FORCE LIMITED RANDOM VIBRATION TESTING: THE COMPUTATION OF THE SEMI-EMPIRICAL CONSTANT C^2 FOR A REAL TEST ARTICLE AND UNKNOWN SUPPORTING STRUCTURE

SEMI-EMPIRICAL METHOD

Jaap Wijker

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SEMI-EMPIRICAL METHOD

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Voor Wil

The reason of effects.-Lust and force are the source of all our actions; lust causes voluntary actions, force involuntary ones.

Blaise Pascal (1623-1662)

PREFACE

I have always worked with much pleasure and satisfaction in the field of spacecraft structures design and analysis both at Dutch Space BV (DS) (Airbus Defence and Space Netherlands) and at the Technology University Delft (TUD), Faculty of Aerospace Engineering. After my retirement from DS in 2009 and the TUD 2010, I was asked to join the ECSS working group Space Engineering: "Spacecraft mechanical load analysis handbook", to write chapter 9: "Random vibration and vibro-acoustic environment". After that the idea raised in 2012 to do a PhD. Prof André de Boer of the University Twente, department of Applied Mechanics accepted my request and together with my former colleague at DS Marcel Ellenbroek we discussed a lot about random vibrations in spacecraft structures design. I am very grateful to them for their support as promoter and co-promoter respectively.

I also have to express my gratitude to my wife Wil, who had the patience to stand by me when my brain was overloaded with random vibrations.

*Jacob Job Wijker
Velserbroek, September 7, 2016*

CONTENTS

Summary	xvii
Samenvatting	xix
1 Introduction	1
1.1 General Introduction	1
1.2 Thesis Organization.	4
References	6
2 Design Process & Problem Definition	7
2.1 Introduction	7
2.2 Spacecraft Structures Design and Verification.	7
2.2.1 Introduction	7
2.2.2 Initial Phase	8
2.2.3 Loads Specification and Stiffness Requirement	8
2.2.4 Verification by Analysis	10
2.2.5 Verification by Test.	11
2.3 Introduction Force Limited Vibration with a Numerical Example	12
2.4 Research Question	19
References	20
3 Random Vibration Tests, Modification of Input Spectrum (Notching), An Overview	23
3.1 Introduction	23
3.2 Why Force and Response Limiting	24
3.3 Previous Work.	26
3.4 Response Limited Random Vibration Testing	28
3.4.1 Introduction	28
3.4.2 Maximum Allowable Response Spectra	28
3.5 The QSL design loads and associated reaction forces	34
3.5.1 Response Limited, Mass Participation Factors	34
3.5.2 Enclosing Notching Method of CNES	38
3.5.3 Response Limiting on 1/2 Octave Frequency band with Quasi-Static Load Criterion	40
3.6 Force Limited Vibration.	42
3.6.1 Introduction	42
3.6.2 Force Limit Vibration Testing based on STDFS.	43
3.6.3 Complex Two-Degree of Freedom System (CTDFS)	50
3.6.4 JAXA's Force Limits Estimation Method	56

3.7	Conclusion/Recommendation	59
	References	60
4	Force Limited Vibration, The Semi-Empirical Method	63
4.1	Introduction	63
4.2	Semi-Empirical Method (SEM)	63
4.3	FLVT Literature Review C^2	67
4.4	Conclusions.	68
	References	69
5	Probabilistic Analysis Semi-Empirical Constant C^2	71
5.1	Introduction	71
5.2	Discrete Mathematical Models for Random Response Analysis	73
5.2.1	Coupled System Modal Approach Method (CSMA).	74
5.2.2	Hybrid Mathematical Dynamic Modeling (HMDM)	75
5.3	Mathematical Models Load and Source	77
5.3.1	Mathematical Model Load for CSMA and HMDM Methods	77
5.3.2	Probabilistic Mathematical Model Source, CSMA Method	77
5.3.3	Mathematical Model Source, HMDM Method	80
5.4	Deterministic comparison of CSMA and HMDM models	81
5.5	Probabilistic Analysis	82
5.5.1	The Rosenblueth 2k+1 Point Estimates Probability Moments Method	
	83	
5.5.2	Monte Carlo Simulation	83
5.6	Conclusions.	85
	References	85
6	Applications of Probabilistic Analyses of C^2	89
6.1	Introduction	89
6.2	Haille Problem	89
6.2.1	Response Analysis with CSMA	91
6.2.2	Response Analysis with HMDM	94
6.2.3	Discussion Results Haille Problem	97
6.3	ESA IFLV study	99
6.3.1	Introduction	99
6.3.2	Mass properties of IFLV system	99
6.3.3	Dynamic properties of IFLV system & individual parts	101
6.3.4	Interfaces MIRI/Panel	101
6.3.5	Probabilistic Analysis of C^2	101
6.3.6	Probabilistic computation of C^2 with HMDM	104
6.3.7	Discussion CSMA/HMDM Probabilistic Analysis of C^2	106
6.4	LDU/FSE/FRAM	107
6.4.1	Introduction	107
6.4.2	Dynamic properties LDU and value C^2	107
6.4.3	C^2 from literature	108
6.4.4	Probabilistic computations of C^2 with CSMA	108
6.4.5	Comparison Probabilistic Analysis with use of CSMA	111

6.4.6	Probabilistic computations of C^2 with HMDM.	113
6.4.7	Discussion of Probabilistic Analysis using HMDM	116
6.4.8	Discussion CSMA/HMDM Probabilistic Analysis of C^2	118
	References	120
7	Conclusions, Recommendations and Guidelines	123
7.1	Conclusions.	123
7.1.1	Notching Prediction Methods	123
7.1.2	Probabilistic Analysis of C^2	124
7.1.3	Conclusions Probabilistic Analysis.	126
7.2	How to Apply the Probabilistic Analysis	127
7.3	Guidelines	127
7.3.1	Mathematical model of the Load.	128
7.3.2	Probabilistic Model of the Source	129
7.3.3	Coupled Systems Modal Analysis (CSMA) Method	129
7.4	Recommendations	129
7.5	Applications in other Areas	130
7.5.1	Tractor-Trailer Combination, Trailer Design	130
7.5.2	Tractor-Trailer Combination, Truck Design	131
7.5.3	Ti yres-Vehicle Combination, Tyre Design	132
	References	133
A	Spacecraft Design and Verification	135
A.1	Introduction	135
A.2	Initial Phase.	135
A.3	Preliminary Dimensioning of Spacecraft Structure	137
A.4	Loads Specification and Stiffness Requirement	138
A.5	Test Factors and Factors of Safety	140
A.6	Verification by Analysis	141
A.7	Manufacturing of Spacecraft Structure	142
A.8	Verification by Test	142
A.9	Further Remarks	143
	References	143
B	Design Process Trucks and Tractors	145
B.1	Introduction	145
B.2	Product development.	145
B.3	Design Process Structures.	146
B.4	Dimensioning of Components	148
B.5	Design Loads	148
B.6	Numerical Simulations	149
B.6.1	Creation and Reduction of the Components models	150
B.6.2	Assembly of the Components to Create the Complete Vehicle Model 150	
B.6.3	Generation of Frequency Response Functions	150
B.6.4	Combination of Excitation Auto- and Cross Spectra with FRFs to Yield Response Spectra.	151

B.6.5	Evaluation and Interpretation of Response Spectra	151
B.6.6	Verification of Finite Element Model	152
B.7	Fatigue Evaluation and Verification Chain	152
B.8	Comparison Spacecraft and Truck/Tractor Design Process	153
	References	156
C	Modal Effective Mass	157
	References	161
D	CSMA, Formulation of Coupled Systems	163
	References	169
E	Quasi-Static, Random and Acoustic Loads	171
E.1	Introduction	171
E.2	Quasi-Static Load Specifications	171
E.3	Acoustic and Random Load Specification	172
E.3.1	Introduction	172
E.3.2	Acoustic Loads	172
E.3.3	Random Enforced Acceleration	175
E.4	Random Vibration Test Tolerances	179
	References	179
F	CHARACTERIZATION AND SYNTHESIS OF RANDOM ACCELERATION VIBRATION SPECIFICATIONS	181
F1	INTRODUCTION	182
F2	PREVIOUS WORK.	183
F3	RESPONSE SPECTRUM.	185
F3.1	Shock Response Spectrum (SRS)	185
F3.2	Vibration Response Spectrum (VRS)	185
F3.3	Extreme (Peak) Response Spectrum (ERS)	187
F3.4	Fatigue Damage Spectrum (FDS)	187
F3.5	Pseudo Stationary Random Vibration, Damage Potential	191
F4	SYNTHESIS OF EQUIVALENT RANDOM ACCELERATION VIBRATION SPECIFICATION	193
F5	APPLICATION.	194
F5.1	Random Vibration Measurement during a Acoustic Test	194
F5.2	Synthesis of Random Acceleration Vibration Specification from SRS and ERS	197
F5.3	Synthesis of Random Acceleration Vibration Specification from FDS_d	198
F5.4	Synthesis of Random Acceleration Vibration Specification from FDS_{pv}	199
F5.5	Synthesis of Equivalent Random Acceleration Vibration Specification based on Rayleigh Distribution of Peaks.	200
F5.6	All equivalent random acceleration vibration specifications in one plot	200
F5.7	Influence Number of Fields N_{field}	200

E6 CONCLUSIONS	201
References	202
Curriculum Vitæ	211
List of Publications	213

SUMMARY

Within the framework of the qualification process, the design requirements are verified by analyses and vibration tests i.e. a random vibration test with prescribed random enforced acceleration at the base of the test article (load). To prevent over-testing of the load during the random vibration test an adaptation of the random base acceleration can be applied (notching). Besides that the random force limits at the base of the load can be defined by the force limited vibration test (FLVT). To define the random force limits by the semi-empirical method (SEM) the random acceleration vibration specification, the total mass, the turnover (natural) frequency of the load, and the semi-empirical constant C^2 are key parameters. A number of computational methods of estimating C^2 are described in the literature, i.e. the simple and the complex two degree of freedom system, the STDFS and the CTDFS, respectively. The SEM, several notching approaches, the calculation of C^2 are reviewed and discussed in this thesis.

Stevens presented the coupled systems modal approach (CSMA), where simplified asparagus patch models (parallel-oscillator representation) of load and supporting structure (source) are connected. The asparagus patch model consists of parallel placed single degree of freedom (SDOF) elements. The mass of the SDOF element represents the modal effective masses and the spring stiffnesses are associated with the natural frequency of the fixed load. The supporting structure of the load (source) will be modelled by the asparagus patch model too. When the random acceleration vibration specification is given the CSMA method is suitable to reconstruct interface random accelerations and associated forces between load and source to compute the value of the parameter C^2 .

In this study the Hybrid Mathematical Dynamic Modeling (HMMD) has been developed. The asparagus patch model of the load is coupled with a multi degree of freedom (MDOF) system of the source, consisting of a number in series placed SDOF elements and a very large mass to simulate a fixation, which is not the case with the CSMA method. When the random acceleration vibration specification is provided the HMMD approach is also suitable for reconstructing the interface random accelerations and forces between load and source to compute the value of the parameter C^2 . The system fundamental frequency of the load/source combination is approximated with Dunkerly's method (equation).

When no mathematical model of the source can be made available, estimations of the value C^2 can be found in literature. A number of examples of the values of C^2 were assessed from literature.

During the FLVT both the applied random acceleration and specified force limits are monitored. When interface force grows beyond the force limit specification the enforced random acceleration at the base of load will be adapted (notched). One chapter of this thesis presents an overview of methods of defining notching of the input random acceleration specification and random force limits.

A probabilistic method for the computation has been developed to calculate the statistical moments, distribution function and maximum value of C^2 , when the description of source is unknown. The proposed probabilistic mathematical representation of the unknown (probabilistic) source is modelled using both the CSMA (asparagus patch model) and HMDM (MDOF model) representation of the source. The selected primary probabilistic design variables of the source have prescribed intervals and uniform distributions and are discussed in detail to justify the choice against current design rules in spacecraft structures design. Marchand discussed the formal description to compute the value of C^2 , using the maximum power spectral density (PSD) of the acceleration and maximum PSD of the force at the interface between the load and the source.

The computation of the value C^2 can be performed in conjunction with the CSMA and HMDM methods, knowing the apparent mass of the load and the random acceleration specification at the interface between load and source respectively.

To perform the probabilistic analyses the Rosenblueth Point estimates for probability moments (PEM) and the Monte Carlo Simulation (MCS) are both applied.

Data of three deterministic cases available from literature have been analysed in detail and discussed to get more knowledge about the applicability of the probabilistic approach:

Haille problem The load/source combination is represented by a number of coupled SDOF elements. The load and source consists of coupled SDOF elements too.

MIRI optical unit Information from an ESA/ESTEC study about improvement of force limited vibration testing, with a full test campaign (both sine and random). The test system is composed of an honeycomb panel which is supporting the optical unit (MIRI).

LDU, linear drive unit The LDU is an orbital replacement unit (ORU) for the International Space Station (ISS) launched by the Space Transportation System (STS). Dynamic characteristics of the LDU and deterministic values of C^2 are provided in a paper by Fitzpatrick.

To compute the interface random accelerations and forces and subsequently the semi-empirical constant C^2 , the CSMA method is preferred slightly for modelling the load and probabilistic source.

The dynamic interaction of two elastic systems has been investigated when the dynamic properties one of the systems is unknown and replaced by probabilistic quantities respecting some common design rules in spacecraft structures design such as the separation of natural frequencies.

The probabilistic approach may be applied in other structural disciplines, for example in case of the tractor-trailer combination where interface loads are dependent on the dynamic interaction of both. Two design cases can be identified, for which the probabilistic approach can be applied when the design and analysis of the tractor are performed and the dynamic properties of the trailer are not known or vice versa.

The third design case in which the probabilistic approach may be applied is within the framework of the design of tyres for cars, tractors and trailers.

SAMENVATTING

Ontwerp specificaties van ruimtevaart constructies worden middels analyses en vibratie testen bijv. triltesten met een basis versnelling van stochastische aard (random vibration test) getoetst. Om overbelasting tijdens de random vibration test van het test specimen (load) te voorkomen wordt het vermogensspectrum van de basisversnelling aangepast (notching). Ook kan het vermogensspectrum van de stochastische krachten tussen load en ondersteunende constructie (source) worden voorgeschreven (random force limits). Terry Scharton van het Jet Propulsion Laboratory (JPL) heeft de half empirische methode (semi-empirical method, SEM) ontwikkeld om de random force limits te bepalen. In het kader van de SEM zijn van de load de volgende parameters van belang: de totale massa en de natuurlijke frequentie met bijbehorende dominante modal effectieve massa. De specificatie van het vermogensspectrum van de stochastische basisversnelling is eveneens een belangrijk gegeven. Het vermogensspectrum van de force limits wordt uiteindelijk bepaald (geschaald) met de semi-empirische constante C^2 . Een random vibration test met een voorgeschreven vermogensspectrum van de force limits wordt ook wel Force Limited Vibration Testing (FLVT) genoemd. In de literatuur worden tal van rekenmethodieken, om de constante C^2 te berekenen, genoemd, o.a. met het eenvoudige twee massa veer systeem, respectievelijk het "simple two degrees of freedom" systeem (STDfs) en het "complex two degrees of freedom" system (CTDfs). De SEM en diverse methoden om notching van de basis versnelling te bepalen en om C^2 te berekenen worden in de thesis uitgebreid beschouwd en besproken.

De Coupled Systems of Modal Approach (CSMA) is een gegeven methode om de vermogensspectra van de versnellingen en krachten in het raakvlak (interface) van de load en de source te berekenen. Het model van zowel de load en de source bestaat uit parallel geplaatste simpele massa veer (SDOF) systemen (Asparagus Patch Model). Beide modellen zijn in het raakvlak met elkaar gekoppeld. De discrete massa's van de massa veer systemen vertegenwoordigen de modale effectieve massa's en de veren de natuurlijke frequenties van de ingeklemde load en ingeklemde source. Bij een gegeven vermogensspectrum van de versnellingen op de interface tussen de load en de source worden de dynamische responsies van de versnellingen en verwante krachten gereconstrueerd en vervolgens wordt de waarde van C^2 berekend.

In het kader van mijn onderzoek is de hybride modellering van load en source ontwikkeld (Hybrid Mathematical Dynamic Modeling, HMDM), waarbij het asparagus patch model van de load wordt gekoppeld aan een meer graden van vrijheid model (MDOF) van de source. Het MDOF systeem is opgebouwd uit een aantal in serie geplaatste SDOF elementen met aan de basis een zeer grote massa (large mass approach) om een ondersteuning na te bootsen, dit in tegenstelling tot de CSMA methode, die niet ingeklemd is. Bij een gegeven vermogensspectrum van de basis versnelling in de interface tussen de load en de source wordt net als bij de CSMA methode de waarde van C^2 berekend. De laagste natuurlijke frequentie van de load/source combinatie wordt middels de methode

van Dunkerly bepaald.

Vaak is het wiskundig model van de source niet beschikbaar en dan moet de waarde van C^2 worden geschat, wel gesteund door aanbevolen waarden uit de literatuur. In de thesis wordt een aantal gebruikte waarden van C^2 , gevonden in de literatuur, beschouwd.

Tijdens de FLVT worden zowel de vermogensspectra van de voorgeschreven basisversnelling en krachten gecontroleerd en wordt de basisversnelling eventueel bijgestuurd. Bij een overschrijding van het vermogenspectrum van de voorgeschreven krachten (force limits) wordt het vermogenspectrum van de voorgeschreven basis versnelling vermindert (notched) om een overbelasting van de load te voorkomen. In één van de hoofdstukken van deze thesis wordt een overzicht van een aantal methoden om notching en force limits te bepalen gegeven en uitgebreid beschouwd.

Vaak zijn de dynamische karakteristieken van de source niet bekend, en daardoor is de gedachte ontstaan om een probabilistische methode te ontwikkelen om de statistische momenten van C^2 te berekenen. De voorgestelde probabilistische wiskundige modellering van de source wordt ingebed in het CSMA en het HMDM dynamisch model. Het voorgeschreven vermogenspectrum van de basisversnelling van de load wordt gereconstrueerd en vervolgens wordt het vermogenspectrum van de krachten in de interface tussen de load en de source berekend. De berekening van C^2 gaat volgens de methode van Marchand en is afhankelijk van het maxima van de vermogensspectra van de aangezette versnellingen en krachten. Bij de berekening van het vermogenspectrum van de krachten in de interface is de dynamische massa van de load van groot belang!

De gekozen stochastische ontwerp variabelen zijn gedefinieerd door een voorgeschreven interval en bijbehorende uniforme kansverdeling. De keuze van de ontwerp variabelen met bijbehorende intervallen wordt nauwgezet beschouwd en in relatie tot de ontwerp regels toegepast in het ontwerp van ruimtevaartconstructies.

De probabilistische berekening van C^2 gebeurt zowel met de Rozenblueth Point Estimates methode (PEM) en de Monte Carlo Simulation (MCS) methode. Met de PEM en MCS methoden worden de statistische momenten berekend. De MCS levert eveneens de distributie functie en de maximale waarde van C^2

Gebaseerd op beschikbare gegevens van drie studies zijn berekeningen uitgevoerd om meer inzicht van de voorgestelde probabilistische aanpak te verkrijgen:

Haille probleem De load/source combinatie omvat acht in serie gekoppelde SDOF elementen. Dit probleem is academisch van aard.

MIRI optisch instrument De gegevens van het MIRI optisch instrument komen uit de ESA/ESTEC studie over force limited vibration testing. De MIRI (load) is geplaatst op een sandwich paneel (source). De massa van de source is klein t.o.v. van de massa van het MIRI instrument.

LDU, Linear drive unit De LDU is een vervangbaar systeem voor het ISS (international Space Station) en is door het Shuttle system in een baan om de aarde geplaatst. Dynamische gegevens van de LDU (load) en deterministische waarden van C^2 komen uit een artikel geschreven door Fitzpatrick.

Om de vermogensspectra van de interface versnellingen en krachten te berekenen en vervolgens de waarde van C^2 heeft de CSMA methode een lichte voorkeur boven de HMDM methode.

De dynamische interactie tussen de bekende load en onbekende source is aan de hand probabilistische modellen bestudeerd, rekening houdend met bekende ontwerp regels in ruimtevaartconstructies, bijv. de scheiding van natuurlijke frequenties

Uit deze studie blijkt dat de probabilistische aanpak eventueel ook in andere disciplines toe te passen is, bijvoorbeeld de trekker-trailer combinatie van een vrachtwagen bij het ontwerp van de trekker wanneer de dynamische eigenschappen van de trailer niet bekend zijn of vice versa.

1

INTRODUCTION

1.1. GENERAL INTRODUCTION

During the early phase of the launch (lift-off) a spacecraft will be exposed to severe random acoustic loads. Acoustic loads are the cause of structure-borne random mechanical vibrations inside the spacecraft. Before launch the spacecraft system is tested to be sure it will withstand (survive) the acoustic loads. Additionally, a validated mathematical model has to be developed with which the dynamical structural responses can be calculated.

In the test set-up these acoustic loads (sound pressures) are simulated as a reverberant (diffuse) acoustic sound field, both analytically and during acoustic tests in a reverberant acoustic chamber, which is more or less an approximation of the real acoustic environment under the fairing of the launch vehicle. As an example, a solar wing placed in the reverberant chamber is shown in Fig. 1.1.

Figure 1.1: VAATMLDS BB-wing in IABG Reverberant chamber, 1378 m³ (courtesy Airbus Defence and Space, Netherlands BV)

The mathematical description of a diffuse sound field and the simulation during testing in a reverberant acoustic chamber are well known and, in general, accepted in spacecraft design. Lightweight large areas of structures (solar arrays, antennae, etc.) are very sensitive to the quite severe acoustic loads and within the spacecraft the acoustic loads are transferred into random mechanical vibrations, the so-called structure-borne noise. This is illustrated in Fig. 1.2.

Figure 1.2: Principle acoustical/mechanical vibration transfer

These mechanical random vibrations excite the spacecraft mounted instruments, equipment and other units by random enforced accelerations. In general, the random enforced acceleration specifications are provided to investigate analytically and by test the stiffness and strength characteristics of the subsystems. The random enforced acceleration specifications are derived from the acoustic random response characteristics estimated by fluid/structure response interaction analysis (finite element analysis (FEA), boundary element analysis (BEA) or statistical energy analysis (SEA)) or by an acoustic test in a reverberant chamber.

The dynamic response characteristics between two or more elastic structures show peaks and valleys at resonances and anti-resonances. In practice, the generation of a random enforced acceleration specification means that the peaks are enveloped to achieve a more or less smoothed random acceleration specification as shown in Fig.

1.3. The enveloped random acceleration specification is conservative at anti-resonance frequencies, where valleys in the dynamic responses will be encountered. The random vibration specification will be applied during the design and qualification process of instruments, equipment and units against the random vibration environment. **For convenience the test item (instrument, equipment, unit, etc.) is called the load and the supporting structure (panels, side walls, etc.) the source.**

Figure 1.3: Enveloped random vibration specification of the load

During the random vibration test the load is placed on a shaker system to apply the random acceleration specification. This is illustrated in Fig. 1.4 where the ESA micro-satellite Proba-V is mounted onto a slip table.

Figure 1.4: ESA Micro-satellite Proba-V on slip table (courtesy Interspace, Toulouse, France)

The impedance of the shaker head or shaker slip table is significantly greater than the structural impedance of the source. The boundary condition of the load during the vibration test is quite different from the supported load during launch.

At approximately the natural frequencies of the load (at low modal damping) a large increase of the dynamic mass of the load will occur and the response at the interface of the load/source combination will decrease. This is the same as the principle of the reduction of responses using auxiliary mass spring systems (absorbing vibration [1]). At resonance the source with a normal structural impedance will not be able to put enough

energy in the load to maintain the vibration levels, but the shaker is certainly able to put the required energy into the load. Following the random vibration specification the shaker system will over-test the load considerably. That means the random response acceleration and random force response are higher than may be expected during the launch event.

To avoid over-testing of the subsystem during the random vibration test the specified random enforced acceleration vibration specification will be adapted. That means that at the resonant frequency (frequencies) of the load the input spectrum will be reduced or notched. In general, the adapted (notched) random vibration acceleration spectrum is defined based on either on limiting it to allowable responses (response limited vibration test) or on an allowable force spectrum (force limited vibration test). The force limits are defined at the interface between the load and the shaker. An example of a notched random vibration spectrum is shown in Fig. 1.5. In general, the notches will occur at the natural frequencies of the load when the interface force are beyond the specified force limits in fact, at the anti-resonances in the load/source combination. Many methods for defining the notched spectra are discussed in the literature review in this thesis.

Figure 1.5: Modified input acceleration of random vibration specification of the load

1.2. THESIS ORGANIZATION

In this section an overview of the thesis organization is given.

The problem to be solved is described in chapter 2. This chapter begins with an introduction to the design and verification process of spacecraft structures. A more detailed description of that process is given in Appendix A as well as the design process of Trucks and Tractors in Appendix B. In Appendix E the definition of static, random and acoustic loads is discussed and related to this is the discussion in Appendix F about the characterization, synthesis and simplification of measured random accelerations based on several equivalent response spectra, e.g extreme response spectra, shock response spectrum, fatigue response spectrum, etc [2].

The second part of this chapter 2 is the introduction to the semi-empirical method to obtain the random force limits with aid of a numerical example, where the phenomenon of over-testing is illustrated. This numerical problem is worked out completely to give an insight into why it is necessary to set the force limits in parallel with the random acceleration vibration specification. In this example the dynamic properties of both the load and the source are known.

To compute the random force limits at the interface between the load and the source the dynamical properties of the load are known; they are not known for the source, but are estimated in a probabilistic approach. Therefore the random force limits are calculated statistically.

A number of methods of defining modifications (notching) in the random acceleration vibration specifications are mentioned and discussed in Chapter 3. This modification or reduction in the vibration levels is needed to prevent over-testing of the load on a shaker or shaker table.

The semi-empirical method (SEM) for force limited vibration testing will be discussed in Chapter 4. This method is one of the basic tools in this thesis. The force limits can be established with the SEM to prevent over-testing of the load and to make the test levels as realistic as possible. The key parameters of SEM are the semi-empirical constant C^2 , the fundamental natural frequency of the load (fixed-free) together with associated dynamic properties, the rigid body mass of the load and the random vibration test specification to be applied when testing the load. Except for the semi empirical constant C^2 all other parameters can be made available during the design of the load. The value of C^2 depends on the dynamic behavior of the combination load and source, in particular at the interface. A literature review and overview about the used values of C^2 used are given as well.

In chapter 5 the probabilistic analyses to estimate the semi-empirical constant C^2 are discussed. The constant C^2 is one of the key parameters in the expressions to establish test specification for semi-empirical force limited vibration testing (FLVT) and this FLVT approach will be applied. The dynamic properties (design variables) of the unknown source are described in a probabilistic way prescribing the intervals and distribution functions. The major design variables of the source are the total mass, the lowest natural frequency associated with a dominant modal effective mass and the modal damping ratio applicable to both the load and the source. Additional design variables are derived from the major ones. In particular, the lowest natural frequency and the associated modal effective mass have to obey certain design rules. To perform the random response analyses the deterministic model of the load is coupled with the probabilistic model of the source; either the so-called asparagus patch model or the so-called MDOF model. The analysis procedures and coupled load/source models are described. The semi-empirical constant C^2 is derived from the maximum values of the random response at the interface. When the probabilistic models of the source are established and the (two) coupled probabilistic mathematical models are created, probabilistic analyses are performed to compute the probability distribution and subsequently the mean and standard deviation of the constant C^2 . The Rosenblueth $2k + 1$ point estimation probability moments and the Monte Carlo Simulation methods are used to perform the probabilistic analyses.

In Chapter 6 the probabilistic description of the asparagus patch model of the source and MDOF system is investigated using three cases taken from literature. The outcomes of the probabilistic analyses are presented in Tables and Figures they will be discussed and comments will be made.

In the final Chapter 7 the general conclusions and recommendations are provided.

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2

DESIGN PROCESS & PROBLEM DEFINITION

2.1. INTRODUCTION

The following items will be discussed in this chapter:

1. An introduction of the design & verification process of spacecraft structures. A more detailed discussion can be found in Appendix A.
2. An introduction of the force limited vibration testing method with the aid of a simple numerical example.
3. The problem to be solved in this thesis and the definition of the associated research question.

2.2. SPACECRAFT STRUCTURES DESIGN AND VERIFICATION

2.2.1. INTRODUCTION

The aspects of the design and verification philosophy of structures of spacecraft, instruments, units, etc. are discussed in great detail in [18, 23, 27], but are summarized briefly in this chapter.

The need for a new spacecraft (project) may be generated by scientific, communications, military, commercial, or other interests. In general, a scientific spacecraft is a one off, while communication satellites are made in series.

After preliminary studies a preliminary definition of the orbit of the spacecraft, payload, instrumentation and supporting equipment, geometry and mass will be made and the launch vehicle will be selected.

In the initial phase it is important to set up a good configuration of the spacecraft, which will be used and further optimized in the course of the project until the spacecraft is launched.

The finite element model of the spacecraft structure developed in the early stage of the project will be used to perform a spacecraft/launch vehicle coupled load analysis (CLA). The results of the coupled load analysis may modify the preliminary enveloping sine vibration loads (from launch vehicle user's manual) to be implemented in the subsequent design of the spacecraft and to be used during the sine vibration test to prevent over-testing of the spacecraft.

All imposed requirements and the capability to carry the prescribed loads will be demonstrated in the beginning by analysis and later on when hardware of the spacecraft can be made available by test (static, dynamic, etc.).

2.2.2. INITIAL PHASE

The definition of the spacecraft configuration depends on:

- The system specifications of the launch vehicle
 - Spacecraft total weight (mass)
 - Launch environment (static and dynamic loads)
 - Available volume under the fairing (shroud)
 - Payload adapter (interface between spacecraft and launch vehicle)
 - Payload separation system
 - Launch cost (may be overruled by political reasons)
- Functional requirements of the system (payload) and subsystem requirements (size, mass, position, environment, etc.). It may that some subsystems are already qualified for certain sine and random vibration levels during former projects.
- Orbit and mission duration

The launch vehicle constraints or requirements are prescribed and discussed in the launch vehicle user's manual, e.g. West-European launchers [1–3], however, other launch vehicles may be selected as they are in the USA, Russia, China, India, etc.

2.2.3. LOADS SPECIFICATION AND STIFFNESS REQUIREMENT

The various mechanical loads are not all equally important and depend on the type of the mechanical part of the structure and whether this relates to the primary structure or the secondary structure (solar panels, instruments, etc.). Preparations on the ground, the launch and operations on orbit set various types of design and test requirements, [6, 20].

The static and dynamic loads specified in the User's Manual of the selected launch vehicle are:

STATIC LOADS

Quasi Static Loads (QSL) Loads independent of time or which vary slowly, such that the dynamic response of the structure is not significant. The design of the primary structure is determined to a large extent by the quasi static loads and low

frequency sinusoidal loads. More information on the QSL are discussed in Appendix E

Depressurization During launch the pressure below the fairing will decrease the first 120s from 1 Bar to about 2 mBar just before jettison of the fairing. Structures, instruments, units will be vented.

Temperature induced load The introduction of thermal loads/stresses due to non uniform temperature distribution. On orbit temperature induced loads may jeopardize alignment requirements.

DYNAMIC LOADS

Sinusoidal Excitation Harmonic enforced acceleration at the base of the spacecraft, instrument, unit, etc. The sinusoidal and random base acceleration determine the design of secondary structures like instruments, solar array, antennae, unit and other subsystems. The frequency range is, in general, between 5-100Hz. The analysis results of the coupled load analysis (CLA) as indicated in Fig. A.1 can modify the preliminary enveloping sine loads.

The acceptance level is equal to flight loads, which are the maximum occurring load (with a certain probability) during the life cycle of the spacecraft. The qualification dynamic loads are derived from the flight loads.

Acoustics Loads Random sound pressure fields are design drivers for large areal to mass structures; solar panels, antenna dishes, radiators, etc. In general, the frequency range varies between 20 and 10000Hz. The sound pressures excite the outside of the spacecraft. The sound pressures are transferred into the inside of the spacecraft by the structural responses of the structure. This is illustrated in Fig. 2.1.

Random Enforced Acceleration The sinusoidal and random base accelerations determine the design of secondary structures, instruments, solar array, antennae, unit. The frequency range is between 20 and 2000Hz. In general, random vibration tests are not specified for complete spacecraft because the mechanical random vibrations are covered by the acoustic loads. In general, the random enforced accelerations are specified at the interface between spacecraft and launch vehicle, instrument and spacecraft. **In this thesis the main focus is on enforced random acceleration vibration specifications.**

The mathematical aspects of random vibrations are discussed in Appendix E.

The random vibration specifications are either derived from the acoustic test in a reverberant chamber or by analysis. In general, the measured and analysed structural random acceleration responses are not smooth but have peaks and valleys. Techniques to derive a more smooth random vibration specification are discussed in Appendix F.

Transient Excitation Deterministic loads whose magnitude or direction varies with time and for which the dynamic response of the structure is significant.

Figure 2.1: Spacecraft acoustic/random vibrations environment

High frequency transient excitation, Shocks High frequency transients loads are usually represented by their Shock Response Spectrum (SRS). Separation of launch vehicle stages and the separation of the spacecraft from the last stage will induce high shock loads into the spacecraft.

STIFFNESS

Natural (fundamental) frequencies (modal characteristics) The natural frequency is a governing design requirement for all parts of the spacecraft. This requirement is imposed in order to limit the dynamic coupling of the spacecraft with the launch vehicle.

2.2.4. VERIFICATION BY ANALYSIS

The structural analyses are, in general, performed with the finite element analysis method [5, 14]. The immersion of the analysis details the finite element model (FEM). The finite element analysis is applied to perform the following analyses:

- Strength analysis (yield, failure, buckling, etc.)
- Thermo-elastic analysis (deformations, stresses)
- Dynamic response analysis (modal, sine, random, transients, shocks, etc.)
- Acoustic analysis

STRENGTH ANALYSIS

The strength properties of the spacecraft structure and structural elements will be verified by analyses and later on by tests. The internal stress distribution is calculated and compared to allowable stresses (failure modes) to obtain margins of safety (MoS). For buckling a special buckling (bifurcation) analysis has to be performed. Typical applied load or combination of loads come from the quasi-static load factors.

If only dynamic load factors are applicable, the stress due to dynamic loads will be verified against allowable stress or loads. In case of random loads, in general, the 3σ value of stresses is compared to allowable stress or load.

THERMO-ELASTIC ANALYSIS

The thermal deformation and stresses in the spacecraft structure, due to thermal gradients, will be verified for alignment requirements.

DYNAMIC RESPONSE ANALYSIS

The dynamic response analysis is needed to verify specifications with respect to stiffness and strength requirements. A modal analysis is performed to calculate the modal characteristics of the complete spacecraft clamped at the interface between launch vehicle and spacecraft being natural frequencies, associated real mode shapes, generalized masses and stiffness, modal effective masses. The damping is generally ignored during the eigenvalue analysis, because the modal damping ratio for complete spacecraft is quite low, between 2 and 10%. Later in the spacecraft project the modal properties are verified by a modal survey [7].

The finite and boundary element analysis methods are applied in the lower frequency domain, while the statistical energy analysis (SEA) method can be applied in the high frequency domain, for structures with high modal densities [15, 16].

ACOUSTIC ANALYSIS

Large light weight, area structures mounted outside the spacecraft, e.g. solar arrays and antenna reflectors, are very sensitive to acoustic loads (sound pressures). A combined finite element and boundary element analysis [13, 26, 30] is needed to simulate fluid structure interaction (FSI).

The statistical energy analysis (SEA) method can be applied in the high frequency domain.

2.2.5. VERIFICATION BY TEST

All requirements and load carrying capabilities of the spacecraft structure have already been verified by analysis, however, a number of tests are applied too to qualify the spacecraft structures design. The general applied mechanical qualification tests and associated requirements are presented in Table 2.1.

Within the context of this thesis not all design and test loads are of interest, but the focus will be on the application of random acceleration vibration spectra applied to the test article (load) and the associated random force limited vibration specification, the so-called force limited vibration testing (FLVT) [25].

Table 2.1: Test verification

Type of test	Test verification
Static test	<ul style="list-style-type: none"> • The structural strength particularly in the primary structure and critical interfaces. • Verification (partially) of stiffness matrix.
Modal survey test	<ul style="list-style-type: none"> • Identification of modal properties to support the verification of the finite element model, which is used in load and coupled load analysis.
Sine vibration test (vibration table)	<ul style="list-style-type: none"> • Support the verification of the spacecraft finite element model (amplification from launcher/spacecraft interface input to various spacecraft parts) • Qualification of secondary structures.
Random vibration test (vibration table)	<ul style="list-style-type: none"> • Qualification of units subjected to random (acoustically generated) flight environment
Acoustic test (reverberant chamber)	<ul style="list-style-type: none"> • Verification and qualification of spacecraft system to acoustic environment. which might be experienced by the spacecraft during flight.
Shock test	<ul style="list-style-type: none"> • Spacecraft verification and qualification against high frequency transient loads (pyrotechnics and mechanically induced shocks).

2.3. INTRODUCTION FORCE LIMITED VIBRATION WITH A NUMERICAL EXAMPLE

The expected problem of dynamic over-testing or over-loading a test item (load) placed on the shaker head or shaker table exposed to specified random enforced vibrations will be explained based on a simple dynamic system consisting of the combination of the load and the flexible supporting structure (source). This simple dynamic system is taken from a paper of Haille [12] and is shown in Fig. 2.2. The combination of the coupled load and source is illustrated in Fig. 2.2 and consists of eight discrete masses m_i , $i = 1, 2, \dots, 8$ connected by springs with stiffness k and dampers with damping constant c . Mass 3 defines the interface between the load and the source for the accelerations and the spring between the masses 3 and 4 the interface for the forces.

The discrete dampers c are replaced by the orthogonal (proportional) damping matrix $\mathbf{C}$, which is expressed by the modal damping ratio ζ_i , $i = 1, 2, \dots, 8$, the natural frequency ω_i , $i = 1, 2, \dots, 8$, the total mass matrix $\mathbf{M}$ and eigenvectors ϕ_i , $i = 1, 2, \dots, 8$ [29]

$$\mathbf{C} = \sum_{i=1}^8 2\zeta_i \omega_i \mathbf{M} \phi_i \phi_i^T \mathbf{M}, \quad (2.1)$$

where the modal damping ratio is constant for all modes, $\zeta_i = \zeta$. The total mass of the source is $M_s = m_1 + m_2 + m_3 = 224\text{kg}$ and of the load $M_l = m_4 + m_5 + \dots + m_8 = 31\text{ kg}$, thus the total mass of the 8 DOFs dynamic system $M_{tot} = 255\text{kg}$.

Figure 2.2: Haille problem [12]

The PSD of the limited white noise force is $W_F = 10000 \text{ N}^2/\text{Hz}$ and applied to mass m_1 of the source and ; it ranges from 20 to 2000 Hz.

The random response computations are made with the direct method within a frequency interval of 20-2000 Hz. The power spectra of the accelerations of mass 3 and the forces in the spring between the mass 3 and 4 are presented in Fig. 2.3, (a) and (b), respectively. The specification of the random acceleration spectrum at mass 3 may be

Figure 2.3: Interface random accelerations W_A and forces W_F

considered as the design and test curve for the load, but is too complicated and will (must) be simplified enveloping the exciting random acceleration spectrum. In general, a standard spectrum consists of a frequency interval between 20 and 2000 Hz, a positive slope starting at 20 Hz, a negative slope ending at 2000 Hz, and a domain with a constant

power spectral density [8, 11],.

The new enveloping random vibration specification is shown in Fig. 2.4 (blue line) and in Table 2.2. The new specified enforced random acceleration will be applied to

2

Figure 2.4: Enveloped random acceleration specification (blue line)

Table 2.2: Enveloped random acceleration specification

Frequency range (Hz)	PSD Acceleration/slope
20-44	12 dB/oct
44-60	$0.1548 \text{ g}^2/\text{Hz}$
60-2000	-12 dB/oct
G_{rms}	2.661 g

mass 3 of the load (see Fig. 2.2) a new random response analysis is performed and again the interface random forces in the spring between mass 3 and 4 are computed again. The original (red line) and new random interface force (blue line) are shown in Fig. 2.5. The new computed random interface force is more severe than the original computed random interface force are depicted in in Table 2.3.

The severity of the new computed random interface forces is explained by the following remarks:

- The new specified random acceleration vibration specification envelopes the computed random accelerations at mass 3. This is more or less a standard approach [11] to simplify the specification. It is not common to apply the real computed random accelerations.

Figure 2.5: Original and new computed random interface forces

Table 2.3: Characteristics power spectra interface force

Spectrum	Peak value (N^2/Hz)	Frequency (Hz)	F_{rms} (N)
Original spectrum	$3.973 \cdot 10^4$	44	600.67
New spectrum	$7.758 \cdot 10^5$	67	2870.9
Ratio new/original	19.53		4.78

- At the natural frequency of the dominant mode of the load the random interface accelerations are reduced; this is caused by the vibration absorber effect (redline in Fig. 2.5) [9], which is not considered in the enveloped random acceleration specification.
- The severity of the enveloped specification is shown in Table 2.3. Applying the enveloping random acceleration specification will result in an over-design of the load.

To prevent over-testing the force limited vibration testing (FLVT) was proposed in an early paper by Salter [21] and evaluated further by Chang and Scharton of NASA-JPL ¹ [4, 24, 25].

Scharton proposed in [25] the semi empirical force limited vibration method to cal-

¹Jet Propulsion Laboratory

culate the force limits at the interface between the load and the shaker table

$$\begin{aligned} W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \quad f \leq f_0, \\ W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \left(\frac{f_0}{f} \right)^n \quad f > f_0. \end{aligned} \quad (2.2)$$

These equations will be discussed in more detail in Chapter 4. $W_{FF}(f)$ is the random force limited spectral density, $W_{AA}(f)$ is the specified random acceleration spectral density, M_o is the total mass of the load and C^2 is a dimensionless semi-empirical constant which depends on the dynamic interaction of the combination of the load and the source. f (Hz) is the running frequency and f_0 is the natural frequency of the primary mode with a significant modal effective mass of the load. The factor n can be estimated from the decaying slope of the apparent mass frequency response function of the load, in general, $n = 2$ and $C^2 \approx 1 - 5$ [4].

The maximum value of the PSD of the original random force $W_{F_{max}}$ and the maximum original PSD of the random acceleration $W_{A_{max}}$ at the interface between load and source in combination with total mass of the load M_o are used to calculate the constant C^2 applying the following expression [17]

$$W_{F_{max}} = C^2 M_o^2 W_{A_{max}}. \quad (2.3)$$

The maximum (analytical) values of W_F and W_A can be extracted from Fig. 2.3 (a) and (b) respectively. The computation of C^2 is illustrated in Table 2.4.

Table 2.4: Computation of semi-empirical constant C^2

M_o (kg)	f_o (Hz)	M_{eff} (kg)	$W_{A_{max}}$ $((m/s^2)^2 / Hz)$	$W_{F_{max}}$ (N^2 / Hz)	C^2 (-)	n (-)
31	67.6	28.2821 (91%)	15.2058	$3.9973 \cdot 10^4$	2.7189	2

The semi-empirical random force limits W_{FF} (black line) are compared with the actual interface random forces W_F in the spring between masses 3 and 4 (red line) and the random reaction forces of the fixed load (blue line) caused by the enveloped specified random acceleration applied to mass 3 are shown in Fig. 2.6. The random force limits are an envelop of the original computed interface random forces.

The PSD of both the original random interface forces W_F and the force limited specification W_{FF} are depicted in Table 2.5. The force limited random spectrum W_{FF} is much less severe than the interface random forces based on the enveloped interface random accelerations. As can be noticed from Table 2.5 the peak values of the power spectra are

Table 2.5: Characteristics power spectra force limited specification

Spectrum	Peak value (N^2 / Hz)	Frequency (Hz)	F_{rms} (N)
Original spectrum W_F	$3.973 \cdot 10^4$	44	600.67
Force limited (FL) spectrum W_{FF}	$3.973 \cdot 10^4$	44	1248.5
Ratio FL/original	1.00		2.08

Figure 2.6: Random force limits W_{FF} (black line) compared to random interface forces W_A (red line) and fixed load (blue line)

equal and at the same frequency. However, the ratio of the *rms* values of the force limited spectra to the original spectra of the interface forces decreases to a value 2.08, which is much lower than 4.78 as presented in Table 2.3.

The force limited vibration test of the load can be performed when both the random acceleration W_{AA} and the random force limits W_{FF} are specified. This is illustrated in Fig. 2.7. The associated random acceleration and force limits specifications are given in Fig. 2.8.

The pilot accelerometer(s) control the level of the random acceleration as input to the load. This pilot accelerometer is affixed to the base fixture of the shaker system. Force gauges are placed under the load to control the interface random forces. If the random input accelerations as well as the random forces are controlled a dual control random of the vibration test is performed - a force limited vibration testing- is performed. If force gauges and pilot accelerometers are implemented a dual control can be made, where

$$W_{Aa}(f) \leq W_{AA}(f), \quad (2.4)$$

and simultaneously

$$W_{Fa}(f) \leq W_{FF}(f), \quad (2.5)$$

where W_{Aa} are the actual applied random acceleration and W_{Fa} the actual random measured random interface force. Control strategies are suggested in [24].

When the force limited vibration testing is based on the semi-empirical method (Eq's 2.2) the semi-empirical constant C^2 is a very important parameter for computing the

Figure 2.7: Force limited vibration testing (FLVT)

Figure 2.8: Random acceleration and force specifications

random force limits W_{FF} . The mass M_o of the load is usually known as well as the random acceleration vibration specification $W_{AA}(f)$. C^2 can only be calculated if and only if the dynamic characteristics of both the load and source are known. What can be done if the dynamic properties of the source are unknown? A value of the semi-empirical constant C^2 has to be chosen which should be based on past experiences, papers [19, 22] and data from handbooks, e.g. [24, 25]. The force limit conservatism is dependent on

the chosen constant, C^2 . A high $C^2 = 5$ is used for directly mounted lightweight loads, whereas for heavier strut mounted equipment a low $C^2 = 2$ may be considered [4].

2.4. RESEARCH QUESTION

Figure 2.9: Deterministic load and probabilistic source

The problem to be solved in this thesis is to compute the random force limits Eq. (2.2) when the dynamic properties of the source are not known. The total mass M_o and all dynamical characteristics of the load are known as is the specified random acceleration vibration specification $W_{AA}(f)$ during the design phase of the load. In fact, the semi-empirical constant C^2 has been calculated from the unknown random spectrum of the interface forces between load and source.

Before the computation of random interface forces can even begin the dynamic characteristics of the source have to be estimated. These estimations are based on the observation that, in general, the design rules are followed. This yields a possibility to assume ranges for the most important design variables. Design rules (requirements) in the design of spacecraft structures [10, 27] has to be obeyed to approximate the most important design variables of the source [28]; total mass, lowest natural frequency, associated modal effective mass and modal damping ratio.

The main task is to reconstruct the random accelerations spectrum at the interface and the associated random interface force power spectrum from the specified acceleration spectrum. Knowing both power spectra the semi-empirical constant C^2 can be obtained and subsequently the force limited vibration testing can be performed (Fig.

2.9).

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3

RANDOM VIBRATION TESTS, MODIFICATION OF INPUT SPECTRUM (NOTCHING), AN OVERVIEW

3.1. INTRODUCTION

In this section an overview of a number of methods of defining modifications in the random vibration specifications are mentioned and discussed. This modification or reduction in the vibration levels is needed to prevent over-testing of the test item on a shaker or shaker table.

The dynamic configuration of the test item (spacecraft, instrument, equipment) on the shaker table is totally different from the flight configuration, spacecraft launch vehicle, instrument, spacecraft, radiator, instrument, etc. In the flight configuration the test item will act like a vibration absorber, which is not the case when placed on a shaker table with a very high impedance. When the test item acts like a vibration absorber the vibration in the interfaces will be reduced, which is not the case during the dynamic test. This was already noticed by Blake in [1] where he stated: "The neglect of the effects of impedance generally leads to tests of real equipment which are far more severe than any field conditions".

The need to limit the levels during the vibration tests arises when the specifications for the load are derived from an envelope covering the mechanical load environment generated by a source excited by forces F (see Fig. 3.1). In this case, the specified acceleration levels $\ddot{x}$ are based on the dynamic responses of the combined source and load structure ignoring the minima (valleys) related to the natural frequencies (resonances) of the load as illustrated in Fig. 3.1. Reduction of the test random vibration levels (notching) is called the "response limiting vibration" and a reduction of the random interface

Figure 3.1: Principle of notching [12]

forces is indicated by "force limiting". Both approaches will be discussed.

Notching is the reduction of acceleration input levels around natural (resonant) frequencies, to avoid over-testing. There are two forms of notching

- Primary notching
- Secondary notching

The primary notching is performed to limit the load interface forces to the specified values. This is basically the same as limiting the equivalent accelerations at the centre of gravity of the load. The secondary notching is performed to limit local accelerations with a view to protecting instruments and equipment. In general, instruments and equipment have been qualified to a certain vibration level, which has to be respected.

In this thesis the emphasis is on notching applied to random enforced acceleration during random vibration testing and no distinction between primary and secondary notching need to be made. Sinusoidal vibration testing is, in general, in a frequency range between 5 and 150 Hz. Also over-testing problems may occur and notching will be applied. The nature of acoustic and shocks test is such that adaptation of sound pressure levels (SPL) and shock response spectra (SRS) is not possible [28].

3.2. WHY FORCE AND RESPONSE LIMITING

The need for notching will be explained with a very simple example. The total system is represented by a two degree of freedom system (TDFS). The supporting structure

(source) in the TDFS is represented by mass m_1 , damper c_1 and stiffness k_1 . The test item (load) of interest is mounted on top of the supporting structure and is represented by mass m_2 , damper c_2 and stiffness k_2 . The load is also shown as a simple one degree of free system (SDOF). Both systems are shown in Fig. 3.2. The source represents the supporting structure of the load. The properties of the TDFS and SDOF are shown in Table 3.1. The TDFS will be excited by a constant amplitude enforced sinusoidal acceleration $\ddot{U}(\omega)$ m/s². The SDOF will be excited by a constant amplitude enforced sinusoidal acceleration $\ddot{U}^*(\omega) = \max \{\ddot{X}_1(\omega)\} = A$ m/s². The modal superposition method is used

Figure 3.2: Base-excited TDFS and SDOF

Table 3.1: Properties TDFS and SDOF

Item	Natural frequency (Rad/s)	Damping Ns/m	Amplification factor
Source	$\omega_1 = \sqrt{k_1/m_1}$	$c_1 = 2\zeta_1\sqrt{k_1m_1}$	$Q_1 = 1/2\zeta_1$
Load	$\omega_2 = \sqrt{k_2/m_2}$	$c_2 = 2\zeta_2\sqrt{k_2m_2}$	$Q_2 = 1/2\zeta_2$
Mass ratio $\mu = m_2/m_1$			
Frequency ratio $\phi = \omega_2/\omega_1 = f_2/f_1$			
Frequency ratio $r = \omega/\omega_1 = f/f_1$, ω , f excitation frequency			
Frequency ratio $\bar{r} = \omega/\omega_2 = f/f_2 = r/\phi$, ω , f excitation frequency			

to solve numerically the sinusoidal acceleration responses $\ddot{X}_1$ and $\ddot{X}_2$ of the TDFS system when sinusoidally excited at the base $\ddot{U}(\omega) = 10$ m/s². The following values for the parameters are used: $m_1 = 500$ kg, $m_2 = 10$ kg, $f_1 = 25$ Hz and $f_2 = 35$ Hz. The modal damping ratio is $\zeta = 0.05$ for both modes. The acceleration responses are shown in Figure 3.3. The natural frequencies of the undamped system are 24.5Hz and 35.7Hz and the associated modal effective masses are 501.37kg and 8.63kg, respectively. The first mode will be excited very well and the second mode slightly. This can be seen in Figure 3.3. The maximum acceleration for $\ddot{X}_1 = 97.2$ m/s² and for $\ddot{X}_2 = 188.6$ m/s².

The SDOF system, the load, will be now excited at the base by a conservative constant amplitude sinusoidal enforced acceleration $\ddot{U}^* = \max \ddot{X}_1 = A = 97.2$ m/s². No adapta-

Figure 3.3: Sinusoidal accelerations TDFS and SDOF

tion of $\ddot{X}_1$ has been made for the second natural frequency at 35.7Hz. In fact, the load is placed on a shaker with an infinite impedance. The maximum acceleration response of the load is $\ddot{X}_2 = 1005.8 \text{ m/s}^2$, however, at 35Hz and not 24.5Hz. In fact, the load is over-tested by a factor $1005.8/188.6=5.33$.

The interface forces in the TDFS and SDOF system as indicated in Figure 3.2 are plotted in Figure 3.4. The maximum amplitude of the interface force in the TDFS system is $F_{int} = 1829\text{N}$ and the maximum amplitude of the interface force in the SDOF system is $F_{int} = 9782\text{N}$. The over testing by interface force is a factor 5.35.

The high over-testing factors for both the enforced accelerations and interface forces, during vibration testing of the load by an enveloping enforced acceleration spectrum, make it necessary to adapt the vibration test spectrum in order to prevent over-testing of the load during the vibration test.

3.3. PREVIOUS WORK

In an early paper [1] Blake described the need to reduce or notch the specified base random acceleration specification at the natural frequency of the load (test item). On the shaker table with a very high impedance the test item cannot act as a vibration absorber as in the case where the test item had been coupled on to another structural system.

In general the following methods are discussed in reports and papers from institutes, working groups, conferences and journals.

- *Response limiting methods.* The random vibration specification is modified (notched) in such a manner that the responses will not be beyond a specified acceleration

Figure 3.4: Interface forces F_{int} TDFS and SDOF

level (This may be primary or secondary notching). This is illustrated in Fig. 3.1. One method for defining the modified random test levels is discussed:

- Approximate Computation of Notch Spectrum [5].
- *Quasi Static Loads.* The following methods are discussed:
 - Mass participation factors
 - Enclosure method proposed by CNES¹ (Toulouse) based on quasi-static loads [11, 13]
 - Response Limiting on 1/2 octave frequency band [9, 13] with quasi-static load criterion
- *Force limiting methods.* The random force at the interface between the shaker table and the load will be prescribed to overcome over-testing of structural parts and subsystems. The control system of the shaker system will reduce (modify) the enforced random vibration acceleration specification when the PSD of the random interface force is beyond the specified random interface force limits (primary notching)

The force limited vibration testing (FLVT) was developed and propagated by Schar-ton of the NASA/JPL and over the years he published a lot of papers about the philosophy of the FLVT [22–24]. Coté, Soucy, Sedaghati, Marchand et al of the

¹Centre National d'Etudes Spatiales

Canadian Space Agency (CSA) and several Universities [3, 15, 16, 20, 25] have also published about this subject. An ESA/ESTEC study had been performed to investigate the FLVT [7, 8] in greayer detail. On behalf of JAXA Nagahama reported about their FLVT activities in [18, 19]. The following methods of defining force limits are discussed in this chapter:

- Simple two degree of freedom model (STDFS). The STDFS model is frequently applied to compute the constant C^2 of the force limiting testing, e.g. [10, 16, 18]. Diagrams are provided in [24] .
- Complex two degree of freedom model (CTDFS). The derivation of the equations of motions and successively the constant C^2 is quite complicated and provided only by Davis in [6]. The constant C^2 is provided in tables in [24].
- JAXA enveloping method [19].

The semi-empirical method of force limitation is described by Soucy in [10, 20]. This method will be discussed in chapter 4.

3.4. RESPONSE LIMITED RANDOM VIBRATION TESTING

3.4.1. INTRODUCTION

In this section the maximum allowable random vibration acceleration spectrum (pre-defined specifications). Certain parts of the load are already tested (qualified) before applying a predefined qualifying random vibration acceleration specification spectrum (g^2/Hz) and G_{rms} (g). This pre-qualification is mostly respected by applying secondary notching. If the structural response of the load, caused by the random base acceleration, is beyond the pre-defined random vibration specification, notching of the input spectrum should be applied. The Curtis method [5] determines the depth and width of the notched input of the random vibration specification.

3.4.2. MAXIMUM ALLOWABLE RESPONSE SPECTRA

The philosophy of the reduction of the random acceleration input spectrum W_{in} (blue line, Fig. 3.5) when the structural random acceleration response spectrum W_{res} at a critical location of the test item (instrument, equipment) (black line) is beyond the maximum allowable random acceleration spectrum W_{max} (red line) is more or less illustrated in Fig. 3.5. The random acceleration response spectrum shows a peak beyond W_{max} at the natural frequency f_1 and peaks below W_{max} at the resonant frequencies f_2, f_3 . Hence a modification (notching) of the input spectrum W_{in} is needed at the resonant frequency f_1 (eased black line). The maximum allowable spectrum is probably the outcome of the pre-qualification random vibration test of a certain sensible element (e.g. sensor or switch). In order to prevent the rupture (malfunctioning) of that sensible element an adaptation of W_{in} is required. However, negotiations are always needed and are reflected in the test plan.

The random acceleration response curve W_{res} can be computed using the finite element analysis (FEA) technique. The damping assumptions are quite important, and they may come either from experience from previous projects or from modal or sine excita-

tion tests. The standard frequency range for random vibration tests is between 20 and 2000Hz. The finite element model (FEM) must cover this frequency range.

Figure 3.5: Typical input and response control spectra [5]

APPROXIMATE COMPUTATION OF NOTCH SPECTRUM

The approximate calculations of the notched input spectrum W_{in} are discussed in [5]. It is assumed that the response peaks of concern are well separated and are similar

Figure 3.6: Typical transfer function magnitude

in the region of the natural frequency, as the response peak of one SDOF system with equal maximum and damping ratio. This assumption then permits the calculation of the notched input using only the magnitudes of the transfer function at the response locations where the structural response spectrum is beyond the maximum allowable spectrum at the fixed base natural frequencies. Fig. 3.6 shows the square of the magnitudes of the frequency transfer function $|H(f)|$, at a location where the response should be limited to the maximum allowable spectrum. The peaks of $|H(f)|^2$ are illustrated for three modes. The response had to be limited at the response peak of the i th mode over a bandwidth BW_i (Hz), as a fraction η of the centre frequency (natural frequency) f_i (Hz)

$$BW_i = \eta f_i \quad (3.1)$$

The mean constant PSD of the structural acceleration response within the band width

Figure 3.7: Mean square transfer function within averaging bandwidth, $\overline{|H(f_i)|^2}$

BW_i can be calculated using the following equation

$$\overline{W} = \frac{1}{\eta f_i} \int_{(1-\eta/2)f_i}^{(1+\eta/2)f_i} W_{in}(f) |H(f)|^2 df \approx \frac{W_{in}(f_i)}{\eta f_i} \int_{(1-\eta/2)f_i}^{(1+\eta/2)f_i} |H(f)|^2 df, \quad (3.2)$$

or

$$\overline{W} = W_{in}(f_i) \overline{|H(f_i)|^2}, \quad (3.3)$$

where $\overline{|H(f_i)|^2}$ is the average mean square of the frequency transfer function within the bandwidth BW_i , assuming that the PSD of the enforced (input) random acceleration at the frequency f_i does not vary very rapidly. This implies that η is small. The PSD of the structural acceleration response at the natural frequency f_i is given by

$$W_{res}(f_i) = |H(f_i)|^2 W_{in}(f_i). \quad (3.4)$$

The mean square transfer function with the bandwidth BW_i must be determined as shown in Fig. 3.7. The ratio between the average PSD $\overline{W}$ and the peak PSD $W_{res}(f_i)$ of the structural acceleration can be expressed in the following way

$$\frac{\overline{W}}{W_{res}(f_i)} = \frac{\overline{|H(f_i)|^2}}{|H(f_i)|^2} \quad (3.5)$$

The magnitude of the frequency transfer function will be approximated by the frequency transfer function of the SDOF system given by

$$|H(f)|^2 = \frac{4\zeta_i^2 f^2 + f_i^2}{(f_i - f^2)^2 + 4\zeta_i^2 f^2 f_i^2} = \frac{4\zeta_i^2 r^2 + 1}{(1 - r^2)^2 + 4\zeta_i^2 r^2} = |H(r)|^2, \quad (3.6)$$

where $r = f/f_i$, and the damping ratio is ζ_i . Thus the ratio

$$\frac{\overline{|H(f_i)|^2}}{|H(f_i)|^2} = \left(\frac{4\zeta_i^2}{4\zeta_i^2 + 1} \right) \frac{1}{\eta} \int_{(1-\eta/2)}^{(1+\eta/2)} |H(r)|^2 dr = \frac{\sigma_i^2}{\eta}. \quad (3.7)$$

In Fig. 3.8 σ_i^2/η is plotted versus $\eta/2\zeta_i$. N.B.: when $\eta \rightarrow 0$ the value $\sigma_i^2/\eta \rightarrow 1$.

Figure 3.8: σ_i^2/η versus $\eta/2\zeta_i$

The average PSD $\overline{W}$ in the bandwidth ηf_i is in accordance with (3.7) and can be expressed

$$\overline{W} = W_{in}(f_i) \overline{|H(f_i)|^2} = W_{in}(f_i) |H(f_i)|^2 \frac{\sigma_i^2}{\eta}, \quad (3.8)$$

where $|H(f_i)|^2$ is the square of the magnitude of the original frequency transfer function as illustrated in Fig. 3.6.

The notch factor (NF) is defined by

$$NF = \frac{W_{max}}{\bar{W}} = \frac{W_{max}}{W_{in}(f_i)} \frac{\eta}{|H(f_i)|^2 \sigma_i^2}. \quad (3.9)$$

If the $NF \geq 1$, no notching is permitted. However, when $NF < 1$ the notched input W_{in}^* is defined by

$$W_{in}^* = W_{in}(f_i) NF = W_{max} \frac{\eta}{|H(f_i)|^2 \sigma_i^2}. \quad (3.10)$$

The initial notched input W_{in}^* must be applied all over the bandwidth ηf_i , and η is defined by the engineer who computes the notched input. The Initial shape of the notch is shown in Fig. 3.9. However, the structural responses just left and right of the initial notched input will exceed the maximum allowable PSD spectrum. This is illustrated in Fig. 3.10. To prevent this from exceeding structural responses additional notching of the input is needed. This additional notched input starts at $r_l f_i$ (l-low) and ends at $r_h f_i$ (h-high) as shown in Fig. 3.10. Applying again the frequency response function $|H(f)|^2$ of

Figure 3.9: Initial shape of notch ($BW_i = \eta f_i$)

the SDOF system, r_l , r_h can be calculated using

$$\frac{W_{max}}{W_{in}} = \frac{4\zeta_i^2 r^2 + 1}{(1 - r^2)^2 + 4\zeta_i^2 r^2}. \quad (3.11)$$

Eq. (3.11) has two positive roots r_l , r_h which are presented in Fig. 3.11, where $Q = 1/2\zeta$ is the amplification factor. Proper values of r_l , r_h satisfy the following conditions

$$\begin{aligned} r_l &< 1 - \frac{\eta}{2}, \\ r_h &> 1 + \frac{\eta}{2}. \end{aligned} \quad (3.12)$$

Figure 3.10: Final shape of notch

Figure 3.11: Additional notching parameters r_l , r_h

Assuming logarithmic slopes (dB/oct) of the additional notched input, the slopes can be

calculated by the following expressions

$$\begin{aligned} n^- &= 3 \frac{\log\left(\frac{W_{in}^*}{W_{in}}\right)}{\log\left(\frac{1-\eta/2}{r_l}\right)} = -3 \frac{\log(NF)}{\log\left(\frac{1-\eta/2}{r_l}\right)}, \\ n^+ &= 3 \frac{\log\left(\frac{W_{in}}{W_{in}^*}\right)}{\log\left(\frac{r_h}{1+\eta/2}\right)} = 3 \frac{\log(NF)}{\log\left(\frac{r_h}{1+\eta/2}\right)}. \end{aligned} \quad (3.13)$$

Instead of reducing the PSD of the input concerning the allowable PSD acceleration, one can apply the random spectrum of strains, stresses, forces, etc. The use of frequency response functions remains the same.

3.5. THE QSL DESIGN LOADS AND ASSOCIATED REACTION FORCES

In this section notching methods based on the quasi-static design loads are discussed. The random vibration at the base of the load will introduce random reaction forces. The 3σ value of the random reaction force should not exceed the reaction force associated with the QSL. If it does the random acceleration input spectrum will be adapted or notched. Applications of this approach to define notches are:

- *The mass participation method*, in which the modal effective mass, apparent mass in conjunction with Miles' equation are the basic ingredients to determine notched random acceleration input comparing 3σ reaction loads with the reaction loads caused by the QSL design loads.
- Response limiting method presented by [11] which is more or less an enclosure technique to determine the notched random input. The 3σ reaction loads are compared to reaction loads caused by QSL design loads.
- Response limiting on 1/2 octave frequency band with quasi-static load criterion as presented by Fabries [9]. The 3σ reaction loads are compared to static reaction loads caused by the QSL design loads.

3.5.1. RESPONSE LIMITED, MASS PARTICIPATION FACTORS

The quasi-static design limit load applied for the design of equipment and instruments is mostly based on experience from previous spacecraft projects and is defined, in general, using the mass acceleration curve (MAC). Such a curve can be derived from analytical and flight data, and includes the effects of both transient and mechanically transmitted random vibration [14, 26]. An example of a MAC is presented in Fig. 3.12.

The design loads (factors) are dependent on the mass of the equipment or the instrument. The design factor (inertia load) γ (g) defines the interface load between the equipment, the instruments and the spacecraft to which the boxes are mounted. The maximum random interface loads will occur approximately at the natural frequencies. If the random interface load is higher than the interface loads caused by the design limit load the input acceleration levels may be reduced (notched) at natural frequencies.

Figure 3.12: Representative mass acceleration curve for cryogenic assemblies [21]

Figure 3.13: SDOF system excited by random acceleration

An instrument is represented by a SDOF system enforced by a random acceleration $\ddot{U}$ represented by the PSD function $W_{\ddot{U}}(f)$, is illustrated in Fig. 3.13. The 3σ value of the random base force F_{base} , with a PSD $W_{F_{base}}(f)$ will be less or equal to the total inertia force $m\gamma$ applying at the CoG of the SDOF system. Applying Miles' equation [29] will result in

$$3\sqrt{\int_{f_{min}}^{f_{max}} W_{F_{base}}(f) df} = 3m\ddot{X}_{rms} \approx 3m\sqrt{\frac{\pi}{2} f_0 Q W_{\ddot{U}}(f_0)} \leq m\gamma, \quad (3.14)$$

where $W_{F_{base}}(f)$ is the PSD function of the random interface loads, f_0 (Hz) is the natural frequency, Q is the amplification factor of the SDOF system and m is the mass. f_{min} is the minimum frequency of interest and f_{max} the maximum frequency of interest, most times 20 and 2000 Hz, respectively. γ is the load factor. The probability the random acceleration is beyond or below the $\pm 3\sigma$ value is given by $Prob(|X| >= 3\sigma) = 0.997$. It is common practice in the European space industry to take 3σ values for comparison with QSL

In general, the notch criteria with regard to the quasi-static design load (QSL) is de-

defined as

$$3\sqrt{\int_{f_{min}}^{f_{max}} W_{F_{base}}(f) df} \leq M_{tot}\gamma, \quad (3.15)$$

where M_{tot} the total mass of the equipment or instrument. Eq. (3.15) means that the 3σ ($3 \times rms$ and zero mean) value of the interface force will be less than or equal to the interface load caused by the QSL design loads.

The expected 3σ peak value of the interface force F_{base} is based on the mass participation method

$$F_{base} = 3\sqrt{\sum_{k=1}^n \left[M_{eff}^2(f_k) \frac{\pi}{2} f_k Q_k W_{\ddot{u}}(f_k) \right]} \leq M_{total}\gamma, \quad (3.16)$$

where n is the number of modes taken into account. The sum of all modal effective masses $M_{eff}(f_k)$, $i = 1, 2, \dots, n$ is equal to the total mass M_{tot} of the system. If m ($m < n$) modes are considered, the effect of the residual mass M_{res} will be taken into account

$$M_{res} = M_{tot} - \sum_{k=1}^m M_{eff}(f_k). \quad (3.17)$$

Eq. (3.16) will be as follows:

$$F_{base} = 3\sqrt{\sum_{k=1}^m \left[M_{eff}^2(f_k) \frac{\pi}{2} f_k Q_k W_{\ddot{u}}^*(f_k) \right] + M_{res}^2 \int_{f_{min}}^{f_{max}} W_{\ddot{u}}^*(f) df} \leq M_{tot}\gamma, \quad (3.18)$$

where $W_{\ddot{u}}^*(f_k)$ and $W_{\ddot{u}}^*(f)$ are the adapted enforced random acceleration spectra, $W_{\ddot{u}}(f_k)$ the original specified random acceleration spectrum and f_k , $k = 1, 2, \dots, m$ are the natural frequencies. The random enforced acceleration spectrum may be adapted (notched) when the 3σ random interface loads are higher than the quasi-static interface loads.

The modal power contribution of the vibration mode with natural frequency f_k will be related to the original PSD value of the enforced random acceleration $W_{\ddot{u}}(f_k)$. The modal power is defined by

$$M_{eff}(f_k) \ddot{x}_{rms}(f_k) \dot{x}_{rms}(f_k) = M_{eff}(f_k) \dot{x}_{rms}(f_k) \sqrt{\frac{\pi}{2} f_k Q_k W_{\ddot{u}}(f_k)}, \quad k = 1, 2, \dots, n. \quad (3.19)$$

The modal power contribution is now correlated to the $W_{\ddot{u}}(f_k)$ as follows

$$M_{eff}(f_k) \ddot{x}_{rms}(f_k) \dot{x}_{rms}(f_k) = A W_{\ddot{u}}(f_k), \quad k = 1, 2, \dots, n, \quad (3.20)$$

where $\ddot{x}_{rms}(f_k)$ is the rms acceleration, $\dot{x}_{rms}(f_k)$ is the rms velocity of the vibration mode, A is a constant and $W_{\ddot{u}}(f_k)$ is the original PSD function of the enforced acceleration at the natural frequency f_k . Rewrite Eq. (3.20) as follows

$$M_{eff}(f_k) \frac{\ddot{x}_{rms}^2(f_k)}{2\pi f_k} = A W_{\ddot{u}}(f_k). \quad (3.21)$$

By introducing of Miles' equation for the *rms* acceleration $\ddot{x}(t)$, the following expression can be obtained:

$$M_{eff}(f_k) \frac{\frac{\pi}{2} f_k Q_k W_{\ddot{u}}^*(f_k)}{2\pi f_k} = \frac{M_{eff}(f_k)}{4} Q_k W_{\ddot{u}}^*(f_k) = A W_{\ddot{u}}(f_k). \quad (3.22)$$

The constant A is assumed to be equal for all n vibration modes in the frequency range of interest and can be written as

$$\frac{M_{eff}(f_k) Q_k W_{\ddot{u}}^*(f_k)}{W_{\ddot{u}}(f_k)} = 4A = \bar{A} \quad k = 1, 2, \dots, m. \quad (3.23)$$

If all vibration modes are considered the residual mass $M_{res} = 0$ and Eq. (3.18) becomes

$$3\sqrt{\sum_{k=1}^n \left[M_{eff}^2(f_k) \frac{\pi}{2} f_k Q_k W_{\ddot{u}}^*(f_k) \right]} \leq M_{tot} \gamma. \quad (3.24)$$

After substituting Eq. (3.23) into Eq. (3.24) the following expression will be obtained:

$$3\sqrt{\frac{\pi}{2} \bar{A}} \sqrt{\sum_{k=1}^n [M_{eff}(f_k) f_k W_{\ddot{u}}(f_k)]} \leq M_{tot} \gamma. \quad (3.25)$$

Using Eq. (3.25) the constant $\bar{A}$ can be solved

$$\bar{A} = \frac{(M_{tot} \gamma)^2}{\frac{9\pi}{2} \sum_{k=1}^n [M_{eff}(f_k) f_k W_{\ddot{u}}(f_k)]}. \quad (3.26)$$

With use of Eq. 3.23 all n individual reduced input spectra $W_{\ddot{u}}^*(f_k)$, $k = 1, 2, \dots, n$ can be calculated.

If the residual mass is not negligible (say $M_{res} > 0.2 M_{total}$) Eq. (3.18) must be applied to obtain the reduced PSD of $W_{\ddot{u}}^*(f_k) \leq W_{\ddot{u}}(f_k)$ at the natural frequencies f_k . This is an iterative process.

The choice of width of the notch around f_k (if needed) is not always chosen to be three times the half power width which depends on the amplification factor Q_k :

$$\Delta f_k = \frac{3f_k}{Q_k} \quad k = 1, 2, \dots, m, \quad (3.27)$$

and the slopes p_k of the reduced PSD function $W_{\ddot{u}}^*$ around f_k are may be chosen as follows

$$p_k = \mp Q_k \text{ dB/oct.} \quad (3.28)$$

This shape of the notch is illustrated in Fig. 3.14. The natural frequencies and associated amplification factors may be extracted from a low level sine sweep test with a low sweep rate.

Figure 3.14: Shape of the notch

3.5.2. ENCLOSING NOTCHING METHOD OF CNES

In this section the response limiting method as proposed by CNES (Toulouse) [11, 13] will be explained. This method is presented by Didier Gangloff of CNES in one of the working meetings within the frame of the handbook ECSS-E-HB-32-26A, entitled "Spacecraft mechanical loads analysis handbook".

For linear structures the input-output relation between PSD functions is given by

$$W_X(f) = |H_{X\ddot{U}}(f)|^2 W_{\ddot{U}}(f), \quad (3.29)$$

where $\ddot{U}$ is the enforced acceleration and X is the response. The random response may be acceleration, force, stress, etc. This input-output relation is illustrated in Fig. 3.15.

If the output PSD value at a certain frequency $W_X(f)$ has to be decreased or increased by a factor α , the input PSD value $W_{\ddot{U}}(f)$ has to be altered by the same factor α . This is shown in Fig. 3.16.

Figure 3.15: Input-output relation random vibrations linear structure

Figure 3.16: Input-output relation PSD functions

ENCLOSING NOTCHING METHOD OF CNES

The scaled (notch) random acceleration vibration spectrum can be computed in an iterative manner [11, 13]. The scaling of the random acceleration vibration spectrum is based on the quasi-static design loads (QSL). The methodology applied to define the notched spectrum is the following:

- The finite element analysis (FEA) gives the random resulting force in N_{rms} at the base of the load without notching.
- This force is multiplied by 3 (random vibration limit loads are typically taken as the 3σ load).
- The resulting force ($F_{rms} = 3N_{rms}$) is compared to the N_{QSL} .
- If F_{rms} is higher than the N_{QSL} , it is necessary to apply a notched input (notching).

The following iteration procedure or roadmap to calculate notched input is proposed by CNES:

- **Iterations.** All iterations are based on the original input PSD $W_{\ddot{U}}$ and the original response PSD W_X .

1st iteration Calculate the average response PSD ($W_{average} = \alpha W_X$) of the minimum and the maximum response PSD of W_X . Remove all peaks above the average PSD value and scale the original input PSD $W_{\ddot{U}}$ at the locations of the peaks in accordance with Fig. 3.16.

n^{th} iteration If F_{rms} is higher than $N_{QSL}(1 + \epsilon)$ (ϵ is tolerance parameter), a new average response PSD value is calculated by replacing the maximum response PSD by the average response PSD value calculated in the previous iteration step. Conversely, if F_{rms} is lower than $N_{QSL}(1 - \epsilon)$, the new average response PSD value can be calculated by replacing the minimum response PSD by the average response PSD calculated in the previous step in combination with the maximum response PSD. A better convergence can be achieved by multiplying the average response PSD from the previous step by a factor $(1 + \lambda)$.

Remove all peaks above the average response PSD value and scale the original input PSD $W_{\ddot{U}}$ at the locations of the peaks in accordance with Fig. 3.16.

Last iteration The loop is repeated until obtaining the criterion $N_{QSL}(1 + \epsilon) > F_{rms} > N_{QSL}(1 - \epsilon)$.

A numerical example is worked out in [31].

3.5.3. RESPONSE LIMITING ON 1/2 OCTAVE FREQUENCY BAND WITH QUASI-STATIC LOAD CRITERION

GENERAL

The method described in this section is applied for about a decade by TAS-F² and this notching strategy is applied to all antenna products with a random specification [9]. This method is presented by Cristophe Fabries of TAS-F in one of the working meetings within the frame of the handbook ECSS-E-HB-32-26A, entitled : "Spacecraft mechanical loads analysis handbook". It concerns antenna products from 5 to more than 60 kg (from a little Ka band antenna feed to big multi-beams feeds).

When this methodology was first applied, the contribution of the (reaction) force PSD included the entire frequency band between 20 and 2000Hz. However, this methodology led to too deep notches and unceasing discussion between subsystem and spacecraft partners. Consequently, a compromise was found to limit the computation of the criteria only in the 1/2 octave band, but it should be noted that this approach is more "industrial" than "physical".

INTRODUCTION

This section provides the road map used to reply to a random vibration specification as described below, including the notching strategy to prevent an over-sizing of the equipment.

The proposed notching strategy is to limit the r.m.s. resultant interface force to the defined envelope of the quasi-static load (QSL) resultant interface force. The following considerations are important:

- The notching is applied to the natural frequencies with corresponding vibration modes with high effective masses (taken from FEA prediction or from test measurements).
- The notched frequency band is limited to the 1/2 octave frequency band around the notched natural frequencies. The choice of the 1/2 octave frequency bandwidth is made in a mutual agreement between the subsystem supplier and the spacecraft prime contractor. It is a compromise between the necessity to reduce effective loads and to limit the frequency range of notching.
- The QSL resultant interface force criterion (comparison) is the 3σ value computed on the 1/2 octave frequency band around the notched natural frequencies (criteria issued from FEA prediction or from load cell measurements during test).

²Thales Alenia Space-France

Figure 3.17: Interface force limitation prediction

NOTCHING PREDICTION WITH FE RANDOM VIBRATION RESPONSE ANALYSIS

- The QSL resultant interface force N_{QSL} is computed with an equipment mass of M associated with the acceleration levels defined in QSL load factor specification γ :
 1. QSL load $N_{QSL} = M \times \gamma \times g$. This design value N_{QSL} is compared with 3σ resultant load values $F_{rms} = 3N_{rms}$ computed in a specific bandwidth in the direction of load factor component γ .
- Available notched frequency bands are the vibration modes with high effective mass at the natural frequencies f_n , $n = 1, 2, \dots, N$:
 1. Natural frequencies f_n , $n = 1, 2, \dots, N$
 2. Lower frequency $f_{nl} = 2^{-\frac{1}{4}} f_n$ of 1/2 octave band
 3. Upper frequency $f_{nu} = 2^{\frac{1}{4}} f_n$ of 1/2 octave band
 4. Notching band width $\Delta f_n = f_{nu} - f_{nl}$
 5. Power spectral density in 1/2 octave frequency band is $W_n = N_{rms}^2 / \Delta f_n$
- The PSD of the random interface force is iteratively decreased to obtain an r.m.s. value of the interface load $N_{rms} = N_{QSL}/3$ for each available notched frequency band Δf_n , $n = 1, 2, \dots, N$ if the PSD peak value at frequency f_n is higher than W_n . This is illustrated in Fig. 3.17.

A numerical example is worked out in [30].

SUMMARY QSL BASED NOTCHING

Three notching prediction methods, based on the QSL, are discussed:

Response Limited, Mass Participation Factors: This method is the most straight forward method of predicting notching of the random acceleration vibration specification. An FE model of fixed load is needed to calculate natural frequencies and associated modal effective masses and the residual mass. RSS reaction forces are obtained using Miles' equation approximating r.m.s. modal responses.

Enclosing Notching Method of CNES This method of predicting notches of the random acceleration vibration specification is an iterative process to estimate r.m.s. reaction forces performing a random response FE analysis of the fixed load. This method can be used for all types of spacecraft, instruments and antennae.

Response Limiting on 1/2 Octave Frequency band This method of prediction of the notch levels in the 1/2 octave band of the specified acceleration specification is conservative because too high a load is introduced in spacecraft, instruments and antennae. A complete FE model of the fixed load is needed to calculate r.m.s. reaction forces.

In general, the quality of the FE analysis determines the quality of the modal and random response analysis within a frequency band between 20-2000Hz.

When notching based on QSL is used, It is recommended to start with the mass participation method to get a sense of the location and depth of the notches finalized by the enclosure technique to estimate the location and depth of the notches. Only primary notching can be performed.

3.6. FORCE LIMITED VIBRATION

3.6.1. INTRODUCTION

During the force limited random vibration testing the random interface loads $W_{FF}(f)$ (N^2/Hz) are controlled (limited) to prevent over-testing of the load clamped to the shaker head or shaker slip table of the shaker system. The random vibration acceleration specification $W_{AA}(f)$ (g^2/Hz) is provided. The mass normalized random interface loads $W_{FF}(f)/M_o^2 W_{AA}(f) = C^2$ are limited by a constant C^2 and M_o is the rigid body mass of the load. This force limited random acceleration specification was developed at NASA JPL³, in particular, T.D. Scharton [22–24].

Scharton proposed in [22] methods of estimating the factor C^2 using two different simplified two-coupled SDOF system, denoted by the simple two degree of freedom system (STDFS) and the complex two degree of freedom system (CTDFS). The STDFS approach is the best known method of estimating C^2 .

In 2009 Nagahama of JAXA [19] presented a method of estimating C^2 based on enveloping the apparent masses of source and load.

Quite a number of papers about force limited vibration testing (FLVT) have been published in journals and conferences.

³Jet Propulsion Laboratory, California Institute of Technology

Three methods of estimating C^2 are discussed in this section. The methods of estimating the value of C^2 using the STDFS and CTDFS are well known [22–24], but the JAXA method is a relatively new alternative-and more complex-method where the dynamic mass of both load and source need to be known.

3.6.2. FORCE LIMIT VIBRATION TESTING BASED ON STDFS

INTRODUCTION

This section about the STDFS method is divided into two paragraphs; the first paragraph about the theoretical aspects and the second paragraph about parameter studies to investigate the sensitivities and to generate interesting figures and tables.

Theoretical Aspects The simple two degree of freedom system (STDFS) will be discussed in this section. The STDFS consists of a source and on top of the source a load. With aid of the STDFS method the values of C^2 will be estimated, which dependent on the ratio of the masses of load and source.

In Fig. 3.2 the TDFS and the associated SDOF system (load) are illustrated and both systems are excited at the base to random enforced accelerations $\ddot{U}(t)$ and $\ddot{U}^*(t)$ respectively.

The equations of motion of the TDFS are given by

$$\begin{aligned} m_1 \ddot{X}_1 + c_1(\dot{X}_1 - \dot{U}) + c_2(\dot{X}_1 - \dot{X}_2) + k_1(X_1 - U) + k_2(X_1 - X_2) &= 0, \\ m_2 \ddot{X}_2 + c_2(\dot{X}_1 - \dot{X}_2) + k_2(X_2 - X_1) &= 0, \end{aligned} \quad (3.30)$$

and the equation of motion of the SDOF system

$$m_2 \ddot{X}_2 + c_2(\dot{X}_2 - \dot{U}^*) + k_2(X_2 - U^*) = 0. \quad (3.31)$$

Equations (3.30) may be rewritten as

$$\begin{aligned} \ddot{X}_1 + (2\zeta_1\omega_1 + 2\zeta_2\omega_2)\dot{X}_1 + (\omega_1^2 + \omega_2^2\mu)X_1 - 2\zeta_2\omega_2\mu\dot{X}_2 - \omega_2^2\mu X_2 &= 2\zeta_1\omega_1\dot{U} + \omega_1^2U, \\ \ddot{X}_2 + 2\zeta_2\omega_2\dot{X}_2 + \omega_2^2X_2 - 2\zeta_2\omega_2\dot{X}_1 - \omega_2^2X_1 &= 0, \end{aligned} \quad (3.32)$$

where ω_1^2 and ω_2^2 are the clamped natural frequencies and ζ_1 and ζ_2 are the modal damping ratios of the source and load respectively. The two natural frequencies $\tilde{\omega}_1, \tilde{\omega}_2$ of the STDFS can be obtained by the following characteristic equation

$$\left(\frac{\tilde{\omega}}{\omega_1} \right)^2 = \frac{1}{2} \left[\phi^2 + \mu\phi + 1 \pm \sqrt{(4\mu + 1)\phi^4 - 2\mu\phi^3 + (\mu^2 - 2)\phi^2 + 2\mu\phi + 1} \right]. \quad (3.33)$$

The notations μ , ϕ , ω_1^2 and ω_2^2 are explained in Table 3.1.

In the frequency domain the responses $\ddot{X}_1(\omega)$ and $\ddot{X}_2(\omega)$ are as follows

$$\left\{ \frac{\ddot{X}_1(\omega)}{\ddot{U}(\omega)} \right\} = H_1(\omega), \left\{ \frac{\ddot{X}_2(\omega)}{\ddot{U}(\omega)} \right\} = H_2(\omega), \quad (3.34)$$

where H_1 and H_2 are the frequency response functions with $r = \omega/\omega_1 = f/f_1$, and are given in [5]

$$\begin{aligned} H_1(r) &= \frac{(-r^2 + 2jr\zeta_2\phi + \phi^2)(2jr\zeta_1 + 1)}{|D|}, \\ H_2(r) &= \frac{(2jr\zeta_2\phi + \phi^2)(2jr\zeta_1 + 1)}{|D|}, \\ |D| &= (-r^2 + (1 + \phi^2\mu) + jr(2\zeta_1 + 2\zeta_2\phi\mu))(-r^2 + 2jr\zeta_2\phi + \phi^2) - \\ &\quad \mu(2jr\zeta_2\phi + \phi^2)^2. \end{aligned} \quad (3.35)$$

The equation of motion of the SDOF system, shown in Fig. 3.2, is given by

$$\ddot{X}_2 + 2\zeta_2\omega_2\dot{X}_2 + \omega_2^2 X_2 = 2\zeta_2\omega_2\dot{U}^* + \omega_2^2 U^*. \quad (3.36)$$

The acceleration response $\ddot{X}_2(\omega)$ of that SDOF system can now be expressed as follows

$$\left\{ \frac{\ddot{X}_2(\omega)}{\dot{U}^*(\omega)} \right\} = \frac{2j\bar{r}\zeta_2 + 1}{1 - \bar{r}^2 + 2j\bar{r}\zeta_2}, \quad (3.37)$$

where $\bar{r} = \omega/\omega_2 = f/f_2$.

Parameter Studies The parameters in previous equations will be substituted by values to generate some interesting Figures and Tables.

The dynamic responses of the STDfs and the SDOF will be calculated with $m_1 = 1$, $\mu = m_2/m_1 = 0.01, 1$, $\omega_1 = \omega_2 = \omega_0$ ($\phi = 1$) and $Q_1 = Q_2 = 50$ as used by Scharton in [22]. The frequency response functions of the two degree of freedom system (TDFS) $|H_1|$, $|H_2|$ and the SDOF system $|H_2|$ are shown in Fig. 3.18 and 3.19 for $\mu = 0.01$ and $\mu = 1$, respectively.

The maximum amplitudes of $|H_1|$, $|H_2|$ are well separated due to the dynamic vibration absorber effects of the load. The anti-resonance notch is at the natural frequency of the load.

The interface force $F_{int}(f)$ of the SDOF system (Fig. 3.2) can be obtained by the following expression [27]

$$F_{int}(f) = m_2\ddot{U}^*(f) \left[1 + \frac{\bar{r}^2}{(1 - \bar{r}^2) + 2j\bar{r}\zeta_2} \right] = M_2(f)\ddot{U}^*(f), \quad (3.38)$$

where $M_2(f)$ is the dynamic mass or apparent mass. The normalized dynamic mass $M_2(f)/m_2$ of the SDOF system is the SDOF- m_2 (red curve) frequency response function shown in Fig. 3.18 and 3.19.

The normalized power spectral density function of the interface force $W_{FF}(f)$, for both the STDfs and SDOF system, is given by

$$\frac{W_F(f)}{m_2^2 W_{\ddot{X}_1}(f)} = \frac{W_{FF}(f)}{m_2^2 W_{\dot{U}^*}(f)} = \frac{W_{FF}(f)}{m_2^2 W_{AA}(f)} = \frac{1 + \frac{\bar{r}^2}{Q_2^2}}{(1 - \bar{r}^2)^2 + \frac{\bar{r}^2}{Q_2^2}}. \quad (3.39)$$

Figure 3.18: Frequency response function $\mu = 0.01$

where $\bar{r} = f/f_2$ for the SDOF system and $r = f/f_1 = \bar{r}\phi$ for the TDFS. In case $\phi = 1$ the frequency ratios are equal $r = \bar{r}$. The amplification factor is $Q_2 = 1/2\zeta_2$.

For the STDFS, the maximum response of the load and therefore the maximum interface force $F_{int}(f)$ (Fig. 3.2) occurs when $\phi = f_2/f_1 = 1$, thus $f_1 = f_2 = f_0$. The solution of the characteristic equation Eq. (3.33) then is with $\mu = m_2/m_1$

$$\left(\frac{\tilde{\omega}_i}{\omega_0}\right)_{\phi=1}^2 = \left(\frac{\tilde{f}_i}{f_0}\right)_{\phi=1}^2 = 1 + \frac{\mu}{2} \pm \sqrt{\mu + \frac{\mu^2}{4}}, \quad i = 1, 2. \quad (3.40)$$

The derivation of the force limits using the STDFS (source and load) is based on the frequency shift method, which is discussed in a paper by T.D. Scharton [22]. Applying the frequency shift method the maximum forces in the coupled system (Fig. 3.2) are calculated by evaluating the normalized power spectral density of the interface force $W_{FF}(f)$ (Eq. 3.39) of the coupled system, or shifted natural frequencies $(\tilde{f}_i/f_0)^2$, $i = 1, 2$, from Eq. (3.40) instead of its peak at the uncoupled load (SDOF) natural frequency $f_2 = f_0$. The ratio of the STDFS maximum interface force to the force in a conventional random vibration test (SDOF), the so-called "knock down" factor, is equal to the ratio of the value of the dynamic mass of the load at the shifted frequencies to the peak value at the uncoupled load natural frequency.

The lowest root of Eq. (3.40) is used in Eq. (3.39) to calculate the normalized interface power spectral density $W_{FF}(\tilde{f}_1/f_0)$, since it results in the maximum value of the interface force. The maximum normalized power spectral density of the interface force is shown

Figure 3.19: Frequency response function $\mu = 1$

in Fig. 3.20. The mass ratio varies between $\mu = 0.0001, \dots, 2$ and the amplification factor varies between $Q_2 = 5, 10, 25, 50$. The curves in Fig. 3.20 can be expressed (similar to Eq. (3.39)) in the following way

$$\max \frac{W_{FF}(\tilde{f})}{m_2^2 W_{AA}(\tilde{f})} = C^2. \quad (3.41)$$

If $\mu \rightarrow 0$ the normalized power spectral density of the interface force

$$\lim_{\mu \rightarrow 0} W_{FF} / m_2^2 W_{AA} = Q_2^2, \quad (3.42)$$

and if $\mu \rightarrow \infty$

$$\lim_{\mu \rightarrow \infty} W_{FF} / m_2^2 W_{AA} = 1. \quad (3.43)$$

The power spectral density function of normalized interface force $W_{FF} / m_2^2 W_{AA}$ is calculated for a number of discrete values of $\mu = 0.01, 0.1, 1, 2$, where the ordinate is r , $\phi = 1$ and the amplification factor $Q_1 = Q_2 = Q = 10$. The force limits proposed by Scharton [22] are also drawn in Fig. 3.21 and are given by:

$$\begin{aligned} W_{FF} &= C^2 m_2^2 W_{AA}, \quad f \leq f_0, \\ W_{FF} &= C^2 m_2^2 W_{AA} \left(\frac{f}{f_0} \right)^{-n}, \quad f > f_0 \end{aligned} \quad (3.44)$$

Figure 3.20: Maximum normalized power spectral density of interface force $W_{FF}/m_2^2 W_{AA} = C^2$

The maximum values of $W_{FF}/m_2^2 W_{AA}$ are calculated at the lowest natural frequency of the TDFS for a varying number of mass ratios μ and amplification factors and presented in Table 3.2. The values of $C_{max}^2 = C^2$ computed for $Q = 5, 10, 25, 50$, and $\mu = 0.0001, 0.001, 0.01, 0.1, 1, 2$ can be traced in Fig. 3.20.

Table 3.2: Normalized power spectral density of interface force

μ	C_{max}^2			
	Q=5	Q=10	Q=25	Q=50
0.0001	26.2	101	626	2481
0.001	26.2	101	487	775
0.01	24.6	59.7	95.1	106
0.1	9.64	12.3	13.4	13.6
1	2.53	2.59	2.62	2.62
2	1.84	1.86	1.87	1.87

Another method of estimating the value of C^2 is published by Ceresetti in [2] applying the STDFS, but is less known than the Scharton frequency shift method [22].

The power spectral density of the TDFS responses $W_{\ddot{x}_1}(f)$ and $W_{\ddot{x}_2}(f)$ can be calcu-

Figure 3.21: Normalized power spectral density interface force $W_{FF}/m_2^2 W_{AA}$

lated

$$\begin{aligned} W_{\ddot{X}_1}(f) &= |H_1(f)|^2 W_{\ddot{U}}(f), \\ W_{\ddot{X}_2}(f) &= |H_2(f)|^2 W_{\ddot{U}}(f). \end{aligned} \quad (3.45)$$

The mean square values of the acceleration responses $\ddot{X}_1$ and $\ddot{X}_2$, $\langle \ddot{X}_1^2 \rangle = E(\ddot{X}_1^2)$ and $\langle \ddot{X}_2^2 \rangle = E(\ddot{X}_2^2)$ respectively, are

$$\begin{aligned} E\{\ddot{X}_1^2\} &= \int_0^\infty W_{\ddot{X}_1}(f) df = \int_0^\infty |H_1(f)|^2 W_{\ddot{U}}(f) df \approx W_{\ddot{U}} \int_0^\infty |H_1(f)|^2 df, \\ E\{\ddot{X}_2^2\} &= \int_0^\infty W_{\ddot{X}_2}(f) df = \int_0^\infty |H_2(f)|^2 W_{\ddot{U}}(f) df \approx W_{\ddot{U}} \int_0^\infty |H_2(f)|^2 df, \end{aligned} \quad (3.46)$$

where $W_{\ddot{U}}$ is a white noise process. The white noise solutions of the mean square ensemble averages $E\{\ddot{X}_1^2\}$ and $E\{\ddot{X}_2^2\}$ are given in [4], page 84. Ceresetti defines in [2] the feedback factor Q_{eff} , which is given by

$$Q_{eff}^2 = \frac{E\{\ddot{X}_2^2\}}{E\{\ddot{X}_1^2\}}. \quad (3.47)$$

and in [22] it is stated that

$$\frac{W_{FF}}{m_2^2 W_{AA}} \approx Q_{eff}^2 \Big|_{\phi=1} = C^2. \quad (3.48)$$

The normalized power spectral density of the interface force, based on Q_{eff}^2 , is shown in Fig. 3.22. The frequency ratio $\phi = 1$, the mass ratio varies between $\mu = 0.0001, \dots, 2$ and the amplification factor varies between $Q_1 = Q_2 = Q = 5, 10, 25, 50$. In the limit case when

Figure 3.22: Normalized power spectral density of interface force $W_F/m_2^2 W_A$

$\mu \rightarrow 0$ the square of the feedback factor Q_{eff}^2 approaches value [17]

$$\lim_{\mu \rightarrow 0} Q_{eff}^2 \Big|_{\phi=1} = \frac{Q^2}{2}, \quad (3.49)$$

and if $\mu \rightarrow \infty$

$$\lim_{\mu \rightarrow \infty} Q_{eff}^2 \Big|_{\phi=1} = 1. \quad (3.50)$$

For small values of μ the calculated values of the normalized power spectral density of the interface forces obtained by Eq. (3.39) (frequency shift method) in combination with the lowest natural frequency of the coupled TDFS are a factor two higher than the calculated values of the normalized power spectral density of the interface force calculated by Eq. (3.48). The values of $W_{FF}/m_2^2 W_{AA} = Q_{eff}^2 \Big|_{\phi=1} = C^2$ are calculated at the lowest

natural frequency of the TDFS for a varying number of mass ratios μ and amplification factors and presented in Table 3.3. The values C^2 are computed for $Q = 5, 10, 25, 50$, and $\mu = 0.0001, 0.001, 0.01, 0.1, 1, 2$ can be traced in Fig. 3.22. Comparison of the values of C^2

Table 3.3: Normalized power spectral density of interface force

μ	$C^2 = Q_{eff}^2 _{\phi=1}$			
	Q=5	Q=10	Q=25	Q=50
0.0001	13.46	50.7	304	1112
0.001	13.33	48.6	239	556
0.01	12.14	34.5	77.1	94.1
0.1	6.79	9.72	11.1	11.4
1	2.21	2.30	2.33	2.33
2	1.71	1.74	1.78	1.75

in the Table 3.2 and 3.3 shows that in the higher regions of the mass ratio μ the computed values of C^2 are about the same.

The power n of the roll off ratio $(f/f_0)^n$ is dependent on the apparent mass of the load $M_2(r)$ given by Eq. (3.39)

$$\frac{W_{FF}(r)}{M_0^2 W_{AA}(r)} = \frac{M_2^2(r)}{M_0^2} = \frac{1 + \frac{r^2}{Q_2^2}}{(1 - r^2)^2 + \frac{r^2}{Q_2^2}},$$

where $r = f/f_0$. For small values of $r \rightarrow 0$

$$\lim_{r \rightarrow 0} \frac{W_{FF}(r)}{M_0^2 W_{AA}(r)} \rightarrow 1, \quad (3.51)$$

and for large values of $r \rightarrow \infty$

$$\lim_{r \rightarrow \infty} \frac{W_{FF}(r)}{M_0^2 W_{AA}(r)} \rightarrow \frac{1}{r^2} = \left(\frac{f_0}{f} \right)^2, \quad (3.52)$$

thus $n = 2$. In the NASA Handbook [24] for force limited random vibration testing $n = 2$ (-6dB/oct) is proposed.

3.6.3. COMPLEX TWO-DEGREE OF FREEDOM SYSTEM (CTDFS)

In Fig. 3.23 the so-called complex two-SDOF system (CTDFS) of the source and load is illustrated. This more advanced CTDFS can also be used to estimate the force limits. The capital M represents the residual mass, and the small m^* represents the modal effective mass. The dampers and the springs are given by c^* and k^* respectively. The following theoretical discussion is based on [6]. When $F_{ext} = 0$ is assumed the following equations of motion can be derived

$$\begin{aligned} (M_1 + M_2)\ddot{x} - k_1^*(x_1 - x) - k_2^*(x_2 - x) - c_1^*(\dot{x}_1 - \dot{x}) - c_2^*(\dot{x}_2 - \dot{x}) &= 0, \\ m_1^*\ddot{x}_1 + k_1^*(x_1 - x) + c_1^*(\dot{x}_1 - \dot{x}) &= F_{ext} = 0, \\ m_2^*\ddot{x}_2 + k_2^*(x_2 - x) + c_2^*(\dot{x}_2 - \dot{x}) &= 0. \end{aligned} \quad (3.53)$$

Figure 3.23: Modal effective and residual mass two-SDOF source-load system

3

The first equation of motion in Eq. (3.53) will be decomposed into two symmetric SDOF systems:

$$\begin{aligned} [M_1 \ddot{x} - c_1^* (\dot{x}_1 - \dot{x}) - k_1^* (x_1 - x)] + [M_2 \ddot{x} - k_2^* (x_2 - x) - c_2^* (\dot{x}_2 - \dot{x})] &= 0, \\ F_{source} + F_{load} = F_{int} - F_{int} &= 0, \end{aligned} \quad (3.54)$$

where F_{int} is the interface force between residual masses M_1 and M_2 . The interface force can be expressed as follows using Eq. (3.53)

$$\begin{aligned} F_{int} &= M_2 \ddot{x} - k_2^* (x_2 - x) - c_2^* (\dot{x}_2 - \dot{x}), \\ F_{int} &= M_2 \ddot{x} + m_2^* \ddot{x}_2. \end{aligned} \quad (3.55)$$

Using the third equation of motion, in Eq. (3.53) the acceleration $\ddot{X}_2(\omega)$ can be expressed in the frequency domain in terms of the interface acceleration $\ddot{X}(\omega)$

$$\ddot{X}_2(\omega) = \left(\frac{\omega_2^2 + 2j\zeta_2\omega\omega_2}{-\omega^2 + \omega_2^2 + 2j\zeta_2\omega\omega_2} \right) \ddot{X}(\omega), \quad (3.56)$$

where the square of natural frequency is $\omega_2^2 = k_2^*/m_2^*$, the viscous damping is $c_2^* = 2\zeta_2\omega_2 m_2^*$, and the damping ratio is $\zeta_2 = c_2^*/2\sqrt{k_2^* m_2^*}$. Substituting Eq. (3.56) into Eq. (3.55) gives the interface force $F_{int}(\omega)$:

$$F_{int}(\omega) = \left(M_2 + m_2^* \frac{\omega_2^2 + 2j\zeta_2\omega\omega_2}{-\omega^2 + \omega_2^2 + 2j\zeta_2\omega\omega_2} \right) \ddot{X}(\omega) = \mathbf{M}_2(\omega) \ddot{X}(\omega). \quad (3.57)$$

The frequency response function $\mathbf{M}_2(\omega)$ is called the load dynamic mass, and

$$\begin{aligned} |\mathbf{M}_2(\omega)|^2 &= M_2^2 \frac{[(1 - \beta_2^2) + \alpha_2]^2 + \frac{\beta_2^2(1 + \alpha_2)^2}{Q_2^2}}{(1 - \beta_2^2)^2 + \frac{\beta_2^2}{Q_2^2}} \\ &= M_2^2 \frac{[(1 - \beta_2^2) + \alpha_2]^2 + 4\zeta_2^2 \beta_2^2 (1 + \alpha_2)^2}{(1 - \beta_2^2)^2 + 4\zeta_2^2 \beta_2^2}, \end{aligned} \quad (3.58)$$

where $\alpha_2 = m_2^*/M_2$ ($\alpha_1 = m_1^*/M_1$), $\beta_2 = \omega/\omega_2$ ($\beta_1 = \omega/\omega_1$), and $Q_2 = 1/2\zeta_2$ ($Q_1 = 1/2\zeta_1$). Because of the symmetry in Eq. (3.54) the expression for the source dynamic mass $\mathbf{M}_1(\omega)$ will have exactly the same form as Eq. (3.58) but all indices 2 must be replaced by 1. Eq. (3.58) must be evaluated for the system's natural frequencies to determine the general force limits.

The governing matrix equations of motion of the dynamic system illustrated in Fig. 3.23 are

$$\begin{pmatrix} M_1 + M_2 & 0 & 0 \\ 0 & m_1^* & 0 \\ 0 & 0 & m_2^* \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} c_1^* + c_2^* & -c_1^* & -c_2^* \\ -c_1^* & c_1^* & 0 \\ -c_2^* & 0 & c_2^* \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} k_1^* + k_2^* & -k_1^* & -k_2^* \\ -k_1^* & k_1^* & 0 \\ -k_2^* & 0 & k_2^* \end{pmatrix} \begin{pmatrix} x \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (3.59)$$

The characteristic equation for the undamped eigenvalue problem is

$$\begin{vmatrix} k_1^* + k_2^* - \omega^2(M_1 + M_2) & -k_1^* & -k_2^* \\ -k_1^* & k_1^* - \omega^2 m_1^* & 0 \\ -k_2^* & 0 & k_2^* - \omega^2 m_2^* \end{vmatrix} = 0, \quad (3.60)$$

which yields

$$\begin{aligned} \omega^2 [(M_1 + M_2)(k_1^* - \omega^2 m_1^*)(k_2^* - \omega^2 m_2^*) \\ + k_2^* m_2^*(k_1^* - \omega^2 m_1^*) + k_1^* m_1^*(k_2^* - \omega^2 m_2^*)] = 0. \end{aligned} \quad (3.61)$$

Eq. (3.61) has one root $\omega = 0$ corresponding to the rigid body motion. The remaining two roots are given by

$$\begin{aligned} (1 - \phi^2 \beta_2^2)(1 - \beta_2^2) + \alpha_1(1 - \beta_2^2) \\ + \mu(1 - \phi^2 \beta_2^2)(1 - \beta_2^2) + \mu\alpha_2(1 - \phi^2 \beta_2^2) = 0, \end{aligned} \quad (3.62)$$

where $\phi = \omega_2/\omega_1$, $\beta_1 = \phi\beta_2$ and $\mu = M_2/M_1$. Eq. (3.62) has two real roots, β_+ and β_- , corresponding to the upper and lower natural frequencies. ϕ is called the *tuning parameter*. The natural frequencies β_2^2 can be obtained as follows [18, 22]

$$\beta_2^2 = \frac{1}{2} \left[-B \pm \sqrt{B^2 - 4C} \right], \quad (3.63)$$

where

$$\begin{aligned} B &= \frac{-(1 + \mu + \alpha_1)/\phi^2 - (1 + \mu + \mu\alpha_2)}{1 + \mu}, \\ C &= \frac{(1 + \mu + \alpha_1 + \mu\alpha_2)}{(1 + \mu)\phi^2}. \end{aligned}$$

At this point, the development of force limits for the CTDFS model becomes more complicated than for the STDFS. In the case of the STDFS it was shown that the interface force and the interface acceleration peak are at the same system natural frequencies

($\omega_1 = \omega_2$). This condition does not always hold for the CTDFS: the maximum interface force may occur at the upper natural frequency while the maximum interface acceleration may occur at the lower natural frequency. Consequently, the natural frequencies at which the respective maxima occur must be identified. The bounding value of the load dynamic mass will then be the ratio of the larger of the two interface forces to the larger of the two interface accelerations. This task is further complicated by the fact that the maximum magnitude of the interface acceleration is dependent upon how the CTDFS is excited.

In [22] Scharton chose an external force $F_0(\omega)$ that is constant over the system's frequency band and applied to the source modal effective mass m_1^* . Expressions for the peak interface acceleration will now be derived based in constant source force excitation.

Figure 3.24: Free acceleration $A_0(\omega)$, interface acceleration $A(\omega)$

Fig. 3.24 can be used to derive an expression relating the free $A_0(\omega)$ acceleration to the interface acceleration $A(\omega)$ for the source residual mass M_1 . The acceleration of both residual masses M_1 and M_2 is given by

$$F_0(\omega) = A(\omega)(\mathbf{M}_1(\omega) + \mathbf{M}_2(\omega)), \quad (3.64)$$

where the dynamic masses $\mathbf{M}_1(\omega), \mathbf{M}_2(\omega)$ (Eq. 3.57) are placed in series and keeping in mind that for the source the free body acceleration $A_0(\omega)$ is given by

$$F_0(\omega) = A_0(\omega)\mathbf{M}_1(\omega), \quad (3.65)$$

The force $F_0(\omega)$ can now be eliminated from Eq. (3.64) and Eq. (3.65)

$$\frac{A(\omega)}{A_0(\omega)} = \frac{\mathbf{M}_1(\omega)}{\mathbf{M}_1(\omega) + \mathbf{M}_2(\omega)}. \quad (3.66)$$

The expressions of dynamic mass in Eq. (3.57) can be substituted in Eq. (3.66), leading to a rather involved algebraic expression for the ratio of the interface to the free acceleration, which will be simply denoted by

$$\frac{A(\omega)}{A_0(\omega)} = D(\alpha_1, \alpha_2, \mu, \beta_1, \beta_2, \zeta_1, \zeta_2). \quad (3.67)$$

The modified expressions for the dynamic masses $\mathbf{M}_1(\omega)$ and $\mathbf{M}_2(\omega)$ are given by

$$\begin{aligned}\mathbf{M}_1(\omega) &= M_1 \left[1 + \alpha_1 \frac{\omega_1^2 + 2j\zeta_1\omega\omega_1}{-\omega^2 + \omega_1^2 + 2j\zeta_1\omega\omega_1} \right] = M_1 \left[1 + \alpha_1 \frac{1 + 2j\zeta_1\beta_1}{1 - \beta_1^2 + 2j\zeta_1\beta_1} \right], \\ \mathbf{M}_2(\omega) &= M_2 \left[1 + \alpha_2 \frac{\omega_2^2 + 2j\zeta_2\omega\omega_2}{-\omega^2 + \omega_2^2 + 2j\zeta_2\omega\omega_2} \right] = M_2 \left[1 + \alpha_2 \frac{1 + 2j\zeta_2\beta_2}{1 - \beta_2^2 + 2j\zeta_2\beta_2} \right].\end{aligned}\quad (3.68)$$

The equations of motion Eq. (3.53) can be used to derive an expression for the free acceleration of the source residual mass M_1 . The equilibrium of the external force, applied at the source, with the modal effective mass m_1^* is given by

$$m_1^* \ddot{x}_1 + c_1^* (\dot{x}_1 - \dot{x}) + k_1^* (x_1 - x) = F_{ext}, \quad (3.69)$$

where the variables and parameters are shown in Fig. 3.23. The free body equilibrium of the source residual mass M_1 is

$$M_1 \ddot{x} - c_1^* (\dot{x}_1 - \dot{x}) - k_1^* (x_1 - x) = 0. \quad (3.70)$$

Adding Eq. (3.69) and Eq. (3.70) the damping and stiffness terms can be eliminated:

$$F_{ext}(\omega) = M_1 \ddot{X}(\omega) + m_1^* \ddot{X}_1(\omega). \quad (3.71)$$

The acceleration $\ddot{X}(\omega)$ is in fact the free acceleration $A_0(\omega)$. The acceleration $\ddot{X}_1$ can be expressed in terms of $\ddot{X}$, and (Eq. 3.71) can be written as

$$F_{ext}(\omega) = \left[m_1^* \frac{-\frac{\omega^2}{\alpha_1} + 2j\zeta_1\omega_1\omega + \omega_1^2}{2j\zeta_1\omega_1\omega + \omega_1^2} + M_1 \right] \ddot{X}(\omega), \quad (3.72)$$

or

$$\frac{F_{ext}(\omega)}{m_1^*} = \left[\frac{1}{\alpha_1} + \frac{1 - \frac{\beta_1^2}{\alpha_1} + 2j\zeta_1\beta_1}{1 + 2j\zeta_1\beta_1} \right] \ddot{X}(\omega). \quad (3.73)$$

Using $\ddot{X}(\omega) = A_0(\omega)$ the following expression can be obtained

$$\frac{A_0(\omega)}{\frac{F_{ext}(\omega)}{m_1^*}} = \frac{\alpha_1(1 + 2j\zeta_1\beta_1)}{2j\zeta_1\beta_1(1 + \alpha_1) - \beta_1^2 + \alpha_1 + 1} = G(\alpha_1, \beta_1, \zeta_1), \quad (3.74)$$

and [23]

$$\left| \frac{A_0(\omega)}{\frac{F_{ext}(\omega)}{m_1^*}} \right|^2 = \frac{\beta_1^4(1 + 4\beta_1^2\zeta_1^2)}{\left[(1 - \beta_1^2) \left(1 - \frac{\beta_1^2}{\alpha_1} \right) - 1 \right]^2 + 4\beta_1^6\zeta_1^2 \left(1 + \frac{1}{\alpha_1} \right)^2} = |G(\alpha_1, \beta_1, \zeta_1)|^2. \quad (3.75)$$

The acceleration A_0 can be eliminated from Eq. (3.67) and Eq. (3.75), thus

$$\frac{A(\omega)}{\frac{F_{ext}(\omega)}{m_1^*}} = D(\alpha_1, \alpha_2, \mu, \beta_1, \beta_2, \zeta_1, \zeta_2) G(\alpha_1, \beta_1, \zeta_1) \quad (3.76)$$

The external force $F_{ext}(\omega)$ will be held constant over the frequency band of interest, thus Eq. (3.76) can be evaluated from Eq. (3.62) at the upper and lower system natural frequencies β_+ and β_- to yield the ratio of the corresponding system acceleration peaks

$$\frac{A_+}{A_-} = \frac{D(\alpha_1, \alpha_2, \mu, \beta_{1+}, \beta_{2+}, \zeta_1, \zeta_2) G(\alpha_1, \beta_{1+}, \zeta_1)}{D(\alpha_1, \alpha_2, \mu, \beta_{1-}, \beta_{2-}, \zeta_1, \zeta_2) G(\alpha_1, \beta_{1-}, \zeta_1)}. \quad (3.77)$$

The damping ratio of the source and the load are equal, $\zeta_1 = \zeta_2 = \zeta$. β_1 will be eliminated, and $\phi = \frac{\omega_2}{\omega_1} = \frac{\beta_1}{\beta_2}$, and Eq. can be expressed as Eq. (3.77) as

$$\left| \frac{A_+}{A_-} \right| = |H(\alpha_1, \alpha_2, \mu, \phi, \beta_{2\pm}, \zeta)|. \quad (3.78)$$

The purpose of knowing the ratio of the resonant interface accelerations $|A_+ / A_-|$ is to properly identify the positions of the force and accelerations maxima; the proper "load" dynamic mass will be the ratio of the larger of the two interface forces to the larger of the two interface accelerations, $\mathbf{M}_2 = F_{max} / A_{max}$. Table 3.4 summarizes the possible out-

Table 3.4: Possible ratio's of the maximum force F_{max} to maximum acceleration A_{max}

	Position A_{max}, F_{max}			
	A_-, F_-	A_-, F_+	A_+, F_-	A_+, F_+
F_{max}	$\mathbf{M}_2 - A_-$	$\mathbf{M}_{2+} A_+$	$\mathbf{M}_2 - A_-$	$\mathbf{M}_{2+} A_+$
$\frac{F_{max}}{A_{max}}$	$\frac{\mathbf{M}_2 - A_-}{A_-} = \mathbf{M}_{2-}$	$\frac{\mathbf{M}_{2+} A_+}{A_-}$	$\frac{\mathbf{M}_2 - A_-}{A_+}$	$\frac{\mathbf{M}_{2+} A_+}{A_+} = \mathbf{M}_{2+}$

comes of this ratio for all combinations of natural frequency locations of the force and the acceleration maxima, where the + and the - denote the higher and lower natural frequencies respectively. The last row in Table 3.4 shows the proper force to acceleration ratio will be the maximum of the load dynamic mass $\mathbf{M}_2$ evaluated at either the higher acceleration peak or the lower acceleration peak scaled by the ratio of the lower to higher acceleration. Symbolically, this logic can be expressed as

```

if  $|A_+| > |A_-|$  then
     $|\tilde{\mathbf{M}}_2(\beta_2)| = \max \left\{ |\mathbf{M}_2(\beta_{2+})|, |\mathbf{M}_2(\beta_{2-})| \left| \frac{A_-}{A_+} \right| \right\}$ 
else if  $|A_+| < |A_-|$  then
     $|\tilde{\mathbf{M}}_2(\beta_2)| = \max \left\{ |\mathbf{M}_2(\beta_{2-})|, |\mathbf{M}_2(\beta_{2+})| \left| \frac{A_+}{A_-} \right| \right\}$ 
end if

```

Where $\tilde{\mathbf{M}}_2(\beta) = \mathbf{M}_2(\beta) / M_2$ is the scaled apparent or dynamic mass. It is clear that both $\beta_{\pm}$ and $A_{\pm}$ depend on the tuning parameter $\phi = \omega_2 / \omega_1$. For the STDFS the tuning parameter $\phi = 1$ always yields the maximum value for the "load" dynamic mass unfortunately, this is not generally true for the CTDFS. Consequently, $|\tilde{\mathbf{M}}_2(\omega)|$, in the algorithm above must be maximized by numerically tuning ϕ for all possible combinations of mass, stiffness, and damping for the CTDFS. This procedure is summarized as follows

1. Start procedure

2. Pick $\alpha_1 = m_1^*/M_1$, $\alpha_2 = m_2^*/M_2$ and $\mu = M_2/M_1$, $M_1 = 1$
3. Solve characteristic equation for $\beta_{2\pm}$ ($\beta_2 = \omega/\omega_2$)
4. For a given ϕ , determine $\beta_{1\pm}$ ($\beta_1 = \omega/\omega_1 = \phi\beta_2$)
5. Evaluate the dynamic masses $M_1(\beta_{2\pm})$, $M_2(\beta_{2\pm})$
6. Evaluate $|A_+/A_-|$,
7. Determine F_{max} from $A_{\pm}$ and scaled $M_2(\beta_{2\pm})$
8. if $F_{max,new} < F_{max,old}$ then
Pick new ϕ , goto 4
else
Save $F_{max,new}$, goto 2
end if
9. End of procedure

The magnitude squared of the load dynamic mass $\mathbf{M}_2(\omega)$ is given by Eq. (3.58). The calculation of the random force limit $W_{FF}(f)$ normalized by the "load" mass M_2 and random acceleration $W_{AA}(f)$ is now

$$\frac{W_{FF}(f)}{M_2^2 W_{AA}(f)} = |\tilde{\mathbf{M}}_2|_{max}^2 = C^2. \quad (3.79)$$

Analysis results of the maximum normalized interface force PSD are discussed and reported in [23]. In Fig. 3.25 values for C^2 are shown assuming equal amplification factors $Q_1, Q_2 = Q = 10$. ϕ^2 is varied in steps of 1/16 from 8/16 to 32/16.

In general, the relationship between the normalized force PSD $W_{FF}(f)$ and its controlling input parameters is more complicated for the CTDFS system model than for the simple two-SDOF model. For smaller values of $\mu = M_2/M_1$, the CTDFS model predicted force limits are larger for $\alpha_1 = \alpha_2 = 1$ ($\alpha = m^*/M$) and smaller at $\alpha_1 = \alpha_2 = 0.25$ than are their STDFS counterparts. At larger values for μ , the CTDFS estimated force limits are larger than the estimation made with the STDFS method.

In the limiting case of the load modal effective mass equal to zero (i.e. $\alpha_2 = 0$), the normalized force PSD reduces to 1.

In practice, to implement the above force limits the modal effective mass and residual mass for both the source and the load must be known as a function of frequency.

3.6.4. JAXA'S FORCE LIMITS ESTIMATION METHOD

In a paper presented at the 11th ECSSMMT (2009), Toulouse, France, JAXA⁴ [19] an additional method of estimating force limits was proposed and discussed. This JAXA method is based on enveloping combinations of source and load apparent masses.

The basic structural model consists of a source and the load and is illustrated in Fig. 3.26. This model is quite similar to CTDFS model as shown in Fig. 3.24.

⁴JAXA is Japan Aerospace Exploration Agency

Figure 3.25: CTDFS computed C^2 , $Q=10$

When the source is uncoupled and excited by a force F_B , the response acceleration A^0 , at the location of the interface to the load, is given by

$$A^0 = \frac{F_B}{\mathbf{M}_i^s}, \quad (3.80)$$

where $\mathbf{M}_i^s$ is the apparent mass between force F_B and acceleration A^0 , indicated with a dotted arrow in Fig. 3.26.

The response acceleration A_{int} at the interface is equal for both source and load, while the interface force F_{int} is equal in magnitude but opposite in direction. Thus, when the connected source/load model is excited by the excitation force F_B the interface acceleration A_{int} related to the source can be expressed as follows

$$A_{int} = \frac{F_B}{\mathbf{M}_i^s} - \frac{F_{int}}{\mathbf{M}^s}, \quad (3.81)$$

while the interface acceleration A_{int} , related to the load can be expressed as

$$A_{int} = \frac{F_{int}}{\mathbf{M}^l}, \quad (3.82)$$

where $\mathbf{M}^s$ and $\mathbf{M}^l$ are the apparent masses at the interface of the source structure and load structure, respectively. Both F_{int} and A_{int} can be expressed in the free interface

Figure 3.26: The structural models of source and load [19]

response acceleration A^0 by equating Eq. (3.81) and Eq. (3.82) and applying Eq. (3.80)

$$F_{int} = \frac{\mathbf{M}^s \mathbf{M}^l}{\mathbf{M}^s + \mathbf{M}^l} A^0, \quad (3.83)$$

and

$$A_{int} = \frac{\mathbf{M}^s}{\mathbf{M}^s + \mathbf{M}^l} A^0. \quad (3.84)$$

The value of denominator $\mathbf{M}^s + \mathbf{M}^l$ in Eq. (3.83) and (3.84) becomes a minimum at the natural frequencies of the combined source load system $f_{ci}, i = 1, 2, \dots$. The interface force F_{int} and the interface acceleration have their peak values at the natural frequencies of the combined system.

When the enveloped values of F_{int} and A_{int} are denoted as F_{env} and A_{env} , those values are equal to the functions of the connected peak values of F_{int} and A_{int} at the natural frequency of the connected source/load model. The ratio of the F_{env} and the A_{env} is defined as the enveloped apparent mass M_{env} and can be expressed as follows

$$\mathbf{M}_{env} = \frac{F_{env}}{A_{env}} = \frac{\left(\frac{\mathbf{M}^s \mathbf{M}^l}{\mathbf{M}^s + \mathbf{M}^l} A^0 \right)_{env}}{\left(\frac{\mathbf{M}^s}{\mathbf{M}^s + \mathbf{M}^l} A^0 \right)_{env}}, \quad (3.85)$$

where F_{env} is the maximum interface force specification and A_{env} is defined as the acceleration specification for the vibration test of the load. This illustrated in Fig. 3.27.

The free interface acceleration A^0 in the denominator and numerator in Eq. (3.85) take the same values at natural frequencies of the combined source/load model. There-

Figure 3.27: Interface force and acceleration in connection of source and load [19]

for Eq. (3.85) can be changed into

$$F_{max} = M_{env} A_{spec} = \frac{\left| \frac{\mathbf{M}^s \mathbf{M}^l}{\mathbf{M}^s + \mathbf{M}^l} \right|_{env}}{\left| \frac{\mathbf{M}^s}{\mathbf{M}^s + \mathbf{M}^l} \right|_{env}} A_{spec}. \quad (3.86)$$

The maximum force limit of the source F_{max} can be calculated from (3.86), using the specified acceleration A_{spec} and the apparent masses $\mathbf{M}^s$ and $\mathbf{M}^l$ of source and load.

In case of a random vibration test Eq. (3.86) can be written

$$W_{FF}(f) = \left[\frac{\left| \frac{\mathbf{M}^s \mathbf{M}^l}{\mathbf{M}^s + \mathbf{M}^l} \right|_{env}}{\left| \frac{\mathbf{M}^s}{\mathbf{M}^s + \mathbf{M}^l} \right|_{env}} \right]^2 W_{AA}(f) = \mathbf{M}_{env}^2(f) W_{AA}(f). \quad (3.87)$$

Dividing $W_{FF}(f)$ in Eq. (3.87) by the rigid body mass of the load M_o , squared, and the specified random acceleration spectrum $W_{AA}(f)$ the following expression can be obtained

$$\frac{W_{FF}(f)}{M_o^2 W_{AA}(f)} = \frac{\mathbf{M}_{env}^2(f)}{M_o^2} = C^2(f), \quad (3.88)$$

where $C^2(f)$ is a function of the excitation frequency f . When the maximum value of $C^2(f)$ is taken similar results as for the STDFS and CTDFS can be obtained.

3.7. CONCLUSION/RECOMMENDATION

It is difficult to say which of the mentioned methods for defining adapted test specifications is the best approach. In practice response limited and force limited vibration test

methods are more or less equally applied in the spacecraft structure design and verification approach. A summary of the reviewed notching methods is given in Table 3.5.

Table 3.5: Summary of reviewed notching methods

#	Notching method	Reference(s)	Type	Notching
1	Curtis method	[5]	Response limiting	Primary & Secondary
2	Mass participation	[29]	3σ Force limiting	Primary
3	Enclosure method	CNES, [13]	3σ Force limiting	Primary
4	1/2 octave band	TAS-F, [13]	3σ Force limiting	Primary
5	STDFS	[24]	PSD force limiting	Primary
6	CTDFS	[6]	PSD force limiting	Primary
7	JAXA method	[19]	PSD force limiting	Primary

To compute the notch levels with the methods 1,2,3 and 4 the finite element model (FEM) of the load is needed to calculate modal and response characteristics. Applying method 2 only the natural frequencies and associated modal effective masses of the fixed load are required and therefor very straightforward to use.

The methods 2, 3 and 4 are strongly related to the specified quasi-static loads (QSL) factors.

The application of the methods 5 and 6 to estimate PSD force limits is very simple because the dynamic model of the load-source combination is represented by simplified two degree of freedom systems to estimate values of C^2 . The properties of the STDFS and CTDFS are approximated or derived from FE models and the associated values of C^2 can be extracted from Figures and Tables, e.g. [24]. The use of the STDFS is preferred use of the CTDFS.

The JAXA method is more complex to apply then the STDFS or CTDFS methods. For both the load and the source the dynamic masses at the interface are needed. The dynamic mass can be derived from the FEM of the load.

To perform PSD force limited vibration testing force gauges are needed between the load and the shaker table to measure the interface forces during the vibration test.

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4

FORCE LIMITED VIBRATION, THE SEMI-EMPIRICAL METHOD

4.1. INTRODUCTION

In this chapter the semi-empirical method (SEM) for force limited vibration testing will be discussed. The force limits $W_{FF}(f)$ can be established with the SEM to prevent over-testing of the test item (load) and to introduce the test levels as realistically as possible.

The key parameters of SEM are the semi-empirical constant C^2 , the fundamental natural frequency f_o of the load (fixed-free) together with associated dynamic properties, the rigid body mass M_o of the load and the random vibration test specification $W_{AA}(f)$ to be applied during vibration testing of the load. C^2 is a scale factor between the random acceleration and force at the interface load/source. The definition of C^2 is given in Eq. (3.41), where m_2 is the mass of the load. Except for the value C^2 all other parameters can be made available during the design of the load. The estimation of the value of C^2 is based on past experience of FLVT [16]. A literature review and overview of the values of C^2 used are given and discussed below.

4.2. SEMI-EMPIRICAL METHOD (SEM)

In this section the SEM will be discussed. Using SEM the constant C^2 has to be estimated from past experience or by analysis.

During random vibration the force spectral density $W_{FF}(f)$ at the interface between the load and the source can be expressed in terms of the apparent mass $\mathbf{M}_I(f)$ of the load and the acceleration spectral density $W_{AA}(f)$ at the same interface.

$$W_{FF}(f) = |\mathbf{M}_I(f)|^2 W_{AA}(f). \quad (4.1)$$

In general, the load can be idealized by a set of parallel placed single degree of freedom (SDOF) elements (asparagus patch model), where the discrete masses are the modal effective masses of the load fixed at the interface between load and source. The modal effective mass m_{iI} , $i = 1, 2, \dots, n$ is associated with the natural frequency f_i , $i = 1, 2, \dots, n$

Figure 4.1: Load/source combination with random acceleration and force in the interface

and vibration mode [20] of the load. The spring stiffnesses and the damper constants are related to modal effective masses, hence $\omega_i^2 m_{il}$, $i = 1, 2, \dots, n$ and $2\zeta_i \omega_i m_{il}$, $i = 1, 2, \dots, n$ respectively. The residual mass m_{rl} is associated with a natural frequency outside the frequency range of interest. The total mass of the load is denoted by M_o .

The apparent mass [17, 20] at the interface of the load can be obtained by the following expression

$$\mathbf{M}^l(f) = \sum_{i=1}^n m_{il}(f_i) \left[1 + \left(\frac{f}{f_i} \right)^2 H \left(\frac{f}{f_i} \right) \right] + m_{rl}, \quad (4.2)$$

where the frequency transfer function $H \left(\frac{f}{f_i} \right)$ of the acceleration of a SDOF system with respect to the base is given by

$$H \left(\frac{f}{f_i} \right) = \frac{1}{1 - \left(\frac{f}{f_i} \right)^2 + 2j\zeta_i \left(\frac{f}{f_i} \right)}. \quad (4.3)$$

The apparent mass function $\mathbf{M}_l(f)$ will now be analyzed considering three special intervals (parts) the damping controlled part, the mass controlled part and the stiffness controlled part.

- The damping controlled maximum value of the absolute value of the apparent mass at the natural frequency f_i with a dominant modal effective mass m_{il} is given by

$$|\mathbf{M}_l|_{\max}(f_i) \approx m_{il}Q < M_oQ, \quad (4.4)$$

where Q is the amplification factor at f_i .

- When the excitation frequency f is $f \ll f_i$ the asymptotic mass controlled value of the apparent mass is equal to the rigid body mass of the load

$$|\mathbf{M}_l(f)| \approx \sum_{i=1}^n m_{il}(f_i) + m_{rl} = M_o. \quad (4.5)$$

- If the excitation frequency f is $f \gg f_i$, where f_i is the natural frequency with associated dominant modal effective mass m_{il} , then the asymptotic stiffness controlled decrease of the apparent mass is given by

$$|\mathbf{M}_l(f)| \approx m_{il} Q \left(\frac{f_i}{f} \right) < M_o Q \left(\frac{f_i}{f} \right) \quad (4.6)$$

The asymptote of the magnitude of the load apparent mass is equal to the total mass below the first natural frequency (see Eq. 4.5) and falls off above the first natural frequency (see Eq. 4.6). The asymptotic load apparent mass is in accordance to Salter [10] illustrated in Fig. 4.2.

Figure 4.2: Asymptotic apparent mass of the load

The semi-empirical method (SEM) describes the maximum value of the power spectral density of the force in terms of a constant C^2 times the squared rigid body mass M_o of the load times the maximum power spectral density of the acceleration power spectral density at the interface

$$W_{FF_{max}} = C^2 M_o^2 W_{AA_{max}}. \quad (4.7)$$

The SEM is illustrated in Fig. 4.3. The maximum value of the left-hand side of Eq. (4.1) is $W_{FF_{max}}$ of the force limited spectral density from Eq. (4.7). The value of the right-hand side of Eq. (4.1) may be taken as $W_{AA_{max}}$ the maximum spectral density of the specified random vibration spectrum.

The maximum value of the apparent mass of the load occurs at the natural frequency f_o of the fixed load associated with dominant modal effective mass. The semi-empirical method maintains the ratio between maximum interface force to the maximum interface acceleration during testing. Consequently any conservatism in the test random acceleration specification representing the actual flight random acceleration specification will be maintained by the force spectral density specification too.

Figure 4.3: Principle of SEM

4

To prevent over-testing of the load Scharton proposed in [15, 16] the following well-known semi empirical vibration testing force limits at the interface between the load and the shaker table

$$\begin{aligned}
 W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \quad f \leq f_0, \\
 W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \left(\frac{f_0}{f} \right)^n \quad f > f_0,
 \end{aligned} \tag{4.8}$$

where $W_{FF}(f)$ is the random force spectral density, $W_{AA}(f)$ is the specified random acceleration spectral density, M_o is the total mass of the load and C^2 is a dimensionless semi-empirical constant which depends on the combination of the load and the supporting structure (source). f (Hz) is the running frequency and f_0 is the natural frequency of the vibration mode with the largest (most significant) modal effective mass of the load. The factor n can be estimated from the apparent mass of the load, in general, $n = 2$ (see also Eq. 4.6) and $C^2 \approx 1 - 5$ [1]. The roll-off ratio n is more or less dependent on the shape of the apparent mass of the load Eq. (4.6). Eq. (4.8) is illustrated in Fig. 4.4.

Figure 4.4: Force limited vibration specification

4.3. FLVT LITERATURE REVIEW C^2

In spacecraft structure design force limits are established to prevent over-testing of the test article (load), because its dynamic behavior on the shaker table is different from its dynamic behavior when placed on the actual supporting structure (source).

In [16] the history, the actual status and application guidelines of the force limited vibration testing (FLVT) are discussed and 41 interesting references regarding the FLVT are provided.

During the FLVT both the random acceleration and the random force limits are specified. However, the random acceleration specification may be overruled by the random force limits to prevent over-testing.

Back in 1963 Salter proposed in [9] an upper value for $C^2 = 2.25$ and suggested a practical value $C^2 = 1.44$.

Scharton et al revisited the force limiting vibration testing in a presentation [13] and reviewed the methods of estimation of C^2 using the STDFS.

Dharanipathi main conclusions from [3] are that the range of values of C^2 is between 2 and 5. However, there are several cases where $C^2 = 10 \dots 17$, and he stated that C^2 does not depend on the damping in the structure.

In [18] Soucy et al recommend values for C^2 based on limited available measured (flight) data. It has been observed that in normal conditions $C^2 = 2$ might be chosen for complete spacecraft or strut mounted heavier equipment. $C^2 = 5$ might be considered for directly mounted lightweight loads.

Based on the frequency shift of a two degree of freedom system (compared to a SDOF system) [14] Scharton developed two methods of establishing the value C^2 ; the simple two degree of freedom system (STDFS) [15] and the complex two degree of freedom system (CTDFS) [2]. These two methods are described in the previous chapter 3.

In [6] Gordon proposed a conservative analytical value of $C^2 = 9$, which is based on the STDFS when the load/source ratio is 0.16. This conservative estimation of C^2 will cover model uncertainties and the test configuration remains relatively simple because no force measurement devices are used during the random vibration test.

Salvignol reported in [11] the use of $C^2 = 3$ during the random vibration test of the NIRspec (Near Infrared) instrument, part of the integrated science instrument (ISIM) on the James Webb telescope. The assumed value $C^2 = 3$ was verified by random response analysis.

To compute C^2 with the STDFS, CTDFS, or enveloping method JAXA the dynamic characteristics of both source and load must be made available, simplified or more complex.

In general, the mathematical model (FEM, modal effective masses, ...) of the load is available, because the random vibration test will be conducted under the responsibility of contractor/subcontractor which is responsible for the design of the load as well. The mathematical description of the supporting structure (source) is, in general, lacking. To apply the methods of obtaining the value C^2 , the dynamical properties of the source need to be known.

Some characteristics of the random force limited vibration testing, the value C^2 , the total mass of the source M_s , the total mass of the load M_l , the roll-off frequency f_o and the exponent n representing the apparent mass of the load are presented in Table 4.1.

References are given too.

Table 4.1: Values of C^2 from literature

Ref.	C^2	M_l (kg)	M_s (kg)	f_0 (Hz)	n	Remark
[12]	4	103	-	100	-	Space Shuttle flight
[7]	4	14	8	500	2	Al panel + Box A
[7]	4	5	8	250	4	Al panel + Box B
[8]	9	6	-	376	2	Mars Sample Return Container
[11]	9	200	-	65	2	JWST NIRSPEC
[19]	4	85	-	80-90	2	JWST MIRI
[4]	2.5	114	187	93	2	Linear Drive Unit (X-dir)
[4]	4.8	114	187	75	2	Linear Drive Unit (Y-dir)
[4]	2.7	114	187	59	1.5	Linear Drive Unit (Z-dir)
[5]	2.5	52	-	> 100	2	Safari Instrument, SPICA

Table 4.1 has be reworked applying the STDFS method [15] in combination with Fig. 3.20 and $Q = 10$. The results are given in Table 4.2. The figures in *italic* are reworked.

Table 4.2: Reworked figures using the STDFS method

Ref.	C^2	M_l (kg)	M_s (kg)	Remark
[12]	4	103	285	Space Shuttle flight
[7]	2	14	8	Al panel + Box A
[7]	4	5	8	Al panel + Box B
[8]	9	6	30	Mars Sample Return Container
[11]	9	200	1000	JWST NIRSPEC
[19]	4	85	213	JWST MIRI
[4]	3.8	114	187	Linear Drive Unit (X-dir)
[4]	3.8	114	187	Linear Drive Unit (Y-dir)
[4]	3.8	114	187	Linear Drive Unit (Z-dir)
[5]	2.5	52	52	Safari Instrument, SPICA

4.4. CONCLUSIONS

The key parameter in the semi-empirical method (SEM) to estimate the random spectrum of the force limits is C^2 based on past experience of testing loads and may be vary from load project to load project. It is generally concluded that $C^2 = 2 \cdots 5$, but higher values may be applied. In general, the SEM is applied if no information about the random responses at the interface source/load can be made available.

When the dynamic properties of the load and the source are available the load/source interface random acceleration and force spectra can be computed and the constant C^2 can be calculated by applying the SEM. In the next chapter 5 the dynamic properties of the source source will be estimated in a probabilistic sense.

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5

PROBABILISTIC ANALYSIS SEMI-EMPIRICAL CONSTANT C^2

5.1. INTRODUCTION

In this chapter a probabilistic analysis to estimate the semi-empirical constant C^2 is discussed. This new probabilistic approach is developed within the frame of this thesis. This constant C^2 is one of the key parameters in the expressions Eq. (4.8) to establish the semi-empirical force limited vibration testing (FLVT) test values [17] and is given by:

$$\begin{aligned} W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \quad f \leq f_0, \\ W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \left(\frac{f_0}{f} \right)^n \quad f > f_0, \end{aligned}$$

where $W_{FF}(f)$ is the random force spectral density, $W_{AA}(f)$ is the specified random acceleration spectral density, M_o is the total mass of the test article (load) f (Hz) is the running frequency and f_0 is the natural frequency of the primary mode with a significant modal effective mass of the load. The factor n can be derived from the shape of the apparent mass of the load.

The constant C^2 reflects the random dynamic interaction (accelerations and forces) at the interface between the load and the supporting structure (source), but is defined, either from past experience or computed from the random dynamic responses at the interface load/source.

Within the framework of the probabilistic analysis of C^2 the semi-empirical method (SEM) of the force limited vibration test (FLVT) approach [17] is used. The SEM is described in detail in Chapter 4.

In [11] Marchand provides Eq. (4.7), to compute the value of C^2 at the interface between the source and the load both are, in general, described by finite element models (FEM), such that the random dynamic interface accelerations and forces can be computed. The maximum PSD of the interface force $W_{FF_{max}}$ and the maximum PSD of the

interface acceleration $W_{AA_{max}}$, need not occur at the same frequency, and the value of C^2 can be obtained by Eq. 4.7 and repeated here

$$C^2 = \frac{W_{FF_{max}}}{M_l^2 W_{AA_{max}}},$$

where M_l (M_o) is the total mass of the load.

To calculate the random dynamic responses at the interface between the load and the source, the dynamic characteristics of both must be made available: modal properties, mathematical models, FE models. The (sub)contractor that is responsible for the design and verification of the strength and dynamic behaviour of the load will of course possess a mathematical model of that load. The interface dimensions between the load and the source are known, but the design of the supporting structure is done by another (sub)contractor too, which means that dynamic properties and forces applied are unknown and not available to perform random dynamic response analyses to calculate interface accelerations and forces. Hence the semi-empirical constant C^2 cannot be computed.

5

The dynamic properties (design variables) of the unknown source could be described in a probabilistic way prescribing the intervals and distribution functions of the design variables. The major design variables of the source are: the total mass M_s (kg), the lowest natural frequency f_{1s} (Hz) associated with dominant modal effective mass m_{1s} (kg) and the modal damping ratio applicable to both the load and the source. Minor design variables (e.g. second natural frequency) are derived from the major ones.

In particular the lowest natural frequency associated with the dominant modal effective mass has to obey certain design rules as stated in the ECCS¹ standards and guidelines [5, 8]². Also books about spacecraft structures [14, 16, 21] discuss these design rules in great detail.

The known deterministic load is modeled as an asparagus patch model consisting of parallel placed SDOF elements. The asparagus patch model is discussed in more detail in Appendix C.

The probabilistic model of the source will represent the unknown source. Two probabilistic models are developed:

- Probabilistic asparagus patch model, consisting of the probabilistic design variables, modal effective masses, natural frequencies, total mass and modal damping ratio.
- Probabilistic MDOF model consisting of in series SDOF elements coupled, where the unknown design variables are varied; total mass distributed over the discrete masses, the lowest natural frequency and the modal damping ratio. This model represents a finite element model of the source.

Both probabilistic models are used in parallel with each other.

To perform the random response analyses the deterministic model of the load is coupled with the probabilistic model of the source; either the asparagus patch model or the

¹European Corporation of Space Standardization

²The author of this thesis was one of the members of the ECSS international working groups

MDOF model. The random dynamic accelerations at the interface are scaled such that they touch the prescribed random enforced accelerations $W_{AA}(f)$. The random interface forces are derived from the computed random interface accelerations. The analysis procedures and coupled load/source models are described in the following sections. The semi-empirical constant C^2 is derived from the maximum values of the random responses at the interface.

When the probabilistic models of the source are established and the (two) coupled probabilistic mathematical models are created, probabilistic analyses are performed to compute the probability distribution and subsequently the zero and second moments of the constant C^2 . The Rosenblueth $2k + 1$ point estimation probability moments [15] and the Monte Carlo Simulation [1] methods are used to perform the probabilistic analyses. The maximum value of C^2 will be achieved too. The approaches will be described later in this chapter.

Summarized the following steps are to be established to perform the probabilistic analyses:

- Creation of a deterministic asparagus patch model of the load.
- Generation of the probabilistic models of the source: asparagus patch model and/or MDOF model.
- Coupling of load model and probabilistic model of the source.
- Probabilistic computation of random responses at the interface to calculate C^2 .
- Statistical analysis of the distribution of C^2 : mean, standard deviation, and maximum values.

5.2. DISCRETE MATHEMATICAL MODELS FOR RANDOM RESPONSE ANALYSIS

Two discrete mathematical models of the coupled load/source combination are utilized to perform the random response analyses within the frame of the probabilistic approach to compute C^2 :

- Coupled System Modal Approach (CSMA) [19], where the apparatus patch models of both the deterministic load and probabilistic source are coupled to compute the semi-empirical constant C^2 . The CSMA method is discussed in detail in Appendix D, but will be briefly outlined in this chapter.
- Hybrid Mathematical Dynamic Modeling (HMDM). The load is mathematically described by an asparagus patch model and the coupled source by a probabilistic series of n SDOF elements. The large mass approach has been utilized to implement an additional constraint. The random responses at the interface load/source are used to compute the semi-empirical constant C^2 .

The calculation of C^2 is based on the SEM of estimating the force limits as described in Chapter 4.

In the following sections both discrete mathematical models are described in more detail.

5.2.1. COUPLED SYSTEM MODAL APPROACH METHOD (CSMA)

The CSMA method, proposed by Stevens in [19] and described in greater detail in Appendix D, is one of the methods to reconstruct the random dynamic responses at the interface and subsequently the computation of the force limits for the random vibration testing of the load. The dynamic or apparent mass of the load [6] and the random acceleration test specification are required. The reduced asparagus patch models of both

Figure 5.1: Coupled system in parallel-oscillator representation

source and load are shown in Fig. 5.1. The spring stiffnesses and damper values are given by, respectively, by $k_{kl} = \omega_{kl}^2 m_{kl}$ and $c_{kl} = 2\zeta_k \omega_{kl} m_{kl}$, where ω_{kl} , $k = 1, 2, \dots, n$ are the natural frequency of the load. ζ_k is the modal damping ratio of mode k . The residual mass of the load is denoted by m_{rl} . The notations for the source are similar. The index l is replaced by the index s .

The random acceleration specification $W_{AA}(f)$ at the interface between the source and the load is provided (specified) by the customer (i.e. prime contractor) and is a requirement for the design of the load. In general, this specification is an envelope that is based on data that "smooths over" peaks and valleys. The load is very responsive at the anti-resonance frequencies of the source-load system and acts as a dynamic absorber to reduce the input. Using the CSMA method random dynamic responses of the coupled system between load and source are reconstructed.

To estimate the dynamic random response spectrum at the interface load/source every mass m_{ks} , $k = 1, 2, \dots$ is subsequently loaded by a unit dynamic load W_F with a constant power spectral density (N^2/Hz) over a frequency range of interest. The random acceleration caused by the subsequently applied loading is scaled such that its maximum is equal to the random acceleration specification $W_{AA}(f)$ at the interface. This procedure is illustrated in Fig. 5.2, for example $k = 1$. To calculate the random responses the modal approach is used and the modal damping ratio ζ is equal for both the load and the source and constant for all modes.

After the subsequently loaded mass m_{ks} , $k = 1, 2, \dots$ the scaled PSD acceleration re-

Figure 5.2: Scaled random response spectrum, e.g. W_A applied to m_{is}

sponses $W_A(f)$ at the interface load/source are enveloped to get $W_{Ae}(f)$. This enveloped random acceleration is the maximum random acceleration without exceeding the specified random acceleration specification. The corresponding interface PSD load $W_{Fe}(f)$ can be computed by multiplying the enveloped PSD accelerations $W_{Ae}(f)$ by the absolute value of the apparent mass $\mathbf{M}_I(f)$ squared, hence

$$W_{Fe}(f) = |\mathbf{M}_I(f)|^2 W_{Ae}(f), \quad (5.1)$$

where f is the excitation frequency.

Considering the maximum PSD of the interface force $W_{Fe_{max}}$ and the maximum PSD of the interface acceleration $W_{AA_{max}}$, the value of C^2 can be defined as [11]

$$C^2 = \frac{W_{Fe_{max}}}{M_I^2 W_{Ae_{max}}} = \frac{W_{Fe_{max}}}{M_I^2 W_{AA_{max}}}, \quad (5.2)$$

where M_I is the total mass of the load. Eq. 5.2 represents the SEM force limiting approach.

5.2.2. HYBRID MATHEMATICAL DYNAMIC MODELING (HMDM)

The constrained source is modeled by a series of discrete mass spring elements, the load by an asparagus patch model consisting of a number of p parallel placed SDOF elements (modal effective masses and associated stiffnesses). The source consists of consists of n SDOF elements placed in series, where the discrete masses are $m_i = m$, $i = 1, 2, \dots, n$ and are connected by springs with a spring stiffness k . The fixation of the source is idealized with the large mass approach ($M = 10^9$ kg) [9, 20]. The modeling of the source and the fixation are illustrated in Fig. 5.3. The load and the source are connected via the discrete mass m_1 , which is part of the source. The lowest natural frequency of the load/source combination can be estimated with Dunkerley's method [12, 21], in combination with the alternative Dunkerley's method. The lower bound of the natural frequency of the fixed-free source/load combination can be estimated with the following equation

$$\frac{1}{\omega_1^2} = \frac{n(n+1)m}{2k} + \sum_{i=1}^p \frac{1}{\omega_{il}^2} = \frac{(n+1)M_s}{2k} + \sum_{i=1}^p \frac{1}{\omega_{il}^2}, \quad (5.3)$$

where ω_1 is the lowest natural frequency of the total system (related to the fundamental frequency of the load), $m = M_s/n$, and M_s is the total mass of the source. The spring stiffness k can be obtained using Eq. (5.3)

Considering the maximum PSD of the interface force $W_{F_{e\max}}$ and the maximum PSD of the specified interface acceleration $W_{A_{A\max}}$, Eq. (5.2) is again applied to compute C^2 .

5.3. MATHEMATICAL MODELS LOAD AND SOURCE

In this section the mathematical models of the load and source are discussed. The mathematical model of the load is an asparagus patch model and applicable for random response analysis methods; CSMA and HMDM. The probabilistic mathematical model of the source is described by an asparagus patch model when the CSMA method is used and a series of discrete SDOF elements in case the HMDM method is used.

5.3.1. MATHEMATICAL MODEL LOAD FOR CSMA AND HMDM METHODS

We assume the availability of a mathematical description (finite element model) of the load. An estimation of the modal damping ratio will be made, in general, based on past experiences or measurements. The finite element model degrees of freedom at the interface between the load and source will be fixed. The following modal data of the load are needed to build the asparagus patch model for the CSMA and HMDM method:

- The total mass of the load M_l (kg).
- The undamped natural frequencies f_k , $k = 1, 2, \dots, n$ (Hz) assuming clamped conditions at the interface load/source.
- The associated modal effective masses m_{kl} , $k = 1, 2, \dots, n$ (kg) and the residual mass m_{rl} , in the three translational directions respectively. The cross-coupling is not considered. The theoretical background of the modal effective mass and the residual mass is discussed in Appendix C.
- The estimated or measured modal damping ratios ζ_k , $k = 1, 2, \dots, n$.
- The apparent mass $\mathbf{M}_l(f)$ (kg) of the load with respect to the interface is needed to compute interface forces (see Eq. 5.1).

5.3.2. PROBABILISTIC MATHEMATICAL MODEL SOURCE, CSMA METHOD

Coté stated in his paper [4] that the asparagus patch model of the source (common to the load) can be extracted from a finite element model, experiment or from experience. However, in this subsection we assume that the finite element model or experimental results cannot be made available, so the simplified model will be constructed using engineering design rules (i.e. ECSS³ standards and handbooks).

The dynamic characteristics (design parameters) of the source with respect to the interface between the load and the source are considered to be probabilistic related to the modal properties of the load.

The probabilistic design parameters of the source are related to the mass and modal properties of the load and are discussed in [23] and are defined as follows.

TOTAL MASS

The total rigid body mass of the source M_s will be provided (i.e. by the prime contractor). If the M_s cannot be made available the following total mass variation is assumed:

$$M_s = 0.1 \cdots 10 M_l \quad (5.5)$$

³European Corporation of Space Standardization

NATURAL FREQUENCIES

When the lowest undamped natural frequency of the load is f_l the interface source/load fixed and the assumed undamped natural frequency of the source will vary between

$$f_{1s} = \frac{f_l}{2} \dots \frac{f_l}{\sqrt{2}}. \quad (5.6)$$

This range is based on the design practice that the dynamic interference between load and source is minimized.

This undamped natural frequency of the source is associated with a high modal effective mass m_{1s} .

The following distribution of natural frequencies, with substantial modal effective mass, is defined by:

$$\begin{aligned} f_{2s} &= 2f_{1s}, \\ f_{3s} &= 4f_{1s}, \\ f_{4s} &= 6f_{1s}. \end{aligned} \quad (5.7)$$

5

The proposed distribution of the natural frequencies f_{is} is inspired by the distribution of natural frequencies of a pinned-pinned uniform beam [3]. Torsional vibration modes are usually not of importance in spacecraft structures design.

Force limits typically cover only the first three modes [10]. Therefore, it is usually adequate to specify the force limits of the load only in the frequency regime encompassing a few modes for each axis, which might be up to approximately 100 Hz for a large spacecraft, 500 Hz for an instrument, or 2000 Hz for a small component [17].

MODAL EFFECTIVE MASSES

To get a feeling for the main modal effective mass value, the main modal effective mass will be calculated for simple systems assuming basic mode shapes [22].

A cantilevered beam is representing a clamped spacecraft. The mass per unit of length is m_b and the length of the beam is L (see Fig. 5.4). The total mass is mL . The

Figure 5.4: Simple representations of spacecraft and subsystems

assumed bending mode $\Phi(x)$ (lateral direction) and the assumed longitudinal (launch)

mode $\Psi(x)$ are given by

$$\begin{aligned}\Phi(x) &= 2\left(\frac{x}{L}\right)^2 - \frac{4}{3}\left(\frac{x}{L}\right)^3 + \frac{1}{3}\left(\frac{x}{L}\right)^4, \\ \Psi(x) &= x.\end{aligned}\quad (5.8)$$

A simply supported beam may be a representation of a fixed spacecraft. The assumed mode $\Phi(x)$ is taken to be

$$\Phi(x) = \sin\left(\frac{\pi x}{L}\right), \quad (5.9)$$

and a subsystem may be represented by a simply supported plate with edges a, b and mass per unit of area m_p we assume a vibration mode $\Phi(x, y)$

$$\Phi(x, y) = \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right). \quad (5.10)$$

The simple structural representations of the cantilevered beam, simply supported beam and plate are illustrated in Fig. 5.4.

The corresponding modal effective masses are given in Table 5.1. and in this table the total mass of the structures is denoted by M_s .

Table 5.1: Modal effective mass of simple systems; beam, plate

Structure	Assumed mode shape	M_{eff} (kg)	$M_{eff}(M_s)$ (kg)
Cant. beam	$\Phi(x)$	$\frac{81}{130}m_bL$	$0.62M_s$
Cant. beam	$\Psi(x)$	$\frac{3}{4}m_bL$	$0.75M_s$
S.S. beam	$\Phi(x)$	$\frac{8}{\pi^2}m_bL$	$0.81M_s$
S.S. plate	$\Phi(x, y)$	$\frac{64}{\pi^4}m_pab$	$0.66M_s$

The first undamped natural frequency f_{1s} will be associated with the first significant modal effective mass m_{1s} . The fundamental modal effective masses of simple systems is assumed to be a first approximation of modal effective mass of the source. However, for more complex, real-life structures the dominant modal effective mass is somewhat lower. Therefore the dominant modal effective mass will be assumed in the following mass range:

$$m_{1s} = 0.4 \cdots 0.6M_s. \quad (5.11)$$

The residual mass is the sum of the modal effective masses excited outside the frequency range of interest and the residual mass m_{rs} will be assumed to be 5% of the total mass of the source, such that

$$m_{rs} = 0.05M_s. \quad (5.12)$$

The summed modal effective masses of the computed modes will be about 95% of the total mass of the source M_s .

Further Δm is the sum of the missing modal effective masses that must be still distributed and is defined by

$$\Delta m = M_s - (m_{1s} + m_{rs}). \quad (5.13)$$

The deterministic distribution (best guess) of the modal effective $m_{ks}(f_{ks})$, $k = 2, \dots, 4$ will be descending and the effective masses of the remaining modes are distributed according to the following descending scheme:

$$\begin{aligned} m_{2s} &= 0.5\Delta m, \\ m_{3s} &= 0.3\Delta m, \\ m_{4s} &= 0.2\Delta m. \end{aligned} \tag{5.14}$$

MODAL DAMPING RATIO

In general, in the case of random vibrations in spacecraft structures the modal damping ratio used is $\zeta = 0.05$, therefore a variation of the modal damping ratio in the interval $\zeta = 0.01 \dots 0.1$ is assumed. The modal damping ratio ζ is constant and applicably for all vibration modes of the combination load and source.

5

SUMMARY OF MEAN AND STANDARD DEVIATION OF DESIGN VARIABLES, CSMA METHOD

The probability density function of the stochastic design variables M_s, f_{1s}, m_{1s} and ζ is assumed to be uniform. The changes are then equally distributed.

The summary of mean and standard deviation of the selected design variables, with a uniform distribution is presented in Table 5.2.

Table 5.2: Mean and standard deviation stochastic variables

Description	Symbol	Interval	Mean μ	Std dev. σ
Mass (kg)	M_s	$[0.1 - 10]M_l$	$5.0500M_l$	$2.8579M_l$
Natural frequency (Hz)	f_{1s}	$[0.5 - 0.5\sqrt{2}]f_{1l}$	$0.6036f_{1l}$	$0.0598f_{1l}$
Modal effective mass (kg)	m_{1s}	$[0.4 - 0.6]M_s$	$0.5000M_s$	$0.0577M_s$
Modal damping ratio (-)	ζ	$[0.01 - 0.1]$	0.055	0.0260

5.3.3. MATHEMATICAL MODEL SOURCE, HMDM METHOD

Calculating the random responses at the interface load/source with the HMDM method, the design variables of the discrete model of the source are the total mass M_s , the first natural frequency with dominant modal effective mass f_{1s} and the modal damping ratio ζ . The design variables have the same ranges and density functions as proposed for the probabilistic source for the CSMA mathematical model.

The probability density function of the stochastic design variables M_s, f_{1s} and ζ are assumed to be uniform.

The summary of mean and standard deviation of the selected design variables with a uniform distribution is shown in Table 5.3. The modal effective mass is missing because that is intrinsically in the MDOF system.

Table 5.3: Mean and standard deviation stochastic variables

Description	Symbol	Interval	Mean μ	Std dev. σ
Mass (kg)	M_s	$[0.1 - 10]M_l$	$5.0500M_l$	$2.8579M_l$
Natural frequency (Hz)	f_{1s}	$[0.5 - 0.5\sqrt{2}]f_{1l}$	$0.6036f_{1l}$	$0.0598f_{1l}$
Modal damping ratio (-)	ζ	$[0.01 - 0.1]$	0.055	0.0260

5.4. DETERMINISTIC COMPARISON OF CSMA AND HMDM MODELS

The proposed mathematical models: CSMA and HMDM, to compute the semi-empirical constant C^2 in Eq. (4.8) will be compared with each other performing deterministic analyses. This is only one example to see the trends in the computation of C^2 . The asparagus patch model of the is taken from a paper of Fitzpatrick [7] and is given in Table 5.4. The residual mass has an artificial natural frequency of 2500 Hz, which is outside the frequency region of interest and for random vibration specifications mostly specified in between 20-2000Hz. The total mass of the load is $M_l = 113.85\text{kg}$ and the first natural frequency of the load is $f_{1l} = 59.0\text{Hz}$.

Table 5.4: Asparagus patch model of the load, $M_l = 113.85\text{kg}$

Modal effective mass (kg)	$m_{1l} = 36.0$	$m_{2l} = 18.2$	$m_{rl} = 59.65$
Natural frequency (Hz)	$f_{1l} = 59.0$	$f_{2l} = 92.7$	$f_{rl} = 2500$

The deterministic design variables of the source are specified in Table 5.5. The mass of the source M_s and the first natural frequency (CSMA) or first system frequency (HMDM) f_{1s} will be varied. The significant modal effective mass of the source m_{1s} will not be changed, because in case of the HMDM model there only is one significant modal effective mass of importance, which is associated with the system frequency.

The specified (band-limited) interface random acceleration $W_{AA} = 0.01\text{g}^2/\text{Hz}$ in between the frequency range of interest 20-2000Hz.

Table 5.5: Deterministic design variables source

Description	Symbol	Value
Mass (kg)	M_s	$[0.01, 0.1, 1, 5, 10, 100]M_l$
Natural frequency(system frequency) (Hz)	f_{1s}	$[0.4, 0.5, 0.6, 0.7, 0.8]f_{1l}$
Modal effective mass (kg)	m_{1s}	$0.5M_s$
Modal damping ratio (-)	ζ	0.05

The computations of the semi-empirical constant C^2 are made both with the CSMA and HMDM methods and the results are presented in Table 5.6.

When the values of C^2 are computed with the CSMA asparagus patch models the following aspects are noticed:

Table 5.6: Comparison CSMA/HMDM method

CSMA method C^2					
M_s	$M_s = 0.1M_l$	$M_s = 1M_l$	$M_s = 5M_l$	$M_s = 10M_l$	$M_s = 100M_l$
$f_{1s} = 0.4f_{1l}$	2.55	3.12	5.24	7.08	7.64
$f_{1s} = 0.5f_{1l}$	1.78	1.56	1.51	1.54	1.59
$f_{1s} = 0.6f_{1l}$	1.47	1.68	2.03	2.14	3.11
$f_{1s} = 0.7f_{1l}$	1.83	2.29	3.38	4.04	5.78
$f_{1s} = 0.8f_{1l}$	2.66	3.79	7.20	7.53	7.07
HMDM method ($n = 5$) C^2					
$f_{1s} = 0.4f_{1l}$	0.59	1.10	1.17	1.21	1.53
$f_{1s} = 0.5f_{1l}$	1.10	1.14	1.39	1.51	1.71
$f_{1s} = 0.6f_{1l}$	1.07	1.29	2.14	2.87	6.59
$f_{1s} = 0.7f_{1l}$	1.10	1.87	1.72	0.95	0.80
$f_{1s} = 0.8f_{1l}$	1.36	0.54	0.28	0.31	0.33
HMDM method ($n = 10$) C^2					
$f_{1s} = 0.6f_{1l}$	0.69	1.28	2.08	2.77	6.82

- For the natural frequencies of the load $f_{1s} = [0.4, 0.8]f_{1l}$ the calculated values of C^2 are rather high compared with the calculation done with the HMDM mathematical model. When the system natural frequency is $f_{1s} = 0.5f_{1l}$ the dynamic interaction between load and source shows a minimum. For the HMDM approach values C^2 are very low when the system natural frequency is high because the combination load-source is rather stiff with low dynamic interaction.
- In the frequency range $f_{1s} = [0.5, 0.6, 0.7]f_{1l}$ the computed values of C^2 are on average higher than calculated with the HMDM model.
- Values of C^2 presented in Table 5.6 show that the probabilistic analyses of C^2 may be expected to give higher values when the CSMA model approach is applied than when the HMDM model approach is applied.
- The analysis results are based on one example for which especially design parameter f_{1s} and M_s are varied.

Doubling the number of SDOF elements n in the HMDM mathematical model approach has hardly any influence on the results. On the other hand it has a great impact on the computation times.

5.5. PROBABILISTIC ANALYSIS

Two methods are used to perform the probabilistic/optimization analysis to calculate the semi-empirical constant C^2 :

- The Rosenblueth 2k+1 Point Estimates Probability Moments Method
- Monte Carlo Simulation (MCS)

The design variables are described by uniform distributions with specified ranges (intervals). The outcomes of the probabilistic analyses are statistical values: the mean μ_{C^2} , the standard deviation σ_{C^2} and the maximum value C_{max}^2 of the semi-empirical constant C^2 .

5.5.1. THE ROSENBLUETH $2k+1$ POINT ESTIMATES PROBABILITY MOMENTS METHOD

The Rosenblueth point estimates for probability moments method [13, 15] computes the mean and the variance of the value C^2 in combination with either CSMA or HMDM. If the number of design variables is k , $2k+1$ samples (analysis cases) have to be computed.

The reference value Y_0 of C^2 is computed by substituting the mean values μ for all k design variables in the probabilistic source, Y_{nm} value (m mean minus) of C^2 are computed by substituting for the n^{th} design variable the value $\mu_n - \sigma_n$ and for the other design variables the mean values and the Y_{np} value of C^2 (p means plus) is computed by substituting for the n^{th} design variable the value $\mu_n + \sigma_n$ and for the other design variables the mean values. Index m indicates subtracting the standard deviation and index p indicates adding the standard deviation.

The mean Y_n of two point estimates Y_{np}, Y_{nm} is given by

$$Y_n = |Y_{np} + Y_{nm}| / 2, \quad n = 1, \dots, k, \quad (5.15)$$

and the variance V_n can be obtained by

$$V_n = \left| \frac{Y_{np} - Y_{nm}}{Y_{np} + Y_{nm}} \right|, \quad n = 1, \dots, k. \quad (5.16)$$

When we assume that all design variables are statistically independent the following approximation of the mean $\bar{Y} = \mu_Y$ and the variance $V_Y = \sigma_Y / \mu_Y$ can be made [15]

$$\frac{\bar{Y}}{Y_0} = \prod_{n=1}^{2k+1} \frac{Y_n}{Y_0}, \quad (5.17)$$

and

$$1 + V_Y^2 = \prod_{n=1}^{2k+1} (1 + V_n^2). \quad (5.18)$$

With the Rosenblueth PEM approach no distribution of the statistical variable can be obtained, With the aid of the Camp-Meidell inequality [18] the probability

$$Prob(0 \leq y \leq \mu_y + n\sigma_y) \leq \left(\frac{3}{4n} \right)^2 \quad (5.19)$$

can be estimated.

5.5.2. MONTE CARLO SIMULATION

To predict the results with Monte Carlo Simulation (MCS) quite a number of simulation runs have to be done, varying the design variables randomly. In accordance with [2] the following steps are performed:

- Definition of the mathematical model(s) (CSMA/HMDM)
- Generation of random numbers
- Generation of random design variables
- Evaluation of random responses
- Statistical analysis of resulting behaviour

The random acceleration and force responses at the interface Load/source and subsequently the semi-empirical constant are evaluated by varying the uniform distributed design variables randomly. The stochastic design variables being the total mass of the source M_s , the lowest natural frequency f_{1s} , the modal effective mass m_{1s} corresponding to the f_{1s} and the modal damping ratio ζ are varied as follows:

$$M_s = M_{smin} + X_M(M_{smax} - M_{smin}), \quad (5.20)$$

$$f_{1s} = f_{1smin} + X_f(f_{1smax} - f_{1smin}), \quad (5.21)$$

$$m_{1s} = m_{1smin} + X_m(m_{1smax} - m_{1smin}), \quad (5.22)$$

$$\zeta = \zeta_{min} + X_\zeta(\zeta_{max} - \zeta_{min}). \quad (5.23)$$

where X_M , X_f , X_m and X_ζ are subsequently generated random numbers with uniform distribution function $f_X(x)$ given by

$$f_X(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases} \quad (5.24)$$

In the software packages MatlabTM and OctaveTM the random numbers X are generated using the preprogrammed function `rand`. In every random response analysis loop all design variables are randomly varied. The accuracy of the numerical solution obtained through MCS will improve with the number of analysis loops [1].

After all analysis loops are done the mean and standard deviation and the maximum value of C^2 are calculated. The distribution of C^2 may be expected to match a lognormal distribution of $C^2 \geq 0$. The associated mean $\lambda = E[\ln(C^2)]$ and standard deviation $\xi = \sqrt{Var[\ln(C^2)]}$ are related to the straightforward calculated mean μ_{C^2} and standard deviation σ_{C^2} of C^2 .

$$\lambda_{C^2} = \ln \mu_{C^2} - \frac{\xi_{C^2}^2}{2}, \quad (5.25)$$

$$\xi_{C^2}^2 = \ln \left[1 + \left(\frac{\sigma_{C^2}}{\mu_{C^2}} \right)^2 \right]. \quad (5.26)$$

The lognormal distribution f_X of the random variable x is given by [1]

$$f_X = \frac{1}{\sqrt{2\pi}(\xi x)} \exp \left[-\frac{1}{2} \left(\frac{\ln x - \lambda}{\xi} \right)^2 \right], \quad x \geq 0. \quad (5.27)$$

An example of the lognormal distribution is shown in Fig. 5.5, with $\lambda = 1.065$, $\xi = 0.470$ ($\mu = 3.24$, $\sigma = 1.62$).

Figure 5.5: Lognormal distribution, $\lambda = 1.065$, $\xi = 0.470$

PROBABILISTIC DENSITY FUNCTIONS C^2 TABLE 5.6

The real density and the lognormal density of the results of C^2 , are presented in Table 5.6 and in particular the CSMA results are computed and shown in Fig. 5.6. Due to interpolation/extrapolation of the real density of C^2 negative values occur. These negative values are ignored in Fig. 5.6. The ratio of the areas of the real density and the lognormal density $A_{rd}/A_{logn} \approx 1$.

5.6. CONCLUSIONS

The concept of replacing an unknown source (supporting structure) by a probabilistic source, to compute the semi-empirical constant C^2 , is new. Within that framework two mathematical models are developed to reconstruct the random acceleration and force spectra at the interface between the load and the source. It means that the computations of C^2 are in fact a probabilistic analyses. The Rosenbluth $2k + 1$ PEM and the MCS methods are used to calculate the mean, the standard deviation and the maximum value of C^2 .

Three cases are worked out in detail Chapter 6.

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Figure 5.6: Density functions of C^2

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6

APPLICATIONS OF PROBABILISTIC ANALYSES OF C^2

6.1. INTRODUCTION

The probabilistic analysis of C^2 is performed using the CSMA and HMDM models and three cases taken from literature are analysed:

- The Haille problem: a simple example problem with eight DOFs [7] will be examined to show the analysis results of C^2 compared to the actual source with the probabilistic source. This is a typical benchmark problem.
- The ESA study: "IFLV-Improvement of Force Limited Vibration Testing Methods for Equipment Instrument Unit Mechanical Verification", [5]. The mass ratio load/source is $O(10)$
- The Linear Drive Unit (LDU), which is an Orbital Replacement Unit (ORU) of the International Space Station (ISS) programme, [6]. The mass ratio load/source is $O(1)$

The computations of the semi-empirical constant C^2 are based on an interface random acceleration specification $W_{AA} = 0.01 \text{ g}^2/\text{Hz}$ in a frequency band between 20-2000 Hz.

6.2. HAILLE PROBLEM

A simple example problem with eight DOFs [7] will be used to show the analysis results of C^2 , comparing the actual source with the probabilistic source. This example is illustrated in Fig. 6.1. The value C^2 is calculated at the interface between masses 3 and 4 (mass 3 side) applying (Eq. 4.7), by calculating the maximum PSD acceleration of node 3 and the maximum PSD of the interface force (PSD of force in spring between masses 3 and 4). The total mass of the source (masses 1 to 3) is $M_s = 224 \text{ kg}$ and total mass of the load (masses 4 to 8) is $M_0 = 31 \text{ kg}$. The load and the source are illustrated in Fig. 6.1 too.

Figure 6.1: 8 DOF simple problem [7]

The source consists of masses 1 to 3 and the spring between masses 3 and 4 is cut into two springs with a spring stiffness $2k$, which is fixed at the right side of the interface. Two springs in series with spring stiffness $2k$ is equivalent with one spring with stiffness k . The dynamic properties of the source fixed at the right side of the interface are presented in Table 6.1. The load consists of the masses 4 thru 8 and the spring between masses 3

Table 6.1: Dynamic properties source (fixed between masses 3 and 4)

Mode #	Natural frequency (Hz)	Modal effective mass (kg)	Residual mass (kg)
1	22.57	200.5 (m_{1s})	23.47
2	76.14	12.37 (m_{2s})	11.10
3	141.2	11.10 (m_{3s})	0.00
Total mass (kg)		224.0	

and 4 is cut into two springs with a spring stiffness $2k$, which is fixed at the left side of the interface. The dynamic properties of the source fixed at the left side of the interface are shown in Table 6.2.

Table 6.2: Dynamic properties load (fixed between masses 3 and 4)

Mode #	Natural frequency (Hz)	Modal effective mass (kg)	Residual mass (kg)
1	81.25	24.19 (m_{1l})	6.81
2	177.4	6.10 (m_{2l})	0.71
3	246.6	0.71 (m_{3l})	0.00
4	365.9	0.000	0.00
5	589.6	0.000	0.00
Total mass (kg)		31.00	

6.2.1. RESPONSE ANALYSIS WITH CSMA

The coupled systems modal analysis (CSMA) method will be applied to compute the random responses at the interface between the load and source. Both are idealized using asparagus patch models. The vales of the semi-empirical constant are computed with Eq. (5.2).

DETERMINISTIC SOURCE

The first three modes (modal effective mass and natural frequency) are taken from the Tables 6.1 and 6.2 to build the asparagus patch models of the load and the source respectively. The white noise (arbitrarily) random vibration specification, at the interface, is between 20 and 2000Hz, $W_{AA} = 0.01 \text{ g}^2/\text{Hz}$. The white noise random force applied subsequently to m_{1s}, m_{2s}, m_{3s} is $W_F = 1 \text{ N}^2/\text{Hz}$. The modal damping ratio is constant for all modes, and applicable for the source and load is varying $\zeta = 0.01, 0.05, 0.1$. To investigate the influence of the stiffness of the source on the dynamic responses at the interface between load and source the natural frequencies of the source f_{1s}, f_{2s}, f_{3s} are scaled by a scaling factor $\alpha = 1, 0.2, 0.4, 2.5, 3.6$. The value of the scaling factor $\alpha = 3.6$ represents equal first natural frequencies of load and source. A low scaling factor α represents a very weak source and a high scaling factor represents a very stiff source. The calculated values for C^2 are tabulated in Table 6.3: The results of C^2 in Table 6.3 can be explained

Table 6.3: Deterministic values of C^2

ζ	C^2				
	$\alpha = 1$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 2.5$	$\alpha = 3.6$
0.01	3.39	1.35	4.33	5.47	9.60
0.05	3.23	1.35	4.16	5.47	8.58
0.10	3.21	1.34	3.64	4.96	5.08
$f_{1s}(\text{Hz})$	22.6	4.5	9.0	56.5	81.3

using Fig. 6.2. The black line is the envelope of the reconstructed PSD of the random acceleration at the interface of the load and source. The blue line represents the apparent mass $\mathbf{M}_l(f)$ of the load. The envelope of the PSD of the random force at the interface of the load and the source is given by the magenta colour. If the peaks of the random acceleration and the peaks of the apparent mass coincide ($\alpha = 3.6$) at the same frequencies the random interface forces will be higher than the when the peaks do not coincide. The high random interface force will result in higher values of C^2 .

PROBABILISTIC SOURCE

The deterministic properties of the asparagus patch model of the load are taken from Table 6.2. In this section a Rosenbluth PEM and a Monte Carlo Simulation (MCS) analysis will be performed, where the stochastic design variables M_s, f_{1s}, m_{1s} and ζ are assumed to be uniformly distributed and are given in Table 5.2.

Figure 6.2: Interface responses load/source interface, CSMA method, $\zeta = 0.05$

PROBABILISTIC ANALYSIS USING PEM

The deterministic properties of the asparagus patch model of the load are taken from Table 6.2. With the aid of the Rosenblueth Point Estimates for probability moments [8], the estimates of the mean and the variance of the value C^2 are computed in combination with the CSMA. The number of samples is nine. The four stochastic design variables M_s , f_{1s} , m_{1s} and ζ are assumed to be uniform distributed and were given in Table 5.2.

The design variables will be varied to perform a sensitivity analysis:

- The mass of the source is tailored to the actual mass; $M_s = 7.225M_l$
- The intervals of the lowest source natural frequency f_s will be varied.
- The modal damping ratio is fixed; $\zeta = 0.05$.

The estimates of the statistical values of C^2 are given in Table 6.4.

Table 6.4: PEM computations of mean μ , standard deviation σ , and $\mu + 3\sigma$ value of C^2 , CSMA

$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)				
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	4.50	1.09	7.76
9	7.225	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	5.01	1.15	8.45
9	[0.1, 10]	[0.3, 0.5]	[0.01, 0.1]	6.40	0.53	14.57
9	[0.1, 10]	[0.7, 0.9]	[0.01, 0.1]	9.29	3.59	20.05
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	0.05	4.62	1.11	7.96

PROBABILISTIC ANALYSIS USING MCS

One sample is defined as when all four design variables M_s , f_{1s} , m_{1s} and ζ are varied simultaneously. The statistical values of C^2 are shown in the Table 6.5. The four stochastic

design variables are assumed to be uniformly distributed and were given in Table 5.2.

Table 6.5: Monte Carlo Simulation of Haille problem

$\neq$ Samples	C^2_μ	$C^2_{1\sigma}$	$C^2_{\mu+3\sigma}$	C^2_{max}
100	4.44	1.39	8.60	9.31
1000	4.35	1.50	8.85	14.79
2000	4.41	1.54	9.03	16.44

The MCS with 2000 samples is used to generate the probability density (PDF MCS) and cumulative density (CDF MCS) functions of the semi-empirical constant C^2 . These functions are shown in Fig. 6.3 (a) and (b). The corresponding lognormal distribution of C^2 is shown in Fig. 6.3 (c) and the comparison between the probability density function associated with the MCS and the lognormal distribution of C^2 is shown in Fig. 6.3 (d). The PDF MCS is very similar to the lognormal distribution.

(a) PDF MCS

(b) CDF MCS

(c) Lognormal

(d) Lognormal/PDF MCS

Figure 6.3: Probability and Cumulative Density Functions of C^2 , Haille problem

The probability $Prob(0 \leq C^2 \leq C_\mu^2 + nC_\sigma^2)$ is presented in Table 6.6

Table 6.6: Lognormal probabilities, $C_\mu^2 = 4.41$, $C_\sigma^2 = 1.54$, Eq. (5.27)

n	$Prob(0 \leq C^2 \leq C_\mu^2 + nC_\sigma^2)$
0	0.567
1	0.854
2	0.958
3	0.989
4	0.997

DISCUSSION CSMA RESULTS HAILLE PROBLEM

The deterministic values of C^2 shown in Table 6.3 are significant lower than computed statistical mean values C_μ^2 .

The probabilistic analysis of the semi-empirical constant C^2 computed with the Rosenblueth PEM method and the MCS method are presented in Tables 6.4 (first line) and 6.5 respectively. The mean values C_μ^2 are similar, but the standard deviation C_σ^2 calculated by the MCS method is somewhat higher, which results in higher values for the $C_{\mu+3\sigma}^2$ values of C^2 .

Computations with the Rosenblueth PEM method need much fewer samples than the MCS method, so a sensitivity analysis was performed using this PEM method. Setting the lowest natural frequency range below and above the standard frequency interval $f_s = [0.5, 0.5\sqrt{2}]f_l$ high values of C^2 were computed. A weak or high stiffness of the source results in high interface loads.

When the MCS method is applied the C_{max}^2 can be calculated and the density function of C^2 can be derived from all the computed samples. The density function of C^2 is quite similar to the lognormal distribution of random variables. This is shown in Fig. 6.3.

6.2.2. RESPONSE ANALYSIS WITH HMDM

DETERMINISTIC ANALYSIS

The asparagus model of the load can be built from information given in Table 6.2. The MDOF model of the source is build up with $n = 5$ equal masses m and a large mass M . The HMDM model is shown in Fig. 5.3. The total mass of the source is $M_s = 224\text{kg}$, hence the individual mass $m = M_s/n = 44.8\text{kg}$.

The white noise random vibration specification, at the interface, is between 20 and 2000Hz, $W_{AA} = 0.01 \text{ g}^2/\text{Hz}$. The white noise random force which is applied subsequently to the masses m and the large mass M is given by $W_F = 1 \text{ N}^2/\text{Hz}$. The modal damping ratio is constant for all modes, and the ratio applicable to the source and load is variable; $\zeta = 0.01, 0.05, 0.1$. The lowest system natural frequency f_{1s} of the load/source combination found by multiplying the first natural of the load $f_{1l} = 81.25\text{Hz}$ by the factor α . This factor is variable; $\alpha = 0.6, 0.2, 0.4, 0.5, 0.7, 0.8, 1.0$. The total mass of the load/source combination is 255kg. The calculated values for C^2 are tabulated in Table 6.7.

The outcomes of C^2 listed in Table 6.7 will be explained using Fig. 6.4 a) and b). When the system natural frequency f_{1s} with dominant modal effective mass does not coincide

Table 6.7: Deterministic values of C^2

ζ	$C^2, n = 5$						
	$\alpha = 0.6$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 1.0$
0.01	4.02	2.08	1.95	1.96	3.19	2.42	183,9
0.05	3.94	2.31	1.87	1.96	3.29	2.40	31.9
0.10	3.68	2.84	1.81	1.94	3,26	2.02	8.97
f_{1s} (Hz)	60.3	16.4	35.3	46.9	72.7	79.1	83.2
f_{2s} (Hz)	95.2	48.4	83.3	89.0	112	162	153
f_{3s} (Hz)			121	153		200	181
f_{4s} (Hz)							247
$M_{eff}(f_{1s})$ (%)	71.1	87.3	85.6	82.3	40.4	16.1	
$M_{eff}(f_{2s})$ (%)	17.7	8.9	5.7	7.4	46.9	37.1	
$M_{eff}(f_{3s})$ (%)			5.4	5.8		35.2	7.8
$M_{eff}(f_{4s})$ (%)							84.1

(a) $\alpha = 0.6$

(b) $\alpha = 1.0$

Figure 6.4: Interface responses load/source interface, HMDM method $n = 5$, $\zeta = 0.05$

with the dominant natural frequency of the load with significant modal effective mass m_{1l} , the peak of the load apparent mass will not coincide with the peak of the random acceleration response and the random interface forces will result in moderate values of C^2 (Fig. 6.4 a)). If the natural frequencies f_{1s} and f_{1l} coincide high random interface forces will occur and consequently high values of C^2 (Fig. 6.4 b)).

PROBABILISTIC ANALYSIS USING PEM

The deterministic properties of the asparagus patch model of the load are taken from Table 6.2. The Rosenblueth Point Estimates for probability moments [8], and the estimates of the mean and the variance of the value C^2 are computed in combination with

the CMSA. The number of samples is nine. The three stochastic design variables M_s , f_{1s} and ζ are assumed to be uniform distributed and given in Table 5.3.

Despite the source is consisting of three masses the convergence of the number of masses n in the HMDM approach will be investigated by increasing the number $n = 3, 5, 10, 20, 50$. The computational results are presented in Table 6.8. The effect of in-

Table 6.8: Convergence investigation of number of masses n in HMDM approach

n	HMDM				
	3	5	10	20	50
C_μ^2	4.01	3.86	3.76	3.64	3.60
C_σ^2	2.29	2.19	2.08	2.01	1.96
$C_{\mu+3\sigma}^2$	10.88	10.43	9.99	9.67	9.49

creasing the number of masses n in the HMDM approach is a very slight decrease of the statistical values of C^2 . However, the computation time increases drastically. For all further calculation applying the HMDM approach the number of masses is set to $n = 5$.

The design variables will be varied to perform a sensitivity analysis:

- The mass of the source is tailored to the actual mass: $M_s = 7.225M_l$
- The intervals of the system frequency f_s will be varied
- The modal damping ratio is fixed; $\zeta = 0.05$.

The computations of the variations and the statistical values of C^2 are given in Table 6.9.

Table 6.9: PEM computations of mean μ , standard deviation σ , and $\mu + 3\sigma$ value of C^2 , HMDM, $n = 5$

$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)				
7	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.86	2.19	10.43
7	7.225	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	4.62	2.15	11.09
7	[0.1, 10]	[0.3, 0.5]	[0.01, 0.1]	1.93	0.53	3.52
7	[0.1, 10]	[0.7, 0.9]	[0.01, 0.1]	1.75	1.37	5.88
7	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	0.05	3.94	2.26	10.71

PROBABILISTIC ANALYSIS USING MCS

One sample (loop) is defined as when all three design variables M_s , f_{1s} and ζ are varied simultaneously (Table 5.3). The number of masses in the source model is $n = 5$. The statistical values of C^2 are shown in the Table 6.10.

The MCS with 2000 samples is used to generate the probability density (PDF MCS) function of the semi-empirical constant C^2 . This function is shown in Fig. 6.5 (a). The corresponding lognormal distribution of C^2 and the probability density function associated with the MCS are shown in Fig. 6.5 (b). The PDF MCS is very similar to the lognormal distribution.

The probability $Prob(0 \leq C^2 \leq C_\mu^2 + nC_\sigma^2)$ is presented in Table 6.11

Table 6.10: Monte Carlo Simulation of Haille problem, HMDM, $n=5$

$\neq$ Samples	C_μ^2	$C_{1\sigma}^2$	$C_{\mu+3\sigma}^2$	C_{max}^2
100	3.07	1.54	8.35	8.34
1000	3.23	1.58	7.98	8.84
2000	3.21	1.61	8.08	8.99

Table 6.11: Lognormal probabilities, $C_\mu^2 = 3.21$, $C_\sigma^2 = 1.61$, Eq. (5.27)

n	$Prob(0 \leq C^2 \leq C_\mu^2 + nC_\sigma^2)$
0	0.594
1	0.863
2	0.956
3	0.985
4	0.995

DISCUSSION HMDM RESULTS HAILLE PROBLEM

Increasing the number of masses in the source n will result in a slight decrease of the statistical values of C^2 , but computation times are increasing drastically. To perform the calculations more efficiently the number of masses is set to $n = 5$.

The probabilistic analysis of the semi-empirical constant C^2 computed with the Rosenblueth PEM method and the MCS method are presented in Tables 6.9 (first line) and 6.10 respectively. The mean values C_μ^2 and standard deviations C_σ^2 computed with the PEM method are higher than those computed by the MCS method, which results in higher values for the $C_{\mu+3\sigma}^2$ values of C^2 .

Computations with the Rosenblueth PEM method need much fewer samples than for the MCS method. A sensitivity analysis was performed using this PEM method. Setting a system frequency intervals below and above the standard frequency interval $f_s = [0.5, 0.5\sqrt{2}]f_l$ low values of C^2 were computed, because the peak values of the dynamic mass of the load $\mathbf{M}_l(f)$ do not coincide with the maximum responses of the load/source combination at the interface of both.

When the MCS method is applied the C_{max}^2 can be calculated and the density function of C^2 can be derived from all the computed samples. The density function of C^2 is quite similar to the lognormal distribution of random variables. This is shown in Fig. 6.5 (b).

6.2.3. DISCUSSION RESULTS HAILLE PROBLEM

The statistical values of the design variables for the CSMA and HMDM models are given in Tables 5.2 and 5.3, respectively. The computed values of C^2 of both approaches are presented in Tables 6.12 and 6.13. No variations of the design variables are shown.

The computed deterministic values presented in Table 6.3 are lower than the computed mean values of C^2 .

In general, it may be concluded that for the Haille problem (fig. 6.1) the HMDM

(a) PDF MCS

(b) PDF MCS/Lognormal

Figure 6.5: Probability and lognormal density functions of C^2 , 2000 loops, $n=5$, Hailie problem

approach will result in lower mean values of C^2 , but higher standard deviations. The maximum values found for the CSMA method are higher. The modal effective mass at

Table 6.12: PEM computations of mean μ , standard deviation σ , and $\mu + 3\sigma$ value of C^2 , CSMS and HMDM, $n = 5$

Model	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_I$ (kg)	$\times f_{1I}$ (Hz)				
CSMA	9	[0.1,10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01,0.1]	4.50	1.09	7.76
HMDM	7	[0.1,10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01,0.1]	3.86	2.19	10.43

Table 6.13: Monte Carlo Simulation of Haille problem, CSMA and HMDM, $n=5$

Model	$\neq$ Samples	C_μ^2	$C_{1\sigma}^2$	$C_{\mu+3\sigma}^2$	C_{max}^2
CSMA	2000	4.41	1.54	9.03	16.44
HMDM	2000	3.21	1.61	8.08	8.99

the system frequency is f_s of the HMDM model is very dominant in comparison with the other modal effective masses. The frequency shift between the asparagus patch model of the load and the MDOF model of the source may be significant and will result in lower interface forces, and subsequently lower values of C^2 .

This is more or less a confirmation of the results found from the deterministic analyses and presented in Tables 6.3 and 6.7. The HMDM approach will produce, in general, lower values of C_μ^2 .

6.3. ESA IFLV STUDY

6.3.1. INTRODUCTION

This real life example is taken from the ESA study: "IFLV-Improvement of Force Limited Vibration Testing Methods for Equipment Instrument Unit Mechanical Verification" presented by Destefanis in [5]. The IFLV study facilitated a full test campaign (both sine and random) on a test system composed of a honeycomb panel (source) which supports an optical unit (MIRI) (load) and an electronic box (EBOX) (not considered) (see Fig. 6.6). Force measurement devices (FMDs) were installed at the mechanical interfaces between units and honeycomb plate (1210 mm x 910 mm). The test runs were performed both on the system and on the units (MIRI, EBOX) standalone, therefore collecting experimental evidence of the difference (in terms of mechanical interface forces) between soft mounted and hard mounted configurations. However, the numerical data of the interface forces were difficult to assess.

6.3.2. MASS PROPERTIES OF IFLV SYSTEM

The individual mass properties of the test setup are taken from [5]. These mass properties were extracted from the very detailed finite element models of the EBOX, MIRI, panel and FMD and are presented in Table 6.14, but the EBOX is further not considered in this section. The third column represents the mass properties of the hard-mounted MIRI and FMDs (FMDs between the MIRI and shaker slip table).

(a) Total configuration

(b) MIRI

Figure 6.6: IFLV total and MIRI test setup on shaker slip table, courtesy [5]

Table 6.14: Mass properties of individual items

Mass item	Total configuration (kg)	MIRI (kg)
Optical Unit MIRI	27.945	27.945
Electronic box EBOX	1.257	
Sandwich honeycomb panel	3.166	
Force Measurements Devices & plates	4.266	1.693
Total mass	36.634	29.639

6.3.3. DYNAMIC PROPERTIES OF IFLV SYSTEM & INDIVIDUAL PARTS

Numerical modal analyses were performed on the total test setup (with and without FMD's), the EBOX, the MIRI and the Honeycomb panel hard-mounted. The standard results are the undamped natural frequencies and associated modal effective masses. The modal effective masses are associated with the Z-axis, which is perpendicular to the sandwich panel. In the other directions no information about modal effective masses and associated natural frequencies were made available. The honeycomb panel had been supported at discrete points along the edges. The results of the modal analyses are given in Table 6.15. The values of the natural frequency and the modal effective mass of the honeycomb panel are only given for information, because later on the dynamic properties of the honeycomb panel are estimated in a probabilistic manner and other boundary conditions are applied.

Table 6.15: Mass & Modal properties [5]

Mass item	M (kg)	f_1 (Hz)	m_1 (kg)
Optical Unit MIRI	27.945	104.71	27.47
Sandwich honeycomb panel	3.166	287.74	1.50

6.3.4. INTERFACES MIRI/PANEL

The values of C^2 are applicable in the Z-direction, thus perpendicular to the panel, and in particular between the sandwich panel and MIRI instrument. The C^2 values, computed by the STDFS are taken from [5].

Applying the CSMA method the dynamic properties of the panel are computed with respect to the interface between panel and MIRI instrument. The theoretical natural frequency of the honeycomb panel supported at the midpoint, [3], is approximately 50 Hz. The corresponding modal effective mass varies between 1.50 and 3.0 kg (total mass panel 3.2kg). The deterministic CSMA method gives C^2 values in line with the other methods. The computational results of C^2 are presented in Table 6.16.

Table 6.16: Values of modal effective masses and C^2 (Z-dir) , $Q = 10$ [5]

	MIRI	Honeycomb panel		
load/Source	m_2 (kg)	m_1 (kg)	C^2	Remark
MIRI/Panel	27.47	1.50	1.10	STDFS [9]
	27.47	1.50	2.56	CTDFS [9]
	27.47 (105Hz)	1.50-3.0 (50Hz)	1.70-1.74	CSMA [10]
	Experience gained		2-5	Chang [4]

6.3.5. PROBABILISTIC ANALYSIS OF C^2

The probabilistic analysis of C^2 will be performed by two different probabilistic methods:

- The Rosenblueth $2k + 1$ point estimate moments (PEM) method with k stochastic design variables [8].

- The Monte Carlo Simulation methods [2]. The number of samples will be varied to investigate convergency properties.

STATISTICAL ANALYSIS WITH PEM

The deterministic asparagus patch model of the load (MIRI) is derived from the dynamic properties with respect to the interface between the load and the source (sandwich panel) taken from Table 6.15 and presented in Table 6.17. The residual mass is augmented with an artificial high natural frequency outside the frequency range of 20-2000Hz. The sum of the modal effective and residual masses is equal to the total mass of the MIRI, $M_l = 27.945\text{kg}$. The dynamic properties of the sandwich panel are assumed to be unknown.

Table 6.17: Asparagus patch model MIRI, Z-dir.

Modal effective mass (kg)	$m_{1l} = 27.47$	$m_{rl} = 0.475$
Natural frequency (Hz)	$f_{1l} = 104.71$	$f_{rl} = 2500$
Modal damping ratio ζ (-)	0.01-0.1	

The modal damping ratio is probabilistic and applicable to both the load and the source.

To start the probabilistic computation of C^2 , with the Rosenblueth $2k+1$ PEM method, the uniform distributions of the design variables of the source - the total mass M_s , the first fundamental natural frequency f_{1s} , the first primary modal effective mass m_{1s} and modal damping ζ , as presented in Table 5.2 - are used.

The results of the probabilistic computations, the mean μ , the standard deviation σ and the $\mu + 3\sigma$ of C^2 with additional variations of the distributions of the total M_s , the modal effective mass m_{1s} and the fundamental natural frequency f_{1s} are presented in Table 6.18. The intervals of the design variables m_{1s} , f_{1s} and ζ are well chosen. However, the estimation of the mass of the source M_s will be good as possible.

In Table 6.19 the influence of the modal damping ratio has been numerically investigated. Except for very high damping the modal damping ratio ζ has less influence on C^2 .

Compared to the estimated values of C^2 given in Table 6.16, it can be concluded from the probabilistic computations that a good estimation of the total mass of the source M_s is of importance so as to obtain more reliable figures of $C^2_{\mu+3\sigma}$. The distributions of the other design parameters were well chosen, but the values of $C^2_{\mu+3\sigma}$ are enveloping the STDFS calculations as given in Table 6.16. That means that the load is dynamically uncoupled from the source, and that is one of the major assumptions in the probabilistic approach.

A dynamically stiff source with a high interval of the natural frequency f_{1s} in conjunction with the proposed interval of the modal effective masses m_{1s} will, in general, increase the statistical estimation of C^2 , due to increasing dynamic coupling between load and source.

A low interval of natural frequency f_{1s} associated with a low interval of the modal effective mass m_{1s} of the source will lower the values of $C^2_{\mu+3\sigma}$. That means the load is supported by a flexible source with less dynamic interaction.

Table 6.18: Rosenblueth probabilistic computations of C^2 , Z-dir., $M_l = 27.945\text{kg}$ (ref. means reference values)

Design Variable	Distribution	Mean μ	Stand. dev. σ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
M_s (ref.)	$[0.1M_l, 10M_l]$	$5.05 M_l$	$2.858M_l$	6.34	1.24	10.05
M_s	$[0.1M_l, 1M_l]$	$0.505 M_l$	$0.260M_l$	3.55	0.63	5.44
M_s	$[0.05M_l, 0.15M_l]$	$0.1M_l$	$0.029M_l$	2.85	0.57	4.30
M_s	$0.113M_l$	$0.113M_l$	$0.0M_l$	2.58	0.58	4.32
M_s	$0.01M_l$	$0.01M_l$	$0.0M_l$	2.50	0.54	4.13
M_s (ref.)	$[0.05M_l, 0.15M_l]$	$0.1M_l$	$0.029M_l$			
m_{1s} (ref.)	$[0.4M_s, 0.6M_s]$	$0.5M_s$	$0.0577M_s$	2.85	0.57	4.30
m_{1s}	$[0.6M_s, 0.8M_s]$	$0.7M_s$	$0.0577M_s$	2.66	0.60	4.46
m_{1s}	$[0.2M_s, 0.4M_s]$	$0.3M_s$	$0.0577M_s$	2.24	0.33	3.22
M_s (ref.)	$[0.05M_l, 0.15M_l]$	$0.1M_l$	$0.029M_l$			
m_{1s} (ref.)	$[0.4M_s, 0.6M_s]$	$0.5M_s$	$0.0577M_s$			
f_{1s} (ref.)	$[0.5f_{1l}, 0.707f_{1l}]$	$0.6036f_{1l}$	$0.0598f_{1l}$	2.85	0.57	4.30
f_{1s}	$[0.2f_{1l}, 0.5f_{1l}]$	$0.35f_{1l}$	$0.0866f_{1l}$	2.11	0.24	2.85
f_{1s}	$[0.707f_{1l}, 0.8f_{1l}]$	$0.754f_{1l}$	$0.0268f_{1l}$	5.30	0.96	8.18
M_s (ref.)	$[0.05M_l, 0.15M_l]$	$0.1M_l$	$0.029M_l$			
m_{1s} (ref.)	$[0.4M_s, 0.6M_s]$	$0.5M_s$	$0.0577M_s$			
f_{1s} (ref.)	$[0.5f_{1l}, 0.707f_{1l}]$	$0.6036f_{1l}$	$0.0598f_{1l}$			
f_{2s}, f_{3s}, f_{4s} (ref.)	$2f_{1s}, 4f_{1s}, 6f_{1s}$			2.85	0.57	4.30
f_{2s}, f_{3s}, f_{4s}	$1.25f_{1s}, 1.5f_{1s}, 2f_{1s}$			3.29	0.15	3.75
f_{2s}, f_{3s}, f_{4s}	$1.5f_{1s}, 2f_{1s}, 3f_{1s}$			2.88	0.26	3.66

Table 6.19: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , varying the modal damping ratio ζ

$\neq$	M_s	f_{1s}	m_{1s}	ζ	MIRI Z-direction		
					C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
Samples	$\times M_l$	$\times f_{1l}$	$\times M_s$				
	(kg)	(Hz)	(kg)				
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	6.33	1.24	10.05
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.01]	7.05	1.32	11.02
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	6.47	1.26	10.23
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.1, 0.1]	5.50	0.96	8.39
9	[0.112, 0.114]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.58	0.58	4.32

The selection of the interval of the modal effective mass m_{1s} has less influence on the values of C^2 .

The clustering of the natural frequencies has less influence. In Table 6.19 it can be seen that the influence of the modal damping is not as sensitive as the estimation of the total mass of the source.

The deterministic computation of C^2 with the CSMA method (Table 6.16) is lower than the statistical mean values.

STATISTICAL ANALYSIS WITH MCS

The MCS computations of C^2 are performed varying the number of loops (one loop is four samples) - 100, 500 and 1000 respectively. The results of the probabilistic analysis is presented in Table 6.20.

Table 6.20: MCS computations of mean μ , standard deviation σ and $\mu + 3\sigma$ and maximum value of C^2 , CSMA

# Sampl.	M_s $\times M_l$ (kg)	f_{1s} $\times f_{1l}$ (Hz)	m_{1s} $\times M_l$ (kg)	ζ	MIRI Z-direction			
					C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	C_{max}
400	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	6.25	2.17	12.76	13.68
2000	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	6.02	2.05	12.18	15.75
4000	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	6.04	2.03	12.14	16.98
2000	[0.112, 0.114]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.60	0.60	4.41	4.13

The probability and cumulative density functions of the MCS analysis with 4000 samples are shown in Fig. 6.7, (a) and (b) respectively. The PDF shows high densities in the right tail, which causes a higher mean and standard deviation of C^2 .

The probabilistic analysis results of C^2 with MCS can be very well compared with the probabilistic analyses results obtained with the PEM method as shown in Table 6.19. Except for a tailored mass of the source ($M_s = 0.1132M_l$), no further detailed sensitivity analyses on C^2 will be performed. The additional computations are shown in Table 6.20 too, which are very similar to previous calculated values.

Figure 6.7: Probability density and cumulative density functions of C^2 using CSMA, 4000 samples

6.3.6. PROBABILISTIC COMPUTATION OF C^2 WITH HMDM

The values of C^2 will be estimated using the HMDM method in which the dynamic properties of the constrained source are unknown. The asparagus patch model of the deter-

ministic load has already been given in Table 6.17. The design variables are:

- The mass of the source M_s .
- The lowest system natural frequency f_{1s} .
- The modal damping ratio ζ is assumed to be applicable to both load and source and equal for all modes.

The intervals, the mean values and standard deviations of the design variables are presented in Table 5.3. The probability distribution of the design parameters is uniform. The statistical analyses will be performed with the PEM and MCS methods.

STATISTICAL ANALYSIS WITH PEM

To investigate the sensitivity of the number of SDOF elements n in the MDOF model of the HMDM mathematical model, n will be varied with $n = 5, 10, 20$. The results of the computations are shown in Table 6.21. The values of C^2 , etc decrease slowly by an

Table 6.21: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , HMDM

$\neq$ Samples	n	M_s $\times M_l$ (kg)	f_{1s} $\times f_{1l}$ (Hz)	MIRI Z-direction			
				ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
7	5	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.13	1.36	7.22
7	10	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.03	1.31	6.95
7	20	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.01	1.30	6.90
7	5	[0.113]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.08	0.01	1.13
7	20	[0.1, 10]	[0.4, 0.6]	[0.01, 0.1]	1.89	0.46	3.28
7	20	[0.1, 10]	[0.6, 0.8]	[0.01, 0.1]	3.90	1.80	9.31

increasing number of SDOF elements. There is a significant drop in the value of C^2 when the mass of the source is tailored to its actual value, which had already been noticed using the CSMA mathematical model.

Further the system natural frequency f_{1s} is varied to investigate the influence of that frequency on the values of C^2 . The lower the system frequency the lower C^2 . Increasing the system frequency will increase C^2 too.

STATISTICAL ANALYSIS WITH MCS

In this section the MCS computations of C^2 for the MIRI are performed. The HMDM mathematical model is used to carry out the probabilistic random dynamic response analyses. During one loop three design variables are varied assuming uniform distributions in a prescribed interval. The results for a number of loops-100, 500 and 1000 (three samples per loop mean 300, 1500 and 3000 samples respectively) - are presented in Table 6.22. The probability and cumulative density functions of the MCS analysis with 3000 samples are shown in Fig. 6.8, (a) and (b) respectively. The PDF shows high densities in the right tail, which causes a higher mean and standard deviation of C^2 .

Table 6.22: MCS computations of mean μ , standard deviation σ and $\mu + 3\sigma$ and maximum value of C^2 , HMDM

$\neq$	n	MIRI Z-direction						
		M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	C_{max}
Samples		$\times M_l$ (kg)	$\times f_{1l}$ (Hz)					
300	5	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.23	1.68	8.26	8.27
1500	5	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.02	1.66	8.01	9.47
3000	5	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	3.08	1.65	8.02	9.88
1500	5	[0.112, 0.114]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.08	0.02	1.14	1.13

Figure 6.8: Probability density and Cumulative density functions of C^2 using HMDM, 3000 samples, Haille problem

The computation results of the semi-empirical constant C^2 with both probabilistic methods, PEM and MCS, in combination with the HMDM mathematical model are the same. However, the Rosenblueth PEM is much more efficiently, but no maximum value of C^2 can be computed.

6.3.7. DISCUSSION CSMA/HMDM PROBABILISTIC ANALYSIS OF C^2

In this section the probabilistic computations of C^2 applying the two mathematical models CSMA and HMDM ($n=5$) are compared. Applying the CSMA method the standard intervals of the design variables (M_s , f_{1s} , m_{1s} and ζ) as given in Table 5.2 are used, and applying the HMDM method the standard intervals of the design variables (M_s , f_{1s} and ζ) as shown in Table 5.3 are used.

The computations of C^2 using two different mathematical models do not correlate very well. Deterministic predictions of C^2 presented in Table 5.6 already showed lower estimations using the HMDM method.

The mean values C_μ^2 computed with the CSMA method are, in general, twice the value of C_μ^2 when calculated with the HMDM method.

Table 6.23: PEM/MCS correlation of computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and $\max$ value of C^2 , $n = 5$

Dir.	$\neq$ Samples	Model	Method	C^2_μ	C^2_σ	$C^2_{\mu+3\sigma}$	$C^2_{\max}$
Z	9	CSMA	PEM	6.33	1.24	10.05	
Z	4000	CSMA	MCS	6.04	2.03	12.14	16.98
Z	7	HMDM	PEM	3.13	1.36	7.22	
Z	3000	HMDM	MCS	3.08	1.65	8.02	9.88

6.4. LDU/FSE/FRAM

6.4.1. INTRODUCTION

The Linear Drive Unit (LDU) is an Orbital Replacement Unit (ORU) of the International Space Station (ISS) programme. During the flight of the LDU to the ISS, it is connected to a Space Shuttle Orbiter by an adaptor plate and locking system. The LDU is connected to the adaptor plate by four points, which will be known as interface points. The configuration of the LDU, flight support equipment (FSE) adapter plate and active flight release attachment mechanism (FRAM) together form the integrated model. The integrated model is attached to the Orbiter at seven points, which have various constraint directions. The models are shown in Figure 6.9. The mass of the LDU (load) is $M_l = 113.85$ kg and the remaining FSE and FRAM parts (source) make up $M_s = 187.33$ kg. The modal effective masses of the significant modes and the C^2 of the semi-empirical force limits equations (Eq 4.8) are given in the next sections. The dynamic properties of

Figure 6.9: LDU-FSE-FRAM FEM in launch configuration [6]

the FSE/FRAM (source) are not presented in [6], hence are unknown.

6.4.2. DYNAMIC PROPERTIES LDU AND VALUE C^2

The natural frequencies and associated modal effective masses of the first three dominant modes of the LDU, fixed at the interface between LDU and FSE (see Fig. 6.9), are

taken from the paper of Fitzpatrick [6] and presented in Table 6.24. The Z-direction is perpendicular to the mounting plane

Table 6.24: Dynamic properties LDU, courtesy [6]

Mode $\neq$	Frequency (Hz)	Modal effective mass		
		X-dir (kg)	Y-dir (kg)	Z-dir (kg)
1	59.0	0.4	0.0	36.0
4	75.0	0.1	49.9	0.1
7	92.7	27.9	0.1	18.2

6.4.3. C^2 FROM LITERATURE

The value C^2 derived with the STDFS method and the scaled force power spectral density response at the interface between LDU/FSE were taken from [6] and presented in Table 6.25. The scaled random interface force is computed from the enveloped random acceleration specification multiplied by the squared magnitude of the apparent mass of the LDU (load). The PSD acceleration at the four interface points between the LDU and

6

Figure 6.10: Scaled random interface force procedure, courtesy [6]

FSE/FRAM are represented by the four dotted curves. The final random acceleration specification is the envelope of these four curves. This is illustrated in Fig. 6.10 a). The drawn curve in Fig. 6.10 b) represents the PSD of the interface force and corresponds to the enveloped random acceleration. Applying equation (5.1) the value C^2 can be established.

6.4.4. PROBABILISTIC COMPUTATIONS OF C^2 WITH CSMA

The asparagus patch model of the load (LDU) is derived from the dynamic properties with respect to the interface between the load and the source (FSE/FRAM) taken from Table 6.24 and presented in Table 6.26. The residual mass is augmented with an artificial

Table 6.25: Values of C^2 and n , computed with the STDFS method and FEA method, are taken from [6]

	STDFS (Q=10)	X-dir	Y-dir	Z-dir
C^2	3.48	2.5	4.8	2.7
n	2	2	2	1.5

high natural frequency outside the frequency range of 20-2000Hz. The sum of the modal effective and residual masses is equal to the total mass of the LDU, $M_l = 133.85\text{kg}$.

Table 6.26: Asparagus patch model LDU, X,Y and Z-dir., $M_l = 113.85\text{ kg}$

X-direction			
Modal effective mass (kg)	$m_{1l} = 29.7$		$m_{rl} = 85.95$
Natural frequency (Hz)	$f_{1l} = 92.7$		$f_{rl} = 2500$
Y-direction			
Modal effective mass (kg)	$m_{1l} = 49.9$		$m_{rl} = 63.95$
Natural frequency (Hz)	$f_{1l} = 75.0$		$f_{rl} = 2500$
Z-direction			
Modal effective mass (kg)	$m_{1l} = 36.0$	$m_{2l} = 18.2$	$m_{rl} = 59.65$
Natural frequency (Hz)	$f_{1l} = 59.0$	$f_{2l} = 92.7$	$f_{rl} = 2500$

Two probabilistic methods are applied to compute C^2 :

- The Monte Carlo Simulation (MCS) method [1]
- The Point estimates for probability moments [8]

We start the probabilistic computations of C^2 with the Monte Carlo Simulation (MCS) approach with 2000 runs and in each run four uniformly distributed random samples. Hence in total 8000 samples. The MCS method may be interpreted as an optimization method trying to find the maximum value of C^2 subjected to constraint functions (the intervals of the design variables) [11].

Most of the MCS computations are repeated applying the Rosenblueth point estimation method, which means nine samples. Variations of the design variables - the total mass of the source M_s and the modal viscous damping ratio ζ - are investigated. The presented results of the computations are the mean μ , the standard deviation σ and the $\mu + 3\sigma$ values of C^2 .

In Table 6.27 we present the mean μ , the standard deviation 1σ , the $\mu + 3\sigma$ and maximum values of C^2 when the number of samples is 8000. Applying the Rosenblueth PEM method the probabilistic results with nine samples are given in Table 6.28, Table 6.29, Table 6.30, Table 6.31, Table 6.32 and Table 6.33. The design variables M_s and ζ vary. The total value of the source mass M_s is: a) the standard interval and b) tailored to values of M_s between $[1.64 - 1.65]M_l$ (actual value $M_s = 1.645M_l$). The value of the modal damping ratio ζ varies too: a) the standard interval and b) a constant value. The computations of C^2 are done in X-, Y- and Z-direction. When the mass of the source is tailored to the actual value the values of C^2 are lower than values of C^2 when applying the standard interval of M_s . There is little influence of the modal damping ratio ζ on the C^2 .

Table 6.27: MCS results of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , 8000 samples

						X-dir.	
M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	C_{max}^2
$\times M_l$	$\times f_{1l}$	$\times M_s$					
(kg)	(Hz)	(kg)					
[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	1.76	0.45	3.11	4.70
[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	1.78	0.48	3.21	4.12
[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	1.57	0.24	2.28	2.43
[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	1.58	0.25	2.33	2.31
						Y-dir.	
M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	C_{max}^2
$\times M_l$	$\times f_{1l}$	$\times M_s$					
(kg)	(Hz)	(kg)					
[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.44	0.87	5.06	7.44
[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	2.48	0.93	5.27	7.02
[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.06	0.44	3.39	3.62
[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	2.09	0.46	3.48	3.39
						Z-dir.	
M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	C_{max}^2
$\times M_l$	$\times f_{1l}$	$\times M_s$					
(kg)	(Hz)	(kg)					
[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.13	0.65	4.08	5.92
[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	2.15	0.69	4.22	5.40
[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	1.85	0.34	2.86	3.01
[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	1.87	0.36	2.93	2.90

The distributions of C^2 are calculated with the MCS method and shown in Fig. 6.11. The standard intervals and uniform distributions of the design variables are taken (Table 5.2). The distribution is similar to the Lognormal distribution [1]. Assuming the Lognormal distribution for C^2 the probability $\text{Prob}(C^2 \leq C_{\mu+3\sigma}^2) = 0.9915$. In general, the values $C_{\mu+3\sigma}^2$ do not cover the computed maximum values C_{max}^2 , but the maximum values have a very low density.

The computations of C^2 with the Rosenblueth PEM method are presented in Table 6.28. The number of samples is 9. The mean μ , the standard deviation 1σ and $\mu + 3\sigma$ values of C^2 are computed. No maximum values could be obtained. The results presented in Table 6.28 are comparable to those presented in Table 6.27. The accuracy achieved by the Rosenblueth PEM method is comparable to the MCS method with 8000 samples. The modal damping ratio ζ has a minor influence, but tailoring the total mass of the source M_s is very beneficial and will improve to computations of C^2 .

In Table 6.29 the distribution of the source natural frequency f_{2s}, f_{3s}, f_{4s} is different from the standard proposed values given in eq. (5.7). This leads to an increase of the values of C^2 .

In the Table 6.30 and 6.31 the effect of the variation of the interval of the dominant

(a) X-dir.

(b) Y-dir.

(c) Z-dir

Figure 6.11: MIRI, distributions of C^2 in X-, Y, Z-directions, with standard intervals and distributions of design variables (Table 5.2)

modal effective mass m_{1s} is investigated. Decreasing the interval has minor influence, but increasing the interval compared to eq. (5.11) has a major increasing impact on the values of C^2 . A high modal effective mass will increase dynamic responses at the interface.

In the Tables 6.32 and 6.33 the intervals of the first natural frequency of the source f_s vary. The effect of decreasing or increasing the interval, in comparison with eq. (5.6) leads to an increase of the computed values of C^2 .

It emerged from previous tables that the total mass of the source M_s is an important design variable too. It is worthwhile taking time to find a good estimation of M_s .

6.4.5. COMPARISON PROBABILISTIC ANALYSIS WITH USE OF CSMA

The computation of C^2 with the CSMA method in conjunction with the MCS and PEM method is presented in Table 6.34. The standard design variables and associated intervals are given in Table 5.2. The computations using the CSMA method in combination with the MCS and PEM methods and taking into account the probabilistic nature of

Table 6.28: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2

					X-direction		
$\neq$ Samples	M_s $\times M_l$ (kg)	f_{1s} $\times f_{1l}$ (Hz)	m_{1s} $\times M_s$ (kg)	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	1.80	0.38	2.95
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	1.81	0.39	2.98
9	[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	1.56	0.22	2.23
9	[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	1.57	0.22	2.25
					Y-direction		
$\neq$ Samples	M_s $\times M_l$ (kg)	f_{1s} $\times f_{1l}$ (Hz)	m_{1s} $\times M_s$ (kg)	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.49	0.74	4.72
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	2.54	0.76	4.82
9	[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.04	0.41	3.26
9	[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	2.06	0.42	3.32
					Z-direction		
$\neq$ Samples	M_s $\times M_l$ (kg)	f_{1s} $\times f_{1l}$ (Hz)	m_{1s} $\times M_s$ (kg)	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.21	0.55	3.88
9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	2.23	0.56	3.92
9	[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	1.80	0.30	2.71
9	[1.64, 1.65]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.05, 0.05]	1.83	0.31	2.75

Table 6.29: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $f_{2s} = 1.5f_{1s}$, $f_{3s} = 2f_{1s}$, $f_{4s} = 2.5f_{1s}$

Dir.	$\neq$ Samples	M_s $\times M_l$ (kg)	f_{1s} $\times f_{1l}$ (Hz)	m_{1s} $\times M_s$ (kg)	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
X	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	2.74	1.03	5.84
Y	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	4.20	0.42	5.47
Z	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.4, 0.6]	[0.01, 0.1]	3.73	1.14	7.16

the computations the statistical values of C^2 correlate very well. The Rosenblueth PEM method is a very efficient to perform the probabilistic response analysis using the CSMA method. However, no maximum value of C^2 can be computed. Deterministic values of C^2 were presented in Table 6.25 [6], where in the Y-direction C^2 shows a maximum value. In the case of probabilistic calculations the same tendency is noticed. In general, the probabilistic results of C^2 envelop the values given in Table 6.25.

Table 6.30: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $m_{1s} \in [0.2, 0.4]M_s$

Dir.	$\neq$	M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)	$\times M_s$ (kg)				
X	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.2, 0.4]	[0.01, 0.1]	1.50	0.19	2.08
Y	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.2, 0.4]	[0.01, 0.1]	1.95	0.39	3.12
Z	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.2, 0.4]	[0.01, 0.1]	1.75	0.29	2.62

Table 6.31: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $m_{1s} \in [0.6, 0.8]M_s$

Dir.	$\neq$	M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)	$\times M_s$ (kg)				
X	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.6, 0.8]	[0.01, 0.1]	2.63	1.04	5.75
Y	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.6, 0.8]	[0.01, 0.1]	3.89	1.86	9.48
Z	9	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.6, 0.8]	[0.01, 0.1]	3.33	1.46	7.69

Table 6.32: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $f_{1s} \in [0.3, 0.5]f_{1l}$

Dir.	$\neq$	M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)	$\times M_s$ (kg)				
X	9	[0.1, 10]	[0.3, 0.5]	[0.4, 0.6]	[0.01, 0.1]	1.82	0.65	3.77
Y	9	[0.1, 10]	[0.3, 0.5]	[0.4, 0.6]	[0.01, 0.1]	3.20	1.10	6.50
Z	9	[0.1, 10]	[0.3, 0.5]	[0.4, 0.6]	[0.01, 0.1]	2.58	0.84	5.10

Table 6.33: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $f_{1s} \in [\frac{1}{2}\sqrt{2}, 0.9]f_{1l}$

Dir.	$\neq$	M_s	f_{1s}	m_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)	$\times M_s$ (kg)				
X	9	[0.1, 10]	$[\frac{1}{2}\sqrt{2}, 0.9]$	[0.4, 0.6]	[0.01, 0.1]	2.14	0.89	4.81
Y	9	[0.1, 10]	$[\frac{1}{2}\sqrt{2}, 0.9]$	[0.4, 0.6]	[0.01, 0.1]	4.05	1.32	8.02
Z	9	[0.1, 10]	$[\frac{1}{2}\sqrt{2}, 0.9]$	[0.4, 0.6]	[0.01, 0.1]	3.20	1.19	6.78

6.4.6. PROBABILISTIC COMPUTATIONS OF C^2 WITH HMDM

The values of C^2 will be calculated using the HMDM method in which the dynamic properties of the constrained source are unknown. The asparagus patch model of the deterministic load has already been given in Table 6.26. The design variables are:

- The mass of the source M_s .

Table 6.34: PEM/MCS correlation of computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and $\max$ value of C^2 , CSMA

Dir.	$\neq$ Samples	Model	Method	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	$C_{\max}^2$
X	9	CSMA	PEM	1.80	0.38	2.95	
Y	9	CSMA	PEM	2.49	0.76	4.72	
Z	9	CSMA	PEM	2.21	0.55	3.88	
X	8000	CSMA	MCS	1.76	0.45	3.11	4.70
Y	8000	CSMA	MCS	2.44	0.87	5.06	7.44
Z	8000	CSMA	MCS	2.13	0.65	4.08	5.92

- The lowest system natural frequency f_{1s} .
- The modal damping ratio ζ is assumed to be applicable to both load and source and is equal for all modes.

The intervals, the mean values and standard deviations of the design variables are presented in Table 5.3. The probability distribution of the design parameters is uniform. Two kinds of computations performed to determine the statistical values of C^2 :

- The Rosenblueth point estimates of probability moments (PEM)
- The Monte Carlo Simulation (MCS) method

ROSENBLUETH POINT ESTIMATE MOMENTS METHOD

The Rosenblueth PEM is applied to compute with HMDM the values of the semi-empirical constants C^2 with a number of samples $2k + 1 = 7$ (3 design variables). This is done with the HMDM with 5, 20 and 20 discrete masses, $n = 5, 10, 20$, respectively (see Fig. 5.3). The statistics of the three design variables are given in Table 5.3. The results of the PEM computations are presented in Table 6.35.

Table 6.35: Computed values C^2 , Rosenblueth PEM, $2k+1=7$

Parameter	PEM, n=5			PEM, n=10			PEM, n=20		
	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir	X-dir	Y-dir	Z-dir
C_μ^2	1.60	1.99	2.62	1.59	1.97	2.58	1.58	1.91	2.58
C_σ^2	0.39	0.63	1.31	0.38	0.62	1.24	0.38	0.59	1.24
$C_{\mu+3\sigma}^2$	2.78	3.88	6.56	2.73	3.82	6.31	2.71	3.67	6.31

Table 6.35 shows a slight decrease of the computed semi-empirical constants when the number of degrees of freedom n is increasing. Therefore to perform sensitivity calculations more efficiently later on the number of degrees of freedom of the source is taken to be $n = 5$.

The following sensitivity analyses have been performed:

- The modal damping ratio ζ is a constant; $\zeta_\mu = 0.05$ and $\zeta_\sigma = 0.0$. The results of the computations are given in Table 6.36.

- The actual ratio between the mass of the source/load is constant; $M_s/M_l = 1.645$. The standard deviation is zero. The results of the computations are shown in Table 6.37.
- The interval of the system frequency $f_{1s} = [0.4, 0.6]f_{1l}$, $f_{s\mu} = 0.5f_{1l}$, $f_{s\sigma} = 0.0577f_{1l}$. The results of the computations varying the system frequency are shown in Table 6.38.
- The interval of the system frequency $f_{1s} = [0.6, 0.8]f_{1l}$, $f_{s\mu} = 0.7f_{1l}$, $f_{s\sigma} = 0.0577f_{1l}$. The results of the computations varying the system frequency are given in Table 6.39.

Table 6.36: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $\zeta = 0.05$, $n = 5$

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$	$\times f_{1l}$				
		(kg)	(Hz)				
X	7	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	0.05	1.62	0.40	2.82
Y	7	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	0.05	2.02	0.66	3.99
Z	7	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	0.05	2.72	1.38	6.87

6

Table 6.37: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $M_s = 1.645M_l$, $n = 5$

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$	$\times f_{1l}$				
		(kg)	(Hz)				
X	7	1.645	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.27	0.11	1.60
Y	7	1.645	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.45	0.18	1.99
Z	7	1.645	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.69	0.32	2.55

Table 6.38: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $f_{1s} = [0.4, 0.6]f_{1l}$, $n = 5$

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$	$\times f_{1l}$				
		(kg)	(Hz)				
X	7	[0.1, 10]	[0.4, 0.6]	[0.01, 0.1]	1.22	0.12	1.57
Y	7	[0.1, 10]	[0.4, 0.6]	[0.01, 0.1]	1.39	0.20	2.00
Z	7	[0.1, 10]	[0.4, 0.6]	[0.01, 0.1]	1.42	0.25	2.16

Table 6.39: Rosenblueth PEM computations of mean μ , standard deviation σ and $\mu + 3\sigma$ value of C^2 , $f_{1s} = [0.6, 0.8]f_{1l}$, $n = 5$

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)				
X	7	[0.1, 10]	[0.6, 0.8]	[0.01, 0.1]	1.54	0.59	3.21
Y	7	[0.1, 10]	[0.6, 0.8]	[0.01, 0.1]	2.19	0.85	4.75
Z	7	[0.1, 10]	[0.6, 0.8]	[0.01, 0.1]	2.23	2.09	8.50

DISCUSSION OF PEM RESULTS

In the Z-direction the highest values of C^2 were computed. This is in accordance with values presented in Table 6.25.

The following conclusions can be drawn:

- Keeping the modal damping ratio $\zeta = 0.05$ had hardly any influence on the values of C^2 as shown in Table 6.36.
- Tailoring the total mass of the source to the actual value, the values of C^2 drop considerably!. This is presented in Table 6.37.
- A weak supporting structure (source) shows low values of C^2 , while a stiff source will increase these values, shown in Tables 6.38 and 6.39 respectively.
- The computations are very straightforward when the Rosenblueth PEM method is applied, but no maximum value of C^2 can be obtained.

MONTE CARLO SIMULATION

The Monte Carlo Simulation (MCS) method will be applied to calculate the mean value C_μ^2 , the standard deviation C_σ^2 , the mean plus 3σ value $C_{\mu+3\sigma}^2$ and maximum values C_{max}^2 of the value C^2 given the intervals and distributions of the design variables as provided in Table 5.3. 100, 200 and 1000 loops are performed and within one loop the three design variables are varied using uniform distributed random generator between [0,1]. In fact 300, 600 and 3000 samples are taken. The number of discrete masses of the HMDM modeling of the source is $n = 5$. This was already decided in the previous section.

The MCS analysis results obtained by HMDM method are presented for 100, 200 and 1000 loops in respectively the Tables 6.40, 6.41 and 6.42 respectively.

The distribution functions of the semi-empirical constant C^2 computed with 1000 loops are shown in Fig. 6.12. The tails of the probability density functions show relatively high densities, which result in a high first and second moment of C^2 .

The sensitivity of the mass of the source assuming $M_s = [1.64, 1.65]M_l$ has been investigated and shown in Table 6.43.

6.4.7. DISCUSSION OF PROBABILISTIC ANALYSIS USING HMDM

The probabilistic analyses are done with the HMDM model. The computational results of the statistical values of C^2 with the PEM and MCS methods are compared in Table

Table 6.40: MCS computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and $\max$ value of C^2 , $n = 5$, 100 loops

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	$C_{\max}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)					
X	300	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.67	0.61	3.49	4.09
Y	300	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	2.07	0.84	4.60	6.13
Z	300	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	2.03	0.95	5.03	4.79

Table 6.41: MCS computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and $\max$ value of C^2 , $n = 5$, 200 loops

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	$C_{\max}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)					
X	600	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.61	0.53	3.19	4.00
Y	600	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.96	0.73	4.16	5.45
Z	600	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	2.02	0.81	4.45	4.84

Table 6.42: MCS computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and $\max$ value of C^2 , $n = 5$, 1000 loops

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	$C_{\max}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)					
X	3000	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.66	0.57	3.38	4.39
Y	3000	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.99	0.83	4.50	6.08
Z	3000	[0.1, 10]	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	2.04	0.80	4.44	5.79

Table 6.43: MCS computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and $\max$ value of C^2 , $M_s = 1.645M_l$, $n = 5$

Dir.	$\neq$	M_s	f_{1s}	ζ	C_μ^2	C_σ^2	$C_{\mu+3\sigma}^2$	$C_{\max}^2$
	Samples	$\times M_l$ (kg)	$\times f_{1l}$ (Hz)					
X	300	1.645	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.27	0.12	1.62	1.56
Y	300	1.645	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.43	0.18	1.98	1.87
Z	300	1.645	$[0.5, \frac{1}{2}\sqrt{2}]$	[0.01, 0.1]	1.59	0.36	2.71	2.71

6.44. The standard design variables and intervals are given in Table 5.3. Applying the PEM method gives a maximum $\mu + 3\sigma$ value of C^2 in the Z-direction, which is not in line with the deterministic analysis of C^2 provided in Table 6.25. Probably one of the samples is very dominant and increases the mean and the standard deviation of C^2 considerably.

Figure 6.12: LDU, PDF functions C^2 , HMDM method, X-, Y- and Z-directions

Table 6.44: PEM/MCS correlation of computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and max value of C^2 , $n = 5$

Dir.	$\neq$ Samples	Model	Method	C_{μ}^2	C_{σ}^2	$C_{\mu+3\sigma}^2$	C_{max}^2
X	7	HMDM	PEM	1.60	0.39	2.78	
Y	7	HMDM	PEM	1.99	0.63	3.88	
Z	7	HMDM	PEM	2.62	1.31	6.56	
X	3000	HMDM	MCS	1.66	0.57	3.38	4.39
Y	3000	HMDM	MCS	1.99	0.83	4.50	6.08
Z	3000	HMDM	MCS	2.04	0.80	4.44	5.79

6.4.8. DISCUSSION CSMA/HMDM PROBABILISTIC ANALYSIS OF C^2

In this section the probabilistic computations of C^2 applying the two mathematical models CSMA and HMDM ($n=5$) are compared. Applying the CSMA method the standard

intervals of the design variables (M_s , f_{1s} , m_{1s} and ζ) as given in Table 5.2. Applying the HMDM method the standard intervals of the design variables (M_s , f_{1s} and ζ) as shown in Table 5.3 are used. All these calculated values of C^2 are collected in Table 6.45. In Ta-

Table 6.45: PEM/MCS correlation of computations of mean μ , standard deviation σ , $\mu + 3\sigma$ and max value of C^2 , $n = 5$

Dir.	≠ Samples	Model	Method	C_{μ}^2, C^2	C_{σ}^2	$C_{\mu+3\sigma}^2$	C_{max}^2
X		[6]	deterministic	2.5			
X	9	CSMA	PEM	1.80	0.38	2.95	
X	8000	CSMA	MCS	1.76	0.45	3.11	4.70
X	7	HMDM	PEM	1.60	0.39	2.78	
X	3000	HMDM	MCS	1.66	0.57	3.38	4.39
Y		[6]	deterministic	4.8			
Y	9	CSMA	PEM	2.49	0.76	4.72	
Y	8000	CSMA	MCS	2.44	0.87	5.06	7.44
Y	7	HMDM	PEM	1.99	0.63	3.88	
Y	3000	HMDM	MCS	1.99	0.83	4.50	6.08
Z		[6]	deterministic	2.7			
Z	9	CSMA	PEM	2.21	0.55	3.88	
Z	8000	CSMA	MCS	2.13	0.65	4.08	5.92
Z	7	HMDM	PEM	2.62	1.31	6.56	
Z	3000	HMDM	MCS	2.04	0.80	4.44	5.79

ble 6.25 the deterministic values of C^2 are shown in the X-, Y- and Z-direction and taken from [6]. The highest value of C^2 in the Y-direction. This in accordance with values presented in Table 6.25. In Fig. 6.9 the directions of the axes are shown. The direction of Z-direction is perpendicular to the mounting plane.

In general, the $C_{\mu+3\sigma}$ values of the semi-empirical constant show the same tendency as the deterministic values of C^2 as provided in Table 6.25, except the computations in the Z-direction of the LDU applying the Rosenblueth PEM method. The results of the computations using either the CSMA or HMDM models, and the probabilistic approaches PEM and MCS compare satisfactorily. The results obtained by the CSMA method are common to the deterministic calculated values: the C^2 in the Y-directions is greater than the C^2 in the X- and Z-direction.

The following general comparison between the CSMA and HMDM approach can be made:

- The random acceleration vibration specification W_{AA} (g^2/Hz) is specified at the interface between the load and the source. Using both the CSMA and the HMDM methods the random response characteristics at that interface are reconstructed taking into account the vibration absorber effects of the load.
- For both methods the mathematical description of the load is similar: an asparagus patch model built up of parallel placed SDOF elements of which the discrete masses are the modal effective masses and the spring constants represent the natural frequencies of the load fixed at the interface between load and supporting

structure (source). The asparagus patch models are illustrated in the Fig. 5.1 and 5.3.

- Probabilistic analyses to compute the statistical values of C^2 with the HMDM method are, in general, lower than the computed statistical values with the CSMA method. This may be illustrated by viewing the Tables 5.6, 6.12, 6.13, 6.23 and 6.45. The dominant modal effective mass of the load/source combination is, in general, at the system frequency of the HMDM model and that will result in high random interface accelerations, but shifted from the peak(s) of the apparent mass of the load. The resulting random interface force will be lower than when the peaks of the acceleration at the interface and the apparent mass coincide. This is illustrated in Fig. 6.4.
- The CSMA method is much more efficient regarding the computer time spent to perform the random response analyses with Rosenblueth Point Estimates of Probability Moments (PEM) [8] and the Monte Carlo Simulation (MCS) [2]. The number of SDOF elements n in the HMDM model (Fig. 5.3) had to be minimized. Computer time increases considerably when $n > 5$.
- The asparagus patch model of source is described by four design variables. The lowest natural frequency f_{1s} and associated significant modal effective mass m_{1s} are prescribed. That is not the case when the MDOF model is used within the HMDM approach. The prescribed natural frequency is the lowest natural frequency of the complete combination of the load and the source. The associated significant modal effective mass is not prescribed but a consequence of the mass distribution and vibration mode of the complete system. The direct control about the design variables and the dynamic properties of the probabilistic asparagus patch model of the source are preferred to the more indirect control of the dynamic properties of the probabilistic MDOF model of the source within the HMDM approach. Another advantage of the CSMA approach in comparison with the HMDM approach is that less computer time is needed to perform the random response analysis. This is of importance when the MCS computations are carried out.

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7

CONCLUSIONS, RECOMMENDATIONS AND GUIDELINES

7.1. CONCLUSIONS

7.1.1. NOTCHING PREDICTION METHODS

A number of notching prediction methods have been reviewed and discussed in detail and a summary of the reviewed notching prediction methods is given in Table 7.1. In general, one can say that the accuracy of the prediction methods is strongly dependent on the mathematical models used.

Table 7.1: Summary of reviewed notching methods

#	Notching method	Reference(s)	
1	Curtis method	[3]	Response limiting
2	Mass participation	[10]	3σ Force limiting (QSL)
3	Enclosure method	CNES, [5]	3σ Force limiting (QSL)
4	1/2 octave band	TAS-F, [5]	3σ Force limiting (QSL)
5	STDFS	[8]	PSD force limiting
6	CTDFS	[4]	PSD force limiting
7	JAXA method	[6]	PSD force limiting

It is very difficult to say which of the mentioned notching prediction methods of defining adapted test specifications is the best approach. In practice response limited and force limited vibration test methods are more or less equally applied in the space-craft structure design and verification approach.

The methods 2,3 and 4 are strongly related to the specified quasi-static load (QSL) factors.

To compute the notch levels with the methods 1,2,3 and 4 a finite element model (FEM) of the load is needed to calculate modal and response characteristics. Applying method 2 only the natural frequencies and associated modal effective masses are required and this method is very straightforward to use.

The application of the methods 5 and 6 to estimate PSD force limits is very simple because the dynamic model of the load-source combination is represented by simplified two degrees of freedom systems to estimate values of C^2 . The use of the STDFS is preferred to use of the CTDFS.

The JAXA method is more complex to apply than the STDFS or CTDFS methods. For both the load and the source the dynamic masses at the interface are needed. The apparent masses of load and source can be derived from FE models.

The random acceleration spectrum and random force spectrum at the interface between the load and the source need to be known to calculate the constant C^2 . These random responses at the interface can only be analysed if the dynamic properties of both the load and source can be made available.

To perform PSD force limited vibration testing force gauges are needed between the load and the shaker table to measure the interface forces during the vibration test.

7.1.2. PROBABILISTIC ANALYSIS OF C^2

The constant C^2 can be computed using the semi-empirical method (SEM) in combination with the following expression:

$$C^2 = \frac{W_{FF_{max}}}{M_l^2 W_{AA_{max}}},$$

where M_l is the rigid body mass of the load, $W_{AA_{max}}$ is the maximum value of the random acceleration and $W_{FF_{max}}$ is the maximum random force at the interface between load and source. The random accelerations and forces are strongly dependent on the dynamic interaction between load and source, therefore the dynamic characteristics of both load and source need to be known. However, in most cases the mass and dynamic characteristics of the source are unknown, because the design and verification of the source are under the responsibility of another subcontractor and, in general, not made available.

Within the frame of this thesis the unknown source is replaced by probabilistic models of the source, applying design rules common in the design of spacecraft structures in combination with assumed uniform probability distribution functions and intervals of the design variables, such as mass, natural frequency, modal effective mass and damping. Two probabilistic models are constructed to replace the unknown source:

- the Coupled System Modal Approach (CSMA) [9]
- Hybrid Mathematical Dynamic Modeling (HMDM)

With the aid of the probabilistic models of the source in combination (coupled) with the deterministic asparagus model of the load the specified random interface acceleration can be successfully reconstructed and associated random interface forces can be derived using the apparent or dynamic mass of the load and subsequently C^2 is calculated.

The following general comparison between the CSMA and HMDM approach can be made:

- The random acceleration vibration specification W_{AA} (g^2/Hz) is specified at the interface between the load and the source. Using both the CSMA and the HMDM methods the random response characteristics at that interface are reconstructed taken into account the vibration absorber effects of the load.
- For both methods the mathematical description of the test item (load) is similar; an asparagus patch model built up of parallel placed SDOF elements of which the discrete masses are the modal effective masses and the spring constants represent the natural frequencies of the load fixed at the interface between load and supporting structure (source). The asparagus patch models are illustrated in the Fig. 5.1 and 5.3.
- In general, probabilistic analyses to compute the statistical values of C^2 with the HMDM method give lower values for C^2 than the computed statistical values with the CSMA method. The dominant modal effective mass is at the system frequency f_{1s} of the load/source combination. The maximum value of the apparent mass $|\mathbf{M}_I(f)_{max}$ will occur at the natural frequency of the load with dominant modal effective mass m_{1I} (see Fig. 5.3). The shift in the maximum response of the system at the interface of the load and the source and maximum value of the apparent mass $|\mathbf{M}_I(f)_{max}$ will lower the interface force, thus C^2 . This is illustrated in Fig. 6.4. This may be illustrated by viewing the Tables 5.6, 6.12, 6.13, 6.23 and 6.45. This is also demonstrated by comparing values of C^2 in Tables 6.3 and 6.7.
- The CSMA method is much more efficient regarding the computation time spent to perform the random response analyses with Rosenblueth Point Estimates of Probability Moments (PEM) [7] and the Monte Carlo Simulation (MCS) [1]. The number of SDOF elements n in the HMDM model (Fig. 5.3) had to be minimized. Computation time increases considerably when the number of SDOF elements is $n > 5$.
- Within the CSMA approach the intervals and statistical moments of all design variables of the probabilistic asparagus patch model of the source are given in Table 7.2. The intervals and statistical moments of all design variables of the probabilistic MDOF model of the source are given in Table 7.3. The lowest natural frequency f_{1s} and associated significant modal effective mass m_{1s} are prescribed. That is not the case when the MDOF model is used within the HMDM approach. The prescribed natural frequency is the lowest natural frequency of the complete combination of the load and the source. The associated significant modal effective mass is not prescribed but a consequence of the mass distribution and vibration mode of the complete system. The direct control of the design variables, and the dynamic properties of the probabilistic asparagus patch model of the source (CSMA) is more preferred to the more indirect control of the dynamic properties of the probabilistic MDOF model of the source within the HMDM approach.

- With the MCS method, besides the statistical moments, the distribution function and the maximum value of C^2 can be computed.

Table 7.2: CSMA approach, design variables in spacecraft structures design

Description	Symbol	Interval	Mean μ	St. dev. σ
Mass (kg)	M_s	$[0.1, 10)M_l$	$5.05M_l$	$2.858M_l$
Natural frequency (Hz)	f_{1s}	$[0.5, 0.5\sqrt{2}]f_{1l}$	$0.6036f_l$	$0.0598f_l$
Modal effective mass (kg)	m_{1s}	$[0.4, 0.6]M_s$	$0.5000M_s$	$0.0577M_s$
Modal damping ratio (-)	ζ	$[0.01, 0.1]$	0.055	0.0260
Mod. effect. mass (kg)	$m_{rs} = 0.05M_s, m_{2s} = 0.5\Delta m, m_{3s} = 0.3\Delta m, m_{4s} = 0.2\Delta m$ $\Delta m = M_s - (m_{1s} + m_{rs})$			
Nat. frequencies (Hz)	$f_{2s} = 2.0f_{1s}, f_{3s} = 4.0f_{1s}, f_{4s} = 6.0f_{1s}$			

Table 7.3: HMDM approach, design variables in spacecraft structures design

Description	Symbol	Interval	Mean μ	St. dev. σ
Mass (kg)	M_s	$[0.1, 10)M_l$	$5.05M_l$	$2.858M_l$
Natural (system) frequency (Hz)	f_{1s}	$[0.5, 0.5\sqrt{2}]f_{1l}$	$0.6036f_l$	$0.0598f_l$
Modal damping ratio (-)	ζ	$[0.01, 0.1]$	0.055	0.0260
SDOF elements	n	$[5, 10, 20]$		

7

7.1.3. CONCLUSIONS PROBABILISTIC ANALYSIS

The following conclusions about the probabilistic analysis can be drawn:

- In the previous section it was already discussed and concluded that the Coupled Analysis Modal Analysis (CSMA) method is preferred to the Hybrid Mathematical Dynamic Modeling (HMDM) technique.

When the dynamic properties of the load are known and the dynamic properties of the source are unknown the recommended values for the design variables, intervals and associated uniform distributions, which are presented in Table 7.2, can be applied to perform the probabilistic computations of the semi-empirical constant C^2 . That means that the dynamic characteristics of both the load and the source are properly separated and the dynamic coupling minimized. The design variables presented in Table 7.2 reflect more or less the standard design rules in spacecraft structures design [10]. From the research described in this thesis the following can be concluded:

- Try to estimate the total mass M_s of the source as accurately as possible. The interval of the mass of the source is eliminated and lower statistical moments are obtained.
- An increase of the lowest natural frequency of the source f_{1s} will lead to an increase of C^2 , because the source becomes stiffer and the dynamic coupling between load and source becomes stronger.

- A significant increase of the modal effective mass m_{1s} associated with the fundamental frequency f_{1s} causes higher statistical moments of C^2 .
- Clustering the natural frequencies $f_{1s}, f_{2s}, f_{3s}, f_{4s}$ causes an increase in the statistical moments of C^2 , because the dynamic interaction between load and source becomes more intense.
- The probability distribution of C^2 match well with the lognormal distribution $C^2 \sim LN(\mu_Y, \sigma_Y)$ where $Y = \ln(C^2)$, Eq (5.27), [1].
- It is very straightforward and efficient to perform the probabilistic analyses with the Rosenblueth Point Estimates of Probability Moments (PEM), [7]. The number of samples is $2k + 1$ where $k = 4$ is the number of design variables using the CSMA method and $k = 3$ using the HMDM method. The intervals and statistical moments of the primary design variables are given in the Tables 7.2 and 7.3 respectively. The distribution functions of the design variables are uniform within the specified intervals and zero outside.

7.2. HOW TO APPLY THE PROBABILISTIC ANALYSIS

It is advised to:

- Apply the Coupled System Modal Approach (CSMA) method [9] to perform the coupled load/source random response analysis to calculate statistical moments, and the maximum value (MCS) of the semi-empirical constant C^2 , using Eq. (5.2).
- Calculate the statistical moments of C^2 using the Rosenblueth Point Estimates for Probability Moments (PEM) [7] which is a most efficient probabilistic analysis approach than the MCS approach. However, with the MCS method maximum values can be computed with the MCS method.
- Estimate the mass M_s of the source as accurately as possible. The interval of M_s will be eliminated or reduced resulting in a lower mean value C_μ^2 and standard deviation C_σ^2 of the semi-empirical constant.

7.3. GUIDELINES

The design and (structural) verification against random vibrations of the (sub)systems (load) in spacecraft design is, in general, under the responsibility of one (sub)contractor. During the verification process the random acceleration vibration specification will be established. When force limited random vibration testing (FLVT) is selected to verify structural and functional integrity against random enforced loads [8] the semi-empirical constant C^2 in Eq. (4.8) will be defined or estimated. When the dynamic properties of the supporting structure (source) are unknown a probabilistic estimation of C^2 is to be performed, which is the main research topic of the thesis.

To generate a CSMA mathematical model the asparagus patch model of the load will be built up, consisting of parallel placed SDOF elements, which base represents the interface between the load and the source. At this interface the random acceleration vibration specification is given. After that a probabilistic asparagus patch model of the source has to be developed.

7.3.1. MATHEMATICAL MODEL OF THE LOAD

To build up the asparagus patch model of the load an eigenvalue analysis using, in general, the finite element model (FEM) of the load has to be performed. This FEM can be surely made available by the (sub)contractor, which is responsible for the design and verification of that load. The asparagus patch model of the load is illustrated in Fig. 7.1.

Figure 7.1: Asparagus patch model of the load

The following modal data and mass of the load are needed to build the asparagus patch model;

- The total mass of the load M_l (kg). The mass properties are available from the mass properties report. First and second moments of mass are not considered in the asparagus patch model.
- The undamped natural frequencies $f_{i,l}$, $i = 1, 2, \dots, n$ (Hz) assuming clamped conditions at the interface load/source. The natural frequencies reflect the stiffness (springconstant) of the springs of the SDOF elements, viz. $k_{i,l} = (2\pi f_{i,l})^2 m_{i,l}$, $i = 1, 2, \dots, n$.
- The associated modal effective masses $m_{i,l}$, $i = 1, 2, \dots, n$ (kg) and the residual mass $m_{r,l}$ in the three translational directions respectively. The cross-coupling is not considered because only interface forces or sum of interface forces are mea-

sured during the force limited vibration tests. The required sum of the modal effective masses $\sum_{i=1}^n m_{il} \geq 0.95M_l$ and then $m_{rl} = M_l - \sum_{i=1}^n m_{il} \approx 0.05M_l$.

- There is no need to estimate the damping constants of the discrete dampers, because later on a constant modal damping ratio ζ will be introduced during the probabilistic modal response analyses of the load/source combination.
- Cross coupling in the modal effective mass matrix is not known and therefore not taken into account.

7.3.2. PROBABILISTIC MODEL OF THE SOURCE

To build up the asparagus patch model of the source the intervals and statistical moments of all design variables as mentioned in Table 7.2 are used. The asparagus patch model of the source (which is quite similar asparagus patch model of the load) is shown in Fig. 7.2.

Figure 7.2: Asparagus patch model of the source

An artificial mass $m_{int} = 0.01 \text{ kg}$ has been added to the interface to prevent infinite eigenvalues of the combined load/source system. The m_{int} associated natural frequency will be beyond 2000Hz, outside the spectrum of the random acceleration specification (20-2000Hz).

7.3.3. COUPLED SYSTEMS MODAL ANALYSIS (CSMA) METHOD

The asparagus patch models of both the load and the source are now coupled via its mutual interfaces. The subsequent random response analyses are described in detail in Chapter 5 and Appendix D. The random response analyses are performed to approximate the random acceleration specification W_{AA} at the interface (Fig. 7.1).

7.4. RECOMMENDATIONS

The dynamic interaction of two elastic systems has been investigated when one of the systems is unknown and replaced by a probabilistic elastic system respecting some com-

mon design rules, such as the separation of natural frequencies. Within that framework the following recommendations for further investigation are suggested:

- The theoretical background of the concept of modal effective mass is applicable for a statically deterministic set of six interface degrees of freedom. This concept should be extended to a statically deterministic set of interface degrees of freedom at the interface. Chen proposed in [2] the normalized quasi-static mass for multi-supported motion problems.
- The space agency e.g. ESA or NASA prescribes the values of C^2 of all systems to be tested against random enforced accelerations when the FLVT is applied.

7.5. APPLICATIONS IN OTHER AREAS

In this thesis the dynamic interaction between of a deterministic load and a probabilistic source has been investigated. The source (adjacent subsystem) is described by a number of probabilistic design variables with prescribed intervals and statistical moments. This probabilistic approach can be implemented in other domains where a system is designed against a specified dynamic environment but the adjacent system (structures) are unknown. In the design of complex systems where more subcontractors are involved the characteristics of some of the subsystems can not or will not be made available. Then the unknown subsystems can be described in probabilistic manner as discussed in this thesis.

Such an example is the design of the tractor-trailer combination where interface loads are dependent on the dynamic interaction. Two design cases can be identified, for which the probabilistic approach can be applied:

- In the case of the design and analysis of the trailer the dynamic characteristics of the tractor are unknown
- In the case of the design and analysis of the tractor the dynamic properties of the trailer are not known.

A third case is the design and analysis of a tyre of a car or heavy duty vehicle without knowing the dynamic characteristics of these vehicles.

7.5.1. TRACTOR-TRAILER COMBINATION, TRAILER DESIGN

The design of trailers is influenced by the dynamic characteristics of the tractor, of which the design is unknown. This is illustrated in Fig. 7.3.

The design and development of the tractor and the trailer are in fact two separate worlds. The design of the tractor must be suitable for a wide range of trailers and therefore the design of the tractor is not focused on one specific trailer. Fortunately, the interaction between the tractor and different trailers is very common, but may be strongly influenced by the role features and loading of the trailer.

The dynamic interaction of the tractor and trailer below 3 Hz is a design driver. Beyond 3 Hz the size of the coupling attachment is dependent on the loading conditions of the trailer.

In practice the design of the trailer is based on engineering experience such as bending and torsion stiffness and is more or less independent of the design of the tractor and weather conditions. In general variations in design properties of trailers are not significant, only when a special trailer must be designed.

The strength verification of the frame of the trailer is performed against static loading conditions. Dynamic loads in parts of the frame can be made available from the intensive application of the trailers. Fatigue properties due to vibrations are investigated between 3 and 25Hz.

In conclusion we can state that the design worlds of the tractor and trailer are quite separated and the design and development occurs in parallel. The two worlds should be combined in all facets of the design process. The design and development of spacecraft is independent of that of a launch vehicle, hence comparable to the design and development of a tractor and a trailer.

To combine the design worlds of the tractor and the trailer may be represented by a deterministic tractor and a probabilistic description of the trailer common as discussed in Chapter 5.

Figure 7.3: Tractor with probabilistic trailer

7.5.2. TRACTOR-TRAILER COMBINATION, TRUCK DESIGN

The design of tractor may be influenced by the dynamic characteristics of the trailer, of which the design is unknown. This is illustrated in Fig. 7.4. The trailer may be represented by a probabilistic description common as discussed of Chapter 5. More discussion is provided in previous section.

Figure 7.4: Probabilistic tractor with trailer

7.5.3. TI YRES-VEHICLE COMBINATION, TYRE DESIGN

The design of tyres is influenced by a number of design parameters such as internal pressure, condition of the road, speed of the vehicle, temperatures, and sound radiation, but will be also influenced by the dynamic characteristics of the vehicle for which the tyre is designed. The tires are, in general, tested on rotating drums. Also the tyres are mounted on several bought cars, trucks and trailers to test the tyres under real life road and dynamic conditions. To minimize the number of real life tests, cars, tractors and trailers may be represented by a probabilistic description common as discussed in Chapter 5. This is illustrated in Fig. 7.5. The probabilistic models may be improved using measured data.

Figure 7.5: Tyres with probabilistic tractor

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SPACECRAFT DESIGN AND VERIFICATION

A.1. INTRODUCTION

The aspects of the design and verification philosophy of structures of spacecraft, instruments, units, etc. are discussed in great detail in [16, 18, 22], but are summarized in this appendix.

The need for a new spacecraft (project) may be generated by science, communications, military, commercial, or other interests. In general, a scientific spacecraft is one-off, while communication satellites are made in series.

After preliminary studies a preliminary definition of the orbit of the spacecraft, payload, instrumentation and supporting equipment, geometry and mass will be made and the launch vehicle will be selected. The design and verification process of a spacecraft structure is illustrated in Fig. A.1, [1].

In the initial phase is important to set up a good configuration of the spacecraft, which will be used and further optimized in the course of the project until the spacecraft is launched.

The finite element model of the spacecraft structure developed in the early stage of the project will be used to perform a spacecraft/launch vehicle coupled load analysis (CLA). The results of the coupled load analysis may modify the preliminary enveloping sine vibration loads (from launch vehicle user's manual) to be implemented in the subsequent design of the spacecraft and to be used during the sine vibration test to prevent over-testing of the spacecraft.

All imposed requirements and the capability to carry the prescribed loads will be demonstrated in the beginning by analysis and later on when hardware of the spacecraft can be made available by test (static, dynamic, etc.)

A.2. INITIAL PHASE

The definition of the spacecraft configuration depends on:

Figure A.1: Design flow spacecraft structure design and verification [1]

- The boundary conditions of the launch vehicle
 - Spacecraft total weight (mass)
 - Launch environment (static and dynamic loads, etc.)
 - Available volume under the fairing (shroud)
 - Payload adapter (interface between spacecraft and launch vehicle)
 - Payload separation system
 - Launch cost (may be overruled by political reasons)
- Functional requirements of the system (payload) and subsystem requirements (size, mass, position, environment, etc.). Some subsystems may be qualified against certain sine and random vibration levels during former projects. Re-qualification may cost a lot of effort and money.

- Orbit and mission duration
- ...

The launch vehicle constraints or requirements are prescribed and discussed in the launch vehicle user's manual, e.g. West European launchers [2–4]. However, other launch vehicles may be selected as they are in the USA, Russia, China and India

A.3. PRELIMINARY DIMENSIONING OF SPACECRAFT STRUCTURE

For the initial design of the spacecraft structure, the following aspects should be considered:

- Stiffness requirements
- Design Launch Loads, Quasi Static Loads (QSL)
- Factors of safety [10]
- Materials [7]
- Basic Design (configuration)

Figure A.2: Dimensioning structures and structural components

In Fig. A.2, the dimensioning of the spacecraft structure and structural components are illustrated. The mission requirements (objectives of mission) in combination with the functional requirements (functions of payload) will lead to a geometry and associated estimated mass distribution. The selected launch vehicle(s) user's manual(s) specify stiffness, design loads, geometry constraints of the spacecraft structure. The quasi-static design load factors may be applied respecting minimum natural frequency requirements.

The load factors applied to the mass distribution give an internal load distribution (inertia loads) and these loads are the starting point for dimensioning the spacecraft structural elements. After that, the stiffness of the primary load carrying structure should be checked against launch vehicle stiffness requirements. If the minimum natural frequencies of the total spacecraft, associated with the longitudinal (launch direction) and lateral modes, comply with the launch vehicle requirements, dynamic loads can be applied. The manufacturing and testing aspects will follow later during the project.

A.4. LOADS SPECIFICATION AND STIFFNESS REQUIREMENT

The various mechanical loads are not all equally important and depend on the type of the mechanical part of the structure. Does it relate to the primary structure, the secondary structure (solar panels, instruments, etc.). Preparations on the ground, the launch, and operations on orbit set various types of design and test requirements, such as [8, 17].

For example the static and dynamic loads are specified in the user's manual of the Soyuz launch vehicle are:

STATIC LOADS

Quasi Static Loads (QSL) Loads independent of time or which vary slowly, such that the dynamic response of the structure is not significant. The design of the primary structure is determined to a large extent by the quasi static loads and low frequency sinusoidal loads.

Depressurization During launch the pressure below the fairing will decrease in the first 120 seconds from 1 Bar to about 2 mBar just before jettison of the fairing. Structures, instruments, units are to be vented.

Temperature induced load The introduction of thermal loads/stresses due to non-uniform temperature distribution. On orbit temperature induced loads may jeopardize alignment requirements.

DYNAMIC LOADS

Sinusoidal Excitation Harmonic enforced acceleration at the base of the spacecraft, instrument, unit, etc. The sinusoidal and random base acceleration determine the design of secondary structures; instruments, solar array, antennae, unit and other subsystems. The frequency range is, in general, between 5 and 100Hz. The analysis results of the coupled load analysis (CLA) as indicated in Fig. A.1 can modify the preliminary enveloping sine loads.

The acceptance level is equal to flight loads, which are the maximum loads occurring with a certain probability during the life cycle of the spacecraft. The qualification dynamic loads are derived from the flight loads multiplied by a design factor.

Acoustics Loads Random sound pressure fields are design drivers for large areal to mass structures, such as solar panels, antenna dishes and radiators. In general, the frequency range varies between 20 and 10000Hz. The sound pressures excite the outside of the spacecraft. The sound pressures are transferred into the inside of the

spacecraft by the structural responses of the structure. This is illustrated in Fig. A.3.

Figure A.3: Spacecraft acoustic/random vibrations environment

Random Excitation The random base accelerations determine the design of secondary structures, such as instruments, solar array, antennae and units. The frequency range is between 20-2000Hz. In general, random vibration tests are not required for a complete spacecraft because mechanical random vibrations are covered by the acoustic loads. In general, the random enforced accelerations are specified at the interface between spacecraft and launch vehicle, instrument and spacecraft.

The spacecraft structure will be excited by the random sound pressures (acoustic) caused by engine exhaust noise and turbulences during flight. The acoustic loads are converted into random structural responses.

The mathematical aspects of the random vibration are discussed in Appendix E,

The random vibration specifications are derived either from the acoustic test in a reverberant chamber or by analysis. In general, the measured and analysed structural random acceleration responses are not smooth but have peaks and valleys. Techniques to derive a more smooth random vibration specification are discussed in Appendix F.

Transient Excitation Deterministic loads whose magnitude or direction varies with time and for which the dynamic response of the structure is significant.

High frequency transient excitation, Shocks High frequency transients loads are most times represented by their Shock Response Spectrum (SRS). Separation of launch vehicle stages and the separation of the spacecraft from the last stage will induce high shock loads into the spacecraft.

STIFFNESS

Natural (fundamental) frequencies The natural frequency is a governing design requirement for all parts of the spacecraft. This requirement is imposed in order to limit the dynamic coupling of the spacecraft with the launch vehicle. In general, the lower and lowest natural frequencies are calculated applying clamped (hard mounted) boundary conditions at the interface between spacecraft and launch vehicle. The significance of the vibration mode can be defined by the value of associated modal effective mass. The modal effective mass is discussed in [21] and Appendix C.

A.5. TEST FACTORS AND FACTORS OF SAFETY

A specific ECSS standard "Structural Factors of Safety for Spaceflight Hardware" was created, in which the *test factors* and *factors of safety* (FoS) are discussed in very detail [10]. The test factors and factors of safety are used to account for uncertainties.

All load data given should be considered as flight limit loads, applied to the spacecraft.

In Fig. A.4, page 140, the logic for factors of safety application for spacecraft is shown. *KA* and *KQ* are called the test factors and are applied to define respectively the accep-

Figure A.4: Logic for Factors of Safety application [10]

tance and the qualification test loads. The ultimate and yield design factor of safety, *FoSU* and *FoSY*, are safety factors applied to the design limit load in order to calculate the design ultimate and yield load respectively. Values for the test factors and factors of

safety are provided in the ECSS standard [10]. However, the ECSS values of the test factors and factors of safety may be overruled by the launch vehicle authority, because this authority has the final responsibility of launching the spacecraft.

A.6. VERIFICATION BY ANALYSIS

The structural analyses are, in general, made with the finite element analysis method [5, 6, 13]. The objective of the analysis defines the detail of the finite element model (FEM). The finite element analysis is applied to perform the following analyses:

- Strength analysis (yield, ultimate, buckling, etc.)
- Thermo-elastic analysis (deformations, stresses)
- Dynamic response analysis (modal, sine, random, transients, etc.)
- Acoustic analysis

STRENGTH ANALYSIS

The strength properties of the spacecraft structure and structural elements will be verified by analyses and later on by tests. The internal stress distribution is calculated and compared to allowable stresses (failure modes) to obtain margins of safety (MoS). For buckling special buckling (bifurcation) analysis has to be performed. Typically applied load or combination of loads come from the quasi-static load factors.

If only dynamic load factors are applicable, the stress due to dynamic loads will be verified against allowable stress or loads. In the case of random loads, in general, the 3σ value of stresses will be compared to allowable stress or load.

THERMO-ELASTIC ANALYSIS

The thermal deformation and stresses in the spacecraft structure, caused by thermal gradients, are verified for stress and alignment requirements.

DYNAMIC RESPONSE ANALYSIS

The dynamic response analysis is needed to verify specifications with respect to stiffness and strength requirements. A modal analysis is performed to calculate the modal characteristics of the complete spacecraft clamped at the interface between launch vehicle and spacecraft, namely the natural frequencies, associated real mode shapes, generalized masses and stiffness and modal effective masses. The damping is generally ignored during the eigenvalue analysis, because the modal damping ratio for complete spacecraft is quite low, between 2-10%. Later in the spacecraft project the modal properties are verified by a modal survey [9].

The finite and boundary element analysis methods are applied in the lower frequency domain, while the statistical energy analysis (SEA) method can be applied in the high frequency domain, for structures with high modal densities [14, 15].

COUPLED LOAD ANALYSIS

Early in the project a spacecraft/launch vehicle coupled load analysis is performed by the launch vehicle authority to analyse the dynamic loads during lift-off and launch of the spacecraft. This analysis is carried out to verify in a preliminary phase the design load factors from the launch vehicle user's manual. The load factors given in the user's manual are very generic and enveloping (conservative). A complete, reduced or modal condensed finite element model, in combination with load transformation matrices, should be delivered to the launch vehicle authority. The outcome of the coupled load analysis can modify the design load factors and may influence the spacecraft test levels. In general, the coupled load analysis is repeated in a later phase of the programme. The coupled load analysis is described in every detail in the ECSS Handbook "Spacecraft Mechanical Load Analysis Handbook" [11].

ACOUSTIC ANALYSIS

Large light weight areal structures mounted outside the spacecraft, e.g. solar arrays and antenna reflectors, are very sensitive to acoustic loads (sound pressures). A combined finite element and boundary element analysis [12, 20, 23] is needed to simulate to fluid structure interaction (FSI). The structural behaviour is governed by the finite element analysis calculating the modal base (natural frequencies, vibration modes, stress modes, modal damping ratios, ..) and the influence of the fluid (added mass, radiation damping). The acoustic loads are governed by the boundary element analysis method.

The statistical energy analysis (SEA) method can be applied in the high frequency domain.

A.7. MANUFACTURING OF SPACECRAFT STRUCTURE

The structural parts of the spacecraft structure may be produced by the responsible company or sub-contacted to other companies. Subcontracting structural parts may be based either on the "build to print" principle on the risk sharing principle, which means that the design and manufacturing of the structural part is done under the responsibility of the subcontractor.

The complete structure is assembled mainly at the premises of the company responsible for the spacecraft structure.

Mechanical ground support equipment (MGSE) is to be manufactured to assemble the structural parts into the complete structure or to support the structure during transport.

A.8. VERIFICATION BY TEST

All requirements and load carrying capabilities of the spacecraft structure have already been verified by analysis. However, a number of tests are applied too to qualify spacecraft structures design. The generally applied mechanical qualification tests and associated requirements are presented in Table A.1.

Within the context of this thesis not all design and test loads are of interest, but the focus will be on the application of random acceleration vibration spectra applied to the test article and the associated random force limits, the so-called force limited vibration

Table A.1: Test verification

Type of test	Test verification
Static test	<ul style="list-style-type: none"> • Check qualification and structural strength, in particularly, the primary structure and critical interfaces. • Verification (partially) of stiffness matrix.
Modal survey test	<ul style="list-style-type: none"> • Identification of modal properties to support the verification of the finite element model, which is used in load and coupled load analysis.
Sine vibration test (vibration table)	<ul style="list-style-type: none"> • Support the verification of the spacecraft finite element model (amplification from launcher/spacecraft interface input to various spacecraft parts) • Qualification of secondary structures.
Random vibration test (vibration table)	<ul style="list-style-type: none"> • Qualification of units subjected to random (acoustically generated) flight environment
Acoustic test (reverberant chamber)	<ul style="list-style-type: none"> • Verification and qualification of spacecraft system to acoustic environment. which might be experienced by the spacecraft during flight.
Shock test	<ul style="list-style-type: none"> • Spacecraft verification and qualification against high frequency transient loads (pyrotechnics and mechanically induced shocks).

testing (FLVT). The FLVT is discussed in the NASA handbook [19].

A.9. FURTHER REMARKS

In Appendix B the design process of DAF trucks is compared with the design process of spacecraft structures. It must be concluded that the design and verification process of trucks is similar to the design of spacecraft structures, instruments and units.

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B

DESIGN PROCESS TRUCKS AND TRACTORS

B.1. INTRODUCTION

In this appendix the design process of commercial trucks and tractors, in particular structures, will be briefly discussed. The typical truck and tractor are shown in Fig. B.1.

This appendix is added to this work to provide a comparison with the design process of spacecraft and other space hardware, as discussed in chapter A. The description of the design process of trucks is based on an interview with and documentation provided by René Liebrechts of DAF Trucks N.V., Eindhoven, [1, 3–5].

Figure B.1: DAF truck (left) and DAF tractor (right) (courtesy DAF Trucks N.V.)

B.2. PRODUCT DEVELOPMENT

In Fig. B.2 the product development of trucks and tractors is illustrated, [3]. The design process will focus on "Structures". The V-chart of the development-verification cycle is shown in Fig. B.3 and duration of this cycle is about five years. The development process

Figure B.2: Product development trucks & tractors

of trucks and tractor is evolutionary and depends from the market (customer driven) and governmental and European regulations, e.g. EURO 6 the reduction of submission of harmful parts from exhaust gases into the environment. This norm became applicable in September 2014. Overall or general requirements are flown down from the system to subsystems, components and parts and are verified upstream on each individual level.

B.3. DESIGN PROCESS STRUCTURES

In this section only the structures part of the design of trucks and tractors will be described. In general, the structures are designed against vibrations and fatigue. All vehicle applications are logged in a vehicle application sheet in terms of

- Distance
- Road surface conditions
- Loading
- Number and type of manoeuvres

However, the different types of road surface are translated into the damage equivalent mile age of a reference road surface (Belgian pavé). This allows the ranking of all vehicle applications in terms of structural damage from driving on poor road surfaces.

Figure B.3: V development verification chart

The probability of a severe vehicle failure should be less than 10% within the standard vehicle life, without over-dimensioning, which is one of the most important design requirements of the structure. This probability of failure or being taken out of commission is illustrated by the S-curve in Fig. B.4 and the translation of service life to distance is provided in Table B.1 [5].

Table B.1: Standard service life translated to distance (km)

Type	Class 1	Class 2	Class 3	Class 4
F65	350,000	500,000	800,000	-
F75	450,000	600,000	800,000	-
F85	450,000	600,000	1,000,000	1,200,000
F95	500,000	800,000	1,200,000	1,600,000
Class 1: off-road, kippers, mixers				
Class 2: regional / national distribution				
Class 3: long-distance transportation				
Class 4: extra long-distance transportation (multiple drivers)				

The loads of all sold trucks and tractors are statistically considered and the maximum dynamic loads encountered for the 95th percentile of the cumulative number of sold trucks will be the basis for the design target of the truck and/or tractor. That means that 95% of most severe vehicle application correspond to 7750 km pavé. All other road conditions are scaled to this type of pavé. This is shown in Fig. B.5. Only 5% of the trucks sold have a more severe application.

There are separate targets for driving and manoeuvres.

Figure B.4: Vehicle life [3]

B.4. DIMENSIONING OF COMPONENTS

The definitions for the failure probability for the components (fuel tanks, battery carrier, cabins, etc.) are categorized as follows:

Safety critical components The failure probability should be $< 10\%$ for $2 \times$ the nominal vehicle life, with a confidence level of 90%.

Operational critical components The failure probability should be $< 10\%$ for the nominal vehicle life, with a confidence level of 90%.

Other components The failure probability should be $< 10\%$ for the nominal vehicle life, with a confidence level of 50%.

B.5. DESIGN LOADS

The design loads are extracted from the database of vehicle applications for the 95% vehicle design target for each loading condition; the mileage and number of cycles of the manoeuvres. The principal load cases are:

- Vertical static load
- Misuse wrong torque
- Road irregularities
- Braking
- Crash

Figure B.5: 95% most severe application

- Driveline torque

For every load case a histogram will be generated and scaled to the number of cycles to the requirements of the design target truck or tractor. Peak loading of the component is scaled to standardized operating conditions.

B.6. NUMERICAL SIMULATIONS

The finite element analysis method is applied mostly to linear or linearized models, particularly in the frequency domain [1].

Analysis of a complete truck or tractor is performed using the finite element analysis method and includes the following (analysis) steps:

1. Creation and reduction of component models.
2. Assembly of the components to create the complete vehicle finite element model.
3. Generation of frequency response functions (FRF) from the finite element model.
4. Combination of FRFs and excitation auto and cross power spectral densities to yield response spectral densities
5. Evaluation and interpretation of the results.

B.6.1. CREATION AND REDUCTION OF THE COMPONENTS MODELS

Before any vehicles can be built, the necessary components must be meshed, validated, and reduced to substructures. The Guyan Reduction technique (GR) is mostly applied to generate reduced models [6]. These are then stored in libraries with similar components. One disadvantage of substructures is that it can be difficult to visualize the deformed shape of a component from a collection of retained nodes. Ideally, one would want at least a coarse mesh of shell or line elements associated with these nodes. These elements would have to be "dummy" elements that are not included in the analysis but are included in the post processing

B.6.2. ASSEMBLY OF THE COMPONENTS TO CREATE THE COMPLETE VEHICLE MODEL

Models are built up of substructures and predefined subassemblies (see Fig. B.6). The subassemblies may contain one or more substructures. An example is the rear suspension subassembly, which contains substructures for the frame brackets and air spring brackets along with spring and damper elements and rigid connections.

Figure B.6: Schematic of vehicle model structure [1]

B.6.3. GENERATION OF FREQUENCY RESPONSE FUNCTIONS

The set of transfer functions $H(f)$ is obtained from the finite element model by applying unit sinusoidal force or displacement excitations across the range of frequencies of interest (usually 0.2 - 20Hz) to each input point. These can be displacement or force excitations. Fig. B.7 illustrates this.

Figure B.7: Generation of the frequency transfer functions $H_i(f)$ [1]

In practice, the transfer functions are merely the responses to the unit sinusoidal inputs. The frequency transfer functions are saved for later post processing.

B.6.4. COMBINATION OF EXCITATION AUTO- AND CROSS SPECTRA WITH FRFS TO YIELD RESPONSE SPECTRA

The responses are calculated using a MATLAB toolbox called *TricaT* (Toolbox for Ride Comfort Analysis of Trucks). The toolkit used was developed at DAF trucks.

The following matrix evaluation is done in MATLAB:

$$[W_{yy}(f)] = [H(f)]^T [W_{xx}(f)] [H(f)], \quad (\text{B.1})$$

where

- $[W_{xx}(f)]$ is the matrix of the auto and cross power spectral densities of the vertical tyre displacements
- $[H(f)]$ are the frequency transfer functions at locations of special interest to be calculated by the finite element analysis method
- $[W_{yy}(f)]$ are the matrices of power spectral densities of the responses; accelerations, velocities, forces, stresses.

B.6.5. EVALUATION AND INTERPRETATION OF RESPONSE SPECTRA

The response spectra at locations of interest are evaluated to estimate the spring travel, ride comfort, strength and fatigue life aspects. The ride comfort is obtained by weighting

the acceleration response spectral densities by ISO comfort weighting functions. The fatigue life predictions are based on S-N curves in combination of the Palmgren Miner rule. The S-N curves vary over the component surfaces and are dependent on:

- local mean stress (prestress, residual stress)
- local stress concentration
- local surface conditions (machining, cutting)
- local material influences (welds, plasma cuts)

B.6.6. VERIFICATION OF FINITE ELEMENT MODEL

The mathematical model (reduced finite element model) of the complete vehicle has to be validated:

- within a frequency range of 0-20 Hz (for spacecraft structure the frequency range is normally between 5 and 150Hz)
- for about 60 vibration modes, with a strong modal coupling
- for more than 100 parameters; stiffness, damping, mass and inertia distribution, connections, etc.
- because the complete vehicle response characteristics need to be evaluated
- and a global approach is needed because local optimization fails

The most popular modal correlation method is based on the modal assurance criteria (MAC). However, due the high modal density and high damping the MAC correlation technique is not very suitable to apply and the experimental modal analysis is therefore not very accurate.

The associated frequency assurance criteria (FRAC) is applied for correlation and updating activities of the mathematical model of the complete vehicle. The FRAC is defined as

$$FRAC(f_i, f_j) = \frac{\left| [H(f_i)_{measured}]^T [H(f_j)_{modal}] \right|^2}{\left([H(f_i)_{measured}]^T [H(f_i)_{measured}] \right) \left([H(f_j)_{modal}]^T [H(f_j)_{modal}] \right)} \quad (B.2)$$

The $FRAC(f_i, f_j) = 1$ at full correlation.

B.7. FATIGUE EVALUATION AND VERIFICATION CHAIN

The fatigue evaluation and verification chain of a truck or tractor design is shown in Fig. B.8. The real service conditions are defined by the 95th percentile of the vehicle applications. This design target will be translated into test and simulation specifications. Design and numerical simulation activities will result in build a prototype of the truck or tractor being built. The prototype will be used to measure internal loads to be applied to component testing. The final proof of the pudding is the durability testing to verify fatigue life predictions. (This is not possible at spacecraft level)

Figure B.8: The fatigue valuation and verification chain [4]

B.8. COMPARISON SPACECRAFT AND TRUCK/TRACTOR DESIGN PROCESS

In this section a global comparison between the design processes of spacecraft systems and subsystems (S/C) and Trucks and Tractors (T/T) will be given.

- **Number of Specimens**

S/C In general, the number of spacecraft systems or subsystems is very limited. In most scientific missions one spacecraft will be designed, manufactured, tested and launched. Unmanned spacecraft systems and subsystems have very limited possibilities to maintain or repair.

T/T Contrary to spacecraft one type of truck or tractor will be designed and tested, but many vehicle will be manufactured in series and sold. After sales service and maintenance are important.

- **Design Loads**

S/C The design loads, test loads and test factors are provided by the launch vehicle authority, whose launch vehicle has been selected to launch the spacecraft. The flight loads (maximum loads encountered during the life cycle) have a probability exceeding 99% with a confidence level of 90%. The allowable mass and inertia properties depend on the mission (orbit, trajectory). In general, quasi static loads, depressurization, sinusoidal vibrations and shock

loads at the interface between spacecraft and launch vehicle are specified as well as severe acoustic loads. The loads are flown down to subsystems and equipment. Acoustic loads cause random structural vibration inside the spacecraft. These random acceleration vibrations are a design driver for subsystems and equipment. Load specifications given by the launch authority concern average dynamic and load characteristics of spacecraft and may be conservative. Later on in the course of the project coupled load analyses (CLA) are made to fine tune the quasi-static and sinusoidal vibrations. After launch the in-orbit mechanical loads are low, but thermal-elastic load may occur, especially for subsystems mounted on outside of the spacecraft (solar wing, antennae). The thermal-elastic load depends strongly on the orbit or trajectory.

T/T The design of trucks and tractors is more or less evolutionary, driven by national and European regulations. Quite a number of load measurements have been performed on previous developed and sold trucks and tractors. The 95th percentile vehicle loads will be used as the design target and less than 10% are allowed to fail during vehicle service life for several classes of trucks and translated into distances (km). These loads consists of quite a number of contributors: road loads, manoeuvres (cornering, braking, ..). Measured time histories are converted into load spectra. In general, the load has a random nature.

- **Major (structural) design (structural) requirements**

S/C The basic requirement for the spacecraft structure is to support the payload and spacecraft subsystems with enough strength and stiffness to preclude any failure (rupture, collapse, detrimental deformation) that may keep them from working successfully. Other typical requirements for spacecraft structures are stiffness, strength, structural life, structural response, damping, mass properties and mechanical interface.

T/T The typical requirement for truck and tractor are common to the typical requirements of S/C: strength, vibrations, fatigue life, comfort and structure borne noise.

- **Design and Verification**

The design and verification process of both S/C and T/T are based on the same philosophy flowing down requirements from system, subsystems, component, etc. Both design and verification process are shown in Fig. B.9.

- **Analysis methods**

S/C The finite element analysis (FEA) method is the most important numerical tool to predict the structural response of spacecraft structure - static (inertia and thermal) and both in time (shock) and frequency domain (sinusoidal and random). The accuracy of the predictions depends on the complexity of the structure and the detail in the number of nodes and finite elements. The boundary element analysis (BEA) method will be applied in the case of

Figure B.9: Design and verification process S/C (left) [2], T/T (right) [3]

acoustic pressure loads. The accuracy of the structural response predictions depends on the finite element model. The modal characteristics calculated with the aid of the FEA are transferred into the BEA. In general, the radiation of noise from the vibrating structure is of no interest. In the higher frequency bands the statistical energy analysis (SEA) method will be applied. The analyses are mostly done in the linear domain.

T/T The FEA method is an important tool too, to predict the structural responses in truck and tractor structures caused by static (inertia) and frequency dependent (random) excitations from the tyres. The modal density in truck and tractor configurations is higher than in spacecraft structures. Reduced FEA models are used to calculate frequency transfer functions to be used for random responses. Dedicated software is applied to calculate strength and fatigue life characteristics. The analysis effort for trucks and tractors is comparable to analyses done within the frame of spacecraft structures. In general, non-linear effects are linearized (e.g. spring strokes).

• Test/Analysis Correlation

Both S/C and T/T mathematical models are updated to achieve a good test/analysis correlation, but frequency ranges in which the finite element models are updated are quite different. Mathematical methods of updating finite element models may be similar.

T/T Frequency range between 0 and 20 Hz with a high number of vibration modes.

S/C Frequency range between 5 and 150 Hz with a low number of global vibration modes.

Nothing is said about the design of trailers, designed and built by third-party companies. It is supposed that interface (dynamic) load between the truck and trailer and loads introduced via the tyres of the wheels are provided.

Manufactures of Instruments, equipment and units builders mounted to the spacecraft should design their hardware against a number of specifications, loads, mass, stiff-

ness provided by the main contractor responsible for the design, verification, and manufacture of that spacecraft.

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C

MODAL EFFECTIVE MASS

In 1972 Wada, Bamford and Garba of the Jet Propulsion Laboratory (JPL) published a method of describing large finite element models of spacecraft as an equivalent spring-mass model of each mode of vibration in [6]. The theory of the modal effective masses will be presented and discussed below.

A linear elastic linear structure is represented in a finite element model (FEM) with N degrees of freedom (DOFs). The undamped equations of motion of that MDOF dynamic system are

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = \{F(t)\}, \quad (\text{C.1})$$

where

- $[M]$ is the mass matrix
- $[K]$ is the stiffness matrix
- $\{\ddot{x}(t)\}$ is the acceleration vector
- $\{x(t)\}$ is the displacement vector and
- $\{F(t)\}$ is the external force vector.

The interface DOFs are denoted by $\{x_r(t)\}$ (r-set) and the internal DOFs by $\{x_i(t)\}$ (i-set). The interface DOFs describe the six motions as a rigid body: three translations and three rotations. The six rigid body motions may be introduced in one node with six DOFs. The interface DOFs constitute a determinate interface. Partitioning the equations of motion (C.1) into the r-set and i-set the following equations can be set up

$$\begin{bmatrix} M_{rr} & M_{ri} \\ M_{ir} & M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{x}_r(t) \\ \ddot{x}_i(t) \end{Bmatrix} + \begin{bmatrix} K_{rr} & K_{ri} \\ K_{ir} & K_{ii} \end{bmatrix} \begin{Bmatrix} x_r(t) \\ x_i(t) \end{Bmatrix} = \begin{Bmatrix} F_r(t) \\ F_i(t) \end{Bmatrix}. \quad (\text{C.2})$$

The physical displacements are expressed as a superposition of rigid body modes $[\Phi_r]$ and elastic modes $[\Phi_e]$ multiplied by the associated generalized coordinates $x_r(t)$

(r-set) and $\eta_n(t)$ (n-set) respectively

$$\{x(t)\} = [\Phi_r]\{x_r(t)\} + [\Phi_e]\{\eta_n(t)\} = [\Psi] \begin{Bmatrix} x_r(t) \\ \eta_n(t) \end{Bmatrix}. \quad (C.3)$$

We can obtain the matrix of rigid body motions Φ_r from the stiffness matrix (C.2), ignoring the inertia terms and $F_i(t)$, from the second equation

$$[K_{ri}]\{x_r\} + [K_{ii}]\{x_i\} = 0. \quad (C.4)$$

From (C.4) we can derive a relation between the internal and external interface DOFs if there is no load applied to the internal DOFs, hence $\{F_i\} = \{0\}$

$$\{x_i\} = -[K_{ii}]^{-1}[K_{ri}]\{x_r\} = -[K_{ii}]^{-1}[K_{ri}][I] = -[K_{ii}]^{-1}[K_{ri}] = \{\phi_r\}. \quad (C.5)$$

The interface DOFs $\{x_r\}$ are released one by one, so this vector can be replaced by a diagonal unitary matrix to obtain the rigid body motion $\{\phi_r\}$. The complete set of rigid body modes $[\Phi_r]$ is

$$[\Phi_r] = \begin{bmatrix} I \\ \phi_r \end{bmatrix}. \quad (C.6)$$

If the r-set DOFs are constrained $\{x_r\} = \{0\}$ the elastic vibration modes can be obtained with respect to the external interface DOFs from the eigenvalue problem

$$(-\omega_k^2[M_{ii}] + [K_{ii}])\{\phi_k\} = \{0\}, \quad (C.7)$$

where ω_k^2 is the k-th eigenvalue with the associated eigenvector $\{\phi_k\}$.

The matrix of the elastic vibration modes $[\Phi_e]$ is

$$[\Phi_e] = \begin{bmatrix} 0 \\ \Phi_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \phi_{i1} & \phi_{i2} & \cdots & \phi_{in} \end{bmatrix} \quad (C.8)$$

The introduction of the modal matrix $[\Psi]$ of (C.3) into (C.2) leads to the following equations of motion expressed in terms of the external and generalized DOFs, $\{x_r(t)\}$ and $\{\eta_n(t)\}$

$$\begin{bmatrix} M_o & L^T \\ L & [m] \end{bmatrix} \begin{Bmatrix} \ddot{x}_r(t) \\ \ddot{\eta}_n(t) \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & [k] \end{bmatrix} \begin{Bmatrix} x_r(t) \\ \eta_n(t) \end{Bmatrix} = \begin{Bmatrix} F_r(t) \\ 0 \end{Bmatrix} \quad (C.9)$$

where

- $[M_o] = [\Phi_r]^T[M][\Phi_r]$ is the rigid body mass matrix with respect to the r-set
- $[K][\Phi_r] = [0]$ is the rigid body force vector
- $[L]^T = [\Phi_r]^T[M][\Phi_e]$ are the modal participation factors
- $[m] = [\phi_i]^T[M][\phi_i]$ is the generalized masses
- $[k] = [\phi_i]^T[K][\phi_i]$ are the generalized stiffnesses.

$$\bullet \{F_r(t)\} = [\Phi_r]^T \{F(t)\}$$

Introducing in an ad hoc manner the modal damping ratio in the uncoupled equations of motion of the generalized coordinates $\eta_k(t)$ gives the second equation of Eqs (C.9)

$$\ddot{\eta}_k(t) + 2\zeta_k \omega_k \dot{\eta}_k(t) + \omega_k^2 \eta_k(t) = \frac{-\{L_k\}\{\ddot{x}_r(t)\}}{m_k}, \quad k = 1, 2, \dots, n. \quad (C.10)$$

The number of modes $\{\phi_k\}$, $k = 1, 2, \dots, n$ is in general much smaller than the number of internal DOFs i . The Fourier transform of (C.10), using $\eta(t) = \Pi(\omega)e^{j\omega t}$, can be written as

$$(-\omega^2 + 2j\zeta_k \omega_k + \omega_k^2)\Pi_k(\omega) = \frac{-\{L_k\}\{\ddot{X}_r(\omega)\}}{m_k}, \quad k = 1, 2, \dots, n, \quad (C.11)$$

and if $\Pi_k(\omega)$ is made explicit then

$$\Pi_k(\omega) = H_k(\omega) \frac{-\{L_k\}\{\ddot{X}_r(\omega)\}}{m_k}, \quad k = 1, 2, \dots, n. \quad (C.12)$$

where the frequency transfer function $H_k(\omega) = (-\omega^2 + 2j\zeta_k \omega_k + \omega_k^2)^{-1}$. If we use the following expression $\ddot{\Pi}_k = -\omega^2 \Pi_k$, the first part of equation (C.9), applicable to the external interface DOFs $x_r(t)$ is

$$[M_o]\{\ddot{x}_r\} + [L]^T \{\ddot{\eta}(t)\} = \{F_r(t)\}. \quad (C.13)$$

If (C.11) is substituted in the Fourier transform of (C.13) with $\ddot{\Pi}_k = -\omega^2 \Pi_k$ then

$$\left([M_o] + \omega^2 \sum_{k=1}^n \frac{H_k(\omega)\{L_k\}^T\{L_k\}}{m_k} \right) \ddot{X}_r(\omega) = F_r(\omega), \quad (C.14)$$

or

$$\left([M_o] + \omega^2 \sum_{k=1}^n H_k(\omega)[M_{eff,k}] \right) \ddot{X}_r(\omega) = F_r(\omega), \quad (C.15)$$

where the 6×6 modal effective mass $M_{eff,k}$ is defined as

$$[M_{eff,k}] = \frac{\{L_k\}^T\{L_k\}}{m_k}. \quad (C.16)$$

The mass matrix $[M_o]$ can be expanded as the sum of the modal effective masses $M_{eff,k}$

$$\begin{aligned} [M_o] &= \sum_{k=1}^i [M_{eff,k}] \\ &= [\Phi_r]^T [M] [\Phi_e] ([\Phi_e]^T [M] [\Phi_e])^{-1} [\Phi_e]^T [M] [\Phi_r] \\ &= [\Phi_r]^T [M] [\Phi_r]. \end{aligned} \quad (C.17)$$

The inverse of the modal matrix $[\Phi_e]$ exists, if all modes $\{\phi_k\}$, $k = 1, 2, \dots, i$ are calculated.

Figure C.1: Asparagus patch model [1]

The modal effective mass matrix $[M_{eff,k}]$ of mode k is the fraction of the total static mass $[M_o]$ that can be attributed to this mode.

The residual mass matrix $[M_{res}]$ is defined by

$$[M_{res}] = [M_o] - \sum_{k=1}^n [M_{eff,k}], \quad (C.18)$$

where n is the number of kept modes $n \ll i$, where i is the total number of degrees of freedom.

Finally (C.15) can now be written as

$$\sum_{k=1}^n \{ [M_{eff,k}] (1 + \omega^2 H_k(\omega)) \} \{ \ddot{X}_r(\omega) \} = \{ F_r(\omega) \}, \quad (C.19)$$

Equation (C.19) shows the relation between the reaction forces $\{F_r(\omega)\}$ at the location of the statically determinate supported structure excited by enforced acceleration $\{\ddot{X}_r(\omega)\}$. This equation will be used in combination with Miles' equation to calculate the rms reaction forces of an elastic structure exposed to random dynamic enforced acceleration. The term between the curly brackets is called the dynamic mass matrix

$$[M_d(\omega)] = \sum_{k=1}^n [M_{eff,k}] (1 + \omega^2 H_k(\omega)). \quad (C.20)$$

The concept of modal effective mass can be used to decompose a MDOF structure into a series of parallel placed SDOF systems. The discrete mass reflects the modal effective mass $m_{eff,n}$ which corresponds to angular natural frequency ω_n , spring constant $k_n = \omega_n^2 m_{eff,n}$, modal damping ratio ζ_n and the damping constant $c_n = 2\zeta_n \omega_n m_{eff,n}$. The representing series of SDOF systems is called the asparagus model of the structure [1]. This is illustrated in Fig. C.1. In [7] Witting mentioned the reduced model shown in Fig. C.1 - the Mono-Axial Mass-Spring system.

More about the theory and experimental estimation of modal effective masses is discussed in the papers [2–5, 7].

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D

CSMA, FORMULATION OF COUPLED SYSTEMS

The Coupled Systems Modal Analysis (CSMA) method is discussed by Stevens in [2] and the coupling of subsystems is summarized briefly below. Any system of mass-spring oscillators can be represented as a parallel-oscillator model (asparagus patch model) consisting of a system of SDOF elements connected in parallel. Fig. D.1 illustrates the coupling of two uncoupled MDOF systems. The masses m_{ij} , $i = 1, 2$, $j = 1, 2, 3$ in the parallel oscillator represent the modal effective masses of the subsystems. The spring stiffness k_{ij} , $i = 1, 2$, $j = 1, 2, 3$ are equal to $m_{ij}\omega_{ij}^2$ where ω_{ij} is the undamped natural frequency for mode j of the physical subsystem i . The interface itself may be assumed to have a mass $m_{interface}$.

Figure D.1: Two coupled asparagus patch models load/source

The physical and modal representations as shown in Fig. D.1 have identical eigenvalues, and their eigenvectors are associated in a simple way. For a given base acceleration, the total base reaction force for both representations is identical.

An eigenvalue problem representing the coupled dynamic system composed of two or more subsystems can be set up based on the modal parameters of the subsystems fixed at their interface. The modal parameters that are required are the undamped natural frequency, the associated modal effective mass and the (assumed) modal damping ratio for each vibration mode of each sub-system.

The mass and stiffness matrix for the coupled system in Fig. D.1 are given by

$$\mathbf{M} = \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{interface} \end{bmatrix} \quad (\text{D.1})$$

and

$$\mathbf{K} = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & -k_{11} \\ 0 & m_{12} & 0 & 0 & 0 & -k_{12} \\ 0 & 0 & k_{13} & 0 & 0 & -k_{13} \\ 0 & 0 & 0 & k_{21} & 0 & -k_{21} \\ 0 & 0 & 0 & 0 & k_{22} & -k_{22} \\ -k_{11} & -k_{11} & -k_{13} & -k_{21} & -k_{22} & \sum_{i=1}^2 \sum_{j=1}^{3,2} k_{ij} \end{bmatrix} \quad (\text{D.2})$$

The eigenvalues of this system of equations of the coupled asparagus patch models are identical to the eigenvalues for the coupled physical sub-systems. The eigenvectors of the coupled parallel oscillator systems are not identical to the coupled physical sub-systems. However, the two representations both have the same total base reaction force for any given acceleration.

The random acceleration vibration specification W_{AA} (g^2/Hz) at the interface between the load (subsystem 1) and the source (subsystem 2) is provided to design and test the load against the random vibrations environment. However, this random acceleration specification typically envelops the real dynamical random vibrations environment at the interface of the load/source. The peaks and valley representing the actual random responses are smoothed. This is illustrated in Fig. D.2. The real acceleration response is given by the black line and the envelope is given by the red dashed line, which is normally the provided specification. The peaks occur at the coupled system (load + source) resonances and the valleys at the load anti-resonances. When the envelope of the random vibration specification is applied at the base of the load over-design of the load will happen and during the random vibration test over-testing may be expected.

To prevent over-testing during the random vibration test of the load force limed vibration testing (FLVT) may be applied. A good approximation of the random interface forces W_{FF} (N^2/Hz) is needed. This approximation is the only way to reconstruct the interface acceleration spectrum, because the real excitation forces applied to the source are, in general, not known. The reconstruction of the interface acceleration spectrum can be accomplished in the following manner.

The discrete masses of the asparagus patch model of the source m_{2i} , $i = 1, 2, \dots, n$ are loaded by a white noise force W_F (N^2/Hz). The actual value of that force spectrum is not important, because the response spectrum at the interface W_A is scaled such the

Figure D.2: Random vibration specification

D

the maximum value of W_A touch the specified spectrum W_{AA} and the low and high frequency range response spectrum W_A will not beyond the spectrum W_{AA} . This done for every load application of W_F . This procedure is shown in Fig. D.3. The black dashed lines are the initial response spectra and the blue lines are the scaled response spectra. All scaled response spectra W_A are now enveloped to become the response spectrum W_{Ae} . The enveloped response spectra $W_{Ae}(f)$ will not be greater then the specified spectrum W_{AA} .

Figure D.3: Scaled response interface

The frequency response function apparent mass [1, 3] of the load $\mathbf{M}_l$, at the interface, is given by

$$\mathbf{M}_l(f) = \sum_{i=1}^n m_{li} \left[1 + \left(\frac{f}{f_{li}} \right)^2 H \left(\frac{f}{f_{li}} \right) \right], \quad (\text{D.3})$$

where f is the excitation frequency, f_{li} is the natural frequency of the (fixed) load and $H \left(\frac{f}{f_{li}} \right)$ the frequency transfer function and is given by

$$H \left(\frac{f}{f_{li}} \right) = \frac{1}{1 - \left(\frac{f}{f_{li}} \right)^2 + 2j\zeta \left(\frac{f}{f_{li}} \right)}. \quad (\text{D.4})$$

The maximum achievable interface force spectrum $W_{FF}(f)$ can be calculated with the following equation

$$W_{FF}(f) = |M_I(f)|^2 W_{Ae}(f). \quad (D.5)$$

EXAMPLE

The discussion about the CSMA method will be illustrated by an example. The load consists of three SDOF elements and the source of two SDOF elements. The masses represent the modal effective masses and the frequencies represent the natural frequencies of the fixed load and fixed source respectively. The dynamic properties of the load and source are given in Table D.1.

Table D.1: Dynamic characteristics of load and source (kg, Hz)

Load			Source	
m_{11}	m_{12}	m_{13}	m_{21}	m_{22}
25	7.5	2.5	145	25
f_{11}	f_{12}	f_{13}	f_{21}	f_{22}
75	100	125	40	60

The interface mass is kept very low to prevent interaction with natural frequencies of the coupled system, $m_{interface} = 0.01$ kg. The random acceleration vibration specification is $W_{AA} = 0.01$, $20 \leq f \leq 2000$ g^2/Hz and the excitation force applied to the source masses is $W_F = 1$, $20 \leq f \leq 2000$ N^2/Hz .

The natural frequencies of the coupled system load/source are

Table D.2: Natural frequencies coupled system

Natural frequencies (Hz)				
f_1	f_2	f_3	f_4	f_5
0.0	53.4	68.3	95.3	122.2

The vibration modes associated with the natural frequencies have no physical meaning but only a mathematical implication. The response analysis is done with a modal damping ratio $\zeta = 0.05$.

The apparent mass $\mathbf{M}_I$ of the load is calculated using Eq. (D.3) and is shown in Fig. D.4. At low frequency $\mathbf{M}_I(20) = 35$ kg, the total mass of the load. The first peak of the apparent mass is at the first natural frequency of the load $\mathbf{M}_I(75) \approx m_{11}Q = 250$ kg.

The scaled response spectra W_A derived from the subsequently loaded effective masses of the source are presented in Fig. D.5. The scaled response spectrum corresponding to m_{21} is represented by the blue line. The peak response is at 53.4 Hz. The other peaks occur at the other natural frequencies. The second scaled response spectrum corresponds to the effective mass m_{22} and is shown by the red line. Again the maximum scaled response is at 53.4 Hz. The second high peak occurs at 68.2 Hz.

The enveloped acceleration response spectrum W_{Ae} at the interface load/source is presented in Fig. D.6.

Figure D.4: Dynamic/Apparent mass load

Figure D.5: Response spectra W_A at the interface load/source

The enveloped force spectrum W_{FF} at the interface load/source is presented in Fig. D.7.

Figure D.6: Enveloped response spectra W_{Ae} at the interface load/source

Figure D.7: Enveoped interface force spectrum W_{FF}

In general, the maximum peak values of the enveloped acceleration response spectrum W_{Ae} and the maximum peak value of the enveloped force response spectrum W_{FF} do not coincide at the same frequencies.

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E

QUASI-STATIC, RANDOM AND ACOUSTIC LOADS

E.1. INTRODUCTION

During flight, the spacecraft is subjected to static and dynamic loads. Within the framework of the thesis only the following static and dynamic loads are discussed:

- Quasi-static load
- Acoustic loads
- Random vibrations

The specifications of the quasi-static design loads and the way the acoustics loads and the random vibrations are prescribed need more explanation, which will be given in the following sections.

E.2. QUASI-STATIC LOAD SPECIFICATIONS

Fig. E.1 shows a typical longitudinal static acceleration-time history for the Soyuz launch vehicle during its ascent flight. The highest longitudinal acceleration occurs just before the first-stage cutoff and does not exceed 4.3 g. The highest lateral static acceleration may be up to 0.4 g at maximum dynamic pressure and takes into account the effect of wind and gusts encountered in this phase [2]. The very low frequency dynamic loads, which are expected to have no influence on the dynamic behaviour of the spacecraft are added to the static acceleration. The static acceleration increased by the low frequency dynamic loads is called the quasi-static load (QSL).

The quasi-static design loads are multiplied by the factors of safety to construct design limit loads to design the spacecraft structure and to analyse strength and stiffness characteristics of the spacecraft structure such as permanent deformation, ultimate strength, and buckling.

Figure E.1: Typical longitudinal static acceleration [2]

E

E.3. ACOUSTIC AND RANDOM LOAD SPECIFICATION

E.3.1. INTRODUCTION

The acoustic noise inside the fairing (shroud) of the launch vehicle is assumed to be reverberant (diffuse), however this is a mathematical representation of the real sound field. The reverberant sound field representation is well accepted in the spacecraft structures design community. This sound field under the fairing will excite the outer structure of the spacecraft as well as the subsystem mounted to the outside of the spacecraft, such as solar array wing and antennae dishes. The random sound pressures exciting the outer structure of the spacecraft is transferred into random structural vibrations and acoustic pressures of inside cavities. The interior acoustic noise will excite the internal spacecraft structure and are transferred in random structural vibrations too. This process is illustrated in the Fig. E.2. The random structural vibrations will excite panel mounted equipment and components at the base. In general, small equipment is not sensitive to acoustic noise.

The typical representation of acoustic load and random acceleration specifications is discussed in subsequent sections [10, 11].

E.3.2. ACOUSTIC LOADS

Acoustic loads appear as design specifications for spacecraft and spacecraft attachments such as solar arrays and antennae. Acoustic loads are generated during launch, or in acoustic facilities for test purposes, e.g. reverberation chamber¹. It is very common to specify a reverberant sound field [2], which means that the intensity of the sound is the

¹A reverberation chamber is an enclosure with thick, rigid walls and smooth interior surfaces that strongly reflect sound waves.

Figure E.2: Mechanism to produce random vibrations

E

same for all directions. More about the fundamentals of acoustics or sound can be read in [5, 7, 9].

In general, the acoustic loads are described as *sound pressure levels* (SPL) and specified in decibels (dB) [1, 2]. The SPL is defined by

$$SPL = 10 \log \left(\frac{p}{p_{ref}} \right)^2, \quad (E.1)$$

where p is the rms pressure in a certain frequency band with frequency band width Δf , mostly *one octave* or *one third octave band*, and p_{ref} is the *reference pressure* 2×10^{-5} Pa.

The x -th octave band of two sequential frequencies f_1 and f_2 is given by

$$\frac{f_2}{f_1} = 2^x, \quad (E.2)$$

where $x = 1$ for the octave, and $x = \frac{1}{3}$ when the one-third-octave band; then $\frac{f_2}{f_1} = 2^{\frac{1}{3}} = 1.260$.

The centre frequency f_c (Hz) is defined by

$$f_c = \sqrt{f_{min} f_{max}}, \quad (E.3)$$

where f_{min} (f_{max}) is the minimum (maximum) frequency (Hz).

The frequency bandwidth Δf (Hz) is given by

$$\Delta f = f_{max} - f_{min}. \quad (E.4)$$

With $\frac{f_{max}}{f_{min}} = 2^x$ the bandwidth Δf can be expressed in terms of the centre frequency f_c as follows

$$\Delta f = \left(2^{\frac{x}{2}} - 2^{-\frac{x}{2}}\right) f_c. \quad (E.5)$$

When

- $x = 1$, the one octave bandwidth is $\Delta f = 0.7071 f_c$
- $x = \frac{1}{3}$ the one-third octave band width is $\Delta f = 0.2316 f_c$

The PSD of the pressure field $W_p(f_c)$ (Pa^2/Hz) in the frequency band with centre frequency f_c , bandwidth Δf and rms pressure $p(f_c)$ is defined as

$$W_p(f_c) = \frac{p^2(f_c)}{\Delta f(f_c)}. \quad (E.6)$$

The mean square value p_{rms}^2 of the sound pressure level is given by

$$p_{rms}^2 = \int_0^\infty W(f) df = \sum_{i=1}^k W_i(f_c) \Delta f = \sum_{i=1}^k p_i^2(f_c), \quad (E.7)$$

where k is the number of one octave or one-third octave bands.

The *overall sound pressure level* (OASPL) in dB is defined as

$$OASPL = 10 \log \left(\frac{p_{rms}^2}{p_{ref}^2} \right). \quad (E.8)$$

The following relation determines the conversion of a one third octave band to a one octave band;

$$SPL_{1-octave} = 10 \log \left[\sum_{k=1}^3 10^{\frac{SPL_{\frac{1}{3}-octave}}{10}} \right], \quad (E.9)$$

and the following relation determines the conversion of the a one-octave band to the a one-third-octave-band

$$SPL_{\frac{1}{3}-octave} = SPL_{1-octave} + 10 \log \left[\frac{\Delta f_{\frac{1}{3}-octave}}{\Delta f_{1-octave}} \right]. \quad (E.10)$$

REVERBERANT SOUND FIELD

The cross-power spectral density of the pressure $W_p(f, \mathbf{x})$ (Pa^2/Hz), in a reverberant (diffuse) sound field (acoustic chamber), at position $\mathbf{x}$ and frequency f (Hz), is given by [3]

$$W_p(f, \mathbf{x}) = W_p(f) \frac{\sin k|\Delta \mathbf{x}|}{k|\Delta \mathbf{x}|}, \quad (E.11)$$

where $W_p(f)$ (Pa^2/Hz) is the auto-power spectral density of the (reference) pressure p (Pa), $k = 2\pi f/c$ is the wavenumber, c (m/s) is the speed of sound in air, $|\Delta \mathbf{x}|$ (m) the distance between the positions $\mathbf{x}_1$ and $\mathbf{x}_2$. It is stated in [12] that the spectral density between two distinct points $\mathbf{x}_1$ and $\mathbf{x}_2$ in space is:

- dependent on the relative distance $|\Delta \mathbf{x}|$ between two points only, i.e. it is independent of the absolute coordinates of any of the two points. Hence the sound field is *homogenous*
- independent of the direction of the vector $\Delta \mathbf{x}$ between the two points, i.e. it is the same in all directions. Hence the sound field is *isotropic*

Fig. E.3 shows a plot of the normalized diffuse (reverberant) field spatial cross-spectral density in air ($c = 340\text{m/s}$) for frequencies $f=50, 150$ and 500Hz as a function of the distance Δx . This Fig. E.3 is created using the computer algebra system wxMaxima®, version 11.08.0.

Figure E.3: Normalized diffuse pressure field cross-spectral density, $y = \sin(k\Delta x)/(k\Delta x)$, $f = 50\text{ Hz}$ (blue), $f = 150\text{ Hz}$ (red), $f = 500\text{ Hz}$ (green), $c = 340\text{ m/s}$

A reverberant sound field can be simulated by a combination of (at least 26) plane waves caused by point sources placed equally distributed on a sphere with a large radius [6].

E.3.3. RANDOM ENFORCED ACCELERATION

In most cases the random mechanical loads for spacecraft and subsystems of spacecraft are specified in a very special manner. The power spectral density (PSD) values of the acceleration depend on the frequency (Hz). In general, the frequency range is between 20 and 2000 Hz. The specification must be accompanied by the G_{rms} value of the random acceleration in the frequency range. An example of a typical acceleration specification is given below.

- 20–150 Hz 6dB/oct

Figure E.4: Specification PSD acceleration $G_{rms} = 7.3g$

- 150–700 Hz $W_{\ddot{u}} = 0.04 \text{ g}^2/\text{Hz}$
- 700–2000 Hz -3dB/oct
- $G_{rms} = 7.3 \text{ g}$.

The graphical representation of the random acceleration specification is shown in Fig. E.4.

The *octave band* between f_2 and f_1 is defined by

$$\frac{f_2}{f_1} = 2^1. \quad (\text{E.12})$$

The power of 1 denotes the octave band. The number x is the number of octaves between two frequencies f and the reference frequency f_{ref} can be obtained using

$$\frac{f}{f_{ref}} = 2^x, \quad (\text{E.13})$$

this yields

$$x = \frac{\ln\left(\frac{f}{f_{ref}}\right)}{\ln 2} = \frac{\log\left(\frac{f}{f_{ref}}\right)}{\log 2}. \quad (\text{E.14})$$

The relation between the PSD values depends on the number of dBs per octave n (dB/oct) and the number of octaves between two frequencies f and f_{ref} . The relation in

dB between $W_{\ddot{u}}(f)$ and $W_{\ddot{u}}(f_{ref})$ is given by

$$10 \log \left\{ \frac{W_{\ddot{u}}(f)}{W_{\ddot{u}}(f_{ref})} \right\} = nx = \frac{n \log \left(\frac{f}{f_{ref}} \right)}{\log 2}, \quad (\text{E.15})$$

or

$$\left\{ \frac{W_{\ddot{u}}(f)}{W_{\ddot{u}}(f_{ref})} \right\} = \left(\frac{f}{f_{ref}} \right)^{\frac{n}{10 \log 2}} \approx \left(\frac{f}{f_{ref}} \right)^{\frac{n}{3}}. \quad (\text{E.16})$$

If both the frequency f axis and the axis of the PSD function $W(f)$ have a log scale the angle m (dB/freq) can be obtained by

$$m = \frac{\log W_{\ddot{u}}(f) - \log W_{\ddot{u}}(f_{ref})}{\log f - \log f_{ref}} = \frac{\log \left\{ \frac{W_{\ddot{u}}(f)}{W_{\ddot{u}}(f_{ref})} \right\}}{\log \left(\frac{f}{f_{ref}} \right)} = \frac{n}{3}. \quad (\text{E.17})$$

Finally the derivation of the following expression is obtained, a relation between the PSD functions and the frequencies

$$W_{\ddot{u}}(f) = W_{\ddot{u}}(f_{ref}) \left(\frac{f}{f_{ref}} \right)^{\frac{n}{3}} = W_{\ddot{u}}(f_{ref}) \left(\frac{f}{f_{ref}} \right)^m. \quad (\text{E.18})$$

The total root mean square (rms) value (magnitude) of $\ddot{u}(t)$ is equal to the square root of the area bounded by the PSD function between frequency limits f_1 and f_2 . This can be written as

$$\ddot{u}_{rms} = \sqrt{E \{ \ddot{u}^2(t) \}} = \int_{f_1}^{f_2} W_{\ddot{u}}(f) df \quad (\text{E.19})$$

Substituting (E.18) into (E.19) we obtain the following expression for $\ddot{u}_{rms}$

$$\begin{aligned} \ddot{u}_{rms} &= \sqrt{E \{ \ddot{u}^2(t) \}} \\ &= \sqrt{\int_{f_1}^{f_2} W_{\ddot{u}}(f_1) \left(\frac{f}{f_1} \right)^m df} = \sqrt{\frac{W_{\ddot{u}}(f_1) f_1}{m+1} \left[\left(\frac{f_2}{f_1} \right)^{m+1} - 1 \right]}, \quad f_1 < f_2 \\ &= \sqrt{-\int_{f_2}^{f_1} W_{\ddot{u}}(f_2) \left(\frac{f}{f_2} \right)^m df} = \sqrt{\frac{W_{\ddot{u}}(f_2) f_2}{m+1} \left[1 - \left(\frac{f_1}{f_2} \right)^{m+1} \right]}, \quad f_1 < f_2. \end{aligned} \quad (\text{E.20})$$

The parameters needed to calculate the G_{rms} value of the random acceleration spectrum are illustrated in Fig. E.5.

The specification of the PSD (sometimes called *acceleration spectral density* (ASD)) of the enforced acceleration or base excitation can be divided into three regions:

- Spectrum with a positive slope n_1 (rising)
- Flat spectrum (slope is zero)
- Spectrum with negative slope n_2 (falling)

Figure E.5: Calculation of G_{rms} **SPECTRUM WITH A POSITIVE SLOPE**

Fig. E.5 shows a rising spectrum with a constant slope $n_1 > 0$ between f_1 and f_2 . The constant slope is expressed in decibels per octave. The area A_1 can be calculated as follows

$$A_1 = \left\{ \frac{W(f_2)f_2}{m_1 + 1} \left[1 - \left(\frac{f_1}{f_2} \right)^{m_1 + 1} \right] \right\}, \quad (\text{E.21})$$

where $m_1 = n_1/3$ and $n_1 > 0$ is the increase of the PSD value in decibels per octave.

FLAT SPECTRUM

For a flat spectrum with a zero slope between f_2 and f_3 with $m_1 = 0$ in (E.21) the area A_2 becomes

$$A_2 = W(f_2) [f_3 - f_2], \quad (\text{E.22})$$

as shown in Fig. E.5.

SPECTRUM WITH A NEGATIVE SLOPE

For a falling spectrum of a constant slope $n_2 < 0$ between f_3 and f_4 the constant slope is expressed in decibels per octave. The area A_3 can be calculated as follows

$$A_3 = \left\{ \frac{W(f_3)f_3}{m_2 + 1} \left[\left(\frac{f_4}{f_3} \right)^{m_2 + 1} - 1 \right] \right\}, \quad m_2 \neq -1, \quad (\text{E.23})$$

where $m_2 = n_2/3$, and $n_2 (< 0)$ is the decrease of the PSD value in decibels per octave. Equation (E.23) is not applicable if $m_2 = -1$. In that case we have to calculate the value of A_3 when $\lim_{m_2 \rightarrow -1}$. This limit can be found using L'Hôpital's Rule² [8]. If $u(m_2) = \left(\frac{f_4}{f_3} \right)^{m_2 + 1} - 1$ and $v(m_2) = m_2 + 1$, then *L'Hôpital's rule* gives

$$A_3 = W(f_3)f_3 \ln \left(\frac{f_4}{f_3} \right) = 2.30 W(f_3)f_3 \log \left(\frac{f_4}{f_3} \right), \quad m_2 = -1. \quad (\text{E.24})$$

²Let $u(a) = v(a) = 0$. If there exists a neighborhood of $x = a$ such that (1) $v(x) \neq 0$, except for $x = a$, and (2) $u'(x)$ and $v'(x)$ exist and do not vanish simultaneously, then

$$\lim_{x \rightarrow a} \frac{u(x)}{v(x)} = \lim_{x \rightarrow a} \frac{u'(x)}{v'(x)}$$

whenever the limit on the right exists.

The G_{rms} of the enforced random acceleration specification can be obtained, as illustrated in Fig. E.5, by the following expression

$$G_{rms} = \sqrt{A_1 + A_2 + A_3}. \quad (E.25)$$

E.4. RANDOM VIBRATION TEST TOLERANCES

At the end of this appendix something is said about tolerance on power spectral densities and r.m.s. values of the random enforced accelerations during the random vibration test.

In the ECSS testing standard [4] the allowable tolerances for several kinds of tests specified. For random vibration test the frequency resolution will be better than 10 Hz following tolerances for the amplitude power spectral density are specified

Table E.1: Random vibration test tolerances [4]

Frequency range (Hz)	Amplitude PSD/rms
20-1000	-1/+3 dB
1000-2000	± 3 dB
Overall G_{rms}	± 10 % g

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F

CHARACTERIZATION AND SYNTHESIS OF RANDOM ACCELERATION VIBRATION SPECIFICATIONS

The appendix is the paper presented at:

COMPDYN 2013
4th ECCOMAS Thematic Conference on Computational Methods in Structural
Dynamics and Earthquake Engineering
M. Papadrakakis, N.D. Lagaros, V. Plevris (eds.)
Kos Island, Greece, 12-14 June 2013

ABSTRACT

Random acceleration vibration specifications for subsystems, i.e. instruments, equipment, are most times based on measurement during acoustic noise tests on system level, i.e. a spacecraft and measured by accelerometers, placed in the neighborhood of the interface between spacecraft and subsystem. Tuned finite element models can be used to predict the random acceleration power spectral densities at other locations than available via the power spectral density measurements of the acceleration.

The measured and predicted power spectral densities do represent the modal response characteristics of the system and show many peaks and valleys.

The equivalent random acceleration vibration test specification is a smoothed, enveloped, peak-clipped version of the measured and predicted power spectral densities of the acceleration spectrum.

The original acceleration vibration spectrum can be characterized by a different number response spectra: Shock Response Spectrum (SRS) , Extreme Response Spectrum

(ERS), Vibration Response Spectrum (VRS), and Fatigue Damage Spectrum (FDS). An additional method of non-stationary random vibrations is based on the Rayleigh distribution of peaks. The response spectra represent the responses of series of SDOF systems excited at the base by random acceleration, both in time and frequency domain.

The synthesis of equivalent random acceleration vibration specifications can be done in a very structured manner and are more suitable than equivalent random acceleration vibration specifications obtained by simple enveloping. In the synthesis process Miles' equation plays a dominant role to invert the response spectra into equivalent random acceleration vibration spectra.

A procedure is proposed to reduce the number of data point in the response spectra curve by dividing the curve in a numbers of fields. The synthesis to an equivalent random acceleration spectrum is performed on a reduced selected set of data points. The recalculated response spectra curve envelops the original response spectra curves.

A real life measured random acceleration spectrum (PSD) with quite a number of peaks and valleys is taken to generate, applying response spectra SRS, ERS, VRS, FDS and the Rayleigh distribution of peaks, equivalent random acceleration vibration specifications. Computations are performed both in time and frequency domain.

F.1. INTRODUCTION

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Subsystems (instruments, equipment, boxes) are to be qualified against rather severe mechanical random acceleration vibration test specifications as discussed in [12]. The random acceleration vibration test specifications are, in general, enforced accelerations at the interface between spacecraft and subsystems. The random vibrations are mainly induced by the acoustic loads exposed to the spacecraft during launch and performing acoustic tests, representing the launch environment. The acoustic loads (sound pressures) are assumed to be diffuse and are simulated in a reverberant chamber, like the Large European Acoustic Facility (LEAF) at ESA/ESTEC, Noordwijk, The Netherlands.

The measured random accelerations and or similar predictions are broad-banded and shows many peaks as shown in Fig. F.1 (blue line). These random acceleration measurements and predictions are converted into more or less equivalent smooth random acceleration vibration test specification, which represent as good as possible the underlying measured and calculated random acceleration responses. The equivalent random acceleration vibration test specification shall not lead to under-testing or significant over-testing of the test-item. An enveloping test random acceleration vibration specification is illustrated in Fig. F.1 (black line), which is such severe that over-testing of the test item will certainly occur.

Several methods are available to reconstruct and characterize in a very structured manner the equivalent random acceleration vibration test specification from the measured and predicted random response data. The following characterization methods are discussed in this paper:

- Shock Response Spectrum (SRS) [20], Extreme Response Spectrum (ERS) [22]. Both response spectra are based on extreme responses of SDOF systems excited by enforced random accelerations, the first in the time and and the second in frequency domain.

- Vibration Response Spectrum (VRS) [17]. The VRS is the response spectrum based on the 3σ responses of series of SDOFs, with varying natural frequency, excited at the base by random acceleration in the frequency domain.
- Fatigue Damage Spectrum (FDS) [22]. This damage response spectrum represents the cumulative damage due to the random responses of series of SDOF systems, with varying natural frequency, excited by random acceleration, both in the time and frequency domain.

Another method to synthesize the equivalent random vibration spectra is a method applied to pseudo stationary random vibration [9, 24], and is based on Rayleigh distribution of peaks (damage-potential) [36].

Figure F1: Schematic random acceleration vibration test specification (black) representing measured PSD responses (blue) g^2/Hz

Miles' equation [25] is in the synthesis process of equivalent random acceleration vibration specifications from the different response spectra of great importance.

F.2. PREVIOUS WORK

The SRS was already mentioned by M.A. Biot in 1933 [5] and later in 1941 [6]. The theoretical description of the SRS was done within the frame of earthquake engineering. A mechanical analyzer was developed to predict stresses in structural systems under transient impulse. The concept of SRS of accelerations is nowadays still in use in spacecraft structure engineering to characterize the severity of high frequency transient accelerations, such as separation of launch vehicle stages, shroud and the separation of spacecraft [33]. In SVM-5 [20] the principles and methods to analyze shocks are discussed in depth. Smallwood developed in [30] a very efficient recursive formula to calculate

the SRS. In [2, 32] the Smallwood recursive method (z-transform) was presented as a Matlab[®] or Octave[®] script. This script is applied to calculate the acceleration SRS in the time domain.

Lalanne discussed in his book "Specification Development" [22] the principle of the ERS and FDS for both sinusoidal and random vibration and the FDS for shocks as well.

Halfpenny et al discussed in their paper [14] the application of the SRS, ERS for peak accelerations and the FDS, to represent the cumulative fatigue damage caused by relative displacements (stresses), to describe new methods for vibration qualification of equipment on aircraft. Equivalent ERS and FDS are calculated for the proposed qualification random acceleration vibration test specification, which envelopes the flight ERS and SRS. The ERS is calculated in the frequency domain using numerical integration of the vibration spectrum and/or Miles' equation [25]. The peak values of stationary Gaussian process are discussed in detail in [1, 28]. In [14] the FDS is accurately obtained by numerical integration of the response spectrum or can be approximated applying Miles' equation. The approximate equation is used to calculate the equivalent random acceleration vibration spectrum.

Halfpenny described in his paper [13] the calculation of the the FDS of random vibrations both in the time and frequency domain. The random acceleration vibration spectrum is synthesized into the time domain by the summation of sine waves and the associated frequency, which in turn are applied to calculate the FDS in the time domain. The rain flow counting method is made available in Matlab[®] scripts, however, a number of rain flow counting methods are discussed in [4]. The synthesized random vibration test spectrum is calculated from the FDS, both in time and frequency domain, using the Miles' approximation for the FDS. The FDS is based on the relative displacements of the SDOF systems base excited by the random accelerations.

McNeil described in his paper [24] the application of the FDS, however, not based on the relative displacement, but on the relative pseudo-velocities of the SDOF systems. He stated that at resonance the pseudo-velocity is roughly proportional to the stress. The FDS is calculated both in the frequency and time domain. In the time domain rain flow counting is used to extract amplitudes and associated number of cycles. The equivalent random vibration spectrum is calculated inverting the FDS based on Miles' approximation. The second method discussed in this paper is based upon the Rayleigh distribution of peaks for non stationary random accelerations.

Eaton [11] described in his paper presented on the 2012 SCLV Conference a test tailoring methodology for equipment based on the ERS and FDS.

Dimaggio et al [9] mentioned the Rayleigh distribution of peaks a damage-based approach. The synthesis of the equivalent random vibration spectrum is again based on Miles' equation. Irvine presented on the 2012 SCLV conference a paper [18] a comparison of the damage potential method with the SRS, ERS and VRS methods.

The VRS is described by Irvine in [17], which is used to synthesize random vibration spectra by enveloping the VRS. Miles' equation is applied to obtain synthesized random vibration spectra. The VRS spectrum is very similar to the ERS.

In the previous referenced papers (e.g. [9]) it was recommended to vary the amplification factor and Basquin's exponent of the s-N curve, such that $Q = 10, 25, 50$ and $b = 4, 8, 12$. The worst synthesized random vibration spectra shall be applied to testing.

F.3. RESPONSE SPECTRUM

In this section a number of response spectra will be briefly discussed. The SRS is based on deterministic transient inputs and responses, while the VRS, ERS are based on the random PSD acceleration inputs and r.m.s. responses of SDOF systems. and the FDS is based on the response of SDOF systems both in the time and frequency domain. The random acceleration input is assumed to be weakly stationary and Gaussian. Mean values of responses are assumed to be zero.

F.3.1. SHOCK RESPONSE SPECTRUM (SRS)

The SRS was first introduced by Biot [5, 6] within the frame of earthquake engineering, and has been in existence for a long time.

The SRS is a graphical representation of an arbitrary transient acceleration input, how a single degree of freedom system (SDOF) (mass-spring-damper) responds to that input. Actually it shows the peak acceleration response in principle of an infinite number of SDOF systems, each of which has different natural frequencies ($\omega_i = 2\pi f_i$, $i = 1, 2, \dots$). This illustrated in Fig. F.2, where the SDOF systems are mass normalized. In fact, the SRS analysis is the maximum response of a series of SDOF systems having the same damping to a given transient signal. In practice a damping ratio $\zeta = 0.05$ ($Q = 10$) is assumed.

The analysis procedure to compute the SRS is described in [2, 32], where a MATLAB[®] script has been presented based on the recursive formula proposed by Smallwood [30].

Figure F.2: How a shock response spectrum is developed

F.3.2. VIBRATION RESPONSE SPECTRUM (VRS)

Tom Irvine introduced the VRS in [17], which is similar to the SRS, however, the transient acceleration input is now replaced by random acceleration PSD input $W_{\ddot{u}}(f)$ (g^2/Hz) and the transient maximum responses are replaced by the 1σ absolute responses of the SDOF

systems. In general, the damping ratio is taken $\zeta = 0.05$ ($Q = 10$) and is the same for all SDOF systems.

The graphical representation of the VRS is illustrated in Fig. E3. The SDOF systems are mass normalized. The standard deviation of the response of the SDOF system en-

Figure E3: How a vibration response spectrum is developed

forced at the base by random acceleration input can be calculated by the following expression

$$\sigma_i = \sqrt{\int_0^\infty |H_i(f)|^2 W_{\ddot{u}}(f) df} \approx \sqrt{\sum_{f_{min}}^{f_{max}} |H_i(f)|^2 W_{\ddot{u}}(f) \Delta f}, \quad i = 1, 2, \dots, N, \quad (E1)$$

where the frequency transfer function (FRF) $H_i(f)$ is given by

$$H_i(f) = \frac{1 + 2j\zeta f/f_i}{1 - (f/f_i)^2 + 2j\zeta f/f_i}, \quad (E2)$$

and $f_i = \omega_i/2\pi$ is the natural frequency.

If the input PSD $W_{\ddot{u}}(f)$ is relatively flat at frequencies near the natural frequency f_i equation (E15) can be approximated by Miles' equation [34]:

$$\sigma_i \cong \sqrt{\frac{\pi f_i W_{\ddot{u}}(f_i)(1 + 4\zeta^2)}{4\zeta}} \approx \sqrt{\frac{\pi f_i Q W_{\ddot{u}}(f_i)}{2}}, \quad i = 1, 2, \dots, N. \quad (E3)$$

The VRS is defined as

$$VRS(f_i) = 3\sigma_i, \quad i = 1, 2, \dots, N. \quad (E4)$$

The VRS equivalent random acceleration vibration specification $W_{\ddot{u}}(f_i)$ for the enforced acceleration can be calculated by inverting of (E3)

$$W_{VRS, \ddot{u}}(f_i) = \frac{2(VRS)^2(f_i)}{9\pi f_i Q}, \quad i = 1, 2, \dots, N. \quad (E5)$$

F.3.3. EXTREME (PEAK) RESPONSE SPECTRUM (ERS)

The ERS is similar to the SRS because peak values of the SDOF random response are depicted graphically in the ERS. Lallane already discussed in [22] the ERS for random vibrations. The $1\sigma_i$ response of the SDOF system " i " is multiplied by a random peak factor C_i . The expected value of the peak factor of the random response of the SDOF system is given by [1]:

$$E[C_i] = \sqrt{2\ln(f_i T)} + \frac{\gamma}{\sqrt{2\ln(f_i T)}}, \quad (\text{F.6})$$

and the standard deviation of the peak factor is

$$\sigma[C_i] = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2\ln(f_i T)}}, \quad (\text{F.7})$$

where f_i is the natural frequency of the SDOF system, T the time duration of the random accelerations process and $\gamma = 0.5772$ is the Euler constant [10]. Further it is assumed that $f_i T \gg 1$, and

- the random response X_i of the SDOF system has a Gaussian distribution, and
- the peak values of the random response of the SDOF system are statically independent. The statistically independence assumption is acceptable if $E[C_1]/\sigma[C_1] > 3.5$ and $f_i T > 250$ for a Gaussian process [15].

With a given standard deviation σ_i of the random response of the SDOF system the maximum expected extreme (peak) acceleration response spectrum $ERS_a(f_i)$ can be calculated using:

$$ERS_a(f_i) = E[C_i] * \sigma_i, \quad i = 1, 2, \dots, N. \quad (\text{F.8})$$

The expected peak values for the displacement, the $ERS_d(f_i)$ displacement spectrum, is given by [14]

$$ERS_d(f_i) = 9.81^2 \frac{E[C_i] * \sigma_i}{\omega_i^2}, \quad i = 1, 2, \dots, N, \quad (\text{F.9})$$

in case the PSD input spectrum $W_{\ddot{u}}(f)$ is specified in g^2/Hz .

The ERS equivalent random acceleration vibration specification $W_{ERS,\ddot{u}}(f_i)$ for the enforced acceleration can be calculated with the aid of inverting (F.8)

$$W_{ERS,\ddot{u}}(f_i) = \frac{2ERS^2(f_i)}{E(C_i)^2 \pi f_i Q}, \quad i = 1, 2, \dots, N. \quad (\text{F.10})$$

Because the SRS is also based on peak responses, the equivalent random acceleration vibration specification can be obtained using (F.10),

$$W_{SRS,\ddot{u}}(f_i) = \frac{2SRS^2(f_i)}{E(C_i)^2 \pi f_i Q}, \quad i = 1, 2, \dots, N. \quad (\text{F.11})$$

F.3.4. FATIGUE DAMAGE SPECTRUM (FDS)

In this section the fatigue damage spectrum will be based on the relative displacements FDS_d and pseudo-velocities [24] FDS_{pv} .

FDS_d IN FREQUENCY DOMAIN

The expected cumulative fatigue damage $E[D(T)]$ (in one SDOF system) involves the Palgren-Miner fatigue accumulation rule [21, 23] in combination of the s-N curve $N_k s_k^b = C$ (N_k is the number of allowable cycles at stress level s_k) for a narrow-banded Gaussian process the cumulative damage and is given by [8, 36] for f_i , $i = 1, 2, \dots, N$:

$$E[D(T, f_i)] = \frac{f_i T}{C} (\sqrt{2} \sigma_s)^b \Gamma \left(1 + \frac{b}{2} \right), \quad (\text{E.12})$$

where C is a material constant (Basquin coefficient), b is the fatigue exponent (Basquin's exponent), f_i is the natural frequency of the SDOF system, T the time duration, σ_s is the standard deviation of the stress, and Γ is the gamma function.

The stress s is proportional to the relative displacement of the SDOF multiplied by a constant K . The standard deviation of the stress $\sigma_{s,i}$ can be obtained as follows:

$$\sigma_{s,i} = K \frac{9.81}{(2\pi f_i)^2} \sqrt{\frac{\pi}{2} f_i Q W_{\ddot{u}}(f_i)}, \quad (\text{E.13})$$

because the PSD spectrum $W_{\ddot{u}}$ of the enforced random acceleration is most times specified in g^2/Hz .

The FDS_d corresponding to the natural frequency of the SDOF system " i " is now defined for f_i , $i = 1, 2, \dots, N$:

$$FDS_d(T, f_i) = f_i T \frac{K^b}{C} \left[\frac{9.81^2 Q W_{\ddot{u}}(f_i)}{2(2\pi f_i)^3} \right]^{\frac{b}{2}} \Gamma \left(1 + \frac{b}{2} \right). \quad (\text{E.14})$$

In general, the constants are taken as $K = C = 1$, the exponent $b = 4, 8, 12$, and the amplification factor $Q = 10, 25, 50$.

Instead of using Miles' equation, the standard deviation of the relative displacement response $\sigma_{d,i}$ of the SDOF system can be computed as follows

$$\sigma_{d,i} = \sqrt{\int_0^\infty |H_{d,i}(f)|^2 W_{\ddot{u}}(f) df} \approx \sqrt{\sum_{f_{min}}^{f_{max}} |H_{d,i}(f)|^2 W_{\ddot{u}}(f) \Delta f}, \quad i = 1, 2, \dots, n, \quad (\text{E.15})$$

where the frequency transfer function (FRF) $H_i(f)$ is given by

$$H_{d,i}(f) = \frac{1}{(2\pi f_i)^2} \frac{1}{(1 - (f/f_i)^2 + 2j\zeta f/f_i)}. \quad (\text{E.16})$$

The standard deviation of the stress s_i is $\sigma_{s_i} = 9.81 K \sigma_{d,i}$. The mean value of the stress is assumed to be zero.

The synthesized equivalent random acceleration vibration spectrum $W_{FDS_d, \ddot{u}}(f_i)$ can be calculated by the inverse of (E.14)

$$W_{FDS_d, \ddot{u}}(f_i) = \frac{2(2\pi f_i)^3}{9.81^2 Q} \left[\frac{C FDS_d(f_i)}{K^b f_i T \Gamma \left(1 + \frac{b}{2} \right)} \right]^{\frac{2}{b}}, \quad i = 1, 2, \dots, N. \quad (\text{E.17})$$

FDS_d IN TIME DOMAIN

If the random acceleration is provided in the time domain or is synthesized into the time domain, the the cumulative damage D is given by the Palgren Miner rule, which tells us that cumulative damage D is

$$D = \sum_{k=1}^N \frac{n_k}{N_k}, \quad (\text{F.18})$$

where n_k is the number of stress oscillations at stress level s_k , and N_k is the number of allowable oscillation at stress level s_k given by the $s - N$ curve

$$s_k^b N(s_k) = C. \quad (\text{F.19})$$

Figure F4: How a time domain damage fatigue spectrum is developed

The generation of the FDS_d in the time domain is symbolically illustrated in Fig. F4. The random enforced transient acceleration is applied to the base of each of the SDOF systems. From the calculated random relative displacements $z(t)$, per SDOF system, the numbers of cycles and associated "stress" peaks are extracted using a rain flow counting procedure [3, 27]. Using "stress" peaks and cycles the fatigue damage spectrum can be obtained. Again the stress constant and the Basquin's coefficient are assumed to be $K = C = 1$ and the Basquin exponent may vary $b = 4, 8, 12$.

FDS_{pν} IN FREQUENCY DOMAIN (SPECTRAL METHOD)

in [24] the pseudo-velocity $pν(t)$ of the SDOF system is applied to calculate the FDS_{pν}, because the pseudo-velocity is roughly proportional to stress for many structures, a scale factor exists between the stress $σ$ and the pseudo-velocity $pν$, $σ = Kpν$, [7, 19]. The maximum strain amplitude $ε$ is proportional to the "vibration Mach number" v/c , where v

is the maximum vibratory velocity and c is the speed of sound in the material. The maximum strain is given by $\epsilon = k(v/c)$, where $k = 0.145 - 2.00$ for many different configurations [7].

The pseudo-velocity is defined as the relative displacement response $z(t)$ of the SDOF system multiplied by the natural frequency of that SDOF, $p v(t) = z(t) * (2\pi f_i)$.

The expected cumulative fatigue damage $E[D(T)]$ involves the Palmgren-Miner fatigue accumulation rule [23] in combination of the s-N curve $N_k s_k^b = C$ (N_k is the number of allowable cycles at stress level s_k) for a narrow-banded Gaussian process the expected cumulative damage of FDS and is given by [8, 36], for f_i , $i = 1, 2, \dots, N$

$$FDS_{pv}(f_i) = E[D(T, f_i)] = \frac{f_i T}{C} (\sqrt{2} \sigma_s)^b \Gamma \left(1 + \frac{b}{2} \right) = \frac{f_i T}{C} K^b (2 \sigma_{pv}^2)^{\frac{b}{2}} \Gamma \left(1 + \frac{b}{2} \right). \quad (E.20)$$

The standard deviation of pseudo-velocity $\sigma_{pv,i}$ is expressed as follows:

$$\sigma_{pv,i} = 9.81 \sqrt{\frac{1}{8\pi f_i} Q W_{\ddot{u}}(f_i)}, \quad (E.21)$$

where the PSD function $W_{\ddot{u}}$ is most times given in g^2/Hz .

In general, the constants are taken as $K = C = 1$, the exponent $b = 4, 8, 12$, and the amplification factor $Q = 10, 25, 50$.

Instead of using Miles' equation, the standard deviation of the pseudo-velocity response $\sigma_{pv,i}$ of the SDOF system can be numerically calculated as follows

$$\sigma_{pv,i} = 9.81 \sqrt{\int_0^\infty |H_{pv,i}(f)|^2 W_{\ddot{u}}(f) df} \approx \sqrt{\sum_{f_{min}}^{f_{max}} |H_{pv,i}(f)|^2 W_{\ddot{u}}(f) \Delta f}, \quad i = 1, 2, \dots, n, \quad (E.22)$$

where the frequency transfer function (FRF) $H_{pv,i}(f)$ is given by

$$H_{pv,i}(f) = \frac{1}{(2\pi f_i)} \frac{1}{(1 - (f/f_i)^2 + 2j\zeta f/f_i)}. \quad (E.23)$$

The standard deviation of the pseudo-velocity s_i is $\sigma_{s_i} = K \sigma_{pv,i}$. The mean value of the stress is zero.

The PSD values $W_{FDS_{pv}, \ddot{u}}(f_i)$, $i = 1, 2, \dots, N$ of the random acceleration vibration specification can be obtained inverting (E.20) in combination with (E.21). Thus we get

$$\sigma_{pv}^2(f_i) = \frac{1}{2} \left(\frac{C FDS_{pv}(f_i)}{f_i T K^b \Gamma(1 + \frac{b}{2})} \right)^{\frac{2}{b}}, \quad (E.24)$$

and

$$W_{FDS_{pv}, \ddot{u}}(f_i) = \frac{8\pi f_i \sigma_{pv}^2(f_i)}{9.81^2 Q}, \quad i = 1, 2, \dots, N. \quad (E.25)$$

FDS_{pv} IN TIME DOMAIN

The calculation of FDS_{pv} is identical to calculation of the FDS_d, however, the stress is now proportional to the pseudo-velocity. If the random pseudo-velocity is provided in the time domain the the cumulative damage D is given by the Palgren Miner rule, which tells us that cumulative damage D is

$$D = \sum_{k=1}^N \frac{n_k}{N_k} = \frac{K^b}{C} \sum_{k=1}^N n_k p v_k^b, \quad (\text{F.26})$$

where n_k is the number oscillations at pseudo-velocity level $p v_k$, and N_k is the number of allowable oscillation at pseudo-level level $p v_k$ given by the $s - N$ curve

$$s_k^b N(s_k) = K^b p v^b N(p v) = C. \quad (\text{F.27})$$

The constants are taken $K = C = 1$ and, in general, the exponent $b = 4, 8, 12$.

The calculation procedure for FDS_{pv} is the similar to the calculation of FDS_d as shown in Fig. F.4.

F.3.5. PSEUDO STATIONARY RANDOM VIBRATION, DAMAGE POTENTIAL

The method of pseudo stationary random vibration described in this section was discussed in [9, 24], however, McNeil applied this method using pseudo velocities, as discussed in sections F.3.4 and F.3.4.

The acceleration response of a lightly damped SDOF system excited by a zero-mean, stationary Gaussian white noise excitation is narrow banded and the probability of peak values is a Rayleigh distribution function and given by

$$f(A) = \left(\frac{A}{\sigma^2} \right) \exp \left(\frac{-A^2}{2\sigma^2} \right), \quad A \geq 0, \quad (\text{F.28})$$

where A is the amplitude and σ is the standard deviation. For a zero-mean response the standard deviation is equal to the r.m.s. value of the response. The probability the amplitude is greater than A , $A >$ can be obtained by integrating the probability density function from A to ∞

$$P(A >) = \int_A^\infty f(a) da = \exp \left(\frac{-A^2}{2\sigma^2} \right). \quad (\text{F.29})$$

If we consider a stationary test of duration T_o , the total time $T(A >)$, or cumulative duration, spent during which response cycles with amplitudes exceeding A occur is

$$T(A >) = T_o \exp \left(\frac{-A^2}{2\sigma^2} \right). \quad (\text{F.30})$$

For a SDOF system, with natural frequency f_i , the mean square acceleration response to a broad-band Gaussian excitation with a power spectral density W (g^2/Hz) is approximated by Miles' equation

$$\sigma^2 = \frac{\pi}{2} f_i Q W(f_i). \quad (\text{F.31})$$

Taking the natural logarithme of both sides of (E30) and substituting (E31) yields

$$\ln\left(\frac{T_o}{T(A>)}\right) = \frac{1}{\pi f_i W(f_i)} \frac{A^2}{Q}. \quad (\text{E32})$$

For a particular natural frequency f_i , the function (E32) appears as a straight line on a plot of $\ln T(A>)$ versus linear A^2/Q as shown in Fig. E5. In Fig. E5, A_{max} is the expected maximum amplitude at the exceedence duration of one period $1/f_i$. Prescribing a test

Figure E5: Cumulative duration plot for Rayleigh distributed Maxima

based on an enveloping Rayleigh line guarantees that the damage potential of the test, for a resonance of that frequency, envelops the damage potential of flight loads over the range of damping uncertainty. The Rayleigh line starts at T_o and $A^2/Q = 0$ and goes to a minimum time, the duration of the period $1/f_i$, associated with A_{max}^2/Q . The total time $T_o = n_{A_{min}>} / f_i$, where $n_{A_{min}>}$ the number of cycles with amplitudes $A \geq A_{min}>$. The number of cycles can be obtained by the rain flow counting method. The equivalent power spectral density $W_{DP,\ddot{u}}(f_i)$ corresponding to the the line T_o to A_{max}^2/Q can be calculating by the following expression

$$W_{DP,\ddot{u}}(f_i) = \frac{(A_{max}^2/Q)}{\pi f_i \ln(f_i T_o)}. \quad (\text{E33})$$

To construct the Rayleigh line (Fig. E5) the following step by step procedure is to be done:

1. Translate the power spectral density spectra of the random vibration into a time domain random excitation.

2. Determine the absolute random acceleration response of the SDOF system, with natural frequency f_i , excited by the random excitation in the time domain.
3. Perform a rain flow counting to analyze the spectrum of amplitudes and corresponding number of cycles
4. Select a number of amplitudes A_k , $k = 1, 2, \dots$, analyze the number of cycles $n(A_k >)$, $k = 1, 2, \dots$ and calculate the durations $T(A_k >) = n(A_k >)/f_i$, $k = 1, 2, \dots$. The minimum number of cycles beyond A_{max} is $n(A_{max} >) = 1$, with $T(A_{min} >) = 1/f_i$, and the number of cycles beyond A_{min} is $n(A_{min} >) = T_o/f_i$. This step is illustrated in Fig. F.6.

Figure F.6: The evaluation of $T(A >)$ versus the amplitude A

F.4. SYNTHESIS OF EQUIVALENT RANDOM ACCELERATION VIBRATION SPECIFICATION

It is assumed that the equivalent random acceleration vibration spectra represent the same amount of (fatigue) damage as expected for the original measured or calculated spectrum. The reconstruction or synthesis of an equivalent random acceleration vibration specification $W_{\ddot{u}}(f)$ can be done using the response spectra; SRS, VRS, ERS, FDS_d , FDS_{pv} and the Rayleigh line (damage potential). The response spectra computing in the time domain can be used as well. For that purpose the equations (E.5), (E.10), (E.11), (E.17), (E.25), and (E.33) can be applied. Miles' equation has a key role computing the equivalent random acceleration specifications. The procedure to reconstruct the equivalent random acceleration vibration specification is illustrated in Fig. F.7 (a and b). In order to address the variations in damping and s-N curve fatigue exponent b , the response spectra and Rayleigh line shall be computed with amplification factor values of $Q = 10, 25, 50$ and exponents $b = 4, 8, 12$ for each natural frequency f_i of the SDOF systems. The greatest power spectral density value of $W_{\ddot{u}}(f_i)$ over the 3-9 variations of Q and b is used as the random equivalent vibration specification level at frequency $f = f_i$. Finally, the complete random acceleration vibration specification is established. However, engineering judgement is still needed.

Figure F7: Schemes to synthesize the random acceleration vibration specification

F.5. APPLICATION

F.5.1. RANDOM VIBRATION MEASUREMENT DURING A ACOUSTIC TEST

Within the frame of the ESA/TRP study: "Vibro-Acoustic Analysis Test methods for Large Deployable Structures" (VAATMLDS) [35] acoustic tests were performed on the VAATMLDS Bread Board solar array wing in the acoustic reverberant chamber of IABG [16].

The read-out of accelerometer Acc-4Y (Fig. F.9) is the random acceleration spectrum, for which a smooth and shaker controllable random vibration acceleration test spectrum should be generated, as illustrated in Fig. F.1. The read-out of accelerometer Acc-4Y is in fact the raw random acceleration vibration specification. The number of data points is 1251 and the frequency increment $\Delta f = 2Hz$. The generation of the random acceleration

Figure F8: VAATMLDS bread board 3 panel solar array wing in IABG acoustic chamber

F

Figure F9: PSD measurement of accelerometer 4-Y, Grms=24.98g

vibration test specification will be performed in a more or less structured manner applying the response spectra; SRS, ERS, VRS, FDS and the Rayleigh distribution of peaks. To account for uncertainties in damping and Palgren-Miner cumulative damage rule, the

amplification factor (quality factor) will be varied $Q = 10, 25, 50$ and fatigue exponent $b = 4, 8, 12$. Matlab[®] is applied to perform all computations.

The random acceleration spectrum is synthesized into the time domain (waveform) using the method described in [29], in combination with the Fourier transform [26]. The synthesized equivalent signals in the time domain are shown Fig. F10. It should be noticed that the synthesized time domain accelerations are random and will vary from analysis to analysis, however, having the same mean and standard deviation. This is due to the random frequency shifts in the arguments of the sine waves. The time increment $\delta t = 1/2f_{max} = 2.0 \times 10^{-4}$ s, and the total time $T = 2N\delta t = 0.5004$ s. The mean value of the all time domain signals is $\mu = 0$, and the standard deviation $1\sigma = r.m.s. = 24.98$ g. In case the time domain random acceleration is a Gaussian process the skewness $\lambda = 0$ and the kurtosis $\gamma = 3$ [31]. A skewness $\gamma > 3$ will result in higher peaks and a skewness $\gamma < 3$ will result in lower peaks compared to the ideal Gaussian process.

Figure F10: Synthesized time domain random accelerations, $\mu = 0$, $1\sigma = 24.98$ g, $\lambda = 0.01$, $\gamma = 2.97$ (λ, γ mean values 10 samples) [31]

The measured accelerations of accelerometer Acc-4Y, during the acoustic test, are now available both in the frequency and time domain. The response spectra: SRS, ERS, VRS, FDS_d , FDS_{pv} and the distribution of Rayleigh peaks (damage potential) can now be computed.

The response spectra will be divided into N_{field} fields containing each n_{field} data points. This is illustrated in Fig. F11. The parameter N_{field} is set to $N_{field} = 100$. In each field the maximum value of the spectrum, and corresponding frequency, is detected and used to calculate the synthesized PSD of the random acceleration vibration specification. The local maximum may be an internal point or at the boundaries of the field. The first and last data point of the complete spectrum are considered too. The synthesized PSD's at the detected points are connected to each other assuming the log-log scales for the spectra and corresponding frequencies, respectively. Internal PSD values can be obtained by the following expression

Figure F.11: Division of spectra in fields

$$N = \log\left(\frac{W_2}{W_1}\right) / \log\left(\frac{f_2}{f_1}\right), \quad W_i = W_1 \left(\frac{f_i}{f_1}\right)^N = W_1 \left(\frac{f_i}{f_1}\right)^{\frac{m}{3}}, \quad (\text{F.34})$$

where W_1 and W_2 are the PSD values of the extreme points, f_1 and f_2 are the associated frequencies and m is the slope of the spectrum in dB/octave. Index i indicates a point with frequency f_i in between the extreme points. Equation (F.34) is applied to calculate the area (mean square) under the synthesized equivalent PSD spectrum.

F.5.2. SYNTHESIS OF RANDOM ACCELERATION VIBRATION SPECIFICATION FROM SRS AND ERS

In this section the synthesized equivalent random acceleration vibration specification is calculated based on the SRS and ERS spectra. The SRS and ERS have 1251 and 2481 data points, respectively. Both are ranging from 20-2500Hz. The SRS and ERS are computed in accordance to the methods mentioned in sections F.3.1 and F.3.3. The VRS is common to the ERS, and therefore not considered. The computed ERS and SRS are shown Fig. F.12(a).

The SRS and ERS overlay each other very well. Instead of computing the SRS in the time domain the ERS is a very good representative shock spectrum. Most times in spacecraft structures engineering the 3σ VRS spectrum is used, however, it is recommended to use the ERS as the SRS for a given random acceleration vibration specification.

The SRS and ERS are divided into $N_{field} = 100$ fields. For each field the maximum value of the SRS and ERS is detected and applied to synthesize the reduced number of data points of the SRS and the ERS into PSD of the random acceleration vibration specification using (F.11) and (F.10), respectively. Both random acceleration vibration specifications are shown in Fig. F.12(b). Further, the synthesized random acceleration vibration specifications W_{SRS} and W_{ERS} are compared to the original response spectrum of accelerometer Acc-4Y in Fig. F.12(c). The PSD spectra W_{SRS} and W_{ERS} are computed varying the amplification factor $Q = 10, 25, 50$. The synthesized random accelerations are calculated from the original spectrum as shown in Fig. F.9, although, mean and standard deviation remain the same, but the distribution of the peaks alters in time, due to

Table F.1: Synthesis random acceleration vibration specification from ERS and SRS

	Q		
	10	25	50
W_{ERS} r.m.s.	26.1	26.3	26.9
W_{SRS} r.m.s. μ	24.0	23.1	22.5
W_{SRS} r.m.s. σ	0.8	0.6	0.4
Acc-4Y	25.0 G_{rms}		

random arguments in the sine waves representation of the original PSD spectrum. That means that the SRS and W_{SRS} are more or less random. Therefore the mean and standard deviation of W_{SRS} is calculated from 10 samples. The PSD spectrum W_{ERS} will not change. The r.m.s. values of the PSD spectra W_{SRS} and W_{ERS} are given in Table F.1. The synthesized r.m.s. values of PSD spectra of W_{SRS} underestimate the r.m.s. value of the original PSD spectrum and the synthesized r.m.s. values of the PSD spectrum of W_{ERS} show a higher r.m.s. values compared to the original PSD spectrum of the accelerometer Acc-4Y.

A further simplification of the random acceleration vibration spectra W_{SRS} and W_{ERS} can be achieved dividing the SRS and ERS spectra into less fields, however, this will result in higher r.m.s. values of the equivalent random acceleration vibration specifications.

In Fig. F.12 (d) the equivalent ERS from the equivalent W_{ERS} is computed for $Q = 10$ and compared to the original ERS.

F.5.3. SYNTHESIS OF RANDOM ACCELERATION VIBRATION SPECIFICATION FROM FDS_d

The FDS_d spectrum is computed both in the frequency and time domain. In the time domain the rain flow counting method (RF), as provided in Matlab[®], is used. The number of data points of the FDS_d in the frequency domain is 2481 and in the time domain 1251. The number of data point in the original spectrum is namely 1251 too. The FDS_d is computed varying the amplification factor $Q = 10, 25, 50$ and the fatigue exponent $b = 4, 8, 12$. For $Q = 10$ and $b = 8$ the FDS_d are shown in Fig. F.13(a). To compute the equivalent random acceleration vibration specifications from the FDS_d , the spectra are divided into $N_{field} = 100$ fields. W_{Freq} is the synthesis of the random acceleration vibration specification taken from the FDS_d spectrum and W_{RF} is the synthesis of $FDS_{d,RF}$. For $Q = 10$ and $b = 8$ both synthesized FDS are shown in Fig. F.13(b). In Fig. F.13(c) both synthesized random acceleration vibration specifications are compared to the random acceleration spectrum of Accelerometer Acc-4Y.

All computed data are given in Table F.2. The mean values and the standard deviations of the W_{RF} PSD spectra are calculated using 10 samples. The figures in between the brackets are the standard deviations.

Again it is noticed that the synthesized random acceleration vibration specifications W_{RF} (time domain) are below the original spectrum of accelerometer Acc-4Y.

Table E2: Synthesis random acceleration vibration specification from FDS_d

Q / r.m.s.	<i>b</i>					
	4		8		12	
	W_{Freq}	W_{RF} (RF*)	W_{Freq}	W_{RF}	W_{Freq}	W_{RF}
10	26.1	24.0(0.3)	26.2	23.8(0.5)	26.2	23.1(0.4)
25	26.2	24.2(0.4)	26.7	23.9(0.4)	26.7	22.9(0.5)
50	27.0	24.4(0.4)	27.4	24.0(0.4)	27.4	22.8(0.5)
Acc-4Y	25.0 G_{rms}					

* RF is Rain flow counting

F.5.4. SYNTHESIS OF RANDOM ACCELERATION VIBRATION SPECIFICATION FROM FDS_{pv}

The FDS_{pv} is quite similar to the FDS_d . The FDS_{pv} spectrum is computed both in the frequency and time domain. In the time domain the rain flow counting method (RF) was used. The number of data points of the FDS_{pv} in the frequency domain is 2481 and in the time domain 1251. The FDS_{pv} is computed varying the amplification factor $Q = 10, 25, 50$ and the fatigue exponent $b = 4, 8, 12$. For $Q = 10$ and $b = 8$ the FDS_{pv} are shown in Fig. E14(a). To compute the synthesized random acceleration vibration specifications the FDS_{pv} spectra are divided in $N_{field} = 100$ fields. W_{Freq} is the synthesis of the random acceleration vibration specification taken from the FDS_{pv} spectrum and W_{RF} is the synthesis of $FDS_{pv,RF}$. For $Q = 10$ and $b = 8$ both synthesized FDS are shown in Fig. E14(b). In Fig. E14(c) both synthesized random acceleration vibration specifications are compared to the random acceleration spectrum of Acc-4Y accelerometer.

All computed data are given in Table E3. The mean values and the standard deviations of the W_{RF} PSD spectra are calculated using 10 samples. The figures in between the brackets are the standard deviations.

It is again noticed that the r.m.s. values of the synthesized random acceleration vibration specifications W_{RF} (time domain) are below the r.m.s. value of the original spectrum of accelerometer Acc-4Y.

Table E3: Synthesis random acceleration vibration specification from FDS_{pv}

Q / r.m.s.	<i>b</i>					
	4		8		12	
	W_{Freq}	W_{RF} (RF*)	W_{Freq}	W_{RF}	W_{Freq}	W_{RF}
10	26.1	23.9(0.2)	26.1	24.1(0.4)	26.1	23.2(0.5)
25	26.4	24.3(0.5)	26.4	24.3(0.4)	26.4	23.3(0.3)
50	27.5	25.0(0.6)	27.5	24.5(0.4)	27.5	23.2(0.4)
Acc-4Y	25.0 G_{rms}					

* RF is Rain flow counting

F.5.5. SYNTHESIS OF EQUIVALENT RANDOM ACCELERATION VIBRATION SPECIFICATION BASED ON RAYLEIGH DISTRIBUTION OF PEAKS

Table F4: Equivalent random acceleration vibration specification from Rayleigh distribution of peaks

W_{DP}	Q		
	10	25	50
Mean r.m.s. μ	26.0	25.4	24.8
Std-dev r.m.s. σ	0.7	0.4	0.2
Acc-4Y	25.0 G_{rms}		

The analysis of the Rayleigh distribution of peaks (damage potential) is completely done in the time domain on a transient random acceleration as illustrated in Fig. F10. The rain flow method as provided by Matlab[®], is used to extract from the transient signal the amplitudes and corresponding number of cycles. The minimum value A_{min} of the distribution of peaks is used to calculate the number of positive crossings n_0 through A_{min} . The total time $T_{A>A_{min}} = n_0 / f_i$, $i = 1, 2, \dots$, where f_i is one the frequency in the range of the spectrum (20-2500Hz). The number of crossings (spectrum of crossings) n_0 is shown in Fig. F15(a). In the application the total time is $T = 0.5004s$, in fact the duration of the random transient signal. The total equivalent random acceleration vibration spectrum is computed from (F33). The random acceleration vibration spectrum is divided in $N_{field} = 100$ fields, and maximum values of that spectrum in these fields and associated frequencies are indicated and applied to generate the reduced equivalent random acceleration vibration specification W_{DP} . The full and reduced synthesized equivalent random acceleration spectra are shown in Fig. F15(b). The comparison to the original random acceleration vibration spectrum of accelerometer Acc-4Y is provided in Fig. F15(c). The presentation of Figures F15 are based on an amplification factor $Q = 10$.

The amplification factors are varied $Q = 10, 25, 50$ and the results of the calculation are presented in Table F4. The mean values and standard deviations of the r.m.s values of W_{test} are calculated from 10 samples.

F.5.6. ALL EQUIVALENT RANDOM ACCELERATION VIBRATION SPECIFICATIONS IN ONE PLOT

All synthesized equivalent random acceleration vibration specification from SRS, ESR, FDS_d , FDS_{pv} and from the damage potential (Rayleigh distribution of peaks) are depicted in Fig F16 (a) and (b). It can be seen that all equivalent random acceleration specifications have the same shape and about the same r.m.s values of the spectra, however, the r.m.s. values of the equivalent spectra computed from the frequency domain are somewhat higher than calculated in the time domain.

F.5.7. INFLUENCE NUMBER OF FIELDS N_{field}

The number of fields N_{field} defines to a high extent the details kept in the equivalent random acceleration vibration specification, although in combination with the number of available data points in the response spectra. In previous sections the computations of the equivalent vibration spectra was based on a number of fields $N_{field} = 100$.

The original random acceleration spectra of accelerometer Acc-4Y (Fig. E.9) is rather smoothened in the lower frequency range, but in the higher frequencies the spectra still exhibit a number of peaks and valleys (i.e Fig E.12). If we decrease the number of fields $N_{field} = 50, 25, 10, 5$, the equivalent random acceleration vibration specification becomes more smoothened, however, the r.m.s. value will increase, and at the other hand the vibration specification becomes more severe. This will be illustrated by an example computing the equivalent acceleration vibration specification from the original ERS assuming an amplification factor $Q = 10$. The figures are shown in Table E.5. The mean and standard deviation of the equivalent vibration spectrum W_{SRS} is computed using 10 samples. If the number of fields $N_{field} < 10$, then it becomes difficult to get an

Table E.5: Equivalent random acceleration vibration spectra (r.m.s.) from ERS, SRS with varying N_{field} .

	$N_{field}, Q = 10$				
	100	50	25	10	5
W_{ERS} r.m.s.	26.1	27.2	28.0	30.9	23.5
W_{SRS} r.m.s. μ	24.0	25.0	26.0	27.8	26.6
W_{SRS} r.m.s. σ	0.8	0.9	0.8	1.1	1.6

envelope of the response spectra and a r.m.s. value of the equivalent random acceleration spectrum below the r.m.s. value of the original spectrum (Fig. E.9) will be achieved. It is recommended to keep $N_{field} = 10 - 20$ as a minimum. In Fig. E.17(a) the synthesized equivalent random acceleration vibration specifications, based on the ERS, SRS and $N_{field} = 10$ are shown.

F.6. CONCLUSIONS

In this paper a number of damage spectra are discussed to characterize measured or computed random acceleration vibration spectra, most times defined in PSD (g^2/Hz). The characterization is based on equivalent damage caused by extreme peaks (SRS, ERS, VRS and Rayleigh distribution of peaks or cumulative damage (FDS), using relative displacements and pseudo-velocities. The response spectra are all based on the response of SDOF systems excited to random accelerations, both in the time and frequency domain. The principles to compute the response spectra are illustrated in the Figures E.2, E.3, E.4 and E.5.

For random acceleration vibration it is shown numerically, that the ERS is the same as the SRS (i.e. Fig. E.12(a)). The ERS is calculated in the frequency domain, but the SRS is calculated in the time domain, therefore the computation of the ERS is more straight forward then the calculation of the SRS. Nowadays in spacecraft structure engineering practice the VRS is applied to compare to the SRS, however, it is recommended to use the ERS instead of the VRS., because the ERS match better with the SRS then the VRS.

The FDS_d and FDS_{pv} spectra are quite the same.

Miles' equation fulfills a key role in the synthesis process to generate equivalent random acceleration vibration specifications, however, this equation represents the r.m.s. acceleration response of a SDOF system excited at the base by white noise random acceleration, which is an idealization of the real random acceleration spectrum. The ap-

plication of Miles' equation in the synthesis process is an approximation!

An envelope of the damage response spectrum is achieved by dividing the spectrum into a number of fields N_{field} . The lower N_{field} to more severe and smoother the equivalent random acceleration vibration specification becomes. This process is shown in Fig. E11.

All damage response spectra result in about the same equivalent random acceleration vibration specifications (see Fig. E16).

The methods described in this paper are applied to a real life problem, the random acceleration spectrum (PSD) measured during an acoustic noise test of an accelerometer placed on the outside panel of the VAATMLDS bread board solar array wing.

To handle uncertainties in damping and material properties the amplification factor Q and the Basquin's exponent b in the s-N curve have to be varied, in general, $Q = 10, 25, 50$ and $b = 4, 8, 12$. The worst case equivalent random acceleration vibration specification shall be selected, however, engineering judgement of the engineer is still needed.

The synthesized random transient signal derived from the Acc-4Y random spectrum has an invariant mean and standard deviation, however, the distribution of the peaks and the skewness and kurtosis show small variations.

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(a) SRS/ERS

(b) W_{SRS}, W_{ERS}

(c)

(d) ERS

Figure F.12: Synthesis of equivalent random acceleration vibration specification from SRS/ERS

(a)

(b)

(c)

Figure F.13: Synthesis of random acceleration vibration specification from FDS_d (RF is rain flow counting)

F

Figure F14: Synthesis of random acceleration vibration specification from FDS_{pv} (RF is rain flow counting)

Figure F.15: Synthesis of random acceleration vibration specification from potential damage (Rayleigh distribution of peaks)

(a)

(b)

(c)

F

Figure F16: Presentation of all equivalent random acceleration vibration specifications

(a)

(b)

Figure F17: Equivalent random acceleration vibration spectra, $N_{field} = 10$

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