic algorithm based blending scheme for design of multiple composite laminates. In: Collection of technical papers – AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference; 2009.
 - [36] Jin P, Zhong X, Yang J, Sun Z. Blending design of composite panels with lamination parameters. *Aeronaut J* 2016;120(1233):1710–25. <https://doi.org/10.1017/aer.2016.88>.
 - [37] Jin P, Wang Y, Zhong X, Yang J, Sun Z. Blending design of composite laminated structure with panel permutation sequence. *Aeronaut J* 2018;122(1248):333–47. <https://doi.org/10.1017/aer.2017.132>.
 - [38] Ghiasi H, Pasini D, Lessard L. Optimum stacking sequence design of composite materials part 1: Constant stiffness design. *Compos Struct* 2009;90(1):1–11. <https://doi.org/10.1016/j.compstruct.2009.01.006>.
 - [39] Farzan Nasab F, Geijselaers HJM, Baran I, de Boer A. Generating the best stacking sequence table for the design of blended composite structures. In: Advances in structural and multidisciplinary optimization: proceedings of the 12th world congress of structural and multidisciplinary optimization (WCSMO12); 2018. p. 779–788. doi:10.1007/978-3-319-67988-459.
 - [40] Liu J, Ma Y. A new multi-material level set topology optimization method with the length scale control capability. *Comput Methods Appl Mech Eng* 2018;329:444–63. <https://doi.org/10.1016/j.cma.2017.10.011>.
 - [41] Vese LA, Chan TF. A multiphase level set framework for image segmentation using the Mumford and Shah model. *Int J Comput Vision* 2002;50(3):271–93. <https://doi.org/10.1023/A:1020874308076>.
 - [42] Liu D, Toropov VV, Querin OM, Barton DC. Bilevel optimization of blended composite wing panels. *J Aircraft* 2011;48(1):107–18. <https://doi.org/10.2514/1.C000261>.
 - [43] Kassapoglou C. Design and analysis of composite structures: with applications to aerospace structures. 2nd ed. John Wiley and Sons; 2013. <https://doi.org/10.1002/9781118536933>.
 - [44] Montemurro M, Vincenti A, Vannucci P. A two-level procedure for the global optimum design of composite modular structures-application to the design of an aircraft wing: Part 1: theoretical formulation. *J Optim Theory Appl* 2012;155(1):1–23. <https://doi.org/10.1007/s10957-012-0067-9>.
 - [45] Montemurro M, Vincenti A, Vannucci P. A two-level procedure for the global optimum design of composite modular structures-application to the design of an aircraft wing: Part 2: numerical aspects and examples. *J Optim Theory Appl* 2012;155(1):24–53. <https://doi.org/10.1007/s10957-012-0070-1>.
 - [46] IJsselmuiden S, Abdalla MM, Gürdal Z. Optimization of variable-stiffness panels for maximum buckling load using lamination parameters. *AIAA J* 2010;48(1):134–43. <https://doi.org/10.2514/1.42490>.
 - [47] Montemurro M, Catapano A. A new paradigm for the optimum design of variable angle tow laminates. *Springer Optim App* 2016;116:375–400. <https://doi.org/10.1007/978-3-319-45680-514>.
 - [48] Montemurro M, Catapano A. On the effective integration of manufacturability constraints within the multi-scale methodology for designing variable angle-tow laminates. *Compos Struct* 2017;161:145–59. <https://doi.org/10.1016/j.compstruct.2016.11.018>.
 - [49] Tsai SW, Hahn HT. Introduction of composite materials. Lancaster, PA: Technomic Publishing Company; 1980.
 - [50] Hammer V, Bendsoe M, Lipton R, Pedersen P. Parametrization in laminate design for optimal compliance. *Int J Solids Struct* 1997;34(4):415–33.
 - [51] Reddy JN. Mechanics of laminated composite plates: theory and analysis. Boca Raton: CRC Press; 1997.
 - [52] Arora JS. Introduction to optimum design. 3rd ed. Boston: Academic Press; 2012. <https://doi.org/10.1016/B978-0-12-381375-6.00033-4>.
 - [53] Haftka RT, Gürdal Z. Elements of structural optimization. Springer; 1992.
 - [54] Allemang RJ, Brown DL. A correlation coefficient for modal vector analysis. In: 1st International Modal Analysis Conference; 1982.