On the collective collapse of a large number of gas bubbles in water

By

L. van Wijngaarden

Netherlands Ship Model Basin, Wageningen

1. Introduction

During model tests as well as on full scale ship propellers the phenomenon of propeller blades, bent at the trailing edge towards the pressure side has been observed (Fig. 1).

Fig. 1.
Bent trailing edge of propeller blade

In a recent paper VAN MANEN [1] has discussed this phenomenon.

VAN MANEN showed by analysis of the flow along the propeller, turning through the non uniform wake field behind the ship that, due to the camber induced by the flow, cavitation bubbles can be generated midchord on the blade.

These bubbles travel, growing by gas diffusion and possibly by vaporous cavitation, towards the trailing edge of the blade.

When arriving at the trailing edge, the bubbles collapse in the region where the stagnation pressure is effective.

VAN MANEN suggested that the phenomenon of bent trailing edges is associated with the collapse of these bubbles.

It is the purpose of the present paper to analyse the collective collapse of a large number of small bubbles theoretically, in order to investigate whether this can indeed be a mechanism, responsible for the observed bending of trailing edges.

A considerable amount of information exists concerning the collapse of a single cavitation bubble. It is known that during the implosion of a single bubble high pressures occur and it is generally accepted that at least part of the damage, caused by cavitation, is due to these pressure peaks.

A collapsing bubble can, however, have effect only over a range, comparable in magnitude with the radius of the bubble. The bending of trailing edges must be due to high pressures, effective over a region which is many orders of magnitude larger than the radius of an individual bubble.

We shall therefore attempt to study the interaction of many neighbouring bubbles during a collective collapse.

It is clear that interaction is present when the distance between the bubbles is small enough, because then the pressure field developed around an individual bubble affects the pressure determining the collapse of its neighbours.

As a model for analysis of such a collective collapse, we have chosen the following situation.

2. Formulation of the problem

A flat plate of infinite length and breadth is covered with a region of thickness h , consisting of a mixture of water and bubbles with radius R (Fig. 2). The region $y \geq h$ is occupied by pure water, y being a coordinate normal to the plate.

We assume that the number of bubbles in 1 m^3 of the mixture is n and that these bubbles contain gas.

In practice cavitation bubbles contain gas and vapor, since the bubbles grow initially by gas diffusion and only in a further stage primarily by vaporisation.

We neglect the effect of the vapor content on the dynamic behaviour. It has been shown by PLESSET and ZWICK [2] that heat effects associated with the presence of vapor, do not considerably affect the dynamic behaviour.

We consider the case where at times $t < 0$ the pressure in the mixture and in the fluid is p_0 . All the bubbles are assumed to have radius $R = R_0$ at $t < 0$.

At $t = 0$ the pressure in the fluid, outside the mixture is increased by an amount Δp .

We ask for the average pressure p in the mixture as a function of y and t . The average is taken over a region containing many bubbles. This implies that we shall consider the case where the thickness of the mixture is large in regard to the distance between the bubbles or

$$hn^{1/3} \gg 1. \quad (1)$$

In the following we shall neglect the compressibility of the fluid. This is acceptable as long as the velocity of the bubble walls is smaller than the velocity of sound of the fluid. We shall return to this point in Sec. 5.

The pressure increase Δp causes the bubbles to collapse. The corresponding decrease in bubble volume gives rise to motion of the fluid in the mixture.

Let the average velocity of the fluid in y direction be $v(y, t)$. Further the volume of one bubble is denoted by τ_b . Then the continuity of the fluid requires

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial t} (n\tau_b). \quad (2)$$

Considering a unit volume in the mixture, the equation of motion for the fluid is

$$\rho(1 - n\tau_b) \frac{dv}{dt} + \rho_g n\tau_b \frac{du}{dt} = -\frac{\partial p}{\partial y}, \quad (3)$$

where ρ and ρ_g are the densities of fluid and gas, respectively, and u is the average velocity of the bubbles in the volume element.

For a bubble in the volume element the equation of motion is

$$\rho_g \frac{du}{dt} + 1/2 \rho \left(\frac{du}{dt} - \frac{dv}{dt} \right) = -\frac{\partial p}{\partial y}. \quad (4)$$

If the volume fraction of gas is small or

$$n\tau_b \ll 1, \quad (5)$$

v will be small as well, although the rate of change of the radius of an individual bubble may become large. We assume that $n\tau_b$ is sufficiently small to permit the neglect of convective acceleration terms in (3) and (4). From (3) and (4) follows that u and v are of the same order of magnitude. Since $\rho_g \ll \rho$ and on account of (5), (3) can be reduced to

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y}. \quad (6)$$

The continuity equation for the gas is

$$\frac{\partial n}{\partial t} + n \frac{\partial u}{\partial y} = 0. \quad (7)$$

With the aid of (7), (2) can be written as

$$\frac{\partial v}{\partial y} = n \frac{\partial \tau_b}{\partial t} - n\tau_b \frac{\partial u}{\partial y}.$$

On account of (5) and because $\frac{\partial v}{\partial y} \approx \frac{\partial u}{\partial y}$, this can be reduced to $\frac{\partial v}{\partial y} = n \frac{\partial \tau_b}{\partial t}$ or

$$\frac{\partial v}{\partial y} = 4\pi n R^2 \frac{\partial R}{\partial t}. \quad (8)$$

Fig. 2. Water bubble mixture on flat plate

A relation between the average pressure p and the bubble radius R is provided by considering a single bubble in the mixture.

For an individual bubble the average pressure p is the "pressure at infinity" of single bubble theory.

Therefore we have, neglecting the effects of viscosity and surface tension (see LAMB [3], par. 91^a)

$$\frac{p - p_g}{\rho} = -R \frac{\partial^2 R}{\partial t^2} - \frac{3}{2} \left(\frac{\partial R}{\partial t} \right)^2, \quad (9)$$

where p_g is the pressure of the gas in the bubble. The problem of a single imploding empty cavity was dealt with by LORD RAYLEIGH (LAMB [3], loc. cit.). In that case $p = \text{const}$, $p_g = 0$ and R is a function of t alone, whereas in the present case R is a function of two variables, viz. y and t . In formulating (9) squares and products of u and v are neglected. The pressure p_g is related to τ_b by a relation of the type

$$p_g \tau_b^\gamma = \text{const}. \quad (10)$$

We take here $\gamma = 4/3$ (for isothermal behaviour $\gamma = 1$, for adiabatic behaviour $\gamma = 1.4$, if the bubbles contain air). The value $4/3$ is chosen because this facilitates the calculus.

Inserting (10) into (9) yields

$$\frac{1}{\rho} \left\{ p - \frac{p_0 R_0^4}{R^4} \right\} = -R \frac{\partial^2 R}{\partial t^2} - \frac{3}{2} \left(\frac{\partial R}{\partial t} \right)^2. \quad (11)$$

For the quantities v , p and R we now have obtained the Eqs. (6), (8) and (11).

The boundary and initial conditions are:

$$R = R_0, \quad \frac{\partial R}{\partial t} = 0 \text{ for } t = 0, \quad 0 \leq y \leq h, \quad (12)$$

$$v = 0 \quad \text{for } y = 0, \quad (13)$$

$$p = p_0 + \Delta p \quad \text{for } y = h. \quad (14)$$

The condition (14) amounts to the assumption that the motion in the region $0 \leq y \leq h$ does not affect the pressure in the fluid in $y > h$.

The disturbance of the pressure in this region is $\rho c(v)_{y=h}$, where c is the velocity of sound in the fluid.

The condition (14) is thus valid if

$$\rho c(v)_{y=h} \ll p_0 + \Delta p. \quad (15)$$

Under the conditions envisaged here, the right hand side of (15) is of the order of magnitude of 1 bar $\approx 10^5$ N/m².^{*} Since $\rho = 10^3$ kg/m³ and $c = 1.5 \times 10^3$ m/s, $(v)_{y=h}$ has to be $O(10^{-2}$ m/s).

The validity of this assumption will be discussed in Sec. 5 after results have been obtained.

In the next section we shall derive an approximation for the maximum value of p at $y = 0$.

3. Approximate calculation of the maximum pressure at the plate

Differentiation of (6) with respect to y and of (8) with respect to t yields

$$-\frac{1}{\rho} \frac{\partial^2 p}{\partial y^2} = 4\pi n R \frac{\partial^2 R}{\partial t^2} + 8\pi n R \left(\frac{\partial R}{\partial t} \right)^2. \quad (16)$$

We choose as new independent variables y and R and as dependent variables p and $\frac{\partial R}{\partial t}$.

With $\frac{\partial R}{\partial t} = U(R, y)$, we obtain from (16)

$$-\frac{1}{\rho} \frac{\partial^2 p}{\partial y^2} = 2\pi n R^2 \frac{\partial U^2}{\partial R} + 8\pi n R U^2, \quad (17)$$

while (11) becomes

$$\frac{1}{\rho} \left(p - \frac{p_0 R_0^4}{R^4} \right) = -\frac{1}{2} R \frac{\partial U^2}{\partial R} - \frac{3}{2} U^2. \quad (18)$$

^{*} Rationalised M.K.S. units will be used throughout this paper.

The advantage of introducing U^2 as a new variable is that (17) and (18) are linear equations in p and U^2 , whereas (11) and (16) are non linear.

We introduce dimensionless quantities with the relations

$$R = R_0 x; \quad U^2 = \frac{p_0 Y}{\rho}; \quad \frac{p}{p_0} = P; \quad y = \frac{\eta}{(2\pi n R_0)^{1/2}}. \quad (19)$$

Inserting (19) into (17) and (18), yields

$$\frac{\partial^2 P}{\partial \eta^2} = -x^2 \frac{\partial Y}{\partial x} - 4x Y, \quad (20)$$

$$P - x^{-4} = -\frac{1}{2} x \frac{\partial Y}{\partial x} - \frac{3}{2} Y. \quad (21)$$

The boundary and initial conditions are in terms of the dimensionless variables

$$Y = 0 \text{ for } x = 1, \quad (22)$$

$$\frac{\partial P}{\partial \eta} = 0 \text{ for } \eta = 0, \quad (23)$$

$$P = 1 + \frac{\Delta p}{p_0} = P^* \text{ for } \eta = h(2\pi n R_0)^{1/2} = \bar{\eta}. \quad (24)$$

The condition (23) is equivalent to (13) on account of (6).

We have not been able to find an exact solution for the system (20)–(24).*

It is, however, possible to obtain a first approximation for the maximum value of p at the plate, without solving the equations exactly.

This approximation is obtained in the following way. Extrema of the average pressure p occur when $\frac{\partial P}{\partial \eta} = 0$ and $\frac{\partial P}{\partial t} = 0$. In terms of the new variables x and Y the latter condition is $\frac{\partial P}{\partial x} Y^{1/2} = 0$. At time $t = 0$, $Y = 0$ everywhere. Then all the bubbles have the maximum radius $x = 1$. A minimum radius occurs when $Y = 0$ again. The maximum average pressure is attained at the plate ($\frac{\partial P}{\partial \eta} = 0$) when at this location

$$Y = 0.$$

From (20) and (21) we deduce

$$\frac{\partial^2 (P - x^{-4})}{\partial \eta^2} - 2x(P - x^{-4}) = -x Y. \quad (25)$$

When

$$Y \ll |P - x^{-4}|, \quad (26)$$

the right hand side of (25) may be neglected, which leads to

$$\frac{\partial^2 (P - x^{-4})}{\partial \eta^2} - 2x(P - x^{-4}) = 0. \quad (27)$$

(27) is an equation for $(P - x^{-4})$ under circumstances where (26) holds.

On account of (23) we have to choose the solution

$$P - x^{-4} = A(x) \cosh\{(2x)^{1/2} \eta\}.$$

The function $A(x)$ is determined by (24). Using (24) we obtain

$$A(x) = \frac{P^* - x^{-4}}{\cosh\{(2x)^{1/2} \bar{\eta}\}},$$

and thus

$$x^{-4} - P = (x^{-4} - P^*) \frac{\cosh\{(2x)^{1/2} \eta\}}{\cosh\{(2x)^{1/2} \bar{\eta}\}}. \quad (28)$$

* Author's note added in proof: During the time elapsed since the Congress was held, I have obtained the exact solution of (20)–(24). Accordingly, the results of the following pages, while remaining qualitatively the same, can be corrected quantitatively. The exact solution will be published together with the corrected results in the near future.

For small values of Y , (28) gives for the pressure on the plate

$$x^{-4} - P = \frac{x^{-4} - P^*}{\cosh\{(2x)^{1/2} \bar{\eta}\}}. \quad (29)$$

We obtain from (21), taking (22) into account

$$Y = \frac{2}{x^3} \int_x^1 (P - \xi^{-4}) \xi^2 d\xi, \quad (30)$$

where ξ is an integration variable.

To obtain a first approximation for the value of x for which $Y = 0$, we substitute (29) into (30) and then $x_{\min}$ is evaluated from

$$\int_{x_{\min}}^1 \frac{(P^* - \xi^{-4}) \xi^2}{\cosh\{(2\xi)^{1/2} \bar{\eta}\}} d\xi = 0. \quad (31)$$

Some conclusions can be drawn from (29) in combination with (31).

If $\bar{\eta} \rightarrow 0$, it follows from (29) that $P \rightarrow P^*$. Now $\bar{\eta} = \frac{h}{R_0} (2\pi n R_0^3)^{1/2}$; $\bar{\eta} \rightarrow 0$ corresponds with $n \rightarrow 0$.

In that case we are dealing with the collapse of a single bubble, since there is no interaction between the bubbles and consequently $P = P^*$.

Let us consider the other limit: $\bar{\eta} \rightarrow \infty$.

When $n R_0^3$ is kept constant, this corresponds with $R_0 \rightarrow 0$, $n \rightarrow \infty$. Then the solution of (31) tends to $(P^*)^{-1/4}$ and from (29) follows that for $\bar{\eta} \rightarrow \infty$, $P \approx x_{\min}^{-4} \approx P^*$.

In this case the bubble character of the gas content is lost. The pressure of gas and fluid in the mixture is the same and the maximum pressure at the plate is $0(P^*)$.

The mixture is a homogeneous medium, with density determined by the fluid and compressibility determined by the gas.

The concept of such a medium is well suited for the calculation of the velocity of sound in a fluid-gas mixture below the resonance frequency of the bubbles.

From the above considerations we infer that average pressures $P \gg P^*$ will occur at moderate values of η , ranging from say $O(1)$ to $O(10)$.

Such values are quite possible indeed in the case of small cavitation bubbles on a propeller blade.

We give two numerical examples

1. $R_0 = 10^{-4}$ m; $h = 2 \times 10^{-2}$ m; $n = 10^8$ m $^{-3}$.

We obtain $\frac{h}{R_0} = 2 \times 10^2$; $h n^{1/3} \approx 10$; $(2\pi n R_0^3)^{1/2} = 2.5 \times 10^{-2}$; $\bar{\eta} = 5$.

2. $R_0 = 5 \times 10^{-5}$ m; $h = 1.5 \times 10^{-2}$ m; $n = 10^9$ m $^{-3}$.

In this case $\frac{h}{R_0} = 3 \times 10^2$; $h n^{1/3} = 15$; $(2\pi n R_0^3)^{1/2} = 2.7 \times 10^{-2}$; $\bar{\eta} = 8.1$.

For values of $\bar{\eta} \neq 0$, the integral in (31) cannot be evaluated analytically.

With $2^{1/2} \bar{\eta} = \lambda$, $x_{\min}$ is numerically obtained from

$$\int_{x_{\min}}^1 (\xi^{-4} - P^*) \xi^2 e^{-\lambda \xi^{1/2}} d\xi = 0^* \quad (32)$$

for $\lambda = 5, 8$ and 10 . Anticipating the result, $\lambda \xi^{1/2}$ was expected to be sufficiently large for these values of λ to replace the cosh in (31) by $\frac{1}{2} e^{\lambda \xi^{1/2}}$.

The results for a range of values of P^* are given in the next section.

* For the numerical calculation the author is indebted to Mr. H. LE GRAND of the N.S.M.B. Computer Centre.

4. Results

In practical circumstances the pressure rise will be from various low values of p_0 (initial pressure in the bubbles) to a pressure of about $10^5 \text{ N/m}^2 \approx 1 \text{ bar}$ (stagnation pressure). We have calculated $x_{\min}$ for $p_0 = 2 \times 10^3 \text{ N/m}^2$, 10^3 N/m^2 , $5 \times 10^2 \text{ N/m}^2$, $3.33 \times 10^2 \text{ N/m}^2$.

With $\Delta p = 10^5 \text{ N/m}^2$, we have in all cases $p_0 \ll \Delta p$ so that [cf. (24)] $P^* = \frac{10^5}{p_0}$.

The values of P^* corresponding with the values of p_0 given above, are presented in Table 1 together with the result of the numerical calculation of $x_{\min}$ for $\lambda = 5, 8$ and 10 . The values for $\lambda = \bar{\eta} = 0$ are directly calculated from (31) and are approximately given by $x = \frac{3}{P^*}$.

The value of $\exp\{-2\lambda x^{1/2}\}$ is small in regard to one for all values of $x_{\min}$ ($\lambda \neq 0$) in the table below. The use of (32) instead of (31) is therefore justified. From the results presented in Table 1 the curves of $x_{\min}$ versus λ are drawn in Fig. 3.

$\lambda \backslash P^*$	0	5	8	10
50	0.0600	0.1823	0.2318	0.2551
100	0.0300	0.1319	0.1800	0.2028
200	0.0150	0.0937	0.1382	0.1600
300	0.0100	0.0757	0.1177	0.1389

Fig. 3. $x_{\min}$ versus λ for various P^*

The maximum average pressure at the plate P_m can now be calculated with the help of (29) and the values given in Fig. 3, bearing in mind that $\lambda = 2^{1/2}\bar{\eta}$.

In Fig. 4 $p_m \times 10^{-5} \text{ N/m}^2$, i.e. the physical (dimensional) pressure in terms of 10^5 N/m^2 is given as a function of P^* for $\lambda = 4, 5, 6$ and 8 .

Fig. 4. P_m at the plate for various values of P^* and λ

Fig. 4 shows that for these moderate values of λ pressures in the range 10^6 – 10^7 N/m^2 can be expected.

Fig. 5. Schematic representation of trailing part of propeller blade

These pressures are sufficiently high to be responsible for the phenomenon of bent trailing edges of ship propellers. This can be made plausible as follows:

The trailing edge is schematically represented by a two-dimensional wedge with angle α (see Fig. 5).

Per unit length perpendicular to the plane of the wedge, the bending moment caused by a uniform load p is at distance s from the top

$$M = \frac{1}{2} p s^2.$$

Since $M = \sigma W$, σ being the stress at location s and W being the section modulus, and since $W = \frac{1}{6} s^2 \alpha^2$, we obtain

$$\sigma = 3 p \alpha^{-2}. \quad (33)$$

For a propeller a representative value for α is $1/10$ rad. The tensile strength is about 5×10^8 N/m². In practice a stress of about $1/10$ of this value is allowed. For $p = 10^5$ N/m², (33) gives $\sigma = 3 \times 10^7$ N/m².

If, however, pressures occur of 10^6 to 10^7 N/m², as calculated in the present paper, the corresponding stresses are well beyond 5×10^8 N/m².

We conclude therefore that during the collective collapse of a large number of gasfilled cavitation bubbles, average pressures can be developed at the trailing edge, which are sufficiently high to explain the observed bending of these edges.

Fig. 6.
Enlarged thickness of trailing edge

If it is not possible to change the design (existing propeller), a remedy is found of course in an enlargement of the thickness at the trailing edge by reducing the blade width as shown in Fig. 6.

For other cases (model propellers), VAN MANEN [1] recommends some measures to reduce the risk of bent trailing edges. Generally speaking, these measures imply a reduction of the camber of the blade in order to prevent the generation of cavitation bubbles.

5. Discussion

In this section the validity is discussed of two assumptions made in the previous analysis, viz. the assumption that $\frac{\partial R}{\partial t}$ is not larger than the velocity of sound in the fluid, and the assumption (15).

When Y , which is [cf. (19)] equal to $\frac{\rho}{p_0} \left(\frac{\partial R}{\partial t} \right)^2$, is large, then we can neglect $\frac{\partial Y}{\partial x}$ in (20) and (21), obtaining

$$\frac{\partial^2 (P - x^{-4})}{\partial \eta^2} - \frac{8}{3} x (P - x^{-4}) = 0.$$

This yields analogous to (28)

$$P - x^{-4} = \frac{(P^* - x^{-4}) \cosh \left\{ \left(\frac{8}{3} x \right)^{1/2} \eta \right\}}{\cosh \left\{ \left(\frac{8}{3} x \right)^{1/2} \bar{\eta} \right\}}.$$

Using (30), we obtain for Y under these conditions

$$Y = \frac{2}{x^3} \int_x^1 \frac{(P^* - \xi^{-4}) \xi^2 \cosh \left\{ \left(\frac{8}{3} \xi \right)^{1/2} \eta \right\}}{\cosh \left\{ \left(\frac{8}{3} \xi \right)^{1/2} \bar{\eta} \right\}} d\xi. \quad (34)$$

From this expression follows that $Y_{\max}$ is larger at $\eta = \bar{\eta}$ than at $\eta = 0$. At $\eta = \bar{\eta}$ the collapse is just as in the case of a single bubble, since in the present description the interaction between the bubbles is neglected at the boundary between fluid and mixture.

BROOKE BENJAMIN [4] has shown that for a single collapsing gasbubble $\frac{dR}{dt}$ remains below the velocity of sound of water when $P^* = \frac{\Delta p}{p_0}$ is $O(10^2)$.

This can be verified with the aid of (34). From this expression follows that at $\eta = \bar{\eta}$, $\frac{\partial Y}{\partial x} = 0$ for $x = \frac{4}{P^*}$.

Substituting this value for x in (34) and using the second relation in (19), we obtain

$$U_{\max} = \left(\frac{\partial R}{\partial t} \right)_{\max} = \left(\frac{p_0}{\rho} \right)^{1/2} P^{*2} O(10^{-2}), \quad (35)$$

whereas the velocity of sound in water is given by $c \approx 1.5 \times 10^3$ m/s.

The neglect of the compressibility of the fluid is thus permitted for most of the values of P^* chosen in the present paper.

Finally the assumption is discussed [cf. (15)] that

$$\varrho c(v)_{y=h} \ll p_0 + \Delta p, \text{ when } \frac{\partial R}{\partial t} = 0 \text{ at } y = h.$$

Taking the numerical value of the various quantities into account, this implies

$$(v)_{y=h} \ll 10^{-1}.$$

From (8) we derive

$$(v)_{y=h} = \int_0^h 4\pi n R^2 \frac{\partial R}{\partial t} dy,$$

or using (19),

$$(v)_{y=h} = 2 \left(\frac{p_0}{\varrho} \right)^{1/2} (2\pi n R_0^3)^{1/2} \int_0^{h(2\pi n R_0^3)^{1/2}} x^2 Y^{1/2} d\eta < 2 \left(\frac{p_0}{\varrho} \right)^{1/2} \frac{h}{R_0} (2\pi n R_0^3) \text{ times max. } (x^2 Y^{1/2}).$$

With $\frac{p_0}{\varrho} = O(1)$, $\frac{h}{R_0} = O(10^2)$, $2\pi n R_0^3 = O(10^{-3})$, (see examples in Sec. 3), this equals max. $(x^2 Y^{1/2}) O(10^{-1})$. The maximum value of $x^2 Y^{1/2}$ occurs at $\eta = \bar{\eta}$ for $x = \frac{4}{P^*}$ and [from (35) and the second relation in (19)] $Y^{1/2} = P^{*2} O(10^{-2})$.

Hence

$$(v)_{y=h} < \max(x^2 Y^{1/2}) O(10^{-1}) = O(10^{-2}).$$

The neglect of the pressure disturbance in the fluid in $y > h$ caused by the motion in the mixture, is therefore justified for the orders of magnitude of the various quantities considered here.

The above estimate for $(v)_{y=h}$ is moreover far too large, because $\int_0^{\bar{\eta}} x^2 Y^{1/2} d\eta$ is, when $Y = 0$ at $\eta = \bar{\eta}$, much less than the value used for the estimation.

References

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