A spatial mechanism for pilot laser alignment with four independently controlled degrees of freedom.

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Summary

Alignment mechanism for optical components, such as mirrors for manipulating laser beams, frequently require four degrees of freedom: two translations and two rotations, i.e. a four axis system. When the adjustment of one axis influences the others, as often will be the case, alignment procedures can be rather cumbersome. The mechanism, described in this paper, has four mutually independent adjustments, each controlling just one axis. Rotation takes place around a fixed point, which can be located freely in space. A computer simulation program has been developed to determine first and higher order deviations. This program proved to be a useful designer's tool to optimize the construction with respect to other system parameters.

Introduction

When using a CO₂ laser for materials processing usually a HeNe pilot laser is incorporated to trace the beam. The transversal and angular alignment of the pilot beam to the main beam is an essential step and will, depending on the alignment mechanism take some time. In laboratory situations where components have to be changed regularly a suitable mechanism becomes important. The system, described in this article has been designed to align a pilot system for a pulsed TEA-CO₂ laser with an unstable resonator but has general applicability.

Requirements for the pilot system

A typical configuration is given in Figure 1. The design was destined for two different CO₂ lasers, a 80 J system with glow discharge pre-ionisation¹ and a 500 J E-beam controlled system which is still under construction².

Figure 1. Configuration scheme.

The pilot is coupled into the resonator by a ZnSe window through a small hole in the back mirror of the unstable resonator. To align the pilot system just four degrees of freedom are required, two translations x and y to coincide with the system axis (given by the aperture in the back mirror) and two rotations ψ and θ with respect to a fixed point. For the given configuration this fixed point is the origin of the x - y - z frame.

Table 1: List of requirements.

1. Two orthogonal rotations, both orthogonal to the beam axis.2. Centre of rotations on the beam axis, fixed in space, outside the pilot system.3. Two orthogonal translations of the pole (centre of rotations) both orthogonal to the beam axis.4. All adjustments mutually independent (the alignment of one axis may not influence any other one).5. Rigid and temperature independent system.

Constructions

While many good mechanisms are known which meet almost all requirements except req.2, some special constructions are needed to get the point of rotation outside the system. Two possible solutions are given in the Figures 2 and 3.

Figure 2. Rotation about an outside pivot by means of a spherical bearing.

Figure 3. An outside pivot realized by a bar mechanism with flexible joints.

With the construction of Figure 2 the pilot can rotate with respect to a shifting ring. The ring itself can be translated with respect to the base. The required four degrees of freedom are obtained by cascading the two systems. In fact there is an additional degree of freedom, a rotation about the axis of the beam. Rotation of the shifting ring is not allowed this will result in a rotation of the inner coordinate frame. After the alignment of the beam the position has to be fixed by some clamp screws. A second function of these screws is to remove all tolerances. Fastening these screws however can cause some misalignment. The total construction will be rather expensive due to the difficult shaped high precision components. Another disadvantage is the possibility of stick slip due to friction between the slides.

The latter problem can be solved adequately by using elastic elements as shown in Figure 3. The pilot is coupled on the translating ring by a set of three bar linkages which are directed onto one point, the pole or pivot position. Two orthogonally placed rotation screws can rotate (ϕ and θ directions) the pilot with respect to the translating ring. This ring itself is coupled by three parallel bar linkages on the base and can be translated by the two translation screws (x and y directions). It can be seen easily that only one spring is required to keep all screws in contact. The mechanism has been inspired by a plane four-bar linkage which is well known and widely used in its two-dimensional form. For an extension to three dimensions however we first have to analyse it more thoroughly.

Analyses of a three dimensional bar mechanism

The system is schematically shown in Figure 4. The pilot laser is supposed to be on the bottom of the moving platform with the beam crossing the point P at a height h above the platform. From a mobility analysis³ it can be derived that the platform has three independent degrees of freedom. This subsystem will be indicated as TRIAX.

The coordinate systems as used are indicated in Figure 4. The origin of the Moving (x, y, z) frame is expressed in Fixed (X, Y, Z) coordinates. Two freedoms, the translations X_0 and Y_0 are controlled by the rotation screws. With $\phi = \text{constant}$, it is easily seen that this also fixes the value of Z_0 . The orientation of the moving frame is specified by a sequence of rotations about the X, Y and Z axes.

As in every case of a sequence of rotations, the order in which the rotations are performed determines the final outcome. A handsome set of rotations as shown in Figure 5 are roll, pitch and yaw (RPY) as used on board of ships, space vehicles etc. The RPY-rotation matrix is specified by Paul⁴ as:

$$\text{RPY}(\phi, \theta, \psi) = \text{Rot}(z, \phi) \text{Rot}(y, \theta) \text{Rot}(x, \psi) \quad (1)$$

that is a rotation of ψ about station x , followed by a rotation θ about station y and finally a rotation ϕ about station z . The total rotation matrix is then found to be (eqn.2):

Figure 4. TRIAX system. The moving platform has three degrees of freedom.

Figure 5. Roll, Pitch and Yaw angles.

$$RPY = \begin{pmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi \end{pmatrix} \quad (2)$$

Now any point (x, y, z) in the moving frame can be expressed into fixed coordinates by the Denavit-Hartenberg transformation matrix⁵ (DH-matrix):

$$\begin{pmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{pmatrix} = \begin{pmatrix} RPY & X_0 \\ 0 & Y_0 \\ 0 & Z_0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_i \\ y_i \\ z_i \\ 1 \end{pmatrix} \quad (3)$$

The moving platform, which is the suspension of the pilot laser, has 6 degrees of freedom of which 3 are mutually independent. When X_0, Y_0 and ϕ are chosen as independent variables (control variables) then the others i.e. Z_0, ψ and θ can be found from the constraint equations, following from the given length L of the linkages:

$$\Delta L_1 = (X_{F1} - X_{M1})^2 + (Y_{F1} - Y_{M1})^2 + (Z_{F1} - Z_{M1})^2 - L_1^2 = 0 \quad (4a)$$

$$\Delta L_2 = (X_{F2} - X_{M2})^2 + (Y_{F2} - Y_{M2})^2 + (Z_{F2} - Z_{M2})^2 - L_2^2 = 0 \quad (4b)$$

$$\Delta L_3 = (X_{F3} - X_{M3})^2 + (Y_{F3} - Y_{M3})^2 + (Z_{F3} - Z_{M3})^2 - L_3^2 = 0 \quad (4c)$$

The coordinates of the joints follow from Figure 4 as:

$$\begin{array}{lll} X_{F1} = 0 & Y_{F1} = R & Z_{F1} = 0 \\ X_{F2} = -.5 R/\sqrt{3} & Y_{F2} = -.5 R & Z_{F2} = 0 \\ X_{F3} = .5 R/\sqrt{3} & Y_{F3} = -.5 R & Z_{F3} = 0 \\ x_{M1} = 0 & y_{M1} = r & z_{M1} = 0 \\ x_{M2} = -.5 r/\sqrt{3} & y_{M2} = -.5 r & z_{M2} = 0 \\ x_{M3} = .5 r/\sqrt{3} & y_{M3} = -.5 r & z_{M3} = 0 \end{array} \quad (5)$$

The coordinates of the joints M1 until M3 on the moving platform, which are given with respect to the moving frame, should be translated to fixed coordinates (eq. 3) before they can be substituted in eq. 4. After solving for z_0 , ϕ and θ the deviations of the point P are found from equation 6 where Z_0' represents the initial value of Z_0 .

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ 1 \end{pmatrix} = \begin{pmatrix} & & \\ & \overline{DH} & \\ & & \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ h \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ h + Z_0' \\ 1 \end{pmatrix} \quad (6)$$

Numeric procedure

Since the equations are non-linear, an iterative procedure has been used. The process starts with the independent variables (X_0, Y_0 and ϕ) and estimates for Z_0 , ϕ and θ . Then the transformation matrix is calculated. Next the absolute coordinates of the moving point M1 (eqns. 5 and 3), and the distance ΔL_1 (eq. 4a) are calculated. Last distance is assumed to be only a function of ϕ so a simple Regula-Falsi iteration gives a value of ϕ corresponding to $\Delta L_1 = 0$. In the same way θ and Z_0 are found from $\Delta L_2(\theta) = 0$ and $\Delta L_3(Z_0) = 0$ respectively. The iteration is repeated until $\sum \text{abs}(\Delta L_i) < \epsilon$. With an appropriate damping on the variables the process converges rapidly to $\epsilon < 10^{-6}$ in just a few steps. The TRIAX program was written in Turbo-Pascal for use with MS-DOS computers. One run on an Olivetti M21 personal computer takes about 10 seconds.

Results of the simulations

For a given geometry, and control variables X_0, Y_0 and ϕ , the values ϕ, θ and Z_0 are calculated. From this result displacements $\Delta P = (\Delta X, \Delta y, \Delta Z)$ of the apparent pivot (Figure 1,3,4) are determined and given in Figure 6. The best results follows when $\phi = 0$, which means that the square sum of the deviations is minimum. For this reason ϕ has been set zero for all other calculations. The real rotations (ϕ and θ) follow from the numeric procedure while the linearized rotations are directly found from Figure 4 as:

$$\phi' = \frac{Y_0}{h} \quad \text{and} \quad \theta' = -\frac{X_0}{h} \quad (7)$$

The deviations $\Delta\phi = (\phi - \phi')$ and $\Delta\theta = (\theta - \theta')$ are given in Figure 7.

Crosstalk

From eq.1 follows that the rotations are not fully independent, giving rise to crosstalk. Although it is not possible to avoid this (req.4) the effect can be reduced to 1% or less by choosing proper geometrical parameters. The crosstalk terms:

$$\frac{\partial \phi}{\partial \theta}, \quad \frac{\partial \theta}{\partial \phi}, \quad \frac{\partial \phi}{\partial \phi} \quad \text{and} \quad \frac{\partial \theta}{\partial \phi}$$

are calculated at each run. From the overview (Figure 8) it can be concluded that a rotation ϕ about the z-axis influences the alignments ϕ and θ considerable which means that ϕ should be kept constant.

This analysis gives a good explanation for the derivations as measured at a first prototype where ϕ was a free parameter. Based on the numerical results the model has been changed with no rotational freedom about the z-axis. The Figures 6,7 en 8 are calculated for a configuration with:

$$R = 47 \text{ mm}, \quad h = 120 \text{ mm}, \quad Z_0 = 55 \text{ mm} \quad \text{and} \quad \phi = 0.$$

Figure 6. Aberrations of the beam at the pivot position P.

Figure 7. A-linearities of the TRIAX system.

Figure 8. Crosstalks.

Experimental verification

The design as shown in Figure 3 and analyzed in the previous section has been built and thoroughly tested. The measurement set-up is shown schematically in Figure 9. A four quadrant photocell detector can be moved on an xy-table in the measurement plane, which is located on a distance z' from the pole. The centering detector is used only as a null detector to obtain a maximum sensitivity (better than $0.1 \mu\text{m}$). The maximum accuracy follows from the micrometer screws on the XY-table. The detector displacements X_d and Y_d can be read in μm 's. At the mechanism we distinguish two translations X_t and Y_t of the whole system and the movements X_0 and Y_0 of the pilot platform which result in the rotations θ and ψ respectively. The position of the pole was found at $h = 115.5 \text{ mm}$ instead of the design value of 120 mm . The difference is due to fabrication tolerances and the finite length of the elastic joints which consist of 3 mm long high quality spring steel with 1 mm diameter. The behaviour of the mechanism is described in terms of translation, rotation, position of the pole and stability.

Figure 9. Measurement system.

- a) Translations: The transfer coefficients were found as $X_d/X_t = 1.0067$ and $Y_d/Y_t = 1.026$. When the translation mechanism is properly used i.e. to bring the pole on the desired position, this is accurate enough. The standard deviations in both directions amount $3 \mu\text{m}$, corresponding with the reading accuracy of the (small) micrometer screws on the alignment system. Unexpectedly a cross-talk of 7% was found.
- b) Rotations: A linear regression analysis was used to determine the constants b from the linear relations $Y_d = a + b Y_o$. From Figure 9 follows $b = z'/h$. Because the origin of the moving frame has no measurable reference both values h and z' must be determined experimentally. The constant h was found to be 115.3 mm for x - and 115.8 mm for the y -direction. The differences are due to construction tolerances of the bar linkages. To calculate θ and ϕ from X_o and Y_o the mean value has been used. With this value the standard deviations between the linerized and the measured rotations were found as $\sigma(\theta) = 60 \mu\text{rad}$ and $\sigma(\phi) = 47 \mu\text{rad}$, about twice of the maximum reading accuracy of the X_o and Y_o screws which can be read until $3 \mu\text{m}$ ($25 \mu\text{rad}$). With the standard deviations taken into account, the a-linearities are in accordance with the values as predicted by Figure 7. The crosstalks however were significantly higher than the values as predicted by Figure 8. We found 5% in X - and 7,5% in Y -direction. This phenomenon could be explained afterwards by an assembly error. There are 2 cross edges, between the base, the translation ring and the commuting platform respectively, which prevent rotations in ϕ -direction. Any misalignment of these cross edges will lead to crosstalk as can be derived from Figure 4. A computersimulation with $\phi = -25 \text{ mrad}$ delivers the same crosstalks as measured.
- c) Position of the pole: The main effect of a rotation ϕ is to cause a cross deviation of the pole. The deviations ΔX due to variation of X_o and ΔY due to Y_o are completely in accordance with Figure 6. The cross deviations however are more than ten times higher, as found both experimentally and by the computer simulation. At longer distances the influence on the rotation decreases. For $z' = 9 \text{ m}$ the cross errors are reduced until 4%.

Figure 10. The calibration set-up

d)Stability: The stability of the beam has been measured during a couple of weeks. The standard deviation of the pole was found $3\text{ }\mu\text{m}$ in both directions and the directional stability measured at a distance of 500 mm, was found $10\text{ }\mu\text{rad}$ also in both directions. During warming up a maximum deviation of $25\text{ }\mu\text{rad}$ was found only in ϕ -direction

Figure 11. The mechanism as used with the 80 J TEA CO₂ laser

Conclusion

Using the mechanism as described before, the alignment of a pilot beam has been proved very convenient. The design of the system is very simple. A necessary modification to prevent ϕ -rotations brought some complications. It is expected that a re-design using bellows in stead of cross edges will improve the performance furthermore. The computer simulation has been proved a useful tool to optimize the design and was necessary to understand the behaviour of the TRIAX mechanism.

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