OPTIMAL DOWNLINK RATE ALLOCATION IN MULTICELL CDMA NETWORKS

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Abstract. We study downlink rate allocation for a three cells CDMA system. Based on the discretized cell model, the rate optimization problem that maximizes the total downlink rate allocation is formulated. We propose an approximation procedure for obtaining a rate allocation in three cells case. Via numerical examples, we show that this procedure gives a good approximation of the optimal downlink rate allocation.

Key words and Phrases: Rate Allocation Optimization, Discretized Cell Model, Approximation Algorithm, CDMA Wireless Networks Model

1. INTRODUCTION

One of the most important features of future wireless communication systems is their support of different user data rates. As a major complicating factor, due to their scarcity, the radio resources have to be used very efficiently. In Code Division Multiple Access (CDMA) systems, transmissions of different terminals are separated using (pseudo) orthogonal codes. The impact of multiple simultaneous calls is an increase in the interference level, that limits the capacity of the system. The assignment of transmission powers to calls is an important problem for network operation, since the interference caused by a call is directly related to the power. In the CDMA downlink, the transmission power is related to the downlink rates. Hence, for an efficient system utilization, it is necessary to adopt a rate allocation scheme in the transmission powers assignment.

The downlink rate assignment problem has been extensively studied in the literature [2, 5, 11, 13, 15]. In [5], Duan et al. present a procedure for finding the

power and rate allocations that minimizes the total transmit power in one cell. In [11], Javidi analyzes several rate assignments in the context of the trade-off between fairness and overall throughput. The rates are supposed to be continuous and the algorithms proposed for the rate allocation are based on solving the Lagrangean dual. Another approach for joint optimal rates and powers allocation, based on Perron-Frobenius theory, is proposed by Berggren [2] and by O'Neill et al. [13]. Berggren [2] describes a distributed algorithm for assigning base station transmitter (BTSs) powers such that the common rate of the users is maximized, while in [13] multiple rates are considered. Again, both algorithms assume continuous rates. In [7], Endrayanto et al. present a model for characterizing downlink and uplink power assignment feasibility, for a single data rate. Boucherie et al. [3] extended this model to two cells. They propose a downlink rate allocation scheme which approximates very close the maximum of a generally chosen utility function.

This paper is an extension of the results in [4]. We consider three cells of a CDMA system in the plane. Our goal is to assign rates to users in each cell such that the total utility of the system is maximized. We propose a discretized model for the three cells case by projecting the location of the users into the closest line between two BTSs. Moreover, we develop an heuristic rate allocation procedure for the three cells system. The heuristic procedure is based on solving a set of multiple choice knapsack problems.

The remainder of this paper is organized as follows. In Section 2 we present the model of three cells. In Section 3 we formulate the rate allocation optimization problems and present an approximation procedure for finding a near optimal solution. Some numerical examples will be presented in section 4. We conclude our work and present ideas for further research in Section 5.

2. MODEL

Consider a CDMA system with three cells X, Y and Z with no soft-handoff area as in Fig. 1. For the moment, let assume that users are only located in area inside the triangle. Let $[X, Y]$, $[Y, Z]$, $[X, Z]$ denote the line between BTS X and BTS Y, BTS Y and BTS Z, BTS X and BTS Z. Let B_{X_1}, B_{Y_2} be the points where the borders of cells X and Y intersect $[X, Y]$; B_{X_2}, B_{Z_1} be the points where the borders of cells X and Z intersect $[X, Z]$; B_{Y_1}, B_{Z_2} be the points where the borders of cells Y and Z intersect $[Y, Z]$. Suppose each line is divided in small segments, numbered as follows: the segments between (X, B_{X_1}) are numbered $1, 2, \ldots, I_1$ and the segments between (X, B_{X_2}) are numbered $I_1 + 1, I_1 + 2, \ldots, I_1 + I_2$; the segments between (Y, B_{Y_1}) are numbered $1, 2, \ldots, J_1$ and the segments between (Y, B_{Y_2}) are numbered $J_1 + 1, J_1 + 2, \ldots, J_1 + J_2$; and the segments between (Z, B_{Z_1}) are numbered $1, 2, \ldots, K_1$ and the segments on (Z, B_{Z_2}) are numbered $K_1 + 1, K_1 + 2, \ldots, K_1 + K_2$.

Let I be the incenter of the triangle ΔXYZ. We approximate the location of a users in the interior of ΔXYZ as follows: the users in ΔXIY, ΔXIZ, respectively

ΔZYI are projected on the segments $[X, Y]$, $[X, Z]$, respectively $[Y, Z]$. Consider e.g., a segment i on $[X, Y]$ (see Fig. 1). All the users situated in the polygon bounded by segment i, the perpendiculars on $[X, Y]$ at the end points of segment i, and $[X, I]$ will be projected on segment i. We approximate the initial system with the one in which all users are projected on the lines of ΔXYZ as described above (see Fig. 2). Note that it is not necessary to know exactly the position of each user, it suffices to know approximately in which segment its location will be projected. This approximation is good enough under the deterministic path loss model (see [1, 8, 9]), i.e., the received power of a user located in a segment i at distance d_i from a BTS is equal to $P_i l_{i,X}$, where P_i is the transmission power towards segment i and $l_{i,X} = d_i^{-\gamma}$, $\gamma \geq 0$ is the path loss exponent.
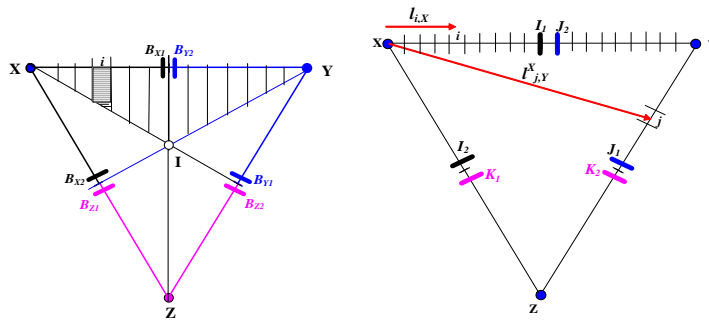

Fig. 1. Three cells cell model.   Fig. 2. Discretized Cell Model

In the approximate system, denote by m_i be the number of users in segment i of cell X, denote by n_j be the number of users in segment j of cell Y and denote by q_k the number of users in segment k of cell Z. We assume that in each segment the users are located in the middle of the segments. We suppose that the users in each segment receive the same rate and power. We measure the satisfaction of a user in a segment i by means of a positive utility function $u_i(R_i)$, where R_i is the rate allocated in segment i. For a presentation of the utility functions commonly used in the literature see [16]. Our goal is to allocate rates from a discrete and finite set $\{R_1, ..., R_L\}$ to users such that the total utility, i.e., the sum of the utilities of all users, is maximized under the condition that the prescribed quality of service is met for all users and that a feasible power assignment exists.

2.1. Downlink Interference Model

Consider the discretized cell model for three cells in Fig. 2. For a user in segment i of cell X, the energy per bit to interference ratio is given by

$$\left(\frac{E_b}{I_0}\right)_i = \frac{W}{r_i} \frac{X_i l_{i,X}}{I_{\text{intra}} + I_{\text{inter}} + N_0},$$

where W is the system chip rate, r_i is the downlink data rate, X_i is the transmit power of BTS X to the user located in a segment i, $l_{i,X}$ is the path loss from BTS X to the user in segment i and N_0 is the thermal noise. The interferences from users in its own cell, $I_{\text{intra}} = \alpha l_{i,X} \left(\sum_{l=1}^{I_1+I_2} m_l X_l - X_i \right)$ and the interference from users in other cells, $I_{\text{inter}} = l_{i,X}^Y \left(\sum_{j=1}^{J_1+J_2} n_j Y_j \right) + l_{i,X}^Z \left(\sum_{k=1}^{K_1+K_2} q_j Z_j \right)$,where α is a non-orthogonality factor, $0 \le \alpha \le 1$; $l_{i,X}^Y$, respectively $l_{i,X}^Z$ are the path loss from BTS Y, respectively BTS Z, to a user in segments i of cell X; Y_j, respectively Z_k are the transmit power of BTS Y, respectively BTS Z, to a user in the segment j of cell Y, respectively segment k of cell Z. Under the perfect power control, the energy per bit to interference ratio of the user in segment i should satisfy the power control equation $\left(\frac{E_b}{I_0} \right)_i = \epsilon^*$, where ϵ^* is the required energy per bit to interference ratio. In practice, the required energy per bit to interference ratio depends on the downlink data rate allocation [10, 12]. Without loss of generality, we assume that ϵ^* is the same for all downlink rate r_i. Then, the required transmit power of BTS X to the user in segment i is

$$X_i = V(r_i) \left(\alpha P_X + g_{i,X}^Y P_Y + g_{i,X}^Z P_Z + \frac{1}{l_{i,X}} N_0 \right), \tag{1}$$

where $V(r_i) = \frac{\epsilon^* r_i}{W + \alpha \epsilon^* r_i}$, $r_i \in \{r_1, r_2, \cdots, r_L\}$, $g_{i,X}^Y = \frac{l_{i,X}^Y}{l_{i,X}}$ and $g_{i,X}^Z = \frac{l_{i,X}^Z}{l_{i,X}}$ is the fraction of path loss to a user in segment i from BTS Y, and respectively BTS Z, to path loss from own BTS, and $P_X = \left(\sum_{i=1}^{I_1+I_2} m_i X_i \right)$, $P_Y = \left(\sum_{j=1}^{J_1+J_2} n_j Y_j \right)$ and $P_Z = \left(\sum_{k=1}^{K_1+K_2} q_k Z_k \right)$ is the total transmit power of BTS X, BTS Y and BTS Z, respectively. Then, the total transmit power of BTS X can be expressed by multiplying Eq. (1) with the number of terminals in segment i, n_i, and then sum it up for all segments. We obtain the following relation of the total transmit power of BTS X, BTS Y and BTS Z for a given users distribution in the cells and for given rate allocations to users in all segments.

$$\left(1 - \sum_{i=1}^{I_1+I_2} \alpha m_i V(r_i) \right) P_X - \left(\sum_{i=1}^{I_1+I_2} m_i V(r_i) g_{i,X}^Y \right) P_Y$$
$$- \left(\sum_{i=1}^{I_1+I_2} m_i V(r_i) g_{i,X}^Z \right) P_Z = \left(\sum_{i=1}^{I_1+I_2} \frac{m_i V(r_i)}{l_{i,X}} \right) N_0, \tag{2}$$

Similarly, we can express the total transmit power for BTS Y and Z. Hence, we can write the transmit power equations in the following matrix form

$$(\mathbf{I} - \mathbf{T}) \mathbf{P} = \mathbf{c}, \tag{3}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{P} = (P_X \quad P_Y \quad P_Z)^T$ is a column vector representing the total transmit power of BTSs in the system;

$\mathbf{c} = N_0 \left(\sum_{i=1}^{I_1+I_2} \frac{m_i V(r_i)}{l_{i,X}} \quad \sum_{j=1}^{J_1+J_2} \frac{n_j V(r_j)}{l_{j,Y}} \quad \sum_{k=1}^{K_1+K_2} \frac{q_k V(r_k)}{l_{k,Z}} \right)^T$ is a column vector representing the right hand side of the system of equations and

$$\mathbf{T} = \begin{pmatrix} I_X & I_{XY} & I_{XZ} \\ I_{YX} & I_Y & I_{YZ} \\ I_{ZX} & I_{ZY} & I_Z \end{pmatrix}. \tag{4}$$

where

- $I_X = \alpha \sum_{i=1}^{I_1+I_2} m_i V(r_i)$, $I_Y = \alpha \sum_{j=1}^{J_1+J_2} n_j V(r_j)$ and $I_Z = \alpha \sum_{k=1}^{K_1+K_2} q_k V(r_k)$ are related to the intracell interference of cell X, Y and Z;

- $I_{XY} = \sum_{i=1}^{I_1+I_2} m_i V(r_i) g_{i,X}^Y$ and $I_{XZ} = \sum_{i=1}^{I_1+I_2} m_i V(r_i) g_{i,X}^Z$ are related to the intercell interferences from cell X to cell Y, and respectively cell Z;

- $I_{YX} = \sum_{j=1}^{J_1+J_2} n_j V(r_j) g_{j,Y}^X$ and $I_{YZ} = \sum_{j=1}^{J_1+J_2} n_j V(r_j) g_{j,Y}^Z$ are related to the intercell interferences from cell Y to cell X, and respectively cell Z;

- $I_{ZX} = \sum_{k=1}^{K_1+K_2} q_k V(r_k) g_{k,Z}^X$ and $I_{ZY} = \sum_{k=1}^{K_1+K_2} q_k V(r_k) g_{k,Z}^Y$ are related to the intercell interferences from cell Z to cell X, and respectively cell Y.

2.2. Downlink Transmit Power Feasibility

In this paper, we are interested in finding a feasible solution for system (3). The feasible transmit powers of all BTSs are guaranteed when the Perron-Frobenius eigenvalue of matrix $\mathbf{T}$ is smaller than *one* [14], i.e.,

$$\mathbf{P} \geq \mathbf{0} \text{ and } \mathbf{P} = (\mathbf{I} - \mathbf{T})^{-1}\mathbf{c} \quad \Leftrightarrow \quad \lambda(\mathbf{T}) < 1. \tag{5}$$

This provides a clear motivation for discretizing the cells into segments, since it facilitates obtaining an analytical model for characterizing the transmit power feasibility for a certain rate allocation and a certain user distribution. In the next section, based on $\lambda(\mathbf{T})$, we propose a downlink rate allocation scheme that approximates the maximum of the total utility.

3. THE RATE ALLOCATION PROBLEM

Let $R = \{R_1, R_2, ..., R_L\}$ be the set of admissible rates, where $R_1 < R_2 < ... < R_L$. The decision of dropping the users of a segment is equivalent with assigning zero rate to the respective segment, case in which $R_1 = 0$. The problem of allocating rates from the set R to users such that the total utility of the users is

maximized, under the condition of ensuring the required energy per bit to interference ration and a feasible power assignment, can be formulated as follows:

$$\max \sum_{i=1}^{I_1+I_2} u_i(r_i) + \sum_{j=1}^{J_1+J_2} u_j(r_j) + \sum_{k=1}^{K_1+K_2} u_k(r_k)$$

$$(P): \text{ s.t. } \left(\frac{E_b}{I_0}\right)_i (r,p) = \epsilon^*, \text{ for each user } i$$

$$r_i \in \{R_1,...,R_L\}, \ p_i \geq 0,$$

$$\text{for each user in segment } i \text{ of cell } X, Y \text{ and } Z,$$

where r_i, respectively p_i represent the rate, respectively the power allocated to segment i and ϵ^* is the threshold for the the energy per bit to interference ratio. For the case of two cells, using the Perron-Frobenius eigenvalue, the feasibility condition leads to a nice separation of the rate allocation optimization problem (see [3]). However, in the case of three cells we do not obtain a nice separation as in the two cells cases. Therefore we propose an approximation procedure based on the rate allocation problem for two cells. Given a rate allocation in a cell, we solve the rate allocation in the other two cells. The main idea of the heuristic procedure is as follows: we first consider cell X and cell Y. Once a rate allocation has been determined for cells X and Y, consider cell Y and cell Z and incorporate the interference from cell X as noise. Now consider cell X and cell Z and the interference from cell Y as noise, etc. This procedure may be followed until sufficient convergence is reached. We show in the next lemma that the iteration of the procedure leads to an increasing value of the total utility as defined as the objective function of optimization problem P.

Lemma 1 *The total utility of the $(N+1)^{th}$ iteration is at least equal to the total utility of the Nth iteration.*

Proof. Let $\{r_i\}_X^N$ be the rate allocation of cell X that is being fixed in the Nth iteration. Let, $\{r_j\}_Y^N$ and $\{r_k\}_Z^N$ be the optimal rate allocations for cell Y and cell Z in the Nth iteration. Consider the next iteration with the rate allocation in cell Y being fixed $\{r_j\}_Y^{N+1} = \{r_j\}_Y^N$. It is clear that the sets $\{r_i\}_X^N$ and $\{r_k\}_Z^N$ are also feasible rate allocations in the $(N+1)^{th}$ iteration. Hence, the total utility of the $(N+1)^{th}$ iteration is at least equal to the total utility of the Nth iteration ∎

The lemma above shows that we can guarantee that the approximation procedure leads to an increasing value of the total utility. The details of the approximation procedure are formulated in the following section.

3.1. Approximation for Three Cells Rate Allocation

Let $\{r_k\}_Z$ be the set of a feasible rate in Z, i.e., $r_Z = \left(r_1, r_2, \ldots, r_{(K_1+K_2)}\right)$. Given the rate allocation in cell Z, we are interested in finding the optimal rate

allocation of cell X and cell Y. Thus, the presence of cell Z as a noise power has to be incorperated in the formulation of the rate allocation problem. This is done as follows:

1. First, using (2), we express P_Z as a function of the other parameters:

$$P_Z = A_Z P_X + B_Z P_Y + K_Z, \tag{6}$$

   where $A_Z = \frac{I_{ZX}}{(1-I_Z)}$, $B_Z = \frac{I_{ZY}}{(1-I_Z)}$ and
   $K_Z = \left(\sum_{k=1}^{K_1+K_2} \frac{q_k V(r_k)}{l_{k,Z}} \right) \Big/ (1 - I_Z)$.

2. By replacing P_Z with (6) in system (3), we obtain a new system, depending only on P_X and P_Y. The new system can be rewritten in the same form as (3) as follows:

$$\left(\mathbf{I} - \tilde{\mathbf{T}} \right) \tilde{\mathbf{P}} = \tilde{\mathbf{c}}, \tag{7}$$

   where $\tilde{\mathbf{P}} = (P_X \quad P_Y)^T$; $\tilde{\mathbf{c}} = \hat{\mathbf{c}} + K_Z \begin{pmatrix} I_{XZ} \\ I_{YZ} \end{pmatrix}$ with $\hat{\mathbf{c}}$ be the column vector obtained by deleting the third row of matrix $\mathbf{c}$ and

$$\tilde{\mathbf{T}} = \hat{\mathbf{T}} + \begin{pmatrix} I_{XZ} \\ I_{YZ} \end{pmatrix} \begin{pmatrix} A_Z & B_Z \end{pmatrix}. \tag{8}$$

   with $\hat{\mathbf{T}}$ be the 2×2 submatrix of $\mathbf{T}$ obtained by deleting the entries of third row and the third column. From (8), we notice that the feasibility of the system with two cells under the presence of the third cell is smaller or equal to the the feasibility of the system of two cells only, i.e., $\lambda\left(\tilde{\mathbf{T}}\right) \leq \lambda(\hat{\mathbf{T}}) < 1$.

3. System (7) imply that the positive transmit power P_X and P_Y exist if and only if $\lambda\left(\tilde{\mathbf{T}}\right) < 1$, which is equivalent to the following three conditions (for details see [3]):

$$I_X + I_{XZ} A_Z < 1,$$
$$I_Y + I_{YZ} B_Z < 1,$$

$$\frac{1 - (I_X + I_{XZ} A_Z)}{I_{XY} + I_{XZ} B_Z} > \frac{I_{YX} + I_{YZ} A_Z}{1 - (I_Y + I_{YZ} B_Z)}.$$

   The first equation is related to the pole capacity constraint of cell X and the effect of the interference from cell Z. The second equation is related to the pole capacity constraint of cell Y and the effect of the interference from cell Z. Moreover, in the third inequality, the left part is related to cells X and Z only and the right part is related to cells Y and Z only.

Hence, given a rate allocation in cell Z, the rate allocation problem of cell X and cell Y can be formulated as follows

$$\max \sum_{i=1}^{I_1+I_2} u_i(r_i) + \sum_{j=1}^{J_1+J_2} u_j(r_j)$$

$$P(Z): \quad \text{s.t.} \sum_{i=1}^{I_1+I_2} \left[\alpha + g_{i,X}^Z A_Z \right] m_i V(r_i) < 1,$$

$$\sum_{j=1}^{J_1+J_2} \left[\alpha + g_{j,Y}^Z B_Z \right] n_j V(r_j) < 1,$$

$$\frac{1 - \sum_{i=1}^{I_1+I_2} \left[\alpha + g_{i,X}^Z A_Z \right] m_i V(r_i)}{\sum_{i=1}^{I_1+I_2} \left[g_{i,X}^Y + g_{i,X}^Z B_Z \right] m_i V(r_i)}$$

$$> \frac{\sum_{j=1}^{J_1+J_2} \left[g_{j,Y}^X + g_{j,Y}^Z A_Z \right] n_j V(r_j)}{1 - \sum_{j=1}^{J_1+J_2} \left[\alpha + g_{j,Y}^Z B_Z \right] n_j V(r_j)},$$

$$r_i, r_j \in \{R_1, ..., R_L\}.$$

As in [3], one can prove that the optimum of problem $P(Z)$ is equal to $\max_{t_Z \in [t_{\min}, t_{\max}]} \left[P_1(t_Z) + P_2(t_Z) \right]$, where $[t_{\min}, t_{\max}]$, $P_1(t_Z)$ and $P_2(t_Z)$ are defined as follows:

$$\max \sum_{i=1}^{I_1+I_2} u_i(r_i)$$

$$P_1(t_Z): \quad \text{s.t.} \sum_{i=1}^{I_1+I_2} \left[\alpha + t_Z g_{i,X}^Y + g_{i,X}^Z \left[A_Z + t_Z B_Z \right] \right] m_i V(r_i) < 1,$$

$$\sum_{i=1}^{I} r_i > 0, \quad r_i \in \{R_1, ..., R_L\},$$

and

$$\max \sum_{j=1}^{J_1+J_2} u_j(r_j)$$

$$P_2(t_Z): \quad \text{s.t.} \sum_{j=1}^{J_1+J_2} \left[\alpha + \frac{g_{j,Y}^X}{t_Z} + g_{j,Y}^Z \left[\frac{A_Z}{t_Z} + B_Z \right] \right] n_j V(r_j) < 1,$$

$$r_i \in \{R_1, ..., R_L\},$$

where

$$[t_{\min}, t_{\max}] = \left[\left[\min_{r_j \in R^L} t_{Z_1}(r_j) \right]^+, \quad \left[\max_{r_i \in R^L, r_i \neq 0} t_{Z_2}(r_i) \right]^+ \right] \text{ with}$$

$$t_{Z_1}(r_j) = \frac{\sum_{j=1}^{J_1+J_2} \left[g_{j,Y}^X + g_{j,Y}^Z A_Z \right] n_j V(r_j)}{1 - \sum_{j=1}^{J_1+J_2} \left[\alpha + g_{j,Y}^Z B_Z \right] n_j V(r_j)}, \tag{9}$$

$$t_{Z_2}(r_i) = \frac{1 - \sum_{i=1}^{I_1+I_2} \left[\alpha + g_{i,X}^Z A_Z \right] m_i V(r_i)}{\sum_{i=1}^{I_1+I_2} \left[g_{i,X}^Y + g_{i,X}^Z B_Z \right] m_i V(r_i)}. \tag{10}$$

Note that $P_1(t_Z)$ and $P_2(t_Z)$ are multiple knapsack problems, so they can be easily solved (see [4, 6]). Thus, given the rate allocation in cell Z, we can find the optimal rate allocation in cell X an cell Z. Moreover, from the above formulations we observe that for a rate allocation in cell Y, $\{r_j\}_Y$, and given rate allocation in cell Z, we can calculate the value of $t_{Z_1}(r_j)$ using formula (9). Similarly, we can calculate the value of $t_{Z_2}(r_i)$ using formula (10) for a rate allocation in cell X and given rate allocation in cell Z. Thus, the number of t_Z values that we have to analyze is at most the number of rate allocations in cells X and Y.

Based on the above observations, we develop a heuristic procedure for finding the optimal rate allocations for three cells in the next section.

3.2. Heuristic Procedure

1. Initialize with zero the rate allocation in cell Z. Perform the following algorithm to find optimal rate allocations for cell X and cell Z.

   Algorithm: *Finding optimal rate allocation for cell X and cell Y* :

   - *For any feasible rate allocation in cell Y, find set $\{t_{Z_1}\}$ from Eq. (9) and for any feasible rate allocation in cell X find set $\{t_{Z_2}\}$ from Eq. (10)*
   - *For all $t_Z \in \{t_{Z_1}\} \cup \{t_{Z_2}\}$, solve $P_1(t_Z)$ and $P_2(t_Z)$ with an algorithm for* the multiple choice knapsack problem.
   - *Choose the $t_Z \in \{t_{Z_1}\} \cup \{t_{Z_2}\}$ for which* $\max\limits_{t_Z}\{P_1(t_Z)+P_2(t_Z)\}$ *is attained.*

2. Given the rate allocation in cell X obtained in step 1, we determine the optimal rate allocation for cell Y and Z using the algorithm above where cell X and cell Z change roles.

3. Given the rate allocation in cell Y obtained in step 2, we determine the optimal rate allocation for cell X and Z using the algorithm above where cell Y and cell Z change roles.

4. Repeat step 1 to step 3 until the convergence is found $\left|U^{(N+1)} - U^{(N)}\right| \leq \delta$, for δ small, where U^N is the total utility of the system at N^{th} iteration.

We were not able to prove whether the value obtained by this procedure is a global optimum. Some numerical experiments using this algorithm are presented in the next section.

4. NUMERICAL EXAMPLES

Recall the discretized cell model in Fig. 2. Suppose that the distance between each two BTSs is 2 km and the cell radius of a cell is 1 km, i.e., the case of symmetric downlink cell borders. We divide the line between two BTSs into 8 equal-width segments of 250m each. We have chosen examples with small number of segments for practical reasons: for these examples is easy to calculate the global optimum through enumeration, so we could compare our results.

The system parameters for this example are given as follows: the system chip rate $W = 3.84$ MHz, thermal noise $N_0 = -169$ dBm/Hz, path loss exponent $\gamma = 4$, downlink non-orthogonality $\alpha = 0.3$, QoS required $\frac{E_b}{I_0}$, $\epsilon^* = 5$ dB and downlink transmission rate $r_i \in \{32, 64, 144\}$ kbps [10]. For this paper, we consider the utility of a cell as the total sum of the rates allocated in the segments of the cell, i.e., $u_i(r_i) = n_i r_i$.

In our numerical experiments, we have considered two types of load: homogeneous load and non-homogeneous load.

In the case of homogeneous load, we did 30 experiments where the number of users n in each segment of an instance is generated from uniform distribution on $[1, 50]$, i.e., $n \sim U(1, 50)$. We have observed that in all the examples studied, the solution obtained by the heuristic converged to the optimum (which we obtained by enumerating all possible solutions) in two or three iterations. Moreover, the total utility in each cell converged as well. We also observed that the total utility is almost equally distributed among cells.

In the case of non-homogeneous load, we have conducted 90 experiments, constructed as in Table 1, where the number of users in segment i is generated from a uniform distribution on $[a, b]$, i.e., $n_i \sim U(a, b)$. The algorithm converges very fast, attaining again the optimal solution. However, it can be noted that the time needed for converges increases proportional with the load of the system.

	Convergence after			
Case of Unifom Distribution	2nd	3rd	4th	5th
$X, Y, Z \sim U(0, 8)$	66.7%	33.3%	0%	0%
$X \sim U(0, 10),\ Y, Z \sim U(0, 8)$	73%	13%	14%	0%
$X \sim U(0, 10),\ Y, Z \sim U(0, 5)$	46.7%	46.7%	6%	0.6%
$X, Y, Z \sim U(0, 30)$	20%	60%	18%	2%
$X, Y \sim U(20, 30), Z \sim U(0, 5)$	65%	25%	10%	0%
$X, Y, Z \sim U(20, 30)$	70%	20%	5%	5%

Table 1. Numerical experiments with uniform distribution.

5. SUMMARY AND FURTHER RESEARCH

In this paper, we propose a discretized model for a three cells CDMA system. For this model, we present a heuristic procedure for finding a downlink rate allocation which maximizes the total utility of the system. Tested on small numerical experiments, the heuristic always converged to the optimal solution in few iterations.

It is among our aims for further research to develop a downlink rate algorithm that takes into account mobility of users and limited transmit powers of BTSs.

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